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# ARE REVEALED INTENTIONS POSSIBLE?[1]

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***Abstract***: *This paper asks whether it is possible to design an Intentions Revealing Experiment – that is, an experiment in which the early moves of the decision maker in a dynamic decision problem reveal the intentions of that decision maker regarding later moves in the decision problem. If such a type of experiment is possible, then it will enable economists to test whether individuals have plans and implement them – a basic assumption of all economic theories of dynamic decision making. Unfortunately the main finding of the paper is in the form of two Impossibility Theorems which show that, unless one is prepared to make certain assumptions, such an Intentions Revealing Experiment is impossible. However, the paper does have a positive side – it describes the type of assumptions that one needs to make in order to make an Intentions Revealing Experiment possible.*

***Keywords***: *dynamic decision making, experiments, planning, dynamic consistency.*

***JEL codes***: *D81, C91, D90*

## INTRODUCTION

Is it possible to design an experiment in which early decisions by the participants reveal the intentions or plans of those individuals with respect to later decisions? To answer this question is the purpose of this paper.

We should start with some motivation as to why the answer to this question is of interest. This motivation comes from all economic theories of

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dynamic decision making, in which the economic agent is envisaged, firstly, as having a plan as to what he or she will do later in the decision problem, and secondly, as using this plan to determine the earlier decisions. This follows from the structure of economic models of dynamic decision making. Virtually all such economic theories of dynamic decision making involve two components: a procedure for reducing, either in one move or several, a dynamic decision problem to one or several static decision problems; and a preference functional for determining optimal choice in static decision problems.

There are three main alternative procedures for reducing a dynamic decision problem to a (series of) static decision problem(s): (1) converting the dynamic decision problem into a *strategy* choice problem (where a strategy is a set of conditional decisions at to what to do at each decision node, conditional on having arrived at that node); (2) using *backward induction with reduction* to eliminate choices that will not be taken in the future and then using the principle of the reduction of compound lotteries to simplify the remaining portion of the decision tree; (3) using *backward induction with certainty equivalents* to eliminate choices that will not be taken in the future and using certainty equivalents to replace the eliminated part of the decision tree with a certainty equivalent. Each of these three procedures involves a plan – a set of conditional decisions at each node, conditional on having arrived there. Procedure (1) does this explicitly; procedures (2) and (3) implicitly.

There are many preference functionals in economic theory that attempt to describe optimal decision making in *static* decision problems. The most popular is Expected Utility theory but there are many alternatives and generalisations. Any of these preference functionals can be combined with any of the three procedures (for reducing a dynamic decision problem to a (series of) static decision problem(s)) described above. In general, the three different procedures will generate different plans for tackling any given dynamic decision problem, though in the case of Expected Utility theory this is not so: whichever procedure is used, the plan produced is the same. Many economists regard this as a great normative strength of Expected Utility theory.

If an individual's preference functional is not that of Expected Utility theory, it is possible (though not inevitable) that different procedures (for reducing a dynamic decision problem to a (series of) static decision problem(s)) will result

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in different plans. Because of this, it is possible that a non-Expected-Utility-theory decision maker (henceforth non-EU person) will be dynamically inconsistent – that is, they will want to do something different from their original plan at some point in the decision tree.

There are two ways that a non-EU person can resolve this problem of potential dynamic inconsistency: that of ‘resolution’ and that of ‘sophistication’. The first of these terms was coined by McClennen (1990) and describes the behaviour of an individual who chooses the *ex ante* optimal strategy (out of the set of all possible strategies) and who resolutely implements it without deviation (perhaps he or she leaves instructions with his or her lawyer and then goes away on holiday). The second of these terms (perhaps attributable to Machina (1989)) describes the behaviour of an individual who works by backward induction (either with reduction or with certainty equivalents) and therefore never places him- or her-self in a position of wanting to change his or her mind – change the plan.

Non-EU people who are neither resolute nor sophisticated are described by O’Donoghue and Rabin (1999) as ‘naïve’ – they will typically do something different in the future than they had earlier planned to do. Most economists would describe this kind of behaviour (time inconsistency) as irrational – particularly as the non-EU people who do this kind of thing know in advance that they will do it – and yet disregard this fact. Yet casual empiricism would suggest that such behaviour is not unusual: there seem to be people who either do not have a plan or who have a plan and do not implement it. We are interested to learn whether in fact this is the case.

So the brief is simple: to ascertain whether individuals make plans and whether they implement them. But the execution of the brief is far from simple – as it is difficult to observe whether people have plans and what those plans are. First, there is a methodological problem in that mentioning the word ‘plan’ to individuals may well affect their behaviour. We want to avoid this problem. Secondly, even if we did not, there would be problems in motivating the response of subjects. Suppose, for the moment that individuals do have plans, how do we ask them what those plans are in a way that gives them an incentive to accurately reveal them? If we simply ask them, there is a problem – how do we know whether the reply has any meaning? If, to make their reply to have meaning, we *force* them then to implement whatever plan they have announced, then we have not only *forced them* *to have a plan* but we have also *forced them to implement it*. Which rather spoils the whole purpose of the exercise! We are very sceptical about the value of asking for people's plans: first, because the question itself suggests to individuals that they *ought* to have a plan; secondly, there is no way that we can guarantee that what they say is their plan actually is their plan.

Instead, we have the following suggestion: can we design an experiment in such a way that the earlier decisions of individuals reveal their future intentions? If we can, then we can answer the combination of the two questions above: do individuals have plans and implement them? While we may not have answers to each question individually, it is the answer to the combination that is important to economic theorists and practitioners.

## EXPERIMENTAL DESIGN

We deliberately work with a simple structure – in fact the simplest possible structure for a genuinely dynamic decision problem under risk – a two decision-nodes, two chance-nodes decision problem – as portrayed in Figure 17. Square boxes represent decision nodes and round boxes chance nodes. To make the analysis as simple as possible we restrict the number of decisions at each decision node to *two* - Up or Down, and we restrict the number of possibilities at each chance node to *two* – Up or Down. We also, rather arbitrarily at this point assume that each of these two possibilities are equally likely – so that the probability of moving Up at a chance node and the probability of moving Down are both 0.5.

The tree in Figure 17 1 starts with a decision – whether to move Up or Down at node $S$. Then follows a chance node – either $C_1$ or $C_2$ depending upon the decision taken at $S$. Then there is a second decision node – either $D_1$, $D_2$, $D_3$ or $D_4$ depending on the previous moves by the decision maker and by Nature. There are then choice nodes, labelled $A$ through $H$, and finally there is a payoff, one of the set $P = \{a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2, e_1, e_2, f_1, ,f_2, g_1, g_2, h_1, h_2\}$. For simplicity in what follows, we will number the payoffs in such a way that $x_1$ is at least as large as $x_2$ for all $x$ in the set $a$ through $h$. The participant in the experiment will end up paid one of the payoffs out of this set $P$ – the precise payoff depending upon his or her decisions and upon the moves by Nature. The question is: can we choose the elements of the set $P$ in such a way that the move by the individual at the first decision node, *S*, *reveals* his or her plan as to what he or she will do at (the relevant ones of) the decision nodes $D_1$, $D_2$, $D_3$ or $D_4$?

Obviously how we order the elements of the set is unimportant, so let us be more precise. Let us ask: can we choose the elements of the set *P* in such a way that a decision to move Up at the first choice node (*S*) *reveals the intention* to move Up at whichever of $D_1$ or $D_2$ is actually reached, while a decision to move Down at the first choice node (*S*) *reveals the intention* to move Down at whichever of $D_3$ or $D_4$ is actually reached? We should add to this that we want the experiment to be non-trivial and, in particular, not driven by (first-order) *dominance* – so that all rational subjects all take the same decisions. (Perhaps an extended note is needed at this stage: some of my colleagues have argued that it is of interest to construct a tree where two strategies (one involving Up at the first node and the second involving Down at the first node) dominate all the others, and therefore in which violation of the revealed intentions reveals a violation of dominance. The response is that we are testing to see whether plans are made and implemented. It may be the case that plans are not made and implemented because dominance is violated – but I do not want an experiment in which that is the *only* reason why plans are not made and implemented.) Moreover we want the decision problem to be a genuinely dynamic one so we need that at least one of (*A and B*), (*C and D*), (*E and F*) and (*G and H*) are different – otherwise the second decision would not be a genuine decision. Similarly we need that either (*A and B*) are different from (*C and D*) or that (*E and F*) are different from (*G and H*) – for otherwise the first chance node would not be a genuine chance node. More crucially we do not want the decisions at any node to be driven purely by dominance. In particular we do not want the decisions at the second decision node to be driven purely by dominance. So, for at least one of the pairs (*A and B*), (*C and D*), (*E and F*) and (*G and H*) it must be the case that neither member of the pair dominates the other member of the pair. Dominance in this two-outcome case is clear – for example if $a_1$ is at least as large as $b_1$ and $a_2$ is at least as large as $b_2$ then *A* dominates *B*.

As we will see, the answer to our question depends very much on what we can assume about subjects. We consider various cases, becoming more and more restrictive as we proceed. Before we start, it might be useful to propose a definition of what it is that we are after. This is an *Intentions Revealing Experiment* – defined as an experiment in which some subjects move Up at the first node and some move

Down at the first node, and in which the decision to move Up at the first decision node reveals the intention (for someone who makes plans) to move Up at the second decision node (independently of what Nature does at the first chance node) and in which the decision to move Down at the first decision node reveals the intention (for someone who makes plans) to move Down at the second decision node (independently of what Nature does at the first chance node). In such an *Intentions Revealing Experiment* a decision to move Up at the first decision node and Down at the second, or a decision to move Down at the first and Up at the second, must be a decision by someone who either does not have a plan, or who has a plan but fails to implement it. In other words, such a pattern of decisions reveals a dynamically inconsistent individual.

Before concluding this section let us introduce some notation. A *strategy* is a decision at the first decision node and a decision at both of the possible second decision nodes – depending upon which decision node Nature moves to. We denote a strategy in the following form: *{X ,YZ}* – where each of *X, Y* and *Z* are one of *U* or *D* – indicating Up or Down. The general strategy *{X ,YZ}* indicates the decision to move *X* at the first node and then *Y* at the second if Nature moves Up at the first chance node, or *Z* at the second if Nature moves Down at the first chance node. So one strategy is *{U,UU}* – where the subject moves Up at the first node and then moves Up at the second node, irrespective of what Nature does at $C_1$. Another strategy is *{U ,UD}* - where the subject moves Up at *S* and then moves Up if Nature moves Up at $C_1$ and Down if Nature moves Down at $C_1$. Associated with any strategy is a probability distribution over either penultimate payoffs or final payoffs. For example, the choice of strategy *{U ,UU}* will lead intermediately to one of *A* or *C*, each with equal probability[16], and will lead finally to one of $a_1$, $a_2$, $c_1$ or $c_2$, again each with equal probability. We will denote a gamble with outcomes *a, b, c, d, ...,* each with equal probabilities by *[a,b,c,d, ...J.* The complete set of all possible strategies, and their associated intermediate and final outcomes, in this simple experiment is shown in Table 6.

**Table 6** *The strategies and their implied payoff distributions*

| Strategy number | Strategy | Implied penultimate payoffs | Implied final payoffs |
|---|---|---|---|
| 1 | *{U,UU]* | *[A,C]* | *[a1,a2,c1,c2]* |
| 2 | *{U,UD}* | *[A,D]* | *[a1,a2,d1,d2]* |

[16] Recall that we have assumed that Nature moves Up or Down at each chance node with probability 0.5

| Strategy number | Strategy | Implied penultimate payoffs | Implied final payoffs |
|---|---|---|---|
| 3 | *{U,DU}* | *[B,C]* | *[b1,b2,c1,c2]* |
| 4 | *{U,DD}* | *[B,D]* | *[b1,b2,d1,d2]* |
| 5 | *{D,UU]* | *[E,G]* | *[e1,e2,g1,g2]* |
| 6 | *{D,UD}* | *[E,H]* | *[e1,e2,h1,h2]* |
| 7 | *{D,DU}* | *[F,G]* | *[f1,f2,g1,g2]* |
| 8 | *{D,DD}* | *[F,H]* | *[f1,f2,h1,h2]* |

## NON-EU SUBJECTS

Let me begin with the most general case – in which we are not prepared to make any assumptions about our subjects, other than they have some vaguely sensible preference functional over static decision problems – that they do not violate monotonicity, for example. So they might be EU, they might be Rank Dependent, they might follow Disappointment Aversion theory, and so on. Additionally suppose we are not prepared to make any assumption about which of the three procedures we described above is used by any particular subject. So we neither know their static preference functional nor their procedure for processing a dynamic decision problem.

An Impossibility Theorem is easy to generate under these conditions:

**Theorem 1**: An Intentions Revealing Experiment is impossible under these assumptions.

**Proof of theorem 1:** The proof is simple and revolves around the fact that we do not know which procedure is being used by a particular subject: he or she could be using the strategy method; he could be using backward induction. Suppose first that we have managed to choose the set *P* so that the route[173] followed by a subject using the backward induction method is *either*[18] *{U ,UU} or {D,DD}.* Take a backward inductor subject for whom the preferred route is *{U ,UU}*[19]. Then this subject prefers to move Up at node $D_1$ and Up at node $D_2$. That is, he or she prefers *A* to *B* at $D_1$ and prefers *C* to *D* at $D_2$.

Let us now consider an individual with the same preferences but one who uses the strategy method. Is the information we have obtained above sufficient to prove that he or she prefers the strategy *{U, UU}* to *{U,DU}, {U, UD}* and *{U,DD}*? In

[17] We use this word rather than 'strategy' to avoid confusion. The backward inductor actually implements this as a strategy – but did not derive it as a strategy.

[18] We do not know which, ex ante, as this depends upon their preferences, which we do not know.

[19] The proof in the contrary case follows in a parallel manner.

other words is the fact that *A* is preferred to *B* and that *C* is preferred to *D sufficient* to show that *[A, C]* is preferred to *[B, C], [A,D]* and *[B,D]*?

Unfortunately not. As we show in Appendix Theorem 1, for any *A, B, C* and *D* that satisfy the conditions that we have stated above, we can always find some non-EU preferences for whom *A* is preferred to *B* and *C* is preferred to *D* but either *[B, C]* is preferred to *[A, C]* or *[A,D]* is preferred to *[A, C]* or both. In other words, even if all backward inductors are following the route *{U, UU}* there may be strategy players with the same preferences for whom *{U, UU}* is *not* the best strategy.

## EU SUBJECTS

The problem above is that different procedures for reducing a dynamic problem to a static decision problem may lead to different solutions for individuals with non-EU static preference functionals. This, through Theorem 1, makes an Intentions Revealing Experiment impossible. Let us therefore assume that all our subjects satisfy Expected Utility theory. Does this help us? The answer is 'no' as the next theorem shows.

**Theorem 2:** An Intentions Revealing Experiment is impossible under these assumptions.

**Proof of theorem 2:** Consider *A* and *B*. We have assumed that $a_1 \succ a_2$ and that $b_1 \succ b_2$. Furthermore we do not want our results to be driven by dominance so we want neither that *A* dominates *B* or that *B* dominates *A*. This requires that we have *either* that $a_1 \succ b_1$ and $b_2 \succ a_2$ *or that* $b_1 \succ a_1$ and $a_2 \succ b_2$ . Which way round is irrelevant (as will be seen) and so let us assume that $a_1 \succ b_1$ and $b_2 \succ a_2$. If you like you can interpret this as saying that *A* is *riskier* than *B* – since *A*'s best outcome is better than the best outcome of *B* and *A*'s worst outcome is worse than the worst outcome of *B* – but it is not riskier in the Rothschild and Stiglitz sense, since *A* and *B* do not necessarily have the same mean[20]. However we can say that someone *sufficiently* risk-loving will prefer *A* while someone insufficiently risk-loving will prefer *B*. The question now is: can we choose *A, B, C* and *D* so that some people prefer *A* to *B* and *C* to *D*, while the others prefer *B* to *A* and *D* to *C*? The problem is that the pair (*A and B*) cannot be the same as the pair (*C and D*) – for otherwise the chance node at $C_1$ would not exist - and Appendix

[20] It is riskier in the sense used by Hey and Lambert (1989) in generalizing the results of Rothschild and Stiglitz.

Theorem 2 shows that it does not follow that if *A* is preferred to *B* then *C* is preferred to *D* or vice versa. So *either* even if someone is sufficiently risk-loving to prefer *A* to *B* then it does not follow that they are sufficiently risk-loving to prefer *C* to *D or* even if someone is sufficiently risk-loving to prefer *C* to *D* then it does not follow that they are sufficiently risk-loving to prefer *A* to *B*. The problem is that (*A and B*) must differ from (*C and D*) and if we do not know anything about an individual's utility function other than *either* they prefer *A* to *B or* that they prefer *C* to *D* that is not sufficient to tell us whether they prefer *C* to *D* (given that they prefer *A* to *B*) or whether they prefer *A* to *B* (given that they prefer *C* to *D*).

## EU SUBJECTS WHO ARE EITHER EVERYWHERE RISK-AVERSE OR EVERYWHERE RISK-LOVING

The above result gives us a clue as to what assumptions we might need to design an Intentions Revealing Experiment. Suppose we assume that all our subjects are *either* everywhere risk-lovers *or* everywhere risk-averters. Then we can make *A* and *B* have the same mean – with *A* riskier than *B* – and we can make *C* and *D* have the same mean –with *C* riskier than *D*. Then all the risk-lovers will choose Up at the second decision node in the top part of the tree. This suggests that we design the tree so that the risk-lovers go Up at the first decision node, and continue to play Up thereafter, while all the risk-averters play Down at the first decision node and continue to play Down thereafter. Using the same logic as that used to design the gambles *A, B, C* and *D,* we make *E* and *F* have the same mean but *E* riskier than *F* and we make *G* and *H* have the same mean but *G* riskier than *H*. This means that all risk-averters will play Down at the second decision node in the bottom half of the tree.

We have not finished. We now want to persuade all the risk-lovers to choose Up at the first decision node and all risk-averters to choose Down at the first decision node. How do we do this? Well, we have already set things up so that, at the second decision node, risk-lovers everywhere play Up while risk-averters everywhere play Down. So, as viewed from the first decision node the risk-lovers are choosing between *[A, C]* and *[E, G]* while the risk-averters are choosing between *[B,D]* and *[F,H]*. We therefore want to make *[A, C]* more attractive to risk-lovers than *[E, G]* and we want to make *[F,H]* more attractive to risk-averters than *[B,D]*. At the same time, we want to respect the conditions above: that *A* and $B$ have the same mean but $A$ is riskier; that $C$ and $D$ have the same mean but $C$ is riskier; that $E$ and $F$ have the same mean but $E$ is riskier; and that $G$ and $H$ have the same mean but $G$ is riskier. The argument goes through in a more general case but let us consider a rather special case – in which $B$, $D$, $F$ and $H$ are all *certainties* – that is $b_1 = b_2 = b$, $d_1 = d_2 = d$, $f_1 = f_2 = f$ and $h_1 = h_2 = h$. Then for all risk-averters to prefer to play Down at the first decision node we require that *[F,H]* is more attractive to them than *[B,D]*. We could guarantee that by putting $f + h = b + d$ (thus guaranteeing that the means of *[F,H]* and *[B,D]* are equal) and then by putting $b > f > h > d$ - so that *[F,H]* is less risky than *[B,D]*.

So the secret is to tempt the risk-averters Down by making the two certainties in the bottom half of the tree jointly more attractive to risk-averters than the two certainties in the top half of the tree. This entices the risk-averters Down at the first decision node. We then do a similar thing to tempt the risk-lovers Up at the first decision node – make the risky prospects in the upper part of the tree more attractive than the risky prospects in the bottom half of the tree. We can do this by making them riskier.

An example is presented in Figure 18. It will be seen from this that at all second decision nodes $D_1$ through $D_4$ all risk-lovers will choose Up and all risk-averters will choose Down. So the risk-averters know that if they choose Up at the first decision node they will end up either with 12 or with 8 – each equally likely – whereas if they choose Down at the first decision node they will end up with either 11 or 9 – each equally likely. All risk-averters prefer *[11,9]* to *[12,8]* so all riskaverters will choose Down at the first decision node and will continue to play Down thereafter. Risk-lovers, on the other hand, knowing that they will play Up at any second decision node have a choice between *[14,10,10,6]* by playing Up at the first node and *[12,10,10,8]* by playing Down at the first node. For all risk-lovers the prospect *[14,10,10,6]* is more attractive than the prospect *[12,10,10,8]* – because the former is riskier than the latter – so they will choose Up at the first decision node and will continue to play Up thereafter.

We have seen, therefore, that if we are able to make a sufficiently strong assumption about the preferences of our subjects – in this case, assuming that they are either everywhere risk-loving or everywhere risk-averting – we can design an Intentions Revealing Experiment. (Of course, this cannot work for the dividing case – the individual who is risk-neutral - for he or she is everywhere indifferent.)

## GENERALISATIONS AND CONCLUSIONS

It should be clear that there are obvious generalisations of the 'Possibility Theorem' of section 4. For instance we could take some reference individual – say an individual with utility function *R(.)* and assume that all other individuals are *either* everywhere more risk-averse than the reference individual *or* everywhere more risk-loving than the reference individual. This follows the line of argument of Hey and Lambert (1989) in generalising the results of Rothschild and Stiglitz. Then we amend what we have done above in Figure 18. Note that there the 'reference individual' was the risk-neutral individual. We made all the final choices between gambles with the same mean but the Up decision riskier. In this current section's generalisation we construct the tree so that in all the final choices the reference individual is indifferent – and once again we make the Up decision riskier. So all those agents everywhere more risk-loving than our reference individual will play Up at the first node and continue to play Up thereafter while all those agents everywhere more risk- averse than our reference individual will play Down at the first node and continue to play Down thereafter. (Though, of course, this cannot work for our reference individual – who is everywhere indifferent[21]).

There are clearly countless other generalisations of this form. In Hey (2002) it was supposed that if an individual prefers some gamble $A$ to some other gamble $B$ then he or she will also prefer the gamble $A+d$ to the gamble $B+d$ – where by the notation $A+d$ we mean a gamble which has the same probabilities and outcomes as $A$ except that all outcomes are increased by the constant $d$ (which could be negative). Notice that this assumption has the same type of structure as before: it enables us to divide our subjects into two groups – one sub-group which is always in that sub-group and the rest which are always in some other sub-group. This then enables us to design our tree.

So we have a way of designing an Intentions Revealing Experiment. We need to be able to divide our subjects up into two groups – and be sure that they remain in these two subgroups at all stages in the experiment. This appears to be difficult – without making what would appear to be strong assumptions[22].

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[21] But then he or she has mass zero in the population.

[22] One possibility is that we use a modification of the Binary Lottery incentive mechanism – the payoffs at the end of the tree are probability points to be used in a binary lottery. The trouble with this is twofold: (1) the Binary Lottery mechanism is not viewed with favour by all experimental economists; and (2) the addition of this "sting in the tail" complicates the experiment considerably.

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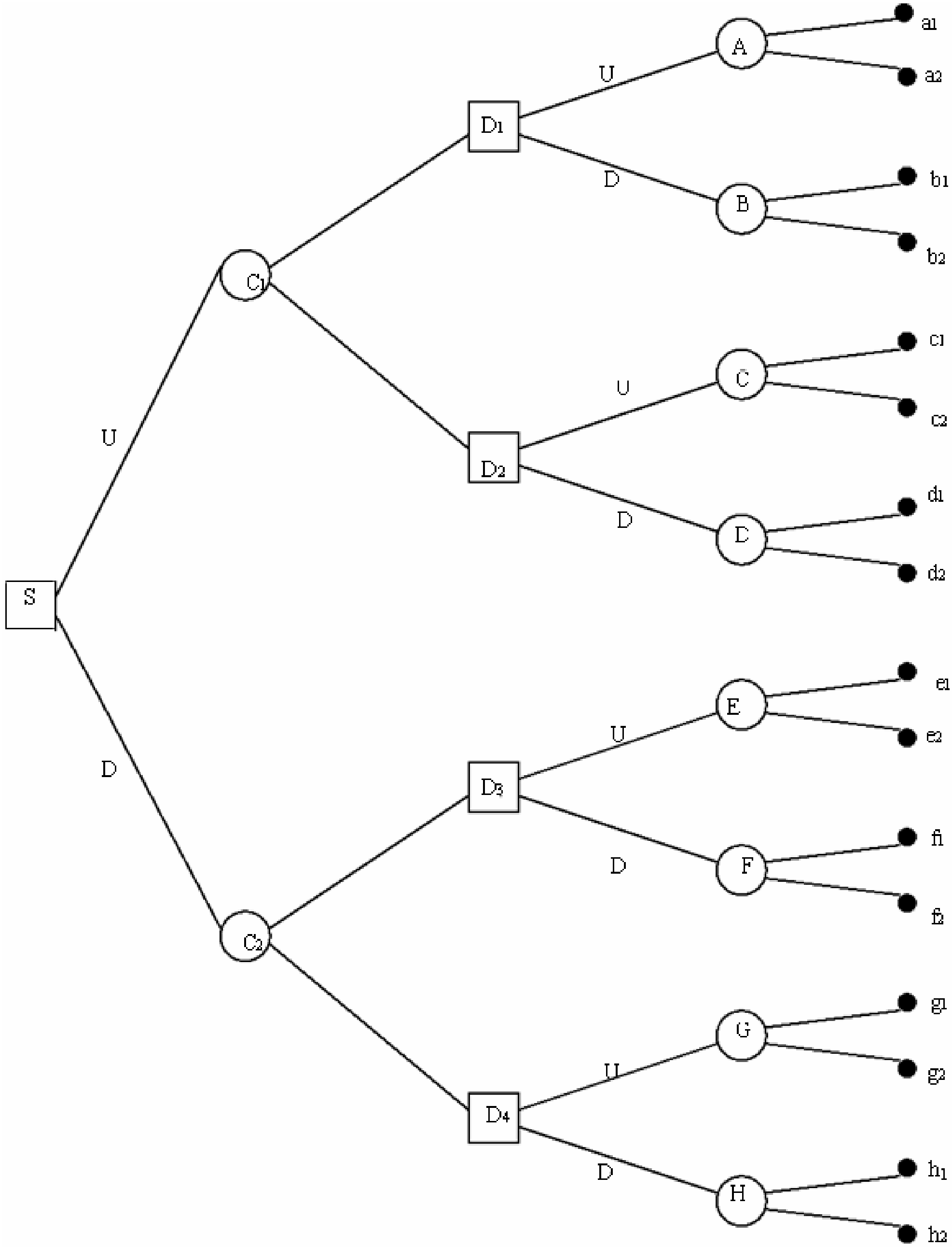


**Figure 17** *The Basic Tree*

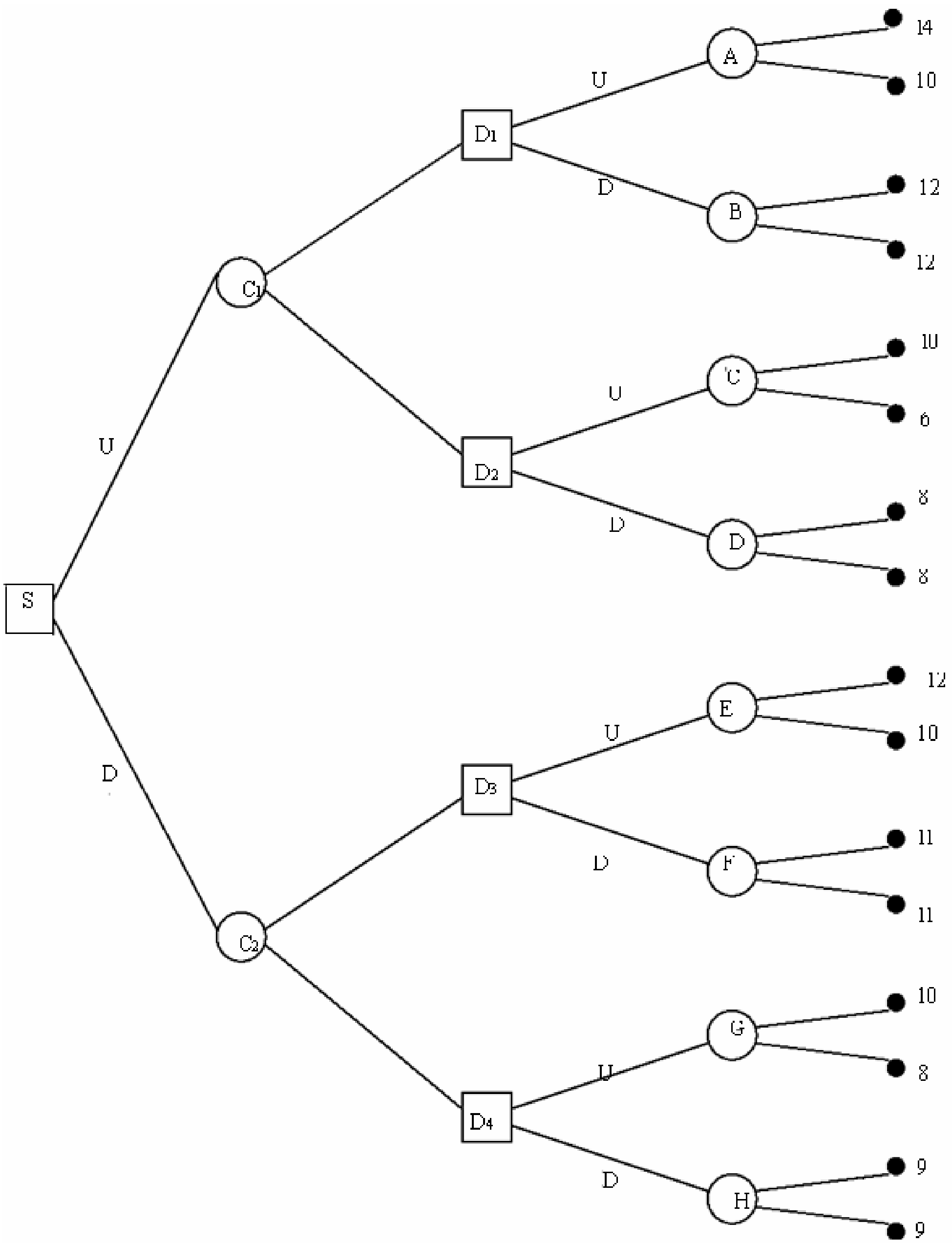


**Figure 18** *A Specific Example*

## Appendix Theorem 1

We do not provide a complete proof – one can be provided on request. The algebra of the proof varies from case to case but the principal is the same. The thing that we want to prove is the following. Suppose we have two risky prospects *C* and *D* and we know that some non-EU person prefers *C* to *D*. Does it follow that this individual prefers *[A, C]* to *[A,D]* where *A* is another distinct risky prospect? The answer is no – since we can always find some non-EU person who prefers *C* to *D*, yet prefers *[A,D]* to *[A, C]*. The key to the proof is finding someone who is *e*-close to indifference between *C* and *D*. Because *A* has to be different there will always be some non-EU preference function for which *[A,D]* is preferred to *[A, C]*.. As we simply have to show that such a person exists we do not need to work with a general preference functional but can take any one that satisfies our requirements.

Let us assume rank *dependent* preferences. Let us suppose *u(.)* is the utility function and that *w(.)* is the cumulative probability weighting function. With all the chance nodes being 50-50 gambles, all risky prospects in our experiment have outcomes which have probabilities of 1/4 or 1/2. We therefore need the value of the probability weighting function at values 0, 1/4, 1/2, 3/4, and 1. We obviously take $w(0) = 0$ and $w(1) = 1$ and use the following notation: $w_1 = w(1/4)$, $w_2 = w(1/2)$ and $w_3 = w(3/4)$.

For any given risky prospect the rank dependent functional ranks the outcomes in order, from the worst to the best, and then evaluates the prospect. So the order of the outcomes is crucial to the evaluation. Accordingly there are many different cases, depending upon the ordering of the 6 outcomes, $a_1$, $a_2$, $c_1$, $c_2$, $d_1$ and $d_2$, though some of these can be eliminated by the restrictions we placed earlier – that is: $a_1$ ? $a_2$, $c_1$ ? $c_2$, $d_1$ ? $d_2$ and $c_1$ ? $d_1$ ? $d_2$ ? $c_2$. The number of cases is increased by the fact that the rank dependent preference functional distinguishes between inequalities and strict inequalities. To save space we consider here just one case. The proof for all the other cases follows a similar path. We take the case $a_1 > a_2 > c_1 > d_1 > d_2 > c_2$.

We start with the supposition that we have an individual who (just) prefers *C* to *D*. It follows that

$$\{u(c_2)(1-w_2) + u(c_1)w_2\} - \{u(d_2)(1-w_2) + u(d_1)w_2\} > \varepsilon \qquad (A1)$$

where ε is an arbitrarily small positive number – reflecting the fact the individual (just) prefers *C* to *D*.

The question now is: can such an individual prefer *[A,D]* to *[A,C]*? The answer is 'yes' if the expression in equation below is negative.

$$\{u(c_2) + [u(c_1) - u(c_2)]w_3 + [u(a_2) - u(c_1)]w_2 + [u(a_1) - u(a_2)]w_1\} -$$
$$\{u(d_2) + [u(d_1) - u(d_2)]w_3 + [u(a_2) - u(d_1)]w_2 + [u(a_1) - u(a_2)]w_1\}$$

We can simplify this. The above expression is negative if the expression below is negative.

$$\{u(c_2)(1-w_3) + u(c_1)(w_3-w_2)\} - \{u(d_2)(1-w_3) + u(d_1)(w_3-w_2)\}$$

We can write this as the difference between two weighted averages – just as *(A1)* above – as follows:

$$\{u(c_2)[(1-w_3)/(1-w_2)] + u(c_1)[(w_3-w_2)/(1-w_2)]\} - \{u(d_2)[(1-w_3)/(1-w_2)] + u(d_1)[(w_3-w_2)/(1-w_2)]\} \qquad (A2)$$

Now examine *(A1)* – it is the difference between a weighted average of $u(c_2)$ and $u(c_1)$, with weights $(1-w_2)$ and $w_2$, and the same weighted average of $u(d_2)$ and $u(d_1)$. Expression *(A1)* says that this difference is (just) positive. Expression *(A2)* is the difference between a weighted average of $u(c_2)$ and $u(c_1)$, with weights $(1-w_3)/(1-w_2)$ and $(w_3-w_2)/(1-w_2)$, and the same weighted average of $u(d_2)$ and $u(d_1)$. The question is: whereas the weights in expression *(A1)* made the difference (just) positive, can we have weights in expression *(A2)* that makes the difference negative? The answer is yes in general. Why? Well, in the case of EU we have $w_1 = 1/4$, $w_2 = 1/2$ and $w_3 = 3/4$, in which case the weights in expression *(A1)* are exactly the same as the weights in expression *(A2)* - and so the expression *(A2)* is (just) negative. In the case of non-EU preferences, we can either put more (less) weight on $c_2$ and $d_2$ and less (more) on $c_1$ and $d_1$ by decreasing (increasing) $w_3$ relative to its EU value of 3/4. By so doing – depending upon the curvature of the utility function - we can make the value of the expression *(A2)* (just) negative. Thus the individual prefers *C* to *D* yet prefers *[A,D]* to *[A,C]*.

**Appendix Theorem 2**

We are given that an EU person prefers *A* to *B* or prefers *B* to *A*[23][9]. For simplicity we take *A* as riskier than *B* in the sense used in the text. That is, $a_1 > b_1 > b_2 > a_2$. The question is: does it follow that we can choose *C* and *D* in such a way that those individuals who prefer *A* to *B* prefer *C* to *D* and those individuals who prefer *B* to *A* prefer *D* to *C*? – subject to the crucial proviso that (*A and B*) are different from (*C and D*). Clearly if we put *C* = *A* and *D* = *B* then we know that the individuals who prefer *A* to *B* must prefer *C* to *D* and those who prefer *B* to *A* prefer *D* to *C* – but this violates the proviso. We should therefore make either *C* a little bit different from *A* and/or *D* a little bit different from *B*. But this makes the difference which makes it possible for there to be an individual who prefers *A* to *B* and *D* to *C*.

You can obviously do it through *dominance* – simply make *C* dominate *A* and *B* dominate *D*. Then it immediately follows that *A* preferred to *B* implies that *C* must be preferred to *D*. But it does not work the other way round – if an individual prefers *B* to *A* then it may be the case that this individual prefers *C* to *D* – because *C* is better than *A* and *D* is worse than *B*. In fact we can guarantee that there is always someone who prefers *B* to *A* but is sufficiently close to indifference so that the preference is reversed when we compare *C* with *D*. The point is that (*A and B*) must be different from (*C and D*). Without defining this formally, let us say that they are *ε-different* – where *ε* is non-zero. We can always find some individual (given that we have a continuum of subjects some of whom prefer *A* to *B* and others who prefer *B* to *A*) who is *ε/2* close to indifference. This individual will switch preference.

Furthermore, if neither *A* nor *C* dominate the other and neither *B* or *D* dominate the other[24] it is even easier to find utility functions for which *A* is preferred to *B* and *D* to *C* and other functions for which *B* is preferred to *A* and *C* to *D*. A formal proof seems unnecessary but can be provided on request.

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[23] Indifferent people are ignored in what follows. Obviously such people are potentially a problem – if an individual is indifferent between all the various decisions in the tree then he or she has no need of a plan.

[24] Recall that we are assuming that $a1 \geq b1$ and $b2 \geq a2$.