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VERY PRELIMINARY

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Abstract

We investigate experimentally whether subjects can learn, from their limited experiences, about relationships between the distribution of votes in a group and associated voting powers in weighted majority voting systems (WMV). Subjects are asked to play two-stage games repeatedly. In the second stage of the game, a group of four subjects bargains over how to divide fixed amount of resources among themselves through the WMV determined in the first stage. In the first stage, two out of four subjects in the group, independently and simultaneously, choose from two options that jointly determine the distribution of a given number of votes among four members. These two subjects face a 2×2 matrix that shows the distribution of votes, but not associated voting powers, among four members for each outcome. Therefore, to obtain higher rewards, subjects need to learn about the latter by actually playing the second stage. The matrix subjects face in the first stage changes during the experiment to test subjects' understanding of relationships between distribution of votes and voting power. The results of our experiments suggest that although (a) many subjects learn to choose, in the votes apportionment stage, the option associated with a higher voting power, (b) it is not easy for them to learn the underlying relationships between the two and correctly anticipate their voting powers when they face a new distribution of votes.

Keywords: experiment, learning, voting power, bargaining
JEL codes: C7, C92, D72, D83

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1 Introduction

The weighted majority voting (WMV) system, which gives different amount of votes to each member, is one of the most popular voting systems in many organizations such as European Union, International Monetary Fund and World Bank. In such a system, the distribution of actual voting power is often quite different from the nominal distribution of voting weights. The relationships between the distribution of voting weights and voting powers are quite complex, and many interesting cases are observed in real organizations.

For example, in their study of Council of Ministers in European Economic Community,[1] Felsenthal and Machover (1998, pp.164-165) concludes that it is "not unreasonable to conjecture that the politicians and officials who designed and re-designed the qualified majority voting rule naively assumed that the voting powers of members would be more or less proportional to their respective weights. If so, the allocation of weights is roughly what was *intended* as a distribution of voting power, whereas the *actual* distribution of voting power is an *unintended* and *unforeseen* outcome of that allocation."

Many power indices have been developed to measure and to analyze the voting powers of members under various WMV systems. The literature started to take form with the seminal works by Penrose (1946), Shapley and Shubik (1954) and Banzhaf (1965). Recently, experimental analyses of weighed majority voting systems are attracting much interests (Montero, Sefton, and Zhang, 2008; Aleskerov, Beliani, and Pogorelskiy, 2009; Drouvelis, Montero, and Sefton, 2010). This new strand of literature emerged to complement the empirical analyses of voting powers in real organiza-

[1] Between 1958 and 1973 the EEC was constituted only by its six founding members: Belgium, France, Italy, Luxembourg, Netherlands and West Germany. The article 148 of the Treaty of Rome (25th March 1957) states the rules to take decisions inside the Council of Ministers, the main organ of the Community. Most of the statements (when decisions have to be taken on proposal by the Commission) are valid if there are at least 12 votes over 17 in favor, with an apportionment of the votes between the states done as follow: Belgium 2, France 4, Italy 4, Luxembourg 1, Netherlands 2 and West Germany 4; other decisions require the same qualified majority but expressed by at least 4 countries, and other ones require unanimity (Felsenthal and Machover (1998)). It appears very simple to check that, in the first case, Luxembourg has no power at all to influence a decision. With only one vote in the Council, it does not exist any coalition including this country that reaches the required majority of 12 votes. The only minimal coalitions that can pass a law are the one formed by the three four votes countries, or the ones formed by two four votes country and both the two votes countries. Luxembourg can add itself to the agreement, but its position never matters in order to get a pass or a fail. This situation can be summarized using Power Indices: both the non-normalized Banzhaf Power Index and the Shapley-Shubik one give zero power to Luxembourg. In addition, in 1981, after several modifications of the way qualifying voting system in the Council of Ministers has been modified, Luxembourg received the same Banzhaf power of Denmark, a country 14 times more populated.

tions. Unlike the analyses using the real data where many unobservable and uncontrollable aspects are present, experimental analyses can provide more precise (empirical) measures of voting power under a given WMV system, and are suited to test theoretical predictions.

In these experimental studies, it is often noted that subjects have learned their voting powers by showing that the averaged payoffs obtained by subjects stabilize after several periods. And the resulting payoffs are presented as observed measures of voting powers. These studies, however, do not question whether subjects have understood the relationships between the way votes and associated voting powers are distributed among group members. This question should not be overlooked, because, as conjectured by Felsenthal and Machover (1998), it is the lack of understanding of such relationships that causes unforeseen and unintended outcomes in designing a WMV system.

To fill this gap in the literature, in this paper, we investigate experimentally whether subjects can learn, from their limited experiences, about underlying relationships between the distribution of votes and the distribution of voting powers in a group. We consider two conditions, one in which veto players are present in majority of the cases and the other in which they are not, to see how the presence of veto players affects the learning by subjects.

This paper is also related to the literature on learning in games. Camerer (2003, p. 474) discusses an example in which a student who participated in an experiment on coordination game and "defected" continuously because the student thought the experiment was about Prisoners' dilemma game, and notes "what game do people think they are playing?" as one of the ten most important open research questions in behavioral game theory.

It is often said that experimental subjects bring-in whatever they have learned elsewhere into the laboratory. In other words, subjects seem to relate what they are currently facing with similar situations they have experienced elsewhere, such as previous laboratory experiments they have participated, and behave in such a way that worked well in similar situations. The majority of existing research on learning in strategic environments analyzes learning how to act in a given environment and not learning about the environments themselves or whether subjects successfully apply what they have learned in one situation to a similar one.

Because of the existence of underlying general relationships between the distribution of votes and the distribution of powers in WMV systems, our experiment allows us to investigate whether subjects learn such relationships from their limited experiences and apply what they have learned in one situation to similar but different ones.

In our experiment, subjects bargain, in a group of four, over a fixed budget under a given WMV system (the points allocation stage). The protocol of this stage basically follows that of Montero, Sefton, and Zhang (2008). Before subjects enter the point allocation stage, there is the votes apportionment stage. This stage looks like a two-players two-action simultaneous move game, in which two out of four subjects of the group, independently and simultaneously, make choices that jointly determine the distribution of votes among four members. We keep the total number of votes to be apportioned and the number of votes required to win the bargaining constant for a given matrix that subjects face. In the votes apportionment stage, what subjects see in the matrices are the distributions of votes and not the associated voting powers. The matrices are made so that there can be non-positive relationship between the two. Therefore, subjects need to learn about the latter to make a choice that lead to higher payoffs, as well as the underlying relationships between the two, via actually playing the points allocation stage that follows the votes apportionment stage. This process is repeated twenty times, not always with the same matrix, to investigate the learning of subjects as well as their ability to generalize in the face of similar but different WMVs.

The results of our experiments show that, as in the previous studies, the averaged payoffs obtained by the subjects in the bargaining process become similar to theoretical measure of power indices, in particular, Shapley-Shubik Index, when there is no veto player in the group. When a veto player is present in the group, outcomes tend to converge to the allocation in the Core instead of what Shapley-Shubik index suggests. As subjects “learn” about the payoffs they can obtain through bargaining, those who make choices regarding the distribution of votes start to choose the one that gives them higher powers.

Our results, however, do not suggest that subjects learned the underlying relationship between the distribution of votes and distribution of powers from their limited experiences. Namely, when subjects faced a new matrix in the votes apportionment stage, even those subjects who have “learned” to choose the option that gives them higher powers in one matrix fail to make the similar choice.

This was particularly so when the presence of the veto player was limited in the matrix they have been facing before they encounter a new one.

The rest of the paper is organized as follows. In Section 2 we present the theoretical background about the concept of power in the literature, as well as the concepts of core and nucleolus. In Section 3 we describe our experimental protocol. The results of experiments are presented and discussed in Section 4, and Section 5 concludes.

2 The concept of power

In WMV systems, the influence of each player is not normally equal to her relative weight. Felsenthal and Machover (1998) presents several historical examples. To analyze the complex relationships between the distribution of weights (votes) and voting powers, various Power indices are proposed in the literature. Two most important power indices in the literature are the non-normalized Banzhaf index, or Penrose measure of power (Banzhaf, 1965), and the Shapley-Shubik index (Shapley and Shubik, 1954).

The Banzhaf index captures the probability for a subject to be decisive in letting the proposal being accepted once that all the possible coalitions are equally probable. It's a measure of influence, or *I-power*, as described in Felsenthal and Machover (1998).

The Shapley-Shubik index, the concept we use in this paper, is, at the contrary, a measure of *P-power*. *P-power* gives the ideal share of a fixed purse that each subject can realistically claim and obtain.[2]

Another measure of *P-power* is the notion of core (Gillies, 1959). In simple games, like weighted voting systems, core is often empty except for the situation with the presence of a veto player. If such a player exists, she should be able to stand strongly in the decisional process and get, at the limit, all the purse. When there is no veto player, the core can be substituted by the nucleolus, which is proved to always exist (Schmeidler, 1969). Its use as a *P-power* measure is studied by

[2] Also the normalized version of the Banzhaf index could be used for this purpose (see Felsenthal and Machover, 1998, pp. 174-175), but it is not very effective as a measure of *P-power* as it can be subjected to several paradoxical behaviors (see Felsenthal and Machover, 1998, pp. 276-277).

Montero (2006).

To briefly see the analytical descriptions of the concepts used in this paper (Shapley-Shubik index, core and nucleolus), let $N = (1, 2...n)$ be the set of players and a_i be the integer number of seats of player i, with $a = \sum_{i=1}^{n} a_i$. A weighted voting game is $G = [q; a_1, ..., a_n]$, where q is the quota, an integer number to be reached by the players in order to get a decision passed.

To compute the Shapley-Shubik index for a player i, ϕ_i, suppose she casts her votes a_i in favour of a proposal in a random order. A voter that will let this proposal win (i.e. $\sum_{i=1}^{pivot} a_i \geq q$) will always exist. This voter is named "pivot". So, as $n!$ is the number of orderings of n players:

$$\phi_i = \frac{\text{Number of orderings in which player } i \text{ is pivotal.}}{n!}. \tag{1}$$

In the case of the Council of Ministers of the EEC quoted in footnote 1 above, France, Germany and Italy had 4 votes, Belgium and the Netherlands 2, and Luxembourg 1. The quota was 12 votes over 17. It's immediate to notice that Luxembourg can never be the pivot, $\phi_{Luxembourg} = 0$.

To define core and nucleolus, let's first define the concept of imputation in simple games. Simples games are game in which coalitions are either winning or losing, so the payoffs are only 1 or 0, without intermediate values. And the imputation is any distribution of payoffs to the players that sums to 1.

Let I be the grand coalition, i.e. a coalition made up by all the players, and S be any particular coalition. Let $p(S)$ be the sum of the payoffs of each player inside a particular coalition S given by a particular imputation, and $v(S)$ be the value of S. In our case, $v(S) = 1$ if this coalition is a winning one and $v(S) = 0$ otherwise.

The core is a set of imputations in which $p(S) \geq v(S)$ for every $S \subseteq I$. If this condition holds, no coalition has an incentive to leave the grand coalition to get a bigger payoff. Without a veto player, or a veto coalition, this set is empty in simple games.

The notion of core can be relaxed: the ε-core is a set in which $p(S) \geq v(S) - \varepsilon$ for every $S \subseteq I$, for any ε (Shapley and Shubik, 1966). In the interception of all these non-empty sets there is the least-core (Maschler *et al.*, 1979).

To check if a particular imputation x is in the nucleolus, we can list its deficit vector having as components all the $d(x)_S = v(S) - p(S)$ for all the possible coalitions $S \subseteq I$. Then, we can order the single deficits from the largest to the smallest. The nucleolus, ν, is the imputation that satisfies $d(\nu) \leq_{lex} d(x)$ for any other imputation x in the lexicographical order.

Because, in our experiment, we let the players divide a fixed purse of money among themselves, we consider the two *P-power* measures described above: the Shapley-Shubik index and the core/nucleolus, in order to analyze their behavior. The same choice has been made in Montero, Sefton, and Zhang (2008).

To compute all the indices described in this section, we use an algorithm described in Derks and Kuipers (1997) and provided as a software in one of the authors' website.[3]

3 An experiment

In order to have a non-obvious relationship between the number of votes and the voting power (while the former is shown to the subjects, the latter is not), we consider a decision making in a group of 4 subjects.

Our game consists of two stages: the first, votes apportionment, stage and the second, points allocation, stage. Below, we first describe the points allocation stage before explaining the votes apportionment stage. We then proceed to describe other aspects of experimental design.

3.1 The points allocation stage

In the point allocation stage, four members decide how to divide the fixed amount of resources (100 points) through the weighted voting system that is determined by the outcome of the votes apportionment stage.

Each player can publicly propose, at any moment during this stage, how to divide 100 points. One can, instead of proposing, approve a proposal by some other members of the group. One is able to withdraw his/her proposal or approval at any time before the end of the stage.[4]

[3] http://www.math.unimaas.nl/personal/jeand/home1.htm.

[4] See the instruction and the screen shots of the experimental software in the Appendix for how subjects can propose

Each player can be in favor of at most one proposal, including his own, at any given moment. Thus, if a subject is proposing but would like to approve other's, he has to withdraw his own before approving the other's. In the same way, one has to withdraw his approval of other's proposal before submitting his own or approving yet other's.[5] When a subject is in favor of a proposal (via submitting one's own or approving other's), all of his votes will be placed on the one he is in favor of. The first proposal to receive the required number of votes will be implemented, and subjects will receive the points accordingly.

There is a time limit for the points allocation stage. The limit is set randomly between 300 and 420 seconds. If none of the proposal receives the necessary number of votes within the time limit, all the members of the group will receive zero points. This procedure follows essentially that of Montero, Sefton, and Zhang (2008).[6] We now turn to the description of the votes apportionment stage.

3.2 The votes apportionment stage

In the votes apportionment stage, two players from a group of four choose, independently and simultaneously, between two alternatives. By allowing only two players to choose between two alternatives, this stage is made to look like a simple 2×2 game.

These two players are fully informed about the majority rules, namely, the total number of votes to be apportioned among four members and the number of votes needed (majority) for a proposal to be implemented in the point allocation stage. They were also informed that their choices will jointly determine how many votes each member will have in the point allocation stage.

There is a time limit for the votes apportionment stage. The limit is set randomly between 120 and 180 seconds for a pair of subjects to make their choices. If a subject fails to make a decision before the stage ends, the group to which this subject belongs does not enter the points allocation stage, and the 100 points will be divided among other members of the group. The subject who

or approve.

[5] See the instruction and the screen shots in the Appendix for how subjects can withdraw his proposal or approval.

[6] There are several minor modifications such as how we randomize the role of each subject within a group, and the way the computer screen looks. See Appendix for the details.

failed to make a choice before the stage ends receives zero point. For example, if one subject fails to choose, the other three members in her group will receive 33 points each. If two subjects from the same group fail to make decisions, remaining two members in their group will each receive 50 points.

3.3 Two treatments and three games

Two main objectives of this experiment are to investigate whether subjects can (a) learn the underlying relationship between the observable keys and the corresponding payoffs (the number of votes and the voting powers in the weighted majority system), and (b) generalize what they have learned from their limited experiences in one situation to similar, but different, situations. Therefore, those who make decisions in the votes apportionment stage need to remain intact throughout the experiment. We randomly select a half of the subjects at the beginning of the experiment to be decision makers in the first stage.

We let the subjects play the game 20 times. We call one play of a game a period, thus an experimental session consists of 20 periods. Subjects will be regrouped every periods, in such a way that (a) the same four subjects are never in the same group, and (b) two out of four subjects are those who make decisions in the first stage.[7] Subjects are given their player IDs every period. These player IDs are re-set at the beginning of each periods when subjects are regrouped. Player IDs are always 1 or 2 for those selected to make decisions in the first stage and always 3 or 4 for those who are not.

During these 20 periods, subjects face three different first stage matrices, called game A, B, and C. There are two treatments, treatment 1 and 2, with different order through which subjects face these three matrices. In session 1 (2), subjects will play game A (C) for period 1 to 16, game B in period 17 and 18, and then game C (A) in period 19 and 20. This is summarized in Table 1.

Before describing these three games in detail, let us first discuss the purpose of these two treatments. In period 1 of the treatment 1 (2) and period 19 of the treatment 2 (1), subjects face game A (C) for the first time. The difference between these two new encounters is that, in the latter, i.e.,

[7] Re-matching is important to exclude the repeated game effect as much as possible.

	Period 1-16		Period 17 -18		Period 19-20
Treatment 1 :	Game A	$\rightarrow$	Game B	$\rightarrow$	Game C
Treatment 2 :	Game C	$\rightarrow$	Game B	$\rightarrow$	Game A

Table 1: Two treatments.

when subjects face the new game in the period 19, subjects have experienced similar situations for 18 previous periods, and have accumulated some, although limited, experiences about possible relationship between the observable keys and associated payoffs. Therefore, by comparing the choices made by subjects in the votes apportionment stage when they face the same game in these two treatments, we test whether subjects successfully generalized what they have learned from one situation to a similar, but, different situation.[8]

We now describe these three games in detail. The total number of votes to be apportioned (and the corresponding majority) are 14 (10), 15 (9) , and 22 (14) in game A, B, and C, respectively. In particular, the 2×2 matrices subjects see in the vote apportionment stage for three games are as shown in the first row of Table 2. These three matrices were selected in order to have as much variety as possible in terms of the relationships between the number of votes and the resulting voting powers while keeping the appearance of matrix in terms of votes as similar as possible. As one can see, these matrices all have a dominant action for those two subjects making decision in the votes apportionment stage.

In particular, for two players who make decisions in the votes apportionment stage, the relationship between the number of own votes and the corresponding voting powers have a non-positive relationship in game A and C. In game B, however, the relationship is non-negative. In addition, while the two measures of voting power, Shapley-Shubik Index (shown in the second row) and the allocation in the Core/Nucleolus (shown in the third row) coincide in three out of four cells in game C, they coincide only in one out of four cells in game A.

Because of the non-negative relationship between the number of own votes and corresponding voting powers, in game B, the Nash equilibrium in the matrix of votes (outcome “BB”, shown in

[8]To avoid framing effect, i.e., subjects just keep choosing the same action regardless of the matrix they face, we inverted the matrix of the original game in period 19 and 20.

In Number of Votes

Game A

		P_2	
		A	B
P_1	A	$P_1:2$ $P_2:2$ $P_3:4$ $P_4:6$	$P_1:1$ $P_2:3$ $P_3:4$ $P_4:6$
	B	$P_1:3$ $P_2:1$ $P_3:6$ $P_4:4$	$\mathbf{P_1:2}$ $\mathbf{P_2:2}$ $P_3:5$ $P_4:5$

Game B

		P_2	
		A	B
P_1	A	$P_1:3$ $P_2:3$ $P_3:4$ $P_4:5$	$P_1:2$ $P_2:4$ $P_3:3$ $P_4:6$
	B	$P_1:4$ $P_2:2$ $P_3:6$ $P_4:3$	$\mathbf{P_1:3}$ $\mathbf{P_2:3}$ $P_3:2$ $P_4:7$

Game C

		P_2	
		A	B
P_1	A	$P_1:4$ $P_2:4$ $P_3:6$ $P_4:8$	$P_1:3$ $P_2:5$ $P_3:7$ $P_4:7$
	B	$P_1:5$ $P_2:3$ $P_3:7$ $P_4:7$	$\mathbf{P_1:4}$ $\mathbf{P_2:4}$ $P_3:5$ $P_4:9$

In Shapley-Shubik Index

Game A

		P_2	
		A	B
P_1	A	$\mathbf{P_1:8.3}$ $\mathbf{P_2:8.3}$ $P_3:25$ $P_4:58.3$	$P_1:8.3$ $P_2:8.3$ $P_3:25$ $P_4:58.3$
	B	$P_1:8.3$ $P_2:8.3$ $P_3:58.3$ $P_4:25$	$P_1:0$ $P_2:0$ $P_3:50$ $P_4:50$

Game B

		P_2	
		A	B
P_1	A	$P_1:17$ $P_2:17$ $P_3:33$ $P_4:33$	$P_1:8.3$ $P_2:25$ $P_3:25$ $P_4:41.7$
	B	$P_1:25$ $P_2:8.3$ $P_3:41.7$ $P_4:25$	$\mathbf{P_1:8.3}$ $\mathbf{P_2:8.3}$ $P_3:8.3$ $P_4:75$

Game C

		P_2	
		A	B
P_1	A	$\mathbf{P_1:17}$ $\mathbf{P_2:17}$ $P_3:33$ $P_4:33$	$P_1:17$ $P_2:17$ $P_3:33$ $P_4:33$
	B	$P_1:17$ $P_2:17$ $P_3:33$ $P_4:33$	$P_1:8.3$ $P_2:8.3$ $P_3:25$ $P_4:58.3$

In Core/Nucleolus solution

Game A

		P_2	
		A	B
P_1	A	$P_1:0$ $P_2:0$ $P_3:0$ $P_4:100$	$P_1:0$ $P_2:0$ $P_3:0$ $P_4:100$
	B	$P_1:0$ $P_2:0$ $P_3:100$ $P_4:0$	$P_1:0$ $P_2:0$ $P_3:50$ $P_4:50$

Game B

		P_2	
		A	B
P_1	A	$P_1:17$ $P_2:17$ $P_3:33$ $P_4:33$	$P_1:0$ $P_2:33$ $P_3:33$ $P_4:33$
	B	$P_1:33$ $P_2:0$ $P_3:33$ $P_4:33$	$\mathbf{P_1:0}$ $\mathbf{P_2:0}$ $P_3:0$ $P_4:100$

Game C

		P_2	
		A	B
P_1	A	$\mathbf{P_1:17}$ $\mathbf{P_2:17}$ $P_3:33$ $P_4:33$	$P_1:17$ $P_2:17$ $P_3:33$ $P_4:33$
	B	$P_1:17$ $P_2:17$ $P_3:33$ $P_4:33$	$P_1:0$ $P_2:0$ $P_3:0$ $P_4:100$

Table 2: Three matrices in the vote apportionment stage, and corresponding voting powers. P_i indicates player i. The total number of votes (the majorities) are 14 (10), 15 (9), and 22 (14), for game A, B, and C, respectively.

bold in Table 2) is the same as the Nash equilibrium in the matrices of voting power indices, both in terms of Shapley-Shubik index and allocation in Core/Nucleolus.

In game A and C, however, Nash equilibria differs across matrices of votes and voting powers due to non-positive relationship between them. In these two games, while the equilibrium is the outcome “BB” in terms of votes, it is the outcome “AA” in terms of Shapley-Shubik Index. In addition, in terms of allocation in Core/Nucleolus, while game A has no equilibrium in pure action, the equilibrium for game C is the outcome “AA”, the same outcome as in the case of Shapley-Shubik Index.

3.4 Payment

At the end of the game, the computer randomly selected 5 out of 20 periods. Subjects were paid according to the points they had obtained in these selected periods, together with show-up fee. All subjects were informed that they will be paid in this manner. We used this design to give subjects incentive to keep paying attention throughout the whole experiment. Let us now turn to the results of the experiment.

4 Results

A computerized[9] experiment took place in the University of Tsukuba, Japan, in February 2010 and University of Montpellier, France, in April and May 2010.

All the experiments proceeded in the same fashion. Upon arrival, a printed instruction (see Appendix) was distributed, which the experimenter read aloud.[10] Subjects then reviewed the instructions and were allowed to ask questions by raising their hands. Subjects were not allowed to communicate with each other throughout the experiment. In order to familiarize subjects with the interface of the experiment, there was a practice period before the real experimental periods. After the twenty periods of the experiment, subjects are asked to answer a questionnaire while experi-

[9] We used the software “Z-tree”, by Fischbacher (2007)

[10] The instruction as well as the computer screens were translated into Japanese and French for experiment in Japan and France, respectively.

Session	Treatment	Location	Date	Number of Subjects
1	1	Tsukuba, JP	Feb, 2010	28
2	2	Tsukuba, JP	Feb, 2010	24
3	1	Montpellier, FR	April, 2010	16
4	1	Montpellier, FR	May, 2010	16
5	2	Montpellier, FR	May, 2010	16
6	2	Montpellier, FR	May, 2010	16

Table 3: List of experimental sessions.

menters were preparing the payments for them.

Total of 116 subjects participated in our experiment. 68.97% of them were male, and around 60% of them were economics or business students. There were 6 sessions independently run in Japan and France, which were lasting about 140 minutes including instruction time. Specifically, in Japan, two sessions were conducted with two different treatments, and involved 28 and 24 subjects respectively. About a half of the subjects were economics or business students, but the majority of the subjects knew, or at least heard, about Prisoners' dilemma. The average payoff in the experiment was JPY 3468 including a JPY 1500 show up fee. In France, four sessions were conducted with two different treatments, and each session included 16 subjects. Around 70% of the subjects were studying in economics or business, but only 45% of them knew, or heard about, Prisoners' dilemma. The average payoff in the experiment was 19.25 euros + different show-up fee depending on whether subjects were the students of the University of Montpellier 1.[11] Table 3 summarizes all the experimental sessions.

4.1 Votes apportionment stage

We first describe the aggregate outcomes in the votes apportionment stage. Figure 1 shows the dynamics of fraction of subjects who have chosen "A", the choice with lower numbers of own votes, but higher voting powers, in the first 16 periods of the experiments. In Treatment 1 (2), which is shown by solid (dashed) line, subjects faced Game A (C) in these 16 periods. While Japanese data

[11]For the students of University of Montpellier 1, show-up fee was 5 euros. For others, it was 10 euros.

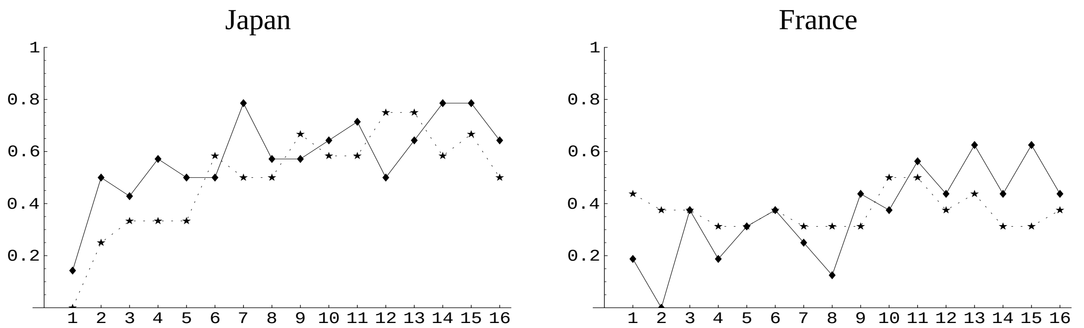

Figure 1: Dynamics of fraction of subjects who have chosen A (one with less votes but a higher power) in the first 16 periods. Results of experiment in Japan (left) and France (right) are shown separately. Solid lines are for Treatment 1 (Game A) and dashed lines are for Treatment 2 (Game C). In the Session 1 (Treatment 1 experiment in Japan), there are several cases in which subjects did not make decisions (1 in period 8, 2 in period 12, and 1 in period 14.)

(left panel) show an upward trend in both treatments, French data (right panel) do not.[12]

For Japanese subjects, we reject the null hypothesis that subjects are choosing between A and B with equal probability in the first period at 1% significance level for both treatment. For French subjects, the null hypothesis is rejected at 5% significance level for Treatment 1, but it cannot be rejected for Treatment 2.

Observation 1: *Inexperienced subjects tend to make a choice that gives them higher numbers of own votes.*

The upward trends shown in Figure 1 suggest, however, that Japanese subjects stopped choosing based on the number of own votes as they gain experiences playing the same game.[13] The rate at which subjects change their choices in the early periods of the experiments is slightly higher, although not significantly different, in the Treatment 1 than in the Treatment 2.

Although the fraction of subjects choosing A is similar in the later periods between two treatments, there is a difference in terms of how frequent subjects switch their choices between A and

[12]There were 5 cases (3 cases by one subject and 2 cases by another), all happened in Session 1 (Treatment 1 of Experiment in Tsukuba, Japan), in which subjects did not make their choices in the votes apportionment stage (1 in period 8, 2 in period 12, and 1 in period 14). These cases are treated as subjects not choosing "A." Such a failure was not observed in other sessions.

[13]Behaviors of Japanese and French subjects may differ not only because of cultural differences between Japan and France, but also the difference in the educational background between the two, or their knowledge about game theory.

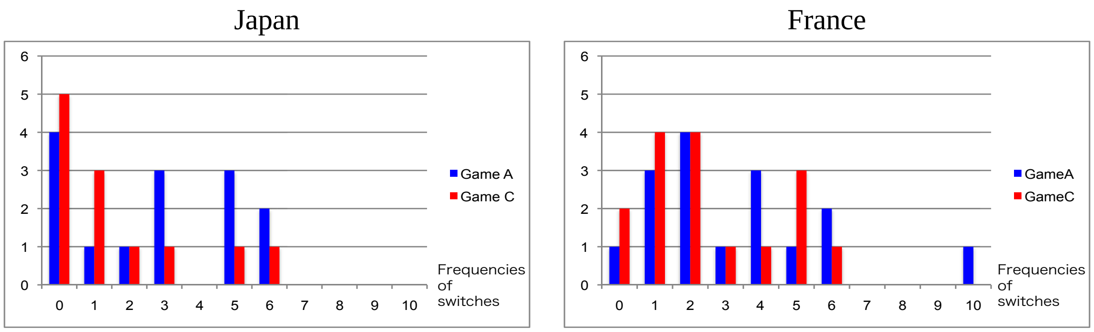

Figure 2: Distribution of frequencies of choice changes between period 6-16. The maximum number of possible switches is 10.

B for Japanese data. Figure 2 shows the distribution of the frequencies of choice changes in two treatments between period 6 and 16. We drop the first 5 periods because many subjects changed their behavior in both treatments during these periods. For the Japanese data shown in the left panel of the figure, while the distribution of the number of switches in Game A (Treatment 1) seems to be bimodal, that of Game C (Treatment 2) is unimodal. The figure shows that, in Game A, about a half of subjects kept switching between choosing A and B in the votes apportionment stage. For our French data, such a difference is not observed between Game A and C. Compared to Japanese subjects, French subjects tend to switch their choices more frequently, particularly for Game C.

Figure 3 shows the corresponding dynamics of frequencies of four outcomes (AA, AB, BA, and BB) in the first 16 periods. Since the matrices in the votes apportionment stages are symmetric, we have summed the number of times we have observed outcomes AB and BA. Because, as noted above, a few subjects did not make their decisions within the time limit in Session 1 (Treatment 1, Japan), in Period 8, 12, and 14, the relative frequencies of three outcomes do not add up to one.

As one would expect from the dynamics of individual choices shown in Figure 1, we see decreasing trends in the frequencies of outcome BB in both Treatment 1 and 2 for our Japanese data. Such trends are not visible in our French data. In addition, as one would also expect from the dynamics of individual choices, the frequencies of outcome BB decline much more quickly in Treatment 1 than in Treatment 2 for the Japanese data. In order to understand the differences in the learning dynamics between the two treatments, as well as between our French and Japanese data, we need to

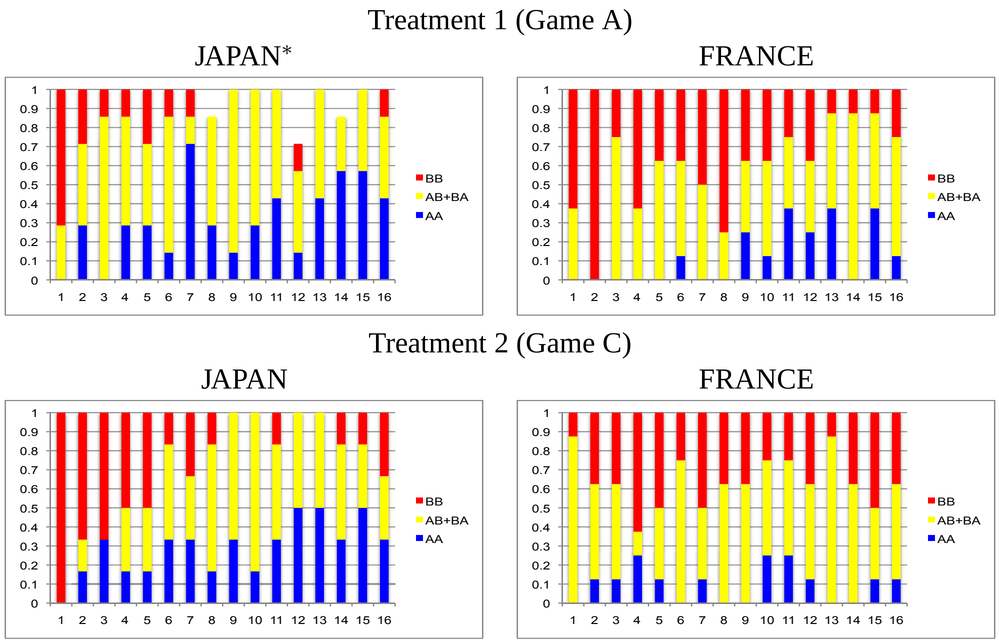

Figure 3: Relative frequencies of outcomes AA, BB, and AB and BA. In Treatment 1 of Japanese experiment (*), there are periods in which subjects did not make their choice within the time limit in votes apportionment stage, and in those periods, frequencies does not add up to one.

look into the outcomes of points allocation stage in detail.

4.2 Points allocation stage

The dynamics of observed behaviors of subjects in the votes apportionment stage cannot be well understood without detailed analysis of the points allocation stage. Because the former dynamics are influenced by the payoffs subjects obtain in the latter. In this subsection, we take a detailed look at outcomes in the points allocation stage.

To facilitate our analyses, we re-label Player IDs into three types, Large, Medium, and Small, according to the number of votes (and associated voting powers) they have[14] (See Tabel 4 for the re-labelling scheme). Two subjects who make decisions in the votes apportionment stage are always Smalls. In presenting outcomes, we will add the points obtained by two Smalls. In addition, since in terms of voting powers, outcomes AA, AB, BA (non-BB outcomes) are the same after re-labeling

[14]When two large players have the same number of votes, we call the player with larger ID number, Large, and the other, Medium.

Game A

		P_2: A	P_2: B
P_1	A	$P_1: 2$ (S) $P_2: 2$ (S) $P_3: 4$ (M) $P_4: 6$ (L)	$P_1: 1$ (S) $P_2: 3$ (S) $P_3: 4$ (M) $P_4: 6$ (L)
	B	$P_1: 3$ (S) $P_2: 1$ (S) $P_3: 6$ (L) $P_4: 4$ (M)	$P_1: 2$ (S) $P_2: 2$ (S) $P_3: 5$ (M) $P_4: 5$ (L)

Game C

		P_2: A	P_2: B
P_1	A	$P_1: 4$ (S) $P_2: 4$ (S) $P_3: 6$ (M) $P_4: 8$ (L)	$P_1: 3$ (S) $P_2: 5$ (S) $P_3: 7$ (M) $P_4: 7$ (L)
	B	$P_1: 5$ (S) $P_2: 3$ (S) $P_3: 7$ (M) $P_4: 7$ (L)	$P_1: 4$ (S) $P_2: 4$ (S) $P_3: 5$ (M) $P_4: 9$ (L)

Table 4: Re-labeling of player IDs according to the number of votes. S, M, and L indicate small, medium, and large players, respectively.

of player IDs into three types, we merge the data from these three outcomes in Game A (Treatment 1) as well as in Game C (Treatment 2).

Figure 4 shows, for Japanese subjects (J-1 to J-4) and French subjects (F-1 to F-4), the dynamics of the average allocations of points among three types of players. The shares suggested by Shapley-Shubik Index are also shown by solid black lines in the figure.[15]

For the outcome BB of Game A (shown in Panel J-1 and F-1 of the figure) and non-BB outcomes of Game C (shown in Panel J-4 and F-4 of the figure), two measures of voting powers, Shapley-Shubik Index and the allocation in Core/Nucleolus, coincide. We expect that the share of points obtained by players in these cases, *at least on average*, to become close to what two measures suggest.

For the outcome BB of Game A, that is indeed what we find. Two veto players (L and M) quickly, from period 2 for Japanese subjects and from period 4 for French subjects, realize their voting powers and begin to divide 100 points equally between themselves, leaving two small players with zero points.

In the non-BB outcomes of Game C, the share of three types of players, on average, becomes close, although they do not coincide perfectly as they do for outcome BB of Game A, to 1/3 each as

[15] The share of the two small players, whey they are positive, is the height of the first line from the bottom, and that of the medium player is distance between two lines, and that of the large player is the remaining. When two small players obtain zero share, as in J-1 and F-1, the height of the line from the bottom represent the share for the medium and the share for the large is represented by the remaining.

the two measures of voting powers suggest. The share of the large player (two small players) is, on average, somewhat less than 1/3 for Japanese (French) data.

For the outcome BB of Game C (Panel J-3 and F-3) and non-BB outcomes of Game A (Panel J-2 and F-2), there exists a veto player and two measures of voting powers disagree about the expected shares. In these cases, we see that the veto players learn to obtain a very large share, particularly so for Japanese data. In fact, the average shares of the large players become, by period 8, higher than what Shapley-Shubik Index suggests (58 points), although not as high as the one suggested by Core (100 points). The previous studies (Montero, Sefton, and Zhang, 2008; Aleskerov, Beliani, and Pogorelskiy, 2009; Drouvelis, Montero, and Sefton, 2010) show similar results in that shares of points obtained by veto players (as well as other players in the group) tend to fall between Shapley-Shubik Index and Core allocation when the two disagree.

Observation 2: *On average, subjects, in later periods, divide the 100 points according to their voting powers when the Shapley-Shubik Index and the Core/Nucleolus solution coincide. When the allocation in the Core and Shapley-Shubik index disagree, the allocations tend to converge toward Core.*

The average shares across subjects presented in Figure 4 does not tell us much about how the implemented allocations of points among three types of players are distributed. For example, we have seen that the average shares for large, medium, and small players approach 1/3 each in Panel J-4 and F-4 as two measures of voting powers suggest. But it is not clear from the averaged results that whether winning coalitions of these three types of players are formed, or something else is going on. How about the average outcome we see in the presence of single veto player in a group shown in Panel J-2, J-3, F-2, and F-3? Why do the average shares of veto players lie between what Shapley-Shubik Index and Core solution suggest? If we take the average across individuals, we do not observe a rich heterogeneity in their behaviors.

Figure 5 and 6 show the implemented allocations of points among three types of players for Game A and Game C, respectively. In each figure, separately for BB (top) and non-BB outcomes (bottom), as well as for Period 1-5 (left), 6-10, (middle), and 11-16 (right), a realized allocation

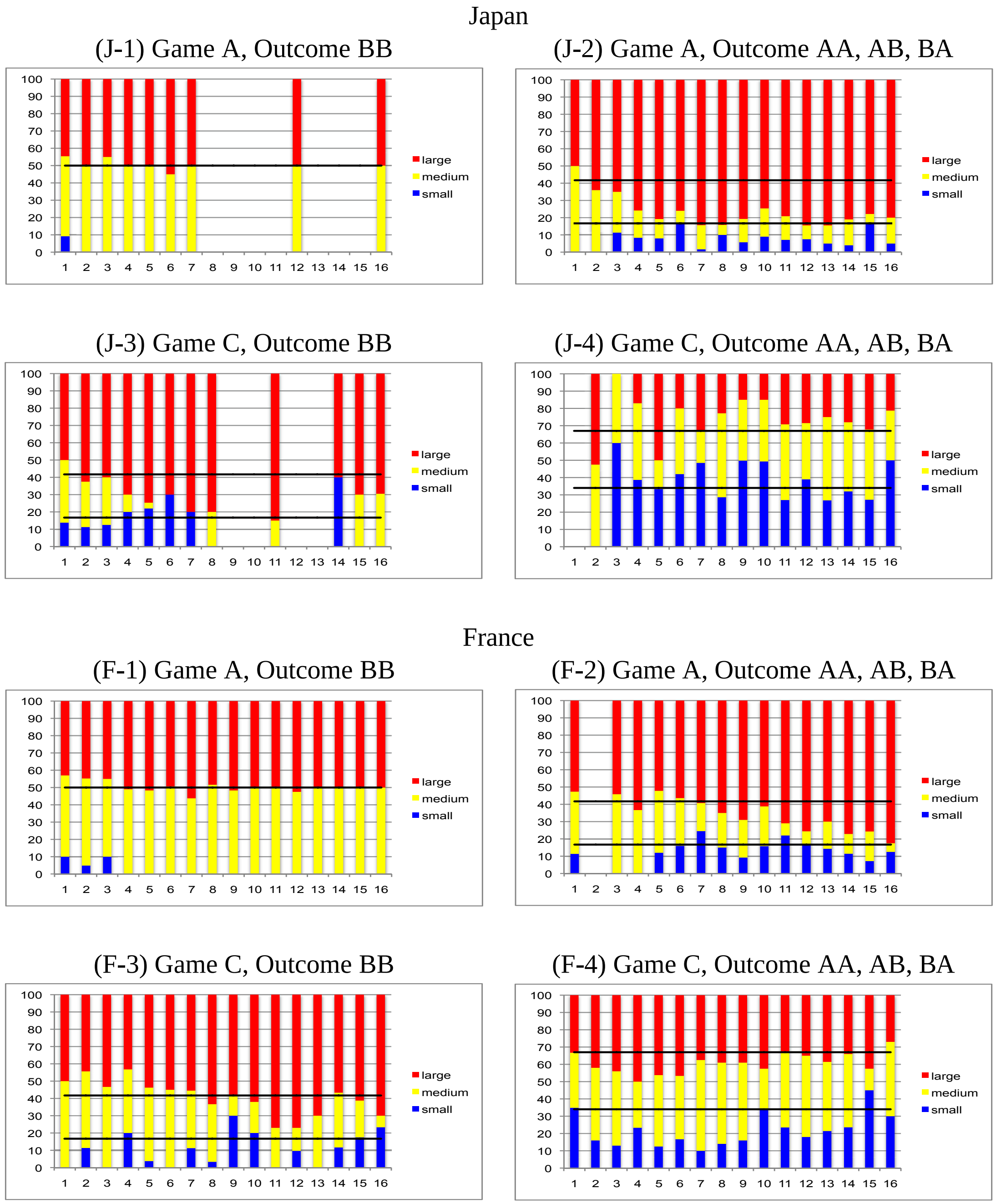

Figure 4: Dynamics of the average allocations of points for Japanese Subjects (Panel J-1 to J-4) and French Subjects (Panel F-1 to F-4). Note that the number of realizations for each outcome changes over time as shown in Figure 3. The powers for small players represent those of two small players together. Black lines in the figure shows the allocation according to the Shapley-Shubik Index.

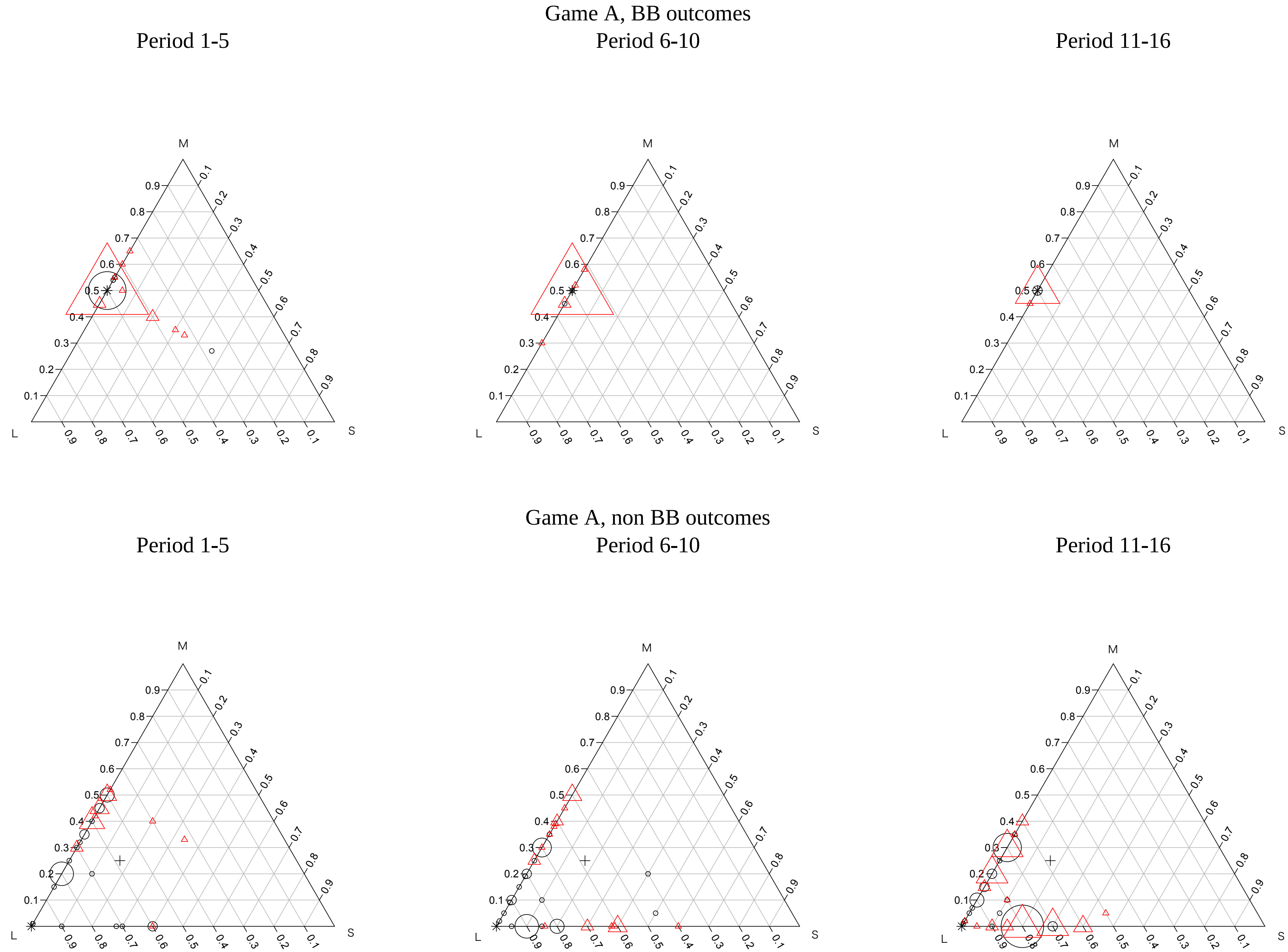

Figure 5: Distributions of implemented allocations of points among three types of players in the points allocation stage for Game A over Period 1-5 (left), 6-10 (middle), and 11-16 (left). Results for BB (non-BB) outcome is shown in top (bottom). Circles (Triangles) represent Japanese (French) data. Size of a circle (triangle) is proportional to the number of observations falling on the same points in the simplex.

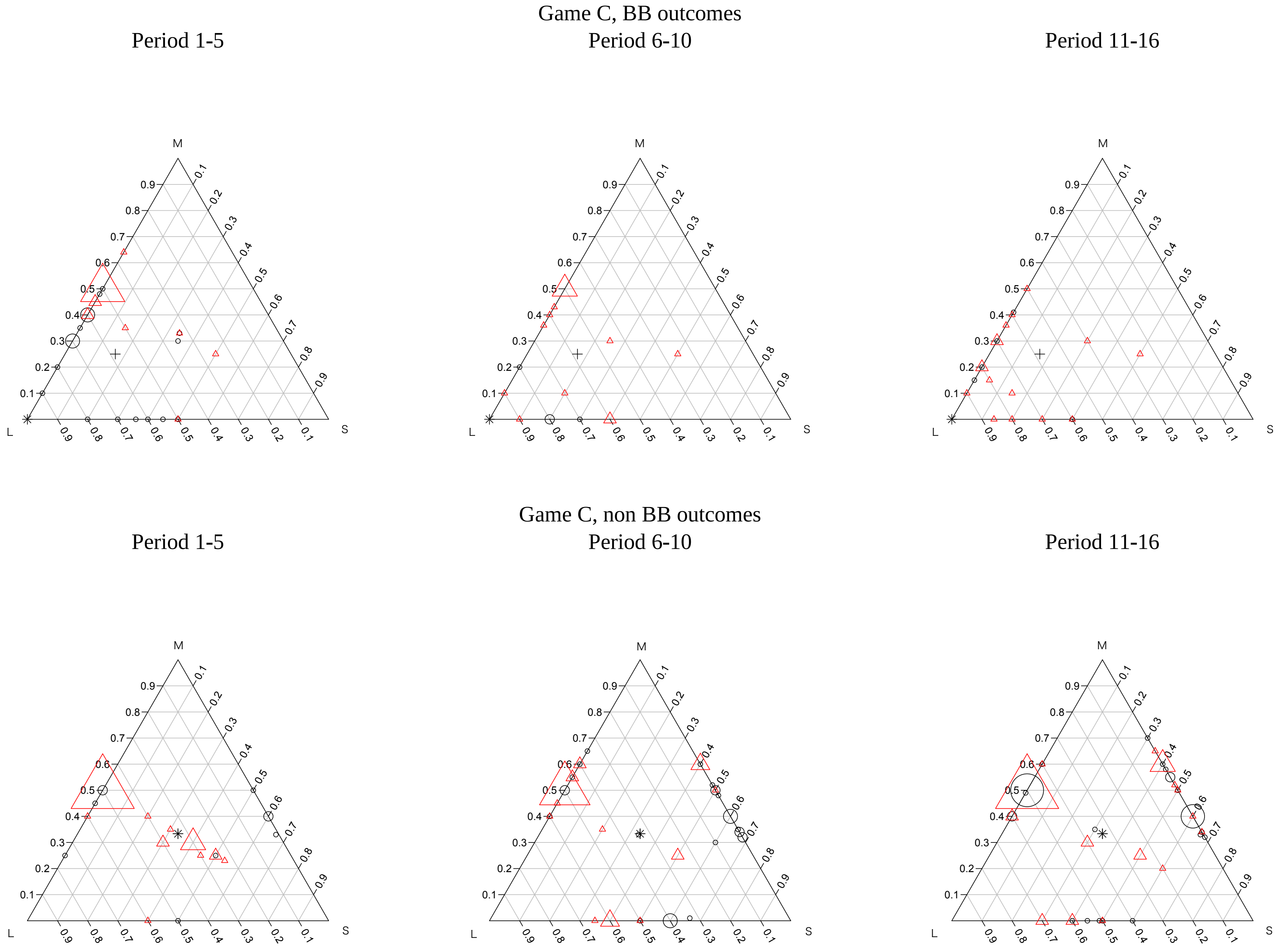

Figure 6: Distributions of implemented allocations of points among three types of players in the points allocation stage for Game C over Period 1-5 (left), 6-10 (middle), and 11-16 (left). Results for BB (non-BB) outcome is shown in top (bottom). Circles (Triangles) represent Japanese (French) data. Size of a circle (triangle) is proportional to the number of observations falling on the same points in the simplex.

is represented by a point in a two dimensional simplex. The obtained share by each type (Large, Medium, or two Smalls together) is represented by the distances between the point and the edge that is opposite to the apex labeled under each of them.[16] For example, if a point is located right on the apex labeled "L" (for Large), the share of Large player is 100 % (i.e., 100 points) and those for other players are 0% (i.e, 0 point). The allocations that correspond to Shapley-Shubik index are shown by $+$ while those under Core/Nucleolus are shown by $*$. Japanese results are represented by circles, while French results are shown with triangles. The size of a circle or a triangle is proportional to the number of observations falling exactly on the same location in a simplex (that is, the same division of points among three types of players).

Figure 5 clearly demonstrates that the two veto players divide all the points among themselves for BB outcomes in Game A (also shown in Panel J-1 and F-1 of Figure 4). In the top three panels of Figure 5, except for Period 1-5 (left) in which some points are located in the middle of the simplex, all the points are located on the edge between apex L and apex M (LM edge). This explains early changes in the behavior of subjects in the votes apportionment stage. Although many subjects started choosing B in Game A, it was quite immediate for them to find out that such a choice, when the other player does the same, results in zero point in the points allocation stage. Hence, they quickly switched to choosing A in the subsequent periods. (French data is somewhat strange in this regard, as even those who have chosen A in period 1 switched to B in period 2. But, from period 3 on, the similar dynamics are observed.)

The difficulty for Smalls in Game A was, however, switching to A did not result in much better outcomes as Panel J-2 of Figure 4 and bottom three panels of Figure 5 clearly show. Even in non-BB outcomes, the share of points two Smalls jointly obtained were often zero (located on LM edge of the simplex). As a results, the subjects kept switching back and forth between choosing A and B in the votes apportionment stage (Figure 2). The bottom three panels of Figure 5 show, in particular the middle one (Period 6-10), that our Japanese data are converging toward apex L (Core allocation) from earlier periods than our French data. And the figure also shows that, as the experiment proceeds, more subjects "learn" to exercise their power when they happen to play the

[16] Or the share of L (M or S), in a particular realization, is represented by the length of the perpendicular line that drawn from the point that represent the realization to the side that confront the vertex labeled L (M or S).

role of the veto player.[17]

Observation 3: *For Game A, in the votes apportionment stage, subjects learn to choose according to the matrix of Core allocation.*

A slower change in the behavior of our Japanese subjects in the votes apportionment stage for Game C compared to Game A can be explained by looking at early outcomes in the points allocation stage. The Panel J-3 of Figure 4 shows that, in early periods, the average shares obtained by the two small players are gradually increasing, while as seen before, the number of subjects choosing B in the votes apportionment stage is declining. A reason for this is that while those Smalls who obtained zero points (those outcomes located on LM edges in the top left panel of Figure 6) in early periods started choosing A, those who obtained positive points (on LS edge or the middle of the simplex) remained choosing B. The latter was possible in early periods because the veto player in their groups did not exercise their strong powers. Once enough subjects started choosing A and non-BB outcomes are reached in Game C, these small players enjoyed much higher points (Panel J-3 of Figure 4 and the bottom-middle panel of Figure 6) and did not change their choices much.

Our French subjects demonstrate quite different behaviors in Game C than the Japanese counter parts. In the votes apportionment stage, we cannot reject the null hypothesis that subjects initially started by choosing randomly between A and B in Treatment 2 (which subjects play Game C for the first 16 periods) of the French experiments, and unlike our Japanese case, many subjects kept switching between two choices even in the later periods (Figure 2). The behavior of later periods can be understood by comparing the top and bottom panels of Figure 6). For Period 1-5, the distribution of the implemented allocations in the points allocation stage show the remarkable similarity between BB and non-BB outcomes for French data (triangles). While there are cases in which Smalls obtained positive share, there are also many cases in which L and M divided all the points between two of them leaving zero points to Smalls both in BB and non-BB outcomes.[18] Unlike the Smalls in Japanese experiments whose experiences where much better in non-BB outcomes than in

[17]There are differences, among Japanese subjects as well, about how early they started exercising their veto power.

[18]This is not because L and M always appear next to each other in the users' interface when subjects submit there proposals. We randomize the order in which subjects' IDs appear in the computer screen of the points allocation stage.

BB outcome for Game C, for many Smalls in French experiments, what ever the choices they make in the votes apportionment stage resulted in the similar situation. This suggests, at least within the 16 periods of the experiment, our French subjects did not learn that choice A was associated with a higher voting power than choice B.

Observation 4: *For Game C, in the votes apportionment stage, while our Japanese subjects learned to choose according to the matrix of voting powers, our French subjects did not.*

Figure 6 also shows that, for non-BB outcomes, as the experiments proceed, most of the points begin to be located on the three edges of the simplex. Thus the allocation suggested by the Shapley-Shubik index and Nucleolus (that are both located in the middle of the simplex shown by $*$) is seldom realized. Since Shapley-Shubik index is a measure of the expected power one can realize over a many repetitions, what we observe may be natural. But when it comes to learning about the voting power, we would like to know at individual level whether over time, on average, each individual obtained the points according to what the measures of voting power suggest. Our null hypothesis is the across periods individual payoffs in Period 6-16 is equal to what Shapley-Shubik Index suggests. Results of t-test shows that among 12 and 16 (12 and 16) subjects who played the role of Small (Large) during Period 6-16 in Treatment 2 of Japanese and French experiments, we reject the null hypothesis for 2 and 4 (1 and 5) subjects at 10% significance level, respectively. Thus, for non-BB outcomes in Game C, we confirm, at the individual level, that subjects have learned to divide the points according to what Shapley-Shubik Index suggests.

4.3 Do subjects learn the underlying relationships between distribution of votes and voting powers?

Except for Treatment 2 in French experiments, subjects seem to learn to make choices that give them higher voting powers according to the allocation in Core/Nucleolus[19] by repeatedly playing the same game for the? first 16 periods.

Did subjects discover the underlying relationships between the distribution of votes and voting

[19]In Treatment 1, therefore, subjects learn to randomize between A and B. In Treatment 2, subjects learn to choose A.

Japan

Treatment	Period 1	Period 17	Period 19
1 $(n = 14)$	0.14^{**} (2) (Game A)	0.36 (5) (Game B)	0.5 (7) (Game C)
2 $(n = 12)$	0.0^{**} (0) (Game C)	0.33 (4) (Game B)	0.17^{*} (2) (Game A)

France

Treatment	Period 1	Period 17	Period 19
1 $(n = 16)$	0.19^{*} (3) (Game A)	0.25^{*} (4) (Game B)	0.19^{*} (3) (Game C)
2 $(n = 16)$	0.44 (7) (Game C)	0.31 (5) (Game B)	0.06^{**} (1) (Game A)

Table 5: Fraction (number) of subjects who have chosen A in the first time subjects have faced each game. $*$ ($**$) indicates that the null hypothesis that subjects have chosen randomly between A and B is rejected at 5% (1%) significance level.

powers? Or what they learned was more limited, i.e., they simply learned that a particular choice resulted, on average, in a higher payoff? We try to answer these questions by looking at the choices that subjects make when they face a similar, but different, game, namely comparing the choices subjects make in the Period 1 and 19 of two treatments.

Table 5 shows, separately for Japanese and French data, the fraction of subjects who have chosen “A” when they faced Game A (left), B (middle), and C (right) for the first time, for two treatments. Before testing whether subjects have learned the underlying relationships between the distribution of votes and voting powers, we test if subjects have chosen randomly between two options when they face a new game for the first time. We have already discussed about their choices in Period 1. Therefore, we focus on Period 17 and 19.

In Period 17, when subjects face Game B for the first time, the null hypothesis that subjects have chosen randomly cannot be rejected at 5% significance level for Japanese subjects. For French subjects, the null hypothesis is rejected at 5% in Treatment 1, but not in Treatment 2.

In Period 19, the majority of subjects again chose option B (the null hypothesis of random choices is rejected at 5% level) in Treatment 2 for Japanese subjects as well as both in Treatment 1 and 2 for French subjects. Only a half of Japanese subjects have chosen B in Treatment 1 and the null hypothesis of random choices is not rejected.

Now we turn to our main question: whether subjects learned the underlying relationships between the way votes are distributed among four members and associated distribution of voting power. From the results shown in Table 5, we can test whether behaviors of subjects were different in Period 1 of Treatment 1 and Period 19 of Treatment 2 and vice versa.

Observation 5: *We do not observe subjects learning the underlying general relationships between the distribution of votes and voting powers.*

A simple χ^2-test shows that in French experiments, we cannot rejects the null hypothesis that subjects behaviors are the same in two treatments when they faced Game A and Game C for the first time. This means that having experienced similar but different situations did not help subjects to make different choices. In case of Japanese experiments, on one hand, we cannot rejects the null hypothesis that behaviors of subjects are the same when they have faced Game A for the first time, but on the other hand, we reject it at 1% significance level for Game C.

Although the Japanese subjects who have experienced playing Game A for 16 periods made different choices when they face Game C for the first time than those who faced Game C without any prior experiences, we cannot conclude that former group has learned the underlying relationship between the distribution of votes and voting powers. As noted above, we fail to reject the hypothesis that subjects randomly choose between A and B in Period 19 of Treatment 1. Subjects have learned through their continuous encounters with a veto player, a random choice is a way to play the votes apportionment stage of Game A. Yet, it seems that they did not learn the importance of detecting the presence of a veto player and avoiding it. Learning to avoid a veto player is very limited compared to learning the underlying relationship between the distribution of votes and voting power, but it is a way to generalize their limited experiences from playing Game A. If subjects were successful in such a generalization, we should expect subjects not to choose B when they see Game C. The results of our experiment do not provide an evidence in favor of such generalizations.

5 Conclusion

We investigate experimentally whether subjects can learn, from their limited experiences, about underlying relationships between the distribution of votes and voting powers in a group. We consider two conditions, one in which veto players are present in majority of the cases and the other in which they are not, to see how the presence of veto players affects the learning by subjects.

In our experiment, subjects bargain, in a group of four, over a fixed budge under a given WMV system (the points allocation stage). The protocol of this stage basically follows that of Montero, Sefton, and Zhang (2008). Before subjects enter the point allocation stage, there is the votes apportionment stage. The votes apportionment stage looks like a two-players two-actions simultaneous move game, in which two out of four subjects of the group, independently and simultaneously, make choices that jointly determine the distribution of votes among four members. We keep the total number of votes to be apportioned and the number of votes required to win the bargaining constant for a given matrix that subjects face. In the votes apportionment stage, what subjects see in the matrices are the distributions of votes and not the associated voting powers. The matrices are made so that there can be non-positive relationship between the two. Therefore, subjects need to learn about the latter to make a choice that lead to higher payoffs, as well as the underlying relationships between the two, via actually playing the points allocation stage that follows the votes apportionment stage. This process is repeated twenty times, not always with the same matrix, to investigate the learning of subjects as well as their ability to generalize what they have learned in one situation when subjects face similar but different WMVs.

The results of our experiments show that initially subjects tend to choose an option that gives them a higher number of own votes. But as subjects “learn” about the payoffs they can obtain in the points allocation stage, they start to choose the option that gives them higher powers, instead of the one gives them a higher number of own votes. As in the previous studies, the averaged payoffs obtained by the subjects in the bargaining process become similar to theoretical measure of power indices when there is no veto player in the group. When a veto player is present in the group, outcomes tend to converge to the allocation in the Core instead of the Shapley-Shubik index.

Our results, however, do not suggest that subjects learned the underlying relationship between the distribution of votes and distribution of voting powers from their limited experiences. Namely, when subjects faced a new matrix in the votes apportionment stage, subjects who have “learned” to choose the option that gives them higher powers in one matrix fail to make the similar choice. The presence of veto players did not make difference in facilitating generalization of limited experiences by subjects.

Our finding suggests a possible answer as to why we kept observing, in real organizations, the distribution of voting weights among members that have been later considered to be bizarre. It is not easy from one’s limited experience from a particular weighted majority voting system to understand the underlying relationship between the distribution of voting weights and corresponding voting powers. Therefore, subjects even if they have learned their voting powers in one system, they may fail to foresee, as conjectured by Felsenthal and Machover (1998) , the changes in the voting powers a new system brings about.

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Appendix

INSTRUCTIONS OF THE EXPERIMENT

Welcome! Thank you very much for taking part in our laboratory experiment.

You are a participant in an experiment in a group decision making. During the experiment, you, as well as other participants in this room, will be making decisions. The experiment will take about two hours.

RECOMMENDATION

We ask you to comply with these rules and respect the instructions of the experimenter. Any communication with other participants is strictly prohibited. During the experiment, you must not talk, exchange notes, watch other participants' actions, and use mobile phones. It is important that during the experiment you remain SILENT. If you have any questions, or need assistance of any kind, RAISE YOUR HAND but DO NOT SPEAK. We expect and appreciate your cooperation.

PROTOCOL

There are twenty rounds in this experiment. In each round, you and three other randomly chosen participants will form a group of four people. Each round comprises of two phases, an allocation phase and a negotiation phase.

In the allocation phase, choices made by two members of the group are going to determine an apportionment of given number of votes among the four members of the group. Given the outcome of the allocation phase, in the negotiation phase, the four players decide how to divide 100 points by the given weighted voting mechanism.

Matching

At the beginning of the experiment, the computer will randomly select a half of you to be decision makers in the allocation phase. If you are selected, your player ID will be either 1 or 2 through out the experiment. If you are not selected, your player ID will be either 3 or 4. All of you will participate in the negotiation phase.

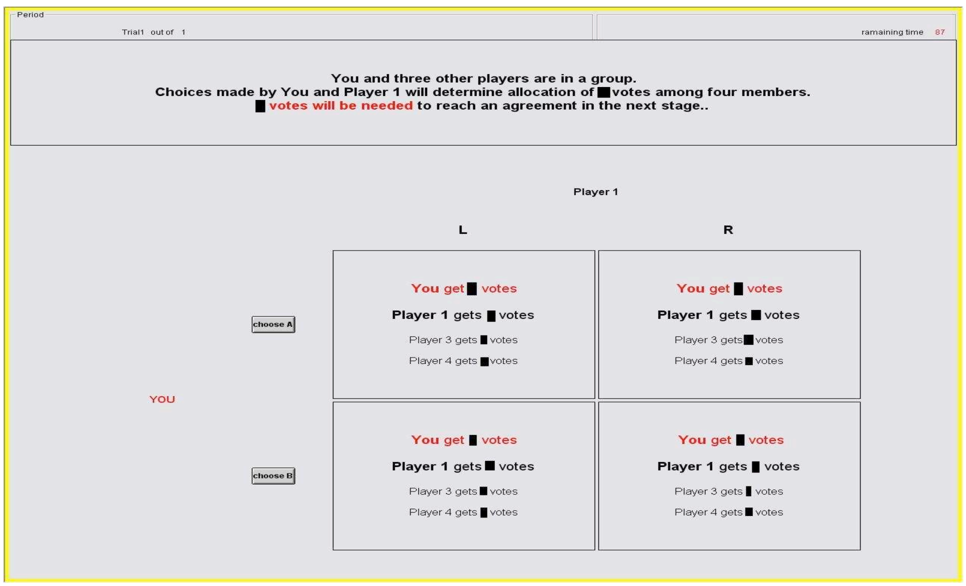
At the beginning of each round, the computer will randomly group four participants, two decisions makers in the allocation phase and two others, into one group. You will not be able to know which participants are in the same group.

Your player ID for the round will also randomly selected to be 1 or 2 if you are a decision maker in the allocation phase, and 3 or 4 otherwise.

You will repeat the same procedure for 20 rounds, but your ID number may change from round to round, the other people in your group also change. In each round, you will be clearly informed on your player ID for that round.

The votes apportionment phase

Once the experiment starts, on the screen of those who are chosen to make a decision in the allocation phase, i.e., players 1 and 2, they will see a screen as follows.



You may choose the strategy “A” or “B”, and the opponent you have been randomly matched to may choose “L” or “R”. You and your opponent make the choice simultaneously, without knowing each other’s decision in advance. The selected outcome depends both on your and your opponent’s choices. Players 3 and 4 ARE NOT active in this phase, but you and your opponent will determine the number of votes four players will have in the negotiation phase. After decisions are made, all participants move to the negotiation phase.

If you are player 3 or 4, you will not see this screen.

There is a time limit in the allocation phase. This time limit will be between 120 and 180 seconds and will be set randomly by the computer at the beginning of each round. You will not be informed of the exact time limit. This means that the allocation phase could end suddenly at any second between 120 second and 180 second after its start.

If you or the other player does not make a choice within the time limit, your group will not enter the negotiation phase and the round will end. The computer will attribute 0 points to you and/or to your opponent in the case he or she did not make a choice. Other players will be given an equal share of 100 points (rounded to be an integer) for that round. That is, if only one of two decision makers in the allocation phase did not make a choice within the time limit, then, that player will receive zero point, while three other players will receive 33 points each. If both two decision-makers in the allocation phase did not make their choices, then they both receive zero point while remaining two players will receive 50 points each.

Caution: The matrix in the allocation phase may change during these 20 rounds. Please pay attention.

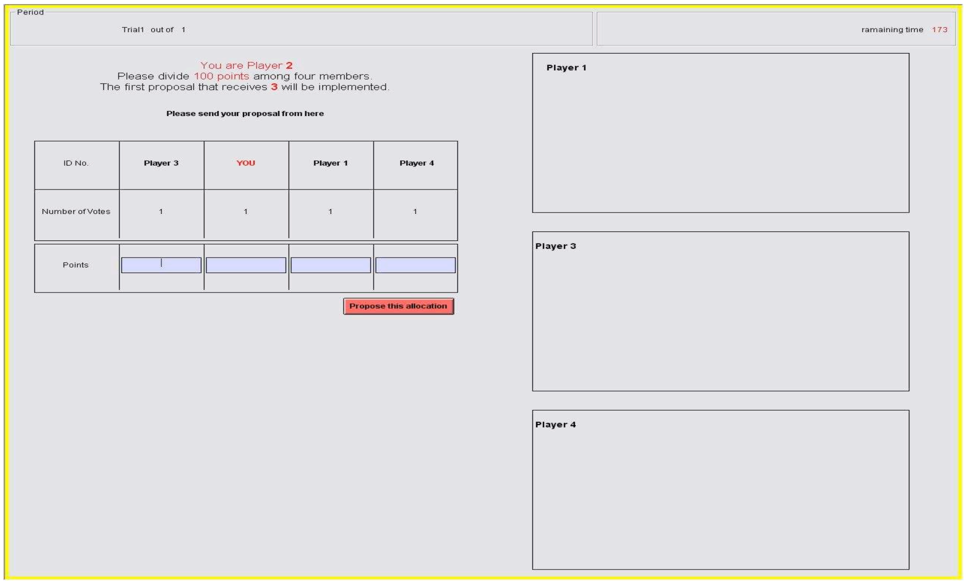
If you have any questions please raise your hand.

The points allocation phase

In the negotiation phases, you will be making a decision in a group with three other people, on how to divide 100 points among four of you.

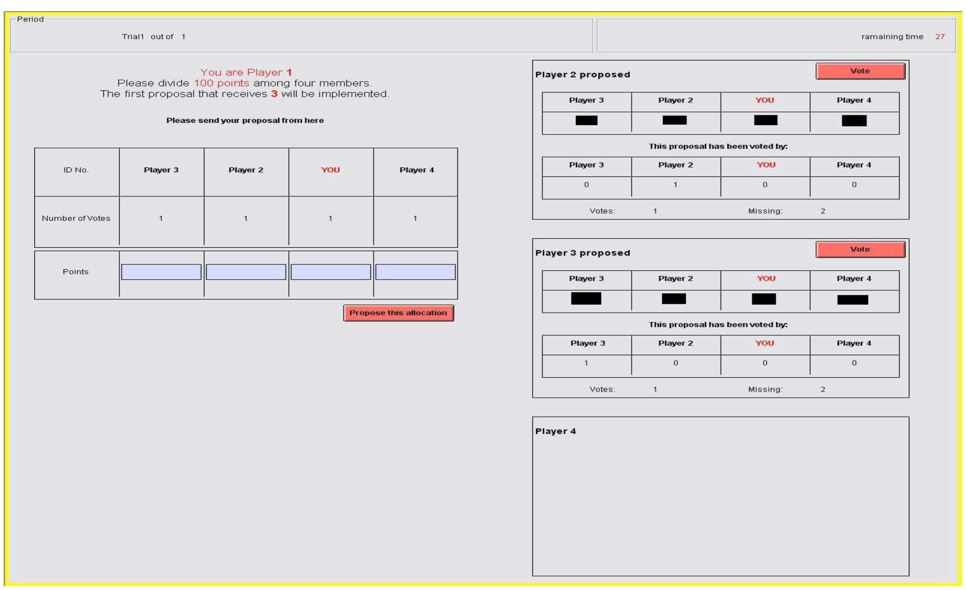
You will not know who the people in your groups are, and the people in your group will change randomly every round. If you are the participant who is chosen to play in the allocation phase, one of your group members will be the one you have played with in the allocation phase.

Each player has a certain number of votes depending on the outcome of the allocation phase. The information will be shown in the table in the left side of the screen.



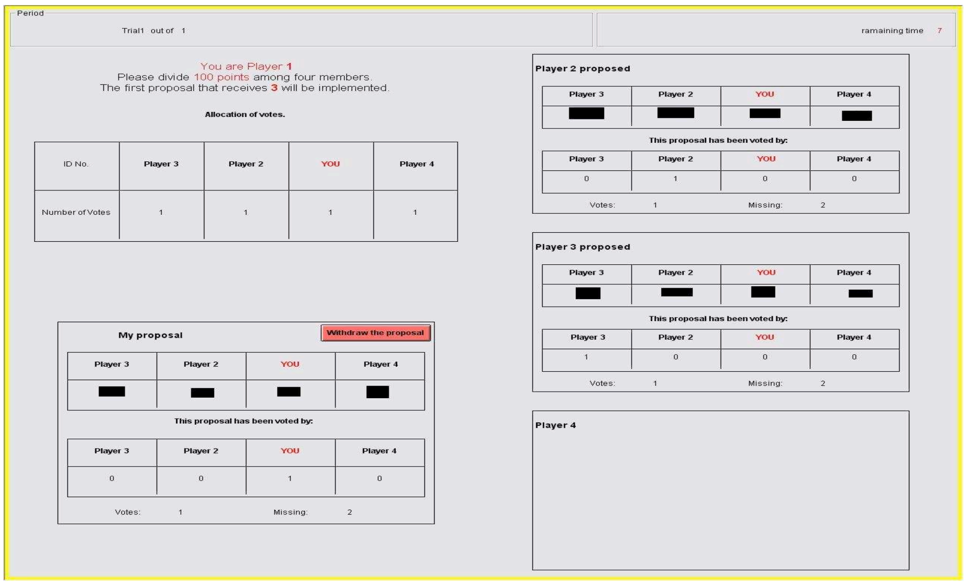
Any member of the group at any moment during the negotiation phase may make a public proposal about how to divide 100 points. To make a proposal, you need to enter 4 numbers in the respective boxes in the left hand side of the screen and press propose.

Any member of a group could also vote for any already submitted proposals. Proposals made by others are shown in the right side of screens. You can vote for a proposal by pressing a "vote" button.

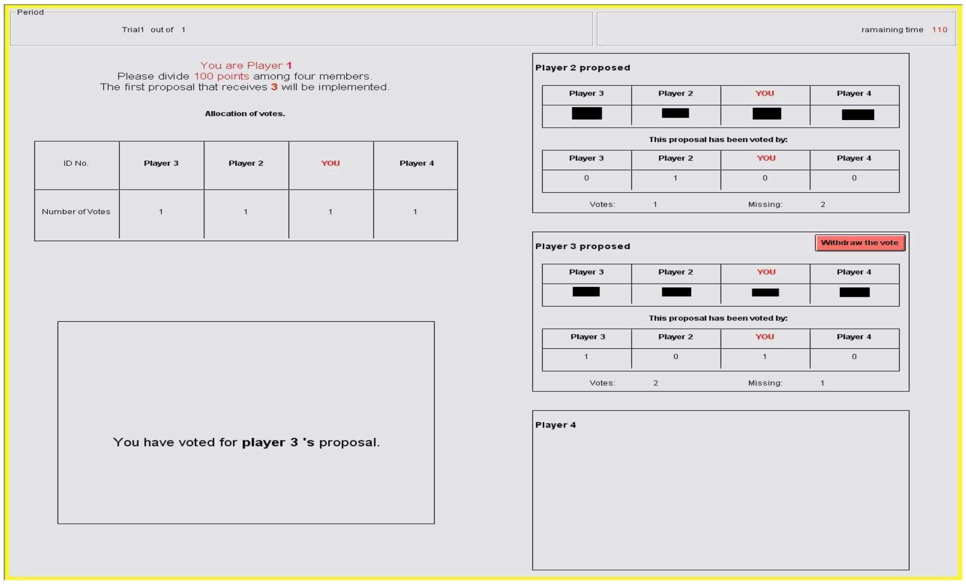


Please remember, you can only be in favor of at most one proposal, including your submitted proposal, at any given time. Even if you have more than one vote, you cannot divide your votes up and support multiple proposals. All your votes will be casted in the proposal that you decide to support. You can change your approval whenever you want during the negotiation phase.

You can withdraw your proposal in order to propose a new one or to vote for other's proposal by pressing "withdraw" button in the left side of your screen.



You can also withdraw your vote for other's proposal to propose or to vote for different proposal by pressing "withdraw" button shown in the right side of the screen.



The first proposal that receives the necessary number of votes (that will be written on screen as public information) will be implemented and the negotiation phase ends. Each of your group members will receive the number of points specified in that proposal.

There is a time limit to the negotiate phase. The time limit will be between 300 and 420 seconds. In each round, before the start of negotiation phase, the computer will randomly set the time limit, and you will not be informed of the exact time limit. This means that the round could end suddenly at any time between 300 seconds and 420 seconds after its start. If none of the proposal has received the necessary number of votes during this time limit, then all the members of your group will receive 0 points in this round.

If you have any questions please raise your hand.

PAYMENT

At the end of the experiment, the computer will randomly select 5 rounds out of 20 rounds. You will be paid only according to the points you have obtained in these selected rounds, and not according to the points of the whole treatment. The total points you have earned in the selected 5 rounds will be converted to cash at the exchange rate of 1 point = 16 euro cents.

In addition to this, you will be paid X euros as a show up fee.

The maximum earning you can make is, therefore, X + 0.16 x 5 x 100 Euros = X+ 80 Euros.

The minimum earning you can make is the show up fee of X Euros.

You will then be paid in cash.

PRACTICE ROUND

In order to make you familiar with the interface and mechanism of the experiment, we now do one round of practice. What you will do in the test will not affect your final payment. The matrix in the allocation phase and resulting apportionments of votes among four members of a group are not related to what you will see in the real experiments to follow.

IF YOU HAVE ANY QUESTIONS, PLEASE RAISE YOUR HAND.