

TECHNICAL WORKING PAPER SERIES

A MONTE CARLO STUDY OF GROWTH REGRESSIONS

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Technical Working Paper 296  
<http://www.nber.org/papers/T0296>

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
January 2004

We thank David McKenzie and seminar participants at Stanford University and at UC Davis for useful comments. The data and programs used in this paper are available upon request, and all errors are ours. The views expressed in this paper are those of the authors and not necessarily those of the National Bureau of Economic Research.

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NBER Technical Working Paper No. 296  
January 2004  
JEL No. O4, O5, C1

### **ABSTRACT**

Using Monte Carlo simulations, this paper evaluates the bias properties of common estimators used in growth regressions derived from the Solow model. We explicitly allow for measurement error in the right-hand side variables, as well as country-specific effects that are correlated with the regressors. Our results suggest that using an OLS estimator applied to a single cross-section of variables averaged over time (the between estimator) performs best in terms of the extent of bias on each of the estimated coefficients. The fixed-effects estimator and the Arellano-Bond estimator greatly overstate the speed of convergence under a wide variety of assumptions concerning the type and extent of measurement error, while between understates it somewhat. Finally, fixed effects and Arellano-Bond bias towards zero the slope estimates on the human and physical capital accumulation variables.

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# 1 Introduction

In the last decade, spurred by the early work of Baumol (1986) and Barro (1991), growth regressions have become an industry. There exists no good alternative for addressing the fundamental question of what accounts for vast observed differences in per capita income across countries. Detailed case studies, while they help identify hypotheses for further testing and theorizing, lack the generality of large sample studies. On the other hand, the detractors of growth regressions have stressed their numerous drawbacks. These include an often excessive distance between measured variables and the theoretical concepts they are meant to capture; poor grounding of estimated functional forms in economic theory, and in particular the prevalence of reduced form relationships from which structural parameters cannot be identified; unjustified claims of causality in explanations of growth; a small number of available observations; and the prevalence of prior-driven data-mining. These are but a few in a growing list, resulting in numerous methodological debates on the proper way to run growth regressions. Many of these debates are yet unresolved, so research evaluating the effectiveness of current methodologies and suggesting improvements to cross-country growth empirics appears necessary.

This paper is such a study. We use simulation methods to evaluate the bias properties of several estimators commonly used in the empirical growth literature.

One contribution of our approach is to consider explicitly the impact of measurement error on estimates of the determinants of growth. Measurement error is likely to be a central problem in cross-country growth empirics. Nonetheless, this issue has received little attention in the literature.<sup>1</sup> In the absence of measurement error and other sources of endogeneity, a fixed-effects estimator unambiguously dominates estimators that use any between-country variation, when omitted variables such as the initial level of technology are correlated with included right-hand side variables. In the presence of measurement error, however, fixed-effects estimators will tend to exacerbate measurement error bias when the right-hand side variables are more time persistent than the errors in measurement. The issue then is

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<sup>1</sup>A notable exception is Barro (1997), chapter 1, page 36, who briefly discusses the possible consequences of measurement error for fixed-effects estimates of the rate of convergence. Consistent with our simulation results, he argues that these estimates of the speed of convergence are likely to be overstated. See also Griliches and Hausman (1986), who made the same point without specifically referring to growth regressions.

whether the gains from reducing omitted variables bias are offset by an increase in measurement error bias under fixed-effects. We lack any guidance from econometric theory to evaluate the resulting net bias in the multivariate context of growth regressions, making Monte Carlo simulations necessary to address this issue.

A related contribution of this paper is to help resolve a long-running methodological debate in growth empirics: whether the appropriate way to control for time invariant cross-country heterogeneity in the level of technology is to use a fixed-effects estimator, thereby identifying parameters solely on the basis of within-country variation, or to retain at least some between-country variation in the data and directly include as right-hand side variables available proxies explaining technological differences.<sup>2</sup> Advancing this debate has profound implications for our understanding of the growth process. Results obtained using fixed-effects differ markedly in two main ways from those obtained by attempting to include additional correlates of growth.

First, fixed-effects lead to estimates of the speed of conditional convergence that are much higher than the conventional 2% obtained in cross sectional studies.<sup>3</sup> For a given history of income shocks, a fast speed of convergence indicates that most countries at any point in time are relatively close to their steady-states, suggesting that incomes can only rise by improving the determinants of steady-state income, a rather pessimistic conclusion. A slow speed of convergence, in contrast, suggests there is still a lot of catching up to do, so that the force of neoclassical convergence alone can be expected to raise per capita incomes of less developed countries over time. Second, the fixed-effects estimator tends to reduce the magnitudes of the estimated coefficients on right-hand variables compared to cross-sectional alternatives, so that it becomes harder to obtain statistically significant estimates on the determinants of steady-state income.<sup>4</sup> This is also a pessimistic finding, as it suggests that given improvements in steady-state determinants such as

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<sup>2</sup>The first approach is associated with the work of Knight, Loayza and Villanueva (1993), Islam (1995) and Caselli, Esquivel and Lefort (1996) among others. Since the mid 1990s, the use of dynamic panel data estimators in growth empirics has become prevalent. The second approach can be likened to a "kitchen-sink" method, in which the unaccounted variation in economic growth was attributed to additional right-hand side variables, meant to capture institutions, policies and economic structures. See Wacziarg (2002) for a broad discussion of fixed-effects versus the kitchen sink approach.

<sup>3</sup>Barro and Sala-i-Martin (1995), chapters 11 and 12.

<sup>4</sup>For example, human capital variables that are highly significant in cross-sectional estimates become insignificantly different from zero in panel-fixed effects applications, and sometimes reverse signs. See Islam (1995) and Benhabib and Spiegel (1994).

capital accumulation will yield smaller steady-state income gains.

To evaluate methodologies for growth empirics, we start with the Solow (1956) growth model, in its human capital-augmented version proposed by Mankiw, Romer and Weil (1992).<sup>5</sup> The Solow model is arguably the only solid theoretical foundation for the specific functional form generally estimated by practitioners, which involves regressing growth rates on the log initial income and a set of steady-state income determinants. Common specifications of growth regressions can be directly derived from this model, and reasonable values for the exogenous parameters of the model can be postulated. Using such values and modelling explicitly the dynamic nature of the Solow growth specification, we generate simulated data with moments resembling those of the empirical data, and perform Monte Carlo simulations to evaluate the performance of several commonly used estimators: fixed-effects, random effects, between (OLS on country means), and the Arellano-Bond estimator first introduced to growth empirics by Caselli, Esquivel and Lefort (1996).<sup>6</sup>

Our results suggest that using a least-squares estimator applied to a single cross-section of variables averaged over time (the between estimator) performs best to estimate the speed of conditional income convergence, though it tends to underestimate it. The fixed-effects estimator, as well as the Arellano-Bond estimator, greatly overstate the speed of convergence under a variety of assumptions concerning the type and extent of measurement error. The random effects estimator also tends to overstate the speed of convergence, though much less drastically than fixed-effects. Finally, fixed-effects seriously biases towards zero the slope estimates on the determinants of the steady-state level of income (the accumulation and depreciation variables of the Solow model), in particular on the human capital accumulation

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<sup>5</sup>A recently proposed alternative to growth regressions has been "levels" regressions, aimed at accounting for variation in the level of income rather than in the growth rate of income. Salient examples include, in chronological order, Hall and Jones (1999), Frankel and Romer (1999) and Acemoglu, Johnson and Robinson (2001). Arguably, these level specifications are even more devoid of theoretical foundations than their growth counterparts, which in most cases can be traced back to some version of the neoclassical growth model. Because this lack of theoretical foundation makes simulation difficult for levels regressions, we focus on growth regression in this paper. These also remain vastly more prevalent in the literature on the determinants of economic development.

<sup>6</sup>In Section 4, we also evaluate the properties of the Seemingly-Unrelated Regression model (SUR or flexible random effects) used for example in Barro and Sala-i-Martin (1995). It turns out that this estimator has properties close to those of random effects. Section 4 also evaluates the bias properties of the cross-sectional estimator used in Mankiw, Romer and Weil (1992), a variant of the between estimator.

rate. In contrast, random effects and between tend to overstate them (bias them away from zero).

Thus, our simulations are able to replicate the basic pattern of coefficients found in the literature using alternative estimators applied to real data. The punchline of our results is that the use of dynamic panel data methods leads to unreliable estimates when measurement error is present: it leads to misleading inferences on the speed of convergence, and to findings that common determinants of the steady-state income level are insignificantly different from zero when this is in fact not the case. In this particular application, old-fashioned OLS on cross-sectional averages performs best.

Perhaps the main contribution of this paper is methodological, and carries broader implications: the type of Monte Carlo exercise we present here should be a systematic rite of passage for studies presenting new estimation methodologies in any field of empirical economics. Estimators that may seem attractive to address a specific econometric problem need to be evaluated in a setting where several sources of bias may coexist. When a potential for omitted variables bias coexists with measurement error, a cure for the first problem can be worse than the disease, as it may exacerbate the second.

This paper is structured as follows: Section 2 discusses theoretical considerations related to the methodology of growth regressions. Section 3 presents our basic simulation methodology and results, contrasting OLS, fixed-effects, random effects and Arellano-Bond GMM estimators. Section 4 discusses extensions of our simulations to country-specific measurement error, regressor-specific measurement error, autocorrelated measurement error and additional estimators. Section 5 concludes by presenting new estimates of the speed of income convergence and of the effect of steady-state determinants using real data, and discusses them in light of our simulation results.

## 2 Theoretical Framework

### 2.1 Growth Regressions and the Solow Model

Mankiw, Romer and Weil (1992, henceforth MRW) and Islam (1995) have shown that the Solow growth model can be transformed in a way that allows its estimation through a simple application of linear regression techniques. This section reviews this well-known derivation and shows how country-specific effects can be entered into the model.

The Solow growth model augmented to include human capital accumulation starts with a simple neoclassical production function:

$$Y(t) = K(t)^\alpha H(t)^\beta (A(t)L(t))^{1-\alpha-\beta} \quad (1)$$

where  $Y$  is output,  $K$  is physical capital,  $H$  is human capital,  $L$  is labor and  $A$  is a labor-augmenting technology parameter.  $L$  and  $A$  are assumed to grow at exogenously determined rates  $n$  and  $g$  such that  $L(t) = L(0)e^{nt}$  and  $A(t) = A(0)e^{gt}$ . MRW and Islam also assume that  $n$  and  $g$  do not vary between countries.

Assume that in a given period a constant fraction of output is saved and devoted to investment in physical and human capital. If we define  $\hat{y} = Y/AL$ ,  $k = K/AL$  and  $h = H/AL$  to be the units per effective unit of labor, changes in physical and human capital can be represented as:

$$\dot{k}(t) = s_k \hat{y}(t) - (n + g + \delta)k(t) \quad (2)$$

$$\dot{h}(t) = s_h \hat{y}(t) - (n + g + \delta)h(t) \quad (3)$$

where  $s_h$  and  $s_k$  are the proportions of output devoted to investment in human and physical capital, respectively, and  $\delta$  is the depreciation rate of both human and physical capital (which is also assumed in the literature not to vary between countries).

The dynamics in equations (2) and (3) imply that the economy converges to steady-state levels of physical and human capital  $k^*$  and  $h^*$ , derived by setting  $\dot{k} = 0$  and  $\dot{h} = 0$ . Substituting these values back into equation (1) and taking logs, we get:

$$\begin{aligned} \log y(t) \equiv \log \frac{Y(t)}{L(t)} &= \log A(0) + gt - \frac{\alpha + \beta}{1 - \alpha - \beta} \log(n + g + \delta) \\ &+ \frac{\alpha}{1 - \alpha - \beta} \log s_k + \frac{\beta}{1 - \alpha - \beta} \log s_h \end{aligned} \quad (4)$$

Equation (4) describes an economy in its steady-state. If one is willing to assume that countries are at their steady-states, this equation can be turned into an econometric specification for a "levels" regression - but the assumption is unlikely to hold.<sup>7</sup>

To derive a growth regression explicitly, we can approximate the model around the steady-state  $y^*$ :

$$\frac{d \log \hat{y}(t)}{dt} = \lambda [\log \hat{y}^* - \log \hat{y}(t)] \quad (5)$$

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<sup>7</sup>However, for recent a paper taking such an assumption seriously, see Bernanke and Gürkaynak (2001). For a critique, see Caselli (2001).

where  $\lambda = (n + g + \delta)(1 - \alpha - \beta)$  is the rate of convergence. That is,  $\lambda$  is the percentage of the gap between a country's steady-state and its current income that will be closed in one period, in the absence of any other shocks.<sup>8</sup> A convergence rate of  $\lambda$  would imply that, given two points in time  $t_1$  and  $t_2$ , we can measure end-of-period output as:

$$\log \hat{y}(t_2) = \left(1 - e^{-\lambda\tau}\right) \log \hat{y}^* + e^{-\lambda\tau} \log \hat{y}(t_1) \quad (6)$$

where  $\tau = t_2 - t_1$ . The higher the convergence rate, the closer we should expect the economy to be to its steady-state at any point in time, all else equal.

Noting that  $\log \hat{y}(t) = \log y(t) - \log A(0) - gt$ , we can substitute equation (4) into  $\log \hat{y}^*$  in equation (6) and get:

$$\begin{aligned} \log y(t_2) &= \left(1 - e^{-\lambda\tau}\right) \frac{\alpha}{1 - \alpha - \beta} \log s_k + \left(1 - e^{-\lambda\tau}\right) \frac{\beta}{1 - \alpha - \beta} \log s_h \\ &\quad - \left(1 - e^{-\lambda\tau}\right) \frac{\alpha + \beta}{1 - \alpha - \beta} \log(n + g + \delta) \\ &\quad + e^{-\lambda\tau} \log y(t_1) + \left(1 - e^{-\lambda\tau}\right) \log A(0) + g(t_2 - e^{-\lambda\tau}t_1) \end{aligned} \quad (7)$$

This equation is the basis for estimating growth regressions in discrete time, as derived from a continuous time Solow growth model. Adding an error term  $\nu_{it}$  with mean zero conditional on all the right-hand side variables, capturing inherent randomness in  $\log y_{it}$ , we can rewrite equation (7) as a fixed-effects panel data regression of the form:

$$\begin{aligned} \log y_{it} &= \gamma_0 + \gamma_1 \log s_{k,it-\tau} + \gamma_2 \log s_{h,it-\tau} + \gamma_3 \log(n + g + \delta)_{it-\tau} \\ &\quad + \gamma_4 \log y_{it-\tau} + \mu_i + \eta_t + \nu_{it} \end{aligned} \quad (8)$$

where  $t$  denotes the end of a time period of duration  $\tau$  and  $t - \tau$  denotes the beginning of that period.<sup>9</sup> The reduced form parameters and error terms

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<sup>8</sup>For example, if  $\lambda = 0.10$ , then the half-life of convergence to the steady state will be approximately  $\log(2)/0.10 = 0.69/0.10 = 6.9$  years.

<sup>9</sup>In our actual empirical application of equation (8) the determinants of the steady-state level of income  $\log s_k$ ,  $\log s_h$ , and  $\log(n + g + \delta)$  are entered as averages over the period  $t - \tau$  to  $t$ , rather than their beginning of period values. This is consistent with the common practice of growth regressions, as in MRW and Islam, where introducing right-hand side variables as period averages is thought to limit the extent of classical measurement error. Note that theory gives us no guidance on this choice, as it considers these regressors to be exogenous and time invariant.



are defined as:

$$\begin{aligned}
\gamma_1 &= (1 - e^{-\lambda\tau}) \frac{\alpha}{1-\alpha-\beta} \\
\gamma_2 &= (1 - e^{-\lambda\tau}) \frac{\beta}{1-\alpha-\beta} \\
\gamma_3 &= - (1 - e^{-\lambda\tau}) \frac{\alpha+\beta}{1-\alpha-\beta} \\
\gamma_4 &= e^{-\lambda\tau} \\
\gamma_0 + \mu_i &= (1 - e^{-\lambda\tau}) \log A_i(0) \quad (\text{an intercept plus a country effect}) \\
\eta_t &= g(t - e^{-\lambda\tau}(t - \tau)) \quad (\text{a time specific effect}) \\
\nu_{it} & \quad (\text{a zero-mean error term, orthogonal to the regressors})
\end{aligned}$$

Equation (8) is the functional form used as the data-generating process for the remainder of this paper. In what follows we will sometimes find it useful to rewrite equation (8) as:

$$\log y_{it} = \gamma' x_{it} + \mu_i + \eta_t + \nu_{it} \quad (9)$$

where we define  $x'_{it} = [1, \log s_{k,it-\tau}, \log s_{h,it-\tau}, \log (n + g + \delta)_{it-\tau}, \log y_{it-\tau}]$  and  $\gamma' = [\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4]$ , with the dimension of these vectors denoted  $Q = 5$ .

## 2.2 Country-level heterogeneity

The  $A(0)$  term constitutes a stumbling block for growth regressions. This term captures the initial level of technology, which can be proxied for using variables such as resource endowments, climate, institutions, government type, and so on. These variables vary widely across countries, so that we can index  $A(0)$  by  $i$ . Hence, we define  $\gamma_0 + \mu_i \equiv (1 - e^{-\lambda\tau}) \log A_i(0)$ , where  $\gamma_0$  is a constant capturing the average level of the initial technology term across countries and  $\mu_i$  is a zero-mean country-specific effect. There have been three basic ways of dealing with country-level heterogeneity (i.e. the  $\mu_i = (1 - e^{-\lambda\tau}) \log A_i(0) - \gamma_0$  term) in estimating growth regressions. These methods are associated with the contributions of MRW (1992), Islam (1995) and Caselli, Esquivel and Lefort (1996), respectively.

**MRW (1992) and Islam (1995)** MRW assumed the  $\mu_i$  term had mean zero conditional on the other right hand side variables. As a result, they ran simple OLS regressions of growth on the log of initial income and time-averaged steady-state determinants (i.e. a single cross-section), including an intercept in the regression to account for the mean  $\gamma_0$  of the  $(1 - e^{-\lambda\tau}) \log A_i(0)$  term. A major drawback of this approach is that it

causes the estimated coefficients to be biased if the orthogonality assumption is untrue - which is likely to be the case in practice.

Assume that we want to estimate the parameters of the panel data regression model of equation (9):

$$\log y_{it} = \gamma' x_{it} + \mu_i + \varepsilon_{it} \quad (10)$$

where  $\mu_i$  is *not* assumed to be independent from  $x_{it}$  and  $\varepsilon_{it} = \eta_t + v_{it}$  is a well-behaved random-noise term.

If a pooled OLS (POLS) regression is run on the stacked data, standard omitted variables bias will result:<sup>10</sup>

$$\text{plim } \hat{\gamma}^{POLS} = \gamma + \Sigma_{xx}^{-1} \Sigma_{\mu x} \quad (11)$$

Define  $X$  as the  $NT \times Q$  matrix that stacks  $x_{it}$  over time periods  $t = 1 \dots T$  and countries  $i = 1 \dots N$ , and  $\mu$  as the  $NT \times 1$  vector that similarly stacks  $\mu_i$ . Then, in equation (11),  $\Sigma_{xx}$  refers to the  $Q \times Q$  covariance matrix of columns of  $X$ , and  $\Sigma_{\mu x}$  is the  $Q \times 1$  vector of the covariances of  $\mu$  with the columns of  $X$ .

Equation (11) implies that slope estimates will be biased if the country-specific effect is correlated with the regressors. In our particular application, the Solow model states that the omitted term captures some positive multiple of the initial level of technology  $\log A_i(0)$ . The observed data in  $X$  are initial income, rates of human and physical capital accumulation and population growth. While the Solow model strictly speaking is silent about the correlation between the  $\log A_i(0)$  term and the right-hand side variables, there is a strong presumption that these four variables will be potentially highly correlated with  $\log A_i(0)$ . Hence, estimated coefficients could be significantly biased when the correlation between  $\log A_i(0)$  and the steady-state determinants is ignored.

This is the point originally made by Islam (1995) in advocating the use of fixed-effects estimation instead of OLS on country means. Islam averaged annual data from the available sample of countries across time, into 5-year periods.  $\mu_i$  is a time-invariant effect if  $\lambda$  is treated as a constant and  $\tau$  does not vary with time (i.e. the panel involves equally spaced periods). Hence, it can be represented as a country-fixed effect in a panel regression, while the term  $g(t - e^{-\lambda\tau}(t - \tau))$  is a time effect. Using a fixed-effects estimator,

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<sup>10</sup>This bias is also known as heterogeneity bias. In what follows, we will use the terms heterogeneity bias and omitted variables bias interchangeably.

Islam found the estimated rate of convergence to be much higher than had been estimated by MRW, and the effect of some right-hand side variables smaller (particularly human capital).

**Caselli, Esquivel and Lefort (1996).** Going one step further, Caselli, Esquivel and Lefort (1996, henceforth CEL) pointed out the *necessary* correlation between the country-specific effect  $\mu_i$  and the log of initial income resulting from the dynamic nature of the specification. The strong presumption that  $A_i(0)$  will be correlated with some right-hand side variable is in fact a necessity. Note that we can rewrite equation (9) as:

$$\log y_{it} = \gamma_0 + \gamma'_s w_{it} + \gamma_4 \log y_{it-\tau} + \mu_i + \eta_t + v_{it} \quad (12)$$

where  $\gamma'_s = [\gamma_1, \gamma_2, \gamma_3]$  and  $w_{it} = [\log s_{k,it-\tau}, \log s_{h,it-\tau}, \log (n + g + \delta)_{it-\tau}]$ . Lagging equation (12) by one period, it is evident that  $\log y_{it-\tau}$  contains  $\mu_i$ . Thus,  $\log y_{it-\tau}$  *must* be correlated with the error term unless  $\mu_i$  is appropriately accounted for.

CEL transformed all variables used in the regressions into deviations from period means (thereby removing the need for a time-specific intercept  $\eta_t$ ) and then eliminate the country-specific effects  $\mu_i$  by taking first-differences. Their transformed regression is:

$$\begin{aligned} \widetilde{\log y_{i,t}} - \widetilde{\log y_{i,t-\tau}} &= \gamma'_s (\widetilde{w_{i,t}} - \widetilde{w_{i,t-\tau}}) \\ &+ \gamma_4 \left( \widetilde{\log y_{i,t-\tau}} - \widetilde{\log y_{i,t-2\tau}} \right) + (\widetilde{v_{i,t}} - \widetilde{v_{i,t-\tau}}) \end{aligned} \quad (13)$$

where " $\widetilde{\phantom{x}}$ " denotes deviations of variables from period means. The problem with this specification is that, while  $\mu_i$  and  $\eta_t$  have been differenced away, the term  $\widetilde{\log y_{i,t-\tau}}$  is clearly not independent from  $\widetilde{v_{i,t-\tau}}$ .<sup>11</sup> Hence, some sort of instrumental variables approach is called for. CEL proposed a GMM estimator similar to the Arellano and Bond (1991) estimator to deal with the problems of heterogeneity bias and endogeneity of the differenced lagged income term in equation (13). Their estimator results in a consistent estimator for the unknown parameters under the moment condition  $E[\widetilde{v_{i,t}} \widetilde{v_{i,t-\tau}}] = 0$ . They instrument for the differenced independent variables using all predetermined independent variables (in levels). For example, their panel consists of four time-periods, and their variables  $(\widetilde{\log y_{i,1}} - \widetilde{\log y_{i,0}})$  and  $(\widetilde{w_{i,1}} - \widetilde{w_{i,0}})$  for period 1 are instrumented using  $y_{i,0}$  and  $w_{i,0}$ . Then,

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<sup>11</sup>We will refer to this source of bias as endogeneity bias, to differentiate it from heterogeneity bias, resulting when country-specific effects are not differenced away completely.

$(\widetilde{\log y_{i,2}} - \widetilde{\log y_{i,1}})$  and  $(\widetilde{w_{i,2}} - \widetilde{w_{i,1}})$  are instrumented using  $y_{i,0}$ ,  $y_{i,1}$ ,  $w_{i,0}$  and  $w_{i,1}$  and so on. The exclusion of the current period  $\widetilde{w_{i,t}}$  term from the list of instruments is meant to deal with the possible endogeneity of the variables in  $w_{it}$ , a valid procedure under the assumption that all the instrumental variables are predetermined.<sup>12</sup> Consistent estimates will result even in the presence of measurement error on the right-hand side variables, as long as the instruments are not correlated with the errors in measurement, for example if these are white noise (as in the classical case).

Estimators in the class of Arellano and Bond's GMM estimator may have an advantage since they address several problems with the cross-sectional approach to growth regressions. However, they require losing at least two periods of data in order to implement the IV procedure, which could affect estimates in an unknown direction when  $T$  is small (in our case, we group observations into only eight time periods).

Another recently identified drawback of the Arellano-Bond method is the problem of weak instruments: the first stage relationship between differenced independent variables and lagged level variables may be weak, biasing GMM estimates towards their fixed-effects counterparts. There is now a sizable literature on this point. For example, Stock, Wright and Yogo (2002) show that in a two-stage least squares (2SLS) context, if the instruments in the first stage do not help at all in predicting the endogenous regressors, 2SLS reduces to OLS.<sup>13</sup> Staiger and Stock (1997) provide a rule of thumb for determining whether instruments are weak in the linear IV case with one endogenous regressor: if the first stage F-test for the joint significance of the instruments is smaller than 10, then the instruments are declared to be weak. For the case of multiple endogenous regressors, Stock and Yogo (2003) propose using the Cragg-Donald (1993) test statistic for underidentification, but using appropriately corrected critical values in order to use the statistic for a test of the null hypothesis of weak instruments. We apply this test in Section 3 in order to assess whether the Arellano-Bond estimates of the

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<sup>12</sup>The Solow model, however, treats the  $w_{it}$  variables as exogenous, so the endogeneity of  $w_{it}$  should not be a problem within the strict confines of the model. Moreover, whether these variables are in fact predetermined is subject to debate.

<sup>13</sup>Stock, Wright and Yogo (2002) define the concentration parameter  $\mu^2$  as a measure of goodness of fit of the first stage regression, or equivalently a measure of the strength of the instruments. They state that: "When  $\mu^2 = 0$  (...), the instruments are not just weak, but irrelevant. In this case, the mean of the 2SLS estimator is the probability limit of the ordinary least squares (OLS) estimator,  $\text{plim}(\hat{\beta}^{OLS})$ . (...) When the instruments are relevant but weak, the 2SLS estimator is biased toward  $\text{plim}(\hat{\beta}^{OLS})$ ." (p. 519).

Solow model are subject to the weak instruments problem.

The three estimators discussed here have their own strengths and weaknesses. However, one topic that has received little attention in the previous literature is what happens to these estimators in the presence of measurement error.

## 2.3 Measurement Error and Heterogeneity Bias

### 2.3.1 Measurement error without heterogeneity bias

In this subsection, we examine what happens once we allow for measurement error in the independent variables. In order to deal with various sources of bias one by one, we will ignore for the moment the omitted variables and endogeneity problems identified above, and return to them later. In a univariate setting, when measurement error is white noise, pooled OLS estimates will exhibit attenuation bias. However, in the more general multivariate case, it is impossible to sign the effect of measurement error on the slope estimates.

Klepper and Leamer (1984) point out that in the presence of classical measurement error in a multivariate context, few substantive restrictions can be placed on the sign and magnitude of the resulting bias unless stringent assumptions are made. They consider a regression model where a dependent variable, in our notation  $\log y_{it}$ , is drawn from a normal distribution with mean  $\gamma'x_{it}$  (where  $x_{it}$  is a  $Q \times 1$  column vector) and variance  $\sigma_\varepsilon^2$  conditional on  $x_{it}$ . Thus:

$$\log y_{it} = \gamma'x_{it} + \varepsilon_{it} \quad (14)$$

where  $\varepsilon_{it} = \eta_t + v_{it}$  is a noise term with  $E[\varepsilon_{it}|x_{it}] = 0$ . They assume that  $x_{it}$  cannot be perfectly observed, but rather that we can only observe  $x_{it}^* = x_{it} + d_{it}$ , where  $E[d_{it}|x_{it}] = 0$ .<sup>14</sup> Define:

$$\text{var}[d_{it}|x_{it}] = D = \text{diag}\left\{\sigma_{d_0}^2, \sigma_{d_1}^2, \dots, \sigma_{d_{Q-1}}^2\right\} \quad (15)$$

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<sup>14</sup>In our simulations, we will consider alternative specifications for the form of measurement error. Note that in this section we are abstracting from measurement error in the dependent variable  $\log y_{it}$ . It is well-known that, in the classical measurement error case, this would be equivalent to raising the variance of the error term  $\nu_{it}$ , reducing the efficiency of the estimates without introducing bias. In our simulations, we will need to consider measurement error in  $\log y_{it}$  explicitly because of the dynamic nature of the Solow specification ( $\log y_{it-\tau}$  appears on the right-hand side of the regression). Again, we are ignoring this dynamic characteristic for now.

In our application, we will set  $\sigma_{d_0}^2 = 0$ , i.e. we will not shock the intercept column in  $X$ . Defining  $\bar{x} \equiv E(x_{it})$ ,  $(\log y_{it}, x_{it}^*)$  has a multivariate normal distribution with moments:

$$E(\log y_{it}, x_{it}^*) = (\gamma' \bar{x}, \bar{x}) \quad (16)$$

$$V(\log y_{it}, x_{it}^*) = \begin{bmatrix} \sigma_\varepsilon^2 + \gamma' \Sigma_{xx} \gamma & \gamma' \Sigma_{xx} \\ \Sigma_{xx} \gamma & \Sigma_{xx} + D \end{bmatrix} \quad (17)$$

Now, performing a pooled OLS regression of  $\log y_{it}$  on  $x_{it}^*$ , we can easily show:

$$\text{plim}(\hat{\gamma}^{POLS}) = (\Sigma_{xx} + D)^{-1} \Sigma_{xx} \gamma \quad (18)$$

where  $X^*$  stacks the  $x_{it}^*$  vectors over time and countries and  $y$  does the same for  $\log y_{it}$ . Obviously, if  $D = 0$ , then pooled OLS will produce a consistent estimator. But in the generic case where this assumption does not hold, the estimator will be inconsistent. We cannot say anything more about the sign of the bias unless we can make assumptions about the correlation structure among the various independent variables, and the covariance matrix of measurement error. That is, if we can place restrictions on  $(\Sigma_{xx} + D)$  and  $\Sigma_{xx}$ , based on some knowledge of the nature of the measurement error, then we may be able to make statements on the sign and magnitude of the bias. This is in general a tall order.

### 2.3.2 Measurement error with heterogeneity bias

Assume now that the conditional expectation of  $\log y_{it}$  is  $\gamma' x_{it} + \mu_i$  where  $\mu_i$  is unobserved and not necessarily orthogonal to the variables in  $x_{it}$ .<sup>15</sup> Thus, the true model is:

$$\log y_{it} = \gamma' x_{it} + \mu_i + \varepsilon_{it} \quad (19)$$

where all the variables are still defined as above. In this case, estimating the model using OLS will involve two separate problems: 1) An omitted variables bias problem due to the fact that  $\mu_i$  is potentially correlated with the right hand side variables in  $x_{it}$ . 2) A measurement error problem due to the fact that  $x_{it}$  is imperfectly observed. As argued above it is already difficult to make statements about the sign and magnitude of the bias when only measurement error is present. Such statements become even more difficult

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<sup>15</sup>In the remainder of this section we continue to ignore the problem that arises from using a fixed-effects estimator in the presence of a lagged dependent variable in  $x_{it}$  - i.e. the problem identified in the discussion following equation (13). We will however take this issue into account in our simulations, which explicitly model the dynamic nature of the empirical Solow model.

when correlated country effects are considered in addition. To illustrate this formally, we can derive the probability limit of the pooled OLS estimator in the presence of both measurement error and heterogeneity bias.

The unconditional expectations of  $\log y_{it}$ ,  $x_{it}^*$  and  $\mu_i$  are:

$$E(\log y_{it}, x_{it}^*, \mu_i) = (\bar{x}'\gamma + \bar{\mu}, \bar{x}, \bar{\mu}) \quad (20)$$

where  $\bar{\mu} = E(\mu_i)$  and  $\bar{x} = E(x_{it}) = E(x_{it}^*)$ , and the variance of  $\mu_i$  is denoted  $\sigma_\mu^2$ . Then:

$$V(\log y_{it}, x_{it}^*, \mu_i) = \begin{bmatrix} \sigma_\varepsilon^2 + \gamma'\Sigma_{xx}\gamma + 2\gamma'\Sigma_{\mu x} + \sigma_\mu^2 & \gamma'\Sigma_{xx} + \Sigma'_{\mu x} & \gamma'\Sigma_{\mu x} + \sigma_\mu^2 \\ \Sigma_{xx}\gamma + \Sigma_{\mu x} & \Sigma_{xx} + D & \Sigma_{\mu x} \\ \gamma'\Sigma_{\mu x} + \sigma_\mu^2 & \Sigma'_{\mu x} & \sigma_\mu^2 \end{bmatrix} \quad (21)$$

Suppose that we estimate  $\gamma$  using pooled OLS, with  $x_{it}^*$  as our observed regressor. Then the limiting value of the pooled OLS estimator for  $\gamma$  is:

$$\text{plim } \hat{\gamma}^{OLS} = (\Sigma_{xx} + D)^{-1} \Sigma_{xx}\gamma + (\Sigma_{xx} + D)^{-1} \Sigma_{\mu x} \quad (22)$$

Obviously, this estimator is inconsistent, for two reasons: the first is measurement error bias, and the second is heterogeneity bias. If  $D = 0$ , we would recover equation (11), showing that pooled OLS would not involve measurement error bias. On the other hand, if  $\Sigma_{\mu x} = 0$ , we would recover equation (18), showing that pooled OLS regression would not involve any heterogeneity bias. If neither of these two issues were a problem, pooled OLS would be a consistent estimator for  $\gamma$ .

In a context where both problems coexist, there may be a trade-off between reducing the extent of bias due to measurement error and reducing the bias attributable to heterogeneity. The common way to deal with heterogeneity, as explained above, is to estimate  $\gamma$  using the fixed-effects (FE) estimator  $\hat{\gamma}^{FE}$ . Appendix 1 derives the limiting value of  $\hat{\gamma}^{FE}$  in the presence of measurement error, showing that it gets rid entirely of heterogeneity bias. It also derives the limiting value of the between (BE) estimator  $\hat{\gamma}^{BE}$  obtained by computing country means of the data over time and running OLS regressions on these country means. It is difficult to make general statements about what happens to the bias from measurement error under FE and BE estimation in the multivariate case. Measurement error bias when using FE may or may not be exacerbated. This is the main justification for a Monte Carlo approach to evaluating the properties of these estimators.<sup>16</sup>

<sup>16</sup>In subsection 2.3.3 below, and in the appendix, we discuss conditions under which

There are, however, a few things we can say when comparing the properties of pooled OLS, BE and FE estimators.<sup>17</sup> Appendix 1 derives the probability limits of the latter two estimators:

$$\text{plim } \hat{\gamma}^{BE} = \left( \Sigma_{xx}^B + \frac{1}{T}D \right)^{-1} \Sigma_{xx}^B \gamma + \left( \Sigma_{xx}^B + \frac{1}{T}D \right)^{-1} \Sigma_{\mu x} \quad (23)$$

$$\text{plim } \hat{\gamma}^{FE} = \left( \Sigma_{xx}^W + \frac{T-1}{T}D \right)^{-1} \Sigma_{xx}^W \gamma \quad (24)$$

where  $\Sigma_{xx}^B$  denotes the between country variation in  $X$ ,  $\Sigma_{xx}^W$  denotes the within country variation (as defined in the Appendix 1), and  $\Sigma_{xx} = \Sigma_{xx}^W + \Sigma_{xx}^B$ . Several observations are in order concerning equations (22), (23) and (24):

1). As noted above, FE gets rid entirely of the heterogeneity bias while there is in general both measurement error and heterogeneity bias when using the BE and pooled OLS estimators.

2). The BE estimator tends to reduce the extent of measurement error bias compared to the other estimators due to the averaging of the imperfectly measured variables over time, which reduces the variance of the measurement error relative to the true signal. Moreover, the greater is  $T$ , the smaller the bias from measurement error.

3). Both pooled OLS and BE will involve smaller heterogeneity bias, the greater the extent of measurement error. In other words, there is an interaction between these two sources of bias. This is because all other things equal, measurement error reduces the correlation between the regressors and the country effects, and hence alleviates the omitted variables problem. For this reason, BE will on average involve larger heterogeneity bias compared to pooled OLS, holding constant  $D$ .

4) Comparing FE and BE, if  $\Sigma_{xx}^W$  relative to  $\frac{T-1}{T}D$  is "smaller" in a matrix sense than  $\Sigma_{xx}^B$  is relative to  $\frac{1}{T}D$ , then the bias arising from measurement error will tend to be smaller under BE compared to FE. This is likely to be the case if the within variation is small compared to the between variation (most of the variation in the panel arises from the cross-section

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measurement error is exacerbated under FE in the specific case of univariate regression. We argue these conditions are likely to hold in our particular application, as in most. But only a Monte Carlo simulation can provide definitive answers.

<sup>17</sup>Appendix 1 also derives the plim of the random effects (RE) estimator  $\hat{\gamma}^{RE}$ , which is a matrix-weighted average of the BE and FE estimator.



rather than the time dimension - which is the case in growth applications), or if  $T$  is large.

To conclude, despite these general lessons, we can say little about the net biases to individual parameter estimates as they would result from each estimation method. Given the multivariate nature of growth regressions, only simulations can determine which estimator dominates in terms of bias under alternative assumptions about the covariance structure of the true data  $\Sigma_{xx}$ , the covariances between the true variables and the country-specific effects  $\Sigma_{\mu x}$  and the covariance matrix of the measurement error  $D$ .

### 2.3.3 Autocorrelated Measurement Error: Univariate Example

Appendix 2 analyzes in detail a simple case illustrating the trade-offs identified above in a case where net biases can be signed: the case of  $Q = 2$  (a single regressor  $x_{it}$  plus an intercept). This example is also useful to illustrate what happens when measurement errors are autocorrelated.

The example shows clearly that, under FE estimation, eliminating heterogeneity bias may come at the cost of exacerbating measurement error bias. The greater the time persistence in  $x_{it}$ , the greater the extent to which measurement error bias is exacerbated, as the variance of the true signal gets differenced away relative to the variance of the error in measurement. In the context of growth regressions, where right-hand side variables are highly time persistent, this point is particularly central. We cannot say analytically whether this increase in measurement error bias is worth the elimination of heterogeneity bias unless we know the moments of the true underlying data and of the measurement error.

As argued in Section 2.2, a GMM procedure such as Arellano and Bond's (AB) could in principle deal with both sources of bias if measurement error is white noise, since measurement error in the instruments will be uncorrelated with measurement error in the regressors. However, introducing measurement error weakens the first stage relationship between predetermined regressors and the instruments, potentially making the weak instruments problem worse. Moreover, the validity of this procedure relies heavily on the assumption of non-autocorrelated errors in measurement.

Appendix 2 shows that when measurement error is autocorrelated, where we define  $\rho_d = \text{corr}(d_{it}, d_{it-\tau})$ , FE will exacerbate measurement error bias compared to pooled OLS whenever  $\rho_d < \rho_x$ , where  $\rho_x$  is the autocorrelation coefficient in  $x_{it}$ . In this case, instrumenting for differenced  $x_{it}$  using

its lagged levels values (as in the AB procedure) will no longer get rid of measurement error bias. In words, as long as  $\rho_d > 0$ , we cannot produce a consistent estimator of the desired parameters using AB's GMM approach. We will consider the case of autocorrelated measurement error in our simulations.

## 2.4 Summary

Four factors can cause inconsistent estimates of  $\gamma$  in panel growth regressions. The first is an omitted-variables bias resulting from the possible correlation between country-specific effects and the regressors, affecting the consistency of pooled OLS, BE and RE estimates. The second is the endogeneity problem specific to dynamic panels, identified after equation (13), which will make FE and RE estimates inconsistent.<sup>18</sup> The third is classical measurement error on the independent variables, which affects the consistency of pooled OLS, BE, RE and FE estimator, though the bias tends to be exacerbated in the latter case and partly averaged away under BE. The fourth is possible autocorrelation in measurement errors, which results in inconsistency for all estimators we consider here including the AB estimator. In addition the AB estimator may be subject to a weak instruments problem.

Each of the estimators under consideration involves a trade-off: pooled OLS suffers from heterogeneity bias but limits the incidence of measurement error bias relative to FE; the BE estimator reduces measurement error through time averaging of the regressors, but does not deal with heterogeneity bias; FE addresses the problem of heterogeneity bias, but tends to exacerbate the problem of measurement error. The AB estimator will be inconsistent when instruments are weak or when measurement error is autocorrelated. As a result, we cannot say a priori which estimator will produce the smaller total bias. Simulations are necessary to evaluate the properties of these estimators.

## 3 Monte Carlo Simulations

### 3.1 Simulation Methodology

Since it is impossible to derive analytical results about the extent and sign of omitted variables and measurement error biases in a multivariate context,

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<sup>18</sup>For the sake of space and because this source of bias is well-known, we have abstracted from it in this subsection, but we will take it into account explicitly in our simulations.

we use Monte Carlo simulations to evaluate the bias properties of of FE, BE, RE and AB estimators.<sup>19</sup>

The starting point for our simulations is equation (8). The data-generating process for the true data (that is, the data that is not measured with error) is:

$$\begin{aligned} \log y_{it} = & \left(1 - e^{-\lambda\tau}\right) \frac{\alpha}{1 - \alpha - \beta} \log s_{k,it-\tau} + \left(1 - e^{-\lambda\tau}\right) \frac{\beta}{1 - \alpha - \beta} \log s_{h,it-\tau} \\ & - \left(1 - e^{-\lambda t}\right) \frac{\alpha + \beta}{1 - \alpha - \beta} \log (n + g + \delta)_{it-\tau} + e^{-\lambda t} \log y_{it-\tau} \\ & + \left(1 - e^{-\lambda\tau}\right) \log A_i(0) + g(t - e^{-\lambda\tau}(t - \tau)) + \nu_{it} \end{aligned} \quad (25)$$

### 3.1.1 Simulated Data

**Underlying data.** We define a period by a five year interval of time (i.e.  $\tau = 5$ ). Our underlying data spans 40 years, from 1960 to 2000, and our 8 five year periods are defined as 1960-1965, 1965-1970, ..., 1995-2000. In equation (25), the variables  $\log s_{k,it-\tau}$ ,  $\log s_{h,it-\tau}$ ,  $\log(n + g + \delta)_{i,t-\tau}$  and  $\log y_{it-\tau}$  are simulated data with moments resembling those of the corresponding observed variables. To obtain these moments, we captured  $\log s_k$  using the log of investment rates as a share of real GDP from the Penn World Tables, version 6.1 (Heston, Summers and Aten, 2002 - henceforth, PWT6.1).  $\log s_h$  is the log of the secondary school gross enrollment ratio from Barro and Lee (2000) and  $n$  is the rate of population growth calculated from the PWT6.1 population series. In calculating  $\log(n + g + \delta)$ , we postulated (as above) that  $g + \delta = 0.07$ . Finally,  $\log y_{it-\tau}$  is the log of per capita income in purchasing power parity from PWT6.1, measured at the beginning of the first time period (1960).<sup>20</sup>

We averaged the variables over each of these time periods and arrayed them in a  $N \times (T(Q - 1) + 1)$  matrix. Specifying each variable at separate time periods instead of stacking them over time allows us to simulate explicitly their time persistence characteristics. Finally, since our underlying data

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<sup>19</sup>Results for pooled OLS estimates are available upon request. However, the pooled OLS estimator is rarely used in cross-sectional growth regressions since it is less efficient than random effects.

<sup>20</sup>Since the model is dynamic, subsequent values of the initial income term  $\log y_{it-\tau}$  will be generated by iterating on income using the Solow specification, starting from a drawn value for the first period. We will, however, calibrate the parameters so that subsequent generated values of the income variables bear characteristics resembling those of the corresponding real data, as explained below.

was available in all periods for 69 countries, and we are seeking a balanced panel, we set  $N = 69$  for the rest of this study.

We assumed that these variables are measured without error. In other words, we take their resulting first and second moments to be those of the "true" variables, which we will later shock by adding a white noise measurement error. As the observed data surely incorporates classical measurement error, we will be understating the magnitude of the covariances among the underlying "true" variables and overstating their variances.

**Simulating the fixed effects term  $\mu_i$ .** One difficulty we face is simulating the country fixed effects  $\mu_i = (1 - e^{-\lambda\tau}) \log A_i(0) - \gamma_0$ .<sup>21</sup> Theory provides no guidance as to the values of  $A_i(0)$  for different countries, since it is taken as exogenous. The problem is important because the covariance structure linking the country-specific effects to the observed regressors determines how much heterogeneity bias will be present. To obtain simulated fixed effects and their corresponding covariance structure with the right-hand side variables, we used our observed panel data set to run an FE regression on the specification in equation (25). We computed the fitted fixed effects from this regression. We then used this series and treated it as an additional variable, as if it were observed, to generate the moments of the simulated data.

Obviously, given that the underlying data must incorporate measurement error, this procedure will lead to biased estimates of the country-specific effects, as discussed in Section 2. If this is the case, the moments of  $\mu_i$  and especially its covariance structure with the other right-hand side variables will also be flawed. Since this is the case, we will also present results with alternative assumptions about the covariance structure between the simulated fixed effects and the simulated regressors in  $x_{it}$ : in our simulations we will also set the correlation between  $\mu_i$  and the observed regressors to values other than those implied by the fitted country-effects.

**Moments of the underlying data.** Table 1, panel A presents the matrix of correlations among our  $Q + 1$  variables of interest in the pooled data used to generate our simulated datasets. For example, once stacked over time and countries,  $\log s_{k,it}$  bears a correlation of 0.60 with  $\log s_{h,it}$ . The

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<sup>21</sup>The time fixed effect  $g(t - e^{-\lambda\tau}(t - \tau))$ , which is identical for all countries at each date, was generated for each period  $t$  simply by setting the parameters  $g$  and  $\lambda$  to their assumed values, and  $\tau = 5$ .

estimated country-specific effect bears high correlations with the right hand side variables, suggesting a big scope for heterogeneity bias when using estimators that do not correct for it. For instance, the correlation between initial income  $\log y_{i,t-\tau}$  and  $\mu_i$  is 0.93.<sup>22</sup> Panel B isolates the between correlations among these variables, by taking time means  $(\bar{x}_i, \bar{\mu}_i)$  of all the variables (where obviously  $\bar{\mu}_i = \mu_i$ ) and computing their correlation matrix. The between correlations are quite close to the pooled data correlations, suggesting that cross-sectional variation dominates in our data. For example, the between correlation between  $\log s_{k,it}$  and  $\log s_{h,it}$  is 0.72. Finally, Panel C displays the within correlations, obtained by computing  $(x_{it} - \bar{x}_i, \mu_i - \bar{\mu}_i)$ . These correlations are always much lower than either the pooled or between correlations, again suggesting that the cross-country variation dominates in the pooled data. For example, the within correlation between  $\log s_{k,it}$  and  $\log s_{h,it}$  is 0.21.

**Draws of simulated data.** We are now in the presence of  $N$  observations for  $T(Q - 1) + 2$  variables.<sup>23</sup> We computed the  $(T(Q - 1) + 2) \times 1$  vector of means for these variables, denoted  $\hat{m}_{x,\mu}$  and their variance covariance matrix, denoted  $\hat{\Omega}_{x,\mu}$ .<sup>24</sup> Stacking the data in this way (in wide format) allows us to provide a realistic simulation of the relative weights of between and within variations - by specifying explicitly the autocorrelation structure of the right-hand side variables in addition to their cross-correlations. For each run of our simulation, we then drew  $N$  observations for the  $T(Q - 1) + 2$  variables from a multivariate normal distribution with mean  $\hat{m}_{x,\mu}$  and variance  $\hat{\Omega}_{x,\mu}$ .

The last part of the data generation procedure is to simulate the residuals  $\nu_{it}$ . We opted to let the variance of the residual differ across time periods, and the residuals covary across time periods. To do this, we generated the fitted residuals from the fixed-effects regression using observed data for each period, and arrayed them in a  $N \times T$  matrix. We computed their  $T \times T$  covariance matrix  $\hat{\Omega}_\nu$ . Finally, we generated  $N$  sets of  $T$  normally distributed residuals with mean zero and covariance matrix  $\hat{\Omega}_\nu$ . An interesting aspect

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<sup>22</sup>For the sake of illustration, in Table 1 we used every 5-year time-interval observation between 1960 and 1995 for the real data on  $\log y_{it-\tau}$ . In contrast, in our simulations, we are generating  $\log y_{it-\tau}$  from the model, for all but the first period - due to the dynamic nature of equation (25). The generated data on  $\log y_{it-\tau}$  and their observed counterparts are very closely correlated (these correlations are available upon request).

<sup>23</sup>i.e.  $N$  observations per period for  $\log s_{k,it-\tau}$ ,  $\log s_{h,it-\tau}$ ,  $\log(n + g + \delta)_{i,t-\tau}$ ,  $N$  observations for  $\log y_{it-\tau}$  in 1960 and  $N$  observations on the time invariant country effects.

<sup>24</sup> $\hat{m}_{x,\mu}$  is a  $(T(k - 1) + 2) \times 1$  vector, while  $\hat{\Omega}_{x,\mu}$   $(T(k - 1) + 2) \times (T(k - 1) + 2)$  matrix.

of this exercise is that the variance of the fixed-effects estimated residual term  $\hat{\nu}_{it}$  was found to be a small fraction of the variance of  $\log y_{it}$ , on the order of 1%.

Using all the parameters and simulated data, we computed the simulated dependent variable  $\log y_{i,1965}$  for period 2, using equation (25) and the simulated data on  $\log y_{i,1960}$ . We used this generated value of  $\log y_{i,1965}$  to similarly generate  $\log y_{i,1970}$ , and so on iteratively until we obtained  $\log y_{i,2000}$ .<sup>25</sup> Formally, log income in period  $t$  for country  $i$  was simulated as:

$$\log y_{it} = \gamma_s \sum_{j=0}^{t-1} \gamma_4^j w_{i,t-j-1} + \gamma_4^t \log y_{i0} + \mu_i \sum_{j=0}^{t-1} \gamma_4^j + \sum_{j=0}^{t-1} \gamma_4^j \nu_{it-j} \quad (26)$$

### 3.1.2 Parameter Values

One issue we face is that there is no guarantee that the generated income data on income resembles in any way the underlying real world data. Equation (26) shows that simulated income is a function of past values of the steady-state determinants in  $w_{it}$ , the log of income at the beginning of the first period  $\log y_{i0}$ , the fixed effects  $\mu_i$  and a weighted sum of the current and past residuals  $\nu_{it}$ , as well as the model's reduced form parameters in  $\gamma$ . As  $t$  increases, the moments of the generated values of income might diverge more and more from those observed in the true income data.

To address this issue we calibrated the model's parameters  $\alpha$  and  $\beta$  so that the generated income variables in a typical draw of the data have moments resembling those of the observed variables. We found that we did not need to diverge greatly from commonly assumed values of  $\alpha$  and  $\beta$  to obtain a good calibration: in a typical draw of the data, setting  $\alpha = \beta = 0.27$  delivers moments of generated income variables that look similar to those seen in the PWT6.1 data.<sup>26</sup> These variables are conventionally both set to  $1/3$  in the context of the Solow model (as discussed for instance in Barro and Sala-i-Martin, 1995).

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<sup>25</sup>There are several reasons for implementing a dynamic method for simulating income rather than treating  $\log y_{it-\tau}$  on the right hand side of equation (25) in the same way as we treat steady-state determinants. First, internal consistency requires that income be modelled in conformity with the dynamics of the Solow model. Second, this dynamic method will allow us to isolate and quantify the extent of the endogeneity bias arising under fixed effects, as identified by CEL and discussed above in section 2.2. Third, this is computationally required for the implementation of the Arellano and Bond estimator.

<sup>26</sup>Details of our calibration exercise, including a detailed comparison of the moments of the generated data with those of the observed data, are available upon request.

The other parameters of the structural model,  $g$ ,  $\delta$  and  $n$  are set to their conventional values as in Barro and Sala-i-Martin (1995):

$$g = 0.02; n = 0.01; \delta = 0.05$$

These parameters imply a convergence parameter  $\lambda = (n + g + \delta)(1 - \alpha - \beta) = 3.68\%$ , which is slightly higher than the value of 2.67% implied by the conventional values of  $\alpha$  and  $\beta$ .<sup>27</sup> With these assumed structural parameters and  $\tau = 5$ , the implied reduced form parameters are as follows:

$$\gamma_1 \approx 0.099; \gamma_2 \approx 0.099; \gamma_3 \approx -0.197; \gamma_4 \approx 0.832$$

Note that in empirical applications of the Solow growth model, a small contradiction exists between the theoretically derived estimating equation and the linear specification actually estimated: insofar as the rate of population growth  $n$  enters the equation as a variable (in the term  $\log(n + g + \delta)$ ), then terms such as  $(1 - e^{-\lambda t})$ , where  $\lambda$  depends in part on  $n$ , should not be treated as constant.<sup>28</sup>

### 3.1.3 Measurement Error

The dataset generated above is free from measurement error. If we were to run fixed-effects regressions of  $\log y_{it}$  on  $\log s_{k,it-\tau}$ ,  $\log s_{h,it-\tau}$ ,  $\log(n + g + \delta)_{i,t-\tau}$  and  $\log y_{it-\tau}$  using repeated draws of the simulated data, the only source of bias in the fixed-effects regression would be the endogeneity problem that stems from the dynamic nature of the model, identified in the discussion that follows equation (13).<sup>29</sup> If we were to run between regressions on simulated data, we would obtain estimated coefficients tainted only by heterogeneity bias, i.e. the term  $(\Sigma_x^B)^{-1} \Sigma_{\mu x}$  in equation (23), where  $D$  is set to 0.

To evaluate the merits of various estimators used to estimate growth regressions in the presence of measurement error, we shock our simulated

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<sup>27</sup>We have rerun *all* our simulations assuming  $\alpha = \beta = 1/3$  and all the results were qualitatively unchanged. These results are available upon request.

<sup>28</sup>However, doing so is an acceptable approximation since variation in  $n$  is likely to have a small impact on variation in  $e^{-(1-\alpha-\beta)(n+g+\delta)\tau}$ , so treating  $e^{-\lambda\tau}$  as a constant is likely to be immaterial. On the other hand, variation in  $n$  will have a larger impact on variation in  $\log(n + g + \delta)$ , which justifies not treating this variable as a constant. At any rate, we follow common practice in treating  $e^{-\lambda\tau}$  as a constant and  $\log(n + g + \delta)$  as a variable.

<sup>29</sup>Additionally, if we remove the error term  $\nu_{it}$  from the model (i.e. we simulate the income data without this term), fixed effects would lead us to recover *exactly* the reduced form theoretical parameters  $\gamma$  of the model. We did this as a check on our simulation program, and the corresponding results are available upon request.

variables by adding white-noise.<sup>30</sup> This can be done in several ways. In our baseline simulations, we simply add a normally distributed, zero mean shock with variance equal to some fraction  $F < 1$  of the variance of the underlying variable (we will refer to  $F$  as the error-to-truth ratio). This was applied to simulated variables period-by-period. Formally, consider first the determinants of the steady-state level of income,  $\log s_{k,it-\tau}$ ,  $\log s_{h,it-\tau}$  and  $\log(n+g+\delta)_{i,t-\tau}$ . For independent variable  $x_q$  in period  $t$ , we compute  $x_{q,it}^*$  as:

$$x_{q,it}^* = x_{q,it} + d_{q,it} \quad (27)$$

with  $d_{q,it} \sim N(0, F\hat{\sigma}_{qt}^2)$  for all  $i$  and  $q = 2..4$ , where  $\hat{\sigma}_{qt}^2$  is the sample variance of  $x_{q,it}$  in period  $t$ . We proceeded in exactly the same way for the income variable  $\log y_{it}$ :

$$\log y_{it}^* = \log y_{it} + d_{y,it} \quad (28)$$

where  $d_{y,it} \sim N(0, F\hat{\sigma}_{yt}^2)$  for all  $i$ , where  $\hat{\sigma}_{yt}^2$  is the sample variance of  $\log y_{it}$  in period  $t$ .

In the specifications above, the variance of the measurement error can vary period-by-period insofar as the variance of the underlying true data does. In other words, the  $T(Q-1) \times T(Q-1)$  variance covariance matrix of the errors-in-variables,  $\hat{\Omega}_d$ , is diagonal, with the diagonal elements allowed to differ across regressors and time. The fraction  $F$ , however, is common to all variables in all periods. We relax some of these assumptions on measurement error in the robustness tests presented in Section 4.

The parameter  $F$  is set to four values - 0%, 5%, 10%, and 15%.<sup>31</sup> It is, of course, difficult to know what the appropriate extent of measurement error is in reality. Hence, it is essential to vary  $F$  to assess the robustness of our results. However, clues about whether our chosen range of values for  $F$  is reasonable can be obtained. Notice first that all variables are entered in logs, so even a value of  $F = 5\%$  may imply rather large shocks, especially on the underlying income variable  $y_{it}$ . Our additive term  $d_{y,it}$  translates into some multiplicative term  $e^{d_{y,it}}$  applied to  $y_{it}$  (income in 1996 PPP dollars).

We can display draws of the mismeasured variables pooled across time periods and compute the average absolute value deviation from their true (unshocked) values, as a summary measure of the extent of measurement

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<sup>30</sup>Obviously, we did not shock the fixed-effects  $\mu_i$  nor the intercept.

<sup>31</sup>As an additional check, we also did our simulations with extreme values of  $F$  - 25% and 50%. The qualitative properties of our results were unchanged, but the extent of bias quickly became unreasonably large.



error. Table 2 displays these values as well as the pooled sample averages of the underlying "true" (unshocked) variables, for comparison. To construct Table 2, we drew simulated data for 2,000 countries in the 8-period panel, i.e. 16,000 observations.

Consider first measurement error on income  $y_{it}$ . For  $F = 5\%$ , the average absolute value of measurement error was \$1,332, for  $F = 10\%$  it was \$2,134 and for  $F = 15\%$  it was \$2,703. These are to be compared to the pooled sample mean of simulated income, which is \$4,997.<sup>32</sup> The magnitudes we obtain on the other variables seem more moderate, due to the fact that their values are between 0 and 1. Consider for instance the rate of physical capital accumulation  $s_k$ : for  $F = 5\%$ , the average absolute value of measurement error was 2.03 percentage points, for  $F = 10\%$  it was 2.95 percentage points and for  $F = 15\%$  it was 3.67 percentage points. The pooled sample mean of  $s_k$  was roughly 17%. Similar relative orders of magnitudes hold for  $s_h$  and  $(n + \delta + g)$ , as shown in Table 2. While it is hard to know what the appropriate level of measurement error would be, the range of values displayed in Table 2 does not seem unreasonable.

### 3.1.4 Regressions on Simulated Data

Having generated our simulated true data and shocked it with classical measurement error, we are now in a position to evaluate the bias properties of alternative estimators in the presence of correlated country-specific effects and measurement error. We estimated equation (8) on our draw of simulated data using four estimators: fixed-effects (FE), the between estimator (BE), random effects (RE) and the Arellano-Bond estimator (AB). We stored the estimated slope coefficients from each run, and repeated this procedure 1000 times. We then computed the means of the resulting estimates, and compared those to the known true parameters. The difference between the mean estimates and the corresponding true parameters gives a measure of bias for each estimates of the slope parameters in  $\gamma$ . The average absolute value of these individual parameter biases is used as summary measures of bias across the slope elements of  $\gamma$ . Although our discussion of the results focuses on bias, the standard errors of the estimates from simulated data are also available to examine the efficiency properties of the estimators.

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<sup>32</sup>Since the average absolute value of the shock on income may seem too large, we examine in Section 4.1 what happens when we reduce the extent of measurement error on income without changing it for the other variables.

## 3.2 Baseline Simulation Results

### 3.2.1 10% Measurement Error

In our baseline case, we set the error-to-truth ratio  $F$  equal to 10% for all the right-hand side variables in the model, and the extent of the correlations between the fixed effects and the regressors is as described in Table 1. Table 3, column 3 presents the resulting estimates based on averages over 1000 runs. In terms of the average absolute value of bias on the slope parameters, our results reveal that the BE estimator dominates by a wide margin: average absolute bias is 33%, versus a value in the neighborhood of 200% for the other three estimators. As suggested by econometric theory, estimators that use the within variation exacerbate measurement error bias, and between averages it out.

Turning to individual coefficient estimates, BE tends to bias the estimate of the convergence parameter  $\gamma_4$  upward by 19% - the average simulated coefficient is 0.990 versus a true coefficient of 0.832 (the implied speeds of convergence, i.e.  $\lambda$  parameter, are respectively 3.68% and 0.2%).<sup>33</sup> In contrast, both the FE and AB severely bias this coefficient downwards, with average biases of -78% and -89% respectively, implying very high speeds of convergence (respectively 33.99% and 47.10%). In terms of the pattern of coefficients, this broadly replicates the finding of the literature - where the FE or AB estimates of the convergence speed are an order of magnitude higher than the between estimate. CEL, for example, report a speed of convergence of 10% per year based on the AB estimator - 5 times larger than the 2% cross-sectional estimate in MRW.<sup>34</sup> Our results suggest that the finding of fast convergence in the literature employing fixed-effects estimators may be traceable to the incidence of exacerbated measurement error bias.

Turning to the other slope parameters of the Solow model, interesting

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<sup>33</sup>To calculate  $\lambda$  from the estimate of  $\gamma_4$ , simply compute  $\lambda = -\log(\gamma_4)/\tau$  where  $\tau = 5$ .

<sup>34</sup>The precise extent to which  $\gamma_4$  is biased downwards when using the fixed-effects and Arellano-Bond in our simulations obviously depends on the postulated extent of measurement error and the postulated correlations between  $\mu_i$  and the elements of  $x_{it}$ . If the error-to-truth ratio is brought down to 5%, the estimated speed of convergence is brought down to 22.05% for the fixed effects estimator and to 32.50% for the Arellano-Bond estimator. These values remain higher than those reported in the literature. In Section 4.1 we discuss how to obtain more reasonable values of the fixed effects and Arellano-Bond estimates of the convergence speed by reducing the extent of measurement error on the income term  $\log y_{it}$  - Section 3.2.3 suggested that the average absolute value of the shock to  $y_{it}$  might be too high, on the order of \$2,000 for the pooled sample in the baseline simulation with  $F = 10\%$ . This is to be compared for a pooled sample mean of simulated income equal to roughly \$5000.

results also emerge from our simulations. While all the estimators involve some bias in these parameters, the direction and magnitudes of the biases differs sharply. BE tends to bias the human capital parameter slightly away from zero: the coefficient on  $\log s_{h,it}$  is biased upward by 6%. The corresponding bias when using FE is a downward bias of  $-209\%$  - the coefficient switches signs. Again, our simulation account for differences between estimators found in the literature - where FE typically lead to a coefficient estimate on the human capital accumulation variable that is closer to zero than BE. For example, Islam (1995) shows that the estimated BE coefficient on  $\log s_{h,it}$  is roughly 0.182, and equals  $-0.071$  when using FE. Our corresponding BE estimate is 0.105, and our FE estimate is  $-0.108$ . Similar comparisons would hold when we turn to AB rather than FE - the accumulation parameters are both severely biased towards zero, and the depreciation parameter  $\gamma_3$  is biased away from zero. Again, our results suggest that the finding of smaller effects of the accumulation variables in the fixed-effects literature compared to the cross-sectional literature may be largely attributable to measurement error bias.

AB estimates are very close to the FE estimates, suggesting, as detailed in Section 2, that the weak instruments problem may be prevalent in this application. To evaluate this more formally, we implemented the test of the null hypothesis of weak instruments suggested by Stock and Yogo (2003), using the real world data that serves as a basis for our simulations. This test is based on computing the Cragg-Donald (1993) statistic, a generalization of the first-stage F-test for the case of multiple endogenous regressors.<sup>35</sup> The statistic is then compared to the critical values in Stock and Yogo (2003). The critical values depend on parameter  $b$ , the maximum amount of squared bias that the researcher is willing to accept relative to squared OLS bias (in our case, FE bias). For instance, a value  $b = 0.1$  indicates that the maximal allowable bias of the IV estimates is 10% of the maximum OLS bias. In our case, the value of the Cragg-Donald statistic was 1.513, which is smaller than all critical values whatever the value of  $b$  presented in Stock and Yogo (2003) (these range from 5% to 30%). Thus, we fail to reject the null of weak instruments at the 5% significance level even when we are willing to

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<sup>35</sup>In our application we allow all the right hand side variables to be predetermined - so that all four regressors are instrumented for. In doing this we follow the practice in CEL. Strictly speaking however, in our simulation only lagged income is endogenous. See the discussion after equation (13). Formally, the Cragg-Donald statistic is the smallest eigenvalue of the matrix analog of the F-statistic from the first stage regression. See Stock and Yogo (2003).

accept a high level of GMM bias relative to FE bias.<sup>36</sup>

Finally, Table 3 also reveals that RE performs more poorly than the other three estimators when the summary measure of bias is the average absolute value of the bias (here 235% compared to 33% for the between estimator). This is significant because this estimator is frequently used by growth regression practitioners who wish to retain the panel dimension but are unwilling to discard all the between variation in the data. However, RE does quite well in estimating the convergence parameter, displaying a bias of  $-16\%$ . The other slope parameters are all biased upwards. For instance the coefficient on  $\log s_{k,it}$  is biased upward by  $107\%$ . One possible reason is that, contrary to between, RE does not average measurement error over time, nor does it address the problem of heterogeneity bias. The interaction of these two biases, which is hard if not impossible to characterize analytically, turns out to result in large net biases in this particular application.

### 3.2.2 Varying the Extent of Measurement Error

Increasing the error-to-truth ratio to  $15\%$  or reducing it to  $5\%$  does not generally change the conclusions reached above (Table 3, columns 2 and 4). As expected, the average absolute bias tends to (weakly) increase with the error-to-truth ratio for most estimators, though this is not necessarily true for individual parameter estimates. An interesting feature of our simulations is how increases in the extent of measurement error across columns of Table 3 seem to little affect the BE estimate of the coefficient on lagged income per capita - in fact the upward bias on  $\gamma_4$  remains equal to  $18 - 19\%$  whatever the value of  $F$ .

Interesting lessons can also be learnt when measurement error is shut down entirely. While unrealistic, this exercise allows us to isolate the incidence of heterogeneity bias in the BE estimator, and of endogeneity bias in the FE estimator. Table 3, column (1) presents simulation results when  $F = 0$ . As predicted, BE still tends to create an upward bias on the lagged income coefficient, an upward bias on the effect of human capital, and a small downward bias on the depreciation term. Clearly, the strong positive

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<sup>36</sup>We also implemented the Staiger and Stock (1997) rule of thumb based on the first stage F-statistics. Strictly speaking, the rule of thumb is only valid for the case of one endogenous regressor. But the values of our first stage F-statistics were sufficiently below 10 to reinforce our confidence that the weak instruments problem is important here. The F-statistics for the first-stage regression of lagged first-differenced initial income on its instruments was 4.73, and the corresponding values for savings on physical capital, savings on human capital and the depreciation term were, respectively, 4.03, 2.71 and 2.22.

correlation between the country-specific effect and the lagged income term, built into our simulations, accounts for the upward bias on  $\log y_{it-\tau}$  when country fixed effects are not included in the regression. The results suggest that most of the bias in the BE estimates seen when  $F$  is set to a value different from zero is attributable to heterogeneity bias, as the biases on individual coefficients change little as  $F$  is increased. In other words, BE does a good job at averaging away measurement error.

When  $F = 0$ , FE estimates are also biased. This is due to the endogeneity problem inherent in this type of dynamic panel. Our simulations allow us to quantify this problem. The biases are relatively small, especially on the main parameter of interest  $\gamma_4$  (biased downwards by  $-5\%$ ). This bias quickly gets swamped by measurement error bias when  $F$  is increased. The AB estimator, which is supposed to get rid of endogeneity bias, does display biases of similar magnitudes as FE. Moreover the biases on the various slope parameters are similar in signs and relative magnitudes to the FE biases. As discussed above, this is due to the weak instruments problem, which tends to bias AB estimates towards FE.

To summarize, as predicted by theory, when measurement error is not present, BE is tainted by heterogeneity bias, the other estimators perform better and the AB estimator in particular performs best. However, this case is unrealistic since we are unable to replicate the broad findings obtained across estimators with real data when setting  $F = 0$ . Even in the presence of a small amount of measurement error ( $F = 5\%$ ), large biases appear when using fixed-effects, and the between estimator asserts itself as the dominant estimator in terms of average absolute bias. Moreover, in that case we are able to broadly replicate the pattern of estimates found in the literature across estimators.

### 3.2.3 Varying the Extent of Heterogeneity Bias

We now examine how our results change when we vary the extent of heterogeneity bias, holding measurement error fixed at some baseline level. As described in Section 3.1.2, the assumed correlations between the right hand side variables and the country fixed effect  $\mu_i$  used to draw simulated data were based on estimated values of  $\mu_i$  from an FE regression. We know from econometric theory that in the presence of measurement error, these estimated  $\mu_i$ s will be inconsistent estimates of the true country fixed-effects, therefore their sample correlations with the regressors will themselves be flawed. Hence, it is critical to examine what happens when we change these

assumed correlations.

Table 4 displays simulation results when varying the correlations between the country-specific effects and the regressors, while setting  $F = 10\%$ . Table 1 showed that the correlations used for our baseline simulations were high. For example, in the pooled sample the correlation between our estimated  $\mu_i$  and  $\log y_{it-\tau}$  was 0.93, and the correlation with  $\log s_{h,it-\tau}$  was 0.80. Here we simply multiply all these assumed correlations, variable-by-variable and period-by-period, by a single constant  $C < 1$ , prior to generating the simulated data.<sup>37</sup> We allowed  $C$  to take on the values 0%, 5% and 75%. For the sake of comparison, column (5) of Table 4 also reports the results obtained when  $C = 100\%$  (i.e. column (3) of Table 3).

Table 4 demonstrates that the biases obtained under FE when varying  $C$  do not change drastically, suggesting that most of the FE bias comes from measurement error.<sup>38</sup> Notably, the estimate of  $\gamma_4$  changes relatively little (from  $-56\%$  when  $C = 0\%$  to  $-78\%$  when  $C = 100\%$ ). As expected from econometric theory, the AB estimate of  $\gamma_4$  exhibits an even greater degree of stability across values of  $C$ .

We now turn to the BE estimator in the extreme case where  $C = 0$ . Again, this is an unrealistic assumption, but it allows us to evaluate the incidence of measurement error in isolation from heterogeneity bias. We observe that the average absolute value bias is increased (to 157% compared to 33% when  $C = 100\%$ ), but that the pattern of signs and relative magnitudes for the bias is roughly in line with the results in column (3) of Table 3. The convergence parameter  $\gamma_4$  now exhibits a larger bias (34% rather than 19%). The same pattern holds for all the other slope parameters: as  $C$  rises, the extent of bias is progressively reduced, illustrating nicely a main message of this paper: as the incidence of heterogeneity bias rises, it increasingly mitigates the problem of measurement error for the BE estimator. In this case, the two sources of bias tend to *cancel each other out*.

As  $C$  is increased to 50% and then to 75%, the mean estimated slope parameters from BE progressively converge to the values in column (5), and mean absolute value bias decreases steadily. To summarize, variations in

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<sup>37</sup>That is, we modified the relevant entries of the data covariance matrix  $\hat{\Omega}_{x,\mu}$  used to generate the simulated series.

<sup>38</sup>We should not expect the FE estimates to remain unchanged when varying the value of  $C$ . While it is true that this estimator eliminates the country specific effect by differencing the data from country means, the extent of bias may vary as  $C$  is modified, as such a change will affect the properties of the simulated  $\mu_i$  and thus those of the simulated  $\log y_{it-\tau}$ .

the incidence of heterogeneity bias do not change the lessons of our baseline simulations regarding convergence: even when we change the incidence of heterogeneity bias, BE and RE provides the closest estimates of the speed of convergence. The signs of the biases on the parameters of the steady-state determinants are robust to changes in  $C$  for all estimators. As before FE and AB always bias downwards estimates on the accumulation variables, bias upwards the estimate on  $\log(n + g + \delta)_{it-\tau}$  and result in much too fast an estimated speed of convergence.

## 4 Extensions

This section considers various modifications of our basic simulation method. We consider what happens when we change assumptions on the nature of measurement error. We also examine the properties of two additional estimators frequently used in the empirical literature on economic growth.

### 4.1 Varying measurement error on income

While our baseline simulation results replicate the broad differences in past findings on convergence and the determinants of steady-state income level across estimators, the estimated speed of convergence under the FE and AB estimators was too high relative to the BE estimate - we obtained an FE estimate of  $\lambda = 33.99\%$  and a BE estimate of  $\lambda = 0.21\%$ , while the literature finds values in the neighborhood of 10% and 2%, respectively. Moreover, in Section 3.1.3 we showed that an error-to-truth ratio of  $F = 10\%$  implies an average absolute value error in measurement of roughly \$2,000, while the mean of simulated income was \$5,000. While it is difficult to know what the appropriate extent of measurement error is, this is probably too big.<sup>39</sup> In this subsection we examine whether reducing the extent of measurement error on log income can help match convergence speeds estimated in the literature: we reduce the extent of measurement error on the income term

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<sup>39</sup>In contrast, the extent of measurement error on the other variables, implied by setting  $F = 10\%$ , seemed more realistic. Some have argued that per capita income may be better measured than savings rates on human capital  $s_h$ , physical capital  $s_k$  and the depreciation variable  $(n + g + \delta)$ . In principle population growth  $n$  will be well-measured, but recall that we had to make an assumption of constancy across time and countries for  $\delta$  and  $g$ , which surely introduces error. Similarly,  $s_h$  in the Solow model should be measured by dollars saved per unit of time for the purpose of financing education, but we followed the literature in proxying for this using gross enrollment rates in secondary education. However, it is well-known that different methods of computing price indices and PPP exchange rates can deliver vastly different estimates of PPP income.

( $F_y$ ), while maintaining  $F = 10\%$  on the other variable. We consider values of  $F_y$  equal to 0%, 1%, 2.5%, and 5%.

We might expect reduced measurement error on  $\log y_{it}$  to reduce attenuation bias on its coefficient and thereby improve the performance of the FE and AB estimators (though this of course is not always true in the presence of measurement error on the other variables). Our intuition turns out to be borne out: Table 5 demonstrates that with a value of  $F_y = 1\%$ , we obtain FE and AB estimates of  $\lambda$  that are much closer to those found in the past literature: 9.19% and 10.81%, respectively. With  $F_y = 1\%$ , the average absolute value of the error in measurement on  $y_{it}$  is \$618, which is perhaps a more realistic value than those implied by  $F_y = 5\%$  and more. The BE estimate of  $\gamma_4$  is unchanged compared to our baseline simulations, with a 19% upward bias, confirming our suggestion that most of this bias is attributable to the omission of the country-specific effect  $\mu_i$ .

In general, the average absolute value bias becomes much lower for the AB and FE estimators, largely because the bias on the income term is now reduced. In fact, when we set  $F_y = 0$ , these estimators get convergence almost right, suggesting that measurement error in this variable is important to replicate the pattern of  $\gamma_4$  estimates found in the literature.

To summarize, when we allow for a smaller error-to-truth ratio on income, we are able to obtain FE and AB estimated speeds of convergence that are much closer to those obtained when using real data. The extent of bias on the other parameters is not affected very much by measurement error on income. We still get BE estimates of  $\gamma_4$  that are too large (and therefore BE estimates of  $\lambda$  that are too small) relative to the existing cross-country literature. We will see why in Section 4.4.

## 4.2 Country-specific measurement error

The extent of measurement error probably varies from country to country, and is correlated with country characteristics such as per capita income. Fortunately, we do have some information on the accuracy of statistics used in most empirical growth studies: PWT6.1, acknowledging that all of their PPP adjusted data is probably measured with some error, includes a data quality rating for each country. The scale of the rating runs from A to D, which we recode as running from 1 to 4, so that countries with less accurate data have a higher rating.

We exploited this information by adjusting the error-to-truth ratio to allow it to vary across countries in proportion to the PWT6.1 data qual-



ity rating. Since the extent of measurement error is probably correlated with some of the other variables used as regressors, we added the PWT data quality indicator as another variable for the purpose of computing the covariance matrix of the observed right-hand side variables, thus allowing simulated data quality ratings to covary with the other simulated variables. For example, lower income is associated with lower data quality. This expanded covariance matrix for the actual data was then used to draw the simulated data, including a simulated data quality rating. Finally, after drawing the errors in measurement, we multiplied them by the country's normalized simulated data quality rating. The simulated data quality rating is normalized by its average. So if  $F = 10\%$ , a country with the average level of data quality would have an error-to-truth ratio of 10%, while a country with twice the average data quality rating would have an error-to-truth ratio of 20%.

Results displayed in Table 6 show that this extension does not change our basic results. With  $F = 10\%$ , BE still performs best at estimating the rate of convergence with an upwards bias of 19% on  $\gamma_4$ . On the other hand, FE displays a downwards bias of 78% and AB leads to a downwards bias of 89%. BE still outperforms the other estimators in general, with an average absolute bias of 33% compared to 212% for the fixed-effects estimator and 213% for the AB estimator. In fact, a comparison of the entries of Table 6 with those of Table 3 reveals very little difference. Thus, allowing the variance of measurement error to depend on observables does not change our findings.

### 4.3 Autocorrelated measurement error

So far, we have assumed classical measurement error, i.e. the error in measurement was purely white noise. However, errors in measurement could be autocorrelated across time. For instance, if a country has over-reported the amount of savings in physical capital in one period, it may be more likely to do so in subsequent periods. Hence, measurement error can be expected to persist over time. Moreover, as discussed in Section 2, persistent measurement error invalidates the IV procedure of the AB estimator when it comes to addressing measurement error bias, since the error in measurement in lagged regressors (the instruments) is no longer independent from error in measurement in the regressors. In this subsection, we run simulations where autocorrelation is built into measurement error, as a further robustness check. Specifically, for each regressor  $x_q$  ( $q = 2...4$ ) and for the income

term  $\log y_{it}$ , we set:

$$\begin{cases} d_{q,it} = \rho_d d_{q,it-\tau} + \zeta_{it} \\ d_{y,it} = \rho_d d_{y,it-\tau} + \vartheta_{it} \end{cases} \quad (29)$$

where we still maintain  $d_{q,it} \sim N(0, F\hat{\sigma}_{qt}^2)$  and  $d_{y,it} \sim N(0, F\hat{\sigma}_{yt}^2)$ . In other words, the  $T(Q-1) \times T(Q-1)$  variance covariance matrix of the errors-in-variables,  $\hat{\Omega}_d$ , is now block diagonal, with the diagonal elements identical to what they were before and the off-diagonal elements of each  $T \times T$  block as implied by equation (29). Errors are then drawn from a multivariate normal distribution with mean zero and covariance matrix  $\hat{\Omega}_d$ , as before.

In this case, we do have some theoretical priors as to how persistence in measurement error might affect the results. If measurement error is highly persistent over time, we would expect FE to perform better relative to the case of  $\rho_d = 0$ , for two reasons: first, with persistence in  $d_{q,it}$ , some of the measurement error will be differenced away when the data is differenced from its country means. In other words, the greater the autocorrelation in measurement error, the larger the between component of measurement error and the smaller the within component. Since FE will difference away the between variation, we expect greater autocorrelation in the measurement error term, all else equal, to improve the performance of FE relative to BE. In the limit, when the autocorrelation coefficient  $\rho_d$  is 1, we would expect FE to get rid of all of the measurement error, as it will get entirely differenced away.<sup>40</sup>

We ran simulations when the measurement error autocorrelation term was  $\rho_d = 0.5$ ,  $\rho_d = 0.75$  and  $\rho_d = 0.90$ , setting  $F = 10\%$ . The results in Table 7 show that our theory-driven priors are confirmed by the simulations. Average absolute value bias declines with  $\rho_d$  in FE and AB estimators. When the autocorrelation term is only 50%, the convergence coefficient  $\gamma_4$  exhibits a 20% bias using the BE estimator and a -49% bias using the FE estimator (in the baseline case of  $\rho_d = 0$ , the corresponding numbers were 19% and -78%, respectively). When the autocorrelation term is increased to 75%, the bias on the BE estimate of  $\gamma_4$  rises to only 21%, and the bias on the FE estimate declines to -35%. Finally, when the autocorrelation term is increased all the way to 90%, which is probably much too high, the BE coefficient has a bias of 21% and the FE bias of -25%. Therefore, while we confirm our intuition that the FE estimator improves relative to the BE estimator when we increase persistence in measurement error, BE tends to do better or as well as alternative estimators even as  $\rho_d$  is raised to

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<sup>40</sup>We checked that this is the case, and the results are available upon request.

implausibly high levels. Even high persistence in measurement error does not invalidate the overall conclusions reached in the baseline case.

## 4.4 Other Estimators

### 4.4.1 Flexible random effects (SUR)

In this subsection, we evaluate the bias properties of the Seemingly Unrelated Regressions (SUR) estimator, which is commonly used in the empirical growth literature (see, for example, Barro and Sala-i-Martin 1995, chap. 12).

This estimator is computationally close to the RE estimator in that it also weighs the between and within variations in the data. However, in contrast to RE, the SUR estimator does not assume the within-country correlation in the error term to be the same across subsequent time periods, but instead allows it to vary. For example, the covariance between  $\varepsilon_{i1}$  and  $\varepsilon_{i2}$  is allowed to differ from the covariance between  $\varepsilon_{i2}$  and  $\varepsilon_{i3}$ , a possibility precluded in RE. Thus, we refer to the SUR estimator as a flexible RE estimator, as the residual covariance matrix is less restrictive. This is expected to lead to efficiency gains. Moreover, the weighing of the between and within variations will now differ from the RE weighing scheme, and be a complicated function of the variance of the fixed-effects, the variance of the error term  $\mu_i + \nu_{it}$  and its autocorrelation structure. Thus, the bias properties of SUR may differ from those of random effects if the country-specific effects are correlated with the regressors, since it will weigh the within and between variations differently.

Results in Table 8 show that the estimates are not very different from RE (for the sake of comparison Table 8 also includes the RE results already presented in Table 3), but SUR does better overall than RE across values of  $F$ . For example, the convergence coefficient  $\gamma_4$  displays a bias of  $-4\%$  with SUR and  $-16\%$  with RE when  $F = 10\%$ . In fact, SUR overall is the best estimator when it comes to estimating the speed of convergence  $\lambda$ . In terms of average absolute bias, SUR does better than RE whatever the level of measurement error. Both estimators do particularly well in estimating the speed of convergence, but both tend to greatly bias away from zero the estimates on steady-state determinants.

### 4.4.2 The Mankiw, Romer and Weil Estimator

The BE estimator does not strictly correspond to the cross-sectional estimator often used in the cross-country growth literature. Indeed, it involves the

time averaging of all variables, including the income term on the left-hand side and lagged income on the right-hand side. In contrast, cross-sectional estimators in the class of MRW's OLS estimator are based on the following regression:

$$\log y_{i,2000} = \gamma_0 + \gamma_1 \overline{\log s_{k,i}} + \gamma_2 \overline{\log s_{h,i}} + \gamma_3 \overline{\log (n + g + \delta)_i} + \gamma_4 \log y_{i,1960} + \vartheta_{it} \quad (30)$$

where " $\bar{\cdot}$ " denotes averages computed over the whole period. As in the BE estimator, the determinants of the steady-state level of income appear as averages computed over the whole period. However, income enters as end and beginning of period values (our total period spans 1960-2000). Hence, contrary to BE, measurement error in initial income does not get averaged on the right-hand side. Note also that in this application  $\tau$  is set to 40, requiring a correction to ensure the comparability of the estimated reduced form coefficients with BE estimates.

Table 8 displays the simulation results, with the appropriate correction. For comparability, we also reproduce the BE results from Table 3. The MRW and BE biases are very similar in terms of magnitudes and signs. However, MRW does slightly better than BE in terms of average absolute bias across values of  $F$ . This partly reflects a lower bias on the convergence parameter  $\gamma_4$ . Since lagged income does not get averaged, measurement error on this variable counteracts the upward bias from the omission of  $\mu_i$  to a greater degree than BE. These results confirm that simple OLS cross-sectional estimators are best at limiting net overall bias resulting from heterogeneity and measurement error.

## 5 Conclusion

In this paper, we have used Monte Carlo simulations to evaluate the econometric methods commonly used to estimate growth regressions. Our results suggest that, in the presence of measurement error, fixed-effects and the Arellano-Bond GMM estimator lead researchers to overestimate the speed of convergence and to underestimate the impact of several common determinants of the steady-state level of income, such as human capital. Simple OLS on variables averaged over time provides a closer estimate of the speed of convergence, but overestimates the magnitude of the effect of steady-state determinants. These findings were shown to be robust to changes in the specification of measurement error and to varying assumptions about the incidence of heterogeneity bias.

Until now, differences in speeds of convergence across estimators were interpreted as implying that heterogeneity bias was prevalent in cross-sectional growth regressions, since fixed-effects methods were thought to correct for this bias and led to a speed of convergence roughly 5 times higher than that estimated using a between estimator. This paper has overturned this interpretation, showing instead that the difference in estimated convergence speeds is in fact attributable to greater bias from measurement error when using this class of estimators. The estimated speed from traditional cross-sectional regressions is in all likelihood closer to the true speed of convergence.

Of course, our simulations can only characterize the properties of the estimators. They cannot inform us as to the actual speed of convergence or the impact of steady-state determinants, since we simulated our data by assuming values for these parameters implied by a strict application of the Solow model. This strict application led to postulated parameters that may or may not hold true in actual data. Table 9 displays the results from estimating our basic specification using the new PWT version 6.1 data and updated series for the secondary school enrollment rate from Barro and Lee. We are able to replicate the basic findings of the past literature in this data: the speed of convergence is roughly 5 times larger under FE and AB (respectively 4.6% and 5.3%) compared to BE (0.8%). Our simulation results suggest the latter number is likely to be much closer to the truth. All the other estimators, that do not isolate the within variation in the data (namely MRW, SUR and RE) lead to estimated speeds of convergence that lie between 0.8% and 1.6%, while FE and AB lead to estimates in the neighborhood of 5%.

The speed of convergence we report based on the application of the BE estimator, less than 1%, falls short of the number typically reported in the cross-sectional literature. Barro and Sala-i-Martin (1995) cite a number closer to 2%, based on the previous version of PWT.<sup>41</sup> This difference is not attributable to our use of new and extended data.<sup>42</sup> Past cross-sectional estimates rely on an OLS specification closer to the MRW estimator described above, where the current and lagged income terms do not get averaged over

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<sup>41</sup>This is also true for the (biased) FE estimated speed, which at 5% is about half of the convergence speeds reported in CEL (1996) and Islam (1995).

<sup>42</sup>We removed the 1990-1999 decade to check this, and our results were unchanged. In an effort to maintain a balanced panel and cover as wide a time period as possible, our regressions also feature only 69 countries. This was not responsible either for the reduced speed of convergence, as results obtained with more countries using an unbalanced panel were essentially the same. All these results are available upon request.

time. Implementing this estimator, we obtain a convergence speed of 1.62%, which is closer to existing estimates. Given that BE and MRW have been shown above to somewhat understate the speed of convergence, a number in the neighborhood of 2% for  $\lambda$  does not seem unreasonable.

The slope parameters on the determinants of the steady-state level of income are reduced in magnitude when using FE or the AB estimator. They are similar across estimators that use at least some between variation in the data (BE, SUR, RE and MRW). For example, the impact of the log of the enrollment rate is equal to 0.04 using the BE estimator, and is significant at the 1% level. As in Islam, this estimate switches signs (to  $-0.03$ ) when using the FE estimator, and is statistically insignificant.

Our paper illustrates an econometric second best property: by addressing one source of bias (stemming from omitted variables), the application of a certain class of estimators makes another source of bias worse (measurement error). Unfortunately, simulation exercises such as ours almost always come too late in empirical economics.

## Appendix 1 - Limiting Values of the Between, Fixed-Effects and Random Effects Estimators

This appendix derives the limiting values of the BE, FE and RE estimators in the multivariate case. As in the text, assume that the true model is:

$$\log y_{it} = \gamma' x_{it} + \mu_i + \varepsilon_{it} \quad (31)$$

where all these variables are as defined in Section 2. As in subsection 2.3, we also abstract from the dynamic nature of the model.

Assume also that  $x_{it}$  is imperfectly measured. Instead of observing  $x_{it}$ , we can only see  $x_{it}^* = x_{it} + d_{it}$ , where  $E[d_{it}|x_{it}] = 0$  for all observations and  $\text{var}[d_{it}|x_{it}] = D = \text{diag}\{\sigma_{d_1}^2, \sigma_{d_2}^2, \dots, \sigma_{d_k}^2\}$ . We can derive the following unconditional expectations for three variables of interest:

$$E(y_{it}, x_{it}^*, \mu_i) = (\gamma' \bar{x} + \bar{\mu}, \bar{x}, \bar{\mu}) \quad (32)$$

Their unconditional variance is:

$$V(y_{it}, x_{it}^*, \mu_i) = \begin{bmatrix} \sigma_\varepsilon^2 + \gamma' \Sigma_{xx} \gamma + 2\gamma' \Sigma_{\mu x} + \sigma_\mu^2 & \gamma' \Sigma_{xx} + \Sigma'_{\mu x} & \gamma' \Sigma_{\mu x} + \sigma_\mu^2 \\ \Sigma_{xx} \gamma + \Sigma_{\mu x} & \Sigma_{xx} + D & \Sigma_{\mu x} \\ \gamma' \Sigma_{\mu x} + \sigma_\mu^2 & \Sigma'_{\mu x} & \sigma_\mu^2 \end{bmatrix} \quad (33)$$

In order to analyze the properties of the BE and FE estimators further, it is useful to break down the variation on each variable into within-country variation and between-country variation. Define the between-country variance for  $x_{it}$  as:

$$\Sigma_{xx}^B \equiv E \left[ \left( \frac{1}{T} \sum_{t=1}^T x_{it} \right) \left( \frac{1}{T} \sum_{t=1}^T x'_{it} \right) \right] - E \left( \frac{1}{T} \sum_{t=1}^T x_{it} \right) E \left( \frac{1}{T} \sum_{t=1}^T x'_{it} \right) \quad (34)$$

and the within-country variance to be:

$$\begin{aligned} \Sigma_{xx}^W &\equiv E \left[ \left( x_{it} - \frac{1}{T} \sum_{t=1}^T x_{it} \right) \left( x'_{it} - \frac{1}{T} \sum_{t=1}^T x'_{it} \right) \right] \\ &\quad - E \left[ x_{it} - \frac{1}{T} \sum_{t=1}^T x_{it} \right] E \left[ x'_{it} - \frac{1}{T} \sum_{t=1}^T x'_{it} \right] \end{aligned} \quad (35)$$

It is well-known that:

$$\Sigma_{xx} = \Sigma_{xx}^W + \Sigma_{xx}^B \quad (36)$$

It is also easy to show that, for the covariance between  $x_{it}$  and  $\mu$ ,  $\Sigma_{\mu x} = \Sigma_{\mu x}^B$ . Finally,  $\Sigma_{xx^*}^B$ , the between covariance matrix of the imperfectly observed data  $x^*$ , is defined as:

$$\begin{aligned}\Sigma_{xx^*}^B &\equiv E \left[ \left( \frac{1}{T} \sum_{t=1}^T x_{it} + d_{it} \right) \left( \frac{1}{T} \sum_{t=1}^T x'_{it} + d'_{it} \right) - E[x_{it} + d_{it}] E[x'_{it} + d'_{it}] \right] \\ &= \Sigma_{xx}^B + \frac{1}{T} D\end{aligned}\quad (37)$$

It is also easy to show that:

$$\Sigma_{xx^*}^W = \Sigma_{xx}^W + \frac{T-1}{T} D \quad (38)$$

We are now ready to derive the plims of the BE and FE estimators in the presence of measurement error and in the multivariate case.

First consider the BE estimator (OLS on country means across time). Using standard OLS results, we can derive:

$$\text{plim } \hat{\gamma}^{BE} = \left( \Sigma_{xx}^B + \frac{1}{T} D \right)^{-1} \Sigma_{xx}^B \gamma + \left( \Sigma_{xx}^B + \frac{1}{T} D \right)^{-1} \Sigma_{\mu x} \quad (39)$$

Now consider FE. To eliminate the heterogeneity bias arising through the correlation between the time invariant country-specific effects and the regressors, the most obvious solution is to use the FE estimator. By the Frisch-Waugh theorem, we can show that:

$$\hat{\gamma}^{FE} = (X^{*'} M_c X^*)^{-1} X^{*'} M_c y \quad (40)$$

where:

$$M_c = I - C(C'C)^{-1} C' \quad (41)$$

and  $C$  is an  $(NT \times N)$  matrix that stacks dummy variables for the different countries (with subvectors of  $T$  ones along the diagonals, zero elsewhere).

Then:

$$\text{plim } \hat{\gamma}^{FE} = \left( \Sigma_{xx}^W + \frac{T-1}{T} D \right)^{-1} \Sigma_{xx}^W \gamma \quad (42)$$

Finally, as is well-known, RE is simply a matrix-weighted average of BE and FE estimates:

$$\hat{\gamma}^{RE} = \left( \hat{\Sigma}_{xx^*}^W + \hat{\theta} \hat{\Sigma}_{xx^*}^B \right)^{-1} \left( \hat{\Sigma}_{xx^*}^W \hat{\gamma}^{FE} + \hat{\theta} \hat{\Sigma}_{xx^*}^B \hat{\gamma}^{BE} \right) \quad (43)$$



where  $\hat{\Sigma}_{xx}^W$  and  $\hat{\Sigma}_{xx}^B$  are the sample estimates of  $\Sigma_{xx}^W$  and  $\Sigma_{xx}^B$ , respectively, and  $\hat{\theta}$  is an estimate of  $\theta$  where:

$$\theta = \frac{\sigma_\varepsilon^2}{T\sigma_\mu^2 + \sigma_\varepsilon^2} \quad (44)$$

i.e.  $\theta$  is the weights given to the BE estimator. Then:

$$\begin{aligned} \text{plim } \hat{\gamma}^{RE} &= \left[ \left( \Sigma_{xx}^W + \frac{T-1}{T}D \right) + \theta \left( \Sigma_{xx}^B + \frac{1}{T}D \right) \right]^{-1} \\ &\times \left[ \left( \Sigma_{xx}^W + \frac{T-1}{T}D \right) \gamma^{FE} + \theta \left( \Sigma_{xx}^B + \frac{1}{T}D \right) \gamma^{BE} \right] \quad (45) \end{aligned}$$

Note that when the variance of the error term  $\varepsilon_{it}$  is zero, RE reduces to FE.

To summarize, we have derived the following:

$$\left\{ \begin{array}{l} \text{plim } \hat{\gamma}^{POLS} = (\Sigma_{xx} + D)^{-1} \Sigma_{xx} \gamma + (\Sigma_{xx} + D)^{-1} \Sigma_{\mu x} \\ \text{plim } \hat{\gamma}^{BE} = (\Sigma_{xx}^B + \frac{1}{T}D)^{-1} \Sigma_{xx}^B \gamma + (\Sigma_{xx}^B + \frac{1}{T}D)^{-1} \Sigma_{\mu x} \\ \text{plim } \hat{\gamma}^{FE} = (\Sigma_{xx}^W + \frac{T-1}{T}D)^{-1} \Sigma_{xx}^W \gamma \\ \text{plim } \hat{\gamma}^{RE} = \left[ \left( \Sigma_{xx}^W + \frac{T-1}{T}D \right) + \theta \left( \Sigma_{xx}^B + \frac{1}{T}D \right) \right]^{-1} \\ \quad \times \left[ \left( \Sigma_{xx}^W + \frac{T-1}{T}D \right) \gamma^{FE} + \theta \left( \Sigma_{xx}^B + \frac{1}{T}D \right) \gamma^{BE} \right] \end{array} \right.$$

## Appendix 2 - A Simple Univariate Example

To illustrate the effects at play in the presence of both heterogeneity bias and measurement error, we consider the case where  $x_{it}$  is unidimensional, and contrast estimation by pooled OLS and FE. Consider the following relationship with a single observed regressor and an intercept term:

$$y_{it} = \gamma_0 + \gamma_1 x_{it} + \mu_i + \varepsilon_{it} \quad (46)$$

Suppose that the observed variable  $x_{it}^*$  incorporates measurement error:

$$x_{it}^* = x_{it} + d_{it} \quad (47)$$

where  $d_{it}$  is independent of the true  $x_{it}$ . The variance of the measurement error and of  $x_{it}$  are, respectively,  $\sigma_d^2$  and  $\sigma_x^2$ .

By estimating (46) using pooled OLS, we get both an omitted variables bias due to the fact that  $\mu_i$  is potentially correlated with  $x_{it}$ , and a measurement error bias due to the correlation between  $\varepsilon_{it}$  and  $x_{it}^*$ . The limiting value of the pooled OLS estimate of  $\gamma_1$  is as follows:

$$\text{plim } \hat{\gamma}_1^{POLS} = \frac{\gamma_1}{1 + \frac{\sigma_d^2}{\sigma_x^2}} + \frac{\text{cov}(x_{it}, \mu_i)}{\sigma_x^2 + \sigma_d^2} \quad (48)$$

In equation (48), the two sources of bias appear clearly. The variance of measurement error contributes to lessen the extent of heterogeneity bias, as it appears in the denominator of the expression on the right hand side of (48).

Consider now FE estimation, still in the univariate case. To simplify things and without loss of generality, assume that we difference away the time invariant individual effects by taking first differences, rather than by taking differences from country means of the data. The limiting value of the FE estimate of  $\gamma_1$  is then:

$$\text{plim } \hat{\gamma}_1^{FE} = \frac{\gamma_1}{1 + \frac{\sigma_{\Delta d}^2}{\sigma_{\Delta x}^2}} \quad (49)$$

where  $\sigma_{\Delta d}^2$  is the variance of the first differenced measurement error, and  $\sigma_{\Delta x}^2$  is the same for the "true" regressor  $x_{it}$ .

We have derived formal expressions for our estimate of interest in two cases. The second method, FE, allows us to remove the heterogeneity bias but will exacerbate measurement error bias. To see why, note that the error-to-truth ratio in the denominator of equation (49) will *always* have increased compared to that under pooled OLS:

$$\sigma_{\Delta d}^2 = \text{var } d_{it} + \text{var } d_{it-\tau} - 2 \text{cov}(d_{it}, d_{it-\tau}) = 2\sigma_d^2 \quad (50)$$

$$\sigma_{\Delta x}^2 = \text{var } x_{it} + \text{var } x_{it-\tau} - 2 \text{cov}(x_{it}, x_{it-\tau}) = 2\sigma_x^2(1 - \rho_x) \quad (51)$$

where  $\rho_x = \text{corr}(x_{it}, x_{it-\tau})$  is the autocorrelation of  $x_{it}$ . Thus,

$$\frac{\sigma_{\Delta d}^2}{\sigma_{\Delta x}^2} = \frac{\sigma_d^2}{\sigma_x^2(1 - \rho_x)} > \frac{\sigma_d^2}{\sigma_x^2} \quad (52)$$

In words,  $\sigma_{\Delta x}^2$  will be smaller relative to  $\sigma_x^2$  the greater the time persistence in  $x_{it}$  (i.e. the higher is  $\rho_x$ ).

We have assumed until now that there was no time persistence in measurement error (i.e. we had white noise errors-in-variables). This assumption is problematic in the context of data used for growth regressions, where errors in measurement from one period are likely to carry over to the next. In the case of autocorrelated measurement error, where we define  $\rho_d = \text{corr}(d_{it}, d_{it-\tau})$ , the error-to-truth ratio under FE is:

$$\frac{\sigma_{\Delta d}^2}{\sigma_{\Delta x}^2} = \frac{\sigma_d^2(1 - \rho_d)}{\sigma_x^2(1 - \rho_x)} \quad (53)$$

It is then trivial to show that FE will exacerbate measurement error bias compared to pooled OLS whenever  $\rho_d < \rho_x$ .

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**Table 1 – Correlation Structure Among Regressors and Fixed-Effects**

	$\log s_{k,it-\tau}$	$\log s_{h,it-\tau}$	$\log(n+g+\delta)_{it-\tau}$	$\log y_{it-\tau}$	$\mu_i$
<b>Panel A – Pooled Data</b>					
$\log s_{k,it-\tau}$	1.0000				
$\log s_{h,it-\tau}$	0.6046	1.0000			
$\log(n+g+\delta)_{it-\tau}$	-0.3800	-0.5763	1.0000		
$\log y_{it-\tau}$	0.6220	0.8086	-0.6640	1.0000	
$\mu_i$	0.6248	0.8031	-0.5957	0.9273	1.0000
<b>Panel B – Between Variation</b>					
$\log s_{k,it-\tau}$	1.0000				
$\log s_{h,it-\tau}$	0.7160	1.0000			
$\log(n+g+\delta)_{it-\tau}$	-0.5004	-0.6594	1.0000		
$\log y_{it-\tau}$	0.7107	0.8691	-0.7154	1.0000	
$\mu_i$	0.7096	0.9070	-0.6629	0.9622	1.0000
<b>Panel C – Within Variation</b>					
$\log s_{k,it-\tau}$	1.0000				
$\log s_{h,it-\tau}$	0.2104	1.0000			
$\log(n+g+\delta)_{it-\tau}$	0.0763	-0.2531	1.0000		
$\log y_{it-\tau}$	0.1497	0.5400	-0.3799	1.0000	
$\mu_i$	0.0000	0.0000	0.0000	0.0000	1.0000

**Table 2 – Magnitude of Measurement Error on the Underlying Data  
(based on various values of F)**

		<b>Income (\$)</b>	<b>Investment Rate (% GDP)</b>	<b>Secondary Enrollment Rate (%)</b>	<b>Population growth (%)</b>
<b>Average Value of Unshocked Variable</b>		4997.15	17.17%	51.40%	1.59%
<b>Average Absolute Value of Shock with F=</b>	1%	617.646	0.92%	4.33%	0.06%
	2.5%	1010.391	1.46%	6.52%	0.10%
	5%	1332.400	2.03%	8.84%	0.14%
	10%	2134.202	2.95%	13.94%	0.20%
	15%	2702.867	3.67%	16.05%	0.25%

Averages computed from simulated data for 2,000 countries in the 8-period panel, i.e. 16,000 pooled observations.

Table 3 – Baseline Simulation Results – Average Estimated Coefficients and Bias (1000 runs)

		(1)		(2)		(3)		(4)	
Error-to-Truth Ratio		F=0%		F=5%		F=10%		F=15%	
Variable	True Coeffs	Mean	Bias (%)	Avg Coeff	Bias (%)	Avg Coeff	Bias (%)	Avg Coeff	Bias (%)
<b>Fixed Effects</b>									
$\log S_{k,it-\tau}$	0.099	0.093	-5%	0.026	-74%	0.003	-97%	-0.007	-107%
$\log S_{h,it-\tau}$	0.099	0.076	-23%	-0.095	-197%	-0.108	-209%	-0.103	-204%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.287	46%	-0.991	402%	-1.107	461%	-1.069	442%
$\log y_{it-\tau}$	0.832	0.787	-5%	0.332	-60%	0.183	-78%	0.110	-87%
Avg. Abs. Bias			20%		183%		211%		210%
Implied $\lambda$	3.68%	4.79%	30%	22.07%	500%	33.99%	824%	44.11%	1099%
<b>Between</b>									
$\log S_{k,it-\tau}$	0.099	0.081	-18%	0.079	-19%	0.079	-20%	0.078	-21%
$\log S_{h,it-\tau}$	0.099	0.114	16%	0.110	12%	0.105	6%	0.100	1%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.031	-84%	-0.033	-83%	-0.027	-86%	-0.031	-84%
$\log y_{it-\tau}$	0.832	0.983	18%	0.986	18%	0.990	19%	0.993	19%
Avg. Abs. Bias			34%		33%		33%		31%
Implied $\lambda$	3.68%	0.35%	-91%	0.29%	-92%	0.21%	-94%	0.14%	-96%
<b>Random Effects</b>									
$\log S_{k,it-\tau}$	0.099	0.110	12%	0.163	65%	0.204	107%	0.234	137%
$\log S_{h,it-\tau}$	0.099	0.171	74%	0.268	172%	0.322	226%	0.347	252%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.182	-8%	-0.899	355%	-1.361	590%	-1.666	744%
$\log y_{it-\tau}$	0.832	0.924	11%	0.789	-5%	0.699	-16%	0.639	-23%
Avg. Abs. Bias			26%		150%		235%		289%
Implied $\lambda$	3.68%	1.58%	-57%	4.75%	29%	7.17%	95%	8.96%	143%
<b>Arellano-Bond</b>									
$\log S_{k,it-\tau}$	0.099	0.095	-4%	-0.038	-139%	-0.072	-173%	-0.083	-185%
$\log S_{h,it-\tau}$	0.099	0.068	-31%	-0.280	-384%	-0.254	-357%	-0.221	-325%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.243	23%	-0.695	252%	-0.642	225%	-0.553	180%
$\log y_{it-\tau}$	0.832	0.795	-4%	0.207	-75%	0.095	-89%	0.046	-94%
Avg. Abs. Bias			16%		213%		211%		196%
Implied $\lambda$	3.68%	4.58%	24%	31.52%	756%	47.10%	1180%	61.45%	1570%

Table 4 - Varying Heterogeneity Bias (alternative correlations between  $\mu_i$  and the regressors) – F = 10%, 1000 runs.

		(1)		(2)		(3)		(4)	
FE Correlation:		C=0%		C=50%		C=75%		C=100%	
Variable	True Coeffs	Mean	Bias (%)	Mean	Bias (%)	Mean	Bias (%)	Mean	Bias (%)
<b>Fixed Effects</b>									
$\log s_{k,it-\tau}$	0.099	0.070	-29%	0.037	-63%	0.022	-78%	0.003	-97%
$\log s_{h,it-\tau}$	0.099	0.049	-50%	-0.018	-118%	-0.061	-162%	-0.108	-209%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.234	19%	-0.577	192%	-0.845	328%	-1.107	461%
$\log y_{it-\tau}$	0.832	0.363	-56%	0.275	-67%	0.228	-73%	0.183	-78%
Avg. Abs. Bias			39%		110%		160%		211%
Implied $\lambda$	3.68%	20.27%	451%	25.85%	602%	29.60%	704%	33.99%	824%
<b>Between</b>									
$\log s_{k,it-\tau}$	0.099	0.010	-90%	0.027	-73%	0.043	-57%	0.079	-20%
$\log s_{h,it-\tau}$	0.099	-0.069	-170%	-0.026	-127%	0.019	-81%	0.105	6%
$\log(n+g+\delta)_{it-\tau}$	-0.197	0.462	-334%	0.349	-277%	0.221	-212%	-0.027	-86%
$\log y_{it-\tau}$	0.832	1.118	34%	1.092	31%	1.059	27%	0.990	19%
Avg. Abs. Bias			157%		127%		94%		33%
Implied $\lambda$	3.68%	-2.23%	-161%	-1.77%	-148%	-1.14%	-131%	0.21%	-94%
<b>Random Effects</b>									
$\log s_{k,it-\tau}$	0.099	0.090	-9%	0.134	36%	0.165	67%	0.204	107%
$\log s_{h,it-\tau}$	0.099	0.078	-21%	0.172	75%	0.239	142%	0.322	226%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.196	0%	-0.595	201%	-0.927	370%	-1.361	590%
$\log y_{it-\tau}$	0.832	0.854	3%	0.802	-4%	0.758	-9%	0.699	-16%
Avg. Abs. Bias			8%		79%		147%		235%
Implied $\lambda$	3.68%	3.15%	-14%	4.41%	20%	5.54%	51%	7.17%	95%
<b>Arellano-Bond</b>									
$\log s_{k,it-\tau}$	0.099	0.011	-89%	-0.049	-150%	-0.061	-162%	-0.072	-173%
$\log s_{h,it-\tau}$	0.099	0.032	-67%	-0.120	-221%	-0.195	-298%	-0.254	-357%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.494	150%	-0.613	211%	-0.646	228%	-0.642	225%
$\log y_{it-\tau}$	0.832	0.060	-93%	0.003	-100%	0.038	-95%	0.095	-89%
Avg. Abs. Bias			100%		170%		196%		211%
Implied $\lambda$	3.68%	56.25%	1429%	114.83%	3020%	65.32%	1675%	47.10%	1180%



Table 5 - Varying Measurement Error on Initial Income, 1000 runs (F=10% on the other variables)

Variable	True Coeffs	F <sub>v</sub> =0%		F <sub>v</sub> =1%		F <sub>v</sub> =2.5%		F <sub>v</sub> =5%	
		Mean	Bias (%)	Mean	Bias (%)	Mean	Bias (%)	Mean	Bias (%)
<b>Fixed Effects</b>									
log s <sub>k,it-τ</sub>	0.099	0.066	-33%	0.052	-47%	0.035	-64%	0.018	-81%
log s <sub>h,it-τ</sub>	0.099	0.030	-69%	-0.003	-103%	-0.038	-139%	-0.072	-173%
log(n+g+δ) <sub>it-τ</sub>	-0.197	-0.140	-29%	-0.371	88%	-0.606	207%	-0.844	328%
log y <sub>it-τ</sub>	0.832	0.773	-7%	0.632	-24%	0.486	-42%	0.337	-59%
Avg. Abs. Bias			35%		66%		113%		160%
Implied λ	3.68%	5.15%	40%	9.19%	150%	14.42%	292%	21.74%	491%
<b>Between</b>									
log s <sub>k,it-τ</sub>	0.099	0.078	-21%	0.079	-20%	0.079	-19%	0.079	-20%
log s <sub>h,it-τ</sub>	0.099	0.107	8%	0.106	7%	0.106	7%	0.105	6%
log(n+g+δ) <sub>it-τ</sub>	-0.197	-0.031	-84%	-0.031	-84%	-0.029	-85%	-0.034	-83%
log y <sub>it-τ</sub>	0.832	0.989	19%	0.989	19%	0.988	19%	0.989	19%
Avg. Abs. Bias			33%		33%		33%		32%
Implied λ	3.68%	0.23%	-94%	0.22%	-94%	0.23%	-94%	0.22%	-94%
<b>Random Effects</b>									
log s <sub>k,it-τ</sub>	0.099	0.104	5%	0.116	18%	0.134	36%	0.160	62%
log s <sub>h,it-τ</sub>	0.099	0.127	29%	0.151	53%	0.186	89%	0.239	142%
log(n+g+δ) <sub>it-τ</sub>	-0.197	-0.155	-22%	-0.298	51%	-0.516	161%	-0.850	331%
log y <sub>it-τ</sub>	0.832	0.951	14%	0.922	11%	0.876	5%	0.808	-3%
Avg. Abs. Bias			17%		33%		73%		135%
Implied λ	3.68%	1.00%	-73%	1.62%	-56%	2.65%	-28%	4.27%	16%
<b>Arellano-Bond</b>									
log s <sub>k,it-τ</sub>	0.099	0.064	-36%	0.036	-63%	-0.007	-107%	-0.042	-143%
log s <sub>h,it-τ</sub>	0.099	0.016	-84%	-0.048	-149%	-0.128	-230%	-0.198	-301%
log(n+g+δ) <sub>it-τ</sub>	-0.197	-0.123	-38%	-0.319	62%	-0.448	127%	-0.556	182%
log y <sub>it-τ</sub>	0.832	0.770	-7%	0.582	-30%	0.386	-54%	0.223	-73%
Avg. Abs. Bias			41%		76%		130%		175%
Implied λ	3.68%	5.24%	42%	10.81%	194%	19.04%	417%	29.97%	714%

Table 6 – Country-Specific Measurement Error, 1000 runs

Error-to-Truth Ratio:		F=5%		F=10%		F=15%	
Variable	True Coeffs	Mean	Bias (%)	Mean	Bias (%)	Mean	Bias (%)
<b>Fixed Effects</b>							
$\log s_{k,it-\tau}$	0.099	0.022	-77%	0.003	-97%	-0.015	-115%
$\log s_{h,it-\tau}$	0.099	-0.100	-201%	-0.110	-211%	-0.096	-198%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-1.019	417%	-1.106	461%	-1.035	425%
$\log y_{it-\tau}$	0.832	0.328	-61%	0.183	-78%	0.110	-87%
<b>Avg. Abs. Bias</b>			189%		212%		206%
<b>Implied <math>\lambda</math></b>	3.68%	22.28%	506%	33.93%	822%	44.21%	1101%
<b>Between</b>							
$\log s_{k,it-\tau}$	0.099	0.080	-19%	0.079	-20%	0.077	-22%
$\log s_{h,it-\tau}$	0.099	0.109	11%	0.104	6%	0.101	2%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.025	-87%	-0.025	-87%	-0.029	-85%
$\log y_{it-\tau}$	0.832	0.987	19%	0.990	19%	0.993	19%
<b>Avg. Abs. Bias</b>			34%		33%		32%
<b>Implied <math>\lambda</math></b>	3.68%	0.27%	-93%	0.20%	-95%	0.15%	-96%
<b>Random Effects</b>							
$\log s_{k,it-\tau}$	0.099	0.163	66%	0.204	107%	0.230	134%
$\log s_{h,it-\tau}$	0.099	0.268	172%	0.321	225%	0.350	255%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.893	353%	-1.357	588%	-1.666	744%
$\log y_{it-\tau}$	0.832	0.789	-5%	0.700	-16%	0.638	-23%
<b>Avg. Abs. Bias</b>			149%		234%		289%
<b>Implied <math>\lambda</math></b>	3.68%	4.73%	29%	7.14%	94%	9.00%	145%
<b>Arellano-Bond</b>							
$\log s_{k,it-\tau}$	0.099	-0.045	-145%	-0.071	-172%	-0.091	-192%
$\log s_{h,it-\tau}$	0.099	-0.286	-390%	-0.257	-361%	-0.215	-318%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.710	260%	-0.651	230%	-0.526	167%
$\log y_{it-\tau}$	0.832	0.200	-76%	0.091	-89%	0.042	-95%
<b>Avg. Abs. Bias</b>			218%		213%		193%
<b>Implied <math>\lambda</math></b>	3.68%	32.23%	776%	47.91%	1202%	63.45%	1624%

Table 7 – Autocorrelated Measurement Error (F=10%, 1000 runs)

Variable	True Coeffs	$\rho_d=50\%$		$\rho_d=75\%$		$\rho_d=90\%$	
		Mean	Bias (%)	Mean	Bias (%)	Mean	Bias (%)
<b>Fixed Effects</b>							
$\log s_{k,it-\tau}$	0.099	0.029	-71%	0.047	-52%	0.066	-33%
$\log s_{h,it-\tau}$	0.099	-0.060	-161%	-0.035	-135%	-0.002	-102%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.818	315%	-0.627	218%	-0.537	172%
$\log y_{it-\tau}$	0.832	0.425	-49%	0.538	-35%	0.627	-25%
<b>Avg. Abs. Bias</b>			149%		110%		83%
<b>Implied <math>\lambda</math></b>	3.68%	17.13%	366%	12.40%	237%	9.33%	153%
<b>Between</b>							
$\log s_{k,it-\tau}$	0.099	0.075	-24%	0.074	-25%	0.072	-27%
$\log s_{h,it-\tau}$	0.099	0.094	-5%	0.086	-13%	0.078	-21%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.031	-84%	-0.027	-86%	-0.044	-78%
$\log y_{it-\tau}$	0.832	0.998	20%	1.003	21%	1.007	21%
<b>Avg. Abs. Bias</b>			33%		36%		37%
<b>Implied <math>\lambda</math></b>	3.68%	0.05%	-99%	-0.06%	-102%	-0.14%	-104%
<b>Random Effects</b>							
$\log s_{k,it-\tau}$	0.099	0.157	59%	0.132	33%	0.115	16%
$\log s_{h,it-\tau}$	0.099	0.231	134%	0.184	87%	0.151	53%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.826	319%	-0.493	150%	-0.300	52%
$\log y_{it-\tau}$	0.832	0.817	-2%	0.880	6%	0.923	11%
<b>Avg. Abs. Bias</b>			129%		69%		33%
<b>Implied <math>\lambda</math></b>	3.68%	4.05%	10%	2.56%	-30%	1.60%	-57%
<b>Arellano-Bond</b>							
$\log s_{k,it-\tau}$	0.099	-0.017	-118%	0.007	-93%	0.032	-68%
$\log s_{h,it-\tau}$	0.099	-0.177	-279%	-0.146	-248%	-0.107	-208%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.367	86%	-0.202	3%	-0.206	5%
$\log y_{it-\tau}$	0.832	0.426	-49%	0.537	-35%	0.599	-28%
<b>Avg. Abs. Bias</b>			133%		95%		77%
<b>Implied <math>\lambda</math></b>	3.68%	17.09%	364%	12.43%	238%	10.24%	178%

Table 8 – Simulation Results for Alternative Estimators– 1000 runs

Error-to-Truth Ratio:		F=5%		F=10%		F=15%	
Variable	True Coeffs	Mean	Bias (%)	Mean	Bias (%)	Mean	Bias (%)
<b>Random Effects</b>							
$\log s_{k,it-\tau}$	0.099	0.163	65%	0.204	107%	0.234	137%
$\log s_{h,it-\tau}$	0.099	0.268	172%	0.322	226%	0.347	252%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.899	355%	-1.361	590%	-1.666	744%
$\log y_{it-\tau}$	0.832	0.789	-5%	0.699	-16%	0.639	-23%
<b>Avg. Abs. Bias</b>			150%		235%		289%
<b>Implied <math>\lambda</math></b>	3.68%	4.75%	29%	7.17%	95%	8.96%	143%
<b>SUR (Flexible Random Effects)</b>							
$\log s_{k,it-\tau}$	0.099	0.133	35%	0.160	62%	0.183	85%
$\log s_{h,it-\tau}$	0.099	0.210	113%	0.244	148%	0.268	171%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.531	169%	-0.857	334%	-1.092	454%
$\log y_{it-\tau}$	0.832	0.865	4%	0.802	-4%	0.751	-10%
<b>Avg. Abs. Bias</b>			80%		137%		180%
<b>Implied <math>\lambda</math></b>	3.68%	2.91%	-21%	4.40%	20%	5.74%	56%
<b>Between</b>							
$\log s_{k,it-\tau}$	0.099	0.079	-19%	0.079	-20%	0.078	-21%
$\log s_{h,it-\tau}$	0.099	0.110	12%	0.105	6%	0.100	1%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.033	-83%	-0.027	-86%	-0.031	-84%
$\log y_{it-\tau}$	0.832	0.986	18%	0.990	19%	0.993	19%
<b>Avg. Abs. Bias</b>			33%		33%		31%
<b>Implied <math>\lambda</math></b>	3.68%	0.29%	-92%	0.21%	-94%	0.14%	-96%
<b>Mankiw-Romer-Weil (modified Between)<sup>a</sup></b>							
$\log s_{k,it-\tau}$	0.099	0.087	-12%	0.085	-14%	0.083	-16%
$\log s_{h,it-\tau}$	0.099	0.134	35%	0.126	27%	0.121	22%
$\log(n+g+\delta)_{it-\tau}$	-0.197	-0.109	-45%	-0.099	-50%	-0.087	-56%
$\log y_{it-\tau}$	0.832	0.959	15%	0.966	16%	0.971	17%
<b>Avg. Abs. Bias</b>			27%		27%		28%
<b>Implied <math>\lambda</math></b>	3.68%	0.84%	-77%	0.69%	-81%	0.60%	-84%

a: MRW estimates adjusted with  $\tau=5$  instead of  $\tau=40$  to ensure comparability of the point estimates with the other estimators.

**Table 9 – Growth Regressions on Actual Data – Dependent Variable: Log Per Capita Income – 1960-1999.**

	(1)	(2)	(2')	(3)	(4)	(5)	(6)
	BE	MRW	MRW adj. <sup>b</sup>	RE	SUR	FE	AB
<b>log <math>s_{k,it-\tau}</math></b>	0.058 (0.016)**	0.382 (0.104)**	0.062	0.065 (0.012)**	0.066 (0.011)**	0.062 (0.017)**	0.061 (0.023)**
<b>log <math>s_{h,it-\tau}</math></b>	0.041 (0.014)**	0.405 (0.085)**	0.066	0.040 (0.011)**	0.038 (0.010)**	-0.028 (0.020)	-0.087 (0.030)**
<b>log(n+g+<math>\delta</math>)<sub>it-<math>\tau</math></sub></b>	-0.214 (0.111)*	-2.173 (0.734)**	-0.355	-0.385 (0.091)**	-0.284 (0.078)**	-0.421 (0.153)**	-0.016 (0.222)
<b>log <math>y_{it-\tau}</math></b>	0.960 (0.014)**	0.523 (0.078)**	0.922	0.940 (0.011)**	0.948 (0.010)**	0.796 (0.026)**	0.766 (0.039)**
<b># of Observations</b>	-	-	-	552	552	552	414
<b>Number of Countries</b>	69	69	69	69	69	69	69
<b>Implied <math>\lambda</math></b>	0.82%	1.62%	1.62%	1.24%	1.07%	4.56%	5.33%

Standard errors in parentheses

\* significant at 10%; \*\* significant at 1%

All regressions include time effects (except columns 1 and 2). Regressions include a constant term (not reported), where applicable.

a: Income at the end of each 5-year period in all but columns 2 and 2', where the dependent variable is income in 1999.

b: MRW estimates adjusted with  $\tau=5$  instead of  $\tau=40$  to ensure comparability of the point estimates with the other columns.