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6 Further Evidence on Business-Cycle Duration Dependence

Francis X. Diebold, Glenn D. Rudebusch, and
Daniel E. Sichel

Do business cycles exhibit duration dependence? That is, are expansions, contractions, or whole cycles more likely or less likely to end as they grow older? In recent work (Diebold and Rudebusch 1990; Sichel 1991), we argued that understanding business-cycle duration dependence is important for understanding macroeconomic fluctuations, we provided a framework for answering the questions posed above, and we provided some preliminary answers. More generally, we argued that the duration perspective may furnish fresh insight on important and long-standing questions in macroeconomics, such as the existence and the extent of a postwar stabilization of business cycles (Diebold and Rudebusch 1992).

Our earlier findings on the attributes of U.S. business cycles from a duration perspective can be compactly summarized:

1a. Prewar expansions exhibit positive duration dependence.
1b. Postwar expansions exhibit no duration dependence.
2a. Prewar contractions exhibit no duration dependence.

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2b. Postwar contractions exhibit positive duration dependence.
3a. Postwar expansions are longer than prewar expansions, regardless of any shift in duration dependence pattern.
3b. Postwar contractions are shorter than prewar contractions, regardless of any shift in duration dependence pattern.

In this paper, we extend our earlier work in two ways. First, we reassess and elaborate on our earlier findings for U.S. data. We use a parsimonious yet flexible exponential-quadratic hazard model, developed for this paper and potentially applicable in other contexts. This model provides a good compromise between nonparametric hazard estimation procedures, for which the available samples are too small, and commonly used parametric hazard estimation procedures, which may impose undesirable restrictions on admissible hazard shapes.

Second, we confront our earlier findings for prewar U.S. business-cycle duration dependence (points 1a and 2a) with prewar data for three additional countries. This is desirable because there have been only about thirty U.S. business cycles since 1854; therefore, only a limited number of duration observations are available. An obvious strategy for obtaining more information about business-cycle duration dependence is to expand the information set by using the NBER chronologies of business cycles in other countries.[1] Such chronologies are available for France, Germany, and Great Britain during the prewar period.

6.1 Methodology

The distribution function of a duration random variable, $F(\tau)$, gives the probability of failure at or before time τ. The survivor function, defined as

$$S(\tau) = 1 - F(\tau),$$

gives the probability of failure at or after time τ. The hazard function is then defined as

$$\lambda(\tau) = f(\tau)/S(\tau),$$

so that an integral of the hazard over a small interval Δ gives the probability of failure in Δ, conditional on failure not having occurred earlier. If the hazard function is increasing (decreasing) in an interval, then it is said to exhibit positive (negative) duration dependence in that interval.

The obvious reference hazard, to which we shall compare our estimated hazards, is flat. That is,

$$\lambda(\tau) = \lambda, \quad \text{if } \tau > 0,$$

1. Similarly, international data have been used in attempts to refine estimates of macroeconomic persistence (see, e.g., Campbell and Mankiw 1989; and Kormendi and Meguire 1990).

where λ is an unknown constant that will of course be different for expansions, contractions, and whole cycles. The associated duration density, $f(\tau)$, for the constant hazard is exponential.

Various hazard models that nest the constant hazard are in common use and could be used to study business-cycle dynamics. Consider, for example, the hazard[2]

$$\lambda(\tau) = \lambda\alpha\tau^{\alpha-1}, \quad \text{if } \tau > 0.$$

This hazard function nests the constant hazard (when $\alpha = 1$, $\lambda(\tau) = \lambda$). The associated duration density is Weibull; thus, the log likelihood (without censoring) is

$$\ln L(\alpha, \lambda; \tau_1, \ldots, \tau_T) = T\ln(\alpha\lambda) + (\alpha - 1)\sum_{t=1}^{T}\ln(\tau_t) - \lambda\sum_{t=1}^{T}(\tau_t)^{\alpha},$$

on which estimation and inference may be based for a given sample of observed durations, $\tau_1, \tau_2, \tau_3, \ldots, \tau_T$.

However, this hazard model, like other commonly used parameterizations, imposes strong restrictions on admissible hazard shapes. In particular, if $\alpha > 1$, the hazard is monotone increasing, and conversely for $\alpha < 1$. Nonmonotone hazard shapes (e.g., U or inverted U) are excluded. Although such restrictions may be natural in certain contexts, they appear unjustified in the business-cycle context.

Here we discuss a class of hazard models, developed for this paper but potentially more widely applicable, that we feel strikes a good balance between parsimony and flexibility of approximation, and on which we rely heavily in our subsequent empirical work. Consider the hazard

$$\lambda(\tau) = \exp(\beta_0 + \beta_1\tau + \beta_2\tau^2), \quad \text{if } \tau > 0.$$

This parsimonious hazard, which we call the exponential-quadratic hazard, is not necessarily monotone and is best viewed as a low-ordered series approximation to an arbitrary hazard.[3] In particular, the constant-hazard case of no duration dependence occurs for $\beta_1 = \beta_2 = 0$. Nonmonotone hazards occur when $\beta_1 \neq 0$, $\beta_2 \neq 0$, and sign $(\beta_1) \neq$ sign (β_2). The hazard is U shaped, for example, when $\beta_2 > 0$ and $\beta_1 < 0$ and inverted U shaped when $\beta_2 < 0$ and $\beta_1 > 0$.

The precise shape of the hazard is easily deduced. Immediately, $\lambda(0) = \exp(\beta_0)$, and rewriting the hazard as

2. For further details, see Sichel (1991).

3. Kiefer (1988) suggests that future research on hazard models of the form $\exp(\beta_0 + \beta_1\tau + \ldots + \beta_p\tau^p)$ would be useful. The exponential-quadratic hazard is, of course, a leading case of interest ($p = 2$). This hazard is also a special case of the Heckman-Walker (1990) hazard and is similar to the logistic-quadratic hazard of Nickell (1979).

$$\lambda(\tau) = \exp\left[\beta_2\left(\tau + \frac{\beta_1}{2\beta_2}\right)^2 - \frac{(\beta_1^2 - 4\beta_0\beta_2)}{4\beta_2}\right], \quad \beta_2 \neq 0,$$

makes obvious the fact that, when an interior maximum or minimum is achieved (i.e., when $\beta_1 \neq 0$, $\beta_2 \neq 0$, and sign $[\beta_1] \neq$ sign $[\beta_2]$), its location is at

$$\tau^* = -(\beta_1/2\beta_2),$$

with associated hazard value

$$\lambda(\tau^*) = \exp\left[-\frac{(\beta_1^2 - 4\beta_0\beta_2)}{4\beta_2}\right].$$

Before constructing the likelihood, we record a few familiar definitions that will be used repeatedly. First, by definition of the survivor function, we have

$$d \ln S(\tau)/d\tau = -f(\tau)/[1 - F(\tau)],$$

so that

$$\lambda(\tau) = -d \ln S(\tau)/d\tau.$$

We also define the integrated hazard as

$$\Lambda(\tau) = \int_0^\tau \lambda(x)dx,$$

which is related to the survivor function by

$$S(\tau) = \exp[-\Lambda(\tau)].$$

It is interesting to note that, for a hazard $\lambda(\tau)$ to be proper, it cannot be negative on a set of positive measure (otherwise, the positivity of probabilities would be violated) and it must satisfy $\lim_{\tau \to \infty} \Lambda(\tau) = \infty$ (otherwise, the distribution function would not approach unity). Thus, certain parameterizations of the exponential-quadratic hazard do not, strictly speaking, qualify as proper hazard functions. This is of little consequence for the results presented below, however, in which the exponential-quadratic hazard is used only as a *local* approximation.[4]

Construction of the log likelihood allowing for right censoring (as, e.g., with the last postwar trough-to-trough duration) is straightforward. Let $\beta = (\beta_0, \beta_1, \beta_2)'$. Then

$$\ln L(\beta; \tau_1, \ldots, \tau_T) = \sum_{t=1}^{T} \{d_t \ln[f(\tau_t; \beta)] + (1 - d_t)\ln[1 - F(\tau_t; \beta)]\},$$

where d_t equals one if the tth duration is uncensored, and zero otherwise. The form of the log likelihood is a manifestation of the simple fact that the contri-

4. Moreover, Heckman and Walker (1990) argue that, in certain contexts, it may be economically reasonable to place positive probability mass on durations of ∞.

bution of a noncensored observation to the log likelihood is the log density while the contribution of a censored observation to the log likelihood is the log survivor. But

$$f(\tau_t;\ \beta) = \lambda(\tau_t;\ \beta)[1 - F(\tau_t;\ \beta)],$$

so

$$\ln L(\beta,\ \tau_1,\ \ldots,\ \tau_T) = \sum_{t=1}^{T} \{d_t \ln[\lambda(\tau_t;\ \beta)] + \ln[1 - F(\tau_t;\ \beta)]\}.$$

Moreover,

$$[1 - F(\tau_t;\ \beta)] = \exp\left[-\int_0^{\tau_t} \lambda(x;\ \beta)dx\right],$$

insertion of which in the log likelihood yields

$$\ln L(\beta;\ \tau_1,\ \ldots,\ \tau_T) = \sum_{t=1}^{T} \left\{d_t \ln[\lambda(\tau_t;\ \beta)] - \int_0^{\tau_t} \lambda(x;\ \beta)dx\right\}.$$

Differentiating, we obtain the score

$$\partial \ln L/\partial\beta = \sum_{t=1}^{T} \{[d_t/\lambda(\tau_t;\ \beta)][\partial\lambda(\tau_t;\ \beta)/\partial\beta] - \int_0^{\tau_t} \partial\lambda(x;\ \beta)/\partial\beta dx\}$$

and the Hessian

$$\begin{aligned}\partial^2 \ln L/\partial\beta\partial\beta' = \sum_{t=1}^{T} \Big\{&[d_t/\lambda(\tau_t;\ \beta)][\partial^2\lambda(\tau_t;\ \beta)/\partial\beta\partial\beta'] \\ &- [d_t/\lambda^2(\tau_t;\ \beta)][\partial\lambda(\tau_t;\ \beta)/\partial\beta][\partial\lambda(\tau_t;\ \beta)/\partial\beta'] \\ &- \int_0^{\tau_t} \partial^2\lambda(x;\ \beta)/\partial\beta\partial\beta' dx\Big\}.\end{aligned}$$

Thus, specialization to the exponential-quadratic case yields the log likelihood

$$\ln L(\beta;\ \tau_1,\ \ldots,\ \tau_T) = \sum_{t=1}^{T} [d_t(\beta_0 + \beta_1\tau_t + \beta_2\tau_t^2) - \int_0^{\tau_t} \exp(\beta_0+\beta_1 x+\beta_2 x^2)dx].$$

The derivatives of the exponential-quadratic hazard are

$$\partial\lambda(\tau_t;\ \beta)/\partial\beta = \begin{bmatrix}1\\ \tau_t\\ \tau_t^2\end{bmatrix} \exp(\beta_0 + \beta_1\tau_t + \beta_2\tau_t^2)$$

and

$$\partial^2\lambda(\tau_t;\ \beta)/\partial\beta\partial\beta' = \begin{bmatrix}1 & \tau_t & \tau_t^2\\ \tau_t & \tau_t^2 & \tau_t^3\\ \tau_t^2 & \tau_t^3 & \tau_t^4\end{bmatrix} \exp(\beta_0 + \beta_1\tau_t + \beta_2\tau_t^2).$$

Insertion of the exponential-quadratic hazard derivatives into the general score and Hessian expressions yields the exponential-quadratic score and hazard

$$\partial \ln L/\partial\beta = \sum_{t=1}^{T} \left\{ \left(d_t \begin{bmatrix} 1 \\ \tau_t \\ \tau_t^2 \end{bmatrix}\right) - \int_0^{\tau_t} \begin{bmatrix} 1 \\ x \\ x^2 \end{bmatrix} \exp(\beta_0 + \beta_1 x + \beta_2 x^2) dx \right\}$$

and

$$\partial^2 \ln L/\partial\beta\partial\beta' = -\sum_{t=1}^{T} \int_0^{\tau_t} \begin{bmatrix} 1 & x & x^2 \\ x & x^2 & x^3 \\ x^2 & x^3 & x^4 \end{bmatrix} \exp(\beta_0 + \beta_1 x + \beta_2 x^2) dx.$$

Although construction of the likelihood, score, and Hessian is straightforward, it is not clear that *maximization* of the likelihood will be numerically tractable, owing to the lack of a closed-form likelihood expression and the resulting necessity of numerically evaluating thousands of integrals en route to finding a likelihood maximum. It happens, however, that (1) the evaluation of the required integrals presents only a very modest computational burden, (2) the expressions derived earlier for the score and Hessian facilitate likelihood maximization, and (3) the likelihood is globally concave, which promotes speed and stability of numerical likelihood maximization and guarantees that any local maximum achieved is global.

First, consider the requisite integral evaluation. This is done in standard fashion by approximating the integrand by a step function with steps at each integer duration value and adding the areas in the resulting rectangles. Thus, for example, the integral

$$\int_0^{\tau_t} x \exp(\beta_0 + \beta_1 x + \beta_2 x^2) dx$$

is evaluated as

$$\sum_{j=1}^{\tau_t} [x_j \exp(\beta_0 + \beta_1 x_j + \beta_2 x_j^2) + x_{j-1} \exp(\beta_0 + \beta_1 x_{j-1} + \beta_2 x_{j-1}^2)](x_j - x_{j-1})/2,$$

where $x_j = j$.

Second, consider numerical likelihood maximization. Given our ability to compute the likelihood value for any parameter configuration β, we climb the likelihood via the Newton-Raphson algorithm,

$$\beta^{(i+1)} = \beta^{(i)} - [\partial^2 \ln L^{(i)}/\partial\beta\partial\beta']^{-1} \partial \ln L^{(i)}/\partial\beta.$$

Convergence is deemed to have occurred if the change in the log likelihood from one iteration to the next is less than 0.01 percent.

Finally, global concavity of the likelihood (i.e., $\partial^2\lambda(\tau;\beta)/\partial\beta\partial\beta' < 0$, for all β in R^3) is easily established. To prove global concavity, let H denote the Hessian of the exponential-quadratic model. We must show that $y'Hy \leq 0$, with equality, if and only if $y = 0$. Now,

$$y'Hy = -\sum_{t=1}^{T} \int_0^{\tau_t} y' \begin{bmatrix} 1\ x\ x^2 \\ x\ x^2 x^3 \\ x^2 x^3 x^4 \end{bmatrix} y \exp(\beta_0 + \beta_1 x + \beta_2 x^2) dx$$

$$= -\sum_{t=1}^{T} \int_0^{\tau_t} [(a'y)^2 \exp(\beta_0 + \beta_1 x + \beta_2 x^2)] dx,$$

where $a = (1, x, x^2)' >> 0$, and $y = (y_1, y_2, y_3)'$. Note that the integrand is nonnegative and zero if and only if $y = 0$. But the integral of a nonnegative function is nonnegative, as is the sum of such integrals. Thus, the entire expression is nonpositive and zero if and only if $y = 0$.

Finally, we note that we have obtained various generalizations and specializations of our results, which are not of particular interest in the present application but may be of interest in others. All are treated in the appendix. First, confidence intervals for the true but unknown hazard function may be computed. Second, models with covariates, Z, may be entertained, such as

$$\lambda(\tau, Z; \beta, \gamma) = \exp(\beta_0 + \beta_1 \tau + \beta_2 \tau^2 + Z\gamma).$$

Third, if it can be maintained that (locally) $\beta_2 < 0$, then the log likelihood can be written as a function of integrals of standard normal random variables, and numerical integration is not required.

6.2 Empirical Results

We take as given the NBER chronologies of business-cycle peaks and troughs for the prewar and postwar United States as well as for prewar France, Germany, and Great Britain, which are shown in tables 6.1 and 6.2.[5] The tables show durations of expansions, contractions, and whole cycles measured both peak to peak and trough to trough. The U.S. chronology in table 6.1 includes a ninety-month duration for the last expansion, a 106-month duration for the last peak-to-peak cycle, and a ninety-eight-month duration for the last trough-to-trough cycle. In the empirical work that follows, we treat them as right censored; that is, they are taken as lower bounds for the true durations, the values of which are as yet unknown.[6]

We are limited to prewar samples with the French, German, and British data because of the scarcity of true recessions, involving actual declines in output, in Europe during the 1950s and 1960s. After the devastation of Europe during World War II, there was a reconstruction of extraordinary pace; thus, it is often impossible to identify the classic business cycle in the early postwar period in the European countries. In the postwar period, growth cycles, which refer to periods of rising and falling activity relative to trend growth, have

5. These dates are taken from Moore and Zarnowitz (1986), which are the same as those in Burns and Mitchell (1946, 78–79), with minor revisions for some of the U.S. dates.

6. Thus, we assume that the great expansion of the 1980s ended no sooner than May 1990 and that the subsequent contraction ended no earlier than January 1991.

Table 6.1 Business-Cycle Chronology and Durations: United States

Trough	Peak	Contractions	Expansions	Trough to Trough	Peak to Peak
			Prewar		
December 1854	June 1857	...	30	...	...
December 1858	October 1860	18	22	48	40
June 1861	April 1865	8	46	30	54
December 1867	June 1869	32	18	78	50
December 1870	October 1873	18	34	36	52
March 1879	March 1882	65	36	99	101
May 1885	March 1887	38	22	74	60
April 1888	July 1890	13	27	35	40
May 1891	January 1893	10	20	37	30
June 1894	December 1895	17	18	37	35
June 1897	June 1899	18	24	36	42
December 1900	September 1902	18	21	42	39
August 1904	May 1907	23	33	44	56
June 1908	January 1910	13	19	46	32
January 1912	January 1913	24	12	43	36
December 1914	August 1918	23	44	35	67
March 1919	January 1920	7	10	51	17
July 1921	May 1923	18	22	28	40
July 1924	October 1926	14	27	36	41
November 1927	August 1929	13	21	40	34
March 1933	May 1937	43	50	64	93
June 1938	...	13	...	63	...
			Postwar		
	February 1945	...	...	...	...
October 1945	November 1948	8	37	...	45
October 1949	July 1953	11	45	48	56
May 1954	August 1957	10	39	55	49
April 1958	April 1960	8	24	47	32
February 1961	December 1969	10	106	34	116
November 1970	November 1973	11	36	117	47
March 1975	January 1980	16	58	52	74
July 1980	July 1981	6	12	64	18
November 1982	?	16	90	28	106
?		...	...	98	...

been identified for the European countries (see Moore and Zarnowitz 1986). However, the timing, and hence duration dependence, of these cycles is not comparable with the prewar business cycles.

Summary statistics, including the sample size, minimum observed duration, mean duration, and standard error, for each of the four samples from each country, are displayed in table 6.3. Also included in table 6.3 are summary statistics from pooled samples of all expansions, contractions, and

Table 6.2 Prewar Business-Cycle Chronologies and Durations: Germany, France, and Great Britain

Trough	Peak	Contractions	Expansions	Trough to Trough	Peak to Peak
		France, 1865–1938			
December 1865	November 1867	. . .	23	. . .	. . .
October 1868	August 1870	11	22	34	33
February 1872	September 1873	18	19	40	37
August 1876	April 1878	35	20	54	55
September 1879	December 1881	17	27	37	44
August 1887	January 1891	68	41	95	109
January 1895	March 1900	48	62	89	110
September 1902	May 1903	30	8	92	38
October 1904	July 1907	17	33	25	50
February 1909	June 1913	19	52	52	71
August 1914	June 1918	14	46	66	60
April 1919	September 1920	10	17	56	27
July 1921	October 1924	10	39	27	49
June 1925	October 1926	8	16	47	24
June 1927	March 1930	8	33	24	41
July 1932	July 1933	28	12	61	40
April 1935	June 1937	21	26	33	47
August 1938	. . .	14	. . .	40	. . .
		Germany, 1879–1932			
February 1879	January 1882	. . .	35	. . .	. . .
August 1886	January 1890	55	41	90	96
February 1895	March 1900	61	61	102	122
March 1902	August 1903	24	17	85	41
February 1905	July 1907	18	29	35	47
December 1908	April 1913	17	52	46	69
August 1914	June 1918	16	46	68	62
June 1919	May 1922	12	35	58	47
November 1923	March 1925	18	16	53	34
March 1926	April 1929	12	37	28	49
August 1932	. . .	40	. . .	. . .	77
		Great Britain, 1854–1938			
December 1854	September 1857	. . .	33	. . .	. . .
March 1858	September 1860	6	30	39	36
December 1862	March 1866	27	39	57	66
March 1868	September 1872	24	54	63	78
June 1879	December 1882	81	42	135	123
June 1886	September 1890	42	51	84	93
February 1895	June 1900	53	64	104	117
September 1901	June 1903	15	21	79	36
November 1904	June 1907	17	31	38	48
November 1908	December 1912	17	49	48	66

(continued)

Table 6.2 (continued)

Trough	Peak	Contractions	Expansions	Trough to Trough	Peak to Peak
September 1914	October 1918	21	49	70	70
April 1919	March 1920	6	11	55	17
June 1921	November 1924	15	41	26	56
July 1926	March 1927	20	8	61	28
September 1928	July 1929	18	10	26	28
August 1932	September 1937	37	61	47	98
September 1938	. . .	12	. . .	73	. . .

whole cycles. We shall not conduct our empirical investigation, however, on pooled samples. Although it might be appealing to pool durations across countries to expand the sample, the conformity of business-cycle timing across countries suggests that the observations across countries are not independent.[7] Hence, simple pooling would be inappropriate. Estimation and testing procedures that control for the degree of interdependence are likely to be very complicated, particularly because so little is known about the transmission of business cycles from one country to another.

There is one area, however, in which we *do* pool information from the four countries, namely, in the specification of a lower bound on admissible durations. This lower-bound criterion, which is denoted t_0, is necessary because, *by definition,* the NBER does not recognize an expansion or a contraction unless it has achieved a certain maturity. The exact required maturity is not spelled out by the NBER, but, in describing the guidelines enforced since Burns and Mitchell (1946), Moore and Zarnowitz (1986) indicate that full cycles of less than one year in duration and contractions of less than six months in duration would be very unlikely to qualify for selection.[8] Because this is a criterion of the NBER definition of business cycles, the choice of t_0 should be, not country specific, but uniform across countries. In particular, we set t_0 for expansions, contractions, or whole cycles equal to one less than the minimum duration actually observed in any of the four countries. We also require t_0 to be identical for peak-to-peak and trough-to-trough cycles, given evidence that the NBER makes no distinction between these two types of whole cycles (see Diebold and Rudebusch 1990). Operationally, the minimum duration criterion is incorporated into estimation of the hazard functions by subtracting t_0 from each of the observed durations before implementing the methodology described in section 6.1.

Let us first consider the United States, for which we can contrast the prewar

7. For qualitative descriptions of the conformity of international business cycles, see Moore and Zarnowitz (1986) and Morgenstern (1959).

8. Note that Geoffrey Moore and Victor Zarnowitz are two of the eight members of the NBER Business Cycle Dating Committee.

Table 6.3 Business-Cycle Summary Statistics

Sample	Sample Size (*N*)	Minimum Duration	Mean Duration	Standard Error
	Prewar			
France, 1865–1938:				
F1: Expansions	17	8	29.2	14.8
F2: Contractions	17	8	22.1	15.9
F3: Peak to peak	16	24	52.2	25.3
F4: Trough to trough	17	24	51.3	23.0
Germany, 1879–1932:				
G1: Expansions	10	16	36.9	14.2
G2: Contractions	10	12	27.3	18.1
G3: Peak to peak	10	34	64.4	27.5
G4: Trough to trough	9	28	62.8	25.5
Great Britain, 1854–1938:				
GB1: Expansions	16	8	37.1	17.8
GB2: Contractions	16	6	25.7	19.4
GB3: Peak to peak	15	17	64.0	32.9
GB4: Trough to trough	16	26	62.8	28.6
United States, 1854–1938:				
US1:Expansions	21	10	26.5	10.7
US2: Contractions	21	7	21.2	13.6
US3: Peak to peak	20	17	47.9	20.3
US4: Trough to trough	21	28	47.7	18.1
All countries:				
Expansions	64	8	31.5	14.8
Contractions	64	6	23.5	16.3
Peak to peak	61	17	55.7	26.7
Trough to trough	63	24	54.7	23.9
	Postwar			
United States, 1945–present:				
US1′: Expansions	9	12	49.9	29.0
US2′: Contractions	9	6	10.7	3.2
US3′: Peak to peak	9	18	60.6	28.2
US4′: Trough to trough	9	28	60.7	30.9

and postwar experiences. We start with prewar half-cycle hazards, estimates of which are graphed in figure 6.1. Each graph in this figure—and those in all subsequent figures—consists of three superimposed estimated hazards: the exponential constant ($\exp[\beta_0]$), exponential linear ($\exp[\beta_0 + \beta_1\tau]$), and exponential quadratic ($\exp[\beta_0 + \beta_1\tau + \beta_2\tau^2]$). These may be viewed as progressively more flexible approximations to the true hazard and are useful, in particular, for visually gauging the conformity of business-cycle durations to the constant-hazard model. The numerical values underlying the figures are given in tables 6.4–6.6, along with maximum-likelihood estimates of the underly-

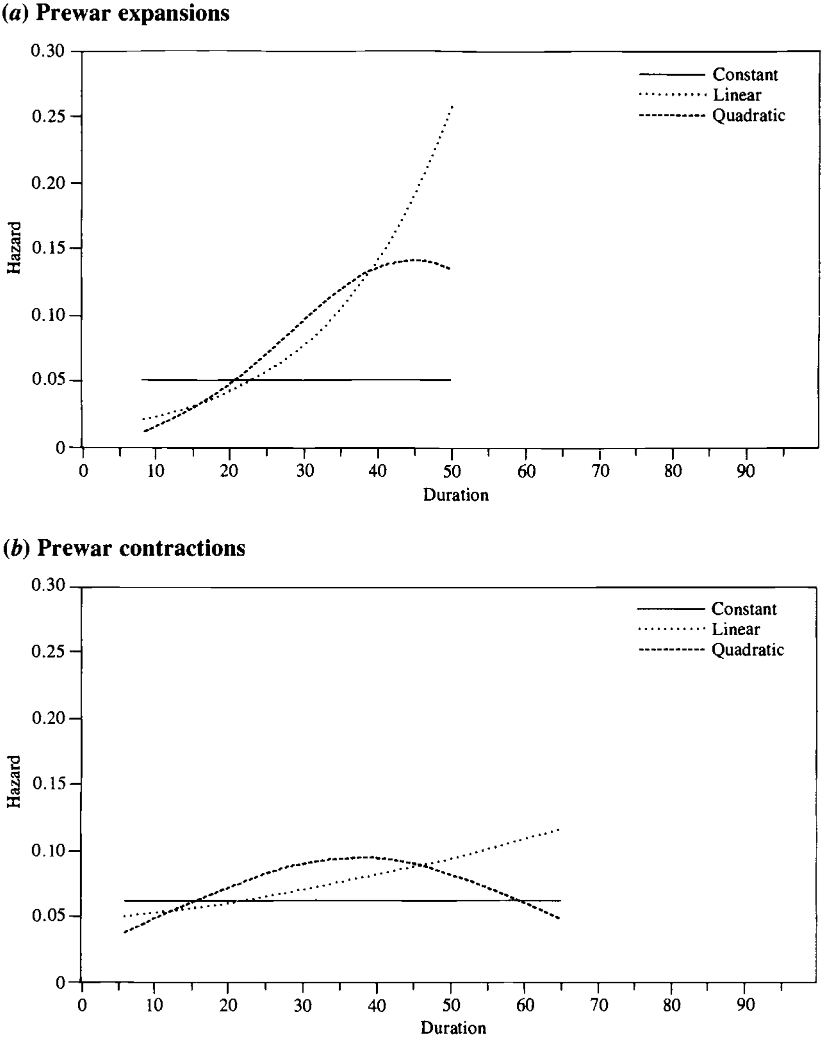

Fig. 6.1 Estimated hazard functions, United States

ing hazard function parameters. In keeping with our interpretation of the exponential hazard as a local approximation, the ranges of the tabled and graphed hazard functions have been chosen to reflect observed historical maximum durations.

Prewar U.S. expansions display strong evidence of duration dependence. The estimated exponential-linear expansion hazard rises sharply, from .03 to .25 after fifty months. The estimated exponential-quadratic expansion hazard rises more sharply at first, but subsequently less sharply, reaching .15 after

Table 6.4 Estimated Exponential-Constant Hazard Functions

Sample	β_0	$\exp(\beta_0)$	Sample	β_0	$\exp(\beta_0)$
	Prewar Expansions			Prewar trough to trough	
F1	−3.099	.045	F4	−3.564	.028
G1	−3.398	.033	G4	−3.875	.021
GB1	−3.405	.033	GB4	−3.846	.021
US1	−2.969	.051	US4	−3.457	.032
	Prewar Contractions			Postwar	
F2	−2.840	.058	US1′	−3.871	.021
G2	−3.105	.045	US2′	−1.735	.177
GB2	−3.030	.048	US3′	−3.910	.020
US2	−2.787	.062	US4′	−3.904	.020
	Prewar Peak to Peak				
F3	−3.589	.028			
G3	−3.850	.021			
GB3	−3.871	.021			
US3	−3.464	.031			

Note: For sample descriptions, see table 6.3.

fifty months. The p-values in table 6.7 indicate that we can soundly reject the constant-hazard null; the p-value for the null that $\beta_1 = 0$ in the exponential-linear model (p_1), for example, is .001.[9] The evidence against the linear-quadratic model, however, is less strong; the p-value for the null hypothesis that $\beta_2 = 0$ in the exponential-quadratic model (p_2) is .18.

Conversely, prewar U.S. contractions do not show strong evidence of duration dependence. The estimated exponential-linear expansion hazard rises only slowly, from .06 to .12 after seventy months. The estimated exponential-quadratic contraction hazard is inverted-U shaped, achieving a maximum of .09 after thirty-six months, but dropping back to .03 after seventy-two months. The p-values indicate that the constant-hazard null is hard to reject; p_1 is .17, and p_2 is .20.

The postwar U.S. results provide striking contrast. Postwar U.S. expansions display no duration dependence, while postwar U.S. contractions display strong positive duration dependence. In short, postwar duration dependence patterns, cataloged in figure 6.2 and tables 6.4–6.6, are precisely *opposite* those of the prewar period!

9. We report asymptotic p-values associated with the Wald statistics in the exponential-linear and exponential-quadratic models. The p-values give the probability of obtaining a sample test statistic at least as large in absolute value as the one actually obtained, under the null of no duration dependence. Small p-values therefore indicate significant departures from the null. p_1 is the p-value for the null hypothesis that $\beta_1 = 0$ in the exponential-linear model. p_2 is the p-value for the null hypothesis that $\beta_2 = 0$ in the exponential-quadratic model.

Table 6.5 **Estimated Exponential-Linear Hazard Functions**

			Duration in Months						
Sample	β_0	β_1	12	18	24	36	48	72	96
			Prewar Expansions						
F1	−3.76	.035	.028	.034	.042	.065	.099	.231	...
G1	−4.93	.065	.010	.015	.022	.047	.102	.481	...
GB1	−4.66	.050	.012	.016	.022	.041	.074	...	...
US1	−3.91	.060	.027	.039	.055	.113	.231	...	...
			Prewar Contractions						
F2	−2.95	.007	.055	.057	.060	.065	.070	.083	...
G2	−3.48	.019	.035	.039	.044	.055	.069	.108	...
GB2	−3.14	.005	.045	.047	.048	.051	.055	.062	.071
US2	−2.99	.014	.055	.060	.065	.077	.091	.127	...
			Prewar Peak to Peak						
F3	−4.06	.015	...	.018	.019	.023	.028	.040	.058
G3	−4.61	.020	...	.010	.012	.015	.019	.030	.048
GB3	−4.59	.018	...	.011	.012	.014	.018	.027	.041
US3	−4.05	.022	...	.018	.021	.027	.035	.060	.101
			Prewar Trough to Trough						
F4	−4.27	.024	...	.015	.017	.023	.030	.053	.093
G4	−5.35	.038	...	.005	.006	.010	.016	.040	.100
GB4	−4.55	.018	...	.011	.012	.015	.019	.029	.046
US4	−4.17	.028	...	.016	.019	.027	.037	.073	.142
			Postwar						
US1′	−4.20	.010	.016	.017	.018	.020	.022	.028	.035
US2′	−2.65	.195	.278	.897	...	...	...	...	...
US3′	−4.20	.008	...	.015	.016	.018	.019	.024	.029
US4′	−4.36	.013	...	.013	.014	.017	.019	.027	.036

Note: For sample descriptions, see table 6.3

The estimated exponential-linear and exponential-quadratic hazard functions for postwar U.S. expansions are hardly distinguishable from each other or from the estimated exponential-constant hazard, rising from .02 to only .03 after ninety-six months. Moreover, the p-values indicate that the data conform closely to the exponential-constant model ($p_1 = .23$, $p_2 = .43$). Conversely, the estimated hazards for postwar U.S. contractions rise extremely sharply. The estimated exponential-linear and exponential-quadratic hazards cannot be distinguished from each other but are readily distinguished from the constant hazard, rising from .07 to .29 in just twelve months. The deviation from constant-hazard behavior is highly statistically significant, with $p_1 = .03$.

It is important to note that the differences between prewar and postwar ex-

Table 6.6 **Estimated Exponential-Quadratic Hazard Functions**

				Duration in Months						
Sample	β_0	β_1	β_2	12	18	24	36	48	72	96
				Prewar Expansions						
F1	−3.80	.039	−.0001	.027	.034	.043	.066	.099	.207	...
G1	−5.13	.083	−.0003	.009	.014	.022	.050	.103	.340	...
GB1	−3.74	−.041	.0016	.020	.018	.019	.027	.063	...	...
US1	−4.44	.132	−.0017	.022	.041	.067	.124	.139	...	...
				Prewar Contractions						
F2	−2.98	.011	−.0001	.055	.058	.061	.066	.070	.074	...
G2	−3.24	−.014	.0006	.037	.036	.038	.047	.070	.270	...
GB2	−3.14	.006	−.0001	.045	.047	.048	.051	.055	.062	.070
US2	−3.28	.056	−.0009	.053	.067	.080	.093	.085	.033	...
				Prewar Peak to Peak						
F3	−4.54	.050	−.0004	...	.012	.016	.025	.035	.051	.047
G3	−5.16	.052	−.0003	...	.006	.008	.014	.022	.039	.048
GB3	−4.17	−.008	.0002	...	.015	.015	.015	.016	.022	.040
US3	−4.73	.075	−.0007	...	.010	.015	.030	.049	.069	.045
				Prewar Trough to Trough						
F4	−4.29	.026	.0000	...	.014	.017	.023	.030	.053	.089
G4	−5.06	.021	.0002	...	.007	.008	.010	.015	.037	.114
GB4	−5.14	.049	−.0003	...	.006	.009	.014	.021	.038	.050
US4	−5.10	.101	−.0010	...	.007	.013	.031	.055	.077	.034
				Postwar						
US1′	−4.32	.018	−.0001	.015	.016	.018	.021	.024	.029	.032
US2′	−2.72	.235	−.0034	.287	...	...	...	...	...	...
US3′	−4.17	.006	.0000	...	.016	.016	.018	.019	.023	.029
US4′	−4.76	.040	−.0003	...	.009	.012	.017	.023	.032	.033

Note: For sample descriptions, see table 6.3

pansion and contraction hazards are not limited to average *slopes*, although, as we have stressed, the slope changes are large and important. In particular, differences between the overall level of prewar and postwar expansion and contraction hazards exist—expansion hazards are higher in the prewar period, whereas contraction hazards are higher in the postwar period. These insights from the conditional perspective of hazard analysis—also noted in Sichel (1991)—lead to a deeper understanding of the unconditional distributional shifts documented in Diebold and Rudebusch (1992).[10]

10. Using exact finite-sample procedures, Diebold and Rudebusch (1992) also document the high statistical significance of the prewar-postwar change in business-cycle dynamics and estab-

Table 6.7 **p-Values for Null Hypotheses That Hazard Parameters Equal Zero**

Sample	p_1	p_2	Sample	p_1	p_2
	Prewar Expansions			Prewar trough to trough	
F1	.017	.472	F4	.015	.480
G1	.002	.425	G4	.004	.375
GB1	.002	.055	GB4	.010	.161
US1	.001	.181	US4	.002	.049
	Prewar Contractions			Postwar	
F2	.330	.468	US1′	.223	.433
G2	.169	.319	US2′	.027	.460
GB2	.328	.496	US3′	.264	.484
US2	.172	.201	US4′	.149	.295
	Prewar Peak to peak				
F3	.048	.176			
G3	.037	.245			
GB3	.024	.203			
US3	.011	.090			

Note: We report asymptotic p-values associated with the Wald statistics in the exponential-linear and exponential-quadratic models. p_1 is the p-value for the null hypothesis that $\beta_1 = 0$ in the exponential-linear model. p_2 is the p-value for the null hypothesis that $\beta_2 = 0$ in the exponential-quadratic model. For sample descriptions, see table 6.3.

Evidence of duration dependence in U.S. whole cycles, whether measured peak to peak or trough to trough, is also present in the prewar data. Moreover, the p-values indicate significance of the quadratic hazard term in the U.S. case. Finding duration dependence in prewar whole cycles is not surprising, in light of our finding of duration dependence in prewar expansions.[11] It is rather surprising, however, not to find significant duration dependence in postwar whole cycles, in light of our finding of significant duration dependence in postwar contractions. This may be due to low power, related to the fact that postwar whole-cycle behavior is dominated by expansion behavior (more than 80 percent of the postwar period was spent in the expansion state, as opposed to approximately 50 percent of the prewar period).

Now let us consider the evidence for France, Germany, and Great Britain. The estimated international exponential-constant, exponential-linear, and ex-

lish the robustness of that conclusion to issues of prewar data quality, the definition of *prewar,* and allowance for heterogeneity.

11. In fact, as pointed out by Mudambi and Taylor (1991), whole cycles may be expected to show duration dependence even in the absence of half-cycle duration dependence because the distribution of the time to *second* failure is not exponential when the distribution of the time to first failure is. (Moreover, the failure probabilities are of course different in expansions and contractions.)

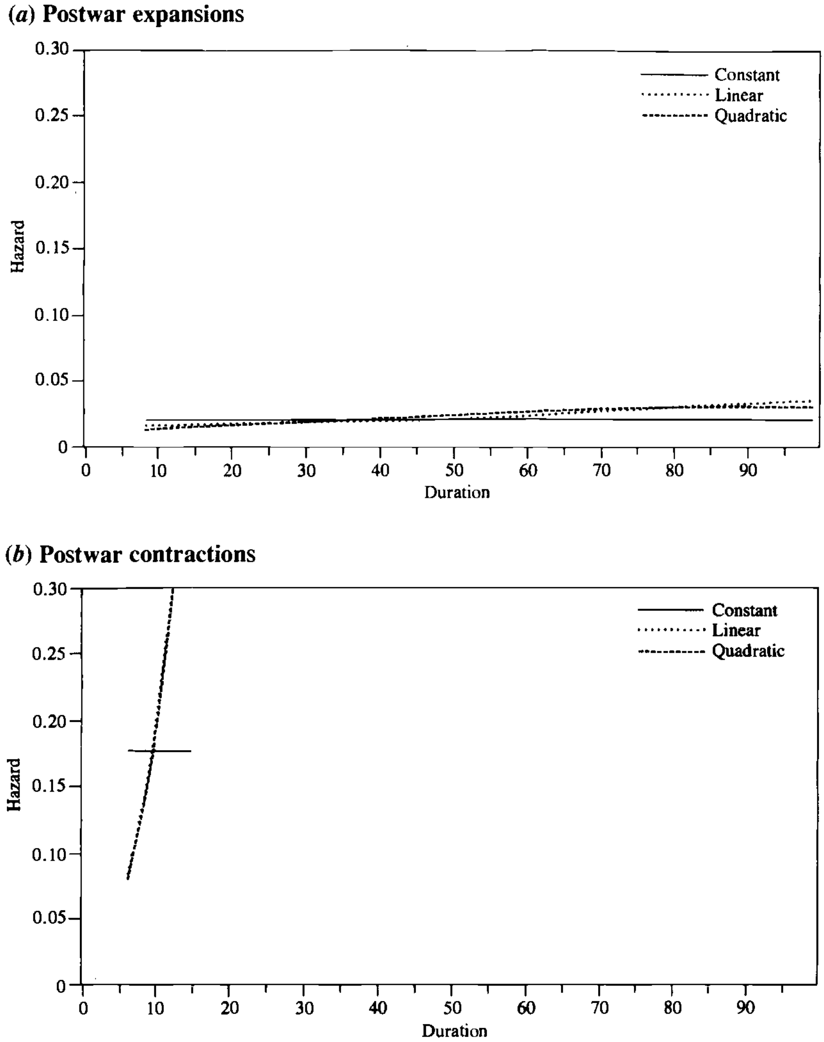

Fig. 6.2 Estimated hazard functions, United States

ponential-quadratic prewar hazard functions, shown in figures 6.3–6.5 and tables 6.4–6.6, indicate striking cross-country conformity in prewar business-cycle duration dependence patterns. All expansion hazards show strong positive duration dependence. The estimated hazard for German expansions, for example, rises from near zero after twelve months to .34 after seventy-two months. France and Great Britain also show substantial slope in their expansion hazard functions. Like that of the U.S. hazard, the departures of the French, German, and British hazards from constancy are highly signif-

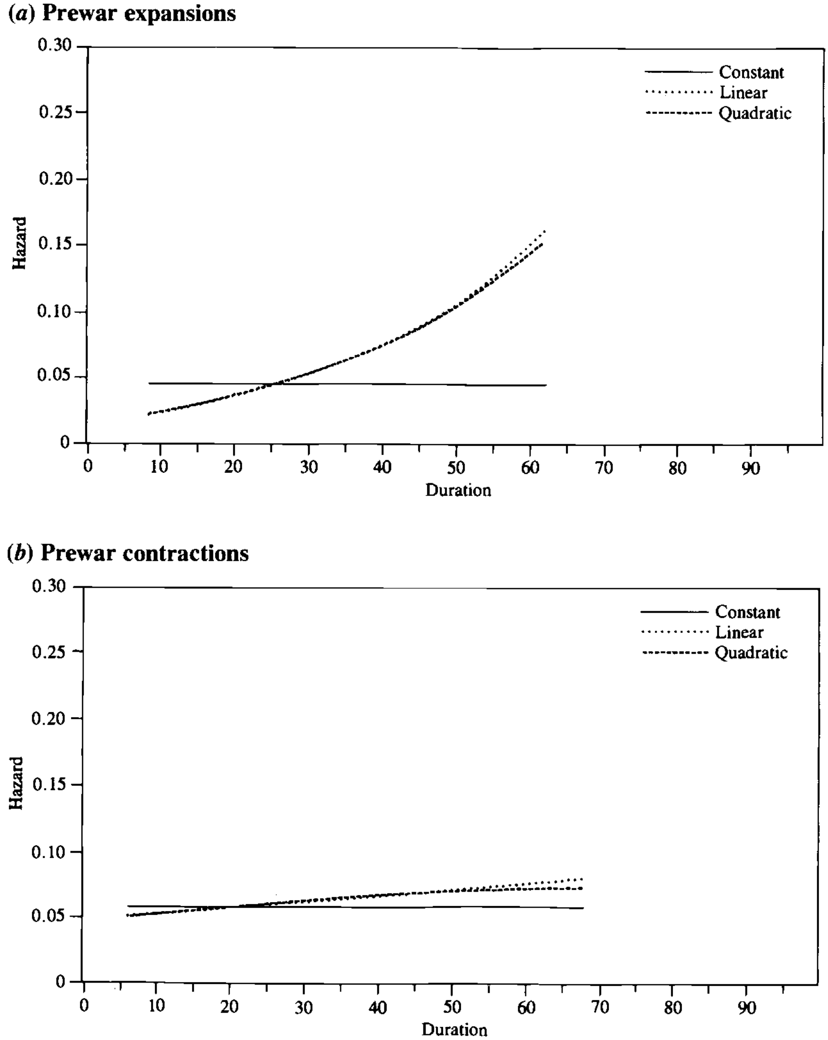

Fig. 6.3 Estimated hazard functions, France

icant, the respective values of p_1 being .02, .00, and .00. Also like the U.S. hazard, the quadratic term does not play a very important role, the respective values of p_2 being .47, .43, and .06.

For contractions, the U.S. prewar findings are again mimicked in France, Germany, and Britain: no evidence of duration dependence is found. All estimated contraction hazards are nearly constant, and the deviations from constancy are never significant. In contrast to the estimated expansion hazards, which start near zero and grow relatively quickly (and at increasing rates), the (*a*) **Prewar expansions**

(*b*) **Prewar contractions**

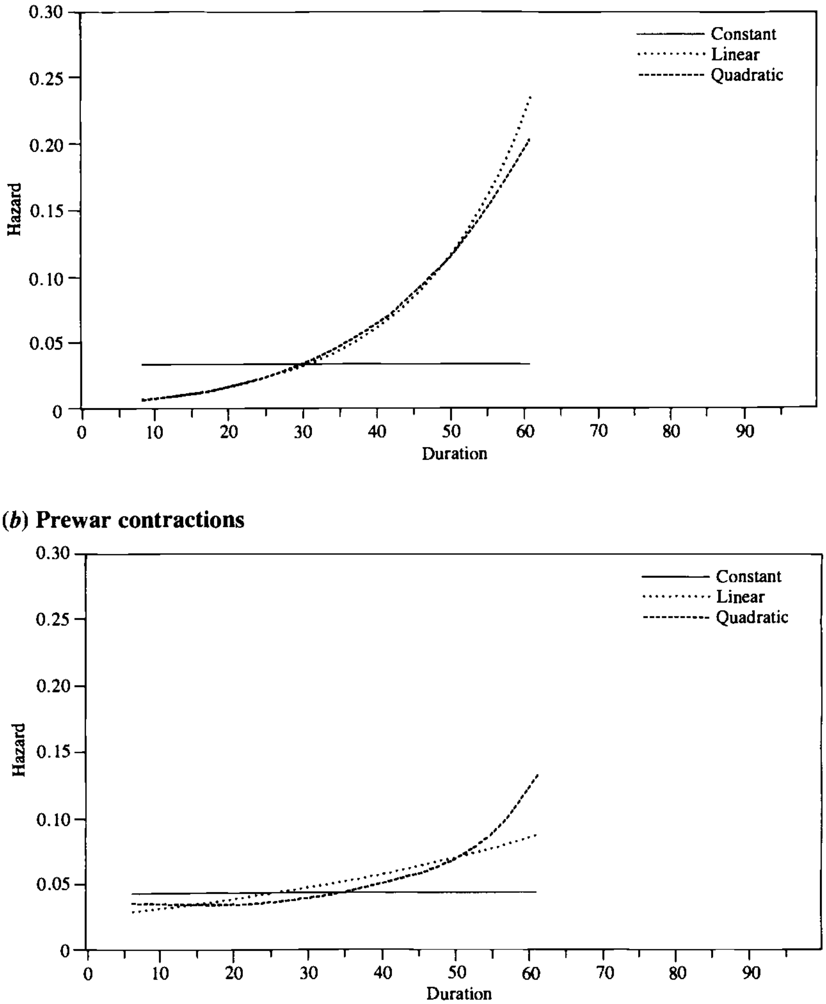

Fig. 6.4 Estimated hazard functions, Germany

estimated contraction hazards start near .05 and grow less quickly (and at decreasing rates).

Evidence for duration dependence in prewar whole cycles, which is strong in the U.S. samples, is also strong in the French, German, and British samples. For both peak-to-peak and trough-to-trough samples, all values of p_1 are less than .05. As in the United States, it would appear that the significant international prewar whole-cycle duration dependence is a manifestation of the significant half-cycle (expansion) duration dependence.

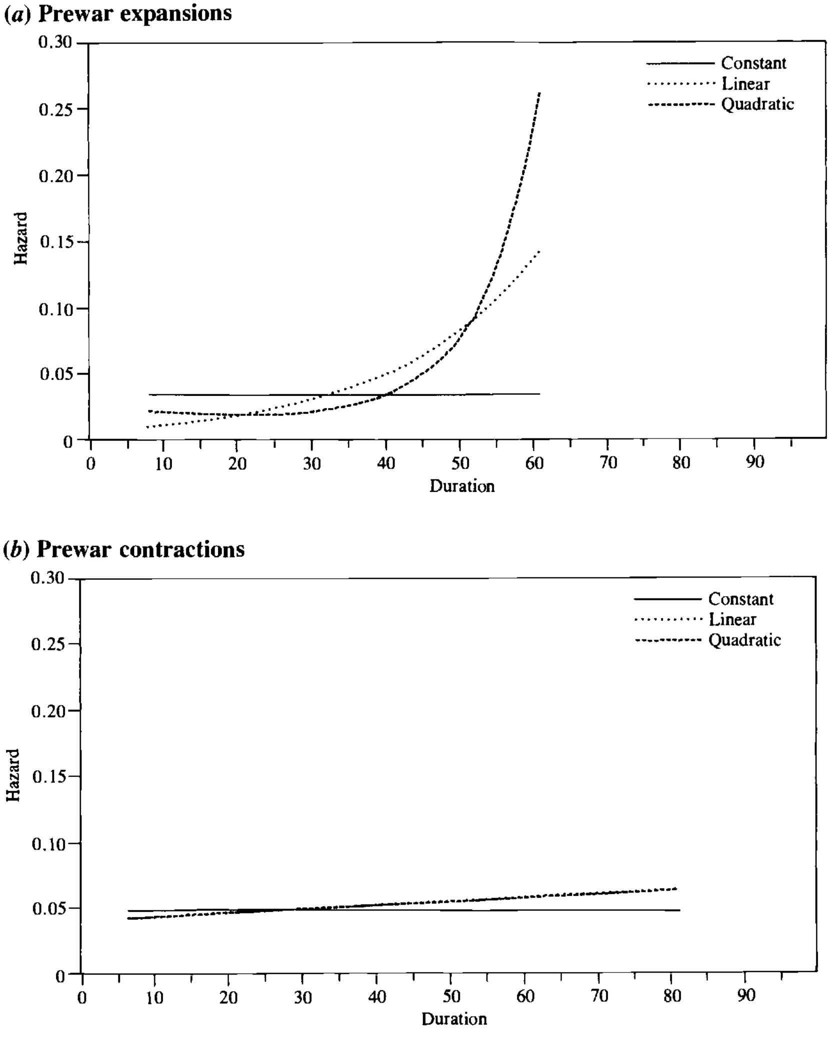

Fig. 6.5 Estimated hazard functions, Great Britain

6.3 Concluding Remarks

We began this paper by asking whether expansions, contractions, or whole cycles are more likely or less likely to end as they grow older, a question whose answer is of importance both methodologically and substantively. Methodologically, for example, the answer has implications for the proper specification of empirical macroeconomic models, such as the Markov-switching models proposed recently by Hamilton (1989). Substantively, for example, the answer has implications for turning-point prediction and business-cycle dating, as pointed out by Diebold and Rudebusch (1989, 1991).

Here we have investigated the patterns of duration dependence in U.S. prewar and postwar business cycles using a parsimonious yet flexible hazard model, deepening our understanding of the nature of postwar stabilization documented in Diebold and Rudebusch (1992). We presented evidence of a postwar shift in U.S. business-cycle duration dependence patterns: postwar expansion hazards display less duration dependence and are lower on average, while postwar contraction hazards display more duration dependence and are higher on average.

Moreover, we compared our prewar U.S. results with those obtained using prewar data from France, Germany, and Great Britain. We found that, for prewar expansions, all four countries exhibit evidence of positive duration dependence. For prewar contractions, none of the countries do. The results paint a similar prewar picture for each country; statistically significant and economically important positive duration dependence is consistently associated with expansions and never associated with contractions. The similarities in the prewar pattern of duration dependence across countries suggest conformity across countries in the characteristics of business cycles.

The empirical results in this paper and in our earlier papers pose substantial challenges for the construction of macroeconomic models; we hope that our measurement stimulates fresh theory. Obvious questions abound: What types of economic propagation mechanisms induce duration dependence in aggregate output, and what types do not? What are the theoretical hazard functions associated with the equilibria of various business-cycle models, and how do they compare with those estimated from real data? What types of models are capable of generating equilibria with differing expansion and contraction hazard functions, and how do they relate to existing linear and nonlinear models? How can we explain and model secular variation in the degree of duration dependence in expansions and contractions? Some recent work has begun to address various of these questions (e.g., Murphy, Shleifer, and Vishny 1989 develop a model in which cyclical duration is influenced by the stock of durables), but much remains to be done.

Appendix
Specialization and Generalization of the Exponential-Quadratic Hazard Model

Confidence Intervals

Confidence intervals for the true but unknown hazard may be obtained in straightforward fashion. Taylor series expansion of $\lambda(\tau_t, \hat{\beta})$ around $\lambda(\tau_t, \beta)$ yields

$$\lambda(\tau_t; \hat{\beta}) \approx \lambda(\tau_t; \beta) + \partial\lambda(\tau_t; \beta)/\partial\beta'(\hat{\beta} - \beta),$$

where $\hat{\beta}$ denotes the maximum likelihood estimate of β. Mean squared error is therefore approximated by

$$E[\lambda(\tau_t; \hat{\beta}) - \lambda(\tau_t; \beta)]^2 \approx \partial\lambda(\tau_t; \beta)/\partial\beta' E[(\hat{\beta} - \beta)(\hat{\beta} - \beta)']\partial\lambda(\tau_t; \beta)/\partial\beta.$$

By asymptotic unbiasedness of the maximum likelihood estimate, $E[(\hat{\beta} - \beta)(\hat{\beta} - \beta)']$ is asymptotically just $\text{cov}(\hat{\beta})$, which we estimate in standard fashion as $-(\partial^2 \ln L/\partial\beta\partial\beta')^{-1}$ evaluated at $\beta = \hat{\beta}$. Thus, as $T \to \infty$,

$$E[\lambda(\tau_t; \hat{\beta}) - \lambda(\tau_t; \beta)]^2 \to \text{var}[\lambda(\tau_t; \hat{\beta})].$$

For the exponential-quadratic hazard, recall that the first derivate of the hazard is

$$\partial\lambda(\tau_t; \beta)/\partial\beta = \begin{bmatrix} 1 \\ \tau_t \\ \tau_t^2 \end{bmatrix} \exp(\beta_0 + \beta_1\tau_t + \beta_2\tau_t^2)$$

and that the Hessian is

$$\partial^2 \ln L/\partial\beta\partial\beta' = -\sum_{t=1}^{T} \int_0^{\tau_t} \begin{bmatrix} 1 & x & x^2 \\ x & x^2 & x^3 \\ x^2 & x^3 & x^4 \end{bmatrix} \exp(\beta_0 + \beta_1 x + \beta_2 x^2)dx,$$

thus producing the asymptotic variance of the estimated hazard

$$\text{var}[\lambda(\tau_t; \hat{\beta})] \approx \exp[2(\beta_0 + \beta_1\tau_t + \beta_2\tau_t^2)][1, \tau_t, \tau_t^2] \left\{\sum_{t=1}^{T} \int_0^{\tau_t} \begin{bmatrix} 1 & x & x^2 \\ x & x^2 & x^3 \\ x^2 & x^3 & x^4 \end{bmatrix} \exp(\beta_0 + \beta_1 x + \beta_2 x^2)dx\right\}^{-1} \begin{bmatrix} 1 \\ \tau_t \\ \tau_t^2 \end{bmatrix}.$$

The Likelihood Function for the Model with Negative Quadratic Coefficient

The log likelihood in hazard form is

$$\ln L(\beta; \tau_1, \ldots, \tau_T) = \sum_{t=1}^{T} [d_t \ln \lambda(\tau_t) - \Lambda(\tau_t)],$$

which in the exponential-quadratic hazard case is

$$\text{(A1)} \qquad \ln L(\beta; \tau_1, \ldots, \tau_T) = \sum_{t=1}^{T} [d_t(\beta_0 + \beta_1\tau_t + \beta_2\tau_t^2) - \int_0^{\tau_t} \exp(\beta_0 + \beta_1 x + \beta_2 x^2)dx],$$

evaluation of which requires evaluation of the integrated hazard. The integration must be done numerically. Under the assumption that $\beta_2 < 0$, however, the integration may be greatly simplified because, as we shall show, the likelihood may be rewritten in terms of the standard normal cumulative density function (c.d.f.). The standard normal c.d.f. has been extensively tabulated and is available, for example, as a primitive function in many FORTRANs. We proceed by noting that

$$\text{(A2)}\quad \Lambda(\tau_t) = \int_0^{\tau_t} \exp(\beta_0 + \beta_1 x + \beta_2 x^2)dx$$

$$= \int_0^{\tau_t} \exp\left\{\beta_2[x + \beta_1/(2\beta_2)]^2 - \beta_1^2/(4\beta_2) + \beta_0\right\}dx$$

$$= \exp[\beta_0 - \beta_1^2/(4\beta_2)] \int_0^{\tau_t} \exp\left[\left(-1/\{2[-1/(2\beta_2)]\}\right)[x + \beta_1/(2\beta_2)]^2\right]dx$$

$$= \exp[\beta_0 - \beta_1^2/(4\beta_2)](2\pi)^{1/2}[-1/(2\beta_2)]^{1/2}$$

$$\int_0^{\tau_t} (2\pi)^{-1/2}[-1/(2\beta_2)]^{-1/2} \exp\left[\left(-1/\{2[-1/(2\beta_2)]\}\right)[x + \beta_1/(2\beta_2)]^2\right]dx,$$

which contains an integral of a normal density function with mean $-\beta_1/(2\beta_2)$ and variance $-1/(2\beta_2)$. (Recall our assumption that $\beta_2 < 0$, which is needed to ensure positivity of the variance.)

The integral may be rewritten as the difference of two integrals with left integration limit $-\infty$; that is,

$$\int_0^{\tau_t}(2\pi)^{-1/2}[-1/(2\beta_2)]^{-1/2} \exp\left[\left(-1/\left\{2[-1/(2\beta_2)]\right\}\right)[x + \beta_1/(2\beta_2)]^2\right]dx$$

$$= \int_{-\infty}^{\tau_t} (2\pi)^{-1/2}[-1/(2\beta_2)]^{-1/2} \exp\left[\left(-1/\left\{2[-1/(2\beta_2)]\right\}\right)[x + \beta_1/(2\beta_2)]^2\right]dx$$

$$- \int_{-\infty}^{0} (2\pi)^{-1/2}[-1/(2\beta_2)]^{-1/2} \exp\left[\left(-1/\left\{2[-1/(2\beta_2)]\right\}\right)[x + \beta_1/(2\beta_2)]^2\right]dx.$$

By standardizing appropriately, we can rewrite the difference of integrals as

$$\text{(A3)}\quad [-1/(2\beta_2)]^{-1/2}\Phi\left\{[x + \beta_1/(2\beta_2)]/[-1/(2\beta_2)]^{1/2}\right\}$$

$$- [-1/(2\beta_2)]^{-1/2}\,\Phi\left\{[\beta_1/(2\beta_2)]/[-1/(2\beta_2)]^{-1/2}\right\}$$

$$= [-1/(2\beta_2)]^{-1/2}\Phi\left\{[x + \beta_1/(2\beta_2)]/[-1/(2\beta_2)]^{1/2}\right\}$$

$$- [-1/(2\beta_2)]^{-1/2}\Phi\left\{[-\beta_1/(-2\beta_2)]^{-1/2}\right\},$$

where

$$\Phi(x) = \int_{-\infty}^{x} (2\pi)^{-1/2} \exp(-y^2/2)dx$$

denotes the standard normal c.d.f. Insertion of (A3) into (A2) yields

$$\Lambda(\tau_t) = \exp[\beta_0 - \beta_1^2/(4\beta_2)](2\pi)^{1/2}$$
$$\left(\Phi\left\{[x + \beta_1/(2\beta_2)]/[-1/(2\beta_2)]^{1/2}\right\} - \Phi\left\{[-\beta_1/(-2\beta_2)]^{1/2}\right\}\right),$$

which, when evaluated for $t = 1, 2, \ldots, T$ and inserted into (A1), yields the log likelihood function.

The Likelihood Function for the Model with Covariates

Consider the introduction of a vector of covariates into the hazard function; that is, consider

$$\lambda(Z_{\tau_t + s_t}, \tau_t; \beta),$$

where $s_t = \Sigma_{j=1}^{t-1} \tau_j$. Note that the total period used for estimation is $\Sigma_{t=1}^{T} \tau_t$. The log likelihood is

$$\ln L(\beta; \tau_1, \ldots, \tau_T) = \sum_{t=1}^{T} \left\{d_t \ln[\lambda(Z_{\tau_t + s_t}, \tau_t; \beta)] - \int_0^{\tau_t} \lambda(Z_{x + s_t}, x; \beta)dx\right\}.$$

The score is

$$\partial \ln L/\partial\beta = \sum_{t=1}^{T} \left\{[d_t/\lambda(Z_{\tau_t + s_t}, \tau_t; \beta][\partial\lambda(Z_{\tau_t + s_t}, \tau_t; \beta)/\partial\beta] - \int_0^{\tau_t} \partial\lambda(Z_{x + s_t}, x; \beta)/\partial\beta dx\right\},$$

and the Hessian is

$$\partial^2 \ln L/\partial\beta\partial\beta' = \sum_{t=1}^{T} \left\{[d_t/\lambda(Z_{\tau_t + s_t}, \tau_t; \beta)][\partial^2\lambda(Z_{\tau_t + s_t}, \tau_t; \beta)/\partial\beta\partial\beta'] - [d_t/\lambda^2(Z_{\tau_t + s_{tt}}, \tau_t; \beta)][\partial\lambda(Z_{\tau_t + s_t}, \tau_t; \beta)/\partial\beta'] - \int_0^{\tau_t} \partial^2\lambda(Z_{x + s_t}, x; \beta)/\partial\beta\partial\beta' dx\right\}.$$

In the exponential-quadratic case, we have

$$\lambda(Z_{\tau_t + s_t}, \tau_t; \beta) = \exp(\beta_0 + \beta_1\tau_t + \beta_2\tau_t^2 + Z_{\tau_t + s_t}\gamma),$$

where both $Z_{\tau_t + s_t}$ and γ are vectors, so that the score and Hessian are

$$\partial \ln L/\partial\beta = \sum_{t=1}^{T} \left\{ \left(d_t \begin{bmatrix} 1 \\ \tau_t \\ \tau_t^2 \\ Z_{\tau_t+s_t} \end{bmatrix} \right) - \int_0^{\tau_t} \begin{bmatrix} 1 \\ x \\ x^2 \\ Z_{x+s_t} \end{bmatrix} \exp(\beta_0 + \beta_1 x + \beta_2 x^2 + Z_{x+s_t}\gamma)dx \right\}$$

and

$$\partial^2 \ln L/\partial\beta\partial\beta' = -\sum_{t=1}^{T} \int_0^{\tau_t} \begin{bmatrix} 1 \\ x \\ x^2 \\ Z_{x+s_t} \end{bmatrix} (1, x, x^2, Z_{x+s_t}) \exp(\beta_0 + \beta_1 x + \beta_2 x^2 + Z_{x+s_t}\gamma)dx.$$

Each integration may be evaluated numerically as discussed in the text. Thus, for example,

$$\int_0^{\tau_t} Z_{x+s_t} \exp(\beta_0 + \beta_1 x + \beta_2 x^2 + Z_{x+s_t}\gamma)dx$$

is evaluated as

$$\sum_{j=1}^{\tau_t} [Z_{x_j+s_t} \exp(\beta_0 + \beta_1 x + \beta_2 x_j^2 + Z_{x_j+s_t}\gamma) + Z_{x_{j-1}+s_t} \exp(\beta_0 + \beta_1 x_{j-1} + \beta_2 x_{j-1}^2 + Z_{x_{j-1}+s_t}\gamma)](x_j + x_{j-1})/2,$$

where $x_j = j$.

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Comment

Bruce E. Hansen

Let me pose the following question. Suppose that the economy is in an expansionary phase. What would be a reasonable estimate of the probability of entering a contractionary phase in the near future? What factors would your answer depend on? Is one factor the length (duration) of the current expansion? Similarly, if the economy is in a contractionary phase, does it seem reasonable that the probability of the contraction ending may depend on its past duration?

Quite frankly, I do not find it easy to come up with an intelligent answer to these questions. This is largely because the statistical models that are typically used to study aggregate output do not lend themselves easily to their analysis. A new approach has been proposed by Diebold, Rudebusch, and Sichel. In a series of papers, these authors have argued for the direct analysis of business-cycle duration data. This provides a statistical framework in which questions such as those listed above can be answered in a straightforward and easily interpretable manner.

Measuring the Business Cycle

The starting point for Diebold et al.'s analysis is dating the business cycle. The authors follow the NBER Business Cycle Dating Committee in assigning

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the label *expansion* or *contraction* to each quarter. It is understood that the committee observes a large set of variables. Let us denote the vector of observables by Y_t and its history by $\{Y_t\}$. On the basis of a set of informal rules and internal discussion, the committee determines the appropriate label for each quarter, which we can denote by S_t, for *state of the economy*. The Business Cycle Dating Committee is in effect inducing a mapping from the observed series $\{Y_t\}$ to the reported labels $\{S_t\}$. If the committee's methods are stable over time, we can write this mapping as

$$S_t = \text{NBER}\ (\{Y_t\}).$$

I call this the NBER business-cycle filter.

Since the authors base their study on the series $\{S_t\}$, one has to think about the nature of the NBER filter that generated it. Does the Business Cycle Dating Committee impose some sort of prior reasoning on how it assigns the label *contraction* or *expansion* to a particular economic quarter? If so, then the dependence in the series $\{S_t\}$ may be a mixture of the committee's prior and the "true" dependence in the underlying economy. In order to justify working with $\{S_t\}$, we must be able to argue that the data are sufficiently informative to outweigh the prior beliefs of the committee members. Could small biases in the committee's dating conventions induce significant changes in the inferences made by the authors in their work? This is a difficult question, but it suggests that, if the questions raised in these papers are indeed important, then more in-depth empirical research needs to be done.

Are Business Cycles Duration Dependent?

The current paper reinforces the authors' past findings of duration dependence in business-cycle data. The general finding is that, regardless of country, time period, or measure of the business cycle (contractions, expansions, or full cycle), durations display *constant* or *increasing hazard*. The data suggest no significant evidence of decreasing hazard. So, the longer the economy has been in a state, the more likely a transition will occur. This suggests that some simple models of the business cycle are misspecified. For example, the Markov-switching model of Hamilton (1989) assumes a constant hazard. The finding by Diebold et al. of positive duration dependence suggests that it may be a useful avenue of research to generalize the Markov-switching model to allow for an increasing hazard. This poses some tricky econometric problems. Identification of the Markov-switching probabilities is known to be problematic in Hamilton's specification. A more complicated specification may suffer even deeper identification problems. Researchers who attempt to generalize Hamilton's approach in this direction should be aware of this problem before they begin and take it seriously when making inferences.

Has the Nature of the Business Cycle Changed?

Diebold et al. use their estimated duration model to argue that the stochastic nature of the business cycle changed after the Second World War. This claim is important for several reasons. If the distribution of business-cycle durations is the same in the prewar and postwar periods, then we can use the combined sample for learning about the nature of the business cycle. Since there are about twice as many cycles before the war as after, this may make a dramatic difference in the precision of estimation.

The authors make the claim that the business-cycle process changed during the war years by performing an informal sample split test. The model is separately estimated over the prewar and postwar periods and the parameter estimates informally compared. This approach, while suggestive, may lead to incorrect inferences. The problem is with the selection of the sample split point. The choice of the war period as the point at which to split the sample is not exogenous to the data. Since the choice has been made after (informally) examining the data, the tendency is to select a sample split point that is particularly tough on the null hypothesis. The critical values used implicitly by the authors to justify rejecting a constant model are too low, and a spurious rejection may have occurred.

The way to think about this is as follows. We want the correct distribution of the test statistic, under the null hypothesis of a constant model. Data generated from a constant-parameter model have a tendency to produce periods in which it appears as if the model is not constant over that period. An applied researcher who examines the data and then "tests" for model stability, conditioning on a sample split point at which the model looks particularly bad, will tend to overreject the hypothesis of constancy.

Recent developments in econometric methods allow us to circumvent this problem. Andrews (1990) and Hansen (1990) develop a unified theory of testing parameter constancy in parametric models. These tests are quite simple to apply, especially in maximum likelihood estimation (the framework used by the authors). In general, suppose that the log likelihood can be written as

$$L_n(\theta) = \sum_{i=1}^{n} l_i(\theta).$$

First, estimate the model over the full sample (prewar and postwar combined), yielding the parameter estimates $\hat{\theta}$. Then form a partial sum process in the estimated scores,

$$S_t = \sum_{i=1}^{t} \frac{\partial}{\partial\theta} l_i(\hat{\theta}),$$

and sequential estimates of the second derivative,

$$V_t = \sum_{i=1}^{t} \frac{\partial^2}{\partial\theta\partial\theta'} l_i(\hat{\theta}).$$

Then the statistic

$$L_C = \frac{1}{n} \sum_{t=1}^{n} S_t' V_n^{-1} S_t$$

is the Lagrange multiplier statistic for the test of the null of parameter stability against the alternative that the parameters follow a random walk. Asymptotic critical values are given in Nyblom (1989) and Hansen (1990, table 1). The statistic

$$\text{SupLM} = \max_{(t/n)\in\Pi} S_t'\left[V_t - V_t V_n^{-1} V_t\right]^{-1} S_t$$

is the Lagrange multiplier statistic for the test of the null of parameter stability against the alternative of a single structural break of unknown timing. Asymptotic critical values are given in Andrews (1990, table 1) for $\Pi = [.15, .85]$.

Both statistics are easy to calculate, and both have power against a much wider range of alternatives than that for which they were designed.

Table 6C.1 reports parameter estimates and asymptotic standard errors for the United States over the joint prewar and postwar periods. The model is the exponential-quadratic model advocated by Diebold et al. The likelihood was programmed in GAUSS386, and the calculations were performed on a 486/33 computer. Using numerical first and second derivatives, the model converged in only a few seconds, so I did not program the analytic derivatives, as recommended by the authors. The test statistics were also calculated using numerical derivatives.

These formal tests confirm the informal finding of Diebold et al. that the models for contractions and expansions are not stable over the joint sample. The SupLM statistic rejects parameter constancy for both contractions and expansions. The L_C statistic rejects parameter constancy for expansions. There are several possible interpretations of these findings. One, advocated by the authors, is that a regime change took place, possibly induced by changes in government macroeconomic policy. If this were indeed the case, we would expect that the SupLM statistic would be maximized for a break point during the war years. Unfortunately for this thesis, the statistic for expansions found the "break point" to be in 1969. We do not have standard

Table 6C.1 **Estimated Quadratic Hazard Functions, United States, 1854–1990**

	β_0	β_1	β_2	L_C	SupLM
Expansions	−3.92	.046	−.0004	1.62**	16.0*
	(.44)	(.031)	(.0004)		
Contractions	−2.79	.032	−.0006	.72	24.8**
	(.36)	(.045)	(.0010)		
Peak to peak	−4.51	.051	−.0004	.68	6.8
	(.55)	(.034)	(.0005)		
Trough to trough	−4.94	.078	−.0007	.51	5.6
	(.60)	(.037)	(.0005)		

Note: Asymptotic standard errors are given in parentheses.

* Significant at the asymptotic 5% level.

** Significant at the asymptotic 1% level.

errors for this estimate of the break point, but it is not encouraging for the authors' thesis.

An alternative interpretation is that there is nothing particularly special about splitting the sample at the war years: instead, the finding of parameter instability is simply evidence against the hypothesis that the duration data come from a stationary distribution. The "parameter change" need not take the form of a simple regime shift. Instead, the distribution may be slowly shifting over time as the economy (or the "NBER filter") changes. We know that the underlying GNP process is nonstationary. Is it obvious that the NBER filter applied to this nonstationary process will process a stationary output? The finding of parameter instability is evidence against this hypothesis and is a reasonable interpretation of the evidence.

The formal tests also confirm the finding of Diebold et al. that full-cycle durations can be described well by a stable process over the entire sample. Neither the L_C nor the SupLM statistic is large for the peak-to-peak or trough-to-trough durations. This is indeed an interesting finding, when placed in contrast to the strong rejection of constancy by the contraction and expansion durations.

Questions for Future Research

The analysis contained in the paper by Diebold, Rudebusch, and Sichel implicitly assumes that the business cycle is well described by a two-state system. This assumption is also made by Hamilton (1989). It is not immediately apparent that this assumption is valid. Are the probabilities of leaving expansions and/or contractions dependent only on duration, or are they also dependent on amplitudes? That is, do these probabilities depend on the strength of an expansion or the severity of a contraction? I would expect so. If so, then the authors are inefficiently ignoring available information and, more important, are possibly distorting correct inferences. It is quite possible that, once amplitude is conditioned on, then the finding of positive duration dependence could disappear. A fruitful avenue for future research may be to explore how business-cycle durations depend as well on other variables.

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