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# Robot traders can prevent extreme events in complex stock markets

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## **Abstract**

If stock markets are complex, monetary policy and even financial regulation may be useless to prevent bubbles and crashes. Here, we suggest the use of robot traders as an anti-bubble decoy. To make our case, we put forward a new stochastic cellular automata model that generates an emergent stock price dynamics as a result of the interaction between traders. After introducing socially integrated robot traders, the stock price dynamics can be controlled, so as to make the market more Gaussian.

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*Keywords:* Stock markets; Robot traders; Financial regulation; Econophysics

## **1. Introduction**

The recent crisis caught the economists in a misplaced consensus of “one instrument one target.” The instrument was the nominal interest rate, and the monetary policy focused only on an implicit inflation target. Stock market bubbles were dismissed as unimportant. After the bursting of the subprime housing bubble, the collapse of the theory prompted a rebirth of old-style Keynesianism, along with talks of an elusive “macroprudential” regulation. However, such approaches fail to recognize the basic fact that stock markets are complex systems.

Prior to the financial crisis, on one side of the controversy on whether central banks should respond to stock price movements, Bernanke and Gertler [1] argued that as a central bank was committed to inflation targeting, it was not generally desirable

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that monetary policy should react to stock price movements (refer [2] and references therein for an overview of the debate). This point of view coincided with that of the US Federal Reserve. The Fed thus believed that monetary policy should not preemptively react to an asset bubble that was uncertain and thus difficult to track and “prick,” but it should strongly react after the bubble had burst. After the deep recession in the aftermath of the crash, the Fed seems to have abandoned this perspective. However, there is still a vacuum in theory about the modus operandi of financial regulation.

The main problem here is that the regulation of a complex system such as the stock market cannot be achieved using conventional policy tools. Although economists are hardly prepared to recognize this fact [3], here one has to resort to the control theory of self-organized systems [4]. Collective behavior based on self-organization has been shown in animals living in groups as well as in humans. This prompted engineers to devise autonomous robots that relied on self-organization as a main coordination mechanism. The controller of individual robots is designed using reactive, behavior-based techniques. Socially integrated autonomous robots, perceived as congeners by the group, and acting as interactive decoys will be able to control self-organized choices [5].

Self-organization is a feature of complex stock markets [6, 7], and robot trading is ubiquitous. We take these facts into account to make a case for the robots to counteract the imitative behavior of human traders, which occasionally leads to bubbles. If correctly engineered, this offers an alternative to monetary policy and conventional regulation, to stabilize stock markets.

Simple software-based traders have been around for many years (some even blamed the first generation of robots for the crash of 1987 [8]), but they are now becoming far more sophisticated, and make trades worth tens of billions of dollars every day. They already appear to be outperforming their human counterparts in the equity markets, where they are buying and selling shares. It is almost impossible for an exchange to tell whether a person or an algorithm is issuing trades. Under such circumstances, why not use the robot traders as an anti-bubble decoy?

To show how this can be accomplished, we put forward a new stochastic cellular automata model that generates an emergent stock price dynamics as a result of the interaction between traders. After introducing socially integrated robot traders, the stock price dynamics can be controlled, so as to make the market more Gaussian.

The rest of this article is organized as follows. Section 2 sets up our cellular automata model of the stock market where imitation plays a key role. Section 3 describes the market price dynamics of the model. Through parameter calibration, the model is made empirically relevant by matching its properties with those of the real world stock market, the Sao Paulo Stock Exchange (Bovespa, for short). Section 4 introduces the robot traders into the model, and Section 5 assesses their impact on the stock price dynamics. Section 6 concludes the study.

## 2. The model

We set the new stochastic cellular automata model to study the stock price dynamics where the interactions between the market participants play a key role (see also reference [9]). Initially, there are only human traders, and subsequently, robot traders enter the market. The traders are represented by cells on a two-dimensional  $L \times L$  grid. There are  $N$  traders who can either buy or sell only one share, and these are two mutually exclusive states. At any given time step  $t$ , the population of traders  $N$  is divided into two distinct groups of buyers  $N_B(t)$  and sellers  $N_S(t)$ .

The stock market dynamics emerges as a result of the synchronous update of cells, according to a local probabilistic rule. Here, traders consider the information related to the behavior of their neighbors and also that related to the “fundamentals.” Imitative behavior can be motivated by the fact that a trader can attempt to extrapolate from their neighbors’ views the information they are lacking [10]. The same trader can either imitate or behave consistently with the fundamentals. This is now discussed in greater detail.

The probability of a trader to choose to buy at time  $t$ ,  $\pi^B(t)$ , is

$$\pi^B(t) = S_I^B(t)^{\omega(t)} \cdot S_F^B(t)^{1-\omega(t)}, \quad (1)$$

where  $S_I^B(t) \in [0,1]$  and  $S_F^B(t) \in [0,1]$ , respectively, are the probabilities of buying, based on imitation and on the fundamentals; and  $\omega(t) \in [0,1]$  is the weight ascribed to imitation. When  $\omega(t) = 0$  the choice is based only on the fundamentals; when  $\omega(t) = 1$  the choice is based only on imitation; and when  $\omega(t) \in (0,1)$  the choice mixes both

strategies. The probability of a trader choosing to sell,  $\pi^S(t)$ , is the probability of the complement of (1).

Some recent literature on the understanding of collective decision making has emphasized the importance of quorum responses, where the probability of exhibiting a particular behavior is an increasing function of the number of actors already performing the behavior [11]. Here, we consider this insight and assume that the willingness to buy (sell) at  $t$  increases as a function of the number of neighbors who have already bought (sold) at the previous step  $t-1$ . Thus, the probability of imitation at time  $t$ ,  $S_t^B(t)$ , is given by

$$S_t^B(t) = \left[ \frac{(N_B^H(t-1))^\kappa}{(N_B^H(t-1))^\kappa + (N_S^H(t-1))^\kappa} \right], \quad (2)$$

where  $N_B^H(t-1)$  and  $N_S^H(t-1)$ , respectively, are the number of buying neighbors and selling neighbors at  $t-1$ ; and  $\kappa \in [1, \infty)$  is a parameter controlling the intensity of the response. When  $\kappa = 1$ , the probability of buying is proportional to the number of neighbors who have previously bought; this characterizes a weak linear response. When  $\kappa > 1$  there is a quorum response, because the probability of buying increases once the quorum is met. Here, the quorum size is determined by the number of selling neighbors at the previous time step.

When choosing is based on the fundamentals, we assume that the traders will consider the difference between the fair value of the stock in terms of the situation of the company selling it (the fundamental value), and the stock market price at the previous time step. Without loss of generality, we assume that the fundamental value  $\bar{F}$  is a positive constant. We also assume that all the traders perceive such a fundamental value identically, at each time step. The probability of buying based on the fundamentals is then given by,

$$S_F^B(t) = \frac{e^{\lambda(\bar{F}-P(t-1))}}{e^{\lambda(\bar{F}-P(t-1))} + e^{-\lambda(\bar{F}-P(t-1))}}, \quad (3)$$

where  $P(t-1)$  is the stock price at  $t-1$ ; and  $\lambda$  is a positive parameter modulating a trader's response, based on the price difference. Parameter  $\lambda$  can also be thought of as the degree of uncertainty facing the traders [12]; for a higher degree of uncertainty,  $\lambda$  is shorter, and vice versa. If a trader perceives the stock price as being lower than the fundamental value  $\bar{F} - P(t-1) > 0$ , he tends to buy; and vice versa. When  $\bar{F} - P(t-1) = 0$ , the decisions of buying and selling are equally probable.

The weight ascribed to either strategy,  $\omega(t)$  (that is, imitating or following the fundamentals), is endogenous, and depends on the size of the deviation of the current stock price from its fundamental value [13]:

$$\omega(t) = \frac{1}{1 + \mu(P(t-1) - \bar{F})^2}, \quad (4)$$

where  $\mu$  is a positive parameter, tracking the speed at which the strategy switches from imitation to fundamental. As  $(P(t-1) - \bar{F})^2 \rightarrow \infty$ ,  $\omega(t) \rightarrow 0$ , and the fundamental strategy grows in importance; as  $(P(t-1) - \bar{F})^2 \rightarrow 0$ ,  $\omega(t) \rightarrow 1$ , and imitation is preferred.

Equations (1) to (4) describe the individual behavior that leads to the emergent stock price as a result of the interactions between traders. As choices are also constrained by what the group collectively does, we use an excess demand function  $D(t)$  to model such a constraint:

$$D(t) = \frac{N_B(t) - N_S(t)}{N}. \quad (5)$$

This constraint is equivalent to assuming that the price adjustment process is explained by the action of a market maker balancing demand and supply [14]. A hyperbolic tangent functional form has been shown to work well here [15], that is,

$$P(t+1) = P(t)(1 + \tanh D(t)), \quad (6)$$

where  $\tanh(\cdot) \in (-1, 1)$ . This closes the model. The next section describes the stock price dynamics that this model generates. Thereafter, we introduce robot traders into the model, to evaluate their effects on the dynamics.

### 3. Market price dynamics

As usual, we have considered log returns rather than the prices, that is,

$$R(t) = \ln P(t+1) - \ln P(t), \quad (7)$$

which are standardized, so that they have a zero mean and unit standard deviation. To make our model empirically relevant we have calibrated its three parameters  $\lambda$ ,  $\kappa$ , and  $\mu$ , to match the statistical properties of the log returns of the Bovespa index. For all the experiments below we have set  $\bar{F} = 1$ . We have considered the daily data of the index from the period July 5, 1994 to June 30, 2009, which comprises 3,710 data points. The data have also been standardized to present the zero mean and unit standard deviation.

The random seed was kept the same for all the experiments. The initial distribution and the density of buyers (or sellers) in the two-dimensional lattice was kept the same across the experiments, that is, 50 percent randomly distributed buyers and sellers. The neighbors of influence were defined by a “nine-neighbor square,” known as the Moore neighborhood (Figure 1). The lattice size was defined to have  $100 \times 100$  cells, totaling 10,000 traders, each simulation was run for 3,800 periods, and the first 90 observations were discarded.

The adjustment of the model to the empirical data can be quantified by a two-sample Kolmogorov-Smirnov goodness-of-fit measure:

$$D_M = \sup_x |F_M(x) - F_B(x)|, \quad (8)$$

where  $F_B(x)$  is the observed cumulative probability distribution of the Bovespa log returns, and  $F_M(x)$  is the cdf of the log returns generated by the numerical simulations of the model. We found  $D_M = 0.0256$  ( $p$ -value = 0.1755) for the parameter values  $\lambda = 1$ ,  $\kappa = 9$ , and  $\mu = 2 \times 10^{10}$ . Thus, the null hypothesis that both samples were

generated by the same distribution cannot be rejected at the 10 percent significance level.

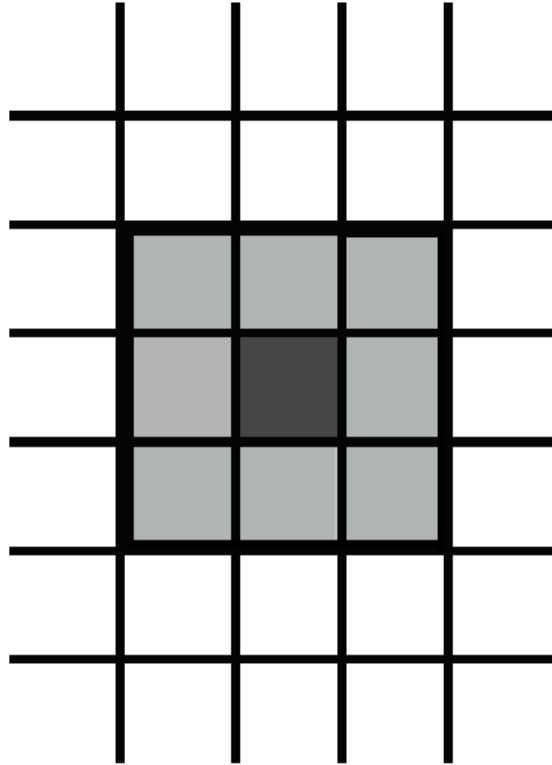


Figure 1. Moore neighborhood

After running the model several times, we found the best fit for the data using equation (8) for the parameter values  $\lambda = 1$ ,  $\kappa = 9$ , and  $\mu = 2 \times 10^{10}$ . The top of Figure 2 shows the time evolution of the Bovespa index log returns, and the bottom shows the log returns generated by the model. Figure 2 shows that the positive and negative returns that exceed by about five times the sample standard deviation occur quite regularly in both the empirical data and the data generated by the model. In the model, no external noise is needed to make this happen. Such behavior is unlikely to be Gaussian. Figure 3 shows the corresponding volatilities.

Figure 4 shows the probability density functions of both the empirical data and the data generated by the numerical simulations of the model. A good quality non-Gaussian fit can be seen, in particular its leptokurtic nature. The Gaussian pdf of zero mean and unit standard deviation is also plotted for comparison.



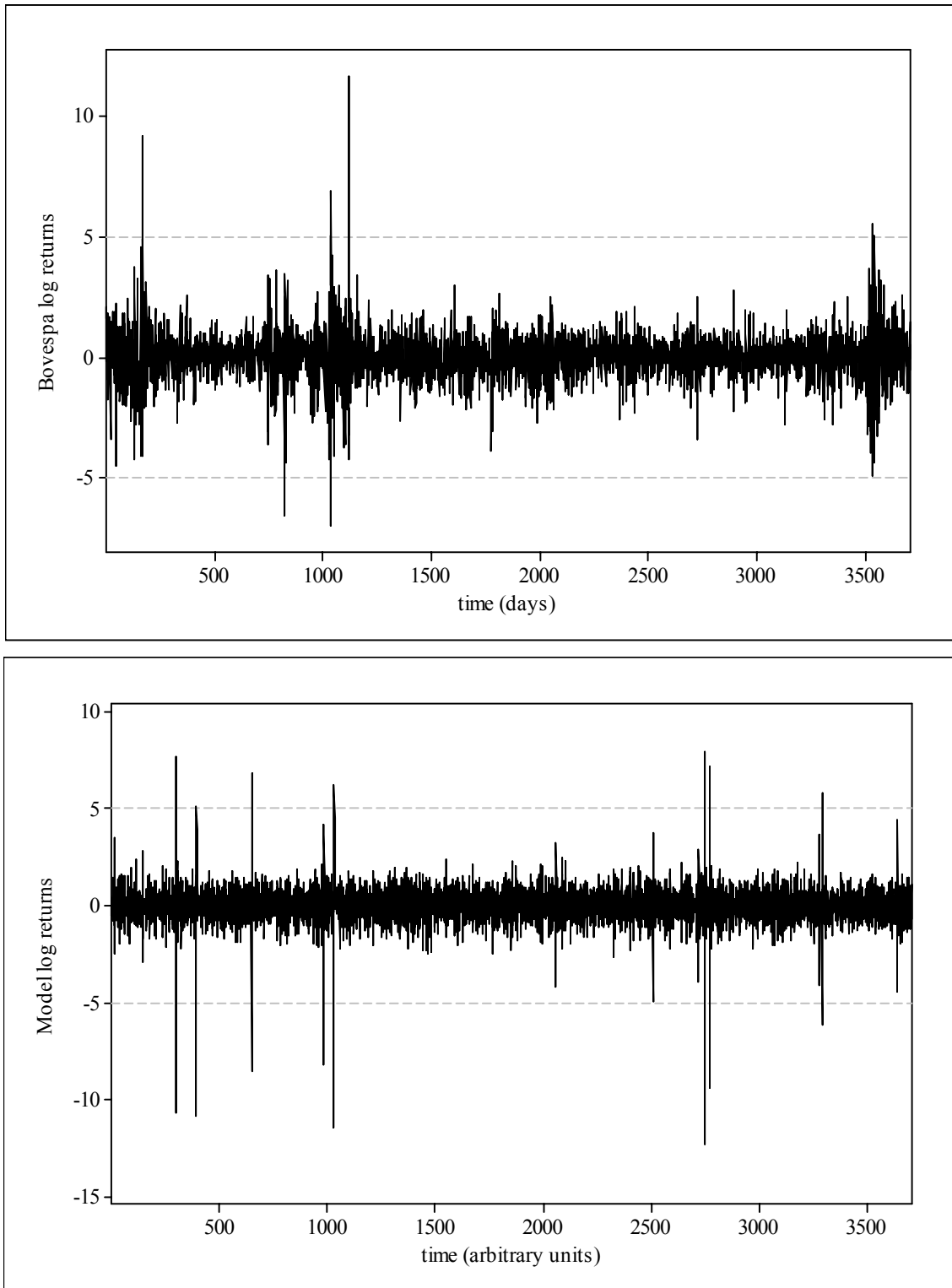


Figure 2. Top: time evolution of the standardized Bovespa log returns. Bottom: standardized log returns generated by the model using  $\lambda = 1$ ,  $\kappa = 9$ , and  $\mu = 2 \times 10^{10}$

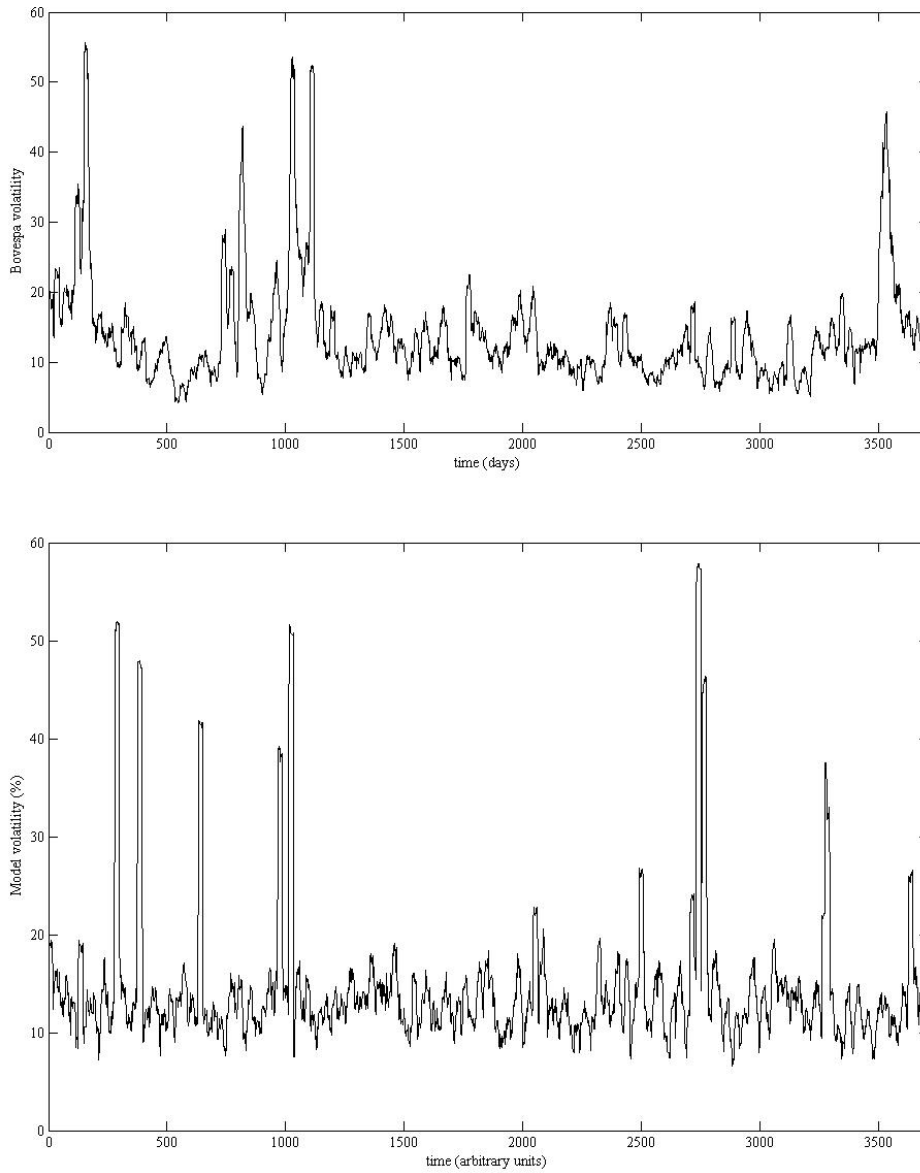


Figure 3. Top: Bovespa annualized volatility for the time window of 20 days and time horizon of one day. Bottom: same for model using  $\lambda = 1$ ,  $\kappa = 9$ , and  $\mu = 2 \times 10^{10}$

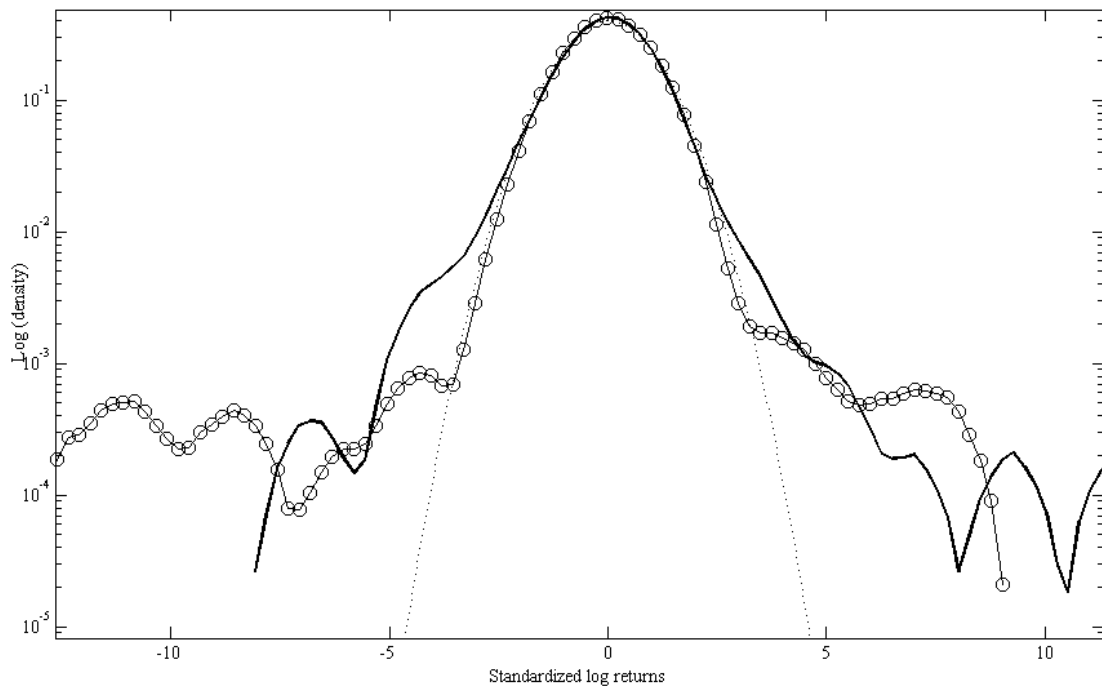
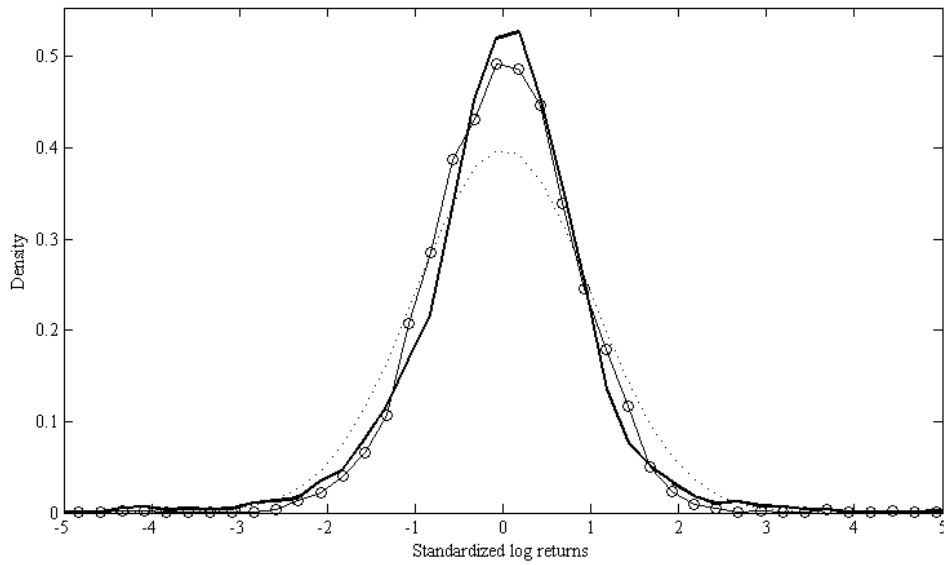


Figure 4. Top: pdfs of the standardized Bovespa log returns (solid line) and of the standardized log returns generated by the model (open circles) using  $\lambda = 1$ ,  $\kappa = 9$ , and  $\mu = 2 \times 10^{10}$ . The leptokurtic nature of the empirical data is replicated by the model. A standardized Gaussian pdf (dotted line) is also plotted for comparison. Bottom: semi-log plot

Table 1 shows the results of three tests for normality. The results do confirm that the Gaussian pdf poorly describes both types of data. All the  $p$ -values are found so close to zero that the Gaussian pdf can be rejected for any reasonable significance level. Excess kurtosis (above 3) is also present. Thus, the model fairly replicates the dynamic behavior of the Bovespa log returns, mainly its leptokurtic pdf.

Table 1. Normality tests

<i>Standardized log returns</i>	<i>Lilliefors</i>	<i>Cramer-von Mises</i>	<i>Anderson-Darling</i>	<i>Kurtosis</i>
Bovespa index	0.06530 (0.0000)	6.15621 (0.0000)	38.8904 (0.0000)	13.87
Model	0.05656 (0.0000)	5.12857 (0.0000)	38.6997 (0.0000)	29.98

Note:  $p$ -values shown in brackets. All the  $p$ -values are close to zero so that the null of Gaussianity is rejected for any standard (0.01, 0.05, 0.10) significance level

One reasonably established result of the econophysics literature is the existence of an inverse cubic power law for extreme events [16]. Thus, we studied the decay of our pdfs' fat tails, that is, the occurrences of large positive or negative log returns. One simple (although robust) technique of estimating tail exponents [17] is to run an ordinary least squares regression for the sizes of the extreme returns ranked from top to bottom ( $R_{(1)} \geq \dots \geq R_{(t)}$ ), that is,

$$\log\left(t - \frac{1}{2}\right) = a - \zeta_R \log R_{(t)}. \quad (9)$$

The  $\zeta_R$  is an estimate of the Pareto exponent, and the asymptotical standard-error of the exponent is given by  $\sqrt{\frac{2}{t}}\zeta_R$ .

Table 2 presents the results for  $\zeta_R$  using regression (9) and tail size defined by the top 10 percent ranked returns, that is,  $t = 1, \dots, 371$ . The estimate of the upper tail index of the model using  $\lambda = 1$ ,  $\kappa = 9$ , and  $\mu = 2 \times 10^{10}$  is not statistically different from 3 at the one percent significance level. Thus, the results are in good agreement with the inverse cubic law (Figure 5).

Table 2. Estimates of the tail index

<i>Standardized log returns</i>	$\zeta_R$	$\zeta_R$ (lower tail)	$\zeta_R$ (upper tail)
Bovespa index	2.9659 (0.2177)	2.8190 (0.2069)	2.5899 (0.1901)
Model	2.4686 (0.1812)	2.5094 (0.1842)	2.9369 (0.2156)

Note: standard-error in brackets

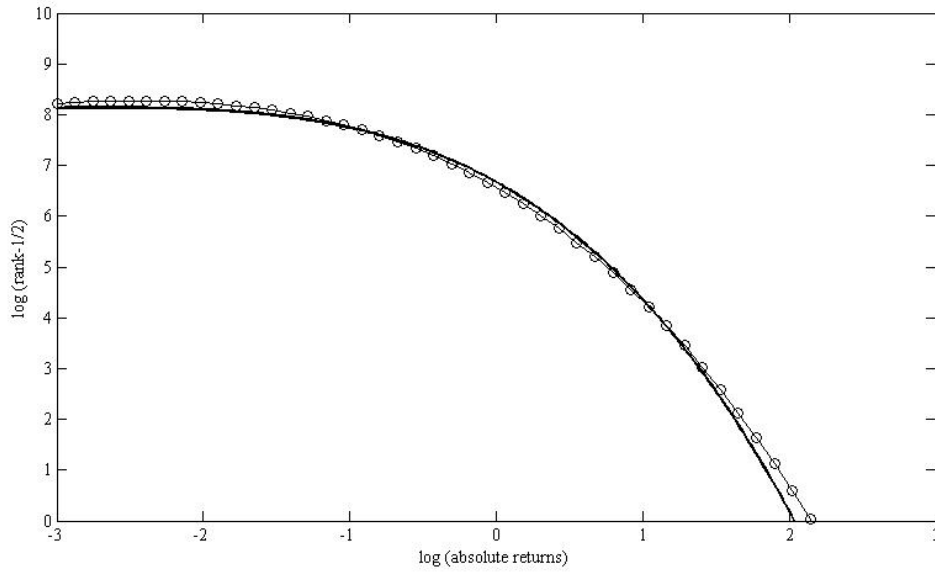


Figure 5. Log of absolute returns versus  $\log(\text{rank} - \frac{1}{2})$  (that is, locally weighted scatter plot smoothing) of the Bovespa index (solid line), and of the model using  $\lambda = 1$ ,  $\kappa = 9$ , and  $\mu = 2 \times 10^{10}$  (open circles). The slope corresponds to the estimate of the coefficient  $\zeta_R$  in regression (9)

Figure 6 shows three snapshots of the grid configuration of the simulations. During the normal session (left) the traders are distributed uniformly between buyers and sellers, that is, half the agents are buyers (black cells) and the other half are sellers (white cells). There is no sharp difference between demand and supply. The situation is different during a bull market (center), where most traders are buying (predominance of black cells). During a crash (right), most traders are selling, and the white cells are in the majority.



Figure 6. Left: snapshot of the grid during a normal trading period. The black cells are the buyers, while the white ones are the sellers. The parameters used in this simulation are  $\lambda = 1$ ,  $\kappa = 9$ , and  $\mu = 2 \times 10^{10}$ . Center: bull market, where buyers predominate. Right: the same simulation during a crash

#### 4. The model with robot traders

We now introduce robot traders into the model. The robots are socially integrated into the group of human traders to control self-organized market returns. We assume that humans and robots are perceived as congeners and influence one another in the same way. Robot behavior is intentionally set to an anti-imitation rule to counteract the imitative human trader behavior as described by equation (2). The robots adopt such a contrarian behavior using the majority principle after considering the neighboring cells at the previous time step. Thus, the probability,  $\pi_R^B$ , of a robot trader to choose to buy at time  $t$  is

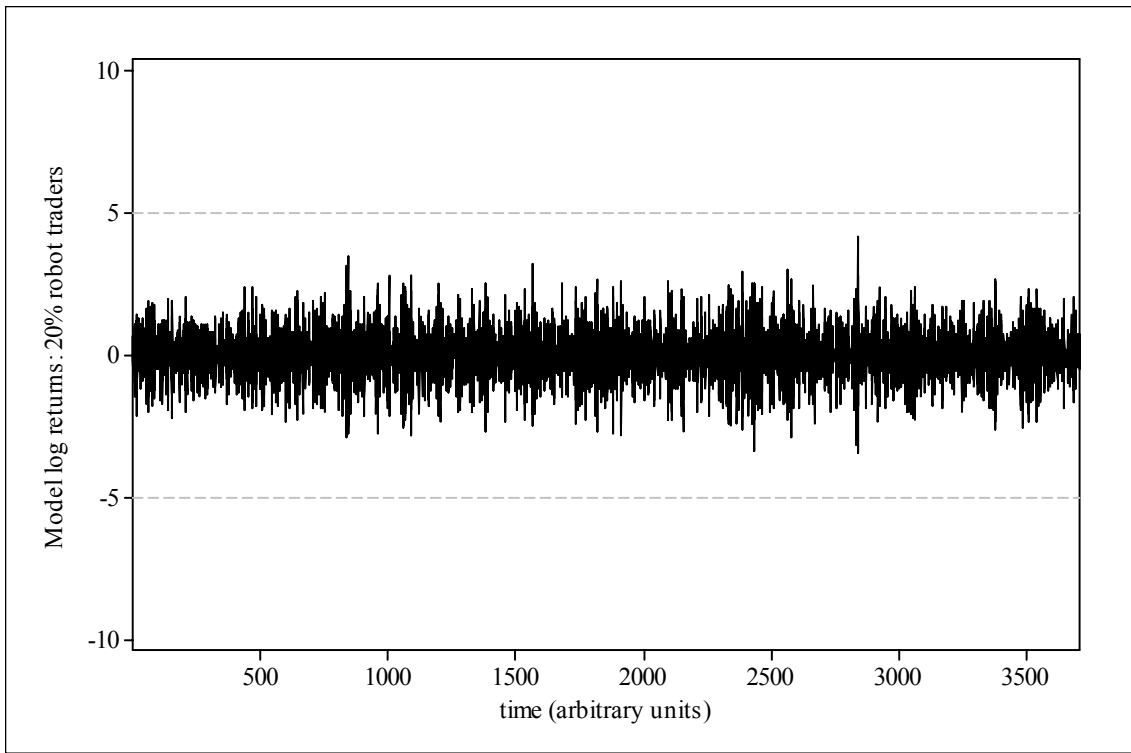
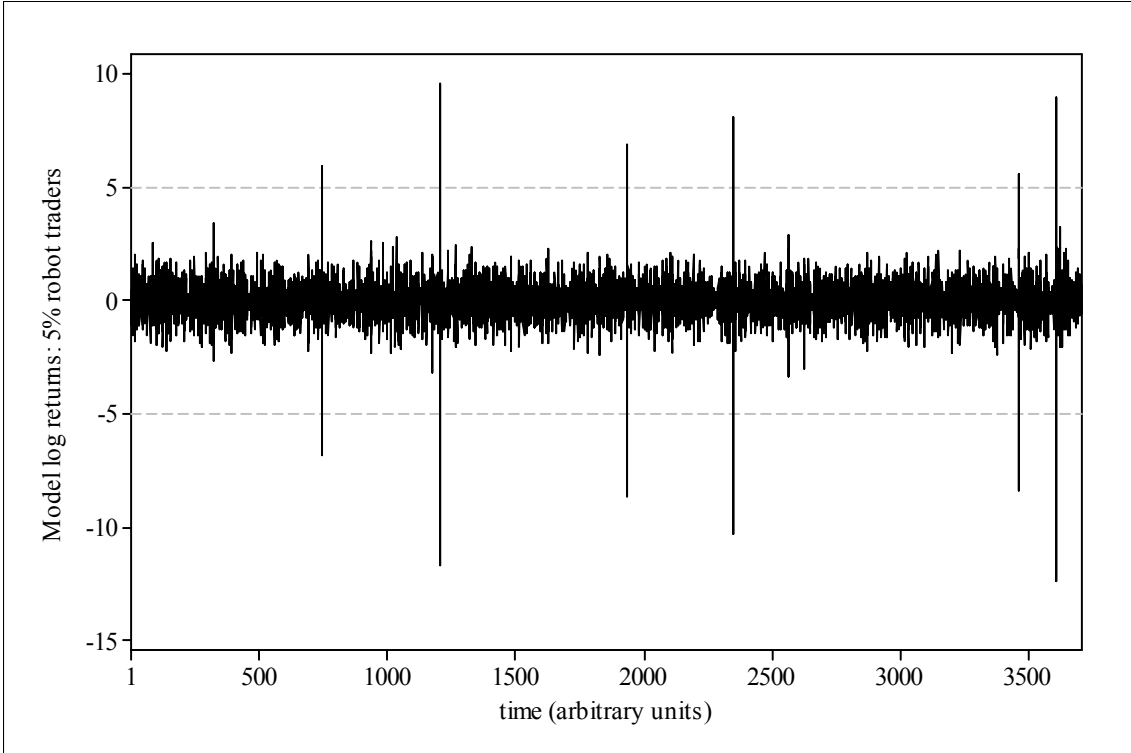
$$\pi_R^B(t) = \left[ \frac{N_S^H(t-1)}{N_B^H(t-1) + N_S^H(t-1)} \right], \quad (10)$$

where  $N_B^H(t-1)$  and  $N_S^H(t-1)$ , respectively, are the number of buying neighbors and selling neighbors at  $t-1$ . According to (10), if  $\pi_R^B \geq \frac{1}{2}$  a robot trader will choose to buy at time  $t$ ; if  $\pi_R^B < \frac{1}{2}$  the robot trader will choose to sell.

#### 5. Market price dynamics after the introduction of robots

To run the experiments with the robots, we adopted the same setup of the basic model: the structure of the cellular automata and the initial conditions were retained, that is,  $\lambda = 1$ ,  $\kappa = 9$ , and  $\mu = 2 \times 10^{10}$ . However, we still needed to specify how the robot traders are inserted into the market. At the start of each simulation, we randomly distributed a fixed number of robots in the grid. In the initial state, half the robots were buyers and the other half were sellers.

Our findings show that even when in the minority (5 and 20 percent of the total traders), the robots could interfere in the self-organized choices and thus in the emergent properties of the market price. Comparing Figure 7 and Figure 8 with Figures 3 and 4, one can see that the volatility abates and the series becomes Gaussian.



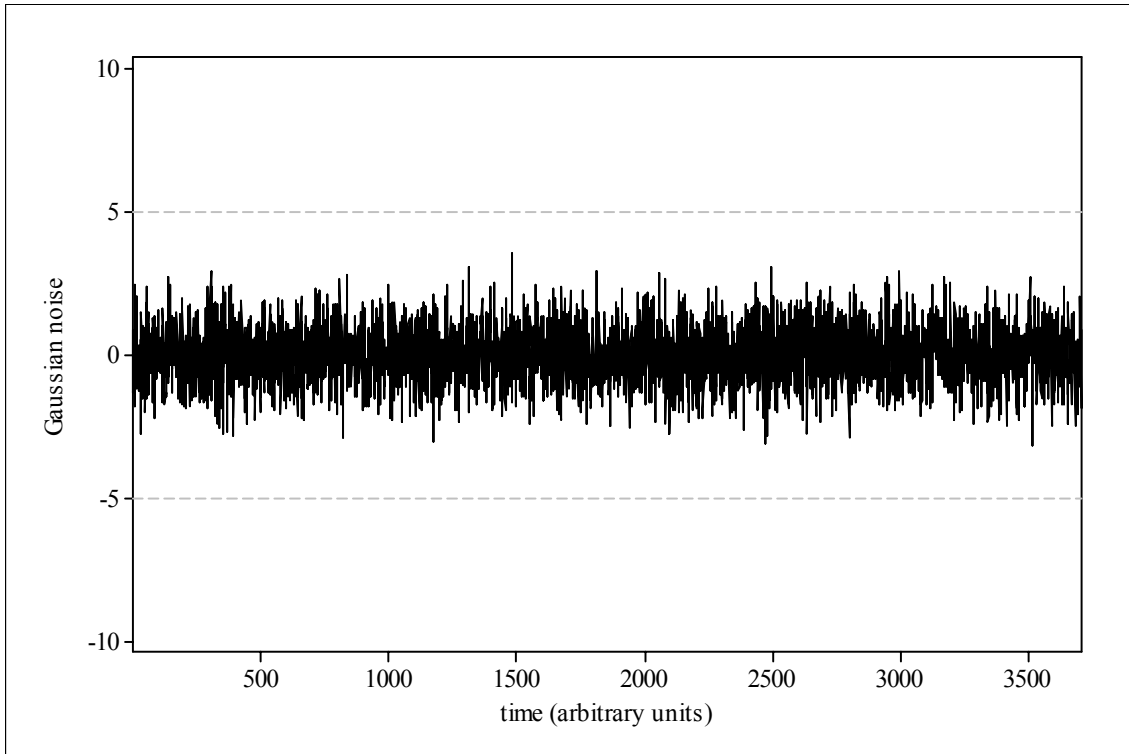


Figure 7. Top: time evolution of the standardized log returns generated by the model with five percent robot traders, using  $\lambda = 1$ ,  $\kappa = 9$ , and  $\mu = 2 \times 10^{10}$ . Center: time evolution of the standardized log returns generated by the model with 20 percent robot traders, and with the same parameters. Bottom: time evolution of a Gaussian series using the same parameters from the model with robot traders

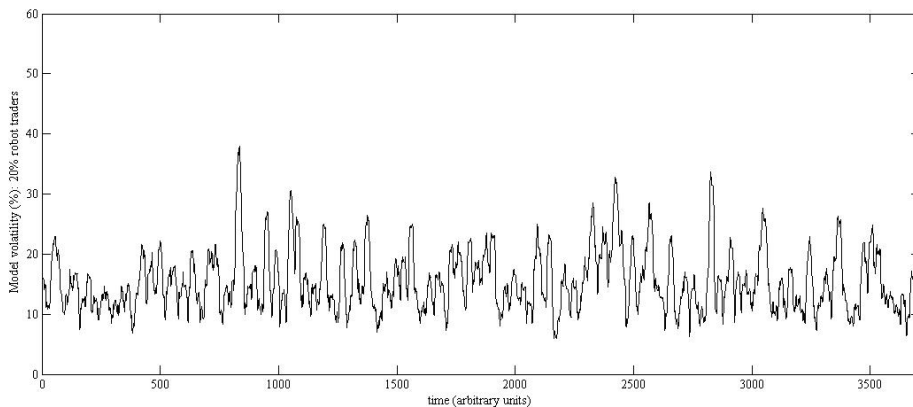


Figure 8. Annualized volatility for the time window of 20 days and time horizon of one day for model with 20 percent robot traders using  $\lambda = 1$ ,  $\kappa = 9$ , and  $\mu = 2 \times 10^{10}$

Figure 8 shows the pdf of the series generated by the model with the robot traders. The Gaussian and empirical pdf of returns are also plotted for comparison. As can be seen, the presence of robots in the stochastic cellular automata model pushes the collective behavior away from the leptokurtic nature of the empirical data (Figure 4) and becomes Gaussian.



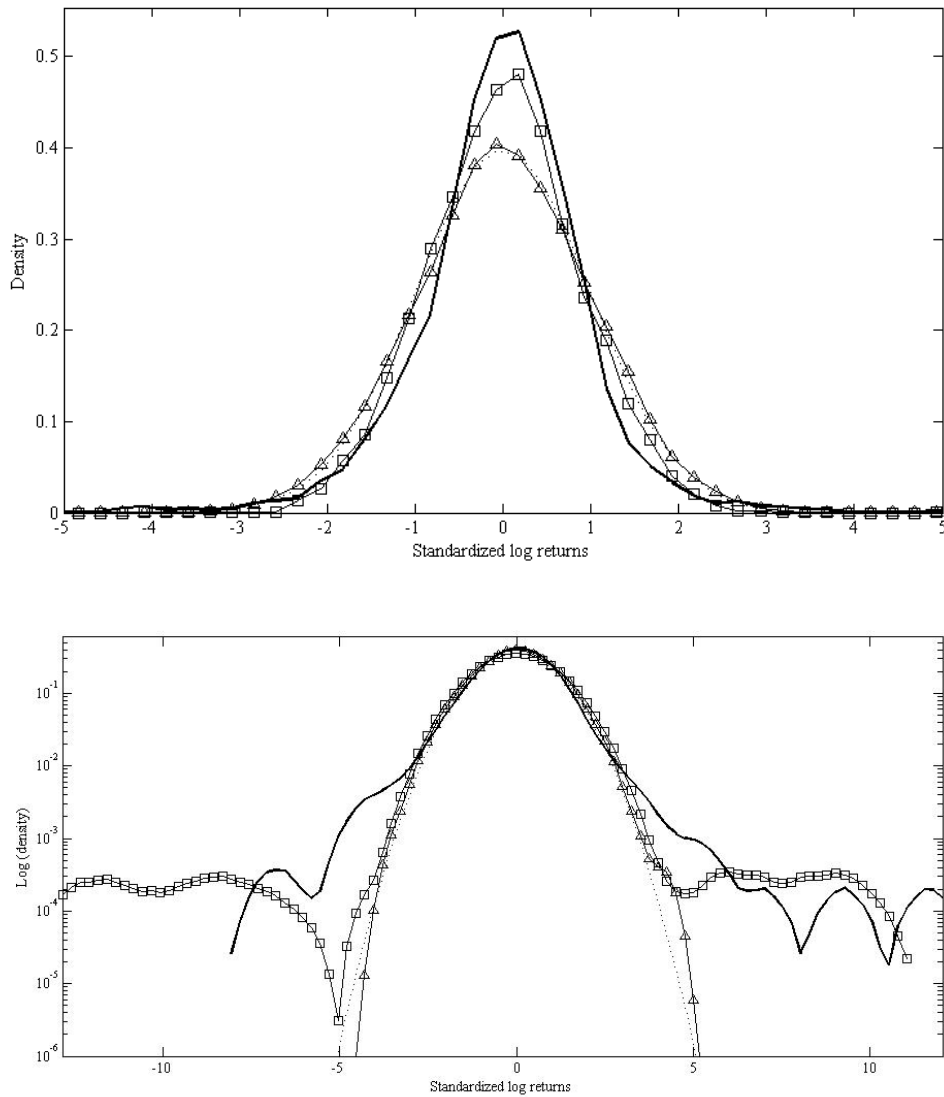


Figure 8. Top: pdf of the Bovespa index returns (solid line), pdf of the series generated by the model with five percent robot traders (squares), pdf of the series generated by the model with 20 percent robot traders (triangles), and the Gaussian pdf (dotted line). The presence of robots changes the leptokurtic nature of the empirical data and makes the collective behavior Gaussian. (Compare it with Figure 5.). Bottom: semi-log plot

Table 3 shows the results of normality tests from the model with robots. The results from the model with five percent robot traders indicate that the normality can be rejected for any standard significance level. However, the Gaussianity of the series generated from the model with 20 percent robot traders cannot be rejected at the 10 percent significance level. Clearly, our simulation results show that the stock returns become more Gaussian in the presence of a minority of robot traders adopting an anti-imitation behavior. Even when in the minority, the robots can modulate the collective decision-making process and produce a market pattern not observed in their absence.

This exemplifies how intelligent autonomous devices can be successfully used to control self-organized choices in the stock market.

Table 3. Normality tests

<i>Standardized log returns</i>	<i>Lilliefors</i>	<i>Cramer-von Mises</i>	<i>Anderson-Darling</i>	<i>Kurtosis</i>
Model with 5% robot traders	0.03900 (0.0000)	2.47612 (0.0000)	19.3567 (0.0000)	25.96
Model with 20% robot traders	0.01186 (> 0.15)	0.05325 (0.4644)	0.32302 (0.5266)	3.006

Note:  $p$ -values shown in brackets. The  $p$ -values fall above the standard significance levels of 0.01, 0.05, and 0.10, and thus the null of Gaussianity cannot be rejected for the model with 20 percent robot traders. Also, the excess kurtosis abates toward the Gaussian value.

One key result of the econophysics literature is that real world stock returns cannot be described by a Gaussian. However, independence and finite variance are also properties of actual data. Thanks to the central limit theorem, this means an asymptotical convergence to the Gaussian, that is, leptokurtosis abatement [18]. From this perspective, our model suggests that robot trading can deliver rapid convergence.

Our simple model of only three parameters is elegant and rich, but the use of robots as a practical tool for financial regulation is not fully developed within this framework. For example, there is no role for who—regulatory bodies or central banks—should be responsible for the management of robots. There are no monetary transactions either, and thus the issue of the financial resources involved in stabilizing the stock market is not addressed. One straightforward implication of our analysis is that 20 percent of the total market turnover would give the necessary resources, because 20 percent of robots are enough for stabilizing; but this is too expensive and thus unfeasible in practice. However, we believe the current model could still be slightly changed so as to remove this limitation by considering an extra constraint on robot trades, which will make them dynamically sustainable; but this should be clearly addressed by future research. Moreover, imitation in financial markets may arise not just by direct and local contact by traders (as modeled by our cellular automata model) but also by means of some central signal like the price. Future research may also wish to address the latter situation using a different framework.

## 6. Conclusion

We devised a new stochastic cellular automata model of the stock market and calibrated its key parameters to fit the empirical data properties of an actual stock market, the Bovespa. We showed that its non-Gaussian profile could be replicated by the model.

The imitation rule of the traders was then counteracted with an anti-imitation rule intentionally ascribed to robot traders. The model with robots was able to modulate the collective decision-making process and produce a market pattern not observed in their absence. In the emergent price behavior, a Gaussian profile emerged.

We then exemplified the use of an intelligent autonomous device to control a self-organized stock market. This alternative to monetary policy and conventional regulation, to prevent bubbles and crashes, can be justified, as the complex nature of the stock markets is recognized.

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