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9 Subjective Survival Curves and Life Cycle Behavior

Michael Hurd, Daniel McFadden, and Li Gan

9.1 Introduction

Many economic models are based on forward-looking behavior on the part of economic agents. Although it is often said that "expectations" about future events are important in these models, more precisely it is the probability distributions of future events that enter the models. For example, consumption and savings decisions of an individual are thought to depend on what he or she thinks about future interest rates, the likelihood of dying, and the risk of substantial future medical expenditures. According to our theories, decision makers have probability distributions about these and other events, and they use them to make decisions about saving. This implies that data on these distributions should be used in estimation.

In a few microeconomic models, we have data on probability distributions that may plausibly be assumed to approximate those required by the models. Life cycle models of consumption in which mortality risk helps determine saving are the leading example, so we will put our discussion in the context of such models.

Suppose that instantaneous utility, or felicity, is given by

$$u(c_t) = \frac{c_t^{1-\gamma}}{1 - \gamma},$$

that the real interest rate r is constant, and that lifetime utility is time separable. Then, in a common formulation, the first-order conditions imply

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$$\frac{1}{c_t} \cdot \frac{dc_t}{dt} = \frac{-h_t + r - \rho}{\gamma},$$

where ρ is the subjective time rate of discount and h_t is mortality risk at time t. Thus, the rate of change of consumption depends on mortality risk h_t and will be negative if h_t is large. Furthermore, from variations in h_t, the risk aversion parameter γ is econometrically identified. Notice that if there is unobserved heterogeneity in h_t, the coefficient on h_t will tend toward zero, and γ will be estimated to be large. That is, the consumption decisions of individuals will appear not to be responsive to variations in mortality risk.

There have been two approaches to the problem of obtaining mortality risk data to be used in estimation based on the first-order condition above. In the first, an individual is assumed to believe his mortality risk is the same as that contained in a life table, adjusted for age, and possibly sex and race. Under this assumption, if the individual chooses consumption based on his beliefs about mortality risk, the analyst can use data from life tables to explain saving behavior (Hurd 1989b).

The assumption that individuals have reasonable knowledge of the population mortality risk is considerably stronger than in typical demand analysis, which has only the reasonable requirement that individuals know their incomes and that they observe prices. Furthermore, in demand analysis, consumers have observed past variation in prices, and they have had the experience of choosing consumption through repeated trials. It is much less obvious how they would learn about the level of mortality risk in the population. Even if individuals do know about population average mortality risk, the average mortality risk of a cohort may not be well approximated by the life tables because of changing risk. For example, members of a younger cohort may forecast mortality improvements, so the life tables overstate their mortality risk.

A second approach to finding data on survival probabilities is especially pertinent for studies of life cycle behavior. It is based on the well-known variation in mortality rates by economic status (Kitagawa and Hauser 1973; Shorrocks 1975; Hurd and Wise 1989; Jianakoplos, Menchik, and Irvine 1989; Feinstein 1992). If the subjective probability distributions of individuals of differing economic status vary in the same way as the observed mortality rates, using standard life tables in the model estimation will cause the parameters to be misestimated. A further consequence will be that a forecast of the distribution of economic status will be incorrect: for example, poorer individuals who believe their mortality risk is higher than average will dissave faster than what is predicted by the model, causing future poverty rates to be underestimated by the model. Thus, the model will not be able to uncover a possible explanation of the high poverty rates of the oldest old: the poorest at retirement dissaved fastest because of their subjective probability distributions of mortality risk. Variation in mortality risk according to observable characteristics can, in principle, be accounted for by estimating the variation in mortality outcomes in panel data that have been linked to the national death index as in Lillard and Weiss (1997). Although this method is undoubtedly an improvement over using unconditional life tables, it often depends on functional form assumptions for identification. To see this, suppose that life table survival probabilities have been adjusted with covariates such as wealth in a model of life cycle wealth change in panel data. A typical estimating equation would be

$$w_{t+1} = f(w_t, \{q_t\}),$$

where w_t is wealth at time t and $\{q_t\}$ is the path of survival rates. If $\{q_t\}$ depends on wealth, then identification depends to a certain extent on functional form assumptions. This would be true of any covariate that is used to adjust $\{q_t\}$ and that also appears elsewhere in the utility maximization problem. An additional implication is that utility cannot be allowed to depend on age because it is the main determinant of $\{q_t\}$.

Even if adjustments can be made to life tables by using observed covariates, individuals are likely to have subjective probability distributions that are related to unobservable as well as observable variables. It is these subjective probability distributions that should enter life cycle models of saving, so that any models that rely on observed probability distributions have intrinsic limitations.

The importance of accounting for individual-level evaluation of mortality risk is shown in the following example. It has been observed that there is a great deal of heterogeneity both in the results of saving (wealth) and in observed saving rates. From this point of view, the life cycle model of saving is inadequate: for example, it cannot say why apparently similar individuals reach retirement with very different wealth levels (Hurd and Wise 1989), and why they save at different rates following retirement. It could be that most of the variation in saving behavior is due to taste differences across individuals or to forces we do not understand. An alternative explanation is that there is a great deal of variation at the individual level in mortality risk, but we do not usually observe this variable.

9.2 Data

Our data come from the survey of the Asset and Health Dynamics among the Oldest Old (AHEAD). This is a biennial panel of individuals born in 1923 or earlier and their spouses. At baseline in 1993, it surveyed 8,222 individuals representative of the community-based population, except for oversamples of blacks, Hispanics, and Floridians. The main goal of AHEAD is to provide panel data from the three broad domains of economic status, health, and family connections (Soldo et al. 1997). This is reflected in the questionnaire sections and average interview timings as follows:

A. Demographics (3.3 minutes)
B. Health (7.3)
C. Cognition (4.5)
D. Family (8.2)
E. Health care and costs (11.9)
F. Housing (3.8)
G. Job status and history (4.0)
H. Expectations (3.3)
J. Income (5.7)
K. Assets (3.2)
R. Insurance (3.2)

Our main interest in this paper is in the data from the expectations section and its relationship to personal characteristics, particularly cognition. The survey has eight measures of subjective probabilities. In this paper we will give some descriptive statistics on them, but our main attention will be on the subjective probability of survival. We will show that it has informational content, but that it cannot be used without modification as a right-hand variable in a model of decision making because of cognition and observation error. We will propose and estimate a model of cognition error and then apply the model to life tables and to data from AHEAD to produce usable subjective probabilities of survival.

Subjects were asked the following series of questions about the likelihood of future events:

> [Using any] number from 0 to 100 where "0" means that you think there is absolutely no chance and "100" means that you think the event is absolutely sure to happen . . . What do you think are the chances that:
>
> 1. You will have to give major financial help to family members during the next 10 years?
> 2. You will receive major financial help from family members during the next 10 years?
> 3. You will leave a financial inheritance?
>
> *If the response was in the range [1, 100] a follow-up question was asked:*
>
> a. You will leave an inheritance of at least $10,000?
>
> *If the response was in the range [31, 100], a further follow-up question was asked:*
>
> b. You will leave an inheritance of at least $100,000?
>
> 4. You will move to a nursing home over the next five years?
> 5. You will move during the next five years?
> 6. Medical expenses will use up all your savings sometime during the next five years?
> 7. Your income will keep up with inflation during the next five years?
>
> *In the following question asked of respondents (R) of age less than 90, A is 80 for R of age less than 70, 85 for R aged 70–74, 90 for R aged 75–79, 95 for R aged 80–84, and 100 for R aged 85–89:*
>
> 8. You will live to be at least A?

The expectations questions were not asked in proxy interviews in AHEAD; we have 7,393 responses to these questions. We treat responses to these questions as if they are subjective probabilities of the events, up to possible reporting error.

The remaining variables employed in the analysis of subjective survival probabilities are quite standard, except for measures of cognitive ability. AHEAD measures cognitive status in a battery of questions that aim to test a number of domains of cognition (Herzog and Wallace 1997): learning and memory are assessed by immediate and delayed recall from a list of 10 words that were read to the subject; reasoning, orientation, and attention are assessed from serial 7s (in which the subject is asked to subtract 7 from 100, and then to continue subtracting 7 from each successive difference for a total of five subtractions), counting backward by 1, and the naming of public figures, dates, and objects. We aggregate these responses into an indicator for cognitive disability that is one if the number of correct answers to all the cognitive questions falls below a threshold level. This identifies, approximately, the lowest quartile in cognitive function.

Subjective probabilities. Subjective survival probabilities measured in the Health and Retirement Survey (HRS) provide a benchmark for AHEAD responses. The HRS subjects were aged 51–61. Average survival probabilities to age 75 were 0.65, which is very close to a weighted average from a 1990 life table of 0.68 (Hurd and McGarry 1995). The survival probabilities vary with risk factors in the same way as mortality outcomes in the population. For example, those with higher socioeconomic status (measured by education, income, or wealth) give higher survival probabilities; smokers give lower probabilities, moderate drinkers give higher probabilities than either teetotalers or heavy drinkers, and those whose parents survived to old age give higher survival probabilities. These subjective survival probabilities correlate with actual mortality experience of subgroups of the HRS population.

In the HRS subjects were asked about their probability of working past age 62 or 65. These probabilities vary with financial and job characteristics in the same way as actual retirement outcomes. For example, those with defined benefit pension plans that offer early retirement give low probabilities of working past 62, those with employer-paid retiree health insurance give low probabilities, and those on jobs where it is usual to retire early give low probabilities (Hurd and McGarry 1993). We take these results to be good evidence that the HRS respondents understood questions about subjective probabilities and gave appropriate responses. However, the AHEAD population is older and has higher levels of cognitive impairment, so it may be that its responses are less appropriate.

Table 9.1 shows the average and median survival probabilities from AHEAD and from 1992 life tables for the target ages used in the AHEAD survival question (e.g., 85 for subjects aged 70–74, 90 for subjects aged 75–79, with minor

Table 9.1 **Survival Probabilities**

	Target Age			
	85	90	95	100
Means				
AHEAD	0.51	0.38	0.31	0.29
Life table	0.50	0.33	0.16	0.05
Medians				
AHEAD	0.50	0.40	0.20	0.10
Life table	0.55	0.37	0.18	0.06

Source: Hoynes, Hurd, and Chand (1995).

spillover from timing of birthdays and interviews). As in the HRS, the AHEAD respondents at the younger ages (70–79) have average subjective survival probabilities that are close to averages from life tables, but at older ages the averages are substantially higher. Besides the obvious explanation that cognitive function declines with age, there are several other explanations. First, at baseline the AHEAD was a community-based survey: those in nursing homes and other institutional care facilities were excluded. Thus AHEAD represents a healthier population than is represented by life tables, with the implication that if the populations were the same the difference in survival probabilities would be reduced. Second, the questions about subjective probabilities were asked only in self-interviews, not in proxy interviews. Subjects who are interviewed by proxy have worse health, and because the frequency of proxy interviews increased with age the population of self-interviews has better health than the complete AHEAD population. Third, even among self-interviews the frequency of nonresponse to the questions about subjective probabilities increases with age, and nonrespondents have worse health than respondents.

We have not attempted any analysis of the magnitude of the bias that results from these levels of sample selection, but they could be substantial: the overall rates of nonresponse to the question about survival probabilities are 12.2, 15.2, 19.5, and 19.3 percent in our four age groups. We have no way to assess the bias resulting from the exclusion of the institutionalized population. Even with these kinds of adjustments, however, it is unlikely that in the older two age groups the means would be reduced to the levels of the life tables.

As shown in table 9.1, the medians from AHEAD and from the life tables are much closer than the means. Apparently, a few AHEAD respondents gave very high probabilities of survival, increasing the mean substantially. This is verified in figures 9.1 through 9.4, which show the distributions. The figures are noteworthy because even at advanced ages a number of respondents give survival probabilities of 1.0. Particularly in the oldest age group, even a fairly small number of such responses will increase the mean because the life table means are so small. The figures show a leftward shift with age in the distribution as is expected. But in all the age groups a large fraction of respondents give what we call focal-point responses: 0.0, 0.5, or 1.0. The prevalence of

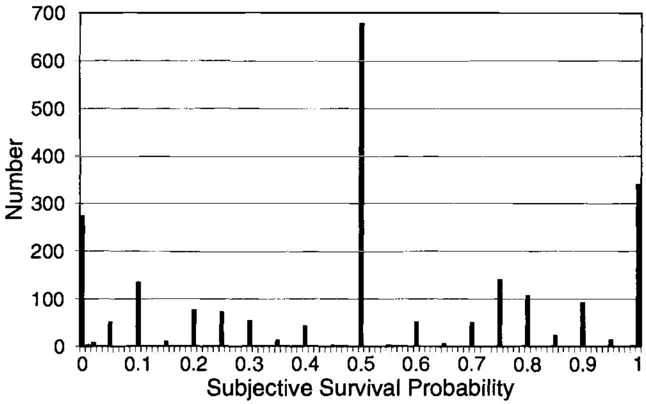

Fig. 9.1 Survival probabilities to age 85 among 70–74-year-olds

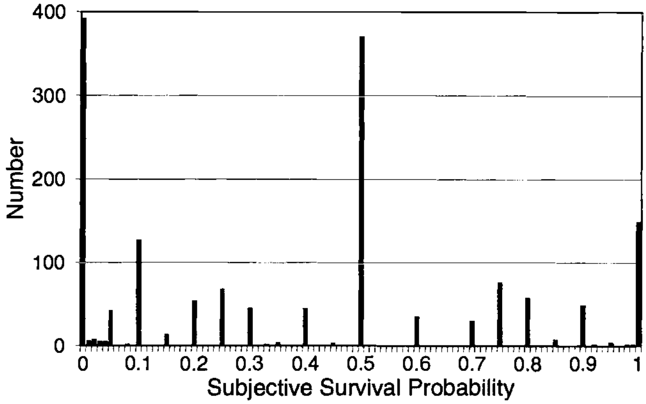

Fig. 9.2 Survival probabilities to age 90 among 75–79-year-olds

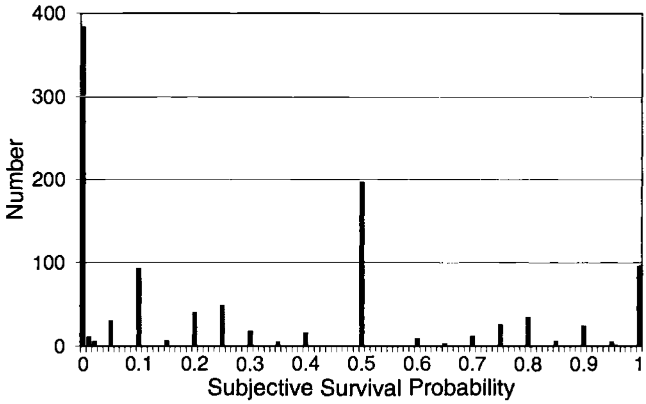

Fig. 9.3 Survival probabilities to age 95 among 80–84-year-olds

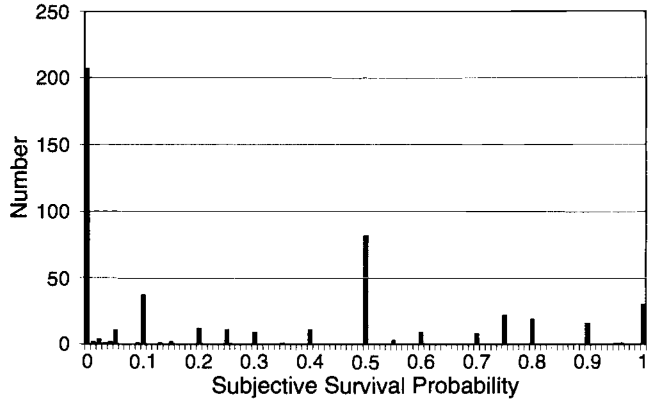

Fig. 9.4 Survival probabilities to age 100 among 85–89-year-olds

focal-point responses shows that the measure of subjective probabilities in AHEAD cannot represent the true probabilities, both because the distribution of true probabilities should be continuous and because the true probabilities cannot be literally either zero or one. A major focus of this paper is to learn about the determinants of the likelihood that a respondent will give a focal-point response and to specify and estimate a model of cognition that will account for the observed tendency for focal-point responses.

9.3 Determinants of Focal-Point Responses

In this section, we investigate the propensity to give a focal response (0.0, 0.5 or 1.0) on one or more of the eight measures of subjective probabilities. Our view is that the stated subjective probabilities, including focal responses, have informational content, but it may not be accurate to take them at face value. To investigate the question of informational content, we use as a standard of comparison the view that they are simply independent random responses to a request to name a number between 0 and 100.

Figure 9.5 shows the distribution of respondents according to the number of responses of zero to the probability questions. About 11 percent of the respondents to the subjective questions gave no zero responses, and the remainder gave a modal number of three. Of course, a response of zero can be appropriate depending on the event because some of the events have almost no stochastic element for some respondents. For example, the probability of receiving major financial help could be zero for someone with no family connections. Similarly, moving is controlled by the respondent, and the probability of moving could be close to zero. However, 59 respondents answered zero to seven of the subjective probability questions. In that the events are mostly controlled

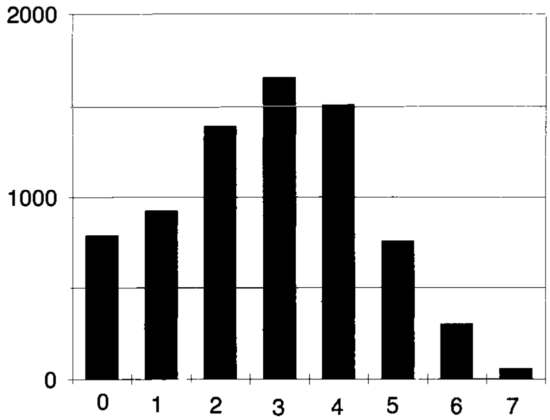

Fig. 9.5 Focal subjective probabilities: distribution by number of zeros

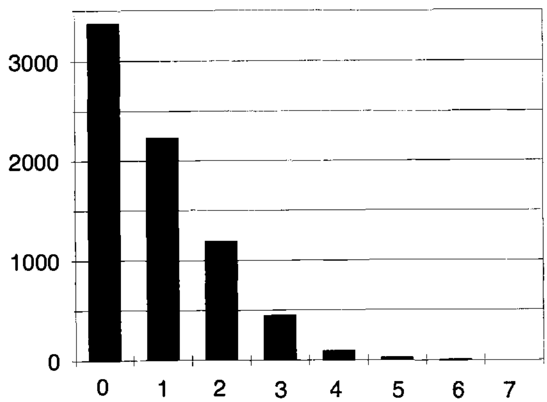

Fig. 9.6 Focal subjective probabilities: distribution by number of 0.5s

stochastic processes, with a mixture of level of control, it is hard to see how a well-informed assessment of the true probabilities could so often be zero. More likely these respondents did not understand the nature of the question or were uncooperative.

Figure 9.6 gives a similar distribution with respect to responses of 0.5. The distribution is quite different from the distribution of zeros: 51 percent gave none, and the distribution declines sharply. Thus, although there is overall a fairly high propensity to give a 0.5, it is mainly concentrated among a few respondents and to a few events. The distribution of 1.0s has a similar shape (fig. 9.7) except that the left-hand part of the distribution is heavier. For example, just 3.2 percent of the respondents gave responses of 1.0 on three or

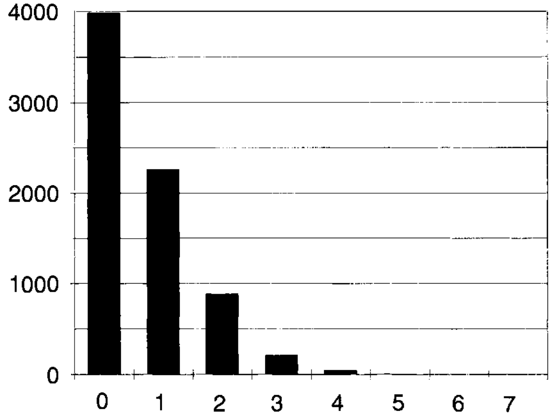

Fig. 9.7 Focal subjective probabilities: distribution by number of 1.0s

more of the subjective probabilities, whereas 7.2 percent gave responses of 0.5 on three or more. The conclusion is that the focal point of zero attracts the most responses, followed by 0.5.

The distributions of focal-point responses indicate that some individuals often give focal-point responses to the subjective probability questions, suggesting that there is an individual-level propensity to give a focal-point response that may be independent of the event that is queried. To examine this we study the probability of giving a focal-point response to the question about the likelihood of survival as a function of the number of focal-point answers given to the other questions about subjective probabilities. For example, we specify that $P(S = 0) = f(n_0)$, where $S = 0$ means the survival probability is reported to be zero and n_0 is the number of zeros on the other subjective probabilities. If the likelihood of giving a focal-point response to the survival question is independent of whether focal-point responses were given to the other subjective probability questions, we should find no relationship between n_0 and $P(S = 0)$.

Figure 9.8 shows the unconditional probability as a function of n_0. About 16 percent of the respondents report a survival probability of zero. Among those who have no zero responses on the other subjective probabilities, just 1 percent gave a zero probability of survival. The likelihood of giving a zero for the survival probability increases in the number of zeros on the other subjective probabilities, so that among those who have zeros on all six of the other subjective probabilities, 30.4 percent gave a zero on the survival probability. One possible explanation for this result is that there are individual characteristics that make the probability of all these events truly low, approaching zero. However, the nature of the questions is that some of the events are desirable and would be positively correlated with socioeconomic status and other character-

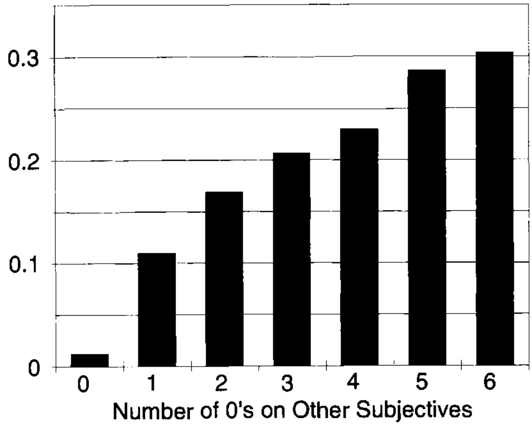

Fig. 9.8 Focal subjective responses: proportion with survival response zero

istics, and some are undesirable and would be negatively correlated. For example, a rough judgment would put them into the following classification:

Positive events
Your income will keep up with inflation during the next five years?
You will live to be at least A? (where A is the target age)
You will leave a financial inheritance?

Negative events
You will move to a nursing home in the next five years?
Medical expenses will use up all your savings sometime during the next five years?
You will have to give major financial help to family members during the next 10 years?

Neutral events
You will receive major financial help from family members during the next 10 years?
You will move during the next five years?

Someone with a small probability of using all his savings on medical expenses is likely to be in good health and to have adequate resources. Such a person is likely to have good survival chances and to leave an inheritance and, therefore, should give high probabilities to the questions about them. Indeed the raw correlation coefficient between the probability of medical expenses and the probability of survival is .13 and between medical expenses and leaving a financial inheritance is .18.

Figure 9.9 has similar results where the focal point of the survival probability is 0.5. The average frequency of giving 0.5 is about 0.21, but among those who give no 0.5s on the other subjective probability questions the frequency

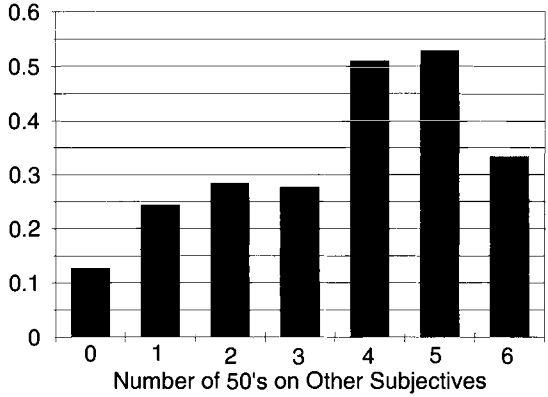

Fig. 9.9 Focal subjective responses: proportion with survival response 0.5

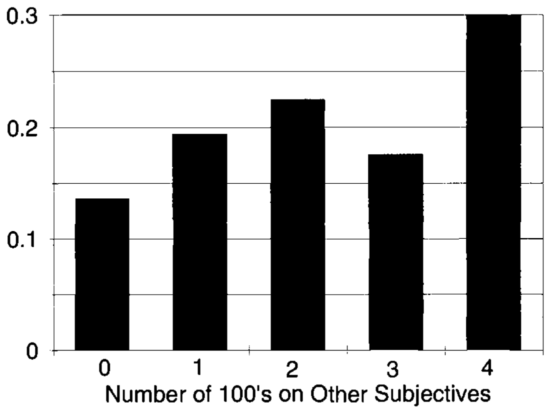

Fig. 9.10 Focal subjective responses: proportion with survival response 1.0

is just 0.13. It increases to 0.51 among the subjects who answered 0.5 to four other subjective probability questions. Figure 9.10 shows that the unconditional frequency of giving 1.0 as the survival probability is about 0.17. Except for the fourth entry, which is based on 19 observations, there is a monotonic increase in the frequency as the number of 1.0s given on the other subjective probability questions increases, reaching 0.3.

We interpret these results to be good evidence of an individual-level propensity to give focal-point responses. Although it seems unlikely that the patterns could be due to covariates or personal characteristics that are related to the probabilities of actual outcomes, we investigate this by a regression (logits) of the probability of giving a focal-point answer on the number of other focal-point answers (as above) and on a number of personal characteristics. That is, we estimate

Table 9.2 **Determinants of the Probability of Giving a Focal-Point Response (linear probability model for survival probabilities)**

	Focal of 0.0		Focal of 0.5		Focal of 1.0	
Variable	Coefficient	S.E.	Coefficient	S.E.	Coefficient	S.E.
Intercept	−0.026	0.025	0.215	0.024	0.159	0.020
$n = 1$	0.154	0.019	0.115	0.013	0.133	0.011
$n = 2$	0.246	0.017	0.170	0.017	0.245	0.017
$n = 3$	0.292	0.016	0.192	0.027	0.346	0.040
$n = 4$	0.308	0.016	0.258	0.051	0.381	0.075
$n = 5$	0.369	0.018	0.602	0.089		
$n = 6$	0.411	0.025	0.271	0.147		
$n = 7$	0.459	0.040	0.852	0.415		
Age						
70–74	0.019	0.020	0.026	0.020	−0.011	0.016
75–79	0.111	0.020	−0.027	0.021	−0.076	0.017
80–84	0.170	0.021	−0.063	0.022	−0.093	0.018
85–89	0.112	0.022	−0.120	0.024	−0.155	0.020
Male	−0.014	0.011	0.000	0.011	−0.001	0.010
Health						
Excellent	−0.074	0.019	−0.006	0.019	0.102	0.016
Very good	−0.058	0.014	0.000	0.015	0.040	0.012
Fair	0.073	0.014	−0.033	0.015	−0.025	0.013
Poor	0.122	0.017	−0.089	0.020	−0.032	0.017
Health change						
Better	−0.010	0.015	−0.007	0.016	0.005	0.013
Worse	0.031	0.013	−0.024	0.015	−0.024	0.013
Wealth						
Quartile 2	0.043	0.015	0.036	0.017	−0.021	0.014
Quartile 3	0.057	0.015	0.043	0.017	−0.042	0.014
Quartile 4	0.011	0.016	0.028	0.017	−0.076	0.014
Cognitive impairment	−0.026	0.011	−0.035	0.013	0.024	0.010

Source: Authors' calculations from AHEAD.

Notes: n is the number of focal points with the same value on other subjective probabilities. The reference is $n = 0$, age 65–69, female, good health, lowest wealth quartile, no cognitive impairment. S.E. = standard error.

$$P(S = 0) = f(n_0) + X\beta,$$

where X is a vector of personal characteristics. Table 9.2 shows the results of the linear regressions. The first two columns have the coefficients and standard errors from the regression of a variable that takes the value one if the subjective survival probability is zero and takes the value zero otherwise. The important regressors are categorical variables (the ns) that represent the number of zeros on the other seven subjective probabilities, age intervals, self-assessed health categories, health change, wealth quartiles, and a categorical variable that may indicate cognitive impairment.

The table shows the same pattern by n as figure 9.8: those with the fewest

Table 9.3 Logit Probability of a Focal-Point Response (survival probabilities, base case except variable indicated)

Variable	Focal At Zero	Focal At One
$n = 0$	0.003	0.143
$n = 1$	0.057	0.357
$n = 2$	0.103	0.509
$n = 3$	0.130	0.632
$n = 4$	0.140	0.642
$n = 5$	0.160	
$n = 6$	0.207	
$n = 7$	0.240	
Age		
65–69	0.140	0.143
70–74	0.162	0.133
75–79	0.272	0.081
80–84	0.351	0.067
85–89	0.280	0.025
Female	0.140	0.143
Male	0.130	0.140
Health		
Excellent	0.083	0.264
Very good	0.096	0.193
Good	0.140	0.143
Fair	0.206	0.110
Poor	0.267	0.096
Health change		
Better	0.134	0.148
Same	0.140	0.143
Worse	0.173	0.107
Wealth		
Quartile 1	0.140	0.143
Quartile 2	0.184	0.115
Quartile 3	0.196	0.098
Quartile 4	0.148	0.073
No cognitive impairment	0.14	0.143
Cognitive impairment	0.125	0.175

Source: Authors' calculations from AHEAD.

Note: n is the number of focal points with the same value on other subjective probabilities. The base case is $n = 0$, age 65–69, female, good health, health same, lowest wealth quartile, no cognitive impairment.

responses of zero on the other subjective probabilities have the lowest probability of having a zero subjective survival probability: for example, if someone gave zeros on all seven of the other subjective probabilities, the likelihood of giving a zero on the survival probability is 0.459 greater than if he had no zeros on the other subjective probabilities. The other covariates show reasonable patterns. The probability of giving a zero increases with age and increases both at lower levels of self-assessed health and with worsening health.

The next two columns have similar results where the left-hand variable takes the value one if the response to subjective survival is 0.5, and zero otherwise. Again the probability is strongly increasing in n, verifying the results of figure 9.9. Unlike the case when the left-hand variables indicates a zero response, we have no particular prior beliefs about the pattern with age: the likelihood of responding with a 0.5 could increase with age because of increasing cognition difficulties, but it could decrease with age because the true probability of survival (as measured by life tables) falls rapidly toward zero. Indeed, the empirical outcome is that the probability of giving a focal response of 0.5 decreases in age and in poor health status and worsening health. The last two columns have the results for a focal response of 1.0. Again the probability is increasing in n. The age pattern is as expected: the older respondents are less likely. The variation by health level and health change is consistent with our other results. Our cognitive impairment indicator shows, when viewed across the three probabilities, that having an impairment increases the likelihood of giving a focal response of 1.0 compared with a focal response of zero or 0.5. To the extent that cognitive impairment is an additional indicator of underlying health status the effect should be the opposite. That is, cognitive impairment increases the likelihood of making an objectively incorrect assessment of the probability of survival.

Table 9.3 shows fitted probabilities from logistic estimation of the same relationships. The pattern as n varies is the same as in table 9.2, but the magnitude of the variation is attenuated. Now it is quite close to the variation shown in figures 9.8 through 9.10. For example, the probability of a focal response of zero varies by about 0.21 as n varies from 0 to 6; in figure 9.8 the (unconditional) variation is 0.27. The table shows that the probabilities of focal responses vary substantially with the other covariates: as health varies from excellent to poor the likelihood of a focal response of zero increases by 0.184 whereas the likelihood of a focal response of 1.0 falls by 0.168. Having a cognitive impairment increases the likelihood of a focal response of 1.0 compared with the likelihood of a response of zero by about 0.05, which is not realistic.

9.4 A Model for Personal Survival Curves

Each individual faces a survival curve, $q(t \mid a,z,\varepsilon)$, giving the probability that remaining life will exceed t years. This curve will depend on the current age a of the individual and may depend on observed and unobserved covariates, denoted by z and ε, respectively. A rational individual who engages in life cycle planning will utilize subjective beliefs about this survival curve. For example, a life cycle optimizer who has a time-separable felicity function $u(c_a)$ of consumption at age a, a discount rate ρ, and no bequest motive and faces no uncertainty other than date of death will seek to maximize

$$\int_a^{+\infty} u(c_{a+t}) \cdot e^{-\rho t} \cdot q(t \mid a, z, \varepsilon)\, dt.$$

The survival curve q is now interpreted as the subjective belief of the individual at age a. Suppose that the covariates (z,ε) influencing these beliefs are time invariant, and that the beliefs are intertemporally consistent, so that $q(t+\tau \mid a,z,\varepsilon) = q(t \mid a,z,\varepsilon) \cdot q(\tau \mid a+t,z,\varepsilon)$. Then the optimization does not involve strategic consideration of possible changes in beliefs. The optimization is carried out subject to a given initial wealth W_a and a condition that future wealth be nonnegative. The equation of motion of wealth is

$$\nabla_t W_{a+t} = rW_{a+t} + y_{a+t} - c_{a+t},$$

where y_{a+t} is annuity income and r is the interest rate. When wealth is positive over an interval $[a,a+t)$, the optimal consumption stream satisfies

$$u'(c_a) = u'(c_{a+t}) \cdot e^{(r-\rho)t} \cdot q(t \mid a, z, \varepsilon).$$

The individual will display decreasing consumption, implying decreasing wealth, if $r - \rho - h(t \mid a,z,\varepsilon) < 0$, where $h(t \mid a,z,\varepsilon) \equiv -\nabla_t \log q(t \mid a,z,\varepsilon)$ is the mortality hazard rate. More generally, the larger $q(t \mid a,z,\varepsilon)$, the lower consumption and the larger net saving. Rising mortality hazard should then eventually lead among survivors to declining consumption and negative saving.

A standard formulation of the life cycle savings model assumes that all individuals of the same age have a *common* survival curve that coincides with national life tables, and that the individuals know this curve, so there is no variation in subjective beliefs about survival. Then a parameterization of $u(c)$ such as the constant relative risk aversion function $u(c) = c^{1-\gamma}/(1 - \gamma)$ allows the model above to be estimated from panel data. A qualitative characterization of the estimation results of Hurd (1990) and others is that there is less dissaving than might be expected with commonly assumed levels of risk aversion and no bequest motive, and substantially more variability in saving rates than a model with homogeneous preferences and survival curves would suggest. Explanations that have been offered for the relatively low rates of dissaving include strong bequest motives, high risk aversion toward the end of life, and unanticipated taste changes, due say to health, that reduce the marginal utility of consumption. Another possible explanation, which also explains some of the high variability in saving rates, is that survival curves are heterogeneous, and selection progressively removes individuals with low survival probabilities and high rates of dissaving, so that average wealth holdings of survivors do not decline rapidly with age. Heterogeneity in the degree of risk aversion and in bequest motives would also contribute to variability in saving behavior.

In this section, we start from the assumption that there is a personal survival curve known to the individual, with the econometrician observing some but not all of the covariates that personalize this curve. We will assume that questions about survival probabilities to specified ages provide information, not necessarily exact, on the personal survival curve. We use this information, plus covariates, to fit an estimated personal survival curve for each individual in the AHEAD sample. In following sections, we investigate the link between this survival curve and saving behavior.

Two critical assumptions will provide the foundation for our model of personal survival curves. First, assume the personal survival curve of an individual can be represented by a *Cox proportional hazards* survival curve at elapsed time t from initial age a,

$$q(t \mid a, v, z, \varepsilon) = \exp(-(\Lambda_v(t + a) - \Lambda_v(a))e^{z\beta - \sigma\varepsilon}),$$

where a is starting age, t is elapsed time, $\Lambda_v(a)$ is an *integrated baseline hazard* function at age a for an individual born in year v, measured starting from age zero, z are covariates, β are parameters, ε is a disturbance idiosyncratic to the individual that is normalized to have zero mean and unit variance, and σ is a scale parameter. The second critical assumption regards the perceptual and reporting errors that may enter stated subjective survival probabilities. We allow for the possibility that individuals may be systematically optimistic or pessimistic by introducing *time scale distortion,* or *accelerated failure time,* in which individuals view their personal clocks as running faster or slower than the chronological clock. We also consider the possibility of focal responses in reporting subjective probabilities. The details of these assumptions are given later.

9.4.1 The Algebra of Heterogeneous Personal Survival Curves

Selection determines the relationship between the personal survival curve $q(t \mid a,v,z,\varepsilon)$, the expected survival curve $Q(t \mid a,v,z)$ of individuals of age a with observed covariates z, and the population mean survival curve $Q(t \mid a,v)$. Let $f(\varepsilon \mid 0)$ denote the density at birth of the unobserved factor ε, and let $g(z \mid 0)$ denote the density at birth of the observed covariates z. Then

$$Q(a \mid 0, v, z) = \int q(a \mid 0, v, z, \varepsilon) f(\varepsilon \mid 0)\, d\varepsilon,$$

$$Q(a \mid 0, v) = \int Q(a \mid 0, v, z) g(z \mid 0)\, dz,$$

the density of ε among survivors of vintage v at age a, given z, is

$$f(\varepsilon \mid a, v, z) = f(\varepsilon \mid 0) \cdot q(a \mid 0, v, z, \varepsilon) / Q(a \mid 0, v, z),$$

and the density of z among survivors of vintage v at age a is

$$g(z \mid a, v) = g(z \mid 0) \cdot Q(a \mid 0, v, z) / Q(a \mid 0, v).$$

Therefore,

$$
\begin{aligned}
&Q(t \mid a, v, z) \\
&= \int q(a+t \mid a, v, z, \varepsilon) \cdot f(\varepsilon \mid a, v, z)\, d\varepsilon \\
&= \int q(a+t \mid 0, v, z, \varepsilon) \cdot f(\varepsilon \mid 0)\, d\varepsilon / Q(a \mid 0, v, z) \\
&= Q(t+a \mid 0, v, z)/Q(a \mid 0, v, z),
\end{aligned}
$$

and

$$
\begin{aligned}
&Q(t \mid a, v) \\
&= \int Q(t \mid a, v, z) g(z \mid a, v)\, dz \\
&= \int\int q(a+t \mid 0, v, z, \varepsilon) \cdot f(\varepsilon \mid 0) \cdot g(z \mid 0)\, d\varepsilon\, dz / Q(t \mid 0, v) \\
&= Q(t+a \mid 0, v)/Q(a \mid 0, v).
\end{aligned}
$$

Let $s = e^{-\sigma\varepsilon}$, and let $k(s)$ be the density of s induced by the density $f(\varepsilon \mid 0)$; that is, $k(s) = f(-\log(s)/\sigma \mid 0)/s\sigma$. Let $\psi(r) = \int_0^{+\infty} e^{-rs} \cdot k(s)\, ds$ be the Laplace transform of k. Then

$$
\begin{aligned}
Q(a \mid 0, v, z) &= \int_{-\infty}^{+\infty} f(\varepsilon \mid 0) \cdot \exp(-\Lambda_v(a) e^{z\beta - \sigma\varepsilon}\, d\varepsilon \\
&= \int_0^{+\infty} k(s) \cdot \exp(-\Lambda_v(a) e^{z\beta} s)\, ds = \psi(\Lambda_v(a) e^{z\beta}).
\end{aligned}
$$

The density of s given a, z then satisfies

$$k(s \mid a, v, z) = k(s) \cdot \exp(-\Lambda_v(a) e^{z\beta} s)/\psi(\Lambda_v(a) e^{z\beta}).$$

The moment-generating function $m(r)$ for the density $k(s \mid a,v,qz)$ is

$$m(r) = \mathbf{E}\{e^{rs} \mid a, v, z\} = \psi(\Lambda_v(a) e^{z\beta} - r)/\psi(\Lambda_v(a) e^{z\beta}).$$

From this, $\mathbf{E}\{s \mid a,z\} = -\psi'(\Lambda_v(a) e^{z\beta})/\psi(\Lambda_v(a) e^{z\beta})$ and one has the moments

$$
\begin{aligned}
\mathbf{E}\{\log q(t \mid a, v, z, \varepsilon) \mid a, v, z\} &= [\Lambda_v(t+a) - \Lambda_v(a)] \cdot e^{z\beta} \cdot \frac{\psi'(\Lambda_v(a) e^{z\beta})}{\psi(\Lambda_v(a) e^{z\beta})}, \\
\mathbf{V}\{\log q(t \mid a, v, z, \varepsilon) \mid a, v, z\} &= [\Lambda_v(t+a) - \Lambda_v(a)]^2 \cdot e^{2z\beta} \cdot \mathbf{M}(z),
\end{aligned}
$$

with

$$\mathbf{M}(z) \equiv \frac{\psi''(\Lambda_v(a) e^{z\beta})}{\psi(\Lambda_v(a) e^{z\beta})} - \left(\frac{\psi'(\Lambda_v(a) e^{z\beta})}{\psi(\Lambda_v(a) e^{z\beta})}\right).$$

Similarly, let $\tau = e^{z\beta - \sigma\varepsilon}$, and let $j(\tau)$ be its density induced by the joint density $f(\varepsilon \mid 0) \cdot g(z \mid 0)$ of ε and z; that is,

$$j(\tau) = \frac{1}{\sigma\tau}\int f\left(\frac{z\beta - \log(\tau)}{\sigma}\Big|0\right) \cdot g(z|0)\,dz.$$

Let $\vartheta(r)$ denote the Laplace transform of j. Then

$$\begin{aligned} Q(a|0,v) &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\varepsilon|0)g(z|0) \cdot \exp(-\Lambda_v(a)e^{z\beta-\sigma\varepsilon})\,d\varepsilon\,dz \\ &= \int_0^{+\infty} j(\tau) \cdot \exp(-\Lambda_v(a)\tau)\,d\tau = \vartheta(\Lambda_v(a)) \end{aligned}$$

and

$$Q(t|a,v) = Q(t+a|0,v)/Q(a|0,v) = \vartheta(\Lambda_v(a+t))/\vartheta(\Lambda_v(a)).$$

Summarizing, selection thins the left tail of $f(\varepsilon \mid a,v,z)$ relative to the right tail, as individuals with unfavorable draws of ε die out. As a result, $Q(t \mid a,v,z)$ declines less rapidly with t than does $q(t \mid a,v,z,\varepsilon)$ for any fixed ε. Similarly, selection thins the regions of z that elevate mortality risk, so that $Q(t \mid a,v)$ declines less rapidly with t than does $Q(t \mid a,v,z)$ for any fixed z. Further, selection induces a correlation of ε and z, so that combinations of z that elevate mortality risk are among survivors positively associated with ε. A completely consistent analysis of personal mortality risk that combines individual data and life table data for persons of different ages has to handle these selection effects. We do this by parameterizing $j(\tau)$.

A fundamental identification question is what can be learned about $\Lambda_v(a)$ in the presence of the unknown function ϑ. For any increasing function $\rho(r)$ such that $\vartheta(\rho(r))$ continues to have the properties of a Laplace transform, one clearly cannot distinguish the model with ϑ and $\Lambda_v(a)$ from the model with ϑ^* and $\Lambda_v^*(a)$ that satisfies $\vartheta^*(r) = \vartheta(\rho(r))$ and $\Lambda_v^*(a) = \rho^{-1}(\Lambda_v(a))$. Consequently, any econometric specification that attempts to estimate $\Lambda_v(a)$ nonparametrically in combination with a parameterization of ϑ that allows monotonically varying alternatives must fail.

With this limitation in mind, we consider the parametric assumption that τ has the gamma density $j(\tau) = \omega^\omega\tau^{\omega-1}e^{-\omega\tau}/\Gamma(\omega)$ with mean one and variance $1/\omega$. The Laplace transform of this density is $\vartheta(r) = (1 + r/\omega)^{-\omega}$. Consider a quadratic spline approximation to $\Lambda_v(a)$:

$$\begin{aligned} \Lambda_v(a) &= \alpha_0 a + \sum_{i=1}^{I}\alpha_{i0}\mathbf{1}(a > A_i)(a - A_i)^2 + \sum_{j=1}^{J}\alpha_{0j}(v - V_j) \\ &\quad + \sum_{i=1}^{I}\sum_{j=1}^{J}\alpha_{ij}\mathbf{1}(a > A_i)\mathbf{1}(v > V_j)(a - A_i)^2(v - V_j), \end{aligned}$$

where the αs are parameters, the A_i are ages at five-year intervals defined so there are life table sample points below the lowest and above the highest A_i, and the V_j are vintages at 20-year intervals defined so there are life tables for vintages below the lowest and above the highest V_j. This form then yields a piecewise linear drift in hazard rates with vintage and with age. The estimation task is then to use life tables for different ages and vintages to determine the relationship

$$Q(a \mid 0, v) = \begin{cases} [1 + \Lambda_v(a)/\omega]^{-\omega} & \text{if } \omega > 0, \\ \exp(-\Lambda_v(a)) & \text{if } \omega = 0. \end{cases}$$

Recognizing that this model corresponds to a Box-Cox transformation,

$$\frac{Q(a \mid 0, v)^{-\omega} - 1}{\omega} = \Lambda_v(a) + \xi,$$

a computationally efficient way to carry out the estimation is by pseudomaximum likelihood, treating the disturbance ξ as normal. The results of this estimation are that the likelihood is maximized over the interval $\omega \geq 0$ consistent with the Laplace transform at the boundary value $\omega = 0$, corresponding to an absence of unobserved heterogeneity. In light of the previous discussion of identification, this provides no real evidence for or against the presence of heterogeneity in mortality hazard, but rather indicates that the spline approximation to the integrated hazard is sufficiently flexible to capture the effects of heterogeneity, so that the additional parameter ω is not needed to characterize the life tables. In the subsequent analysis, we use the fitted baseline integrated hazard function obtained from the regression of log $Q(a \mid 0,v)$ on $\Lambda_v(a)$. Keep in mind that this characterization then includes the average effect of population heterogeneity.

9.4.2 Subjective Survival Probabilities

If an individual knows that he or she has a personal survival curve given by the Cox proportional hazards form, with known covariates z and ε, and with the baseline integrated hazard $\Lambda_v(a)$, and is fully rational, then this curve will enter life cycle savings decisions and may also provide the basis for reported subjective survival probabilities. Alternatively, individuals may not be fully rational and instead may be systematically optimistic or pessimistic about survival. We will parameterize this by allowing individuals to distort the scale of chronological time. An individual of age a who contemplates survival for an interval t is assumed to convert this to an equivalent value

$$T_a(t) = (t + 1)^{\alpha_1 + \alpha_2 \cdot a} - 1,$$

where α_1 and α_2 are parameters. This equivalent value will replace the chronological interval t in the subjective survival curve. If $\alpha_1 = 1$ and $\alpha_2 = 0$, there is no systematic bias about survival. If $\alpha_1 + \alpha_2 \cdot a < 1$, then individuals are systematically optimistic, underestimating mortality risk over a time interval t. If $\alpha_2 < 0$, then individuals become more optimistic as they age. This specification is a parametric specialization of what is termed an accelerated failure time model.

Time scale distortion may appear in individuals' beliefs if they are not fully rational and may influence behavior. Thus, a systematically optimistic individual will be reluctant to dissave, since wealth will have to be spread over a long anticipated remaining life. However, time scale distortion may be more superficial, affecting responses to survey questions on mortality without altering beliefs. Context and framing effects appear in survey responses much less personal than survival and seem to be related to the persona individuals choose to project as well as to psychometric illusions. It is not difficult to imagine that these effects could distort reported survival probabilities. Analysis of subsequent waves of the AHEAD panel should reveal the extent to which systematic distortion in stated survival probabilities infects behavior.

Suppose the subjective probability p^* of surviving for elapsed time τ is known for an individual. Then $p^* = \exp(-(\Lambda_v(a + T_a(\tau)) - \Lambda_v(a))e^{z\beta-\sigma\varepsilon})$ determines

$$\sigma\varepsilon = -\log(-\log p^*) + z\beta + \log(\Lambda_v(a + T_a(\tau)) - \Lambda_v(a)).$$

Substituting this in the survival curve,

$$q(t \mid a, p^*) = (p^*)^{\left[\frac{\Lambda_v(a + T_a(t)) - \Lambda_v(a)}{\Lambda_v(a + T_a(\tau)) - \Lambda_v(a)}\right]}.$$

In subsequent analysis, we shall assume that the density at birth of the unobserved factor $s = e^{-\sigma\varepsilon}$ is gamma with mean one and variance $1/\kappa$, so that the subjective probability p^* of survival over elapsed time τ satisfies

$$V \equiv \mathbf{E}\{\log p^* \mid a, z\} = -[\Lambda_v(a + T_a(\tau)) - \Lambda_v(a)] \cdot e^{z\beta} \cdot \kappa/[\kappa + \Lambda_v(a)e^{z\beta}],$$

$$\lambda^2 \equiv \mathbf{V}\{\log p^* \mid a, z\} = [\Lambda_v(a + T_a(\tau)) - \Lambda_v(a)]^2 \cdot e^{2z\beta} \cdot \kappa/[\kappa + \Lambda_v(a)e^{z\beta}]^2.$$

Note that this distributional assumption on s is distinct from, although consistent with, the earlier assumption that the entire proportional hazard term in the Cox survival function at birth had a gamma distribution. In particular, if the integrated hazard function were free of the effects of heterogeneity, then one would expect the parameter κ in the formula above to be larger than the parameter ω in the formula for the life table probabilities. However, lack of identification makes this consistency check impossible.

If the subjective probability p^* were observed without error, then the nonlinear regression equation

$$\begin{aligned} -\log p^* &= V - \lambda\xi \\ &\equiv [\Lambda_v(a + T_a(\tau)) - \Lambda_v(a)] \cdot e^{\beta_0 + z_1\beta_1} \cdot \kappa/[\kappa + \Lambda_v(a)e^{z\beta}] - \lambda\xi, \end{aligned}$$

which has $\mathbf{E}\xi = 0$ and $\mathbf{E}\xi^2 = 1$ by construction, could be used to estimate the parameters α, β, and κ. Note that positive ξ is associated with larger survival probabilities.

9.4.3 Reporting Errors

We anticipate that stated subjective probabilities p will deviate from true (latent) subjective probabilities p^* due to two types of reporting errors, in addition to systematic time distortion, which may be a reporting effect. First, we observe concentrations of responses at the focal points 0, 1/2, and 1 that appear to be the result of gross classification behavior by respondents. Second, there may be reporting noise in nonfocal responses. We now describe a model that includes these reporting errors. The model allows for the possibility of correlated unobserved factors that influence both the latent survival probability and the propensity to give focal responses. Small p^* is associated with V large positive, and hence with ε large negative. There is a latent selection model

$$w^* = z\gamma + \rho\xi + \sqrt{1-\rho^2}\,v$$

that determines whether the individual gives a continuous nonfocal response or a focal response; ρ is a parameter that permits unobserved factors to influence both "frailty" and the propensity to give a focal response. If $w^* > 0$, then the individual reports a continuous response p that satisfies

$$\begin{aligned} v &\equiv -\log p = V - \lambda\xi + \delta\eta \\ &\equiv [\Lambda_v(a + T_a(\tau)) - \Lambda_v(a)] \cdot e^{z\beta} \cdot \kappa/[\kappa + \Lambda_v(a)e^{z\beta}] - \lambda\xi + \delta\eta, \end{aligned}$$

where η is a disturbance arising from reporting noise that is assumed to have mean zero and variance one and δ is a scale parameter. If $w^* \le 0$, then the individual reports a focal response determined by threshold parameters ψ_0 and ψ_1, with $\psi_0 \le \psi_1$ and

$$p = \begin{cases} 0 & \text{if } v^* > -\psi_0, \\ 1/2 & \text{if } -\psi_0 \ge v^* \ge -\psi_1, \\ 1 & \text{if } v^* < -\psi_1. \end{cases}$$

For further analysis, the disturbances ξ, v, and η are assumed to be independent standard normal. Note that this specification for ξ is an approximation that cannot be exact because of the effects of selection. However, since the true ξ matches the first two moments of the standard normal, we expect this approximation to have no effect on the consistency of parameters estimated by nonlinear least squares and do not believe it will have any significant economic effect on the final estimated survival curves.

9.4.4 Selection of Focal versus Nonfocal Response

The marginal probability of a focal response, given z, is

$$P(w^* \le 0 \mid z) = \Phi(-z\gamma).$$

Then, defining d_+ to be an indicator for a nonfocal response, the marginal log likelihood for selection between focal versus nonfocal responses is

$$l_s = d_+ \cdot \log \Phi(z\gamma) + (1 - d_+) \cdot \log \Phi(-z\gamma).$$

The Likelihood of a Nonfocal Response

Given ε, a nonfocal response $v = -\log p$ is observed if $v > -(z\gamma + \rho\xi)/\sqrt{1-\rho^2}$ and $\eta = (v - V - \lambda\xi)/\delta$. Unconditioning ξ, the density of a nonfocal response v is then

$$\int_{-\infty}^{+\infty} \phi(\xi) \cdot \Phi\left(\frac{z\gamma + \rho\xi}{\sqrt{1-\rho^2}}\right) \cdot \frac{1}{\delta} \cdot \phi\left(\frac{v - V - \lambda\xi}{\delta}\right) d\xi$$

$$= \frac{1}{\sqrt{\lambda^2 + \delta^2}} \cdot \phi\left(\frac{v - V}{\sqrt{\lambda^2 + \delta^2}}\right) \cdot \Phi\left(\frac{z\gamma\sqrt{\lambda^2 + \delta^2} - \rho\lambda(v - V)}{\sqrt{(1-\rho^2)\lambda^2 + \rho^2\delta^2}}\right).$$

Then, the log density of p, conditioned on a, τ, z, and a nonfocal response, is

$$l_N = -\log\left(p \cdot \Phi(z\gamma) \cdot \sqrt{\lambda^2 + \delta^2}\right) - \log(-\log p)$$
$$- \frac{1}{2}\left[\frac{v - V}{\sqrt{\lambda^2 + \delta^2}}\right]^2 + \log \Phi\left(\frac{z\gamma\sqrt{\lambda^2 + \delta^2} - \rho\lambda V}{\sqrt{(1-\rho^2)\lambda^2 + \rho^2\delta^2}}\right).$$

When $\rho = 0$, this log density simplifies to

$$l_N = -\log\sqrt{\lambda^2 + \delta^2} - \log p - \log(-\log p) - \frac{1}{2}\left[\frac{v - V}{\sqrt{\lambda^2 + \delta^2}}\right]^2.$$

In this case, consistent asymptotically normal estimates of the parameters are obtained by computing nonlinear least squares estimates for the regression

$$-\log p = [(\Lambda_v(a + T_a(\tau)) - \Lambda_v(a)] \cdot e^{z\beta} \cdot \kappa/[\kappa + \Lambda_v(a)e^{z\beta}] + \zeta,$$

ignoring heteroscedasticity, then retrieving estimated residuals $\hat{\zeta}$ and fitted values $\hat{V}$, and finally applying ordinary least squares to the regression

$$\hat{\zeta}^2 = \hat{V}^2/\kappa + \delta^2 + \varphi,$$

where φ is a mean zero disturbance. The consistency of this procedure does not require that the disturbances ζ be normal.

If $\rho \neq 0$ and ξ, v are standard normal, a Heckman-type consistent estimator of β, γ, α, and κ can be obtained by considering

$$\mathbf{E}_N(-\log p) = V - \lambda \cdot \mathbf{E}\{\xi | w^* > 0\}$$
$$= [\Lambda_\nu(a + T_a(\tau)) - \Lambda_\nu(a)] \cdot e^{z\beta} \cdot \kappa/[\kappa + \Lambda_\nu(a)e^{z\beta}]$$
$$- \rho\lambda \cdot \phi(z\gamma)/\Phi(z\gamma),$$

where $\mathbf{E}_N$ denotes an expectation conditioned on nonfocal response. Recalling that $\lambda^2 = V^2/\kappa$, the regression can be rewritten

$$-\log p = V \cdot [1 - \left(\rho/\sqrt{\kappa}\right) \cdot \phi(z\gamma)/\Phi(z\gamma)] + \zeta.$$

The inverse Mills ratio term in this regression comes from

$$(v | w^*) \sim N(V - \rho\lambda(w^* - z\gamma),\ \delta^2 + (1 - \rho^2)\lambda^2),$$

and $\mathbf{E}\{w^* - z\gamma \mid w^* > 0) = \phi(z\gamma)/\Phi(z\gamma)$. Calculation of the variance yields

$$\mathbf{V}(v | w^* > 0) = \delta^2 + V^2/\kappa + V^2(\rho^2/\kappa) \cdot \frac{\phi(z\gamma)}{\Phi(z\gamma)} \cdot \left[z\gamma - \frac{\phi(z\gamma)}{\Phi(z\gamma)}\right].$$

A consistent test of $\rho = 0$ can be carried out using the T-statistic on the term $\rho/\sqrt{\kappa}$ in the second term of the regression; a White robust estimator for the standard errors is used in calculating this statistic since the equation is heteroscedastic.

The Likelihood of a Focal Response

We next obtain the conditional log likelihood of an observed focal point p, given a focal response. Define the expressions

$$b_0 = \sqrt{\kappa}(1 + \psi_0/V), \qquad A_0 = \Phi(b_0),$$
$$b_1 = \sqrt{\kappa}(1 + \psi_1/V), \qquad A_1 = \Phi(b_1),$$
$$A_3 = 1 - A_1, \qquad A_4 = A_1 - A_0.$$

The probability of the event $w^* \leq 0$ and $V > -\psi_0$, or a response $p = 0$, conditioned on the event of a focal response, is

$$P_0 = \int_{-\infty}^{b_0} \phi(\varepsilon) \cdot \Phi\left(\frac{-z\gamma - \rho\xi}{\sqrt{1 - \rho^2}}\right) d\varepsilon / \Phi(-z\gamma)$$
$$= A_0 \cdot \int_0^1 \Phi\left(\frac{-z\gamma - \rho\Phi^{-1}(tA_0)}{\sqrt{1 - \rho^2}}\right) / \Phi(-z\gamma)\, dt.$$

When $\rho = 0$, this probability reduces to $P_0 = A_0$. The second form of the integral, obtained by the transformation of variables $t = \Phi(\varepsilon)/A_0$, is convenient for numerical integration.

Similarly, the probability of the event $w^* \leq 0$ and $V < -\psi_1$, or focal response $p = 1$, conditioned on the event of a focal response, is

$$P_1 = \int_{b_1}^{+\infty} \phi(\varepsilon) \cdot \Phi\left(\frac{-z\gamma - \rho\varepsilon}{\sqrt{1-\rho^2}}\right) d\varepsilon / \Phi(-z\gamma)$$

$$= A_3 \cdot \int_0^1 \Phi\left(\frac{-z\gamma - \rho\Phi^{-1}(A_1 + A_3 t)}{\sqrt{1-\rho^2}}\right) / \Phi(-z\gamma)\, dt,$$

which reduces to $P_1 = A_3$ when $\rho = 0$.

Finally, the probability of the event $w^* \leq 0$ and $-\psi_0 \geq V \geq -\psi_1$, or focal response $p = 1/2$, conditioned on the event of a focal response, is

$$P_{1/2} = \int_{b_0}^{b_1} \phi(\varepsilon) \cdot \Phi\left(\frac{-z\gamma - \rho\varepsilon}{\sqrt{1-\rho^2}}\right) d\varepsilon / \Phi(-z\gamma)$$

$$= A_4 \cdot \int_0^1 \Phi\left(\frac{-z\gamma - \rho\Phi^{-1}(A_0 + A_4 t)}{\sqrt{1-\rho^2}}\right) / \Phi(-z\gamma)\, dt,$$

which reduces to $P_{1/2} = A_4$ when $\rho = 0$.

Let d_0, $d_{1/2}$, and d_1 be indicators for the events that observed p takes on the focal point values 0, 1/2, and 1, respectively. Then the conditional log likelihood of the observed focal point, given a focal response, is

$$l_f = d_0 \cdot \log P_0 + d_1 \cdot \log P_1 + d_{1/2} \cdot \log P_{1/2}.$$

When $\rho = 0$, this is an ordered probit model.

9.4.5 Prediction of Personal Survival Curves

The final step of the analysis, once the parameters of the model are estimated, is to estimate a subjective personal survival curve for each sample person. It is convenient to work with the log of the survival curve. Recall that

$$\log q(t \mid a, v, z, p) = \frac{\Lambda_v(a + T_a(t)) - \Lambda_v(a)}{\Lambda_v(a + T_a(\tau)) - \Lambda_v(a)} \cdot \log p^*.$$

To forecast this quantity, we replace $\log p^*$ by its expectation, given z and given the stated subjective probability p. For the alternative that time distortion is interpreted as pure reporting error, $T_a(t)$ in this formula would be replaced by t. Consider the case $\rho = 0$. First consider nonfocal respondents. The conditional distribution of $-\log p^*$ given v is normal with mean $(\delta^2 V + \lambda^2 v)/(\lambda^2 + \delta^2)$ and variance $\lambda^2\delta^2/(\lambda^2 + \delta^2)$. Then the predicted personal survival curve is given by

$$-\log \hat{q}(t \mid a, v, z, p) = \frac{\Lambda_v(a + T_a(t)) - \Lambda_v(a)}{\Lambda_v(a + T_a(\tau)) - \Lambda_v(a)} \cdot \frac{\delta^2 V + \lambda^2 v}{\lambda^2 + \delta^2}.$$

It is of interest to single out two extreme cases. If $\delta = 0$, there is no reporting noise in nonfocal responses, and they identify the individual ξ effects. Then

$$\log \hat{q}(t \mid a, v, z, p) = \frac{\Lambda_v(a + T_a(t)) - \Lambda_v(a)}{\Lambda_v(a + T_a(\tau)) - \Lambda_v(a)} \cdot \log p.$$

Alternatively, if $\lambda = 0$, corresponding to $\kappa = +\infty$, and the disturbance in the regression that maximizes I_N is due to pure reporting noise, then

$$-\log \hat{q}(t \mid a, v, z, p) = [\Lambda_v(a + T_a(\tau)) - \Lambda_v(a)] \cdot e^{z\beta}.$$

Finally, consider focal respondents in the case $\rho = 0$. These individuals have

$$\mathbf{E}\{\xi \mid p, z\} = \frac{1}{P_p}\int_{b'(p)}^{b''(p)} \xi\phi(\xi)\,d\xi = -\frac{\phi(b''(p)) - \phi(b'(p))}{\Phi(b''(p)) - \Phi(b'(p))},$$

where $b'(p)$ and $b''(p)$ are the bounds giving the focal response p, so that $(b'(p),b''(p))$ is $(-\infty,b_0)$ for $p = 0$, (b_0,b_1) for $p = 1/2$, and $(b_1,+\infty)$ for $p = 1$. Then, for focal responses, the estimated personal survival curve is

$$-\log \hat{q}(t \mid a, v, z, p) = \frac{\Lambda_v(a + T_a(t)) - \Lambda_v(a)}{\Lambda_v(a + T_a(\tau)) - \Lambda_v(a)} \cdot V\left[1 + \frac{1}{\sqrt{\kappa}} \cdot \frac{\phi(b''(p)) - \phi(b'(p))}{\Phi(b''(p)) - \Phi(b'(p))}\right].$$

In the case of pure reporting noise, $\kappa = +\infty$, this formula reduces to the same estimated proportional hazards model that applied to the nonfocal respondents. Thus, in the case of pure reporting noise and $\rho = 0$, the stated survival probabilities are used only to calibrate the proportional hazards model in observable covariates.

Finally, consider prediction of personal survival curves when $\rho \neq 0$. First consider nonfocal respondents. The joint density of $-\log p^*$, v, and w^* is

$$\begin{bmatrix} -\log p^* \\ v \\ w^* \end{bmatrix} \sim N\left(\begin{bmatrix} V \\ V \\ z\gamma \end{bmatrix}, \begin{bmatrix} \lambda^2 & \lambda^2 & -\rho\lambda \\ \lambda^2 & \delta^2 + \lambda^2 & -\rho\lambda \\ -\rho\lambda & -\rho\lambda & 1 \end{bmatrix}\right).$$

Then

$$(-\log p^* \mid v, w^*)$$
$$\sim N\left(\frac{\delta^2 V + \lambda^2(1 - \rho^2)v}{\delta^2 + \lambda^2(1 - \rho^2)} - \frac{\rho\lambda\delta^2(w^* - z\gamma)}{\delta^2 + \lambda^2(1 - \rho^2)}, \frac{\lambda^2\delta^2(1 - \rho^2)}{\delta^2 + \lambda^2(1 - \rho^2)}\right).$$

Then the predicted survival curve is given by

$$-\log \hat{q}(t \mid a, v, z, p)$$
$$= \frac{\Lambda_v(a + T_a(t)) - \Lambda_v(a)}{\Lambda_v(a + T_a(\tau)) - \Lambda_v(a)} \cdot \left(\frac{\delta^2 V + \lambda^2(1 - \rho^2)v}{\lambda^2(1 - \rho^2) + \delta^2} + \frac{\rho\lambda\delta^2\phi(-z\gamma)/\Phi(-z\gamma)}{\delta^2 + \lambda^2(1 - \rho^2)}\right).$$

For focal respondents, $-\log p^* = V - \lambda\xi$, so that

$$\mathbf{E}\{\xi \mid w^* \leq 0, b'(p) \leq -\log p^* \leq b''(p)\}$$

$$= \Phi(-z\gamma) \cdot \int_{b'(p)}^{b''(p)} \xi \cdot \phi(\xi) \cdot \Phi\left(\frac{z\gamma - \rho\xi}{\sqrt{1-\rho^2}}\right) d\xi / P_p \equiv \frac{V}{\sqrt{\kappa}} \cdot e_p$$

with

$$e_p = \frac{1}{P_p \cdot \Phi(z\gamma)} \cdot$$

$$\left\{\phi(b''(p)) \cdot \Phi\left(\frac{-z\gamma - \rho b''(p)}{\sqrt{1-\rho^2}}\right) - \phi(b'(p)) \cdot \Phi\left(\frac{-z\gamma - \rho b'(p)}{\sqrt{1-\rho^2}}\right)\right.$$

$$\left. -\rho \cdot \phi\left(-z\gamma\sqrt{1-\rho^2}\right) \cdot \left[\Phi\left(\frac{b''(p) + z\gamma\rho}{\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{b'(p) + z\gamma\rho}{\sqrt{1-\rho^2}}\right)\right]\right\}.$$

The numerical integration formulas introduced for this case are required for evaluation of the denominator in this expression. Then

$$-\log \hat{q}(t \mid a, v, z, p) = \frac{\Lambda_v(a + T_a(t)) - \Lambda_v(a)}{\Lambda_v(a + T_a(\tau)) - \Lambda_v(a)} \cdot V \cdot \left(1 - \frac{e_p}{\sqrt{\kappa}}\right).$$

9.5 Estimation Results

This section gives estimates of the model developed in section 9.4. Table 9.4 describes the covariates used in this analysis and gives their descriptive statistics. The estimates for the binomial probit model for focal response are given in table 9.5. Effects that increase the propensity for a focal response have positive coefficients. We find that cognitive disability, fair or poor health, unmarried status, and missing data on other subjective probability questions are all associated with significantly higher propensities to give a focal response, and high wealth and education are associated with a significantly lower propensity. Thus, focal responses appear to be associated with a lack of aptitude for, or interest in, the survey.

Table 9.6 reports the results of estimating the regression model

$$-\log p = [\Lambda_v(a + T_a(\tau)) - \Lambda_v(a)] \cdot e^{z\beta} \cdot \kappa/[\kappa + \Lambda_v(a)e^{z\beta}] + \zeta$$

on the subsample of individuals who do *not* report a focal response of 0, 1/2, or 1. The integrated hazard function Λ_V is the quadratic spline approximation to life tables, quadratic in age with linear drift, as discussed earlier, with separate curves for males and females. A negative coefficient in this table is associ-

Table 9.4 **Definitions of Explanatory Variables**

Mnemonic	Description	Sample Mean	Sample Standard Deviation	Sample Maximum	Sample Minimum
MALE	Sex of respondent (male = 1, female = 0)	0.3717	0.4833	0.0000	1.0000
COGN	Indicator for cognitive disability	0.2860	0.4519	0.0000	1.0000
BLACKS	Race of respondent (black = 1, nonblack = 0)	0.1062	0.3081	0.0000	1.0000
HEXCEL	Indicator for self-reported excellent health	0.1210	0.3262	0.0000	1.0000
HVGOOD	Indicator for self-reported very good health	0.2510	0.4336	0.0000	1.0000
HGOOD	Indicator for self-reported good health (omitted category)	0.3128	0.4636	0.0000	1.0000
HFAIR	Indicator for self-reported fair health	0.2147	0.4106	0.0000	1.0000
HPOOR	Indicator for self-reported poor health	0.1005	0.3007	0.0000	1.0000
HBETTER	Indicator for improvement in health, last two years	0.1357	0.3425	0.0000	1.0000
HSAME	Indicator for no change in health, last two years (omitted category)	0.6757	0.4681	0.0000	1.0000
HWORSE	Indicator for decline in health, last two years	0.1886	0.3912	0.0000	1.0000
COLLEGE	Indicator for college graduate	0.3018	0.4591	0.0000	1.0000
PAPAGE	Father's (expected) age at death PAPAGE1 = PAPAGE*MALE, PAPAGE0 = PAPAGE*(1 − MALE)	74.0691	15.9661	18.0000	109.0000
MOMAGE	Mother's (expected) age at death MOMAGE1 = MOMAGE*MALE, MOMAGE0 = MOMAGE*(1 − MALE)	72.0878	14.8299	20.0000	107.1111

SMNOW	Indicator for smoker now	0.1088	0.3114	0.0000	1.0000
SMOLD	Indicator for previous smoker	0.4379	0.4962	0.0000	1.0000
MARRIED	Indicator for married	0.5731	0.4947	0.0000	1.0000
NURSE	Subjective probability of nursing home admission	13.0966	22.5391	0.0000	100.0000
MEDX	Subjective probability of major medical expenditures	26.0126	31.7783	0.0000	100.0000
INFLAT	Subjective probability of income lagging inflation	35.2017	34.7054	0.0000	100.0000
NURSMIS	Missing nursing home response	0.0656	0.2477	0.0000	1.0000
MEDXMIS	Missing medical expenditure response	0.0932	0.2907	0.0000	1.0000
INFLAMIS	Missing inflation response	0.0961	0.2948	0.0000	1.0000
WQUART2	Indicator for 25–50 percent wealth quartile	0.2281	0.4196	0.0000	1.0000
WQUART3	Indicator for 50–75 percent wealth quartile	0.2769	0.4475	0.0000	1.0000
WQUART4	Indicator for 75–100 percent wealth quartile	0.3315	0.4708	0.0000	1.0000
AGE1	Age/70	1.0780	0.0871	0.5429	1.2857
AGE2	$(\mathrm{AGE1})^2$	1.1697	0.1861	0.2947	1.6531
AGE3	$(\mathrm{AGE1})^3$	1.2771	0.3026	0.1600	2.1254

Note: Sample size = 6,139.

Table 9.5 **Binomial Probit Model for Focal Response**

Variable	Coefficient	Standard Error	*T*-Statistic
Constant	4.5812	6.1242	0.748
MALE	−0.2286	0.2288	−0.999
COGN	0.0241	0.0112	2.147
BLACKS	−0.0838	0.0550	−1.523
HEXCEL	0.0218	0.0558	0.391
HVGOOD	−0.0470	0.0437	−1.077
HFAIR	0.0945	0.0468	2.017
HPOOR	0.2172	0.0643	3.377
HBETTER	0.0139	0.0490	0.283
HWORSE	0.0543	0.0467	1.161
COLLEGE	−0.1238	0.0379	−3.261
PAPAGE1	−0.0011	0.0017	−0.632
PAPAGE0	−0.0014	0.0013	−1.088
MOMAGE1	0.0010	0.0018	0.531
MOMAGE0	−0.0015	0.0014	−1.092
SMNOW	−0.0014	0.0561	−0.025
SMOLD	0.0253	0.0371	0.682
MARRIED	−0.0921	0.0400	−2.305
NURSE	−0.0001	0.0008	−0.150
MEDX	−0.0006	0.0006	−1.038
INFLAT	−0.0002	0.0005	−0.308
NURSMIS	−0.0424	0.0730	−0.581
MEDXMIS	0.1798	0.0659	2.730
INFLAMIS	0.1518	0.0637	2.384
WQUART2	−0.0196	0.0502	−0.391
WQUART3	−0.0443	0.0512	−0.866
WQUART4	−0.1807	0.0569	−3.178
AGE1	−11.9112	18.5949	−0.641
AGE2	11.3407	18.6604	0.608
AGE3	−3.38169	6.18114	−0.547
Focal response (%)	59.6		
N	6,144		
Log likelihood	−4,095.87		

Notes: Dependent variable: focal = 1, nonfocal = 0. Estimation method is maximum likelihood estimation.

ated with lower mortality hazard and a higher subjective probability of survival. We estimate this model both with and without a correction term for selection, which involves an inverse Mills ratio. Overall, we find a strong relationship between personal survival probabilities and covariates, generally in the expected direction. We find that males are more optimistic than females. Blacks are more optimistic than nonblacks. Married individuals are slightly more optimistic than nonmarried ones. This may reflect both the objective fact that married individuals live longer and in many cases the impact on optimism of the death of a spouse. We do not find a significant relationship between optimism about survival and either an index of cognition or an index of education. There is a strongly significant relationship between self-rated health status and survival expectations: those with better than good health (the omitted category) have sharply higher subjective probabilities, and those in worse than good health have sharply lower subjective probabilities. Changes in health status also influence optimism in the expected directions. Improvements in health status make respondents significantly more optimistic; declines in health status go the other way, but they are not statistically significant.

Conventional wisdom is that individuals weigh the longevity of the same-sex parent heavily in forming their own survival expectations. We include the age of death of father and mother or, if surviving, the expected age of death conditioned on the age attained, calculated from standard life tables. These variables are interacted with the sex of the respondent; then PAPAGE1 and MOMAGE0 are the same-sex parental longevity variables. In all cases, greater parental longevity is associated with greater optimism. However, the only statistically significant effect is that female optimism is higher when father's longevity is higher. These results then do not support the conventional wisdom on the effects of parental longevity.

If the individual has been a smoker in the past, or is a smoker now, then the subjective survival probability is lower. However, the effects are not statistically significant. These results go in the direction of clinical evidence, but do not appear to be strong enough to account rationally for the effect of smoking. This suggests that denial of mortality risk, or an attitude of imperviousness to danger, may be part of smoking behavior. It is also possible that self-rated health status captures some of the effects of smoking.

Subjects gave subjective probabilities of moving to a nursing home within five years, of incurring medical costs within five years that would wipe out their savings, and of seeing inflation within five years that would erode their income. For the inflation variable, we find that a higher subjective probability of the event is associated with greater optimism about survival. This suggests that we are seeing in respondents' behavior a rather carefully articulated calculation of the probability of being at risk for these events as a result of survival, rather than heterogeneity in generalized optimism. For medical expenditure and nursing home variables, decreased optimism about mortality is associated with increased pessimism about the likelihood of nursing home admission or, less significantly, major medical costs. This is consistent with the conventional wisdom that individuals systematically overestimate the probability that their last days will be spent in a nursing home or extended hospital stay. There were a significant number of subjects with missing responses on these subjective probability questions; these events are flagged with dummy variables. If responses are missing at random, then the dummy variable coefficient should equal the coefficient for the variable in the case of nonmissing responses times the average level of the variable. For the nursing home question, the estimated coefficient is 0.3456, compared with the value 0.0576 that would be expected

Table 9.6 **Regression Model for Nonfocal Responses**

	Model without Heckman Correction			Model with Heckman Correction		
Variable	Coefficient	S.E.	*T*-Statistic	Coefficient	S.E.	*T*-Statistic
CONSTANT	3.5436	0.4186	8.465	3.3485	0.5776	5.797
MALE	−1.5323	0.5001	−3.064	−1.5705	0.5322	−2.951
COGN	0.0070	0.0201	0.348	0.0031	0.0224	0.140
BLACKS	−0.2376	0.1069	−2.222	−0.2210	0.1159	−1.907
HEXCEL	−0.6387	0.1253	−5.096	−0.6431	0.1292	−4.976
HVGOOD	−0.4884	0.0950	−5.142	−0.4809	0.0954	−5.039
HFAIR	0.2677	0.0913	2.932	0.2464	0.0993	2.480
HPOOR	0.3418	0.1336	2.559	0.2888	0.1616	1.787
HBETTER	−0.3107	0.0990	−3.138	−0.3110	0.0998	−3.116
HWORSE	0.0790	0.0838	0.942	0.0667	0.0907	0.735
COLLEGE	0.0471	0.0673	0.700	0.0720	0.0851	0.846
PAPAGE1	−0.0009	0.0028	−0.323	−0.0007	0.0029	−0.232
PAPAGE0	−0.0117	0.0031	−3.735	−0.0114	0.0034	−3.396
MOMAGE1	−0.0049	0.0029	−1.676	−0.0049	0.0029	−1.668
MOMAGE0	−0.0036	0.0028	−1.282	−0.0033	0.0029	−1.136
SMNOW	0.0602	0.1003	0.600	0.0564	0.1080	0.522
SMOLD	0.0233	0.0684	0.340	0.0212	0.0682	0.311
MARRIED	−0.0889	0.0718	−1.239	−0.0788	0.0760	−1.037

NURSE	0.0044	0.0016	2.711	0.0044	0.0018	2.444
MEDX	0.0010	0.0011	0.921	0.0012	0.0013	0.938
INFLAT	−0.0078	0.0014	−5.571	−0.0077	0.0014	−5.343
NURSMIS	0.3456	0.1536	2.250	0.3429	0.1593	2.153
MEDXMIS	−0.0305	0.1368	−0.223	−0.0552	0.1550	−0.356
INFLAMIS	−0.4753	0.1454	−3.268	−0.4956	0.1789	−2.770
WQUART2	0.2046	0.1002	2.043	0.2090	0.1099	1.902
WQUART3	0.1024	0.0987	1.037	0.1119	0.1050	1.066
WQUART4	0.0875	0.1024	0.854	0.1208	0.1310	0.922
α_1, male	1.3137	0.1755	7.487	1.3691	0.2004	6.832
α_2, male	−0.0068	0.0021	−3.203	−0.0075	0.0024	−3.111
α_1, female	1.2742	0.0692	18.426	1.2829	0.0842	15.232
α_2, female	−0.0071	0.0012	−5.990	−0.0072	0.0014	−5.235
$1/\kappa$	0.1771	0.0418	4.240	0.2023	0.1053	1.922
σ	0.9568	0.0266	35.993	0.9569	0.0266	35.993
Inverse mills ratio				−0.1680	0.4162	−0.404
Sample size	2,591			2,591		
Multiple correlation	.1370			.1370		

Notes: Dependent variable is $-\log p$. Estimation method is nonlinear least squares. S.E. = standard error.

if the variable were missing at random. Thus, missing data here is associated with significantly more pessimism. For the inflation question, the estimated coefficient is -0.4753, compared with the expected value -0.2746. Thus, missing data on inflation is associated with significantly more optimism. This suggests that missing responses to these questions may be associated with an unwillingness to articulate pessimistic beliefs.

The wealth of individuals is identified by quartile, with the lowest quartile as the omitted category. We find a clear, although not consistently statistically significant, pattern that moving from the second to the third to the top quartile increases optimism about survival. This pattern agrees with the observation at the aggregate level that increased wealth is associated with increased longevity, and with the life cycle model implication that individuals with higher subjective survival probabilities should, other things equal, hold more assets. On the other hand, the lowest quartile, whose coefficient is implicitly zero, is more optimistic than the second quartile. This is inconsistent both with the observed negative correlation of mortality risk with wealth and with the prediction of the life cycle model that optimism will be positively correlated with wealth accumulation. One interpretation of the weak statistical relationship between wealth and stated survival probability is that a simple correlation of wealth and longevity in the population reflects in part the contribution of covariates such as health status and behavioral choices such as smoking that are accounted for in the model. Further, individuals in the lowest wealth quartile may be there in part because of beliefs that "fate" will provide not only long life but also the resources needed to live.

The parameter κ in this model determines the spread of unobserved heterogeneity at birth in the population. The estimated value $\kappa = 5.6460$ implies that 90 percent of the values of s from the density $k(s)$ describing unobserved heterogeneity lie in the interval (0.421,1.870). If this factor indeed measures heterogeneity in unobserved relative mortality risk, rather than a reporting effect, and this factor is known to the consumer, then it has a potentially large economic effect on behavior.

Consider the time-scaling function $T_a(t) = (1 + t)^{\alpha_1+\alpha_2 \cdot a} - 1$. For males, the estimated parameters $\alpha_1 = 1.3137$ and $\alpha_2 = -0.0068$ imply that individuals over age 46 are systematically optimistic about survival, and that optimism increases with age. The degree of optimism is substantial. For example, a 70-year-old male has $T_a(t) = (1 + t)^{0.8377} - 1$, and a time interval of 15 years is scaled to an equivalent of 9.2 years in the standard life table. An 80-year-old male has $T_a(t) = (1 + t)^{0.7697} - 1$, and an interval of 15 years is scaled to an equivalent of 7.4 years in the standard life table. For females, the estimated parameters $\alpha_1 = 1.2742$ and $\alpha_2 = -0.0071$ imply that individuals over age 39 are optimistic; a 70-year-old scales a 15-year interval to 7.6 years, and a 80-year-old scales a 15-year interval to 5.6 years. If these are genuine beliefs about mortality hazard, then it is not surprising that individuals hold on to their wealth to cover their expected remaining life.

Table 9.7 **Ordered Probit Model for Focal Points**

Variable	Coefficient	Standard Error	T-Statistic
Low threshold	-1.394	0.023	-60.530
High threshold	-0.439	0.021	-20.824
$\sqrt{\kappa}$	1.407	0.051	27.475
Log likelihood	$-3{,}486.25$		
	3,553		

Note: Dependent variable: 0, 50, 100 for responses of 0, ½, 1, respectively.

When the model is estimated with a correction term for selection between nonfocal and focal responses, we find no statistical evidence for a common unobserved effect that causes the subjective survival probability to fall and the propensity for a focal response to rise. Further, the coefficients on covariates in the model are relatively unaffected, and the patterns of significant effects are unchanged. We have chosen to maintain the hypothesis $\rho = 0$ for subsequent estimation and prediction tasks.

An estimate of δ^2 and a second estimate of κ are obtained by regressing squared residuals from the model in table 9.6 (without the correction for selection) on a constant and the square of the fitted value of the equation, $\hat{V}^2$. The estimate of δ^2 is 0.5401 (S.E. = 0.0476) and the estimate of $1/\kappa$ is 0.2385 (S.E. = 0.0261), implying an estimate $\kappa = 4.1921$. Then the heteroscedasticity in this regression equation appears to be consistent with the theoretical model, yielding an estimate of κ that is not far from the previous estimate.

Estimates for the ordered probit model for observed focal points among those giving focal responses are given in table 9.7. In this estimation, the probability of the observed focal point is given by $\Phi(b''(p)) - \Phi(b'(p))$, where

$$(b'(p),b''(p)) = \begin{cases} (-\infty, b_0) & \text{if } p = 0, \\ (b_0, b_1) & \text{if } p = 1/2, \\ (b_1, +\infty) & \text{if } p = 1, \end{cases}$$

and letting $\hat{V}$ denote the predicted value of V from the regression on nonfocal responses,

$$b_0 = \sqrt{\kappa}(1 + \psi_0/V) \approx \sqrt{\kappa}(1 + \psi_0/\hat{V}),$$

$$b_1 = \sqrt{\kappa}(1 + \psi_1/V) \approx \sqrt{\kappa}(1 + \psi_1/\hat{V}).$$

The parameters ψ_0, ψ_1, and $\sqrt{\kappa}$ are estimated, using $\hat{V}$ as the covariate. This implies an estimate of 1.9796 for κ, compared with the regression model estimate of 5.6460. This difference is statistically significant (under the maintained hypothesis $\rho = 0$), using a standard error for $\sqrt{\kappa}$ that is not corrected for the use of an estimated variable. This result may then be a statistical artifact,

Table 9.8 Fitted Survival Probabilities

Age Group	Target Age	Life Table	Nonfocal Respondents		Focal Respondents		All Respondents	
			Predicted	Stated	Predicted	Stated	Predicted	Stated
Female								
70–74	85	0.588	0.532	0.500	0.481	0.516	0.503	0.509
75–79	90	0.425	0.454	0.440	0.363	0.353	0.400	0.388
80–84	95	0.224	0.364	0.369	0.295	0.267	0.319	0.303
Male								
70–74	85	0.397	0.481	0.473	0.492	0.538	0.487	0.509
75–79	90	0.250	0.391	0.371	0.382	0.392	0.386	0.382
80–84	95	0.113	0.352	0.350	0.336	0.320	0.342	0.332

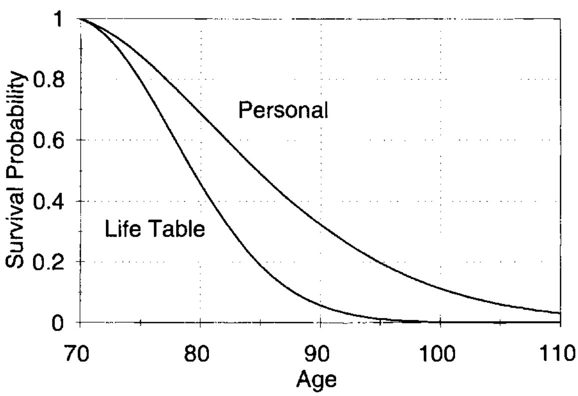

Fig. 9.11 Survival curves: males aged 70

or may indicate a specification problem in the model for focal-point choice or the impact of $\rho \neq 0$.

Personal survival curves are predicted from the models estimated under the maintained hypothesis $\rho = 0$. We produce estimates for the general case in which there is both unobserved heterogeneity in latent personal survival curves (e.g., $\kappa < +\infty$) and reporting error in nonfocal responses ($\delta > 0$). The value of λ^2, obtained from the formula $\lambda^2 = V^2/\kappa$ using estimates from the model in table 9.6 without correction for selection, is approximately 0.7713, which with the estimate $\delta^2 = 0.5401$ implies that the general case predictor places about 59 percent weight on the stated survival probability and 41 percent weight on the fitted proportional hazards model. Thus, this model indicates substantial, but imperfect, information in the stated probabilities. Table 9.8 summarizes these results. In figures 9.11 through 9.14, we plot average predicted personal survival curves for age 70 and age 80 females and males in the sample. For

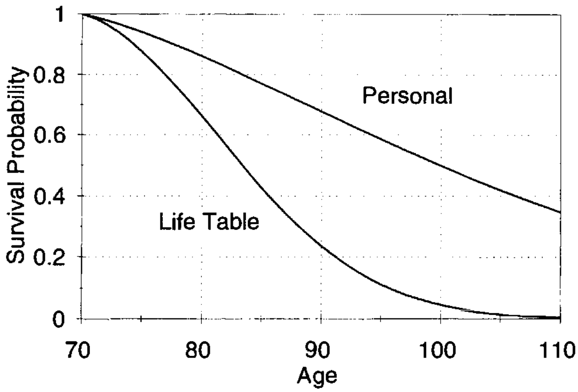

Fig. 9.12 Survival curves: females aged 70

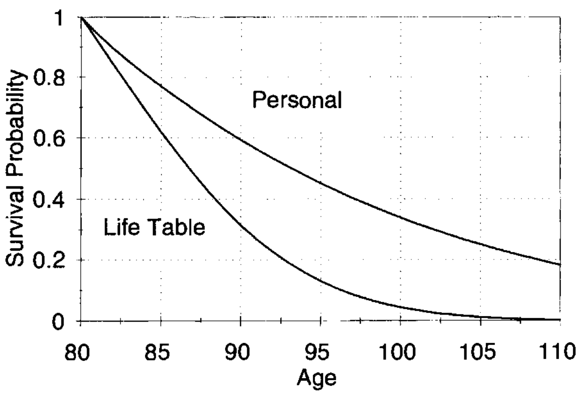

Fig. 9.13 Survival curves: males aged 80

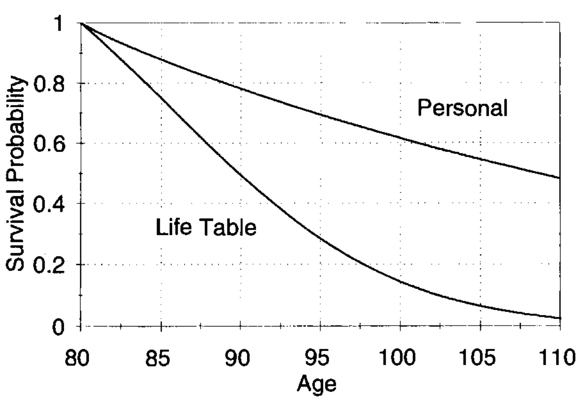

Fig. 9.14 Survival curves: females aged 80

Table 9.9 Subjective Relative Risks (for a 70-year-old with a 0.54 probability of living to age 85)

Risk Factor	Relative Risk (%)
Cognitive disability	1.87
Self-reported health (relative to good health)	
Excellent	−24.73
Very good	−21.50
Fair	25.25
Poor	48.60
Change in self-reported health (relative to no change)	
Better	−11.49
Worse	9.66
College education (relative to none)	−3.42
Father's age at death (20 years longer than average)	
For son	−2.94
For daughter	−12.20
Mother's age at death (20 years longer than average)	
For son	−4.75
For daughter	−4.65
Smoker now (relative to never smoked)	3.45
Previous smoker (relative to never smoked)	1.91
Married (relative to not currently married)	−9.70
Subjective probabilities of events	
Nursing home admission	13.29
Major medical expenditure	1.09
Inflation exceeding income growth	−18.16
Wealth quartile (relative to lowest quartile)	
Second quartile	9.28
Third quartile	−1.78
Fourth quartile	−3.94

comparison, we plot the standard life table survival curves in each case. The predicted personal survival curves become progressively more optimistic as duration increases. Since the effects of selection should lead an average personal survival function to decline more steeply than the life table curve, this illustrates the increasing optimism with duration that is present in personal beliefs.

To summarize the estimation results for the effects of covariates, table 9.9 gives the *perceived* relative risks associated with various risk factors. The computation is done for an individual aged 70 who has a perceived probability of 0.54 of living to age 85. The perceived relative risk is calculated from the formula

$$\mathrm{RR} \equiv \log \frac{q(t \mid a, v, z+\Delta z, \varepsilon)}{q(t \mid a, v, z, \varepsilon)} = q(t \mid a, v, z, \varepsilon) \cdot (e^{\Delta z \beta} - 1),$$

where z, ε correspond to a base case, here specified so that $q(t \mid a,v,z,\varepsilon) = 0.54$, and Δz is the change in covariates from the base case. The table shows, first, strong relative risks associated with self-reported health status, compared to the baseline of good health, and also with changes in health status. Other economically significant relative subjective risk factors are father's age of death (for women), marital status, subjective probability of nursing home admission or inflation, and low but not bottom wealth quartile.

9.6 Subjective Mortality Risk and Saving Behavior

Subjective measures of mortality risk appear to be useful in forecasting the survival probabilities of individuals. Cumulative experience with mortality in the AHEAD panel will determine whether the heterogeneity in perceived risk has a real counterpart. The primary *economic* interest in these stated beliefs is whether they influence, or at least vary with, economic behavior. There are a number of areas where these beliefs might matter, ranging from willingness to go to a doctor, and discretionary adjustments in exposure to risk factors such as smoking, to estate planning and saving behavior.

In this paper, we take a preliminary, and simplistic, look at the relationship between stated saving behavior and beliefs about survival. It is well known that saving is positively related to income and wealth, with substantial variability. The life cycle model predicts that under most circumstances, saving rate should be positively correlated with objective probabilities of survival for various intervals, or with life expectancy, since remaining wealth needs to be spread over a longer remaining lifetime. The AHEAD wave 1 survey collects qualitative information on whether each respondent is (a) a net saver, (b) a zero saver, or (c) a net dissaver. Saving is defined to exclude contributions to trusts; this is appropriate for assets intended as bequests but is arguably inappropriate for trusts that are revocable or that are established to shelter unintended bequests. (Continuous responses on saving are highly erratic, with many cases of missing or implausible data, and have not been used.) The survey questions are ambiguous on whether saving is net of current interest, dividends, and real capital gains. Thus, an individual who consumes asset income while keeping real principal intact may report dissaving, even though there is none from a life cycle point of view.

We use a simple ordered probit to ask whether, in addition to the usual income and wealth effects, saving varies systematically with the predicted subjective survival probability that takes into account individual stated perceptions of mortality risk. We are particularly interested in the nuance of this hypothesis that says individuals respond behaviorally to perceptions rather than to life table survival probabilities. The estimates are given in tables 9.10

Table 9.10 **Survival Probability and Saving for Individuals (ordered probit)**

	Model 1			Model 2			Model 3			Model 4			Model 5		
Variable	Coeff.	S.E.	*T*-Stat.	Coeff.	S.E.	*T*-Stat.	Coeff.	S.E.	*T*-Stat.	Coeff.	S.E.	*T*-Stat.	Coeff.	S.E.	*T*-Stat.
WQUART2	−0.0297	0.0451	−0.6586	−0.0247	0.0451	−0.5472	−0.0253	0.0452	−0.5593	−0.0176	0.0452	−0.3902	−0.0243	0.0451	−0.5392
WQUART3	0.2820	0.0561	5.0291	0.2866	0.0561	5.1085	0.2886	0.0561	5.1427	0.2894	0.0561	5.1558	0.2882	0.0561	5.1359
WQUART4	0.6402	0.0562	11.4002	0.6475	0.0562	11.5155	0.6431	0.0563	11.4313	0.6417	0.0563	11.4047	0.6458	0.0562	11.4823
AIQUART2	−0.1331	0.0474	−2.8057	−0.1375	0.0475	−2.8979	−0.1338	0.0475	−2.8169	−0.1312	0.0475	−2.7616	−0.1350	0.0475	−2.8424
AIQUART3	−0.0915	0.0533	−1.7184	−0.0926	0.0533	−1.7371	−0.0937	0.0533	−1.7572	−0.0910	0.0533	−1.7060	−0.0918	0.0533	−1.7220
AIQUART4	0.1957	0.0576	3.3965	0.2008	0.0577	3.4830	0.1936	0.0577	3.3537	0.1942	0.0577	3.3642	0.1991	0.0577	3.4523
WHI&AILO	−0.0671	0.0661	−1.0155	−0.0640	0.0661	−0.9676	−0.0694	0.0662	−1.0484	−0.0670	0.0662	−1.0118	−0.0675	0.0661	−1.0203
LIFESP				−0.2106	0.0747	−2.8182	−0.1085	0.0814	−1.3331	−0.0979	0.0816	−1.1992	−0.1133	0.0822	−1.3789
SUBJSP − LIFESP							0.7142	0.0550	3.1668	0.2167	0.0594	3.6502			
SUBJSP/LIFESP													0.0243	0.0086	2.8348
RPTSP − SUBJSP										−0.2434	0.1303	−1.8681			
CONSTANT	0.6746	0.0439	15.3559	0.7584	0.0531	14.2871	0.7148	0.0549	13.0310	0.7037	0.0552	12.7542	0.6836	0.0592	11.5399
THRESHOLD1	1.5594	0.0231	67.4741	1.5607	0.0231	67.4664	1.5626	0.0232	67.4476	1.5632	0.0232	67.4449	1.5621	0.0232	67.4574
Log likelihood	−5,797.8			−5,493.8			−5,488.8			−5,487.1			−5,489.7		
N	5,725			5,725			5,725			5,725			5,725		
Dependent variable															
Share of −1	0.1906			0.1906			1.1906			0.1906			0.1906		
Share of +1	0.2721			0.2721			0.2721			0.2721			0.2721		

Notes: Dependent variable is negative (−1), zero, or positive (+1) saving. S.E. = standard error.

AIQUART2, AIQUART3, and AIQUART4 are indicators for quartiles of the distribution of annuity income.

WHI&AILO is an indicator for above-median wealth and below-median annuity income.

LIFESP is the life table survival probability, 12.5 years ahead, for the individual.

SUBJSP is the predicted subjective survival probability, 12.5 years ahead, for the individual.

RPTSP is the reported subjective survival probability, 12.5 years ahead, for the individual.

THRESHOLD1 is the threshold parameter for the +1 saving category; the threshold for the −1 category is normalized to zero.

through 9.12. Table 9.10 gives results for all individuals in the sample, with single-person households and each person in two-person households treated as individual observations. The explanatory variables are household annuity income; household wealth; the life table probability for the individual of surviving 12.5 years beyond current age; the predicted subjective survival probability for the individual in 12.5 years, computed using his or her covariates and stated survival probabilities, under the maintained hypothesis that $\rho = 0$, as outlined in section 9.4 and used in the construction of table 9.9; and the "raw" stated subjective survival probability (to an age that is 10 to 15 years ahead, depending on current age). Income and wealth are classified by quartile to reduce the effects of measurement error and spurious correlation. This model takes no account of the interdependence of saving decisions of different family members. We find that the probability of saving rises significantly with increased income or wealth. One would expect less savings when wealth is high and annuity income is low; the coefficient has the expected sign but is insignificant. Higher life table survival probabilities are associated with *less* saving, contrary to the predictions of the life cycle model (model 2). However, in models that contain both the life table survival probability and the predicted survival probability based on the individual's subjective response, it is the latter variable that has explanatory power, with the expected sign. In particular, when the subjective survival probability is entered as a deviation from the life table survival probability, or as a ratio to this probability, then only the difference or ratio is statistically significant (models 3 and 5). Thus, these results suggest strongly that saving behavior is responding to subjective beliefs about personal mortality risk, rather than to life table hazards. The relationship is economically as well as statistically significant. The proportion of individuals selecting positive saving will vary by 22.0 percent as the subjective probability of survival varies from its upper limit of one to its lower limit of zero (model 3). The regressions establish that the "raw" subjective response has no added explanatory power, once the fitted survival probability is included in the model (model 4). This provides at least weak evidence that focal responses are a reporting bias rather than a true belief that drives behavior, and it supports our approach of estimating latent beliefs for focal respondents. An important research implication is that the subjective survival curves appear to have some power to explain saving behavior. A possible policy implication is that the upward bias in subjective survival probabilities, increasing with age, will retard dissaving. This could lead to what appears to be large bequest or precautionary saving motives and may provide a partial explanation for the stylized fact that people accumulate too little and then save too much to be consistent with the most simplistic life cycle model.

An analysis that pools individuals whether they are single or members of a couple overlooks the interdependence of decisions within a household. More critically, it fails to account for the rather different joint survival calculations facing a couple compared with those facing a single person, or for the possibility that preferences of singles and couples are different. Table 9.11 looks only at single-person households. Saving of these individuals increases significantly as a function of income or wealth, with significantly less saving from individuals with high wealth and low annuity income. Saving by males is higher than that of females, other things equal. This effect may be due to the problems faced by widows in reconciling consumption habits with reduced income following the death of the spouse. We find no significant effect of either life table or subjective survival probabilities, although the coefficient on the subjective survival probability is positive, as predicted by the life cycle model. The economic effect of subjective mortality risk, yielding a 19.0 percent swing in the probability of positive saving when the subjective probability varies from the extreme of one to the extreme of zero, goes in the direction consistent with the life cycle model and is not strikingly different from this percentage for all pooled individuals.

Table 9.12 examines the saving behavior of couples, where each couple is treated as a single decision-making unit. Again, income and wealth are significantly positively correlated with saving. To analyze the effects of mortality hazards, it is necessary to take account of the probabilities that both members will survive *and* that at least one will survive. For survival for a specific future time interval, one has

$$\begin{aligned}
\text{Prob(Both survive)} &= \text{Prob(Husband survives)}\cdot\text{Prob(Wife survives)},\\
\text{Prob(Exactly one survives)} &= \text{Prob(Husband survives)}\cdot\text{Prob(Wife dies)}\\
&\quad + \text{Prob(Husband dies)}\cdot\text{Prob(Wife survives)},\\
\text{Prob(One or more survives)} &= 1 - \text{Prob(Husband dies)}\cdot\text{Prob(Wife dies)}.
\end{aligned}$$

Saving behavior may depend on one or more of these joint probabilities. In reality, the saving calculation may be even more complex, as the order of death is likely to influence the annuity income stream. Of course, treatment of the life cycle mortality risk as a dynamic stochastic programming exercise also complicates the analysis.

In table 9.12, model 2 shows that saving increases with the subjective probability that at least one member of the household will survive 12.5 more years; this effect is significant, and the life table probability of this event is insignificant. The economic impact is substantial: as the subjective probability of at least one survivor varies from the extreme of zero to one, the probability of positive saving varies by 30.6 percent. In model 3, the survival probabilities of the male and female are entered separately. Again, the life table probabilities are insignificant, while the subjective probabilities are jointly significant at the 95 percent level. (The coefficients are not individually significant due to their high correlation.) Model 4 distinguishes the events that both members survive for 12.5 years, that the male only survives, and that the female only survives.

Table 9.11 **Survival Probability and Saving for Singles (ordered probit)**

	Model 1			Model 2			Model 3			Model 4			Model 5		
Variable	Coeff.	S.E.	*T*-Stat.	Coeff.	S.E.	*T*-Stat.	Coeff.	S.E.	*T*-Stat.	Coeff.	S.E.	*T*-Stat.	Coeff.	S.E.	*T*-Stat.
WQUART2	−0.0519	0.0588	−0.8833	−0.0503	0.0588	−0.8557	−0.0444	0.0589	−0.7541	−0.0430	0.0590	−0.7297	−0.0495	0.0588	−0.8421
WQUART3	0.3530	0.0903	3.9083	0.3553	0.0903	3.9328	0.3604	0.0904	3.9860	0.3606	0.0904	3.9882	0.3559	0.0903	3.9392
WQUART4	0.7132	0.0954	7.4734	0.7167	0.0955	7.5064	0.7169	0.0955	7.5075	0.7156	0.0955	7.4921	0.7168	0.0955	7.5070
AIQUART2	−0.0505	0.0573	−0.8811	−0.0501	0.0573	−0.8732	−0.0484	0.0574	−0.8436	−0.0476	0.0574	−0.8297	−0.0495	0.0574	−0.8629
AIQUART3	−0.0571	0.0743	−0.7674	−0.0527	0.0744	−0.7078	−0.0553	0.0745	−1.7430	−0.0548	0.0745	−0.7364	−0.0520	0.0744	−0.6983
AIQUART4	0.2293	0.0993	2.3101	0.2373	0.0995	2.3860	0.2308	0.0996	2.3179	0.2310	0.0996	2.3199	0.2375	0.0995	2.3874
WHI&AILO	−0.2499	0.1002	−2.4947	−0.2483	0.1002	−2.4785	−0.2532	0.1003	−2.5257	−0.2516	0.1003	−2.5091	−0.2497	0.1002	−2.4914
MALE	0.1343	0.0558	2.4066	0.1087	0.0593	1.8340	0.1031	0.0594	1.7370	0.1036	0.0594	1.7441	0.1065	0.594	1.7918
LIFESP				−0.1694	0.1316	−1.2865	−0.0926	0.1405	−0.6590	−0.0659	0.1443	−0.4569	−0.1402	0.1435	−0.9771
SUBJSP − LIFESP							0.1268	0.0812	1.5618	0.1651	0.0940	1.7563			
SUBJSP/LIFESP													0.0064	0.0126	0.5115
RPTSP − SUBJSP										−0.1874	0.2318	−0.8083			
CONSTANT	0.7152	0.0531	13.4805	0.7819	0.0742	10.5328	0.7503	0.0769	9.7502	0.7364	0.0788	9.3423	0.7168	0.0845	8.4855
THRESHOLD1	1.7980	0.0368	48.8895	1.7988	0.0368	48.8815	1.8001	0.0368	48.8677	1.8004	0.0368	48.8666	1.7990	0.0368	48.8799
Log likelihood	−2,315.1			−2,314.3			−2,313.1			−2,312.8			−2,314.2		
N	2,562			2,562			2,562			2,562			2,562		
Dependent variable															
Share of −1	0.2014			0.2014			0.2014			0.2014			0.2014		
Share of +1	0.1862			0.1862			0.1862			0.1862			0.1862		

Notes: Dependent variable is negative (−1), zero, or positive (+1) saving. S.E. = standard error.

AIQUART2, AIQUART3, and AIQUART4 are indicators for quartiles of the distribution of annuity income.

WHI&AILO is an indicator for above-median wealth and below-median annuity income.

LIFESP is the life table survival probability, 12.5 years ahead, for the individual.

SUBJSP is the predicted subjective survival probability, 12.5 years ahead, for the individual.

RPTSP is the reported subjective survival probability, 12.5 years ahead, for the individual.

THRESHOLD1 is the threshold parameter for the +1 saving category; the threshold for the −1 category is normalized to zero.

Table 9.12 **Survival Probability and Saving for Couples (ordered probit)**

	Model 1			Model 2			Model 3			Model 4		
Variable	Coeff.	S.E.	*T*-Stat.	Coeff.	S.E.	*T*-Stat.	Coeff.	S.E.	*T*-Stat.	Coeff.	S.E.	*T*-Stat.
WQUART2	0.0019	0.1069	0.0174	0.0016	0.1072	0.0154	0.0043	0.1074	0.0396	0.0080	0.1081	0.0745
WQUART3	0.2906	0.1123	2.5877	0.2854	0.1128	2.5306	0.2849	0.1129	2.5248	0.2906	0.1134	2.5629
WQUART4	0.6202	0.1109	5.5945	0.6125	0.1117	5.4846	0.6101	0.1118	5.4577	0.6145	0.1123	5.4723
AIQUART2	−0.3379	0.1387	−2.4369	−0.3468	0.1393	−2.4897	−0.3434	0.1395	−2.4619	−0.3421	0.1396	−2.4515
AIQUART3	−0.2359	0.1338	−1.7633	−0.2332	0.1341	−1.7389	−0.2493	0.1345	−1.8536	−0.2515	0.1346	−1.8681
AIQUART4	0.0579	0.1349	0.4292	0.0573	0.1349	0.4247	0.0479	0.1351	0.3544	0.0462	0.1352	0.3416
WHI&AILO	0.0395	0.1532	0.2576	0.0442	0.1533	0.2882	0.0329	0.1534	0.2146	0.0270	0.1537	0.1757
LIFESP12				−0.0275	0.1436	−0.1912						
SUBJSP12 − LIFESP12				0.2890	0.1262	2.2908						
LIFESPM							−0.3049	0.2176	−1.4012			
LIFESPF							0.0767	0.1215	0.6309			
SUBJSPM − LIFESPM							0.1929	0.1116	1.7287			
SUBJSPF − LIFESPF							0.2107	0.1232	1.7101			
LIFESPB										−0.1703	0.2920	−0.5832
LIFESPM1										−0.4324	0.3987	−1.0844
LIFESPF1										−0.0574	0.3086	−0.1859
SUBJSPB − LIFESPB										0.4215	0.1550	2.7197
SUBJSPM1 − LIFESPM1										0.1062	0.1808	0.5874
SUBJSPF1 − LIFESPF1										0.1146	0.1983	0.5778
CONSTANT	0.6938	0.1339	5.1796	0.7101	0.1608	4.4156	0.7592	0.1563	4.8563	0.8071	0.1897	4.2544
THRESHOLD1	1.3619	0.0430	31.6871	1.3657	0.0431	31.6783	1.3684	0.0432	31.6741	1.3687	0.0432	31.6739

Log likelihood	−1,483.1	−1,479.8	−1,477.4	−1,477.2
N	1,498	1,498	1,498	1,498
Dependent variable				
Share of −1	0.1816	0.1816	0.1816	0.1816
Share of +1	0.3498	0.3498	0.3498	0.3498

Notes: Dependent variable is negative (−1), zero, or positive (+1) saving. S.E. = standard error.

AIQUART2, AIQUART3, and AIQUART4 are indicators for quartiles of the distribution of annuity income.

WHI&AILO is an indicator for above-median wealth and below-median annuity income.

LIFESP12 is the life table survival probability, 12.5 years ahead, for one or both members of the couple.

SUBJSP12 is the predicted subjective survival probability, 12.5 years ahead, for one or both members of the couple.

LIFESPM is the life table survival probability, 12.5 years ahead, for the male.

SUBJSPM is the predicted subjective survival probability, 12.5 years ahead, for the male.

LIFESPF is the life table survival probability, 12.5 years ahead, for the female.

SUBJSPF is the predicted subjective survival probability, 12.5 years ahead, for the female.

LIFESPB is the life table survival probability, 12.5 years ahead, for both members of the couple.

SUBJSPB is the predicted subjective survival probability, 12.5 years ahead, for both members of the couple.

LIFESPM1 is the life table survival probability, 12.5 years ahead, of the male only.

SUBJSPM1 is the predicted subjective survival probability, 12.5 years ahead, of the male only.

LIFESPF1 is the life table survival probability, 12.5 years ahead, of the female only.

SUBJSPF1 is the predicted subjective survival probability, 12.5 years ahead, of the female only.

THRESHOLD1 is the threshold parameter for the +1 saving category; the threshold for the −1 category is normalized to zero.

Saving is found to rise significantly with the subjective joint survival probability, and to increase insignificantly with the probabilities of the remaining two events. These results suggest that saving behavior of couples fails to give the life cycle planning of a surviving spouse as much weight as the planning in the event of joint survival. A possible explanation is that saving decisions may be dominated by males who are unwarrantedly optimistic about their own survival and thus underestimate the probability of the event that their widows will have substantial life cycle requirements.

Several cautions should be kept in mind in assessing the results in tables 9.10 through 9.12. As noted, the definition of positive or negative saving is somewhat ambiguous and may be misinterpreted by some subjects. We have not accounted for factors that may influence bequest motives, such as the number and economic status of relatives. We have not taken into account the possibility of reverse causality, where poor health that lowers the personal survival probability is associated with medical expenditures that require dissaving. There are substantial questions about the accuracy of reported saving and wealth data in elderly populations. Selection is potentially a severe problem, as the sample selects individuals who do not have sufficient impairments to require proxy respondents, and who as a consequence may have higher survival probabilities and fewer current medical expenditures that drain savings. More definitive tests of whether there is informational value added in personal probabilities, beyond that contained in life tables, will rely on changes in wealth over time. As further waves of AHEAD become available, these tests will become possible.

9.7 Conclusions

This paper has examined the characteristics of survival probabilities stated by AHEAD respondents, particularly their relationship to standard life tables and their relationship to stated saving behavior. We find that stated probabilities are distorted by focal points. The evidence from the model is that there is in addition reporting error in nonfocal responses, but that this error is small relative to variation in individual heterogeneity. We conclude that nonfocal responses can be used with relatively minor adjustments to predict personal survival curves. More substantial adjustments are required to predict survival curves for focal respondents. With or without adjustment, subjective survival probabilities show expected variations with known relative risks and increasing optimism with increasing age. Future waves of the AHEAD survey will reveal the actual information content of these probabilities for survival. However, it is clear that in aggregate it will be necessary to adjust personal survival probabilities down as age rises in order to track aggregate survival statistics.

Our preliminary analysis of the relationship between personal survival probabilities and saving suggests that there is a significant positive correlation, and that consumers are responding to subjective beliefs about mortality rather than to life table probabilities. This tie, combined with the optimistic bias about survival that increases with age, gives one explanation for the fact that saving rates do not fall as rapidly with age as a classical life cycle model would suggest. Thus, this phenomenon may be in part due to a bias in perception, rather than to strong precautionary or bequest motives.

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Comment Axel Börsch-Supan

When I saw the title of this paper I was reminded of a menu in a three-star restaurant, headed by three chefs, so I was very curious and forward looking. And, alas, the first course did what an hors d'oeuvre is supposed to: generate appetite. First, it showed the general importance of the subject. Almost all our behavioral models use expectations. This is particularly stark in life-cycle–based models. As inputs, we need the paths of expected income streams and expected major fixed expenditures; and we need expected lifetime.

The conventional way is to employ population averages for these expectations, sometimes stratified or taken as predictions from regressions, for example, earnings regressions. In terms of expected life span, this amounts to using (un)conditional life tables. The use of population averages can be motivated by learning. The authors of this paper stress learning from one's own experience, relevant for, say, repeated purchases of consumer goods, and they are therefore skeptical about using life tables as approximations for expected life span—a singular event for anybody. However, one can also learn from other peoples' experiences, even from other peoples' life spans. Manski has provided a formal proof of the conditions that generate consistent expectations by learning from others: the authors are certainly aware of this literature, but I think it is relevant also for this paper.

The authors take a different route and propose a—at least for economists—rather unconventional way to infer peoples' expectations. They exploit what people answer when they are asked about their expectations. Economists have been rather skeptical about this method. First, subjects may just be reluctant to reveal their expectations, specifically about their own life expectancy, partly because of superstition, partly because of "Verdrängung." Second, cognitive dissonance between own beliefs and own actions may lead to biased answers vis-à-vis individual behavior: people may act according to population averages although they do not concede the truth to themselves (and the interviewer). Everybody feels like an above average car driver, for example. Finally, people may give strategic answers when they are asked to reveal their beliefs. I actually think that this problem is of least importance, but it creates the most fun for economists.

The authors try to disprove this skepticism by correlating the survey responses to life table and epidemiological data. Preliminary evaluation of Health and Retirement Survey (HRS) responses shows an astounding coincidence with such life tables. Even more important, the covariance with risk factors such as smoking and drinking corresponds precisely with the epidemiological evidence.

Whether these HRS results also obtain in the data from the survey of Asset

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and Health Dynamics among the Oldest Old (AHEAD) is a nontrivial question because the AHEAD sample is much older. Indeed, as table 9.1 shows, the younger ages' subjective survival probabilities are in line with life tables, while the older ages' are far too optimistic. This reflects a common problem in empirical analysis: if we find coincidence with the life table data, the subjective probabilities have little added information; if we do not, we do not know how reliable the new information is. At the least, we need more data and more waves in order to believe in the stability of the results of table 9.1. Better, we need to perform more experiments to find out what typical response patterns are. I will come back to this point later.

The authors are careful to address two statistical problems that may hinder a direct interpretation of the answers to questions about subjective probabilities. First, sample selectivity goes in the same direction as exaggerated optimism because people who have private information about an above average life expectancy will survive longer. The second statistical issue is the frequency of focal-point responses, that is, survival probabilities of exactly zero, 0.5, and 1.0. Incidentally, such responses are also frequent in the HRS but do not bias the averages there! The authors provide some insight in the correlation among focal-point answers and a selection of covariates. However, they do not analyze the relation between the number of focal-point answers and the respondent's characteristics, say, in a count data model. Because there may be an unobserved trait generating focal-point answers, the analysis in table 9.2 may be biased because it takes the number of other focal-point answers as exogenous.

It is very important for the analysis later in the paper to know whether focal-point answers are to be treated as round-off errors or as an expression of cognitive dissonance. To strengthen the econometrician's belief in the informational content of the beliefs of those being economeasured, the data collectors in AHEAD should do more testing and retesting, for example, by asking the same question in different sections of the questionnaire and by asking questions once in a positive and once in a negative formulation. It is important to verify that the focal points switch accordingly and that the subjective probabilities are consistent. Of course, this cannot be done in the entire AHEAD sample. Psychometricians have done a lot of work in experiments on those issues, and economists are beginning to realize that one can learn a lot from these experiments. This literature is—inappropriately—completely absent in this paper. The statistical problems of sample selectivity and focal points require econometric treatment. The main point of this paper is to show that the raw data need some smoothing before they can fruitfully be used in economic analyses.

This brings me to the second hors d'oeuvre. A second hors d'oeuvre is usually a very light one, a sorbet or—fashionable in these days in my region—a glass of vinegar made from Trockenbeerenauslese. The one here is anything but light. It is a structural model of how to relate the observed subjective probabilities to covariates, where the subjective probabilities are interpreted as points on each individual's own survival curve. The task is to fit these personalized survival curves taking account of observed heterogeneity—using the covariates provided in the data—as well as unobserved heterogeneity creating selectivity.

The methodology is fairly involved. The statistical model has two components. The first component models the selection process that describes true beliefs, while the second component links true beliefs to those measured with error and/or as focal-point answers. The selection model starts with unobserved heterogeneity that is fixed at the time of birth. People have different traits that make them once and for all more or less resilient. Modeled as gamma distributed, this heterogeneity generates a likelihood of observing a nonfocal-point answer not subject to measurement error. I am not sure the authors convinced me that predisposition is what fixes the survival probability of an entire life. I would rather have it modeled as a random walk in which shocks hit individuals, say, drawn from a Poisson distribution with covariates such as smoking, and thereby select individuals out of the sample once a certain threshold is passed.

The second component models the transformation of an exact continuous probability response to one that is measured with error or truncated to a focal point. The authors' model is very general and permits a rather flexible correlation pattern between measurement errors and classification errors into focal points, although they do not estimate the model in its full generality.

The model arrives at three pieces to estimate: a binomial probit model that separates focal-point from nonfocal-point responses, a nonlinear regression model of (potentially mismeasured) nonfocal-point responses, and an ordered probit model describing the three focal points.

The regression model is derived from three assumptions: a Cox proportional separation of baseline and individual hazard, a Weibull baseline modified by the survival selection process, and the result of the parametric unobserved heterogeneity fixing resiliency at birth. Taking logarithms yields the nonlinear regression equation, the main empirical result of the paper, table 9.6.

The results in this table show some reasonable covariation, for example, with health. Also the wealth pattern conforms to our priors: the wealthier have higher subjective survival rates. But table 9.6 also contains a host of surprises. Males and blacks have higher survival probabilities, in contradiction to the evidence. The ages of mother and father do not matter even though we know that they are powerful predictors of life expectancy that appear to be widely used as such.

The authors are aware of this. However, they change their interpretation by now talking about "optimism about survival" rather than "survival" as such. The careful reader recognizes that this is less than consistent with the first part of the paper in which responses in the HRS were proudly taken at face value.

The credibility of this second part of the paper would improve if the authors would give the reader some idea of how well this procedure works. Since the main problem is determining the extent of informational content of the subjective probabilities—that is, finding a balance between believing in the subjective survival probabilities as they are stated and massaging them using prior structural assumptions—any kind of validation would be helpful. One way would be to use a hold-out sample for the purpose of validation. Another way, of course, is to be patient and see whether the subjects in the panel will display actual mortality in accordance to their beliefs.

Just as an aside, I would like to raise a flag when I see wealth included in this regression—I will come back to this identification problem during the main course of this feast.

Indeed, the main course promises to be gorgeous: "Subjective Mortality Risk and Saving Behavior." Unfortunately, it comes on a small plate. For all I can get from this glimpse at the stated main purpose of the paper, my taste buds are a little irritated.

The idea behind this main part is straightforward. The authors plug the predicted subjective survival probabilities into the maximand of the life cycle problem, derive the implied saving-to-wealth ratio, and investigate its correlation to the observed saving-to-wealth ratio by a simple regression. If more waves of data had been available, the authors could have fed them directly into the first-order conditions that were spelled out in the first part of the paper and could have compared the predicted with the observed saving paths.

This sounds very reasonable. I have two problems. Unfortunately, the paper is not specific about what subsample is really used for this exercise—focal-point and/or nonfocal-point respondents?—and does not describe what is used as a predictor for the smoothed subjective survival probability.

My second problem relates to the identification problem mentioned above and the authors' criticism of identifying functional form restrictions by other authors at the beginning of the paper. The personalized survival curve is a function of wealth, but wealth is accumulated according to a trajectory determined by expectations about life span. The authors successfully endogenized mortality and described wealth at the same time. Hence, their approach cannot relieve us from identification by functional form. This third and somewhat rudimentary part of the paper should therefore spell out what exactly can be identified, either by functional form or by suitable instruments.

The results in table 9.10 are disappointing. No R^2 is reported, but it appears to be very low. The two factors are insignificant. However, it is important to stress that the set of savings data in this first AHEAD wave is a pure substitute to what an economist wants. The poor performance visible in table 9.10 is likely to reflect this more than anything else.

Thus, after these two mouth-watering hors d'oeuvres, I am particularly hungry and forward looking to a full main course and a selection of desserts once the authors have reliable data that permit them to test the link to their carefully processed subjective survival probabilities.