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AN OVERVIEW OF THE MODEL

THE DETROIT PROTOTYPE consists of several complex and interrelated submodels, each of which replicates an aspect of urban growth and development. The analytical core of the model is the housing market. Most decisions simulated by the model are related to the determinants of housing demand or supply by location. The algorithms which represent this behavior are fairly straightforward constructs that derive from microeconomic theory. The decomposition of the simulation model into submodels, their interrelation, and much of their content are based on a microeconomic theory of the housing market and its participants.

The model is not, however, merely a straightforward mapping from theory to practice, since several characteristics of the housing market diminish the usefulness of existing theory. Externalities, long-lived capital stocks, strong interdependencies among decision makers, and widespread nonprivate decision making are among the most important of these. Indeed, many of the problems confronted in housing markets are a challenge to economic theory in general.

In this chapter we outline the theoretical considerations which underlie the design of the model and indicate how these considerations are reflected in its components. We also indicate how the theoretical difficulties have been handled in the development of the model design.

Theoretical Considerations

The theoretical foundations of the NBER model rest upon two major components of microeconomic theory, the theory of the consumer and the theory of the firm. On the demand side of the model, as in monocentric theories of urban spatial structure, it is postulated that households choose that type and location of housing that maximizes their utility. The design of the supply side is based upon the premise that housing suppliers, in deciding to build new housing and transform the standing stock, seek to maximize profits. Similarly, the algorithms that simulate maintenance decisions by owner-occupants and landlords derive from investment theory.

Although the demand side of the model is based upon utility maximization by households, it does not include utility functions directly. That is, we do not attempt to estimate household utility functions and then maximize aggregate utility directly during each market period. Instead, in the demand component of the model we use demand functions for different types of dwelling units.

The derivation of demand functions from utility functions, prices, and incomes is a well-developed aspect of economic theory and has been dealt with by many authors.[1] Demand functions have been used because they derive from utility-maximizing decisions of households, yet are amenable to empirical estimation since their arguments—prices, quantities, and incomes—are subject to observation. Household utility, on the other hand, is difficult to measure and deal with empirically without resorting to strong assumptions about the form of utility functions or the constancy of utility within classes of households. Furthermore, since prices are used as intermediaries between the supply and demand sides of the market, a demand approach in which prices are incorporated directly is preferable in terms of model consistency.

On the supply side there is a similar choice between directly representing the profit-maximizing behavior of firms and estimating aggregative supply functions of housing suppliers. However, unlike the theory of the consumer—which is based on the maximization of an inherently unobservable quantity, consumer utility—the theory of the firm deals with the maximization of a measurable quantity, profits. Therefore, profit maximization can be represented directly. Firms are assumed to produce housing at the most profitable location and to use the most efficient least-cost technologies. The profitability of supply activities is determined in the NBER model by comparison

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[1] This point is covered by most intermediate-level textbooks on theory. For example, see Henderson and Quandt, *Microeconomic Theory*, p. 20.

of the expected prices of producing units of various kinds at each location with exogenous construction and transformation costs.

A major obstacle to the formulation and implementation of a true market model has been the unavailability of a suitable algorithm which clears the market and produces market prices for use as determinants in both the demand and supply relationships. Prices can be expressed as a function of proxy variables such as population growth or excess demand, but the proxy variables must then be forecast over time and the prices derived from them. With such a procedure it is generally more efficient to use the proxy variables themselves as intermediaries rather than derive secondary variables from them.[2] An alternative approach is to develop an algorithm which replicates the market process and produces market prices directly. The latter approach is employed in the NBER model. Specifically, market prices are based on the shadow prices or dual variables obtained from a series of linear programming solutions.

Structure of the NBER Model

There are three major components to this housing market model: a demand sector, a supply sector, and a price formation sector. These are combined in a recursive structure which replicates the operations of the housing market during each time period simulated by the model. Although the over-all model can be conceptualized rather easily in terms of these three sectors, the structure of the model is somewhat more complicated. The model is composed of seven major submodels, each of which represents a component of one or more of the three sectors.

In the course of a simulated time period each of the seven submodels is encountered sequentially in a recursive structure. The first submodel transforms exogenous changes in employment levels by workplace location and industry into estimates of changes in the socioeconomic and demographic composition of the labor force at each workplace. These estimates are used in the movers submodel to augment the number of moving households and in the vacancy

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[2] Harris, "Uses of Theory."

submodel to generate the vacancies in the housing stock which moving households will leave behind. Locating households are then allocated to dwelling unit types by means of demand equations in the demand allocation submodel.

In the filtering submodel the distribution of available housing units in each period is modified in response to the relative profitability of different maintenance strategies. In response to the demand forecasts provided by the demand allocation submodel, the supply submodel further modifies the stock of available units by carrying out structural modifications and transformations on existing units and by constructing new units. In the seventh and final submodel, the market-clearing submodel, demanders of housing are assigned to available units within each submarket and revisions are made in expected prices for the next simulated time period. A simple block diagram of the seven submodels is shown in Figure 3.1. In Figure 3.2 a diagram is shown in which the submodels are partitioned among the demand, supply, and price formation sectors.

To represent a household's locational decision, four major dimensions are incorporated into the NBER model. First, the model includes several employment locations or workplaces. These can be thought of as zones within the metropolitan area and are represented by the subscript *J*. Second, the model classifies households according to several household characteristics, such as income level, family size, educational level, and age. The household classes are designated by the subscript *H*. Third, the model defines several housing submarkets within which a discrete housing type is bought and sold. Within a submarket, dwelling units are assumed to be homogeneous in all respects except location. These housing types, defined in terms of structural type, lot size, dwelling unit size, and dwelling quality, are denoted by the subscript *K*. Finally, residence location in the model is represented by contiguous residence zones or areas, which are denoted by the subscript *I*.

A complete model of residence choice would have a household, *H*, choose a three-dimensional combination which included a workplace, *J*; a housing type, *K*; and a residence location, *I* so as to maximize its utility. A model of this kind would require a detailed and interactive representation of both the housing and labor markets.

Figure 3.1
Block Diagram of Submodels
as Encountered in the Model

EMPLOYMENT LOCATION SUBMODEL

Revise level and composition of employment at each workplace and by each of 9 industry types. Translate employment changes by industry to changes in employee characteristics.

MOVERS SUBMODEL

Generate households vacating housing units, and modify them to produce households seeking housing this period.

VACANCY SUBMODEL

Generate vacancies in housing stock created by intermetropolitan moves, outmigration, and household dissolution by residence zone and house type.

DEMAND ALLOCATION SUBMODEL

Combine transportation costs from work zones to residence zones with expected housing prices to form gross housing prices. Form expected gross housing prices by workplace for each housing type. Allocate households to housing types with demand equations and expected gross prices.

FILTERING SUBMODEL

Change quality classification of available housing stock according to quality premiums derived from expected prices and exogenous maintenance costs.

SUPPLY SUBMODEL

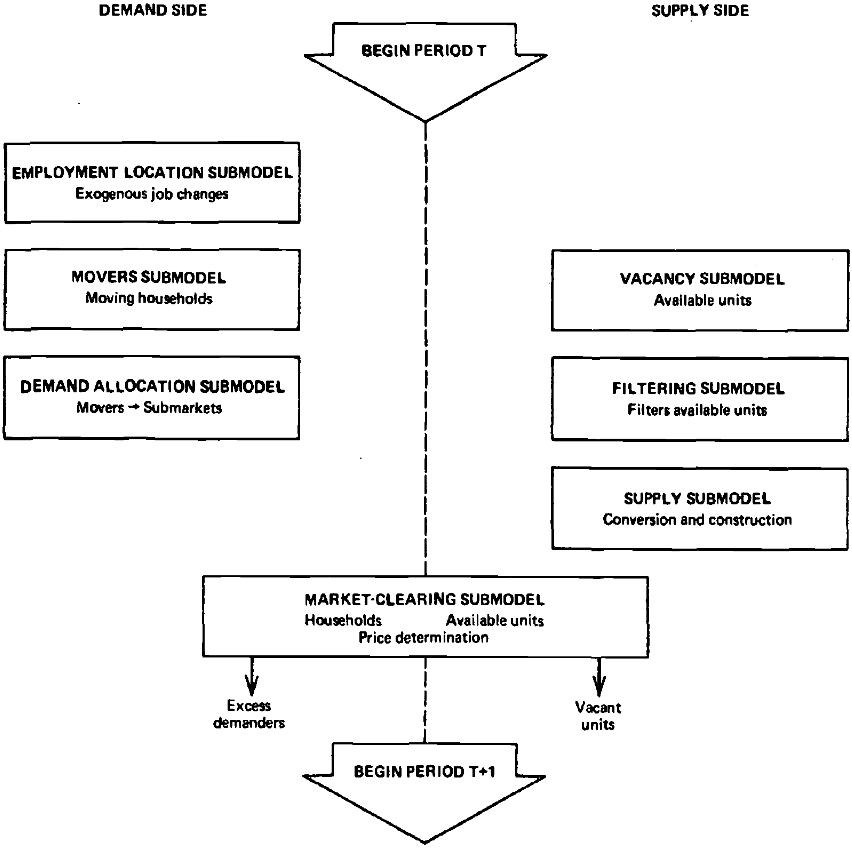
Calculate profitability of construction and transformation activities from expected prices and exogenous building costs. Perform stock transformation according to profit levels and several constraints.

MARKET-CLEARING SUBMODEL

Match moving households to available units of the type chosen by households in the demand allocation submodel. Each house type or submarket is solved separately. Shadow prices are used to generate prices for the next time period. Work trip patterns are updated.

In contrast, the NBER Urban Simulation Model represents the operations of the metropolitan labor market in a limited manner and allows relatively few interactions between the labor and housing markets. The model does simulate some kinds of job change as well

Figure 3.2
Sequence of Submodels Classified by Sector



as alterations in the composition of employment at workplaces, but it describes these processes in a reduced form manner. The Detroit Prototype includes only households with employed heads, and the workplaces of household heads are determined before the households participate in the housing market. The participation of these households in the housing market is then represented in detail. In the NBER model, therefore, households described by household class, *H,* and workplace, *J,* choose a two-dimensional combination that includes a housing type, *K,* and a residence location, *I.*

The model does not obtain a long-run static equilibrium during each market period. Instead it produces an adjustment to a position closer to the current underlying equilibrium. During each time period only a portion of all households changes residence. These households are chosen by the model as movers during a given market period. The moving households are then assigned to new housing types and/or new locations, but only that portion of the total stock which is currently vacant or newly constructed is available to them. Although moving households are located in a manner consistent with utility maximization, they will not usually end up in a long-run-equilibrium position, since they are constrained to locate in available units. However, relocating gives these households an opportunity to move closer to their equilibrium position.

Our view in the model is that long-run equilibrium may never be attained in a metropolitan housing market. Over time, as the characteristics of the population change, as employment locations shift, as the transport system is modified, as new building technology is developed, and as real incomes rise, the equilibrium position keeps shifting in response to these forces and many others. Thus the housing market is perpetually chasing a moving target and is constantly in disequilibrium. In the NBER model we view the adjustment process as being carried out at the margin during each market period. Incremental additions to the housing stock each period, the transformations of the standing stock, and the housing choices made by the subset of households seeking residences during a market period are the vehicles of adjustment.

In describing the theoretical features and difficulties of the model, we now discuss the three major sectors of the model in turn. More detailed discussions of the seven submodels and their interactions are presented in chapters 6 and 7.

The Demand Sector

The demand sector of the market model takes as given during any market period an expected price for each dwelling unit type in each residence zone and the travel cost from each work zone to each residence zone. As is described later in this chapter, these expected prices are based upon the historical price experience for each housing type in each zone. Households seeking new units during the market period consist of intrametropolitan movers, new households, and in-migrants, all of whom are identified by their workplace and

Table 3.1
Annual Rates of Intrametropolitan Mobility by Tenure, Age, and Job Change

	Number of Movers		Rates	
Tenure and Age of Head	No Job Change[a]	Job Change[b]	No Job Change[a]	Job Change[b]
Renters				
30 yrs. or less	657	304	.408	.526
31-60	863	257	.232	.423
60+	106	13	.102	.228
Owners				
30 yrs. or less	56	26	.076	.140
31-60	121	521	.050	.089
60+	69	8	.025	.041

Source: Brown and Kain, "Moving Behavior"; compiled from data in Table 11, which were derived from Bay Area Transportation Study Commission (BATSC) survey.

a. Number of movers and moving rates during one-year period.

b. Job change during twelve months before or six months after mid-point of year of move.

household class. Intrametropolitan movers form the largest group of locating households and are generated endogenously.

Two main reasons for a household move are recognized within the model. First, a household may alter its type of unit in response to a change in income, family size, or family composition; and second, a household may move in response to a change in workplace location that alters its workplace or permits it to acquire a preferred type of housing at lower cost.

The tabulations of mobility rates of San Francisco owner and renter heads of household of various ages, shown in Table 3.1, give some indication of the relative importance of job-related and demographic-related motivations for moving. For the period 1955–65, 72 per cent of all households seeking new residences in San Francisco were intrametropolitan movers. Of the remaining 28 per cent, 25 per cent were in-migrants, most of whom presumably moved for job-related reasons, and 3 per cent were new households.[3]

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[3] Econometric models of moving behavior estimated by H. James Brown provide a clearer statement of the relative effects of various motivations for moving. See Brown, "Changes in Workplace."

Both job-related and demographic-related reasons for moving are reflected in the model. However, the moving algorithm is not designed to replicate the decision-making calculus of individual households, that is, moving costs are not weighed against benefits in determining whether households will move this period or not. Instead the algorithm uses moving rates by household class to determine the number of relocating households during each market period. These moving rates, which vary by household income, family size, age of head, and education of head, reflect many basic determinants of moving behavior. Moving rates are subsequently altered to accommodate changes in the level and composition of employment at specific workplaces. Intrametropolitan movers are supplemented by new households and in-migrants to obtain the total number of households seeking dwelling units during a market period.

After the locating households have been selected, the demand sector allocates households, described by household class and workplace, among available dwelling units, described by unit type and location. These allocations are based upon workplace and income-class–specific housing prices and travel costs.

Since the expected prices of dwelling units are indexed by type and location, the price information defines a price surface over residential zones for each housing submarket. Each price surface may have a different shape, and the relative prices of housing types may vary over residence zones. For households seeking dwelling units, however, the surfaces of expected housing prices do not contain all of the information they need to choose a dwelling unit. The household must also consider the travel costs it will incur if it resides in each possible residence location. These travel costs include the cost of work trips, shopping trips, and social and recreational trips. Because retail and recreational establishments are widely distributed throughout the area, the travel costs of shopping and recreational trips may not vary significantly among residential locations. But the workplace of the head of a household is fixed, and journey-to-work costs will vary systematically with a household's residential location. The relevant constellation of prices which the household should consider when determining its residential choice is the total expenditure required for consuming each housing type in each location. This total expenditure, hereafter termed the "gross price," is the sum of a unit's market price and the household's travel costs for the specific residential location.

In effect, each household constructs a travel cost surface composed of its travel costs if it located in each residence zone, and adds this travel cost surface to each of the market price surfaces described previously. This addition produces gross price surfaces over residence zones which are indexed by housing type, household class, and household workplace zone. Although the total number of price surfaces is quite large, a given household must still consider only one surface for each type of housing.

The problem at this point is to devise an operational procedure wherein the household makes a residence choice using the information in these gross price surfaces. One possibility would be to assume that each housing type in each residence location is a separate good. Following this assumption, demand functions could then be estimated for each good defined by both housing type and location. Unfortunately, if the market model has sufficient housing and locational detail to be interesting, data requirements make this procedure infeasible. For instance, if there are 27 types of dwelling unit and 44 residence zones, as in the Detroit Prototype, there will be 1,188 possible "goods" and, therefore, 1,188 demand equations to be estimated per household class. Clearly the unit choice must be separated from the location choice.

The approach used in this model has been to carry out the demand determination in two steps. Households first select a housing type, and then they select a location or residence zone. The selection could be replicated in the reverse order, with locations being picked before housing types, but there are many reasons for stratifying dwelling units into submarkets rather than stratifying locations into subregions.

Locations are inherently less discrete than dwelling units. There is, for instance, a natural stratification of housing types by the number of units per structure. Thus there are single-family structures, duplexes, row houses, garden apartments, three- to five-story walk-up apartments, and high-rise apartment buildings. Locations can similarly be stratified by political boundaries, but the differences among adjacent boroughs, townships, or counties are often not as significant as the differences among structural types.

A second reason for treating housing types as the major object of household choice is that an operational model will probably have fewer dwelling unit types than residence zones. In recent transportation studies it is not uncommon to use more than a thousand zones, but twenty to forty housing types would probably be sufficient for most purposes. This means that the data requirements are more reasonable if housing types rather than residence zones are used as dependent variables in demand functions.

The above reasoning has undoubtedly occurred to other persons attempting to simulate residence location and housing choices, because earlier models have generally assigned households to housing types before assigning them to locations. The assignments or matching procedures used in these models have had little economic content, however; and entire household classes are often assigned to individual housing types rather than distributed over them.[4] Such matching procedures become untenable when used in models which forecast metropolitan development over periods of twenty or thirty years. Over these longer periods it is unrealistic to assume that the relative prices of various types of dwelling units are invariant, an assumption which is implicit in a static assignment rule.

In order to avoid such pitfalls in assigning households to housing types, we use traditional demand functions which are econometrically estimated. But the assignment is not carried out on a one-to-one basis with each household class choosing only one dwelling unit type. Instead, probabilistic demand functions are used to generate distributions of households over the several housing types. The independent variables in these demand functions are the relative gross prices of the different housing types.

It is, however, difficult to use price surfaces as the independent variables in demand functions; the information in these surfaces must be summarized. Two approaches recommend themselves as possibilities. First, a household with a given workplace could survey its gross price surface for each dwelling unit type and select the lowest point on each surface as the representative price for each housing type. Alternatively, the household could take a weighted average over each surface, where the weights would reflect factors

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[4] Harris, "Quantitative Models," pp. 396–97.

such as the probability of the household's choosing a given location. The first procedure is theoretically the correct one, but it would be prohibitively expensive in practice.

In the Detroit Prototype a two-part weighted average of the price surface is used to form the expected gross price of each type of housing for each workplace. The first part of the weight reflects the distribution of the housing stock; that is, the price in each zone is weighted by the proportion of dwelling units of the given type which are in the zone. This reflects the likelihood that a household will end up in a given zone and suppresses zones which have no units of the appropriate type. Represented in the second part of the weight is the distribution of work trips from the place of work of the household head. The price of each unit type is weighted by the proportion of work trips to the head's work zone which originate in each residence zone. This component reflects past choices made by workers in the household head's work zone and is a proxy for information flow in the housing market. This formulation and particularly the weighting function used are somewhat arbitrary. How best to summarize these gross price surfaces remains very much an open question and is the subject of ongoing research.[5]

The average gross prices generated by the two-part weighting scheme are then used as variables in demand functions which allocate households to dwelling unit types. Because of differences in travel costs, these gross prices will vary by workplace across the metropolitan area. A worker will probably find, for example, that single-family units on large lots are more expensive relative to apartments when he works in the central business district than when he works in the suburban fringe.

Evidence has been available for some time that the housing choices of households vary systematically as the location of employment of

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[5] Further evaluation of this question is an important part of a re-estimation of the submarket demand equations being carried out by the authors for a new version of the NBER model, Pittsburgh II. A much more ambitious attack on the problem of summarizing price surfaces has been mounted by John Quigley. Using the Pittsburgh data base Quigley has generated distributions of observed household choices over gross price surfaces and determined that the majority of households in each submarket reside in locations with minimum gross prices as viewed from their workplaces. See Quigley, "Residential Location."

the household head is shifted away from the central city.[6] Figure 3.3 summarizes some of this evidence. The curves labeled, "Chicago mean" and "Detroit mean," show the mean proportions of white workers employed in each of six workplace rings in Detroit and eight workplace rings in Chicago who reside in single-family units. The comparisons are limited to white workers because racial discrimination distorts the housing choices of black households.[7] These workplace rings are rough semicircles around the central business district, and the proportions are graphed at the ring midpoints. In Detroit, the proportion of white workers residing in single-family units varies from 57.7 per cent for workers employed in the CBD to 79 per cent for workers employed in Detroit's fifth workplace ring, an average of eleven miles from the CBD. In Chicago, the differences in the proportion residing in single-family units by workplace ring are even larger: 36 per cent of Chicago CBD workers reside in single-family units, in contrast to 69 per cent of workers employed in Chicago's eighth workplace ring, an average of twenty-two miles from the CBD. Lower proportions of households choose single-family units in Chicago because the gross price of single-family units relative to other housing types is much higher in Chicago than in Detroit. Similarly, the larger differences reflect the wider variation in gross prices.

The third line in Figure 3.3, labeled "Detroit standardized," summarizes the findings of an econometric model designed to explain the proportion of Detroit workers choosing single-family units for each of 252 workplaces. The explanatory variables used to estimate the proportion of white workers choosing one-family units include family size, sex, number of employed workers, income, and a proxy for variations in relative gross prices. The results are displayed on the assumption that workers at all remaining workplaces have the

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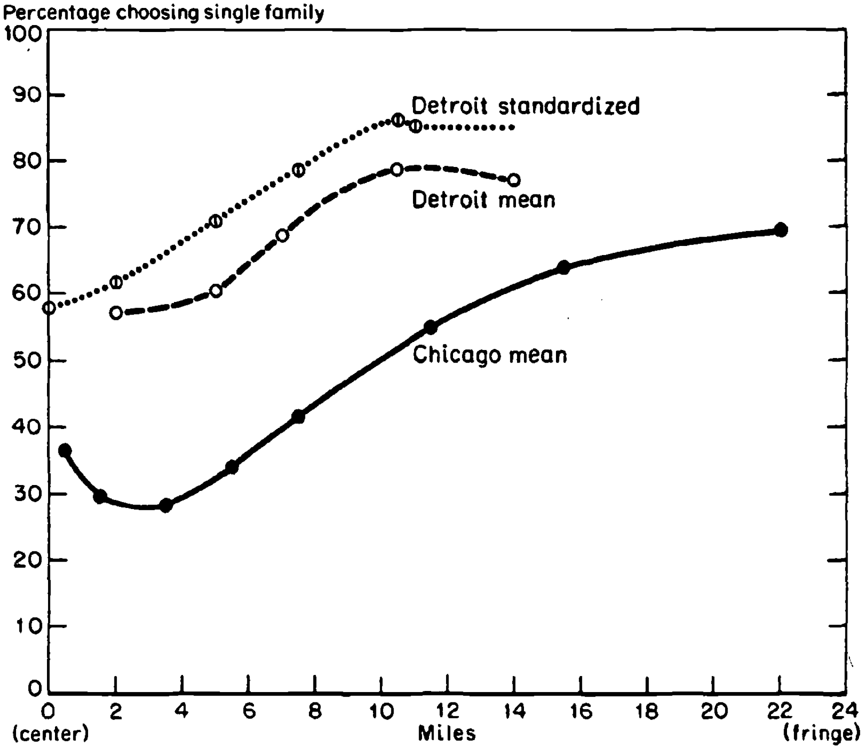
[6] See Kain, "Journey-to-Work," pp. 137–60, and especially Table 12, p. 158; Meyer, Kain, and Wohl, *Urban Transportation Problem*, pp. 119–30; Kain, "Contribution to Urban Transportation Debate," pp. 55–65; Straszheim, "Demand for Housing Services," especially Table 1; and idem, "An Econometric Analysis."

[7] An extensive analysis of the effects of racial discrimination on the housing choices of black households is presented in Kain and Quigley, "Discrimination and a Heterogeneous Housing Stock." See also Meyer, Kain, and Wohl, *Urban Transportation Problem*, Chap. 7; Kain, "Commuting Decisions."

Figure 3.3
Proportion of White Workers Choosing
Single-Family Units, Classified by Workplace Ring



Source: Meyer, Kain, and Wohl, *Urban Transportation Problem*, tables 42 and 43.

same characteristics as those employed in the CBD. This standardization of labor force characteristics indicates that the gross price (workplace) effects are even larger than the simple proportions indicate.

Since the submarket demand functions employed in the NBER model include gross prices by housing type, the demand sector of the model is designed to generate distributions of households over housing types that vary by the workplace of the household head. The central hypothesis in the demand sector is that much of the workplace-specific variation in residential choice is caused by workplace-specific variations in the relative gross prices of different types of dwelling units.[8] Both the empirical estimation of the demand functions used in the demand sector, described in Chapter 8, and the design of the demand sector rest on this hypothesis.

After households are distributed to types of dwelling units by the econometrically estimated demand functions, they must be assigned to locations or residence zones. Within a given housing submarket or dwelling unit type, travel cost is the only attribute which distinguishes locations. Within each submarket, therefore, households are located so as to minimize their aggregate travel costs. These travel costs arise principally from the work-trip requirements of each household, although residence zone–specific travel costs could be included. It must be stressed that this solution does not imply an aggregate travel cost minimization for all households, since it is done separately for each dwelling unit type. Formally, the transport cost minimization is a Hitchcock problem in linear programming and is capable of rapid solution.

Travel costs vary by income class because the value of time varies with income.[9] Therefore, travel cost minimization within a submarket implies that higher-income groups will in effect outbid lower-income groups for more accessible locations. However, high-income workers have longer work trips on average than low-income workers employed at the same workplace because high-income groups tend to choose high-quality, low-density dwelling units, and these are at the periphery of the metropolitan area.[10] But within any given housing submarket high-income groups occupy the most accessible locations. For example, while investigating household residence choice, Hoover and Vernon discerned ". . . a rough tendency for families with higher income to be closer to the center of the Region for any given level of density."[11]

In addition, travel costs on the work trip will vary by mode, and modal choice will be influenced by the value of time. Although

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[8] A diagrammatic representation and early empirical test of this model are presented in Kain, "Journey-to-Work."

[9] Becker, "Allocation of Time."

[10] Hoover and Vernon, *Anatomy of a Metropolis*, pp. 159–61; Meyer, Kain, and Wohl, *Urban Transportation Problem*, pp. 140–43; Kain, "Journey-to-Work."

[11] Hoover and Vernon, *Anatomy of a Metropolis*, p. 161.

high-income workers will commute longer distances to consume housing, they will tend to choose higher-cost and faster modes than low-income workers. The procedure used to assign households to residence locations makes the modal choice for the work trip a concomitant of the household's choice of both housing type and residence location, since travel costs by mode affect the magnitude of gross prices as well as the assignment to residence zones.

Before turning to the supply sector of the market model, it is revealing to contrast the treatment of household demand in the NBER model with the treatment of the same problem in monocentric equilibrium models. The monocentric models cloud the order of causation and hierarchy of choice which households face in the housing market. At equilibrium in monocentric models, residential densities decline uniformly with distance from the center so that a given housing type or density is available at one location or in one ring about the center. In this situation only the first part of the demand sector is necessary. Once a household chooses a type of dwelling unit, it has in effect chosen a location as well. Since locations and densities are perfectly correlated in monocentric models, it makes no sense to separate them as dimensions of household choice. The monocentric metropolitan area thus becomes a special case of the demand sector described here. However, in this demand sector the household's trade-off between travel costs, house prices, and quantities of housing is implicitly represented in the demand functions rather than explicitly reproduced in a utility-maximizing framework.

The Supply Sector

The supply sector of the housing market model simulates two major aspects of supply activity or housing stock adjustment. The first encompasses both new construction and those transformations of the existing stock which alter the structural characteristics of a dwelling unit. Included in this latter category are modifications which enlarge an existing dwelling unit or change it from single-family to multiple-family occupancy, and transformations which involve demolition and reconstruction on the site. The second supply activity simulated is the change in the physical quality of units. This quality change, called "filtering" in the over-all model, does not alter any other attributes of the dwelling unit except its quality level. Units can improve in quality (filter up) because of rehabilitation and maintenance or decline in quality (filter down) because of undermaintenance or disinvestment. Filtering is handled separately from other supply activities, because it involves disinvestment as well as investment. Therefore, it will be described after the main supply activities.

The supply side of the housing market, the residential construction industry, conforms rather well to the economist's image of the ideal competitive industry. For example, there are many small firms in the industry, and even the largest firms produce only a small share of the total output. Because of its low capital requirements, moreover, exit from and entry into the industry are relatively easy. In addition, the labor force in the residential construction industry is highly mobile.[12] As a result, serious bottlenecks and discontinuities should not be prevalent, and the competitive model of the firm, wherein the firm is a price taker and maximizes its profits, should represent the behavior of the supply side of the housing market adequately.

Since the housing stock and housing prices are classified by housing type and residence zone in the NBER model, a disaggregated representation on the supply side is easy to implement. Exogenous to the supply side are expected housing prices in the current period, the cost of performing the various supply activities, zoning constraints which prohibit certain dwelling unit types in some zones, the amount of available input of land and other units, and a forecast of demand for the current market period. Subject to these data and constraints, builders will perform those activities which maximize their profits during the period.

Because the housing stock is disaggregated by type of housing, the supply sector employs an input-output format which specifies the cost of transforming land or existing units into any of the housing types. Supply activities which use vacant land as an input are designated as new construction, while supply activities which use an existing unit as an input are referred to as a transformation of the stock. The cost of carrying out construction or transformation activities is summarized in an array of supply costs which embodies a given

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[12] Muth, "Demand for Non-Farm Housing," p. 44.

technology and fixed factor costs. Since the array of expected prices by housing type and location is an input to the supply sector, the expected profit or loss from each supply activity in each residence zone is calculated by subtracting the cost of the supply activity from the expected selling price of the new or transformed housing unit. Those supply activities that are profitable and satisfy supply constraints are carried out by the supply sector.

Three constraints are placed on supply activities. The first is an exogenous zoning constraint. Zoning is a major policy variable in the supply sector and prohibits certain output types in some zones. For example, in some residence zones, the construction of multiple-family structures may be prohibited or construction of single-family units be permitted only on lots which exceed stipulated minimum sizes.

A second constraint, on available inputs, prevents an unreasonable amount of activity in any zone in each year by allowing only a portion of each zone's vacant land and dwelling units to be available to the construction industry during each market period. These constraints crudely represent a variety of short-term dynamics that limit the scale of activity in any part of the market in a short period. Dwelling units that are available for transformations include units which have been standing vacant for at least one market period and units which are to be vacated in the current market period by relocating households. Recently vacated units are specified by the vacancy submodel. In any period the amount of vacant land available to suppliers is an exogenously specified proportion of the total vacant land in each zone.

The final constraint prohibits suppliers of housing from exceeding the total forecast demand for each type of housing over the entire metropolitan area. There is, however, no requirement that forecast demand be satisfied for each type of housing in each market period. This aggregate demand constraint is provided by the demand sector, where households are allocated to specific types of housing. Total demand for each unit type is calculated and augmented by a normal vacancy rate for each type of housing. The difference between this demand forecast by housing type and the number of available units by type is the demand constraint for each type of housing.

The form of the demand constraint derives from the sensitivity of housing suppliers to vacancies and, therefore, forecast demand.

If housing suppliers supply more units of a given type than are demanded, the vacancy rate of that housing type rises above its normal level, and suppliers will find their inventory of vacant completed units larger than normal. Sherman Maisel has found that such inventory growth discourages construction activity for several reasons:

> Because of carrying costs, any lengthening of the period of sale rapidly erodes the builders' profits. Furthermore, the volume of unsold new units is controlled by limited builders' capital and by the unwillingness of lending agencies to finance additional starts when the builder has a backlog of unsold units.[13]

From his statistical results Maisel concludes that inventories are the "channel of causation" through which many factors influence supply activities, and he argues that "throughout the period [1950–62], vacancies appear to have had far more influence on starts than most observers noted."[14]

Since a profit or loss is calculated for each possible activity, and since there is a set of constraints on supply activities, the supply sector could be formulated and solved as a profit-maximizing linear programming problem. Because of the size of the problem, however, such an approach would be very costly. The Detroit Prototype has 27 types of housing unit as possible outputs and 28 possible inputs (27 types plus land) or 756 possible supply activities in each residence zone. In addition, this version of the NBER model has 44 residence zones and therefore 33,264 possible supply activities to consider each period. Although a linear programming problem of this size could be solved, the computing time required for solution would make the model extremely expensive to operate.

In order to avoid the high cost of a linear programming solution, we developed a ranking and enumeration procedure for the supply submodel that can be solved rapidly. Supply activities are first ranked by their "profit rate," a ratio of the profit amount to the total cost of each activity. This ratio is not the true profit rate, because the total cost of the activity does not equal the investment of the

[13] Maisel, "Theory of Fluctuations in Construction," p. 366.

[14] Ibid., p. 375.

builder. In using total cost as a numeraire, we assume that the risk involved, the seed money required, and the capital committed are proportional to the total cost of an activity. After profit rates are calculated, the supply activities are ranked from most profitable to least profitable, and activity levels consistent with the demand constraints by housing type are assigned to each supply activity beginning with the most profitable one. The method essentially enumerates each vertex of the feasible set of activities and assigns an activity level set by the smallest constraint. Although this approach may be inferior to a programming solution in that trade-offs between activities are not considered and an objective function is not maximized, it is many times faster. Moreover, experiments carried out on small problems indicate that the method of ranking and enumeration provides a fairly close approximation to a linear programming solution. Finally, there is considerable doubt that the linear programming solution would simulate the behavior of housing suppliers any better than the simple ranking algorithm.

Changes in the quality of the housing stock, the second major aspect of housing supply simulated in the model, are also represented in a disaggregated form. Over time the quality of a dwelling unit may be maintained at its original standard or even upgraded by successive owners. Alternatively, a unit may be allowed to decrease in quality and price from its original level, and, as its price declines, the unit may contain households of successively lower economic strata. These changes in the quality of a dwelling unit relative to its earlier level, termed "filtering," result from maintenance and renovation decisions of homeowners and landlords. For this entire discussion of filtering we use the terms "price" and "value" interchangeably to refer to discounted streams of net revenue or the current market value (selling price) of the unit. This is, of course, distinct from the annual payment or rent for the services provided by the unit. Since the services provided by a unit in subsequent years depend on maintenance or investment decisions, current prices of housing capital embody some assumed investment policy, presumably an optimal or conventional one.

In housing market literature there has been much confusion about the precise definition of filtering. The term has been used to refer to a change in the relative price of housing units, to a change in the relative incomes of the occupants of dwelling units, and to an absolute decline in physical condition or quality.[15]

In the NBER model filtering refers to a change in the physical condition of the housing unit. Quality is envisioned as a separable objective characteristic of a dwelling unit that is as observable or measurable as would be the number of rooms in a unit.[16] Unit quality is determined, therefore, by inspecting the unit and scoring it according to some specified standard of quality. For instance, the Bureau of the Census used observable criteria to determine the condition or quality of a unit in its classification of units as sound, deteriorating, or dilapidated. This classification is based upon a range of slight, intermediate, and critical defects that are noted by Census enumerators.[17]

Quality defined in terms of physical condition stems mainly from the maintenance and rehabilitation experience of the dwelling unit over time. To maintain a unit at a particular quality level requires regular outlays for repairs by the owner. Higher outlays can increase the quality of the unit, while outlays below a certain minimum level will eventually cause a perceptible decline in quality. Within the NBER model, the filtering mechanism simulates maintenance decisions for the supply of available units in each period, roughly 20 per cent of the entire housing stock in each year. In an earlier version of the submodel, we attempted to filter the entire stock in each period. However, this procedure produced serious imbalances and bookkeeping problems and was abandoned.

Although maintenance is a less spectacular supply activity than structural conversion or new construction, over time it has perhaps as great an impact upon the housing stock as either of these other activities. In addition, by explicitly representing quality change in the NBER model, we gave it the potential for investigating housing

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[15] Grigsby, *Housing Markets.* In Chapter 3 of this book the author explores several definitions of filtering found in the literature. The definition which is closest to the one in this model is presented by Ira Lowry, who proposed using the percentile rank of a unit's price in the price distribution of all units as the definition of a unit's status. See Lowry, "Filtering."

[16] This view of quality receives support in a recent paper by Kain and Quigley, "Measuring Housing Quality." Their forthcoming monograph provides more evidence on this question; see Kain and Quigley, "Discrimination and a Heterogeneous Housing Stock."

[17] *Census of Housing, U.S. Summary, 1960,* Vol. HC (1), p. LXII.

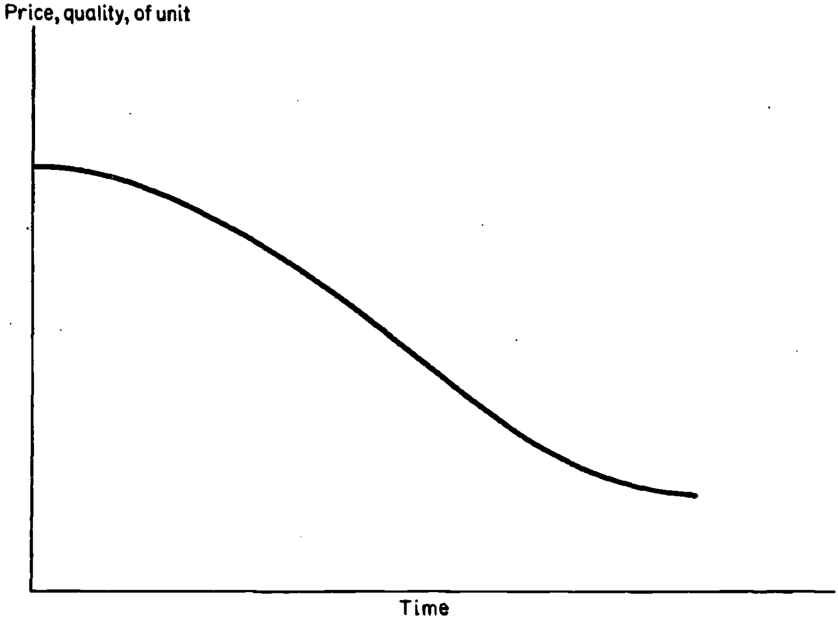
market problems such as stock deterioration, blight, and the phenomenon of abandonment.

Since changes in the quality of the housing stock are assumed to result from investment decisions, the representation of these processes in the model derives from investment theory. When a unit is first constructed, it generally meets a high standard and is in good repair. If no money is spent thereafter to maintain its condition, its price and quality will fall over time. But outlays for maintenance, renovation, and repair at any given time during its life will improve its quality and in most cases will increase the monthly rent the landlord can charge or will increase its market value. How much more the landlord or seller can obtain for the property will depend on neighborhood-specific demand conditions. There is, of course, no requirement that the increase in value be as large as the cost of the improvement. If no further repairs are made, the unit will again begin to decline in value and quality.

Figure 3.4 illustrates the decline in the value of a housing unit over time in the absence of subsequent outlays for renovation and repair. Outlays for maintenance and repair which improve the quality of the unit at discrete intervals will shift the depreciation function upward and produce the pattern of prices over time shown in Figure 3.5. These illustrations assume no change in the structure of prices by housing type and location. Although major improvements would create a saw-toothed effect such as that illustrated, Figure 3.5 undoubtedly exaggerates the discreteness of maintenance and repair outlays. Annual maintenance and repair expenditures typically produce nearly imperceptible year-to-year price discontinuities.

Simulation of maintenance and quality change in the NBER model is based on the concepts illustrated in Figure 3.5. Time is represented by market periods of one year, and maintenance decisions are made by owners and landlords at the beginning of each year. The amount of maintenance in any given period depends upon the difference between the value of the property with no maintenance and its value with different quantities of maintenance. In most situations maintenance expenditures incurred during time period T will increase the expected value of a unit in time period $T + 1$ relative to its expected value in period $T + 1$ if no maintenance expenditures are made in the current period. In terms of Figure 3.5, maintenance

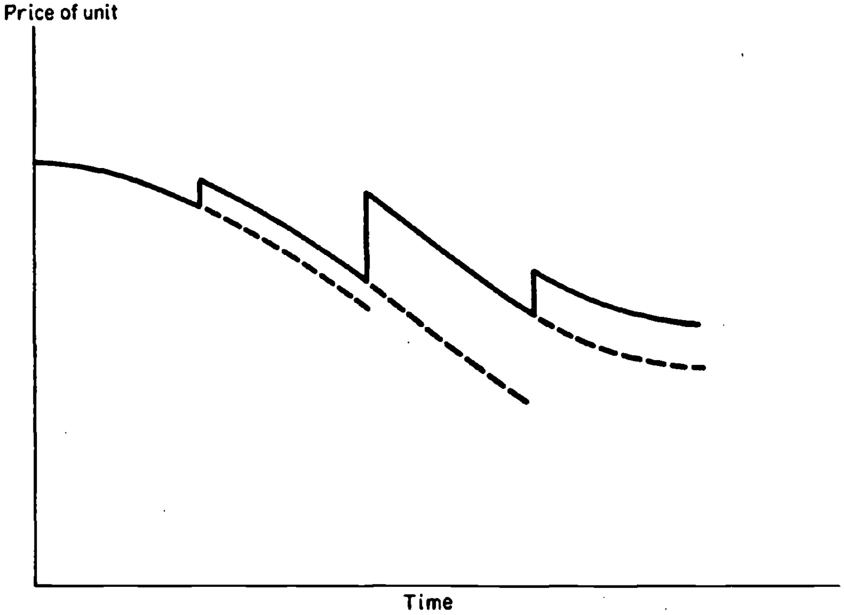
Figure 3.4
Time Path of Price and Quality for a Dwelling Unit Receiving No Maintenance



expenditures during a given time period generally will increase the unit's value and shift it to a higher depreciation function in the next period. But, if no maintenance is performed during the year, the unit's price in the next period will be its present price depreciated at some rate. The difference between this "no maintenance" price, $PNM(T+1)$, and the price of the unit if it were subject to maintenance, $P(T+1)$, is then attributable to the maintenance expenditure. A Fisherian diagram, such as Figure 3.6, may be used to derive the correct maintenance expenditure in the current period. The curve OF is the reaction function of price change when various amounts are spent on maintenance in the current period.

The opportunity cost of funds, or interest rate, R, is represented by a line, such as XY, of slope $1 + R$. By applying the usual marginal equivalencies, the correct maintenance expenditure is shown as point A, the point where the slope of XY is equal to the slope of

Figure 3.5
Time Path of Price for a Dwelling Unit Receiving
Maintenance at Discrete Intervals



OF.[18] In this case the amount of the derived maintenance expenditure will increase the expected price of the dwelling unit relative to $PNM(T+1)$, the expected price when no funds are spent on maintenance.

It should be noted, however, that the relation between the expected price of the dwelling unit in the next period, $P(T+1)$, and the unit's price this period, $P(T)$, is not straightforward. Depending on the period-to-period rate of depreciation, the reaction function of price change to maintenance, and the interest rate, $P(T+1)$ could be greater than, equal to, or less than $P(T)$. That is, the upward shift in the depreciation function caused by maintenance might not be great enough to raise the unit's price in the next period above its price in the current period. Thus funds can be expended on maintenance and

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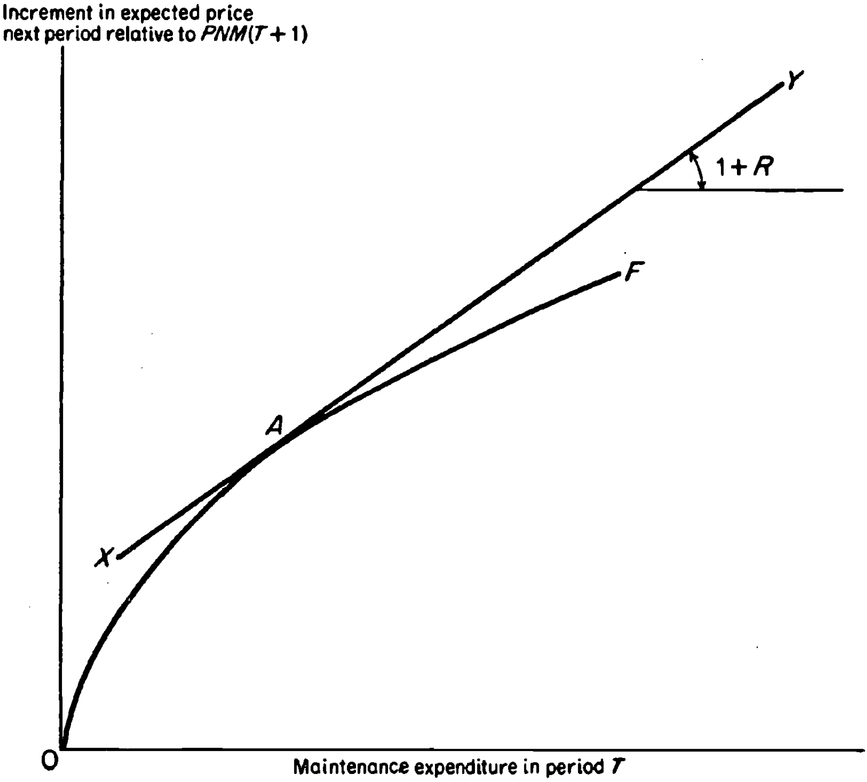
[18] Fisher, *Theory of Interest,* Chap. 10.

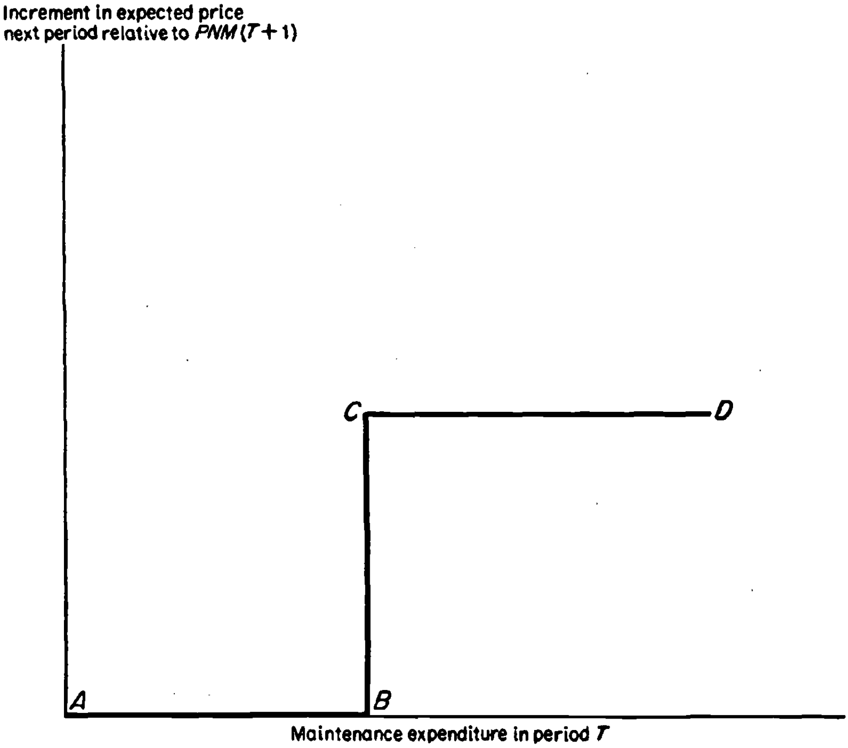
Figure 3.6
A Theoretical Maintenance Response Function



the dwelling unit may still decline in value. And this can be the case even when maintenance expenditures follow a plan which is optimal according to the theory.

It is necessary, however, to simplify the foregoing illustrative maintenance expenditure model to develop an operational representation of these aspects of behavior in the NBER model. The main reason is that in the present version of the model quality is a discrete variable used to define housing submarkets rather than a continuous variable. Therefore, between two quality levels, the reaction function for maintenance expenditures becomes the one shown in Figure 3.7. Line *AB* represents the cost of transforming a

Figure 3.7
Maintenance Response Function of the Model



unit of a given quality level to the next higher quality level, and line *BC* represents the quality premium or the difference in the market price of units differing only in quality.

The fact that opportunity costs of funds, maintenance costs, or perceptions of gains may differ between households is ignored in this representation. A probabilistic or reduced form version of the maintenance model has been adopted so that the net rate of transfer between quality levels is a function of the ratio of the quality premium, *BC*, to the transformation cost, *AB*. When the ratio of *BC* to *AB* exceeds unity, dwelling units are upgraded to the higher quality level. Conversely, when the ratio is less than unity, dwelling units are downgraded from the higher level.

The mechanism producing quality change in the filtering submodel assumes, therefore, that the maintenance decision of owners is a marginal one. The price *differences* which arise from quality change are the determinants of maintenance and, therefore, of dwelling unit quality in this model. This approach can be contrasted with one suggested by Ira Lowry that made the *level* of prices the determinant of maintenance policy and dwelling unit quality.[19] Lowry assumed that a landlord would follow a normal maintenance policy, keeping the quality of his building at a constant level, as long as he covered all of his expenses. When the rents on the structure fell, the landlord would respond by lowering his maintenance expenditure until he again earned a return on his investment. The lower level of maintenance would then reduce the quality of the building until a new equilibrium was reached or the building was withdrawn from the market. All of the landlord's maintenance decisions are thus based on his average costs and average revenues; it is presumed that he fails to recognize that by lowering his maintenance expenditures he may reduce his revenues even further.

In our filtering submodel, on the other hand, it is assumed that building owners invest in maintenance until their marginal costs are equal to their marginal returns. The difference between the two approaches is most obvious from an example. In a situation where the price of a dwelling unit rises because of excess demand, the implication in Lowry's model is that the dwelling unit would not filter down. But in our filtering submodel, if the increase in prices of low-quality units is large enough to reduce the quality premium for the structural type, filtering down will occur. Similarly, if prices fall within a structural type, but the magnitude of the quality premium remains large, our filtering submodel will not produce a lowering of quality whereas Lowry's model will. In those cases where the quality premium changes in proportion to the price level, however, the two models will produce similar results.

The form of the filtering mechanism in our model derives mainly from theory. The data required for testing the hypotheses embodied in this formulation are virtually nonexistent, and no effort has yet been made to verify the formulation empirically.

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[19] Lowry, "Filtering."

The Price Formation Sector

Both the demand and supply sectors of the NBER model make extensive use of the estimates of expected prices by housing type and residence zone during each market period. On the demand side, they are used to form the gross prices used in the household demand functions to allocate households to housing submarkets. On the supply side they are used to calculate profit rates for stock adjustment activities, new construction, and renovation in each residence zone. For the model to operate over time, these prices must be altered during each period in a way that reflects the decisions of households and housing suppliers. The difficulty of devising an operational, yet theoretically defensible, technique of forming prices in a dynamic context may have been the greatest single obstacle to the development of a market model of housing choice and residence location.[20] It is, therefore, not surprising that the design of the price formation sector has posed some of the most difficult theoretical problems encountered in the design and development of the NBER model.

Economists have developed two main ways of generating prices within a dynamic framework. First, a dynamic price adjustment mechanism which converges to the underlying static equilibrium can be explicitly formulated. Such adjustment mechanisms are usually keyed to the excess demand or excess supply in a market.[21] Alternatively, an optimizing technique, such as linear programming or Lagrange multipliers, can be applied to a market in each period, and the dual variables used as a basis for price formation.[22]

The NBER model resorts to the second technique, employing dual variables from linear programming solutions as the basis for price determination. The price formation algorithm is based on the locational assignment generated by the market-clearing submodel, where a linear programming, travel cost minimization solution is used in each housing submarket. Within a given submarket, the dual variables or shadow prices for each residence zone can be used to

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[20] Harris, "Uses of Theory," p. 271.

[21] For a description of five alternative adjustment mechanisms, see Samuelson, *Foundations*, pp. 263–69.

[22] Dorfman, Samuelson, and Solow, *Linear Programming and Economic Analysis*, pp. 166–86. See Lefeber, *Allocation in Space*, for a discussion of price formation in a spatial framework.

form location rents.[23] Location rents can be generated for those zones with available units of the type being considered as well as for residence zones where no units of that type are supplied during the period. This information is needed so that housing prices in all zones can be updated each period.

The shadow price formed by the linear programming solution for a given residence zone and housing type represents the change in over-all travel cost that would occur if an additional unit of that type of housing were added to that zone and if households were reassigned to minimize travel costs. Since the shadow prices change substantially if large supply adjustments are made or if the assignment problem is altered, they must be judiciously interpreted and then only in the context of an appropriate supply adjustment. For these reasons the shadow prices of several periods are used to form expected prices in the NBER model. Persistent shadow price effects are needed for them to have any significant impact on the expected prices in the model.

To transpose the zonal shadow prices for a given submarket into location rents, the dual variables are first transformed into travel savings by changing their sign. Then their magnitudes are adjusted by adding a constant so that the zone with the lowest travel savings in the submarket has a travel saving of zero. The resulting values are location rents for the housing submarket.

Location rents are then used to form one-period prices. These one-period prices, generated from the linear programming dual variables, are not the prices which influence housing demanders and suppliers. Both the supply and demand sectors of the model use expected prices for each period as intermediaries. The one-period prices act as signals which guide the adjustment of the expected prices.

Since the one-period prices are based on linear programming dual variables, they will probably vary significantly from period to period and will rarely be equal to these expected prices. Moreover, the model does not calculate a transaction price for any market period. It is probable, however, that transaction prices in period T lie within the range determined by the expected price in period T and the expected price in period $T + 1$.

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[23] Stevens, "Linear Programming and Location Rent."

Since it is assumed that all units of a given type are perfect substitutes except for location, the absolute magnitude of the differences in location rents among zones for a given type of housing should correspond to the differences in one-period submarket prices among zones.[24] Therefore, the topography of the location rent surface for a given submarket should correspond to the topography of the one-period price surface for the submarket. Forming a one-period price surface amounts to specifying the height of the location rent surface. Since the differences in one-period prices among zones are known for each type of dwelling unit from its location rent surface, specifying the one-period price of a given dwelling unit type in one residence zone also sets the height of the unit type's entire price surface. This fact is utilized to form the one-period prices for each submarket or type of housing.

If there is excess capacity within a submarket, i.e., the number of available units exceeds the number of households, the excess capacity will be found in the zone or zones where location rents are zero. These are the zones in which the marginal units in each submarket are supplied. This assumes that suppliers of available units in the marginal zones break even in whichever supply activity proves feasible and is least-cost. The sum of the cost of the least-cost activity and the expected price of the existing housing units or land used as inputs in the transformation is defined as the one-period price of units in the marginal zone. One-period prices for units in other zones are obtained by adding their respective location rents to the one-period price in the marginal zone. The price formation sector also determines the one-period price of land in each zone. In the Detroit Prototype this land price is the average of the location rents of the housing types which are present in a zone. This averaging process is but one of several alternative techniques that could have been used to form land prices. The choice among them is hard to make on theoretical grounds, since many aspects of the dynamics of the land market are implicitly represented. Fortunately, alternative formulations are easily substituted in the model, and we plan to experiment with other averaging functions in the future.

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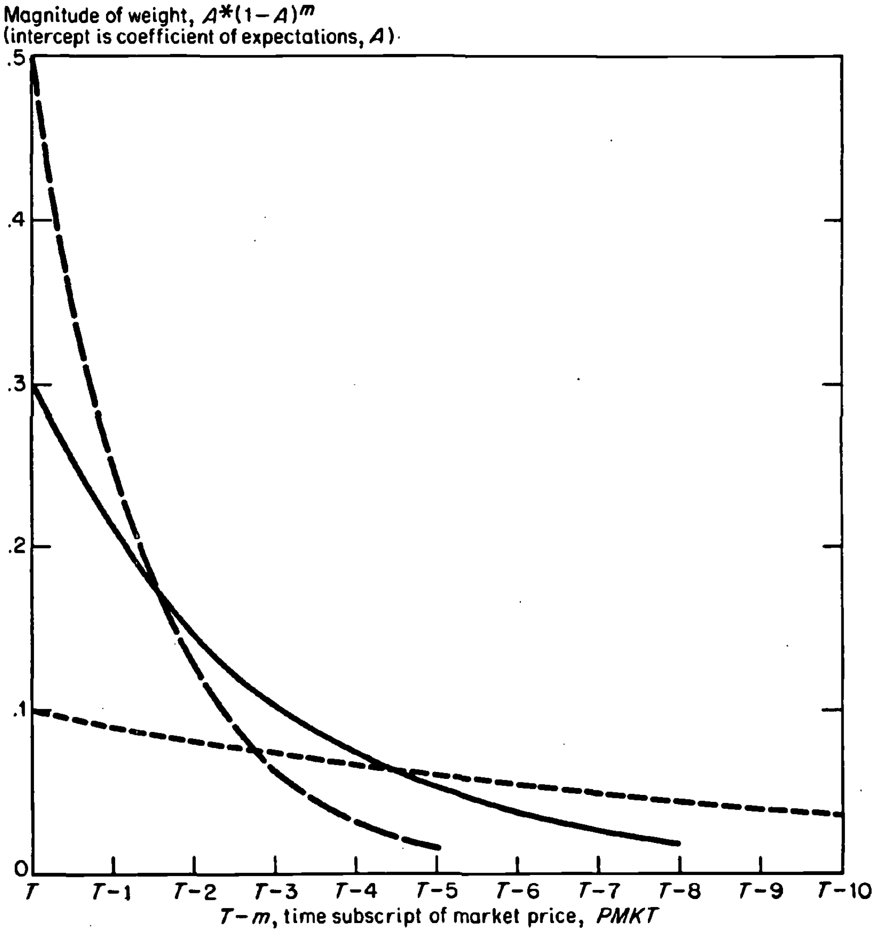
[24] This assumption implies that determinants of price other than location, such as local public service levels, school quality, and air quality, are represented as components of the dwelling unit types.

Figure 3.8
Formulation of Lag of Adaptive Expectations



Note: Expected prices for next period are an average of this period's market price, *PMKT,* and last period's expected price, *PXPCT*:

$$PXPCT(T+1) = A * PMKT(T) + (1-A) * PXPCT(T).$$

This can be transposed to the expectational form:

$$PXPCT(T+1) = PXPCT(T) + A * [PMKT(T) - PXPCT(T)]$$

where A is the coefficient of expectation and $1-A$ is Koyck's λ. The underlying lag is then:

$$\begin{aligned} PXPCT(T+1) = {} & A * PMKT(T) + A * (1-A) * PMKT(T-1) \\ & + A * (1-A)^2 * PMKT(T-2) + \cdots \\ & + A * (1-A)^m * PMKT(T-m) + \ldots. \end{aligned}$$

The one-period prices formed each period from location rents and construction costs are then used to calculate expected prices for the next time period. These expected prices are a weighted average of the one-period prices and the expected prices for the current period. If *PMKT* are the one-period prices, *PXPCT* the expected prices, and *T* a time subscript, then

$$PXPCT(T+1) = A * PXPCT(T) + (1-A) * PMKT(T).$$

The pattern of weights can be expanded by substituting the previous period's expression for the expected price until the weighting system is revealed to be a Koyck lag as shown in Figure 3.8. Thus the expected price in any given period is a weighted average of one-period prices of past periods. This weighting system is an application of the familiar adaptive expectations model where *A* is the coefficient of expectations.[25] Figure 3.8 illustrates how three different values of this coefficient alter the shape of the weighting function. Again, the selection of the proper lag will require considerable experimentation with the entire model.

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[25] Goldberger, *Econometric Theory*, p. 276.