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Document de Travail
n°2010-16

COMPETITIVE PERMIT MARKETS AND VERTICAL STRUCTURES:
The Relevance of Imperfect Competitive Eco-Industries

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May 2010

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Abstract

Permit markets lead polluting firms to purchase abatement goods from an eco-industry, which is often concentrated. This paper studies the consequences of imperfect competition in an eco-industry on the equilibrium choices of the competitive polluting firms. It then characterizes the second best pollution cap. By comparing this situation to a competitive one, we show that Cournot competition on the abatement good market contributes not only to a non optimal level of emission reduction but also to a higher permit price, which reduces the production level. These distortions increase with market power measured by the margin taken by the non competitive firms and suggest a second best less stringent pollution cap.

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Keywords: pollution permit market, eco-industry, imperfect competition.

J.E.L classification numbers: Q58, L13.

1 Introduction

Since the American Acid Rain Program, an increasing number of countries chooses pollution permit markets to challenge pollution problems. Notably, the European Union Emission Trading Scheme (EU ETS) is open since 2005 and the Kyoto Protocol, which came into force in 2006, establishes an international carbon market, both to fight global warming. The EU ETS, for instance, covers over 11,500 energy intensive installations representing nearly half of the EU's emissions of carbon dioxide (Ellerman and al. [7] and [8]). Thus, pollution permit markets appear to be a useful tool to reduce pollution today.

In fact since the seminal contributions of Crocker [3] and Dales [4], it is well known that this kind of instrument - by setting a price signal for pollution - exhorts pollutant firms to reduce their emissions by purchasing abatement goods and services. This specific demand is notably appeared with the Acid Rain Program (Joskow and al. [14]): this latter has led firms to largely invest in scrubbers, a technology which enables to burn coal without releasing sulfur. More recently, the EU ETS leads to an annual spending for pollution reduction representing €2.9 to €3.7 billion which are also mainly invested in this kind of goods. This demand even grows quickly nowadays since firms must be in conformity with more and more stringent environmental policies. This phenomena largely contributes to the development of an "eco-industry"[1] which furnishes these products to polluting firms.

This new industry is ranging from the development of clean technologies to the optimization of methods for monitoring and managing environmental impacts. From that point of view, the eco-industry represents one of the biggest industrial sectors and covers the pollution management and resource management activities: it matches, since 2005, the aerospace and pharmaceutical sectors in size. However, these activities often remain highly concentrated. Vivendi Environment and ODEO are, for instance, the international leaders in wastewater treatment. Even for more general waste treatment sector, one also rapidly identifies CGEA-Onyx, Sarp, Dalkia, Sita, Elyo. The air treatment sector, in which LAB represents the European leader, seems perhaps more competitive but a cooperation with CNIM will maybe challenge this situation.

This points out that perfect competition is a non-suitable assumption to model an "eco-industry". This has several withdrawals. Notably, the price of the abatement goods does not completely reflect the marginal cost. This clearly affects the decision of the polluting firms,

[1] This notion was introduced in an OECD report [17]. It consists of activities that measure, prevent, limit, minimize or correct environmental damages. The reader is also referred to Sinclair-Desgagné [18] for historical facts in Europe and North-America or to the EU reports [9], [10] and [12].

i.e. their production level and their arbitrage between pollution permit purchase and emission reductions. This paper tries to tackle this issue. In fact we consider a vertically related industry composed of an imperfect competitive eco-industry and standard polluting industry which is submitted to a competitive permit market.

This fact is very important for the pollution permit market literature. Studies which analyze pollution permit markets often take as given an emission reduction cost function. By implicitly identifying its marginal cost to the production cost of the abatement good, they assume perfect competition on this market. Under imperfect competition, we can therefore expect that Montgomery's [15] result which claims that pollution permit markets lead firms to choose the optimal level of emission reduction is invalidated. This is well known when the permit market is non competitive (see Hahn [13]). We obtain a similar result with a rather different approach: we maintain pure competition on the permit market but introduce an imperfect competitive eco-industry.

In the best of our knowledge most of the papers which study imperfect competitive eco-industries only consider, as environmental tool, the pigovian tax (see for instance David and Sinclair-Desgagné [5] and [6], Nimubona and Sinclair-Desgagné [16], Canton [1] and Canton and al. [2]). Our vertical structure remains close to the one of David and Sinclair-Desgagné [5] since they also introduce competitive polluting firms and an imperfectly eco-industry. Contrary to their paper, we introduce a third market i.e. the permit one and study the interactions between all these markets.

It appears therefore that there is no available study of a pollution permit market in presence of an eco-industry. Thus, the aim of this paper is to fill this gap in the economic literature. In this article, we consider a sector composed of three markets: an eco-industry, a polluting product and a pollution permit market. As a benchmark, we assume that these markets are competitive. The first result can be exposed, in this case, as "an old wine in a new bottle" because we find a traditional result: emission reductions are efficient, and the first best pollution cap can be implemented. This result is yet new because the emission reduction cost is not given by a cost function but is explicitly deduced from an optimal choice of abatement and service goods.

In a second step, we introduce market power on this market, whereas the two other ones remain competitive. We are then able to analyze the effects of an imperfectly eco-industry on this three market sector equilibrium. As we already states, it first appears that the optimal level of emission reduction is not reached. We next show that market power on the eco-industry reduces the production levels of the polluting goods and of abatement goods. As less abatement is chosen, the consecutive pollution permit price is higher.

Finally, it appears that the welfare is reduced with respect to the competitive case. To improve this situation, we turn out to a second best analysis, to determine the optimal pollution cap in presence of market power. A way to increase welfare under imperfect competition is to globally reduce less pollution than in the first best outcome.

The paper is as follows. Section 2 is dedicated to the presentation of the model. As a benchmark, Section 3 analyses the competitive equilibrium of this three market sector and defines the first best pollution cap. In Section 4, we introduce imperfect competition in the eco-industry and we study this new equilibrium. Section 5 discuses the optimal pollution cap in a second best framework. Finally, some concluding remarks are given in Section 6. The different proofs are relegated to an appendix.

2 A vertically integrated polluting industry

In order to illustrate the problem depicted higher, we consider a competitive polluting industry which faces a standard demand curve. These firms have the opportunity to reduce their emissions by purchasing abatement goods from an upstream eco-industry. Their net emissions must however be covered by pollution permits supplied on a competitive market. The total amount of permits is controlled by a regulator.

The *polluting industry* is composed of a continuum $[0, 1]$ of identical firms which behave competitively. Each member $i \in [0, 1]$ of this industry produces a given output $q(i)$ at a constant marginal cost $c > 0$ sold at price p_Q. This activity is polluting. For simplicity, we identify the level of emission of each firm to its production. This one can nevertheless by reduced by an investment in abatement goods sold by an upstream eco-industry. We denote by $a(i)$ the amount of abatement goods used by firm $i \in [0, 1]$ and measure the reduction of emissions by the function $\alpha(a(i))$. Each firm has access to the same technology which behaves like a standard production function. We even assume that this function has a constant elasticity, i.e. $\alpha(a) = a^{\mu}$ with $\mu \in [0, 1]$. This last assumption gives us the opportunity to capture the effects of the market power exerted by the eco-industry since $(\mu - 1)$ is, as we will see it later, nothing else that the elasticity of the inverse demand for abatement goods.

The *eco-industry*, composed of n members indexed by j, supplies these abatement goods at price p_A. We denote by a_j the production level of firm j and assume that each member of this industry supports the same unit production cost k per unit of abatement good. As the number of firms is finite and typically small, these firms choose their quantities strategically. We are therefore particularly interested by the distortion that is induced with respect to perfect competition, especially concerning the total amount of pollution permits emitted by the regulator.

The *final demand* for the polluting good is noted by $d(p_Q)$. This function is assumed to be downward slopping, i.e. $d'(p_Q) < 0$, and to verify[2] $\lim_{p_Q \to 0} d(p_Q) = +\infty$ and $\lim_{p_Q \to +\infty} d(p_Q) = 0$. As a consequence, the inverse demand curve $p(Q) = d^{-1}(Q)$ is defined for all positive aggregated production level Q. We however also introduce a more technical assumption which states that the elasticity of the marginal demand curve verifies $e_{d'} := \frac{d" \cdot p}{d'} > -1$. This assumption simplifies the study of the second best optimization problem. It is satisfied if, for instance, the demand curve is concave. We therefore claim, in some sense, that the demand curve is not too convex.

The *global emissions* induce a social damage measured by $D(E)$ which increases at an increasing rate, i.e. $D'(E) > 0$ and $D"(E) > 0$. Of course $D(0) = 0$, and we even assume that the marginal damage is small for a low level of emission, i.e. $\lim_{E \to 0} D'(E) = 0$ while it becomes very large for huge emissions, i.e. $\lim_{E \to +\infty} D'(E) = +\infty$.

A *regulator* controls these emissions by organizing a tradeable pollution permit market, which is competitive. The polluting firm must cover their emissions with the corresponding amount of permits. We assume, for simplicity, that the regulator directly sells, at the competitive price p_E, an amount $\bar{E}$ of permits to these firms. This last assumption directly follows from the seminal result of Montgomery [15] which claims that the competitive equilibrium of pollution permit market is independent of the mechanism beyond the permit distribution. We also assume that $\bar{E} \in [0, d(c)[$ since $d(c)$ corresponds to the production level without regulation, and under our assumption, to the unconstrained emission level. The equilibrium condition of the permit market is therefore

$$\bar{E} = Q - \int_0^1 \alpha\left(a(i)\right) di \qquad (1)$$

[2]This strong boundary behavior is essentially introduced for convenience. In fact we want to be sure that both the equilibrium price p_Q and the quantity Q are strictly positive. Our argument also holds if the demand is bounded from above or if there exists a maximal reservation price provided that this quantity is large enough.

3 The competitive case: old wine in a new bottle

The purpose of this section is to study the competitive case. This is of course "old wine" because most of the results are known. The "bottle is nevertheless new": it gives us the opportunity to understand what changes if an eco-industry is introduced and, from that point of view, figures out the distortions induced by an imperfect competitive eco-industry.

3.1 The competitive allocation

In the large polluting firm sector, each producer sets her level of production and her demand for abatement goods in a way to maximize her profit, taking into account the cost induced by her purchase of pollution permits. Moreover, we know that these firms are identical and belong to $[0, 1]$. We can therefore restrict our attention to a representative firm which chooses the aggregated level of production Q and the aggregated demand A for abatement goods. These quantities solve:

$$\max_{(Q,A)\geq 0} \pi(Q, A) := (p_Q - c)Q - p_A A - p_E (Q - \alpha(A))$$

Moreover, it is immediate that this function is linear with respect to Q since $\partial_Q \pi = p_Q - c - p_E$. So if the commodity market clears, we can say that:

Remark 1 *If the polluting good market is competitive, the equilibrium price p_Q^c and the amount of traded goods Q^c are respectively given by $p_Q^c = c + p_E$ and $Q^c = d(c + p_E)$.*

It remains to characterize the competitive demand for abatement goods. This one follows directly from the derivative of the profit with respect to A and is given by[3]:

$$-p_A + p_E \alpha'(A) = 0 \Leftrightarrow \frac{p_A}{\alpha'(A)} = p_E \tag{2}$$

This is simply *old wine* because this condition states that the marginal cost of pollution reduction must be equal to the permit price. In fact, in our setting, a reduction r of the emissions requires $\alpha^{-1}(r)$ abatement goods and costs $c(r) = p_A \alpha^{-1}(r)$. This condition therefore states that

$$c'(r) = \frac{p_A}{\alpha'(\alpha^{-1}(r))} = \frac{p_A}{\alpha'(A)} \tag{3}$$

[3] This condition is necessary and sufficient since the profit function is concave with respect to A, i.e. $\frac{\partial^2 \pi}{\partial A \partial A} = \alpha"(A) < 0$.

But *the bottle is new*. The introduction of an eco-industry gives us the opportunity to explicitly construct the cost $c(r)$ associated to the emission reductions. It emphasizes the technological aspect of this operation and underlines the impact of the abatement good price. This implies that the behavior of the polluting firms in now driven by two price signals: the permit price which transmits some information on the damage and the abatement good price which provides information on the cost of this good. This latter typically becomes important in presence of market power even if other markets remain competitive.

But in any case, this last condition gives us the opportunity to directly spell out the inverse demand function for abatement good[4].

Remark 2 *If the polluting firms act competitively, the inverse demand for abatement good is given by $P_A(A, p_E) = p_E \alpha'(A)$ while the demand is defined by $D_A(p_A, p_E) = (\alpha')^{-1}\left(\frac{p_A}{p_E}\right)$.*

Let us now move to the behavior of the firms in eco-industry. These firms support the same constant marginal production cost k. If they act as pure competitors, a standard market clearing condition imposes that $p_A^c = k$. Moreover, if we restrict our attention to a symmetric equilibrium, each of them produces at equilibrium $a_j^c = \frac{1}{n} D_A(k, p_E)$.

It therefore simply remains to clear the permit market. When both the commodity and the abatement good markets clear, the equilibrium condition (1) becomes:

$$\bar{E} = E^c(p_E) := d(c + p_E) - \alpha\left((\alpha')^{-1}\left(\frac{k}{p_E}\right)\right)$$

So, if this demand $E^c(p_E)$ for permits is decreasing and satisfies suitable boundary conditions, we can claim that:

Proposition 1 *If the regulator fixes the pollution cap to $\bar{E} \in [0, d(c)[$, our three competitive market sector admits a unique symmetric equilibrium. We can even say that:*
(i) the unique permit price p_E^c which solves $E^c(p_E^c) = \bar{E}$ is strictly positive if $\bar{E} < d(c)$,
(ii) the commodity and the abatement good prices are given by $p_Q^c = c + p_E^c$ and $p_A^c = k$,
(iii) the production level $Q^c = d(c + p_E^c) < d(c)$ is reduced with respect to a situation without regulation,

[4] Since the emission reduction technology is concave and verifies the standard Inada conditions, $(\alpha')^{-1}$ and, therefore, the demand for abatement good are properly defined .

(iv) the eco-industry is active since total amount of abatement goods is given by $A^c = D_A(k, p_E^c) = (\alpha')^{-1}\left(\frac{k}{p_E^c}\right) > 0.$

3.2 The optimal choice of the pollution cap

Since the polluting industry induces a negative externality, let us first characterize an efficient allocation. In this case, the regulator sets the production levels of each firm in both sectors and controls for the optimal reduction of the emissions. Her optimal choice maximizes welfare under suitable feasibility conditions.

As the polluting firms share the same linear cost, the regulator can therefore restrict her attention to the aggregated production level $Q = \int_0^1 q(i)di$. Moreover, let us remember that the emission reduction function $\alpha(a)$ is concave and that the social damage $D(E)$ is increasing and convex. This implies that, at an efficient allocation, each polluting firm reduces her emissions by using the same amount A of abatement goods, and this quantity can be identified to the aggregated quantity traded on this market[5]. The allotment of this quantity to the producers does not really matter since each firm in the eco-industry supports the same marginal production cost. From that point of view, we study the welfare properties by restricting our attention to the aggregated variables. More precisely the efficient production level Q^e and the total amount of abatement good A^e solve:

$$(Q^e, A^e) \in \arg\max_{(Q,A)\geq 0} \int_0^Q p(Q)dQ - c\cdot Q - k\cdot A - D\left(Q - \alpha\left(A\right)\right)$$

with the efficient amount of emission given by $E^e = Q^e - \alpha\left(A^e\right)$. We can even claim (see the proof of Proposition 2) that the following first order conditions

$$\begin{cases} D'\left(E^e\right) = p(Q^e) - c \\ D'\left(E^e\right)\alpha'(A^e) = k \end{cases} \tag{4}$$

fully characterize the optimal solution.

This is again an *old wine in a new bottle*. The first condition tells us that the optimal pollution cap must be set in a way which ensures that the marginal damage is equal to the marginal benefit of the consumers which is represented, here, by the marginal net surplus. The second one illustrates the idea that the marginal effect on pollution of the use of an additional

[5] This follows directly from the fact that we have introduce a continuum $[0,1]$ of polluting firms, i.e. $\int_0^1 Adi = A$.

unit of abatement good is equal to its marginal production cost. This is of course equivalent to state that the marginal damage is equal to the marginal cost of emission reduction, i.e. $D'(E) = c'(r) = \frac{k}{\alpha'(A^e)}$. This again underlines the importance of the abatement technology and of the marginal production cost. So, if the price of the abatement good is not equal to its production cost - even when both the commodity and the permit market are competitive - one can expect that the first best allocation is out of reach.

So if we want to summarize our results, we can state that:

Proposition 2 *There exists a unique interior solution* (Q^e, A^e) *to this problem which has the property that:*
(i) the strictly positive pollution cap is given by $E^e = Q^e - \alpha(A^e) > 0$,
(ii) the aggregated production level satisfies $Q^e = d(c + D'(E^c))$,
(iii) the efficient amount of abatement goods is $A^e = (\alpha')^{-1}\left(\frac{k}{D'(E^e)}\right)$.

Let us now compare the efficient production levels Q^e and A^e to those obtained at the competitive equilibrium (see Proposition 1 (iii) and (iv). We obtain the traditional result which claims that the competitive equilibrium is efficient if the permit price is equal to the marginal damage, i.e. $D'(E^e) = p_E$. It is, again, also important to notice that this result holds because the competitive prices are equal to the marginal production costs. This is particularly true for the abatement good price p_A which must be equal to k. Thus, if the firms in the eco-industry take a margin over their costs, one can expect that none of these results hold.

4 An imperfect competitive eco-industry

Let us now move to a situation in which market power is introduced within the eco-industry whereas the two other markets remain competitive. In this case, we essentially show that the first best cannot be reached whatever the level of the pollution cap. In order to obtain this result, we first analyze the optimal strategies of the eco-industry members, and we then compute the equilibrium of this three market sector.

4.1 The optimal strategies

By imperfect competition, we simply mean that firms in the eco-industry act as Cournot players. It therefore becomes important to know the inverse demand for abatement goods. Since the polluting firms maintain their competitive behavior, Remark 2 remains true and the inverse demand curve is given by $P_A(A, p_E) = p_E \cdot \alpha'(A)$. From that point of view, each producer $j = 1, \ldots, n$, composing the eco-industry chooses her production level in order to maximize:

$$\max_{a_j \geq 0} \pi \left(a_j, a_{-j}\right) := \left[p_E \cdot \alpha' \left(\sum_{j=1}^{n} a_j\right) - k\right] \cdot a_j$$

If a Nash equilibrium exists, we can say that the optimal strategies satisfy the following set of First Order Conditions:

$$\forall j = 1, \ldots, n, \qquad \left[p_E \cdot \alpha' \left(A^{ic}\right) - k\right] + p_E \cdot \alpha\text{"} \left(A^{ic}\right) \cdot a_j^{ic} = 0$$

Since the firms in the eco-industry are all symmetric, we can even expect that this Nash equilibrium shares the same property, i.e. $a_j^{ic} = \frac{A^{ic}}{n}$. The preceding set of FOCs can be replaced by the following aggregated condition:

$$p_E \cdot \alpha' \left(A^{ic}\right) = \left(1 + \frac{\varepsilon_{\alpha'}}{n}\right)^{-1} \cdot k \tag{5}$$

where $\varepsilon_{\alpha'}$ stands for the elasticity of α'. With an isoelastic abatement function, this latter is even constant and is given by $\varepsilon_{\alpha'} = \mu - 1$.

Let us now observe that $p_E \cdot \alpha' \left(A^{ic}\right)$ is, by our early definition of the inverse demand curve, the price p_A^{ic} that clears the abatement good market. The quantity

$$m := \left(1 + \frac{\varepsilon_{\alpha'}}{n}\right)^{-1} = \frac{n}{n + \mu - 1}$$

can therefore be viewed as a margin since $p_A^{ic} = m \cdot k$, which measures the degree of the market power of the eco-industry.

To sum up, we can assert that:

Proposition 3 *Whatever the permit price p_E, we can say that:*
(i) there exists a unique symmetric Cournot-Nash equilibrium in the eco-industry,
(ii) the level of abatement goods A^{ic} produced by this industry is given by $A^{ic} = (\alpha')^{-1} \left(m \cdot \frac{k}{p_E}\right)$,
(iii) the equilibrium price p_A^{ic} is obtained by taking a margin over the marginal cost, i.e. $p_A^{ic} = m \cdot k$.

4.2 Equilibrium and inefficiency

Let us now move to the study of the global equilibrium of our three market sector. Since the final good market works competitively, Remark 1 remains true and we simply have to care about the equilibrium on the permit market. Under imperfect competition, this equilibrium condition (see Eq. (1)) becomes:

$$\bar{E} = E^{ic}(p_E) = d\,(p_E + c) - \alpha\left(\left(\alpha'\right)^{-1}\left(m \cdot \frac{k}{p_E}\right)\right)$$

So, if this demand $E^{ic}(p_E)$ for permits is decreasing and satisfies suitable boundary conditions, we can claim that:

Proposition 4 *If the regulator fixes the pollution cap to $\bar{E} \in [0, d(c)[$, our three market sector admits a unique non competitive equilibrium and we even observe that:*
(i) the equilibrium quantities $A^{ic}(m, \bar{E})$ and $p_A^{ic}(m, \bar{E})$ decrease with $\bar{E}$,
(ii) the production level $Q^{ic}(m, \bar{E})$ increases with $\bar{E}$ and verifies that $\frac{\partial Q^{ic}(m,\bar{E})}{\partial E} \in]0, 1[$.

It appears that these last properties are, from a qualitative point of view, similar to those obtained in the competitive case but this does not mean that market power does not matter because it affects the equilibrium levels. Indeed as the eco-industry takes a margin over their costs, the incentives to buy abatement goods are reduced, i.e.

$$\forall m > 1 \quad A^{ic} = \left(\alpha'\right)^{-1}\left(m \cdot \frac{k}{p_E}\right) < \left(\alpha'\right)^{-1}\left(\frac{k}{p_E}\right) = A^c$$

It is therefore obvious that if $m = 1$, there is no market power and we are back to the competitive allocation. This parameter can be used to measure the importance of the distortion induced by imperfect competition. We can say that:

Proposition 5 *As market power increases, we observe that the equilibrium quantities $A^{ic}(m, \bar{E})$ and $Q^{ic}(m, \bar{E})$ decrease while the permit price $p_E^{ic}(m, \bar{E})$ and the abatement good price $p_A^{ic}(m)$ increase. Since for $m = 1$ we are back to perfect competition, we can even say that imperfect competition ($m > 1$):*
(i) reduces the production of abatement goods i.e. $A^{ic}(m, \bar{E}) < A^c(\bar{E})$ and increases their price $p_A^{ic}(m) > p_A^c$,
(ii) increases the permit price i.e. $p_E^{ic}(m, \bar{E}) > p_E^c(\bar{E})$ and reduces the production of the final good $Q^{ic}(m, \bar{E}) < Q^c(\bar{E})$.

Let us now move to the efficiency issue. Market power on the upstream market typically induces cost inefficiencies. In our setting, this market is however linked to the competitive permit market through the behaviors of the polluting firms. We can therefore expect that this inefficiency has two aspects.

As usually, imperfect competition on the abatement good market increases the price and reduces the amount of traded abatement goods (see Proposition 5 (i)). From that point of view, the price which incorporates a margin never transmits a true cost signal and therefore conducts the polluting firms to choose a non optimal level of emission reduction.

Since less abatement goods are traded, imperfect competition also induces an additional demand for pollution permits and contributes to an increase of their price. We recall that the purpose of this price is to transmit a true information on the marginal damage created by one emission unit. We can then expect this market over estimates the damage and therefore that the firms excessively reduce their production of the polluting good (see Proposition 5 (ii)).

In the view of the last proposition, we can also claim that imperfect competitive allocation is never efficient, even for a suitable choice of the pollution cap. To be more precise, we know that the competitive allocation is efficient if the marginal damage is equal to the permit price. Under imperfect competition (i.e. $m > 1$), the production levels are always strictly smaller than those obtained under perfect competition, and this result holds whatever the number of permits available on the permit market. To sum up we can say that:

Proposition 6 *If there is market power on the abatement good market (i.e.* $m > 1$*), even if the permit market remains competitive, we observe that:*

(i) the polluting firms never choose the optimal level of emission reduction,

(ii) the permit price transmits a biased information on the damage,

(iii) it is impossible to find a suitable pollution cap E *which implements the first best.*

So if we want to find the accurate pollution cap, we have to move to a second best analysis.

5 The second best policy

The second best policy tries to correct the inefficiency induced by imperfect competition. However, as noted before, this one produces two effects which are in tension. In fact:

- On the one hand, the regulator may want to make sure that the *permit price provides a non biased information on the damage.* In this case, the pollution cap must ensure that the permit price coincides to the marginal damage, i.e. $p_E^{ic}(m,E) = D'(E)$. However, this price is always greater than the one obtained under perfect competition. This option therefore induces a compliant policy or even a too permissive one.

- On the other hand, she may want to correct the distortion on the abatement good market and, so, to *secure the production of the efficient level of abatement good*. In this case, the pollution cap must satisfy $p_E^{ic}(m,E) = m \cdot D'(E)$. This makes sure that the marginal productivity of the abatement technology is equal to $\frac{k}{D'(E)}$ and therefore that the cost efficiency is restored. This policy however induces a high permit price and conducts to a restrictive emission policy which too seriously depresses the production of the polluting good.

We can therefore expect that the second best policy weighs these two opposite options. In order to verify this point, let us first observe that a second best pollution cap $E^{sb}(m)$ solves:

$$E^{sb}(m) \in \arg\max_{E\in[0,d(c)[} W(m,E) := \int_0^{Q^{ic}(m,E)} p(q)dq - c \cdot Q^{ic}(m,E) - k \cdot A^{ic}(m,E) - D(E)$$

We can even say, under our assumptions, that:

Lemma 1 *The previous concave problem (i.e.* $\frac{\partial^2 W(m,E)}{\partial E \partial E} < 0$*) admits a unique interior solution.*

The optimal second best pollution cap satisfies therefore the following first order condition:

$$\frac{\partial W(m,E)}{\partial E} = p_E^{ic}(m,E^{sb}) \cdot \frac{\partial Q^{ic}(m,E^{sb})}{\partial E} - k \cdot \frac{\partial A^{ic}(m,E^{sb})}{\partial E} - D'(E^{sb}) = 0 \tag{6}$$

Now let us remember that a second best policy is chosen within the set of imperfect competitive equilibria. This means that the permit market always clears so that:

$$\forall m, E \quad E = Q^{ic}(m,E) - \alpha\left(A^{ic}(m,E)\right) \Rightarrow 1 = \frac{\partial Q^{ic}(m,E)}{\partial E} - \alpha'\left(A^{ic}(m,E)\right) \cdot \frac{\partial A^{ic}(m,E)}{\partial E}$$

but this also implies that polluting firms make optimal choices, and that:

$$p_E^{ic}(m,E^{sb}) \cdot \alpha'\left(A^{ic}(m,E)\right) = m \cdot k$$

Given these observations, condition (6) becomes:

$$D'(E^{sb}) = \frac{\partial Q^{ic}(m,E^{sb})}{\partial E} \cdot p_E^{ic}(m,E^{sb}) + \left(1 - \frac{\partial Q^{ic}(m,E^{sb})}{\partial E}\right) \cdot \frac{p_E^{ic}(m,E^{sb})}{m}$$

Since $\frac{\partial Q^{ic}(m,E^{sb})}{\partial E} \in]0,1[$ (see Proposition 4 (ii)), we can effectively say that the regulator selects a pollution cap which combines the two objectives that we have depicted earlier. We can even observe that the permissive policy which consists in transmitting the non biased price signal of the damage is weighed by the marginal effect of an increase of the pollution cap on the supply of goods. So, let us consider a situation in which a raise of the pollution cap largely increases the production of polluting goods. In this case, the regulator favors a permissive environmental policy because the consecutive increase of the marginal damage is compensated by the effect on welfare of an increasing production. In opposite, if this effect is not too important, the planner prefers a strategy which implements an optimal production of abatement goods and, therefore, leads to a lower pollution cap.

The previous formula gives us the level of the second best pollution cap. This however does not indicate how market power affects this pollution cap and provides no information on the gap between the second and the first best policy. However we know that for $m = 1$ we have the competitive, and therefore the first best outcome. So we must go back to the first order condition and look at the second best pollution cap as a function of the mark-up, i.e. compute $E^{sb}(m)$. In fact:

Proposition 7 *We can show that* $\frac{dE^{sb}}{dm} > 0$*, hence:*

(i) the second best pollution cap is greater then the first best one,

(ii) the gap between both increases with the degree of market power.

6 Concluding remarks

The aim of this article was to analyze the relevance of imperfect eco-industry, when a vertical structure and a competitive pollution permit market are considered. In this new framework, the polluting firms deal with two price signals to choose their level of emission reduction: the permit price, which transmits information on the damage value and the abatement good price, which is related to abatement cost. If these price signals reflect the true values, like in perfect competition, the first best can be reached.

These results are challenged if we consider an imperfect competitive eco-industry. Both price signals transmit biased information. The abatement good price includes a margin and the permit price is higher than the marginal damage. Thus, firms do not choose the optimal level of emission reduction. So, we extend, in this article, the seminal work of Hahn [13]. As a consequence, at equilibrium, the production of abatement and polluting goods are reduced with respect to perfect competition. As this equilibrium is not efficient, we turn out to a second best analysis to find the pollution cap. This optimal choice balances two effects: the first tries to restore a true price signal on the damage which leads to a less stringent environmental policy, whereas the second attempts to correct the abatement good price which induces a more stringent policy. We even show that the global emission reduction in the second best is lower than in the first best.

This paper however remains particular on several respects. The reader surely noticed that the market power is measured by the margin taken by the eco-industry. This quantity is constant in our paper, since we have introduced a constant elasticity abatement technology. Even if this simplifying assumption can be very helpful, if would be interesting to look what happens if a more general abatement technology is introduced.

In the paper we also only introduce imperfect competition within the eco-industry. This was enough to underline the double impact of this market structure. It would however be interesting, especially concerning the policy recommendation, to extend this study by considering different sources of market imperfection. We can, for instance, think at imperfect competitive polluting firms which act strategically on the permit or the commodity market or even on both.

It is also well-known, when there is market power on the permit market, that the initial distribution of the pollution permits matters. This is of course not the case in our paper, that why we did not really care about the permit distribution mechanism. So if imperfect competition is also introduced on this market, the policy maker would have another policy instrument to restore efficiency.

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APPENDIX

A Proof of Proposition 1

Step 1: The existence of a unique competitive permit price

Since we have assumed that $\alpha(A)$ is isoelastic[6], we know that the permit demand is given by $E(p_E) = d(c+p_E) - \left(\frac{1}{\mu}\frac{k}{p_E}\right)^{\frac{\mu}{\mu-1}}$. Now let us remember that $d'(p_Q) < 0$, $\lim_{p_Q \to 0} d(p_Q) = +\infty$ and $\lim_{p_Q \to +\infty} d(p_Q) = 0$. Since $\mu \in \,]0,1[$, we can say (i) that $\lim_{p_E \to 0} E(p_E) = d(c) > 0$ and $\lim_{p_E \to +\infty} E(p_E) = -\infty$ and (ii) that $E'(p_E) = d'(c+p_E) - \frac{1}{1-\mu}\left(\frac{1}{\mu}\frac{k}{p_E}\right)^{\frac{1}{\mu-1}} \frac{k}{p_E^2} < 0$. This is why there exists a unique price p_E^c which clears the permit market, i.e. solves $E(p_E^c) = \bar{E}$, for all $\bar{E} \in [0, d(c)]$. The reader even observes that when $\bar{E} = d(c)$ this unique solution is given by $p_E^c = 0$ because $d(c)$ is the level of emission without regulation.

Step 2: Prices and quantities

Since the profit of the representative polluting firm is linear in Q and the members of the eco-industry share the same constant unit production cost, it is immediate that $p_Q^c = c + p_E^c$ and $p_A^c = k$. Now let us observe, by Step 1, that for all $\bar{E} < d(c)$, we have $p_E^c > 0$, this implies that $Q^c = d'(c+p_E) < d(c)$ and that $A^c = (\alpha')^{-1}\left(\frac{k}{p_E^c}\right) > 0$.

B Proof of Proposition 2

Let us remember that an efficient allocation satisfies:

$$(Q^e, A^e) \in \arg\max_{(Q,A)} W(A,Q) := \int_0^Q p(Q)dQ - c \cdot Q - kA - D\left(Q - \alpha\left(A\right)\right)$$

The main question is therefore the existence of a unique solution. We will proceed by steps:

Step 1: $W(A,Q)$ is a strictly concave function

Let us compute $\mathcal{H}\left(W(A,Q)\right)$ the Hessian of $W(A,Q)$. This matrix is given by:

$$\mathcal{H}\left(W(A,Q)\right) = \begin{bmatrix} p'(Q) - D"\left(Q - \alpha\left(A\right)\right); & D"\left(Q - \alpha\left(A\right)\right)\alpha'\left(A\right) \\ D"\left(Q - \alpha\left(A\right)\right)\alpha'\left(A\right); & -D"\left(Q - \alpha\left(A\right)\right)\left(\alpha'\left(A\right)\right)^2 + D'\left(Q - \alpha\left(A\right)\right)\alpha"\left(A\right) \end{bmatrix}$$

Under our assumptions, we can say that:

[6] The reader however observes that the proof of this result can be done without using this specific functional form. In fact our point simply requires that $\alpha(a)$ is an increasing and concave function which satisfies the Inada conditions. We however need this assumption later in order to nicely identify market power.

- $\frac{\partial^2 W(A,Q)}{\partial A \partial A} = p'(Q) - D"(E) < 0$
- $\det(\mathcal{H}(W(A,Q))) = p'(Q)\left[-D"(E)(\alpha'(A))^2 + D'(E)\alpha"(A)\right] - D"(E)D'(E)\alpha"(A) > 0$

or, in other words, that the welfare is a strictly concave function. It follows that the solution, if it exists, is unique and is characterized by the following first order conditions:

$$\begin{cases} \phi_1(Q,A) := \frac{\partial W}{\partial Q} = p(Q) - c - D'(Q - \alpha(A)) = 0 \\ \phi_2(Q,A) := \frac{\partial W}{\partial A} = -k + (D'(Q - \alpha(A))) \cdot \alpha'(A) = 0 \end{cases}$$

Step 2: The construction of $A(Q)$ satisfying $\phi_2(Q, A(Q)) = 0$

Let us first observe that $\forall Q > 0$, $\lim_{A\to 0}\phi_2(Q,A) = +\infty$ and $\lim_{A\to\alpha^{-1}(Q)}\phi_2(Q,A) = -k$. Moreover, since $D'(E), D"(E) > 0$ and $\alpha"(A) < 0$, we can also say that:

$$\frac{\partial \phi_2(Q,A)}{\partial A} = -D"(Q - \alpha(A)) \cdot (\alpha'(A))^2 + D'(Q - \alpha(A)) \cdot \alpha"(A) < 0$$

It follows, by the implicit function theorem, that $\exists A :]0,+\infty[\to \mathbb{R}$ with the property that $\forall Q > 0$, (i) $\phi_2(Q, A(Q)) = 0$, and (ii):

$$\frac{dA}{dQ} = \frac{D"(Q - \alpha(A)) \cdot \alpha'(A)}{D"(Q - \alpha(A)) \cdot (\alpha'(A))^2 - D'(Q - \alpha(A)) \cdot \alpha"(A)} > 0$$

Moreover, under our assumptions, we can even say that $A(Q)$ verifies:

- $\forall Q > 0$, $A(Q) < \alpha^{-1}(Q)$, since $\lim_{A\to\alpha^{-1}(Q)}\phi_2(Q,A) = -k$.
- $\alpha'(A) \cdot \frac{dA}{dQ} \in]0,1[$ because $\alpha"(A) < 0$ and $D'(E), D"(E) > 0$
- $\lim_{Q\to 0} A(Q) = 0$ since $A(Q) \leq \alpha^{-1}(Q)$ and $\alpha(0) = 0$
- $\lim_{Q\to+\infty} A(Q) = +\infty$. In fact, if $A(Q)$ is bounded, then $\lim_{Q\to+\infty}\phi_2(Q, A(Q)) = +\infty$ because $\lim_{E\to+\infty} D'(E) = +\infty$. But in this case we would be able to exhibit a finite Q with the property that $\phi_2(Q, A(Q)) \neq 0$, a contradiction.

Step 3: The existence of a solution

Let us now define $\Phi(Q) := \phi_1(Q, A(Q))$. It is a matter of fact to observe (i) that $\lim_{Q\to 0}\Phi(Q) = +\infty$ since $\lim_{Q\to 0} p(Q) = +\infty$ and $\lim_{Q\to 0} D'(Q - A(Q)) = 0$ and (ii) that $\lim_{Q\to+\infty}\Phi(Q) < -c$ since $\lim_{Q\to+\infty} p(Q) = 0$ and $D'(E) \geq 0$. Moreover

$$\begin{aligned} \frac{d\Phi}{dQ} &= p'(Q) - D"(Q - \alpha(A)) + D"(Q - \alpha(A)) \cdot \alpha'(A) \cdot \frac{dA}{dQ} \\ &= p'(Q) - D"(Q - \alpha(A)) \cdot \underbrace{\left(1 - \alpha'(A) \cdot \frac{dA}{dQ}\right)}_{\in]0,1[} < 0 \end{aligned}$$

It follows that there exists a unique Q^e which solves $\Phi(Q) = 0$ and therefore a unique $A^e = A(Q^e)$ such that (Q^e, A^e) satisfies the FOCs. Moreover, by construction, it is immediate that $E^e = Q^e - \alpha(A^e) > 0$, $Q^c = d(c + D'(E^e)) < d(c)$ and $A^e = (\alpha')^{-1}\left(\frac{k}{D'(E^e)}\right) > 0$.

C Proof of Proposition 3

For a given pollution permit price p_E, a Cournot equilibrium on the abatement good market is typically given by:

$$\forall j = 1, \ldots, n, \quad a_j^{ic} \in \arg\max_{a_j} \pi\left(a_j, a_{-j}^{ic}\right) := \left[p_E \cdot \alpha'\left(a_j + \sum_{k=1, k\neq j}^{n} a_k^{ic}\right) - k\right] \cdot a_j$$

<u>Step 1</u>: *$\pi\left(a_j, a_{-j}^{ic}\right)$ is strictly concave in a_j*

By computation we have:

$$\frac{\partial^2 \pi\left(a_j, a_{-j}\right)}{\partial a_j \partial a_j} = p_E \cdot a"(A)\left[2 + \varepsilon_{\alpha"} \cdot \frac{a_j}{A}\right]$$

where $\varepsilon_{\alpha"}$ denotes the elasticity of $\alpha"(A)$. Since we have also assumed that $\alpha(A) = A^{\mu}$, this expression becomes:

$$\begin{aligned} \frac{\partial^2 \pi\left(a_j, a_{-j}\right)}{\partial a_j \partial a_j} &= -p_E \cdot \mu \cdot (1-\mu) \cdot A^{\mu-2} \cdot \left[2 + (\mu - 2) \cdot \frac{a_j}{A}\right] \\ &< -p_E \cdot \mu^2 \cdot (1-\mu) \cdot A^{\mu-2} < 0 \end{aligned}$$

<u>Step 2</u>: *The individual equilibrium strategies*

If a Cournot equilibrium exists, we know by Step 1, that the individual production levels satisfy the following set of FOCs:

$$\forall j = 1, \ldots, n, \quad \frac{\partial \pi\left(a_j, a_{-j}\right)}{\partial a_j} = \left[p_E \cdot \alpha'(A) - k\right] + p_E \cdot \alpha"(A) \cdot a_j = 0$$

This implies that:

$$\forall j = 1, \ldots, n, \quad a_j = \frac{\left[p_E \cdot \alpha'(A) - k\right]}{p_E \cdot \alpha"(A)}$$

and by summation on j we obtain that:

$$A = n \cdot \frac{\left[p_E \cdot \alpha'(A) - k\right]}{p_E \cdot \alpha"(A)} \Leftrightarrow \phi(A) := \alpha'(A) \cdot \left(1 - \frac{\varepsilon_{\alpha'}}{n}\right) = \frac{k}{p_E} \tag{7}$$

where $\varepsilon_{\alpha'}$ denotes the elasticity of $\alpha'(A)$. So if there exists a unique A which solves the previous equation, we can say that there exists a unique Cournot equilibrium and this one is symmetric.

<u>Step 3</u>: *Existence of a unique aggregated production level A*

Since $\alpha(a) = a^{\mu}$, we obtain that $\phi(A) = \mu \cdot A^{\mu-1}\left(\frac{n-\mu+1}{n}\right)$. We also observe that $\lim_{A\to 0} \phi(A) = +\infty$, $\lim_{A\to\infty} \phi(A) = 0$ and

$$\phi'(A) = -\mu \cdot (1-\mu) \cdot A^{\mu-2}\left(\frac{n-\mu+1}{n}\right) < 0$$

It follows that there exists a unique A^{ic} which solves $\phi(A^{ic}) = \frac{k}{p_E}$. Moreover by equation (7), we can say, by definition of the margin, that

$$A^{ic} = \left(\alpha'\right)^{-1}\left(\left(1 - \frac{\varepsilon_{\alpha'}}{n}\right)^{-1} \frac{k}{p_E}\right) = \left(\alpha'\right)^{-1}\left(m \cdot \frac{k}{p_E}\right) = \left(m \cdot \frac{k}{p_E \cdot \mu}\right)^{\frac{1}{\mu-1}}$$

D Proof of Proposition 4

<u>Step 1</u>: *The existence of an imperfect competitive equilibrium*

It simply remains to show that there exists a positive permit price p_E^{ic} which clears this market, i.e.

$$E^{ic}(p_E^{ic}) = d\left(p_E^{ic} + c\right) - \alpha\left(\left(\alpha'\right)^{-1}\left(m \cdot \frac{k}{p_E^{ic}}\right)\right) = d\left(p_E^{ic} + c\right) - \left(\frac{m}{\mu} \cdot \frac{k}{p_E^{ic}}\right)^{\frac{\mu}{\mu-1}} = E$$

So let us first observe that $\lim_{p_E \to 0} E^{ic}(p_E) = d(c) > 0$ since $\mu \in]0,1[$ and that $\lim_{p_E \to +\infty} E^{ic}(p_E) < 0$ since $\lim_{p_E \to +\infty} d\left(p_E^{ic} + c\right) = 0$. Moreover:

$$\frac{dE^{ic}}{dp_E} = d'\left(p_E^{ic} + c\right) - \frac{m \cdot k}{(1-\mu) \cdot p_E^2}\left(\frac{m}{\mu} \cdot \frac{k}{p_E}\right)^{\frac{1}{\mu-1}} < 0$$

we can therefore say that $\forall m \in]1, +\infty[$, $\forall E \in [0, d(c)]$, there exists a unique $p_E^{ic}(m, E)$ which clears this market.

Step 2: The effect of a change in the pollution cap E

We know that, $p_E^{ic}(m, E)$, $A^{ic}(m, E)$ and $Q_m^{ic}(m, E)$ solve:

$$\begin{cases} Q_m^{ic}(m,E) - \left(A^{ic}(m,E)\right)^{\mu} = E \\ -Q_m^{ic}(m,E) + d\left(p_E^{ic}(m,E) + c\right) = 0 \\ \mu \cdot p_E^{ic}(m,E) \cdot \left(A^{ic}(m,E)\right)^{\mu-1} = m \cdot k \end{cases} \tag{8}$$

So if we differentiate this system with respect to E, we obtain after simplification:

$$\begin{bmatrix} 1 & -\mu \cdot \left(A^{ic}\right)^{\mu-1} & 0 \\ -1 & 0 & d'\left(p_E^{ic} + c\right) \\ 0 & -(1-\mu) \cdot p_E^{ic} & A^{ic} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial Q^{ic}}{\partial m} \\ \frac{\partial A^{ic}}{\partial m} \\ \frac{\partial p_E^{ic}}{\partial m} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

We can therefore say that:

$$\begin{bmatrix} \frac{\partial Q^{ic}}{\partial E} \\ \frac{\partial A^{ic}}{\partial E} \\ \frac{\partial p_E^{ic}}{\partial E} \end{bmatrix} = \frac{1}{\Delta} \cdot \begin{bmatrix} (1-\mu) \cdot p_E^{ic} \cdot d'\left(p_E^{ic} + c\right) & \mu \cdot \left(A^{ic}\right)^{\mu} & -\mu \cdot \left(A^{ic}\right)^{\mu-1} \cdot d'\left(p_E^{ic} + c\right) \\ A^{ic} & A^{ic} & -d'\left(p_E^{ic} + c\right) \\ (1-\mu) \cdot p_E^{ic} & (1-\mu) \cdot p_E^{ic} & -\mu \cdot \left(A^{ic}\right)^{\mu-1} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{\Delta} \cdot \begin{bmatrix} (1-\mu) \cdot p_E^{ic} \cdot d'\left(p_E^{ic} + c\right) \\ A^{ic} \\ (1-\mu) \cdot p_E^{ic} \end{bmatrix} \quad \text{with the property that} \quad \begin{bmatrix} (>0) \\ (<0) \\ (<0) \end{bmatrix}$$

with $\Delta = (1-\mu) \cdot p_E^{ic} \cdot d'(p_E^{ic} + c) - \mu \cdot \left(A^{ic}\right)^{\mu} < 0$.

E Proof of Proposition 5

Step 1: The effect of a change in the margin m

By a similar argument as in Step 2 of the proof of Proposition 4, we can say that:

$$\begin{bmatrix} 1 & -\mu \cdot \left(A^{ic}\right)^{\mu-1} & 0 \\ -1 & 0 & d'\left(p_E^{ic} + c\right) \\ 0 & -(1-\mu) \cdot p_E^{ic} & A^{ic} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial Q^{ic}}{\partial m} \\ \frac{\partial A^{ic}}{\partial m} \\ \frac{\partial p_E^{ic}}{\partial m} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{k}{\mu} \cdot \left(A^{ic}\right)^{2-\mu} \end{bmatrix}$$

It follows that:

$$\begin{bmatrix} \frac{\partial Q^{ic}}{\partial m} \\ \frac{\partial A^{ic}}{\partial m} \\ \frac{\partial p_E^{ic}}{\partial m} \end{bmatrix} = \frac{1}{\Delta} \cdot \begin{bmatrix} (1-\mu)\cdot p_E^{ic} \cdot d'\left(p_E^{ic}+c\right) & \mu \cdot \left(A^{ic}\right)^{\mu} & -\mu \cdot \left(A^{ic}\right)^{\mu-1} \cdot d'\left(p_E^{ic}+c\right) \\ A^{ic} & A^{ic} & -d'\left(p_E^{ic}+c\right) \\ (1-\mu)\cdot p_E^{ic} & (1-\mu)\cdot p_E^{ic} & -\mu\cdot\left(A^{ic}\right)^{\mu-1} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \frac{k}{\mu}\cdot\left(A^{ic}\right)^{2-\mu} \end{bmatrix}$$

$$= \frac{\frac{k}{\mu}\cdot\left(A^{ic}\right)^{2-\mu}}{\Delta} \begin{bmatrix} -\mu\cdot\left(A^{ic}\right)^{\mu-1}\cdot d'\left(p_E^{ic}+c\right) \\ -d'\left(p_E^{ic}+c\right) \\ -\mu\cdot\left(A^{ic}\right)^{\mu-1} \end{bmatrix} \quad \text{with the property that} \quad \begin{bmatrix} (<0) \\ (<0) \\ (>0) \end{bmatrix}$$

with $\Delta = (1-\mu)\cdot p_E^{ic}\cdot d'(p_E^{ic}+c) - \mu\cdot\left(A^{ic}\right)^{\mu} < 0$.

Step 2: *Perfect versus imperfect competition*

If we now have in mind that for $m = 1$ the solution to (8) is the competitive equilibrium, it follows that $A^{ic}(m,\bar{E}) < A^{c}(\bar{E})$, $Q^{ic}(m,\bar{E}) < Q^{c}(\bar{E})$ and $p_E^{ic}(m,\bar{E}) > p_E^{c}(\bar{E})$ and that these distortions increase with market power.

F Proof of Proposition 6

This proof is obvious since all observations follow from the results obtained in Proposition 5.

G Proof of Lemma 1

Let us study $E^{sb}(m) \in \arg\max_{E\in[0,d(c)]} W(m,E)$. Since $W(m,E)$ is continuous and $E \in [0,d(c)]$ belongs to a compact set, it is obvious that a solution exists. Let us now verify that this solution (i) is an interior one and (ii) is unique.

Step 1: $E^{sb}(m) \in \left]0, d(c)\right[$

In order to verify that the solution is an interior one let us observe that:

(i) $\lim_{E\to 0} \frac{\partial W(m,E)}{\partial E} > 0$

Since we have assumed that $\lim_{E\to 0} D'(E) = 0$, we obtain by computation that:

$$\lim_{E\to 0} \frac{\partial W(m,E)}{\partial E} = p_E^{ic}(m,0)\cdot \lim_{E\to 0}\frac{\partial Q^{ic}(m,E)}{\partial E} - k\cdot \lim_{E\to 0}\frac{\partial A^{ic}(m,E)}{\partial E}$$

where $p_E^{ic}(m,0)$ solves $E(p_E^{ic}(m,0)) = d\left(p_E^{ic}(m,0)+c\right) - \left(\frac{m}{\mu}\cdot\frac{k}{p_E^{ic}(m,0)}\right)^{\frac{\mu}{\mu-1}} = 0$

By applying a similar argument as the one of the Step 1 of the proof of Proposition 4 we can even say that $p_E^{ic}(m,0) > 0$. Moreover by Proposition 4, we know that $\lim_{E\to 0}\frac{\partial Q^{ic}(m,E)}{\partial E} = \lim_{E\to 0}\frac{(1-\mu)\cdot p_E^{ic}\cdot d'\left(p_E^{ic}+c\right)}{(1-\mu)\cdot p_E^{ic}\cdot d'(p_E^{ic}+c)-\mu\cdot\left(A^{ic}\right)^{\mu}}$. Since $p_E^{ic}(m,0) > 0$, the numerator of this expression is different from 0, so that $\lim_{E\to 0}\frac{\partial Q^{ic}(m,E)}{\partial E} > 0$. Finally, since (see Proposition 4) $\forall E > 0$, $\frac{\partial A^{ic}(m,E)}{\partial E} < 0$, we have that $\lim_{E\to 0}\frac{\partial A^{ic}(m,E)}{\partial E} \leq 0$. We can therefore conclude that $\lim_{E\to 0}\frac{\partial W(m,E)}{\partial E} > 0$.

(ii) $\lim_{E\to d(c)}\frac{\partial W(m,E)}{\partial E} < 0$

In this case, by a similar argument as in the proof of Step 1 of Proposition 4, the price which clears the permit market is now $p_E^{ic}(m, d(c)) = 0$. Since $\frac{\partial Q^{ic}(m,E)}{\partial E} \in [0, 1]$, we obtain that:

$$\lim_{E\to d(c)} \frac{\partial W(m,E)}{\partial E} = -k \cdot \lim_{E\to d(c)} \left(\frac{\partial A^{ic}(m,E)}{\partial E}\right) - D'(d(c))$$

Now let us remember that $\frac{\partial A^{ic}(m,E)}{\partial E} = \frac{A^{ic}(m,E)}{(1-\mu)\cdot p_E^{ic}\cdot d'(p_E^{ic}+c)-\mu\cdot\left(A^{ic}(m,E)\right)^{\mu}}$, but when the permit price goes to zero, there is no incentive to buy abatement good, i.e. $\lim_{E\to d(c)} A^{ic}(m, E) = 0$. We can therefore say that $\lim_{E\to d(c)} \left(\frac{\partial A^{ic}(m,E)}{\partial E}\right) = 0$ and conclude that $\lim_{E\to d(c)} \frac{\partial W(m,E)}{\partial E} = -D'(d(c)) < 0$

<u>*Step 2:*</u> $W(m, E)$ is strictly concave

Let us observe that:

$$\frac{\partial^2 W}{\partial E \partial E} = \frac{\partial p_E}{\partial E} \cdot \frac{\partial Q}{\partial E} + p_E \cdot \frac{\partial^2 Q}{\partial E\partial E} - k \cdot \frac{\partial^2 A}{\partial E\partial E} - D"(E)$$

Moreover, if $\Delta := (1-\mu) \cdot p_E \cdot d'(p_E + c) - \mu A^{\mu} < 0$, we obtain by computation that:

$$\begin{aligned}
\frac{\partial p_E}{\partial E}\cdot\frac{\partial Q}{\partial E} &= \frac{(1-\mu)^2\cdot p_E^2\cdot d'(p_E+c)}{\Delta^2} = \frac{1}{\Delta^3}\left[(1-\mu)^3\cdot p_E^3\cdot\left(d'(p_E+c)\right)^2 - (1-\mu)^2\cdot p_E^2\cdot d'(p_E+c)\cdot\mu\cdot A^{\mu}\right]\\
&= \underbrace{\frac{(1-\mu)^3\cdot p_E^3\cdot(d'(p_E+c))^2}{\Delta^3}}_{X<0} + \underbrace{\frac{k\cdot A\cdot(1-\mu)\cdot p_E\cdot d'(p_E+c)}{\Delta^3}}_{Y>0}\cdot(m\cdot(\mu-1))
\end{aligned}$$

since $p_E \cdot \mu \cdot A^{\mu-1} = m \cdot k$ at equilibrium. With the same argument and the definition of the first order derivatives (see Step 2 of Proposition 4), we can also say that:

$$\begin{aligned}
\frac{\partial^2 Q}{\partial E\partial E} &= \frac{1}{\Delta^2}\left(-\mu\cdot A^{\mu}\cdot(1-\mu)\cdot\frac{\partial p_E}{\partial E}\cdot d'(p_E+c)\cdot\left(1+\varepsilon_{d'/p_E}\right) + \mu^2\cdot A^{\mu-1}\cdot\frac{\partial A}{\partial E}\cdot(1-\mu)\cdot p_E\cdot d'(p_E+c)\right)\\
&= \frac{1}{\Delta^3}\left(-m\cdot k\cdot A\cdot(1-\mu)^2\cdot d'(p_E+c)\cdot\left(1+\varepsilon_{d'/p_E}\right) + m\cdot k\cdot\mu\cdot(1-\mu)\cdot A\cdot d'(p_E+c)\right)\\
&= \frac{k\cdot A\cdot(1-\mu)\cdot d'(p_E+c)}{\Delta^3}\cdot m\cdot\left[\mu-(1-\mu)\cdot\left(1+\varepsilon_{d'/p_E}\right)\right]
\end{aligned}$$

and that:

$$\begin{aligned}
\frac{\partial^2 A}{\partial E\partial E} &= \frac{1}{\Delta^2}\left[\frac{\partial A}{\partial E}\cdot\Delta - A\left((1-\mu)\cdot\frac{\partial p_E}{\partial E}\cdot d'(p_E+c)\cdot\left(1+\varepsilon_{d'/p_E}\right) - \mu^2\cdot A^{\mu-1}\cdot\frac{\partial A}{\partial E}\right)\right]\\
&= \frac{1}{\Delta^3}\left[A\cdot(1-\mu)\cdot p_E\cdot d'(p_E+c) - A\cdot(1-\mu)^2\cdot p_E\cdot d'(p_E+c)\cdot\left(1+\varepsilon_{d'/p_E}\right)\right] + \frac{\mu\cdot A^{\mu+1}(\mu-1)}{\Delta^3}\\
&= \frac{A\cdot(1-\mu)\cdot p_E\cdot d'(p_E+c)}{\Delta^3}\cdot\left(1-(1-\mu)\cdot\left(1+\varepsilon_{d'/p_E}\right)\right) + \underbrace{\frac{\mu\cdot A^{\mu+1}(\mu-1)}{\Delta^3}}_{Z>0}
\end{aligned}$$

It follows that:

$$\begin{aligned}
\frac{\partial^2 W}{\partial E\partial E} &= X - k\cdot Z + Y\cdot\left[m\cdot(\mu-1) + m\cdot\left(\mu-(1-\mu)\cdot\left(1+\varepsilon_{d'/p_E}\right)\right) - \left(1-(1-\mu)\cdot\left(1+\varepsilon_{d'/p_E}\right)\right)\right]\\
&= X - k\cdot Z + Y\cdot\left[2\cdot m\cdot\mu - m - 1 - (m-1)\cdot(1-\mu)\cdot\left(1+\varepsilon_{d'/p_E}\right)\right]\\
&= X - k\cdot Z - Y\cdot\frac{\left[(2\cdot n-1) + (1-\mu)\cdot\left(1+\varepsilon_{d'/p_E}\right)\right]\cdot(1-\mu)}{n+\mu-1} < 0
\end{aligned}$$

since we have assumed that $\varepsilon_{d'/p_E} > -1$, $n \geq 1$ and $\mu \in]0,1[$ and we know that $m = \frac{n}{n+\mu-1}$.

H Proof of Proposition 7

Since for $m = 1$, the second best pollution cap coincides with the first best one, i.e. $E^{sb}(1) = E^e$ point (i) and (ii) of this proposition are obvious when $\frac{dE^{sb}}{dm} > 0$. So let us now check this last point by applying the implicit theorem function to the first order condition given by Equation 6. In this case we can say that $\frac{dE^{sb}}{dm} = -\frac{\partial^2 W(m,E)}{\partial E \partial m} \Big/ \frac{\partial^2 W(m,E)}{\partial E \partial E}$. But we also known that $\frac{\partial^2 W(m,E)}{\partial E \partial E} < 0$ (see Lemma 1), it therefore remains to verify that $\frac{\partial^2 W(m,E)}{\partial E \partial m} > 0$. So let us observe that

$$\frac{\partial^2 W}{\partial E \partial m} = \frac{\partial p_E}{\partial m} \cdot \frac{\partial Q}{\partial E} + p_E \cdot \frac{\partial^2 Q}{\partial E \partial m} - k \cdot \frac{\partial^2 A}{\partial E \partial m}$$

We can also say that:

$$\begin{aligned}
\frac{\partial p_E}{\partial m} \cdot \frac{\partial Q}{\partial E} &= \frac{-k}{\Delta^2} \cdot \left[\mu \cdot A^{\mu-1} \cdot (1-\mu) \cdot p_E \cdot d'(p_E + c)\right] \\
&= \underbrace{\frac{k^2 \cdot (1-\mu) \cdot d'(p_E + c)}{\Delta^3}}_{W>0} \cdot m \cdot \left[-(1-\mu) \cdot p_E \cdot d'(p_E + c) + \mu A^{\mu}\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 Q}{\partial E \partial m} &= \frac{1}{\Delta^2}\left[-\mu \cdot A^{\mu} \cdot (1-\mu) \cdot \frac{\partial p_E}{\partial m} \cdot d'(p_E + c) \cdot \left(1 + \varepsilon_{d'/p_E}\right) + \mu^2 \cdot A^{\mu-1} \cdot \frac{\partial A}{\partial m} \cdot (1-\mu) \cdot p_E \cdot d'(p_E + c)\right] \\
&\quad \underbrace{+ \frac{1}{\Delta^2}\left(\mu^3 \cdot A^{2\mu-1} \cdot \left(-\frac{\partial A}{\partial m}\right)\right)}_{V>0} \\
&= \frac{\mu \cdot (1-\mu) \cdot A^{\mu-1} \cdot d'(p_E + c)}{\Delta^2} \cdot \left[-A \cdot \frac{\partial p_E}{\partial m} \cdot \left(1 + \varepsilon_{d'/p_E}\right) + \mu \cdot \frac{\partial A}{\partial m} \cdot p_E\right] + V \\
&= \frac{k^2 \cdot (1-\mu) \cdot d'(p_E + c)}{p_E \cdot \Delta^3} \cdot m \cdot \left[\mu \cdot A^{\mu} \cdot \left(1 + \varepsilon_{d'/p_E}\right) - \mu \cdot p_E \cdot d'(p_E + c)\right] + V
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 A}{\partial E \partial m} &= \frac{1}{\Delta^2}\left[\frac{\partial A}{\partial m} \cdot \Delta - A\left((1-\mu) \cdot \frac{\partial p_E}{\partial m} \cdot d'(p_E + c) \cdot \left(1 + \varepsilon_{d'/p_E}\right) - \mu^2 \cdot A^{\mu-1} \cdot \frac{\partial A}{\partial m}\right)\right] \\
&= \frac{1}{\Delta^2}\left[\frac{\partial A}{\partial m} \cdot (1-\mu) \cdot p_E \cdot d'(p_E + c) - A \cdot (1-\mu) \cdot \frac{\partial p_E}{\partial m} \cdot d'(p_E + c) \cdot \left(1 + \varepsilon_{d'/p_E}\right) - \mu(1-\mu) \cdot A^{\mu} \cdot \frac{\partial A}{\partial m}\right] \\
&= \frac{k \cdot (1-\mu) \cdot d'(p_E + c)}{\Delta^3} \cdot \left[-p_E \cdot d'(p_E + c) + \mu \cdot A^{\mu} \cdot \left(2 + \varepsilon_{d'/p_E}\right)\right]
\end{aligned}$$

By substitution we obtain:

$$\frac{\partial^2 W}{\partial E \partial m} = V + X \cdot (m-1) \cdot \left[\mu A^{\mu} \cdot \left(2 + \varepsilon_{d'/p_E}\right) - p_E \cdot d'(p_E + c)\right] > 0$$