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ABSTRACT

Standard practice for the estimation of dynamic stochastic general equilibrium (DSGE) models maintains the assumption that economic variables are properly measured by a single indicator, and that all relevant information for the estimation is summarized by a small number of data series. However, recent empirical research on factor models has shown that information contained in large data sets is relevant for the evolution of important macroeconomic series. This suggests that conventional model estimates and inference based on estimated DSGE models might be distorted. In this paper, we propose an empirical framework for the estimation of DSGE models that exploits the relevant information from a data-rich environment. This framework provides an interpretation of all information contained in a large data set, and in particular of the latent factors, through the lenses of a DSGE model. The estimation involves Markov-Chain Monte-Carlo (MCMC) methods. We apply this estimation approach to a state-of-the-art DSGE monetary model. We find evidence of imperfect measurement of the model's theoretical concepts, in particular for inflation. We show that exploiting more information is important for accurate estimation of the model's concepts and shocks, and that it implies different conclusions about key structural parameters and the sources of economic fluctuations.

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1 Introduction

Recent macroeconomic research has devoted considerable efforts to the development and estimation of dynamic stochastic general equilibrium (DSGE) models that are internally consistent, and based on first principles. Some recent micro-founded DSGE models, which involve numerous frictions and various types of shocks, appear to replicate the data in important dimensions (see, e.g., Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2003, 2004)). For instance, Smets and Wouters (2004) report that a DSGE model with a wide range of shocks fits the data well and performs well in terms of out-of-sample forecasts. Motivated by these promising results, such models are now increasingly perceived as a valuable input to policy making.[1]

In estimating these models, researchers have so far maintained the assumption that *all* relevant information for the estimation is adequately summarized by a relatively small number (i.e., between three and seven) of data series.[2]

This is at odds, however, with the fact that central banks and financial market participants monitor and analyze literally hundreds of data series. Moreover, there is growing empirical evidence suggesting that a large set of macroeconomic variables may in fact be crucial to properly capture the economy's dynamics. In a macroeconomic forecasting context, Stock and Watson (1999, 2002) and Forni, Hallin, Lippi and Reichlin (2000) among others find that factors estimated from large data sets of macroeconomic variables lead to considerable improvements over small scale VAR models.[3] Bernanke and Boivin (2003) and Giannone, Reichlin and Sala (2004) show that this large information set appears to matter empirically to properly model monetary policy. Bernanke, Boivin and Eliasz (2005) argue that inference based on small-scale VARs, by omitting relevant

[1]For instance, the Bank of Canada is "completing the development of a new projection model—a sticky-price dynamic stochastic general-equilibrium (DSGE) model of the Canadian economy" (see http://www.bankofcanada.ca/en/fellowship/highlights_res.htm). Papers that study optimal policy in estimated medium-sized DSGE models include Juillard, Karam, Laxton and Pesenti (2005), Levin, Onatski, Williams and Williams (2005).

[2]The only exception we are aware of is Adolfson, Laséen, Lindé and Villani (2005) which estimates a relatively large model of a small open economy using fifteen data series. Leeper and Sims (1994) estimated their model with three and ten data series, but report that they were unable to obtain a reasonable fit with ten series.

[3]Stock and Watson (1999), comparing a wide range of inflation forecasting exercises, found that their best-performing forecast involves a composite index of aggregate activity based on 168 individual activity measures. They argue that the forecasting gains from using this index are economically large and statistically significant over the 1970-1996 sample period. Similar evidence has been found in Forni et al. (2001), Stock and Watson (2002), Bernanke and Boivin (2003) and Boivin and Ng (2003), among others.

information, may be importantly distorted. For instance, they provide evidence in favor of Sims' (1992) explanation of the "price puzzle," — i.e., the finding based on small-scaled VARs that prices tend to increase following a monetary policy tightening — according to which conventional VARs do not account for the information that the central bank has about future inflation. They show that the information from a large set of indicators is indeed important to properly identify the monetary transmission mechanism. These empirical models with large data sets remain however largely non-structural. This limits our ability to determine the source of economic fluctuations, to perform counterfactual experiments, or to analyze optimal policy.

Why would such information be relevant in the context of available DSGE models? If the model of the economy is well specified and all theoretical concepts are directly observed by the agents and the econometrician, there is no scope for using additional data series in the estimation of DSGE models. But if some of the key concepts of the model are imperfectly observed or if the data are informative about some exogenous shocks or other state variables, exploiting the information from additional series could be important.

While the assumption that some or all theoretical concepts are observed by the econometrician is routinely made in the estimation of DSGE models, it may not be realistic. The upper panel of Figure 1 plots three detrended quarterly measures of employment (in logs) from 1964 to 2002.[4] Figure 2 reports the (de-meaned) quarterly growth rates of popular price measures: the GDP deflator, the personal consumption expenditures (PCE) deflator, and the consumer price index (CPI). While these series display broadly similar patterns, they reveal noticeable differences from one quarter two the next. In fact, as indicated in the lower panels of the figures, the coherences – i.e., the correlations in the frequency domain – between any two of the series are high at low frequencies, but they are markedly lower, though nonzero, at higher frequencies. If all of the fluctuations in these indicators were attributable to fundamental macroeconomic shocks, one would expect for instance these indicators of inflation to move in sync. However, most high-frequency inflation spikes are not common to all three series considered.

Gaps between several indicators of macroeconomic variables may reflect measurement error.

[4] The measure SW is taken from Smets and Wouters (2004), and is described in section 3.2 below. The other two indicators represent employment numbers based respectively on the household and the establishment surveys.

For employment, the systematic discrepancies between its two main measures — one obtained from the establishment survey and the other from the population survey — which have received a lot of attention in the aftermath of the 2001 recession,[5] underscore the fact that employment is imperfectly measured.[6] Aggregate prices are also notoriously difficult to measure. One of the most commonly used measure, the CPI, has undergone various changes in methodology since the 1996 Boskin commission, to mitigate important shortcomings. But recent research emphasizes that the current CPI might still be subject to important biases, stemming for instance, from the difficulty of measuring quality improvements or properly adjusting for outlet substitution.[7] Imperfect measurement may also affect real output, consumption, investment, real wages.

At another level, there may be conceptual differences between the model variables and the data used to measure them. One could of course imagine macroeconomic models to be sufficiently detailed so as to specify a separate role for, e.g., each of the available price indices (such as the GDP deflator, PCE deflator, CPI, core-CPI, and so on). In practice, however, this distinction is rarely made, as there are advantages to analyzing relatively simple models. If follows that researchers often pick a particular price index in a more or less arbitrary way.

Failing to account empirically for the imprecise link between theoretical concepts and observable macroeconomic data can invalidate model estimation and the assessment of whether a particular theory fits the facts. Following Sargent (1989), this has led some researchers to recognize explicitly the presence of measurement error in their empirical framework.[8] However, even when they allow for measurement error, all existing studies that estimate structural models are, to our knowledge, based on at most a *single* (and sometimes arbitrary), observable time series corresponding to each variable of the model. That is, whether or not one considers measurement error in the model estimation, it is typically assumed that a small number of data series contain *all* available information about concepts of the model such as output and inflation.

[5] See, e.g., Bernanke (2004).

[6] The BLS actually reports standard errors for the employment measures based on both surveys in the Employment Report. The non-farm payroll employment number, being based on a larger sample, is statistically more precise. But it is also subject to biases, such as the double-counting of jobs.

[7] See Hausman (2003), Hausman and Leibtag (2004) and Bils (2004).

[8] See, e.g., Altuğ (1989), McGrattan (1994), Anderson, Hansen, McGrattan and Sargent (1996), McGrattan, Rogerson and Wright (1997), Schorfheide (2000), Fernández-Villaverde and Rubio-Ramírez (2004). Another practical motivation for adding measurement error is to avoid the stochastic singularity problem that arises when there are fewer theoretical shocks than observable series.

Once one acknowledges that the data provides only an imperfect indicator of the concept, it is plausible to think that other data series carry useful additional information. Viewed in this light, existing estimations of DSGE models appear to be based implicitly on an arbitrary choice of data. Given each indicator-specific idiosyncrasy, properly exploiting the information from several indicators — rather than from a single one — should help to better separate an estimate of the economic concept (such as employment or inflation) from the indicator-specific "measurement error."[9] This should also provide us with a better estimate of the underlying economic shocks.

Moreover, some exogenous shocks or other state variables, that are typically assumed to be completely unobserved by the econometrician, might in fact be partially observed. One example is the productivity shock underlying many DSGE models. In existing estimations, it is treated as completely latent, which amounts to assuming implicitly that no observable measure contains independent information about this shock, beyond the handful of variable used in the estimation. But measures of labor productivity, oil prices, or commodity prices may all be correlated with total factor productivity, and thus serve as noisy indicators containing independent information.[10] In principle, since this could be the case for all exogenous shocks, many more indicators could carry important information for the estimation.

In this paper, we propose a general empirical framework to estimate DSGE models that exploits the information from a potentially large panel of data series in a systematic fashion. We relax the common assumption that theoretical concepts are properly measured by a *single* data series, and instead treat them as unobserved common factors for which observed data series are merely imperfect *indicators*. We also include information from indicators that potentially have an unknown relationship with the state variables of the model.

The proposed empirical framework has several advantages. First, as a consequence of the factor structure, the latent model concepts and the series-specific component (or "measurement errors") can be consistently identified from the cross-section of macroeconomic indicators, and not exclusively from the dynamic structure implied by the DSGE model. Consequently, unlike in the

[9] In the same spirit, Prescott (1986) used these two indicators to calibrate the labor elasticity of output in his RBC model.

[10] This is in part the rationale for the inclusion of commodity prices in VARs to "fix" the price puzzle (see Sims (1992)).

standard treatment mentioned above, allowing for measurement error does not necessarily help the model fit the data. Using multiple indicators in the model estimation also allows us to consider a potentially large number of imperfectly measured concepts without restricting the number of structural shocks that can be identified within the model. Rather than taking a stance on whether "measurement errors" *or* structural shocks should be part of the model, we can remain agnostic and determine empirically their relative importance. A by-product is an empirical assessment of the information content of each indicator.

Second, we can exploit the information from indicators that are not directly and unambiguously linked to a specific concept of the model. If the additional information considered is relevant, it should make our estimation more efficient. This is particularly important to determine more accurately the state of the economy, and helps in forecasting.

Third, our framework can be interpreted as a dynamic factor models à la Stock and Watson (1999, 2002), Forni et al. (2000), Forni, Lippi and Reichlin (2003), Bernanke, Boivin and Eliasz (2005), and Giannone, Reichlin and Sala (2004), in which we impose the full structure of the DSGE model on the transition equation of the latent factors. This has the added benefit of allowing us to provide a very clear economic interpretation of all estimated latent factors: in our setup, these factors correspond to state variables of the model (i.e., predetermined variables or shocks). In contrast, in empirical studies of factor models, the latent factors do not have a clear interpretation, since they are identified only "up to a rotation." Taken as a whole, the set of the factors obtained in empirical studies spans the space of common components in the data, but each factor does not uniquely characterize a common component. Our framework, in contrast provides an interpretation of all information contained in a large data set through the lenses of a DSGE model.

The estimation involves Markov-Chain Monte-Carlo (MCMC) techniques which deal effectively with the dimensionality problem by working with marginal densities and avoiding gradient methods. Because of the large dimension of models in a data-rich environment, direct estimation by maximum likelihood is usually infeasible in practice. The specific algorithm that we propose extends the standard implementation of Bayesian MCMC methods[11] to account for the relationship between a potentially large number of indicators and a relatively small number of model concepts.

[11] See, e.g., Geweke (1999), Schorfheide (2000), Smets and Wouters (2003, 2004), Levin et al. (2005).

We apply our estimation procedure to a state-of-the-art DSGE model based on microeconomic foundations. The model is taken from Smets and Wouters (2004), which builds on the model of Christiano, Eichenbaum and Evans (2005). One important finding is that by considering information from a larger data set in our model estimation, and by relaxing the link between some indicators and the model's concepts, we are able to considerably improve the estimates of the model's latent concepts such as inflation, of state variables and shocks. Our results suggest that the additional information provided by the data-rich environment is highly relevant for the model estimation. Estimates of critical model parameters such as a pseudo-elasticity of intertemporal substitution in consumption, the degree of habit formation, the degree of inflation indexing as well as estimated variances of exogenous shocks differ importantly depending on the assumed link between theory and data. This arises even though the estimated latent variables display patterns generally consistent with the indicators typically used to measure them. The different estimates also imply very different conclusions about the sources of economic fluctuations. As more data series are used in the model estimation, we find that fewer shocks are necessary to explain economic fluctuations, and that shocks to the efficiency of investment goods become a main source of business cycle fluctuations.

The rest of the paper is structured as follows. Section 2 lays down the formal setup for an arbitrary linear(ized) DSGE model. It explains how we relate the structural model to the large data set, and discusses implications of the setup for a canonical real business cycle (RBC) model. The section then proceeds with a description of the general estimation methodology. Detailed information about the estimation is left in an appendix. Section 3, presents an application of our approach in the context of a state-of-the-art DSGE model, the model of Smets and Wouters (2004), and discusses the estimation results and their implications. Section 4 concludes.

2 Data-Rich Environment

We now present a formal framework that merges a general class of dynamic general equilibrium models with a data-rich empirical model. We then discuss the implications of this framework, both in general terms and in the context of a canonical RBC model.

2.1 General Framework

Let us consider a general linear (or linearized) rational expectations model of the form

$$AE_t \begin{bmatrix} z_{t+1} \\ Z_{t+1} \end{bmatrix} = B \begin{bmatrix} z_t \\ Z_t \end{bmatrix} + Cs_t \tag{1}$$

$$s_t = Ms_{t-1} + \varepsilon_t \tag{2}$$

where $E_t[x] \equiv E[x|\mathcal{I}_t]$ denotes the expectation of some variable x conditional on the information set $\mathcal{I}_t$ available at date t, z_t is a vector of non-predetermined endogenous variables, Z_t is a vector containing predetermined endogenous variables or lagged exogenous variables (i.e., satisfying $E_t Z_{t+1} = Z_{t+1}$), s_t is a vector of exogenous variables following the process (2), ε_t is a vector of mean-zero unforecastable exogenous disturbances (such that $E_t \varepsilon_{t+j} = 0$ for all $j > 0$) with a diagonal variance-covariance matrix Q, and A, B, C and M are conformable matrices of coefficients. Below, we will consider examples of structural dynamic general equilibrium models based on microeconomic foundations that can be cast in the form (1)–(2). Models with additional lags, lagged expectations, or expectations of variables father in the future can be written as in (1) by expanding the vectors z_t and Z_t appropriately. We assume that the information set in period t is $\mathcal{I}_t = \{z_\tau, Z_{\tau+1}, s_\tau, \varepsilon_\tau, \text{ for } \tau \leq t; A, B, C, Q\}$ so that all agents considered in the model are assumed to know the model, its parameters, and the realizations of all variables determined in the present and past.[12] We solve the model using standard numerical techniques,[13] and express the solution as

$$z_t = DS_t \tag{3}$$

$$S_t = GS_{t-1} + H\varepsilon_t, \tag{4}$$

[12] This can be generalized in various ways at the expense of complications for the estimation problem described below. One can for instance assume that some or all of the agents in the model also face imperfect information about the state of the economy, and thus need to solve a filtering problem (see, e.g., Pearlman, Currie, and Levine, 1986, Svensson and Woodford, 2003, 2004) that may or may not be the same as the one of the econometrician described below. The model could still be written in the form (1), except that the vectors z_t, Z_t, and s_t would include also estimates on the part of agents of the respective variables. We leave an analysis of imperfect information on the part of economic agents for future work.

[13] See, e.g., Blanchard and Kahn (1980), King and Watson (1998), Klein (1997), McCallum (1998), Sims (2000).

where

$$S_t \equiv \begin{bmatrix} Z_t \\ s_t \end{bmatrix}$$

is the state vector and the matrices D, G, H are function of the underlying model's structural parameters.

In many applications, the system (1) contains identities and Z_t includes redundant variables such as lags of variables in z_t. We will be interested in a subset F_t of the variables in z_t, S_t (all known at date t), which refers only to variables characterizing the economy in period t. The $(n_F \times 1)$ vector F_t will typically include endogenous variables of interest for which indicators are observable. Specifically, we define

$$F_t \equiv F \begin{bmatrix} z_t \\ S_t \end{bmatrix}$$

where F is a matrix that selects the appropriate elements of the vector $[z_t', S_t']'$. Given (3), we can rewrite the variables of interest as a linear combination of the state vector

$$F_t = \Phi S_t, \tag{5}$$

where

$$\Phi \equiv F \begin{bmatrix} D \\ I \end{bmatrix} \tag{6}$$

is entirely determined by the model parameters and the selection of variables in F_t. The evolution of F_t is given by (4)–(6).

In order to estimate the model we consider n_X observable macroeconomic variables collected in a vector X_t. We collect in a $n_{XF} \times 1$ subvector $X_{F,t} = \left[x_{F,t}^1, ..., x_{F,t}^{n_{XF}}\right]'$ the indicators of the variables of interest $F_t = \left[f_t^1, ..., f_t^{n_F}\right]'$, where $n_{XF} \geq n_F$, and assume that the observed indicators relate to the variables of the model according to

$$x_{F,t}^i = \lambda_F^i f_t^j + e_{F,t}^i \tag{7}$$

for $i = 1, ..n_{XF}$, $j = 1, ...n_F$, where for each i, λ_F^i is a coefficient, and $e_{F,t}^i$ denotes a mean-zero indicator-specific component, which may be viewed as representing measurement error or conceptual differences between the theoretical concept f_t^j and the respective indicator $x_{F,t}^i$. We omit throughout a constant to simplify the notation. We assume that these indicator-specific components are potentially serially correlated, but that they are uncorrelated across indicators. The set of equations (7) can be rewritten in matrix form as

$$X_{F,t} = \Lambda_F F_t + e_{F,t}, \tag{8}$$

where $e_{F,t}$ is a $n_{XF} \times 1$ vector of mean-zero indicator-specific and potentially serially correlated components, and Λ_F is an $(n_{XF} \times n_F)$ matrix of coefficients. As each element of $X_{F,t}$ is supposed to be an indicator of one of the elements of F_t, each row of the matrix Λ_F will have at most one nonzero element. However, to the extent that each variable in F_t can be imperfectly measured by many indicators, each column of Λ_F can have many nonzero elements.

The observation equation (8) is appropriate in the case that several observable indicators relate directly to the same variable of interest, and that each of the indicator-specific components is uncorrelated with that of other indicators. For instance, if inflation based on the personal consumption expenditure deflator and the CPI correspond to the same concept of inflation in the model, then one may want to include both indicators in $X_{F,t}$. However, if these indicators refer actually to different concepts, then at least one of them should not be included in $X_{F,t}$. Such an indicator, even though it does not relate directly to any variable in F_t should still depend on the evolution of the state vector S_t.

More generally, to the extent that the theoretical model is true, a potentially very large number of indicators observed — e.g., asset prices, commodity prices, monetary aggregates and so on — should depend on the state vector S_t. Again, it may be useful to consider such indicators in the estimation, as they may be informative about the state of the model economy. To exploit the information provided by such indicators in the model estimation, we assume that the remaining data series of X_t which do not correspond to any particular variable of F_t are collected in a $n_{XS} \times 1$

vector $X_{S,t}$ and are related to the state vector according to

$$X_{S,t} = \Lambda_S S_t + e_{S,t}, \tag{9}$$

where $e_{S,t}$ is a $n_{XS} \times 1$ vector of mean-zero components that are not related to the model's state vector, and Λ_S is an $(n_{XS} \times n_S)$ matrix of coefficients. Equation (9) allows all indicators not associated with any particular variable of the model to potentially provide information about the state vector S_t. We propose to capture the information from the data in $X_{S,t}$ in a non-structural way, letting the weights in Λ_S be determined by the data.

While the weights Λ_F relating the variables of interest to their indicators can be interpreted as structural — i.e., policy invariant — the weights Λ_S relating the state vector to all other indicators do not need to be so.[14] Even though (9) may not be reliable to determine the effects of alternative policies on the variables in $X_{S,t}$, information about these variables can be very useful for the estimation of the state vector and model parameters under *historical* policy. Once the state vector and model parameters are correctly estimated — using the information provided by (9) — counterfactual exercises can legitimately be performed for all variables F_t, S_t, $X_{F,t}$, without using (9) any more.

Combining (8)–(9) and using (5), we obtain the *observation equation*

$$X_t = \Lambda S_t + e_t \tag{10}$$

where

$$X_t \equiv \begin{bmatrix} X_{F,t} \\ X_{S,t} \end{bmatrix}, \qquad e_t \equiv \begin{bmatrix} e_{F,t} \\ e_{S,t} \end{bmatrix}, \qquad \Lambda \equiv \begin{bmatrix} \Lambda_F \Phi \\ \Lambda_S \end{bmatrix}.$$

We assume that indicator-specific components $e_{F,t}$ and $e_{S,t}$ are uncorrelated across indicators but

[14] In fact the weights Λ_S mix the weights that the variables in $X_{S,t}$ would attribute to their theoretical counterpart, with the coefficients that relate these theoretical concepts to the state vector S_t.

serially correlated, so that

$$e_{F,t} = \Psi_F e_{F,t-1} + v_{F,t} \tag{11}$$

$$e_{S,t} = \Psi_S e_{S,t-1} + v_{S,t} \tag{12}$$

where the vectors $v_{F,t}$ and $v_{S,t}$ are assumed to be normally distributed with mean zero and variance R_F and R_S, respectively, and where the matrices R_F, R_S and Ψ_F, Ψ_S are assumed to be diagonal.[15]

Our empirical model consists of the transition equation (4) — which is fully determined by the underlying DSGE model —, the selection equation (5), and the observation equation (10)-(12) which relates the model's theoretical concepts to the data. It contains as an important special case the measurement error framework proposed by Sargent (1989). In the latter framework, each variable in F_t corresponds to a unique observable indicator in $X_{F,t}$, so that the observation equation reduces to $X_t = F_t + e_t = \Phi S_t + e_t$. In this case $n_{XS} = 0$, $\Lambda_F = I_{n_F}$, $\Lambda = \Phi$. A further trivial special case is one in which model variables are assumed to be directly measured, so that the observation equation reduces to $X_t = F_t = \Phi S_t$, as in most existing estimations of DSGE models.

The key innovation here is to generalize Sargent (1989)'s framework to the case where the vector of observables, X_t, may be much larger than the vector F_t of variables in the model, i.e. $n_X >> n_F$, and that their exact relationship, summarized by Λ, may be partially unknown. The interpretation is that this large number of macroeconomic variables are noisy indicators of model concepts and thus share some common sources of fluctuations. This implies an observation equation with a factor structure similar to the one assumed in the recent non-structural empirical literature which uses a large panel of macroeconomic indicators. However, an important difference with this literature is that, in the present framework, the evolution of the unobserved common components obeys the

[15] We may allow the vector $e_{S,t}$ to be correlated across indicators, as we may want to include in the vector $X_{S,t}$ indicators that are driven by some common factors which are not included in the model's vector of state variables. This could happen for instance if several indicators included in $X_{S,t}$ are part of a same category of indicators, but that their theoretical counterpart is not fully fleshed out in the model. In this case we would assume that the component of these indicators which is not correlated with the model's state vector has the following factor structure

$$e_{S,t} = \Gamma S_{e,t} + \tilde{e}_{S,t}$$

where $\tilde{e}_{S,t}$ is a $n_{XS} \times 1$ vector of mean-zero indicator-specific (i.e., uncorrelated across indicators) and potentially serially correlated components, and $S_{e,t}$ is a vector of common components in the set of indicators $X_{S,t}$, which are uncorrelated with the model's state vector S_t.

structure of a DSGE model.

The use of large information sets provides our framework with two important advantages over the existing implementation of DSGE model estimation. First, as the latent variables and the measurement can be identified from the cross-section of macroeconomic indicators, it allows one to identify a much richer pattern of "measurement errors," even in the presence of many structural shocks. This reduces the risk of biased estimation. Second, it has the potential to yield a more efficient estimation procedure. To illustrate these points, consider the following special case of the framework presented above. Suppose that, according to theory, a variable of interest, f_t, satisfies

$$f_t = \rho f_{t-1} + \eta_t, \tag{13}$$

where $|\rho| < 1$ and the exogenous disturbance η_t is iid.[16] Suppose moreover that we observe an indicator x_{1t} of f_t. In the case that x_{1t} constitutes a perfect measure of f_t, i.e., that the observation equation (10) is trivially $x_{1t} = f_t$, the variable of interest f_t is known, and the parameter ρ can easily be estimated by OLS or maximum likelihood. Suppose instead that x_{1t} is a noisy indicator of f_t and that the observation equation takes the form

$$x_{1t} = f_t + e_{1t} \tag{14}$$

where e_{1t} is iid.[17] In the case that $\rho \neq 0$, standard techniques such as proposed Sargent (1989) can be applied to estimate $\hat{f}_t$ and disentangle it from the "measurement error," using the Kalman filter. For this to work, however, we need the stochastic process of f_t to be different from the one that drives the measurement error. In contrast, when $\rho = 0$, standard techniques cannot be applied to recover the variable of interest f_t, as $x_{1t} = \eta_t + e_{1t}$ is the sum of two variables with the same stochastic process.[18] However, if one or more additional indicators

$$x_{it} = f_t + e_{it} \tag{15}$$

[16] This is a special case of (4)–(5), where $f_t = F_t = S_t$, $\varepsilon_t = \eta_t$, $\Phi = 1$, $G = \rho$ and $H = 1$.

[17] This is a special case of (10) where $X_t = x_{1t}$, $\Lambda_F = 1$, $\Lambda = \Phi = 1$, and $e_t = e_{1t}$.

[18] The likelihood function in this case involves the sum of the variances of η_t and e_{1t}, so that each variance cannot be identified separately.

for $i = 2, .., n_X$ are available, then it is possible to estimate f_t even if it is serially uncorrelated. In fact, f_t is a common factor that can be identified through the cross section, on the basis the observation equations (14)–(15), while the dynamic model (13) is used for identification of the shocks η_t.

More generally, when no more than one indicator is used for any concept of the model — i.e., when $n_X = n_F$, as in existing implementations — both the structural shocks and the unobserved variables have to be identified entirely from the restricted dynamics of the DSGE model, summarized by equations (4)–(5). In that case, having more structural shocks in the model limits the number of independent sources of measurement errors that can be contemplated and it is difficult to formally test whether the resulting model is properly identified or not. Typically, researchers avoid these problems by assuming either no measurement error or few structural shocks. But as argued in the introduction, measurement error or conceptual differences between the measured indicators and the theoretical variables might be quite prevalent, and if so, ignoring them would lead to biased inference.

In contrast, one key feature of factor models with multiple indicators is that the factors can be identified by the cross-section of macroeconomic indicators alone. This implies that in our framework with a factor structure, the large number ($n_X >> n_F$) of indicators provides enough restrictions to identify the latent variables, and the series-specific terms from the observation equation (10). As a result, we can allow for a large amount of measurement errors without restricting in any way the number of structural shocks that can be identified in the model. Rather than taking a stance on which source of variations should be part of the model, we can remain agnostic and determine empirically their importance.

Even when the factors can be identified solely from the model dynamics, as in Sargent (1989), considering the information from the large data set provides another important advantage, namely efficiency of the factor estimation. A key property of factor models is that the variances of the factor estimates are of order $1/n_X$ where n_X is again the number of indicators in X_t. A consistent estimate of the factors can thus be obtained as $n_X \longrightarrow \infty$ (see Forni et al. (2000), and Stock and Watson (2002), Bai and Ng (2004).) This suggests that exploiting information from a large number of macroeconomic indicators can reduce considerably the uncertainty in the estimated latent variables, which in turn implies a more efficient estimation of model parameters. Estimation efficiency is then important, in particular for forecasting exercises and policy analysis, as forecasting performance is directly related to precision in model estimates.

It is important to note that by expanding the vector X_t of indicators we are not facilitating the model's ability to fit the data. To the contrary, given the factor structure, the more indicators we have in X_t, the more we require the state variables (here $\hat{k}_t$ and a_t) to explain the common components in the data series, while at the same time satisfying their law of motion given by (20).

2.2 An Illustrative Example: A Simple RBC Model

To clarify how the empirical framework just discussed can be applied to the estimation of a DSGE model, we first discuss a simple example, the canonical RBC model (see, e.g., King, Plosser and Rebelo (1988)). This model allows us also to relate to much of the literature on estimated DSGE models which has often considered variants of the basic RBC model. In section 3, we estimate a more elaborate model that adds numerous frictions to a RBC model of this kind. In the basic RBC model considered here, households maximize their lifetime utility which depends on consumption, c_t, and leisure, $1 - l_t$,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + v \log(1 - l_t)\right], \qquad 0 < \beta < 1,\ v > 0 \tag{16}$$

subject to the following restrictions

$$e^{a_t} k_t^{1-\alpha} l_t^{\alpha} = c_t + k_{t+1} - (1-\delta) k_t, \qquad 0 < \alpha < 1,\ 0 < \delta < 1 \tag{17}$$

$$a_t = \rho a_{t-1} + v_t, \qquad 0 < \rho < 1 \tag{18}$$

where the exogenous productivity shock a_t follows a mean-zero AR(1) process. Equation (17) indicates that output, which is generated using the capital stock k_t (chosen at date $t-1$), hours worked, l_t, and total factor productivity, a_t, is the sum of private consumption and gross investment. Solving this household problem yields a set of first-order necessary conditions which, together with (17) and a transversality condition, characterize the equilibrium evolution of the variables c_t, l_t, and k_t, for given exogenous disturbances and an initial value of the capital stock. As is well known, this model admits a unique deterministic steady state in which all endogenous variables remain constant. As a closed-form solution does generally not exist, the model is commonly log-linearized around the steady state. The model's approximate dynamics around the steady state can be written as

$$\begin{bmatrix} \hat{l}_t \\ \hat{c}_t \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ a_t \end{bmatrix} \quad (19)$$

$$\begin{bmatrix} \hat{k}_t \\ a_t \end{bmatrix} = \begin{bmatrix} g & h\rho \\ 0 & \rho \end{bmatrix} \begin{bmatrix} \hat{k}_{t-1} \\ a_{t-1} \end{bmatrix} + \begin{bmatrix} h \\ 1 \end{bmatrix} v_t \quad (20)$$

which is of the form (3)–(4) with $z_t = \left[\hat{l}, \hat{c}_t\right]'$, $S_t = \left[\hat{k}_t, a_t\right]'$, and the matrices D, G, H are function only of the model parameters. Here, the circumflex denotes percent deviations from the steady state (e.g., $\hat{c}_t \equiv \log(c_t/\bar{c})$).

To illustrate the richness of our empirical framework, we consider several variants of the *observation* equation (10).

No measurement error. A common approach to the estimation of DSGE models is to suppose that we have perfect indicators of the variables of interest. In the case that an indicator $X_t = hours_{1t}$ (e.g., based on the establishment survey) is viewed as measuring perfectly the concept $\hat{l}_t$, we may write the selection equation (5) as $F_t = \hat{l}_t = [d_{11},\ d_{12}]\, S_t$, so that the observation equation (10) reduces to

$$hours_{1t} = \hat{l}_t = d_{11}\hat{k}_t + d_{12}a_t. \quad (21)$$

In this case, estimation of the model (19)–(20) with the above observation equation would attribute all variations in the indicator $hours_{1t}$ to the only source of exogenous fluctuations, the productivity shock.[19]

[19] Given that the RBC model considered here has only one source of exogenous fluctuations, using more than one observable series would result in the model rejection, in the absence of measurement error. In fact, as Ingram et al. (1994) point out, since the number of exogenous disturbances is smaller than the number of endogenous variables, one can find particular combinations of endogenous variables that are deterministic, so that their variance-covariance matrix is singular. The model is said to be stochastically singular in this case. As this is not true in the data, the model is sure to be rejected.

Standard treatment of measurement error. In the case that $hours_{1t}$ is considered as a noisy indicator of hours, the observation equation needs to be augmented with a measurement error term,

$$hours_{1t} = \hat{l}_t + e_{1t} = d_{11}\hat{k}_t + d_{12}a_t + e_{1t},$$

and the standard approach proposed by Sargent (1989) is commonly applied.[20] According to this approach, the restrictions of the dynamic model and the Kalman filter are used to estimate the unobserved variables $\hat{l}_t, \hat{k}_t, a_t$ and the measurement error. However, as illustrated in the previous simple example, such an approach may have trouble disentangling the structural disturbances — the innovations to a_t — from the measurement error, e_t, and thus may not be able to identify the latent variable of interest, $\hat{l}_t$. Unfortunately, it is difficult to test in practice whether or not the latent variables and the model parameters are actually identified.

An alternative treatment of noisy indicators: Using multiple indicators of given concepts. Once one recognizes that the data often contains *noisy indicators* of the concepts that one seeks to measure, there is scope for using additional indicators to get better estimates of the model's parameters and concepts. This can be done generally and systematically in our empirical framework. It suffices to include all relevant indicators in the vector X_t, and to let them be related to the respective concepts in F_t. For instance, while the establishment survey may provide a good indicator of the concept of hours worked ($hours_{1t}$), it is likely to include measurement error that is uncorrelated with measurement error in the hours as implied by the household survey ($hours_{2t}$). Accounting for the information contained in these two measured series may thus help us get a better estimate of the concept of hours worked and the model parameters.[21] The observation (10) then takes the form

$$\begin{bmatrix} hours_{1t} \\ hours_{2t} \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda \end{bmatrix} \hat{l}_t + e_t = \begin{bmatrix} d_{11} & d_{12} \\ \lambda d_{11} & \lambda d_{12} \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ a_t \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}.$$

[20] See, e.g., Altuğ (1989), McGrattan (1994), Hall (1996), McGrattan, Rogerson and Wright (1997), and Ireland (2004).

[21] In the same spirit, Prescott (1986) used these two indicators of hours worked to calibrate the labor elasticity of output in his RBC model.

All indicators of hours are thus assumed to have one common factor, $\hat{l}_t$, on which they "load" with a particular weight. We typically normalize one of the loading coefficients to 1 so as to normalize the scale of the fluctuations in the latent variable $\hat{l}_t$ to be of the same order of magnitude as the respective indicator, but leave the other loading coefficient free to be estimated, in case the fluctuations in the second indicator are of a different magnitude. By exploiting both the cross-sectional and the time series characteristics of the data, and noting that the latent variables are assumed to generate the common variation in both indicators, while the "measurement error" is specific to each series, we can more easily estimate the latent variables here than in the standard treatment of measurement error. The observation equation mentioned here only exploits information about employment, but one could easily augment it with indicators of other variables.

Using information to estimate the state vector through an unknown link. So far, we have assumed that Λ_S is a zero matrix. We have thus implicitly assumed, as do current estimations of DSGE models, that the data series in $X_{S,t}$, which do not measure any specific variable of the vector F_t — here, hours worked — do not contain any additional information about the remaining latent variables.[22] However, if the theoretical model is true, all economic data series should at least partly determined by the state vector. Data on stock prices, commodity prices, oil prices, monetary aggregates and so on could thus be informative about the current state of the economy, even though the model does not explicitly specify model such concepts. Exploiting their information content should result in a more efficient estimation. In our simple example, if oil prices ($Poil_t$) are systematically related to the state vector of the model economy, we can augment our observation equation as follows

$$\begin{bmatrix} hours_{1t} \\ hours_{2t} \\ Poil_t \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ \lambda d_{11} & \lambda d_{12} \\ \lambda_{S1} & \lambda_{S2} \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ a_t \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix}$$

where the new parameters λ_{S1} and λ_{S2} are to be estimated.

[22] Several studies, including Christiano (1988), Altuğ (1989) and McGrattan (1994), assume that the capital stock is observed, so that it would be in F_t. They however assume that other variables are latent. McGrattan (1994), for instance, using a more elaborate variant of the RBC model presented here assumes that output, investment, government purchases, hours of work, the capital stock and various tax rates are observed, while housing starts and past hours (weighted) are assumed to be latent.

2.3 Estimation Procedure

We now discuss the general procedure for the estimation of the parameters and the latent variables (in z_t, Z_t, s_t) of the structural model (1). This model results in an equilibrium characterized by (3)–(5). We suppose that the observation equation takes the form (10), where we allow X_t to potentially contain a rich set of macroeconomic indicators, and where Λ involves possibly few *a priori* restrictions. Doing so obviously comes at a cost. The high-dimensionality of the problem and the presence of unobserved variables considerably increase the computational burden of the estimation. In particular, methods that rely on explicitly maximizing the likelihood function or the posterior distribution appear impractical (see Bernanke, Boivin and Eliasz (2005)).

To circumvent this problem, we consider a variant of a Markov Chain Monte Carlo (MCMC) algorithm.[23] There are two key general features of these simulation-based techniques that help us in the present context. First, rather than working with the likelihood or posterior directly, these methods approximate the likelihood with empirical densities, thus avoiding gradient methods. Second, by exploiting the Clifford-Hammersley theorem, these methods sample iteratively from a complete set of conditional densities, rather than from the joint density of the parameters and the latent variables. This is particularly useful when the likelihood is not known in closed form, as it is the case in our application. Moreover, by judiciously choosing the break up of the joint likelihood or posterior distribution into the set of conditional densities, the algorithm deals effectively with the high dimensionality of the estimation problem.

Like in existing Bayesian implementations of the MCMC algorithm, the structural parameters of equation (1) are drawn using a Metropolis step, since their distribution conditional on the unobservable state variables and the parameters of equations (3)–(4) are not known in closed form. The unobservable states are drawn using Carter and Kohn (1994) forward-backward algorithm. The remaining parameters are drawn directly from their known conditional distributions. The precise description of the algorithm is provided in Appendix A.

[23] See Johannes and Polson (2004) for a survey of these methods and Geweke (1999). Recent applications to the estimation of DSGE models include DeJong, Ingram and Whiteman (2000), Schorfheide (2000), Otrok (2001), Smets and Wouters (2003, 2004), Fernández-Villaverde and Rubio-Ramírez (2004), Justiniano and Preston (2004), and Justiniano and Primiceri (2006).

3 Application: Estimating a DSGE Model

3.1 Model

We now apply the data-rich environment just described to a state-of-the-art DSGE model based on microeconomic foundations. The model that we consider is taken from Smets and Wouters (2004). It builds on the canonical RBC model presented in the previous section, as well as Rotemberg and Woodford (1997), Christiano, Eichenbaum and Evans (2005) and others, by adding various frictions and allowing for nine different types of exogenous disturbances. The canonical RBC model can be viewed as a special case of the Smets and Wouters (2004) model in the absence of frictions and of shocks, except for the total factor productivity shock. The Smets and Wouters model has received much attention recently, in part because of its success in fitting actual data, both in the U.S. and in the Euro area (see Smets and Wouters, 2003, 2004). As Smets and Wouters (2004) report, this micro-based model performs also surprisingly well in terms of out-of-sample predictions, in some cases outperforming standard VAR and Bayesian VAR models.

A derivation of the non-linear model from first principles can be found in Smets and Wouters (2004). Here, we merely summarize the important log-linearized equilibrium conditions of the model. The model involves optimizing households that consume goods and services, supply specialized labor on a monopolistically competitive labor market, rent capital services to firms, and decide how much capital to accumulate. Firms choose the desired level of labor and capital inputs, and supply differentiated products on a monopolistically competitive goods market. Prices and wages are re-optimized at random intervals as in the Calvo (1983) model. When they are not re-optimized, prices and wages are partially indexed to past inflation rates. While Smets and Wouters (2004) assume an exogenously moving inflation target, so as to allow in a crude way for changes in the monetary policy rule and in average inflation over the 1957 - 2002 period, we do not consider such time-varying target, as we estimate the model using data from 1982 to 2002.

More precisely, the model assumes that there exists a continuum of households who derive utility from consumption and leisure. The utility function is non separable in consumption and leisure as in King, Plosser and Rebelo (1988) and Basu and Kimball (2000), to allow for a steady state growth path driven by labor-augmenting technological progress, and involves consumption in excess of an external, time-varying habit stock. While households may be heterogenous regarding their wage profile and hours worked, there exists a complete set of state-contingent securities which allows households to pool their risks, so that they all make the same consumption and investment decisions. The Euler equation for optimal consumption decisions log-linearized around the deterministic steady state with constant growth and zero inflation is given by

$$\begin{aligned} C_t &= \frac{h}{1+h} C_{t-1} + \frac{1}{1+h} E_t C_{t+1} + \frac{\sigma_c - 1}{\sigma_c (1+\lambda_w)(1+h)} (L_t - E_t L_{t+1}) \\ &\quad - \frac{(1-h)}{(1+h)\sigma_c} (i_t - E_t \pi_{t+1}) + \varepsilon_t^b \end{aligned} \tag{22}$$

where C_t and L_t represent percent deviations of consumption and hours worked from their respective steady state, i_t denotes deviations of the quarterly nominal interest rate from its steady-state level, and π_t is quarterly inflation. The parameter $h \in (0,1)$ measures the degree of habit formation and $\sigma_c > 0$ indicates the curvature of the utility function with respect to consumption, and $(1 + \lambda_w)$ is the steady-state markup of the real wage due to market power on the labor market. In the absence of habit formation, (22) states that consumption depends negatively on the ex-ante real interest rate with a coefficient σ_c^{-1} (corresponding to the elasticity of intertemporal substitution) and positively on expected future consumption. When $h > 0$, current consumption is also higher the higher past consumption. When $\sigma_c > 1$, hours worked and consumption are complementary.[24] Finally the exogenous disturbance ε_t^b is a preference shocks that affects the entire utility function and that is assumed to follow and AR(1) process with degree of serial correlation ρ_b.

On the labor market, households are assumed to re-optimize their wages given the demand for their labor services, with a probability $1 - \xi_w$. When choosing their optimal wage they take into account the probability that wages will not be re-optimized for some periods. Whenever they cannot re-optimize their wages, they index them to lagged inflation with a degree of indexation $\gamma_w \in (0,1)$. Optimal wage setting by households results in the following aggregate linearized equation for the

[24] See Basu and Kimball (2000) for details.

real wage

$$\begin{aligned} w_t &= \frac{\beta}{1+\beta}E_t w_{t+1} + \frac{1}{1+\beta}w_{t-1} + \frac{\beta}{1+\beta}E_t \pi_{t+1} - \frac{1+\beta\gamma_w}{1+\beta}\pi_t + \frac{\gamma_w}{1+\beta}\pi_{t-1} \\ &\quad - \frac{\lambda_w (1-\beta\xi_w)(1-\xi_w)}{(1+\beta)(\lambda_w + (1+\lambda_w)\sigma_L)\xi_w}\left[w_t - \sigma_L L_t - \frac{\sigma_c}{1-h}(C_t - hC_{t-1}) + \varepsilon_t^L\right] + \eta_t^w \end{aligned} \tag{23}$$

where w_t is the percent deviation of the real wage from the steady state path, ε_t^L is a shock to the disutility of labor, which follows an AR(1) process with degree of serial correlation ρ_L, and η_t^w is an iid shock to the wage mark-up. The parameter $\beta \in (0,1)$ is the subjective discount factor, $\sigma_L^{-1} > 0$ is the elasticity of work effort with respect to the real wage. The term in square brackets corresponds to the gap between the actual real wage and the real wage that would prevail in the case of flexible prices and flexible wages. A positive gap tends to reduce the actual real wage, and the effect is stronger the smaller the degree of wage rigidity, ξ_w, the lower demand elasticity for specialized labor, $(1+\lambda_w)/\lambda_w$, and the higher the elasticity of labor supply with respect to the real wage, σ_L^{-1}.

Households choose the capital stock which they rent to firms. To increase the supply of capital services, they can either invest in future capital, or increase the utilization rate of installed capital. Investment in capital takes one period to be installed and involves adjustment costs which assumed to be function of the change in investment, as in Christiano, Eichenbaum and Evans (2005). As in Greenwood, Hercowitz and Krusell (1998) and Fisher (2002), the relative efficiency of investment goods is also assumed to be affected by an exogenous shock ε_t^I which follows an AR(1) process with degree of serial correlation ρ_I. The log-linearized Euler equation for optimal investment is given by

$$I_t = \frac{1}{1+\beta}I_{t-1} + \frac{\beta}{1+\beta}E_t I_{t+1} + \frac{1/\varphi}{1+\beta}\left(Q_t + \varepsilon_t^I\right) \tag{24}$$

where I_t denotes real investment and Q is the real value of capital, in percent deviations from steady state, and φ is a measure of adjustment costs. The real value of capital follows in turn

$$Q_t = -(i_t - E_t\pi_{t+1}) + \frac{1-\tau}{1-\tau+\bar{r}^k}E_t Q_{t+1} + \frac{\bar{r}^k}{1-\tau+\bar{r}^k}E_t r_{t+1}^k + \eta_t^Q, \tag{25}$$

so that the real value of capital relates negatively on the ex-ante real rate of interest, and positively on the expected future real value of capital and the expected future rental rate of capital r_t^k. The mean rental rate of capital $\bar{r}^k$ and the depreciation rate of capital, τ, are assumed to satisfy $\beta = 1/\left(1 - \tau + \bar{r}^k\right)$. The exogenous shock η_t^Q, assumed to be iid, is meant as a shortcut for changes in the external finance premium. The capital accumulation equation then involves both the flow of investment, and its relative efficiency

$$K_t = (1 - \tau) K_{t-1} + \tau I_{t-1} + \tau \varepsilon_{t-1}^I. \tag{26}$$

There is a continuum of firms that hire aggregates of labor and capital (adjusted for effective utilization) as inputs, combine them using a Cobb-Douglas production function with constant returns to scale, and a capital share $\alpha \in (0, 1)$, and supply a differentiated intermediate good on a monopolistically competitive market. In producing their goods, all intermediate firms face a fix cost and a common stationary technology shock, ε_t^a, assumed to be AR(1) with degree of serial correlation ρ_a, and labor augmenting technological progress growing at a constant rate. Intermediate goods are then aggregated into a single final good used for consumption or investment. Minimizing the firms' cost of production results in the linearized demand for labor

$$L_t = -w_t + (1 + \psi) r_t^k + K_{t-1}. \tag{27}$$

This implies that for a given stock of capital, the labor demand depends negatively on the real wage and positively on the capital stock and the rental rate of capital, where $\psi > 0$ is the inverse of the elasticity of the capital utilization cost function.

Similarly to households on the labor market, firms are assumed to re-optimize their prices given the demand for their goods, with a probability $1 - \xi_p$. When they cannot re-optimize their prices, they index them to lagged inflation with a degree of indexation $\gamma_p \in (0, 1)$. Optimal price setting by firms results in the following aggregate linearized equation for inflation

$$\pi_t = \frac{\beta}{1 + \beta\gamma_p} E_t \pi_{t+1} + \frac{\gamma_p}{1 + \beta\gamma_p} \pi_{t-1} + \frac{\left(1 - \beta\xi_p\right)\left(1 - \xi_p\right)}{\left(1 + \beta\gamma_p\right)\xi_p} \left[\alpha r_t^k + (1 - \alpha) w_t - \varepsilon_t^a\right] + \eta_t^p. \tag{28}$$

As in the canonical New Keynesian supply equation, actual inflation depends on expected future inflation and on the marginal cost, here represented by the expression in brackets. The marginal cost depends in turn on the real rental rate of capital, the real wage, the productivity shock. To the extent that prices are indexed, current inflation is also affected by lagged inflation. The exogenous shock η_t^p is assumed to be iid and refers to exogenous fluctuations in the price mark-up.

The linearized goods market equilibrium condition can then be written as

$$\begin{aligned} Y_t &= (1 - \tau k_y - g_y) C_t + \varepsilon_t^G + \tau k_y I_t + \bar{r}^k k_y \psi r_t^k && (29) \\ &= \phi \left[\varepsilon_t^a + \alpha K_{t-1} + \alpha \psi r_t^k + (1 - \alpha) L_t \right] && (30) \end{aligned}$$

where k_y is the steady-state capital-output ratio, g_y is the steady-state government spending-output ratio, and ϕ is one plus the share of fixed cost in production, and ψ is again the inverse of the elasticity of the capital utilization cost function. Government spending (in percent deviation from steady state, times g_y), ε_t^G, is assumed to evolve exogenously and to follow and AR(1) process with serial correlation ρ_G. While the first equation corresponds to the aggregate demand side for output, the second equation results from aggregate production.[25]

The model is closed with a specification of an empirical monetary policy reaction function. Here, we assume that monetary policy follows the generalized Taylor rule

$$i_t = (1 - \rho) \left[r_{\pi 0} \pi_t + r_{\pi 1} \pi_{t-1} + r_{y0} Y_t + r_{y1} Y_{t-1} \right] + \rho i_{t-1} + \eta_t^i \qquad (31)$$

where η_t^i is an iid monetary policy shock. The specification considered here differs slightly from the one in Smets and Wouters (2004): while we suppose that the central bank responds to actual output fluctuations (in deviations from the steady-state trend), Smets and Wouters (2004) assume that the central bank responds to deviations of output from the output that would obtain in the case of flexible prices and flexible wages.[26]

The model is thus summarized by the ten equations (22)–(31). It involves ten endogenous

[25] In the first equation, we corrected the equilibrium condition indicated in Smets and Wouters (2004), adding the term $\bar{r}^k k_y \psi r_t^k$ as in Onatski and Williams (2004).

[26] Their "output gap" may be considered more appropriate as it corresponds to the welfare relevant output gap, in the context of this model. It however differs substantially from empirical measures of "output gap" or the CBO's measure. In addition, their measure of output gap requires the specification of a significantly larger model, as the flexible-price, flexible-wage counterpart to the equations mentioned above need to be adjoined to the model, to determine the flexible-price, flexible-wage level of output.

variables Y_t, C_t, I_t, L_t, K_t, Q_t, r_t^k, π_t, w_t, i_t, and nine exogenous disturbances, five of them autocorrelated (ε_t^a, ε_t^b, ε_t^G, ε_t^L, ε_t^I) and four iid ($\eta_t^Q, \eta_t^w, \eta_t^p, \eta_t^i$). The system can then be written as in (1), and can be solved using numerical techniques to obtain a solution of the form (3)–(4), where z_t is a vector of endogenous non-predetermined variables, Z_t contains predetermined endogenous variables as well as lagged exogenous variables, and ε_t is the vector of innovations to the 10 shocks. In the estimation, we will use indicators of the following vector of variables of interest

$$F_t = [i_t, Y_t, C_t, I_t, \pi_t, w_t, L_t]'.$$

This vector is related to the state vector

$$S_t = \left[i_{t-1}, Y_{t-1}, C_{t-1}, I_{t-1}, \pi_{t-1}, w_{t-1}, K_{t-1}, \varepsilon_t^a, \varepsilon_t^b, \varepsilon_t^G, \varepsilon_t^L, \varepsilon_t^I, \eta_t^Q, \eta_t^p, \eta_t^w, \eta_t^i, \varepsilon_{t-1}^I\right]'$$

through (5)-(6). This state vector follows a law of motion of the form (4).

3.2 Implementation of the Estimation

3.2.1 Data

Smets and Wouters (2004) estimate their model using quarterly U.S. data starting in 1957. However, given the evidence provided about the instability of interest rate rules of the form (31), especially around the end of the 1970's and early 1980's (see, e.g., Clarida, Galí and Gertler (2000), Boivin (2005), Boivin and Giannoni (2006)), we estimate the model starting in 1982:1 and ending in 2002:3. Our large data set contains 91 macroeconomic indicators.[27] Details are provided in Appendix B.

Seven data series, included in a vector $X_{1,t}$ are however worth emphasizing as they are used to normalize the seven concepts of the model included in the vector F_t. They are the series used by Smets and Wouters (2004) for the estimation of their model. For each of these series, we normalize the corresponding weights in Λ to 1. Output (Y_t) is normalized to real GDP. Consumption (C_t) and investment (I_t) are normalized respectively to personal consumption expenditures and fixed

[27] These are indicators of real output, compensation and wages, employment and hours, consumption, investment, interest rates, money, credit, prices, and some miscellaneous indicators.

private domestic investment.[28] The labor input (L_t) corresponds to hours worked per person.[29] All preceding series are expressed in per capita terms by dividing with the population over the age of 16. The real wage (w_t) is normalized with the hourly compensation for the nonfarm business sector, divided by the GDP deflator. We express all these series in natural logs and remove a linear trend, so that they are expressed in percentage deviations from the trend, consistently with the model concepts. Inflation (π_t) is measured as the quarterly percentage change in the GDP deflator. The nominal interest rate (i_t) is the Federal funds rate. Both inflation and the interest rate are demeaned, to be consistent with the model's concepts.

Smets and Wouters (2004) assume that in steady-state, the above series are all growing at the rate of labor-augmenting technological progress, and they estimate their model imposing the common trend. This assumption is unfortunately rejected by the data (see Del Negro, Schorfheide, Smets and Wouters, 2004). To circumvent this issue, we detrend all series before the model estimation, so that the model parameters are estimated on the basis of deviations from the steady state.

3.2.2 Prior distributions of the parameters

As mentioned above, we estimate the DSGE model using Bayesian MCMC methods. We assume the same prior distributions as in Smets and Wouters (2004). These priors are summarized in Table 1 and are discussed in more details in Smets and Wouters (2004). Six of the structural parameters are calibrated, as they are difficult to estimate from percent deviations from the steady state. The discount rate β is set at 0.99, the quarterly depreciation rate τ is set at 0.025, the share of consumption $(1 - \tau k_y - g_y)$ and investment (τk_y) are set at 0.65 and 0.17, which implicitly define g_y and k_y. The capital-income share in the production function, α is set at 0.24. The parameter λ_w is fixed at 0.5. One difference with respect to Smets and Wouters (2004) involves the parameters of the policy rule which we assume takes the form of a generalized Taylor rule. The (long-run) response of the (annualized) federal funds rate to (annualized) inflation is assumed to be normally

[28] The nominal series for consumption and investment are deflated with the GDP deflator, as in Altig, Christiano, Eichenbaum and Lindé (2003), and Smets and Wouters (2004).

[29] As in Smets and Wouters (2004), average hours of the nonfarm business sector are multiplied with the civilian employment to account for the limited coverage of the nonfarm business sector, compared to GDP.

distributed with a mean of 1.5 and a variance of 0.5, and the response to detrended output is assumed to have a mean of 0.5 and variance of 0.2. The degree of inertia in monetary policy, or the response to the lagged interest rate is beta distributed with a mean of 0.75 a standard deviation of 1. Finally, fairly loose priors are assumed on the degree of measurement error. More details are provided in Appendix A.

3.2.3 Alternative specification of the observation equation: Four cases

We now proceed with the model estimation. To assess the importance of measurement error and of additional information, we consider four cases, each involving different restrictions on the observation equation (10), i.e., on the link between the model concepts and the data.

- Our first case, denoted *Case SW* corresponds to the standard estimation with a small set of data series, assuming that there is no measurement error. This case effectively attempts to replicate the results of Smets and Wouters (2004).[30] The seven key model variables included in F_t are assumed to be perfectly observed, and only the associated time series mentioned above — included in $X_{1,t}$ — are used in the estimation. In terms of our general notation, the *observation* equation (10) reduces to $X_{1,t} = F_t = \Phi S_t$.

As argued above, it is plausible that the indicators in $X_{1,t}$ measure only imperfectly the model concepts. If this is true, the estimates of the model parameters and of the shocks should be distorted. We thus consider three different cases in which we allow the indicators collected in $X_{1,t}$ to include a series-specific component (or measurement error) that is unrelated to the actual economic concept, F_t. We however maintain throughout the assumption that the nominal interest rate is perfectly observed.

- In our benchmark case with imperfect measurement, denoted *Case A*, we reestimate the model with the same seven data series but allowing for "measurement error" (except for the nominal interest rate). The observation equation is thus $X_{1,t} = F_t + e_{1,t} = \Phi S_t + e_{1,t}$ where

[30] As mentioned above, our estimation differs slighlty from the baseline case of Smets and Wouters (2004) for the following reasons: We consider a slightly different policy rule, we detrend the data before estimating the model parameters instead of estimating a common trend (the growth rate of technology) with the rest of the model, we assume that the inflation target is fixed, and we use a shorter sample.

the first element of $e_{1,t}$ is set equal to zero. The setup corresponds to the one in Sargent (1989), where the restrictions of the dynamic model are used to estimate the latent variables in F_t.

As discussed in Section 2, Case A is likely to be affected by identification problems due to the difficulty in disentangling the structural disturbances ε_t from the measurement errors $e_{1,t}$, in the face of a large number of shocks and measurement errors. Such problems can be addressed by considering a larger data set which can more easily identify the latent variables of interest by separating the series-specific components from the common factors, F_t. In the next cases, we thus maintain the observation equation $X_{1,t} = F_t + e_{1,t}$, but append to it another observation equation including additional indicators.

- In *Case B*, we add seven new indicators collected in a vector $X_{2,t}$, which have a known link to the variables in F_t. These additional indicators are selected on the grounds that they cannot be a priori rejected as indicators of the variables of interest.[31] The observation equation for this second set of indicators is of the form $X_{2,t} = \Lambda_2 F_t + e_{2,t}$, where each element of $e_{2,t}$ is allowed to follow an AR(1) process. The matrix Λ_2, which we estimate, is restricted to have as many nonzero elements per column as there are new indicators in $X_{2,t}$ of the corresponding variable in F_t. It has however no more than one nonzero element per row as each indicator is assumed to load on only one variable.

- Our final case, *Case C*, exploits the information from our entire data set in a flexible way. The fourteen primary indicators contained in $X_{1,t}$ and $X_{2,t}$ remain linked to the model's concepts as in case B, but we augment the vector $X_{2,t}$ with eight additional indicators of inflation which we link to the variable π_t, as documented in Appendix B. In addition, we introduce in $X_{3,t}$ information from all other indicators and assume that $X_{3,t}$ is related to the state vector S_t in a nonstructural way, according to the observation equation $X_{3,t} = \Lambda_S S_t + e_{S,t}$. The elements of $X_{3,t}$ are the 25 principal components of all remaining data series listed in our

[31] For consumption, the new indicator is real personal consumption expenditures excluding food and energy, for investment we add real gross private domestic investment, for inflation we add the indicators based on the deflator for personal consumption expenditures, the CPI, and the CPI less food and energy. For employment, we add the number of employees in the nonfarm business sector, as based on the establishment survey, and the number of workers as based on the household survey.

Appendix B. Each element of $e_{S,t}$ is assumed to follow an AR(1) process, and the loading matrix Λ_S is left unrestricted and is estimated.

The motivation for this last specification comes from the fact that if the theoretical model is true, the data series in $X_{3,t}$ should be related at least in part to the state vector. These indicators are publicly available. They are thus arguably part of the information set according to which economic agents in the DSGE model base their decisions. If these indicators contain information about the state of the economy that is not included in $X_{1,t}$ and $X_{2,t}$, then exploiting this information should help us obtain even more accurate estimates of the state of the economy and of the model parameters.

In case A, we assume the "measurement error," $e_{1,t}$, to be serially uncorrelated, as this restrictions is necessary to identify the model parameters. Whenever we let $e_{1,t}$ to be serially correlated in this case, we obtain estimated model parameters that are perfectly aligned with the prior distributions, suggesting that the data is uninformative, i.e., that the parameters are unidentified. As conjectured in Section 2, this highlights the fact that for models with a large number of structural shocks, and using a small set of observable variables, the extent to which measurement error can be allowed is severely limited using standard techniques. For comparison with this standard approach (case A), we assume that the measurement error in the primary indicators is also iid in cases B and C (even though we can relax this assumption in these cases, and still be able to identify the model parameters). We however allow the measurement errors of the secondary indicators ($e_{2,t}$ or $e_{S,t}$) to be serially correlated. By restricting the indicator-specific error terms to be iid, we may understate the magnitude of the series-specific component in the primary indicators $X_{1,t}$. This guarantees that the departures from the standard setups (cases SW and A) are relatively small. Nonetheless, as we show below, even for such small departures there are important benefits from exploiting information from a richer data set.

3.3 Empirical Results

We now describe the empirical results. We first provide evidence indicating that some of the seven primary indicators contain a nontrivial amount of series-specific idiosyncrasies, so that estimation allowing for it should be warranted. We next argue that estimating the model with a richer data sets provides a number of benefits among which a more accurate estimate of the state of the model economy.

3.3.1 Evidence of indicator-specific component (or “measurement error”)

According to the observation equation (10), each of the indicators can be decomposed into a “macroeconomic component,”ΛS_t, which is informative about the latent macroeconomic concepts, and an indicator-specific component (or “measurement error”), e_t.

Table 2 reports correlations between the indicators collected in $X_{1,t}$, $X_{2,t}$, and the corresponding model concept. Looking at the seven primary indicators, $X_{1,t}$, there is evidence of substantial “measurement error” for the indicator of inflation. The correlation between the growth rate of the GDP deflator and the estimate of inflation ranges between 0.73 and 0.86 depending on the number of indicators used in the model estimation. For other indicators in $X_{1,t}$, the extent of “measurement error” appears small. Figure 3 plots the posterior distribution of these correlations, in case A. Similar figures — not reported — are obtained for cases B and C. Clearly there is considerable “measurement error” in the growth rate of the GDP deflator. But the figure shows also that at any confidence level, the correlations are slightly lower than 1 for all indicators except for the Federal funds rate which is assumed to be perfectly correlated with i_t.

The lower panel of Table 2 suggests that the additional indicators X_{2t} provide relevant information for the model concepts: while their correlations with the corresponding model variables are relatively low in case A, they increase when they are used in the estimation (cases B and C). Interestingly, the PCE deflator provides a better indicator of the concept of inflation than the GDP deflator, when judged on the basis of their correlation with the estimated concept of inflation, in cases B and C.

Figures 4 and 5 report the estimated time series of the seven main endogenous variables for each of the cases SW, A, B and C. In case SW, the estimated series correspond to the primary indicators which are represented with solid lines. In the other cases, the model’s concepts except for the short-term interest rate are estimated latent variables. Overall, these plots confirm the results found in Table 2: the estimated latent variables and most corresponding primary indicators display very similar patterns, but there are wide discrepancies between inflation and the growth rate of the GDP deflator (Figure 5). These gaps, which reach in some quarters more than one percentage point (in annualized inflation).can have very important implications for the assessment of the inflation situation and for monetary policy actions. As we will see below, such differences can also result in different assessments about the sources of fluctuations in inflation.

One important difference between the growth rate of the GDP deflator and our estimated inflation series and is that the latter show more muted high-frequency fluctuations. In case A, estimated inflation is smoother than the data, as iid measurement errors pick up part of the high frequency fluctuations. In cases B and C, however, this occurs because the high-frequency fluctuations in the growth rate of the GDP deflator have little correlation with those of other inflation indicators, as we saw in Figure 2. While the quarterly growth rate of the GDP deflator and the CPI both display high-frequency fluctuations, some of these short-lived spikes do not occur at the same time in both series, so that they are considered as indicator-specific components. Therefore the estimated inflation, which in cases B and C corresponds to a common component of all inflation indicators, does not involve such high frequency fluctuations. Interestingly, the estimated inflation series are similar in cases B and C, suggesting case B includes an important part of the relevant information contained in our entire data set, for the assessment of inflation.

3.3.2 Benefits of richer data sets

As we just argued, at least one indicator often used in the estimation of DSGE models contains significant idiosyncratic fluctuations unrelated to the macroeconomic concepts which we attempt to measure. Failing to account for such indicator-specific noise in model estimation could potentially distort the estimates of all model parameters, of the model concepts, as well as of the shocks. It is therefore important that we acknowledge indicator-specific fluctuations in estimating such models. However even if one accounts for such "measurement error," one still needs to choose how many indicators to use in the model estimation. While we do not attempt to determine the optimal choice of indicators, we now show how using multiple indicators can be valuable.

Precision of model concepts. Once we recognize that the indicators provide only imperfect measurements of the model concepts, we become necessarily uncertain about the state of the economy in any given period. Table 3 reports measures of the uncertainty surrounding the main model variables in cases A, B, and C. In particular, it reports for each variable, the average over time of the standard deviation (taken at each date) of the estimated variable. The standard deviations are based on the empirical distribution of the estimates generated by the estimation draws. For cases B and C, we report the standard deviations relative to those of case A, so that numbers below 1 indicated a reduction in uncertainty. When the model is estimated using only the seven primary indicators, $X_{1,t}$, (case A), the standard deviation in the estimate of (annualized) quarterly inflation amounts to 0.50. However, when we append the second set of indicators $X_{2,t}$ (case B), the uncertainty surrounding this concept drops by 9%. This reduction in uncertainty about actual inflation is particularly valuable given the extent of "measurement errors" in its indicators.

The uncertainty surrounding the other variables is arguably less important given their small "measurement error." Nonetheless, using the second set of indicators $X_{2,t}$ in the model estimation also reduces considerably the uncertainty surrounding the estimate of such concepts. The standard deviation of real output, consumption and hours worked all fall respectively by 7%, 7%, and 24% in case B.

In case C, as all indicators of our data set are used in a nonstructural way for the model estimation, the uncertainty surrounding some of the estimated model concepts shrinks even further. Most notably, the uncertainty about the estimate of inflation is in this case 35% smaller than that of the case A. Figure 6 plots the estimated inflation rate in cases A, B, and C, together with the 5%-95% confidence bands. The latter bands become sensibly tighter as we more from case A to B and C.

Forecasts. Conditional on the DSGE model and the observation equation being correctly specified, the model characterized by (3)–(5) and (10) provides the best possible forecasts of the macroeconomic concepts. Anyone convinced that the model is well specified should thus perform forecasts using the model restrictions.

However, one may be skeptical about the model, and may thus be interested in forecasting not the latent model concepts — which are model-dependent —, but rather the *data* releases of the primary indicators $X_{1,t}$. We thus compare the performance in forecasting these data releases in cases A, B and C. In all cases, we use the model estimated over the entire sample to forecast each indicator one quarter ahead, over the 1982:1-2003:2 period.[32] Table 4 reports the root-mean-squared error (RMSE) of the forecasts. For cases B and C, we report the RMSE relative to case A. The last row provides an overall measure of forecast error: the log determinant of the forecast-error covariance matrix. The table shows a substantial drop in the RMSE of the forecasts from case A to case B for the indicators of real consumption, investment and the growth rate of the GDP deflator. For instance, the RMSE in forecasting actual releases of GDP deflator inflation drops by 5% when we include the indicators in $X_{2,t}$ for the model estimation.[33] As we include all data series in the model estimation (case C), the RMSE for inflation falls another 5%.

One may find it surprising that cases B and C provide such improvements in forecasting even for releases of the GDP deflator. In fact, the estimated inflation series in cases B and C are not designed to explain the GDP deflator, but the behavior of multiple indicators of inflation. It is akin to the common component to all indicators of inflation. As a result, even if the model in case C is able to forecast accurately true inflation, there is no guarantee that it ought to forecast the growth rate of the GDP deflator, given that a large fraction of the fluctuations in this series are estimated to be "measurement error" (see Table 2). More generally, one may be surprised by the fact that cases B and C can forecast so well the seven primary indicators. In fact, while case A is designed to explain the fluctuations in these seven primary indicators, cases B and C are designed to explain fluctuations in a larger set of indicators.

One explanation for the relatively good performance of cases B and C is the fact that by exploiting information from a large number of indicators in the model estimation, these cases are able to estimate more precisely the state of the economy, and thus the key latent variables, as shown in the previous section. More precision in estimating these variables implies then more precise forecasts of the indicators.

[32] Because of the time involved in the estimation of each case and the relatively short sample, we did not re-estimate the model at each period. The end of sample is in fact 2003:2 minus the forecasting horizon considered.

[33] Note that the improved forecasting performance in case B is not due a more flexible measurement error process. In fact, we assume that the "measurement error" of the primary indicators $X_{1,t}$ is iid for all three cases.

As argued, once one recognizes the fact that indicators are measured with error, or contain indicator-specific components unrelated to the macroeconomic variables, it may be very valuable to exploit the information in additional indicators. Such information can help assess the state of the economy and variables of interest more precisely. Such additional information may even be useful in forecasting more accurately actual data releases. Given the fact that we found inflation indicators to be noisy, and given the prominent role that inflation plays in policy analysis, it is crucial to obtain an accurate and precise assessment of inflation. Overall these results support our conjecture that there is a scope to exploit more information in the estimation of DSGE models.

3.3.3 Implications for estimates of structural parameters

We argued that when we estimate the model with a larger set of indicators, we obtain more precise estimates of the model variables. An important remaining question is whether these alternative empirical models lead to different conclusions about the structure of the economy and the source of business cycles.

Table 1 reports the parameter estimates (the median of the posterior distribution) together with the estimated standard errors. Several parameter estimates are stable across all cases considered. However, as we depart from the case SW and we allow for imperfect measurement using the same indicators, there are some notable changes: in case A, the model calls for slightly more habit persistence (h), price stickiness $\left(\xi_p\right)$, wage indexation (γ_w) and inflation indexation $\left(\gamma_p\right)$. In contrast, when we include additional indicators in the model estimation (cases B and C), the parameter estimates indicate much less habit persistence. In addition, the elasticity of real consumption to changes in the real interest rate implied by the consumption Euler equation — which we call the pseudo-EIS — increases from 0.099 in case A to 0.167 in case B.[34] Hence, as we use multiple indicators, we find that consumption becomes more sensitive to fluctuations in the real interest rate. The rigidity in price re-optimization, ξ_p, falls slightly so that that the elasticity of current inflation to marginal costs in (28) reaches 0.012 and 0.018 is cases B and C, compared to only 0.007 in case A. Importantly, the degree of inflation indexing $\left(\gamma_p\right)$, which is critical for the effect of various shocks

[34] In the presence of habit persistence, there is no natural elasticity of intertemporal substitution (EIS) in consumption. We thus refer to $\frac{1-h}{(1+h)\sigma_c}$ as a pseudo-EIS.

on inflation and its persistence, falls considerably when we use multiple indicators: it reaches 0.36 in case C compared to 0.72 in case A.

3.3.4 Sources of business cycle fluctuations

Another important difference among cases A, B, C and the benchmark case SW relates to the estimated variances of the exogenous shocks. Overall, these variances tend to be much smaller when "measurement error" is allowed for in the model estimation, because part of the fluctuations in the individual series is explained by an indicator-specific component. For most shocks, the estimated variances fall as we move from case SW to case C (except for the labor supply shock which is found to be most important in case A). This can be seen further in Figures 7 and 8 which plot the estimated time series for the state variables such as the capital stock and the exogenous shocks. As the figures show, the estimated shocks display generally much smaller and smoother fluctuations when the indicators-specific components are allowed for, i.e., in cases A, B, C. The fluctuations in the exogenous shocks appear also the smallest in the case C, when all data series are used in the estimation.

An appropriate estimate of the shocks is crucial to assess the state of the economy at any given date, and for conducting, e.g., monetary policy. But the figures reveal quite different histories of the shocks depending on the case considered. Changes in these variances across the cases considered yield different conclusions about the drivers of business cycle fluctuations. When the indicators used are assumed to be measuring the theoretical concepts perfectly (case SW), the nine shocks considered play a role in explaining the behavior of seven primary indicators. While the specification of Smets and Wouters (2004) requires all of these shocks to explain the fluctuations in the data, allowing for "measurement error" suggests that some shocks may be less important than previously thought.

For instance, in the benchmark case SW, price markup shocks (or cost-push shocks, η_t^p) account for 86% of the inflation fluctuations at the 1-quarter horizon and are still responsible for 52% of inflation fluctuations after 3 years (Table 5). However, their role is sensibly reduced in case C: such shocks explain only 56% of inflation fluctuations at the 1-quarter horizon and 23% after 3 years. Instead, total factor productivity shocks (ε_t^a) become an important source of inflation fluctuations in case C. They account for 21% of inflation fluctuations after 1 quarter (compared to only 8% in case SW) and for 52% of inflation fluctuations after 3 years (compared to 30% in case SW).

4 Conclusion

Recent DSGE models have achieved important successes in terms of their ability to fit the data and to forecast. As a result, such models have become relevant for policy analysis. Despite the sophistication of these models, existing empirical applications have maintained one important assumption: that a small number of data series is sufficient to estimate the model. This is however at odds both with the fact that market participants and central banks monitor a large number of data series to assess the state of the economy, and with the growing evidence from empirical factor models according to which a large set of macroeconomic variables may in fact be needed to characterize the evolution of the economy.

In this paper, we have proposed a general framework that exploits the information from a data-rich environment for the estimation of a general class of DSGE models. The fact that observed indicators may constitute imperfect measures of economic concepts relevant in the model provides a scope for using additional indicators in the empirical model. A particularly attractive feature of this framework, which we believe is crucial for policy considerations, is that it facilitates the interpretation of observed economic developments through the use of a structural model. It can also provide forecasts conditional on a variety of scenarios about structural disturbances, and on alternative policies.[35] Such advantages of the proposed framework cannot be achieved with existing more reduced-form factor models.

We have used this framework to estimate a state-of-the-art DSGE model that has been recognized for its empirical success. Our results suggest that much more accurate estimates of the model's concepts, in particular of inflation, of the state variables and of the exogenous shocks can be obtained by exploiting more information in the model estimation. A proper estimation of the

[35] Recall that while the empirical framework estimates both structural parameters (i.e., the policy-invariant parameters of the theoretical model), and non-structural parameters (e.g. the loading coefficients), it is well suited to perform counterfactual experiments on the model's concepts, for all model parameters and theoretical concepts can be estimated under historical policy, as in standard DSGE models. The framework may however not be adequate to generate counterfactual evolution of indicators related in a non structural way to the model's concepts.

state of the economy appears in turn to be very useful to improve forecasts of important macroeconomic variables, and even popular data releases such as the growth rate of the GDP deflator. Moreover, we showed that the inference drawn from the estimated model depends crucially on whether additional information is exploited or not. In particular, by comparing the standard implementation of the estimation using few indicators with our preferred specifications involving a larger set of indicators, we reach different conclusions about the kind of shocks that drive business cycles fluctuations.

The results in this paper open the way to many interesting avenues for future research, which we are pursuing. First, while the results reported provide an important scope for using more information in the estimation of DSGE models, more work needs to be done to determine how to optimally choose the indicators to include in the empirical model. We have proposed a couple of specifications (case B and C) that are successful in providing an accurate description of the state of the economy, and for forecasting. But other specifications within the general framework proposed here may perform even better.

Second, a real-time implementation of the proposed empirical framework should be of interest to central bankers, to the extent that it would allow them to process a large amount of information in real-time, assess the state of the economy systematically through the lenses of fully-specified structural model, and to provide forecasts, conditional on a variety of scenarios.

Finally, many researchers have recently given attention to the development of optimal policy rules or optimal target criteria for the conduct of monetary policy, in the context of DSGE models. Such optimal rules often involve forecasts of important macroeconomic variables over the next few quarters.[36] Improved forecasts obtained through better estimates of the state of the economy, using a rich data set, should thus be a key ingredient for such optimal target criteria. This may help making the tools available for the conduct of optimal monetary policy more attractive to policymakers by incorporating their concern for the developments in a large number of data series.

[36] Giannoni and Woodford (2003, 2004), for instance, characterize such optimal policy rules and target criteria for simpler models than the one presented here. They show that the most important forecasts needed as inputs for the implementation of monetary policy are over short horizons.

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A Appendix: MCMC Algorithm

This appendix describes our implementation of the MCMC algorithm used for the estimation of all cases described in Section 3. The general class of rational expectation models we consider can be represented as follows:

$$AE_t \begin{bmatrix} z_{t+1} \\ Z_{t+1} \end{bmatrix} = B \begin{bmatrix} z_t \\ Z_t \end{bmatrix} + Cs_t \tag{32}$$

$$s_t = Ms_{t-1} + \varepsilon_t \tag{33}$$

and the general form of the solution is:

$$z_t = DS_t \tag{34}$$

$$S_t = GS_{t-1} + H\varepsilon_t, \tag{35}$$

where $S_t \equiv [Z_t', s_t']'$ is the state vector, and the matrices D, G, and H are non-linear functions of the parameters in matrices A, B, C and M, obtained through numerical solutions techniques. As explained in the text, the variables of interest, collected in the vector F_t constitute a linear combination of the state variables

$$F_t = \Phi S_t,$$

and the matrix Φ depends on the model parameters and the selection of variables in F_t. The measurement equation is given by

$$X_t = \Lambda S_t + e_t, \tag{36}$$

where

$$\Lambda \equiv \begin{bmatrix} \Lambda_F \Phi \\ \Lambda_S \end{bmatrix}.$$

and

$$e_t = \Psi e_{t-1} + v_t.$$

Equations (35) and (36) form a state-space representation of the solution of the model. The vectors ε_t and v_t are assumed to be normally distributed with mean zero and variance Q and R respectively. The matrices R and Ψ are diagonal.

The goal is to estimate jointly the structural parameters of the theoretical model $\Theta_M = \{A, B, C, M, Q\}$ and the measurement equation parameters Λ, Ψ and R. Let $\Upsilon = \{\Lambda, \Psi, R\}$, $\Theta = \{\Theta_M, \Upsilon\}$ and $\tilde{X}_T = (X_1, X_2, ..., X_T)$. Our problem consists of characterizing the marginal posterior distribution of Θ. Given the high dimensionality of the problem and the need to integrate out the unobservable states, directly maximizing the posterior distribution is difficult and impractical for some models such as the one that we consider in this paper. Instead, the estimation approach that we consider provides an empirical approximation to this joint density. By judiciously breaking up this posterior distribution into the product of conditional densities, and sampling iteratively from the complete set of conditional densities, it effectively deals with the high dimensionality of the problem. As has been shown in the MCMC literature, the empirical distribution of the parameters resulting from the algorithm that we describe below converges to the joint posterior distribution of Θ.

More specifically, provided with an initial value of the parameters, $\Theta^{(0)}$, the algorithm proceeds by drawing iteratively from $p\left(\Upsilon|\Theta_M, \tilde{X}_T\right)$ and $p\left(\Theta_M|\Upsilon, \tilde{X}_T\right)$. We now provide more details on each step of the algorithm.

A.1 Step 1: Drawing from the conditional distribution $p\left(\Upsilon^{(g)}|\Theta_M^{(g-1)}, \tilde{X}_T\right)$

To draw from $p\left(\Upsilon|\Theta_M, \tilde{X}_T\right)$, we exploit the fact that conditional on $\tilde{S}_T$, Υ represents the parameters in a set of linear equations, the posterior distribution of which is known for standard prior distributions. To do so, we draw separately from $p(\tilde{S}_T^{(g)}|\Theta_M^{(g-1)}, \tilde{X}_T)$ and $p\left(\Upsilon^{(g)}|\tilde{S}_T^{(g)}, \Theta_M^{(g-1)}, \tilde{X}_T\right)$.

A.1.1 Step 1a: Drawing from the conditional distribution $p(\tilde{S}_T^{(g)}|\Theta_M^{(g-1)}, \tilde{X}_T)$

We use the forward-backward algorithm of Carter and Kohn (1994). As in Kim and Nelson (1999, p. 191), the conditional distribution of the whole history of factors can be expressed as the product of conditional distributions of factors at each date t as follows:

$$p\left(\tilde{S}_T|\Theta,\tilde{X}_T\right)=p\left(S_T|\Theta,\tilde{X}_T\right)\prod_{t=1}^{T-1}p\left(S_t|S_{t+1},\Theta,\tilde{X}_T\right).$$

This relies on the Markov property of S_t, which implies that $p\left(S_t|S_{t+1},S_{t+2},...,S_T,\Theta,\tilde{X}_T\right)=p\left(S_t|S_{t+1},\Theta,\tilde{X}_T\right)$.

Because the state-space model (35)–(36) is linear and Gaussian, we have

$$\begin{aligned} S_T|\Theta,\tilde{X}_T &\sim N\left(S_{T|T},P_{T|T}\right) \\ S_t|S_{t+1},\Theta,\tilde{X}_T &\sim N\left(S_{t|t,S_{t+1}},P_{t|t,S_{t+1}}\right), \end{aligned}$$

where

$$\begin{aligned} S_{T|T} &= E\left(S_T|\Theta,\tilde{X}_T\right) \\ P_{T|T} &= Cov(S_T|\Theta,\tilde{X}_T) \\ S_{t|t,S_{t+1}} &= E\left(S_t|S_{t+1},\Theta,\tilde{X}_T\right) \\ P_{t|t,S_{t+1}} &= Cov\left(S_t|S_{t+1},\Theta,\tilde{X}_T\right), \end{aligned} \tag{37}$$

and where the notation $S_{t|t}$ refers to the expectation of S_t conditional on information dated t or earlier. To obtain these, we first calculate $S_{t|t}$ and $P_{t|t}$, $t=1,2,...,T$, by Kalman filter, conditional on Θ and the data through period t, $\tilde{X}_t$, with starting values of zeros for the factors and the identity matrix for the covariance matrix (Hamilton, 1994). The last iteration of the filter yields $S_{T|T}$ and $P_{T|T}$, which together with the first line of (37) allows us to draw a value for S_T. Treating this drawn value as extra information, we can move "backwards in time" through the sample, using the Kalman filter to obtain updated values of $S_{T-1|T-1,S_T}$ and $P_{T-1|T-1,S_T}$; drawing a value of S_{T-1} using the third line of (37); and continuing in similar manner to draw values for S_t, $t=T-2,T-3,...1$.[37]

[37] The Kalman filter is implemented to handle the serial correlation in e_t. In particular, in the Kalman filter iterations, the observation equation is rearranged as:

$$X_t^*=(\Lambda F-\Psi\Lambda)\,S_{t-1}+\Lambda u_t+v_t$$

with $X_t^*=X_t-\Psi X_{t-1}$.

A.1.2 Step 1b: Drawing from the conditional distribution $p\left(\Upsilon^{(g)}|\tilde{S}_T^{(g)}, \Theta_M^{(g-1)}, \tilde{X}_T\right)$

Conditional on the observed data and the estimated factors from the previous iteration, a new iteration is begun by drawing a new value of the parameters. With known factors, (36) amounts to a set of regressions with autoregressive errors. We can thus apply the algorithm proposed by Chib (1993).

This conditional model is non-linear in the parameters. However, since conditional on Λ or Ψ, the model is linear, we can characterize this distribution through a complete set of conditional distributions that are linear in the parameters. More precisely, we assume that *a priori* Λ and R are independent of Ψ. Conditional on Ψ and since R is diagonal, we can apply OLS equation by equation to obtain $\hat{\Lambda}_k$ and $\hat{v}_k$. We define $X_{k,t}^* = X_{k,t} - \Psi_{kk} X_{k,t-1}$ and $S_{k,t}^* = S_t - \Psi_{kk} S_{t-1}$, set $R_{kl} = 0$, $k \neq l$, and assume a proper (conjugate) but diffuse Inverse-Gamma (3, 0.001) prior for R_{kk}. Standard Bayesian results (see Bauwens, Lubrano and Richard, 1999, p. 58) deliver posterior of the form:

$$R_{kk}|\tilde{X}_T, \bar{S}_T, \Psi \sim iG\left(\bar{R}_{kk}, J \times T + 0.001\right)$$

where $\bar{R}_{kk} = 3 + \sum_{j=1}^J \hat{v}_k' \hat{v}_k + \sum_{j=1}^J \hat{\Lambda}_k' \left(\tilde{S}_{k,T}^{*\prime} \tilde{S}_{k,T}^*\right)^{-1} \hat{\Lambda}_k - \bar{\Lambda}_k' \bar{M}_k^{-1} \bar{\Lambda}_k$, $\bar{\Lambda}_k = \bar{M}_k^{-1}\left(\sum_{j=1}^J \tilde{S}_{k,T}^{*\prime} \tilde{S}_{k,T}^* \hat{\Lambda}_k\right)$, and $\bar{M}_k^{-1} = M_0 + \sum_{j=1}^J \tilde{S}_{k,T}^{*\prime} \tilde{S}_{k,T}^*$. Here M_0^{-1} denotes the variance parameter in the prior on the coefficients of the k-th equation, Λ_k, which, conditional on the drawn value of R_{kk}, is $N(0, R_{kk} M_0^{-1})$. We set $M_0 = I$. We draw values for Λ_k from the posterior $N\left(\bar{\Lambda}_k, R_{kk} \bar{M}_k^{-1}\right)$.

Conditional on Λ and since R is diagonal, we can apply OLS equation by equation to obtain $\hat{\Psi}_{kk}$. Letting e_k be the vector whose elements are given by $e_{k,t} = X_{k,t} - \Lambda_k S_t$, and its lagged version $e_{k,-1}$, and assuming that the prior on Ψ_{kk} is $N(0,1)$, the posterior distribution of Ψ_{kk} is $N(\bar{\Psi}_{kk}$, $\bar{N}_k^{-1})$ where $\bar{\Psi}_{kk} = \bar{N}_k^{-1}\left(R_{kk}^{-1} \sum_{j=1}^J e_{k,-1}' e_{k,-1} \hat{\Psi}_{kk}\right)$ and $\bar{N}_k = \left(1 + R_{kk}^{-1} \sum_{j=1}^J e_{k,-1}' e_{k,-1}\right)$.

A.2 Step 2: Drawing from the conditional distribution $p\left(\Theta_M^{(g)}|\Upsilon^{(g-1)}, \tilde{X}_T\right)$

The elements of the matrices Θ_M are individually drawn from a proposal scalar Student t-distribution, with mean centered around the previous draws of the parameters, i.e. the corresponding elements of $\Theta_M^{(g)}$, and a variance calibrated to yield appropriate acceptance rates.[38] Let Θ_M^* be the resulting draws. Based on the solution of the model obtained from this last draw, the following ratio is computed:

$$r = \frac{p\left(\Theta_M^* | \Lambda, R, \tilde{X}_T\right)}{p\left(\Theta_M^{(g)} | \Lambda, R, \tilde{X}_T\right)}$$

With probability $\min(1, r)$, $\Theta_M^{(g+1)} = \Theta_M^*$, and otherwise $\Theta_M^{(g+1)} = \Theta_M^{(g)}$.

Steps 1 and 2 are repeated for each iteration g. Inference is based on the distribution of $\left\{\Theta^{(g)}\right\}_{g=b}^{G}$, where b is large enough to guarantee convergence of the algorithm. As noted, the empirical distribution from the sampling procedure should well approximate the joint posterior or normalized joint likelihood. Calculating medians and quantiles of $\left\{\Theta^{(g)}\right\}_{g=b}^{G}$ provides estimates of the model parameters and the associated confidence regions.

[38] See Johannes and Polson (2004) for practical recommendations on the choice of the proposal density and the desired acceptance rate.

Appendix B - Data Description

All series were taken from DRI/McGraw Hill Basic Economics Database or directly from the Bureau of Labor Statistics. The format is: series number; transformation code and series description as appears in the database. The transformation codes are: 1 – no transformation; 2 – Detrended log per capita; 3 – detrended logarithm level 4 – logarithm; 5 – first difference of logarithm; 6 – Adjustement specific to average hours and hourly earnings; 0 – variable not used in the estimation (only used for transforming other variables). A * indicate a series that is deflated with the GDP deflator (series #145).

Cases SW and A

1	1	Interest Rate: Federal Funds (Effective) (% per annum, NSA)
2	2	Real Gross Domestic Product (billions of chained 2000 dollars, SAAR)
3	2	Real Personal Consumption Expenditures (Chained 2000, NIPA)
4	2*	Gross Private Domestic Investment - Fixed Investment (billions of chained 2000 dollars, SAAR)
5	5	Price Deflator - Gross Domestic Product (NIA)
6		Real Wage (Smets And Wouters)
7		Hours Worked (Smets And Wouters)

Case B

8	2	Personal Consumption Expenditures Excluding Food And Energy
9	2*	Gross Private Domestic Investment (billions of chained 2000 dollars, SAAR)
10	5	Price Deflator - Private Consumption Expenditure (NIA)
11	5	CPI-U: All Items Less Food (82-84=100, SA)
12	5	CPI-U: All Items (82-84=100,SA)
13	2	Civilian Labor Force: Employed, Total (Thous.,SA)
14	2	Employees, Nonfarm - Total Nonfarm

Case C: Known link

15	5	Price Index - Personal Consumption Expenditures - Durable Goods (2000=100), SAAR
16	5	Price Index - Personal Consumption Expenditures - Nondurable Goods (2000=100), SAAR
17	5	Price Index - Personal Consumption Expenditures - Services (2000=100) , SAAR
18	5	CPI -U: Durables (82-84=100, SA)
19	5	CPI -U: Commodities (82-84=100, SA)
20	5	CPI -U: Medical Care (82-84=100, SA)
21	5	CPI -U: Transportation (82-84=100, SA)
22	5	CPI -U: Apparel & Upkeep (82-84=100, SA)

Case C: Unknown link

23	2	Real Gross Domestic Product – Services (billions of chained 2000 dollars, SAAR)
24	2	Real Gross Domestic Product – Structures (billions of chained 2000 dollars, SAAR)
25	2	Industrial Production Index - Products, Total
26	2	Industrial Production Index - Final Products
27	2	Industrial Production Index - Consumer Goods
28	2	Industrial Production Index - Durable Consumer Goods
29	2	Industrial Production Index - Nondurable Consumer Goods
30	2	Industrial Production Index - Business Equipment
31	2	Industrial Production Index - Materials
32	2	Industrial Production Index - Durable Goods Materials
33	2	Industrial Production Index - Nondurable Goods Materials
34	2	Industrial Production Index - Total Index
35	1	Capacity Utilization - Manufacturing (SIC)
36	2*	Nominal Total Compensation Of Employees (NIA)
37	2	Personal Income Chained 2000 Dollars (BCI)
38	2	Personal Income Less Transfer Payments (Chained) (#51) (Bil 92$,Saar)
39	6*	Average Hourly Earnings, Production Workers: Manufacturing,
40	6*	Average Hourly Earnings, Production Workers: Construction,
41	1	Unemployment Rate: All Workers, 16 Years & Over (%,SA)
42	1	Unemploy. by Duration: Average(Mean)Duration In Weeks (SA)

43	2	Unemploy. by Duration: Persons Unempl.Less Than 5 Wks (Thous.,SA)
44	2	Unemploy. by Duration: Persons Unempl.5 To 14 Wks (Thous.,SA)
45	2	Unemploy. by Duration: Persons Unempl.15 Wks + (Thous.,SA)
46	2	Unemploy. by Duration: Persons Unempl.15 To 26 Wks (Thous.,SA)
47	2	Employees, Nonfarm - Total Private
48	2	Employees, Nonfarm - Goods-Producing
49	2	Employees, Nonfarm - Mining
50	2	Employees, Nonfarm - Construction
51	2	Employees, Nonfarm - Mfg
52	2	Employees, Nonfarm - Durable Goods
53	2	Employees, Nonfarm - Nondurable Goods
54	2	Employees, Nonfarm - Wholesale Trade
55	2	Employees, Nonfarm - Government
56	2	REAL PERSONAL CONSUMPTION EXPENDITURES (Index 2000=100): Durable Goods (NIPA Table 2.3.3)
57	2	Nondurable Goods
58	2	Services
59	2*	Gross Private Domestic Investment - Fixed Nonresidential , Billions Of Dollars , SAAR
60	4	Housing Starts:Nonfarm(1947-58);Total Farm&Nonfarm(1959-)(Thous.,SA)
61	1	NAPM Inventories Index (Percent)
62	1	NAPM New Orders Index (Percent)
63	1	NAPM Vendor Deliveries Index (Percent)
64	2	New Orders (Net) - Consumer Goods & Materials, 1996 Dollars (Bci)
65	1	Interest Rate: Federal Funds (Effective) (% Per Annum,Nsa)
66	1	Interest Rate: U.S.Treasury Bills,Sec Mkt,3-Mo.(% Per Ann,Nsa)
67	1	Interest Rate: U.S.Treasury Bills,Sec Mkt,6-Mo.(% Per Ann,Nsa)
68	1	Interest Rate: U.S.Treasury Const Maturities,1-Yr.(% Per Ann,Nsa)
69	1	Interest Rate: U.S.Treasury Const Maturities,5-Yr.(% Per Ann,Nsa)
70	1	Interest Rate: U.S.Treasury Const Maturities,10-Yr.(% Per Ann,Nsa)
71	1	Bond Yield: Moody's Aaa Corporate (% Per Annum)
72	1	Bond Yield: Moody's Baa Corporate (% Per Annum)
73	2	Money Stock: M1(Curr,Trav.Cks,Dem Dep,Other Ck'able Dep)(Bil$,Sa)
74	2	Money Stock:M2(M1+O'nite Rps,Euro$,G/P&B/D Mmmfs&Sav&Sm Time Dep(Bil$,
75	2	Money Stock: M3(M2+Lg Time Dep,Term Rp's&Inst Only Mmmfs)(Bil$,Sa)
76	2	Money Supply - M2 In 1996 Dollars (Bci)
77	2	Monetary Base, Adj For Reserve Requirement Changes(Mil$,Sa)
78	2	Depository Inst Reserves: Total, Adj For Reserve Req Chgs(Mil$,Sa)
79	2	Depository Inst Reserves: Nonborrowed,Adj Res Req Chgs(Mil$,Sa)
80	2	Commercial & Industrial Loans Oustanding In 1996 Dollars (Bci)
81	1	Wkly Rp Lg Com'l Banks:Net Change Com'l & Indus Loans(Bil$,Saar)
82	2	Consumer Credit Outstanding - Nonrevolving(G19)
83	5	Gross Private Domestic Investment, Price Deflators (2000=100) , Saar
84	5	CPI-U: All Items Less Medical Care (82-84=100,Sa)
85	5	CPI-U: All Items Less Shelter (82-84=100,Sa)
86	5	CPI-U: Services (82-84=100,Sa)
87	1	NAPM Commodity Prices Index (Percent)
88	1	U. of Michigan Index of Consumer Expectations(BCD-83)
89	3	Composite Cyclical Indicator (1996) - Leading
90	3	Composite Cyclical Indicator (1996) - Lagging
91	3	Composite Cyclical Indicator (1996) - Coincident

Table 1: Priors and estimates of structural parameters

	Prior Distribution			SW	Case A	Case B	Case C
	Type	Mean	St.Err.				
φ	Normal	4	1.5	5.36	5.88	6.17	3.81
				(0.88)	(1.11)	(1.13)	(1.04)
σ_c	Normal	1	0.375	1.54	1.45	1.79	1.63
				(0.24)	(0.23)	(0.44)	(0.44)
h	Beta	0.7	0.1	0.71	0.75	0.54	0.50
				(0.07)	(0.07)	(0.27)	(0.27)
σ_L	Normal	2	0.75	2.34	2.18	2.42	2.41
				(0.60)	(0.65)	(0.69)	(0.68)
ϕ	Normal	1.25	0.125	1.42	1.24	1.37	1.26
				(0.08)	(0.07)	(0.07)	(0.07)
$1/\psi$	Normal	0.2	0.075	0.32	0.27	0.26	0.27
				(0.06)	(0.06)	(0.06)	(0.06)
ξ_ω	Beta	0.75	0.05	0.81	0.77	0.78	0.82
				(0.02)	(0.03)	(0.04)	(0.03)
ξ_p	Beta	0.75	0.05	0.88	0.90	0.88	0.86
				(0.01)	(0.02)	(0.01)	(0.02)
γ_ω	Beta	0.5	0.15	0.39	0.45	0.43	0.48
				(0.12)	(0.14)	(0.14)	(0.14)
γ_p	Beta	0.5	0.15	0.66	0.72	0.50	0.36
				(0.08)	(0.19)	(0.15)	(0.14)
ρ	Beta	0.75	0.1	0.76	0.67	0.72	0.70
				(0.02)	(0.05)	(0.04)	(0.03)
$r_{\pi 0}$	Normal	1.8	0.1	1.78	1.81	1.72	1.66
				(0.08)	(0.10)	(0.10)	(0.09)
$r_{\pi 1}$	Normal	-0.3	0.1	-0.22	-0.22	-0.30	-0.39
				(0.09)	(0.12)	(0.10)	(0.09)
r_{y0}	Normal	0.188	0.05	0.22	0.23	0.24	0.22
				(0.03)	(0.03)	(0.03)	(0.03)
r_{y1}	Normal	-0.063	0.05	-0.13	-0.11	-0.12	-0.12
				(0.03)	(0.03)	(0.04)	(0.03)
Implied parameters							
pseudo EIS: $\frac{1-h}{(1+h)\sigma_c}$				0.110	0.099	0.167	0.204
slope of PC: $\frac{(1-\beta\xi_p)(1-\xi_p)}{(1+\beta\gamma_p)\xi_p}$				0.011	0.007	0.012	0.018

The parameter estimates are given by the median of the posterior distribution
Results are based on 100 000 replications. Standard errors are reported in ().

Table 1 (continued): Priors and estimates of parameters describing shock processes

	Prior Distribution			SW	Case A	Case B	Case C
	Type	Mean	St.Err.				
ρ_a	Beta	0.85	0.1	0.93	0.97	0.98	0.98
				(0.04)	(0.02)	(0.01)	(0.01)
ρ_b	Beta	0.85	0.1	0.62	0.62	0.71	0.69
				(0.06)	(0.07)	(0.11)	(0.12)
ρ_G	Beta	0.85	0.1	0.95	0.97	0.97	0.97
				(0.02)	(0.01)	(0.01)	(0.01)
ρ_L	Beta	0.85	0.1	0.83	0.77	0.84	0.86
				(0.10)	(0.09)	(0.09)	(0.09)
ρ_I	Beta	0.85	0.1	0.80	0.80	0.78	0.86
				(0.05)	(0.05)	(0.05)	(0.03)
σ_a^2	invGam	0.25	2	0.12	0.03	0.02	0.02
				(0.02)	(0.01)	(0.00)	(0.01)
σ_b^2	invGam	0.25	2	0.01	0.00	0.00	0.00
				(0.00)	(0.00)	(0.00)	(0.01)
σ_G^2	invGam	0.25	2	0.20	0.08	0.06	0.06
				(0.04)	(0.02)	(0.02)	(0.02)
σ_L^2	invGam	0.25	2	0.06	58.15	0.31	0.01
				(0.90)	(44.03)	(11.50)	(0.01)
σ_I^2	invGam	0.25	2	7.69	8.59	9.29	2.16
				(4.55)	(4.17)	(5.08)	(0.99)
σ_Q^2	invGam	0.25	2	14.13	2.41	2.86	0.40
				(10.86)	(3.71)	(5.89)	(0.40)
σ_p^2	invGam	0.25	2	0.01	0.00	0.01	0.01
				(0.00)	(0.00)	(0.00)	(0.00)
σ_ω^2	invGam	0.25	2	0.10	0.04	0.09	0.08
				(0.01)	(0.04)	(0.04)	(0.02)
σ_i^2	invGam	0.25	1.5	0.02	0.01	0.02	0.02
				(0.00)	(0.00)	(0.00)	(0.00)

The parameter estimates are given by the median of the posterior distribution
Results are based on 100 000 replications. Standard errors are reported in ().

Table 2: Correlation between estimated latent concepts and observable measures

	Indicator	Concept	Case A	Case B	Case C
	Fed funds rate	R_t	1.00	1.00	1.00
	Real GDP	Y_t	0.99	0.98	0.98
	Real Consumption	C_t	0.98	0.99	0.99
$X_{1,t}$	Real fixed Investment	I_t	0.99	0.99	0.99
	GDP defl. inflation	π_t	0.73	0.86	0.86
	Real wage	w_t	0.99	0.99	0.98
	Hours worked	L_t	0.99	0.98	0.99
	PCE ex. food and Energy	C_t	0.98	0.99	0.98
	Gross Real Investment	I_t	0.94	0.95	0.94
	PCE deflator inflation	π_t	0.70	0.92	0.93
$X_{2,t}$	core-CPI inflation	π_t	0.53	0.82	0.81
	CPI inflation	π_t	0.54	0.83	0.82
	Employment HH Survey	L_t	0.89	0.92	0.92
	Payroll Employment	L_t	0.81	0.85	0.85

The entries are the correlation between the observable indicators and the median estimate of the corresponding estimated latent variable.

Table 3: Uncertainty about estimated model variables

Concept		Case A	Case B	Case C
		st. dev.	Relative to case A	
Interest rate	R_t	0.000	—	—
Output	Y_t	0.342	0.93	1.01
Consumption	C_t	0.450	0.93	1.01
Investment	I_t	0.908	0.94	0.89
Inflation (annualized)	π_t	0.500	0.91	0.65
Real wage	w_t	0.478	1.04	1.06
Hours worked	L_t	0.311	0.76	0.97

The entries for case A are the standard deviations across estimation draws of the latent variables for each date, and averaged over time. For cases B and C, these standard deviations are divided by those in case A, so that numbers below 1 indicate reductions in uncertainty. Standard deviations apply to annualized data series.

Table 4: One-step ahead forecasting errors

Primary indicator	Case A	Case B	Case C
	RMSE	Relative to case A	
Fed funds rate	0.52	1.08	1.12
Real GDP	0.55	1.00	1.02
Real Consumption	0.59	0.93	0.97
Real Investment	1.64	0.97	0.88
GDP defl. inflation	0.20	0.95	0.90
Real wage	0.75	1.03	0.96
Hours worked	0.49	1.02	1.04
Overall	-9.26	-9.38	-9.41

The entries for case A are root mean squared error (RMSE) of the forecasts. For cases B and C, the RMSE are divided by those in case A. Numbers below 1 indicate forecasting improvements. The overall measure is for all cases the log determinant of the variance-covariance matrix of the forecasting errors for the seven primary indicators.

Table 5: Variance decompositions of model variables at 3-year horizon

Shock \ Endog. variable	i	Y	C	I	π	w	L
Case SW							
Productivity	0.21	0.08	0.02	0.11	0.30	0.09	0.22
Preference	0.10	0.24	0.78	0.06	0.02	0.03	0.22
Govt. Expenditures	0.03	0.14	0.04	0.08	0.00	0.00	0.14
Labor supply	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Investment	0.22	0.38	0.02	0.71	0.04	0.07	0.28
Equity premium	0.00	0.01	0.00	0.01	0.00	0.00	0.01
Cost-push prices	0.15	0.05	0.03	0.01	0.52	0.24	0.02
Cost-push wages	0.06	0.04	0.06	0.01	0.11	0.56	0.07
Monetary policy	0.23	0.05	0.06	0.01	0.01	0.01	0.04
Case A							
Productivity	0.17	0.10	0.07	0.05	0.29	0.08	0.09
Preference	0.09	0.17	0.62	0.05	0.02	0.04	0.16
Govt. Expenditures	0.01	0.11	0.07	0.05	0.01	0.00	0.13
Labor supply	0.15	0.10	0.15	0.02	0.24	0.42	0.18
Investment	0.35	0.46	0.01	0.83	0.09	0.10	0.38
Equity premium	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Cost-push prices	0.12	0.04	0.03	0.00	0.31	0.12	0.02
Cost-push wages	0.03	0.02	0.03	0.00	0.04	0.24	0.03
Monetary policy	0.08	0.01	0.01	0.00	0.00	0.00	0.01
Case B							
Productivity	0.17	0.06	0.06	0.02	0.32	0.05	0.09
Preference	0.08	0.19	0.67	0.02	0.02	0.03	0.20
Govt. Expenditures	0.01	0.08	0.08	0.04	0.01	0.00	0.10
Labor supply	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Investment	0.35	0.56	0.02	0.90	0.07	0.10	0.47
Equity premium	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Cost-push prices	0.10	0.03	0.02	0.00	0.44	0.11	0.01
Cost-push wages	0.07	0.03	0.07	0.01	0.13	0.69	0.08
Monetary policy	0.22	0.05	0.07	0.00	0.01	0.01	0.05
Case C							
Productivity	0.30	0.09	0.10	0.02	0.52	0.12	0.09
Preference	0.06	0.21	0.64	0.02	0.01	0.02	0.22
Govt. Expenditures	0.00	0.07	0.08	0.05	0.00	0.00	0.08
Labor supply	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Investment	0.26	0.48	0.02	0.88	0.07	0.07	0.42
Equity premium	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Cost-push prices	0.05	0.02	0.01	0.00	0.23	0.06	0.01
Cost-push wages	0.09	0.04	0.06	0.02	0.16	0.73	0.10
Monetary policy	0.24	0.08	0.08	0.01	0.01	0.01	0.08

Figure 1: Indicators of employment (de-trended) and pairwise coherence.

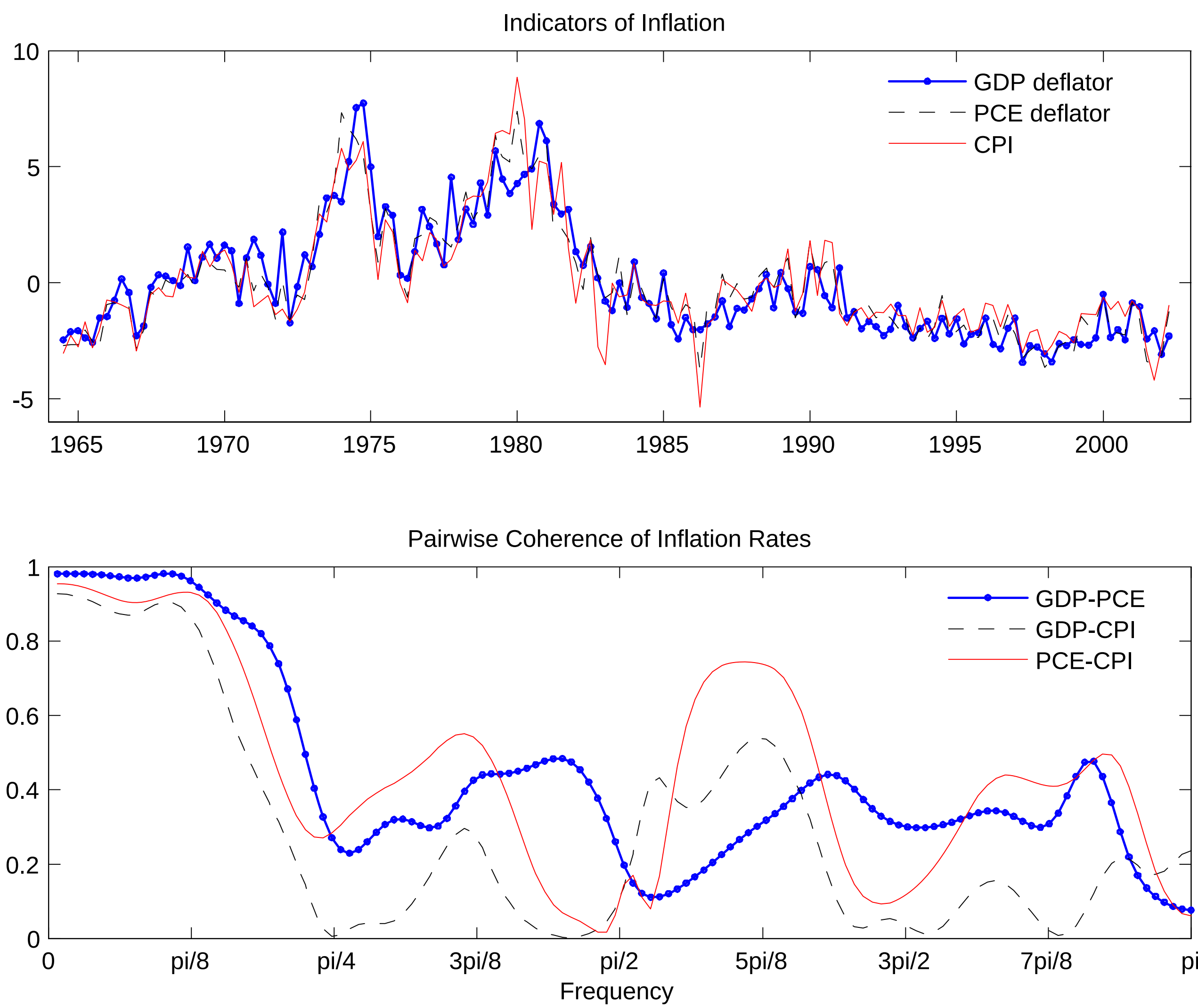

Figure 2: Indicators of quarterly inflation rates (de-meaned) and pairwise coherence.

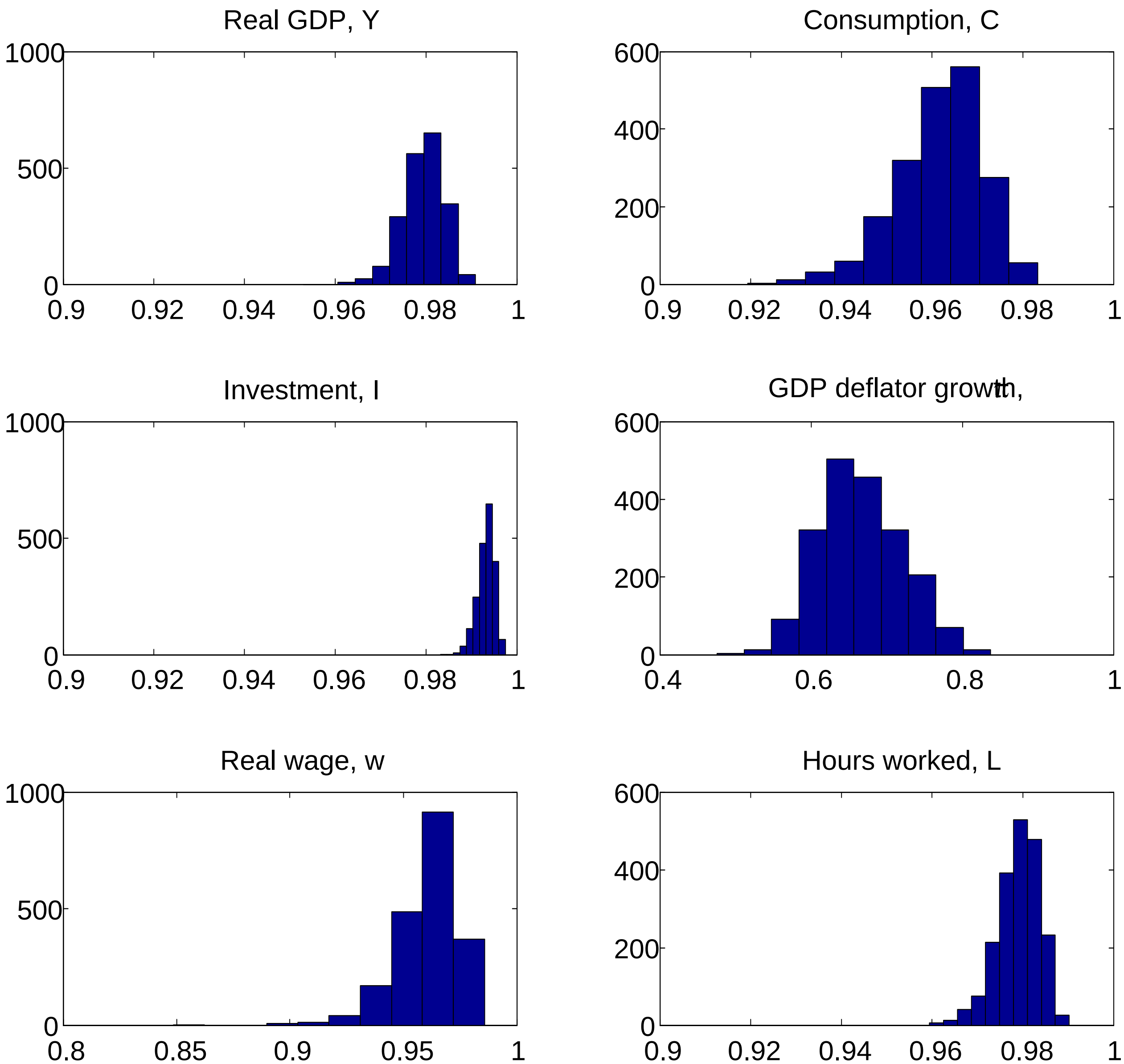

Figure 3: Extent of measurement error. Distribution of correlations between main indicators and corresponding model variables (Case A)

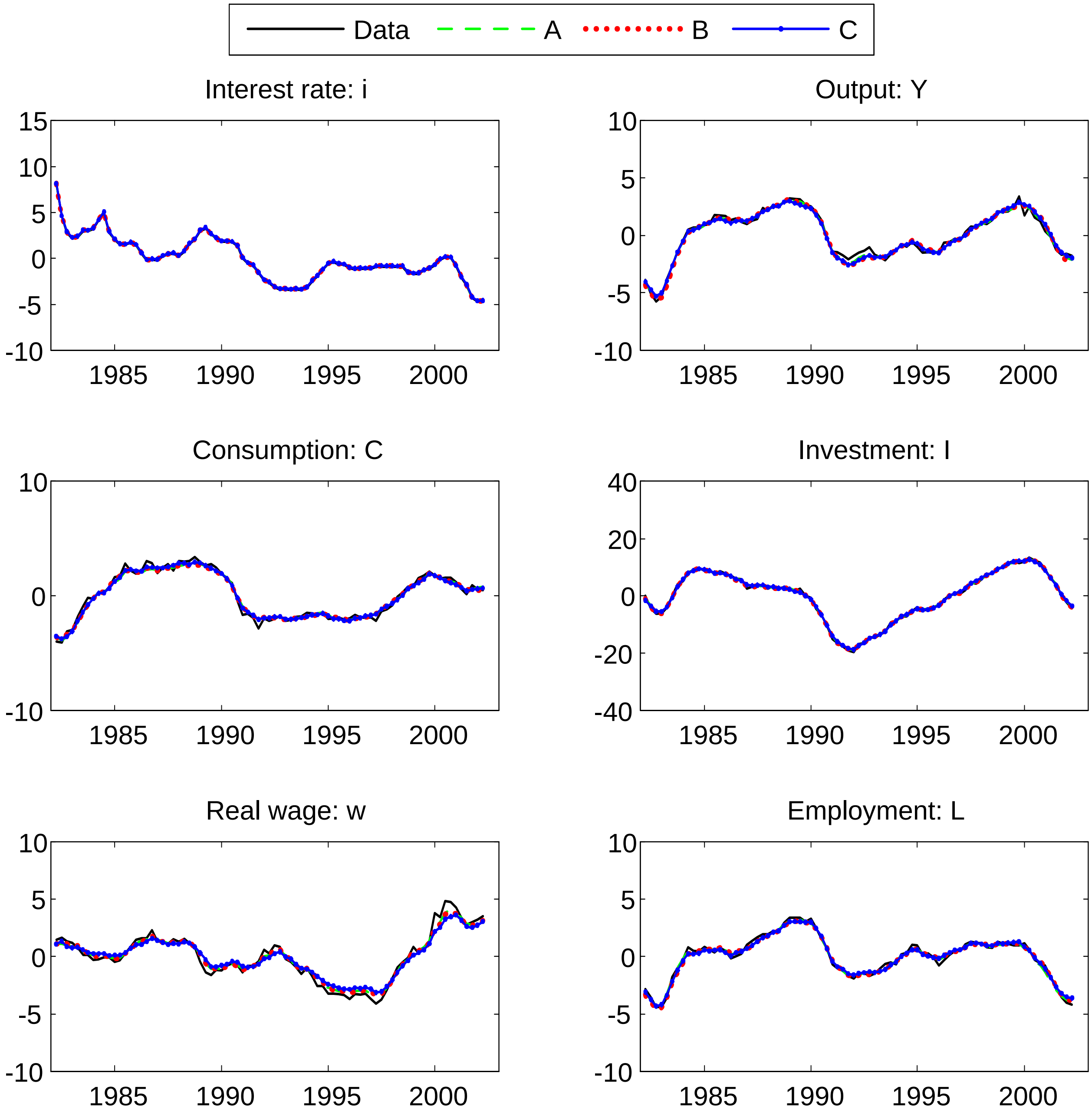

Figure 4: Estimated endogenous variables (Cases SW, A, B, and C)

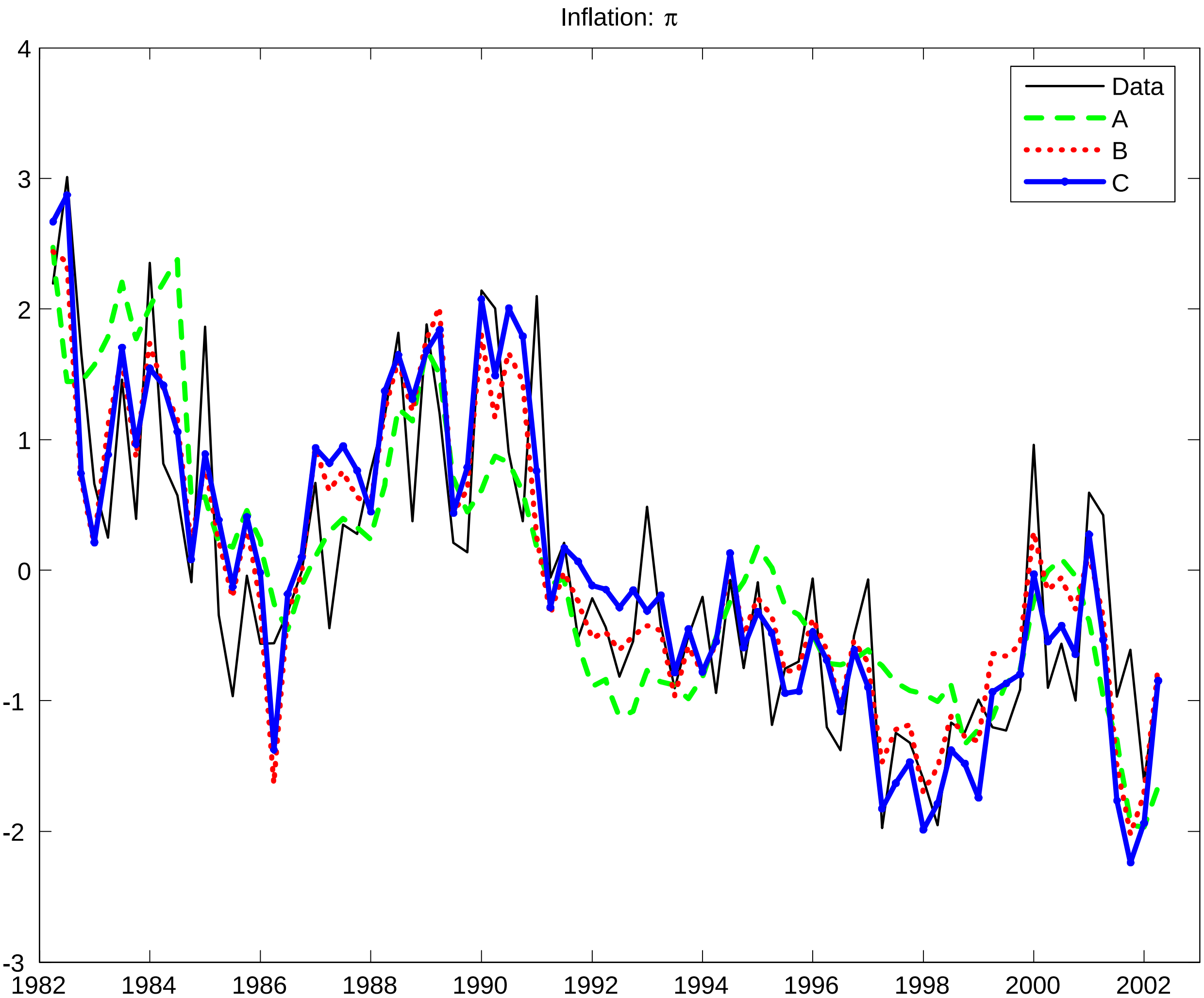

Figure 5: Estimated inflation (de-meaned; Cases SW, A, B and C)

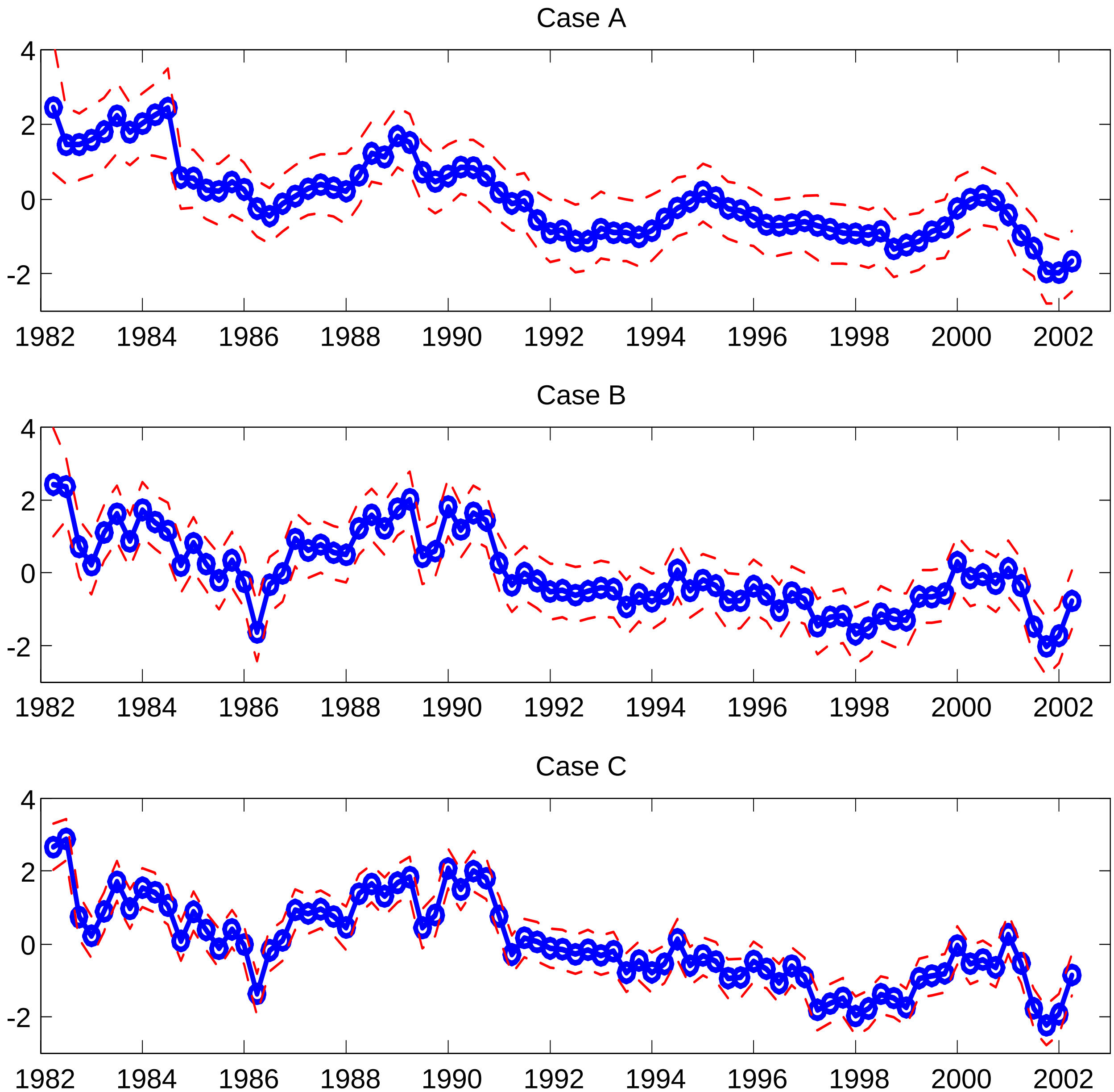

Figure 6: Estimated inflation and 5%-95% confidence band (Cases A, B, C)

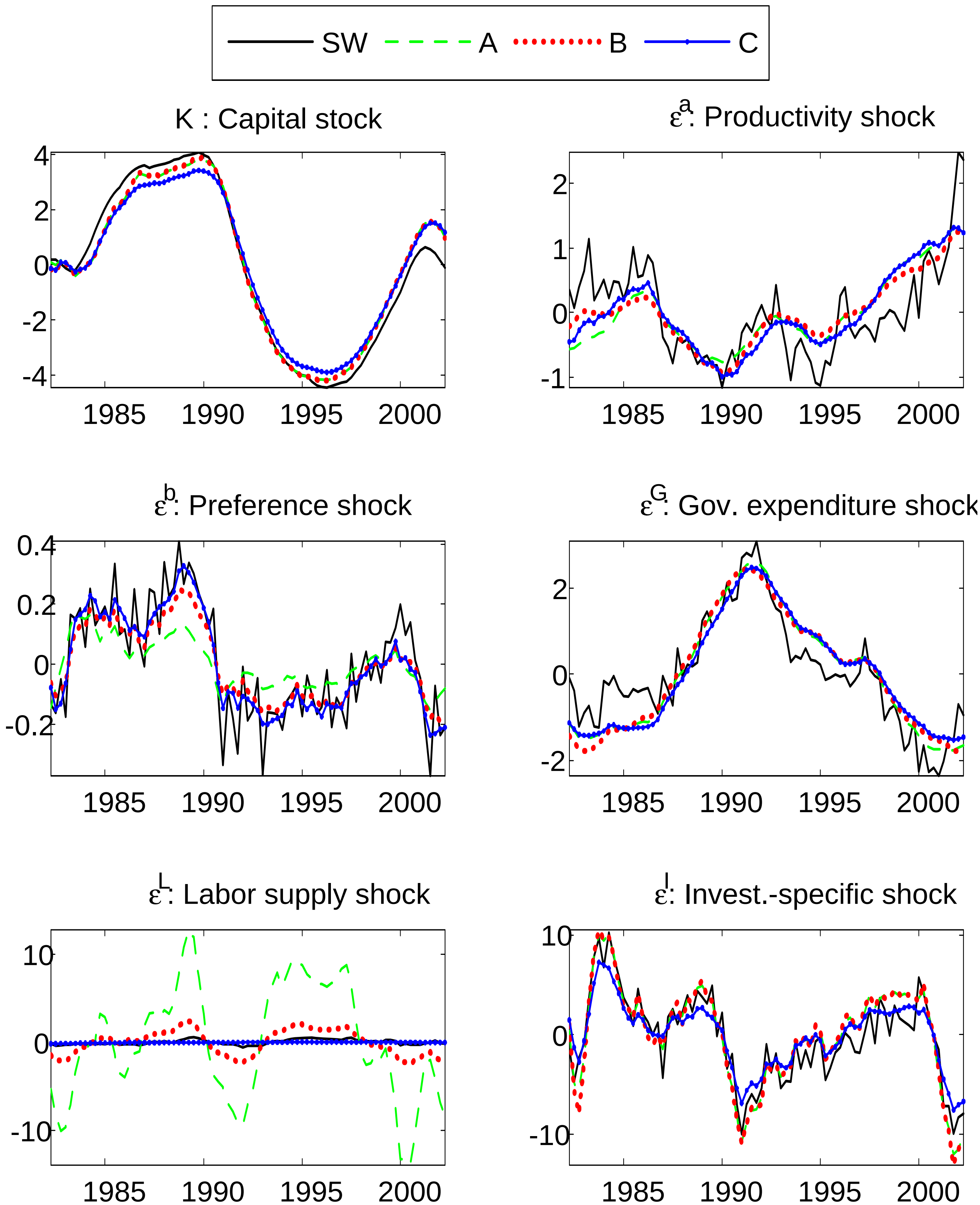

Figure 7: Estimated capital stock and shocks (Cases SW, A, B and C)

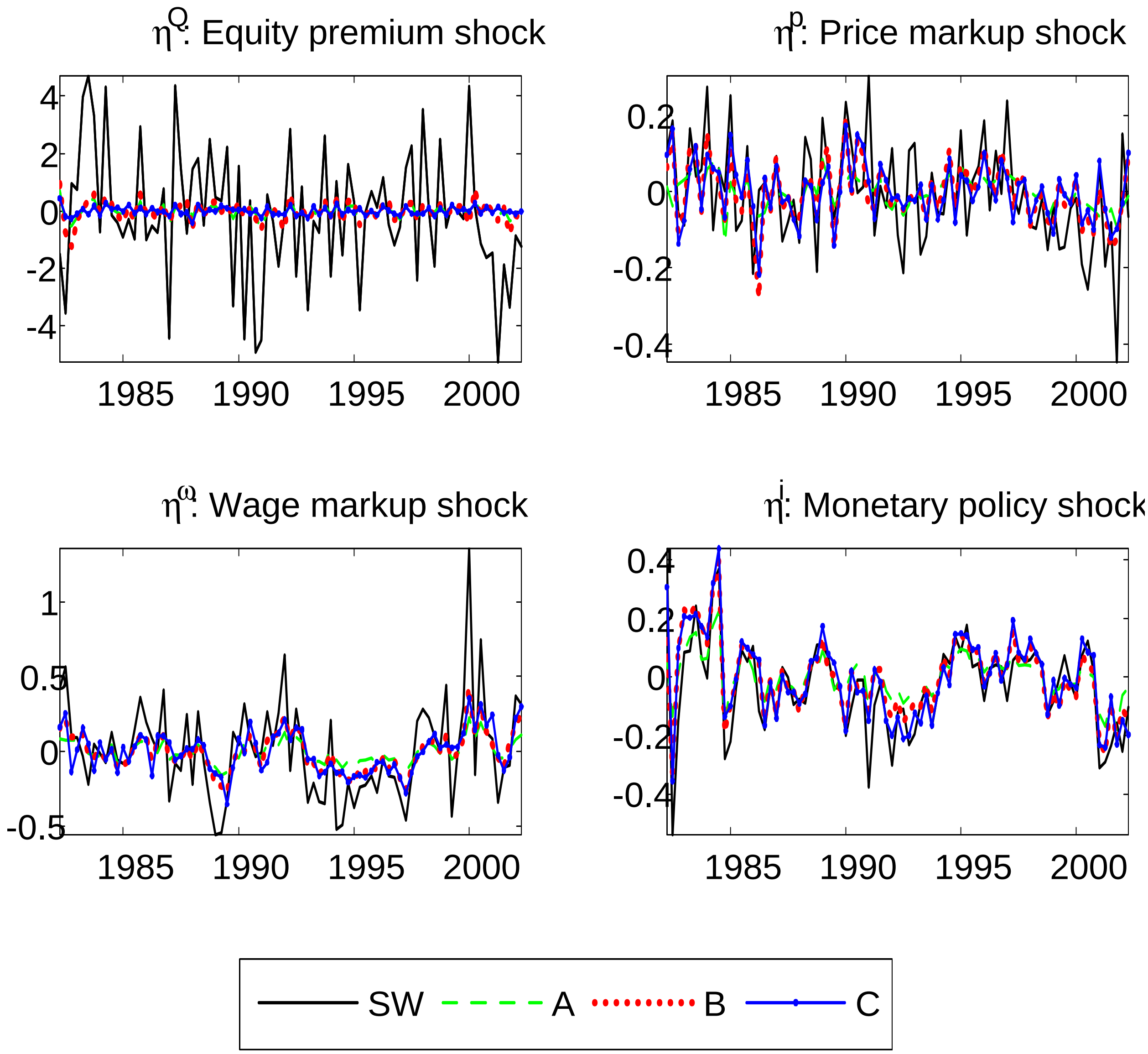

Figure 8: Estimated shocks (Cases SW, A, B, and C)