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CHAPTER 5

The Basic Measures of Cyclical Behavior

IN THIS chapter we explain in greater detail than in Chapter 2 our methods of measuring specific-cycle and reference-cycle behavior. The calculations are described in full for one series, the monthly records of bituminous coal production in the United States (Table 13). This activity conforms closely to business cycles but exhibits a few irregularities not encountered in the coke illustration given in Chapter 2. We also draw on other series to illustrate different steps in the procedure and the results obtained in practice. The bulk of this chapter is devoted to methods of handling monthly data, but Section XI shows how the methods can be adapted to quarterly and annual records.

I Positive and Inverted Specific Cycles

A large majority of the series we have analyzed show a strong tendency to rise during periods of general business expansion and to fall during periods of general business contraction. But series such as bankruptcies and unemployment have as strong a tendency to move in the opposite fashion. In our jargon the latter series have 'inverted' specific cycles. We can often, but by no means always, anticipate whether the specific cycles of a series will be positive or inverted. For example, the specific cycles of oleomargarine production bear a positive relation to business cycles, while the specific cycles of butter production bear an inverted relation. Stocks of raw materials held by manufacturers tend to be related positively to business cycles, stocks of finished products held by manufacturers tend to be related invertedly, and so on.

In 'positive' series we treat specific cycles as units running from trough to trough; in 'inverted' series we treat them as units from peak to peak. Hence it is necessary to decide whether a series is to be classed as 'positive' or 'inverted' before any measures of specific-cycle behavior are made.

Ordinarily, study of the reference-cycle patterns of a series yields a clear verdict: a series rises during most of the reference expansions and falls during most of the reference contractions, or it falls during most of the former and rises during most of the latter phases. More difficult to classify are series that (1) ordinarily rise during the early stages of reference expansion, fall during the later stages, and turn upward again while general business is still shrinking; (2) ordinarily rise from the later stages of reference expansion to the early stages of reference contraction, and fall during the later stages of reference contraction and the early stages of expansion; (3) show marked independence of business cycles.

As explained in Section X, if a series typically moves in the same direction as general business over more reference-cycle stages than it moves in the opposite direction, we class the series as 'positive' and analyze the specific cycles on a trough-to-trough basis. In the opposite case, we class the series as 'inverted' and analyze the specific cycles on a peak-to-peak basis. If the expansions of a series typically run from the middle of reference expansion to the middle of reference contraction, or from the middle of reference contraction to the middle of reference expansion, we arbitrarily analyze the specific cycles on a trough-to-trough basis. Series that move in virtual independence of business cycles are handled in the same way. Thus 'positive', 'neutral', and 'irregular' series are treated on a *positive plan*—that is, their specific cycles are treated as units running from trough to trough; while 'inverted' series are treated on an *inverted plan,* their specific cycles being marked off by peaks.

These rules are instruments of convenience. As will be explained presently, we at times find it desirable to analyze 'neutral' series on both a positive and inverted basis, and sometimes treat a series as positive when strict application of our rules calls for inverted analysis and vice versa. Positive or inverted analysis is a poor guide to the conformity of a series to business cycles. A series may be related to business cycles in many ways, and several measures are necessary to bring out clearly the actual relations.

II Timing of Specific Cycles

One of the principal aims of this investigation is to measure the time sequence in which various economic activities have followed one another in the revivals and recessions of general business activity. Hence we seek to compare the dates of the peaks and troughs of the specific cycles of each series with the reference dates of business cycles in the country to which the series relates. By noting the number of months that the turns of the specific cycles of a series lead or lag behind the reference turns, striking averages of the leads or lags for peaks and also for troughs, and supplementing the averages by average deviations, we can tell what timing has been characteristic of the series in regard to business cycles. By observing the leads or lags of numerous series at successive reference dates, we can determine also in what respects the sequence of upturns has been similar or different in past business-cycle revivals, and in what respects the sequence of downturns has been similar or different in past recessions.

In some series, as in the production of coke or bituminous coal in the United States, the specific cycles correspond closely in time to business cycles. It is fairly easy to fit the turns of the specific cycles of such series into the chronology of business cycles. More often the correspondence between the two sets of cycles is imperfect; sometimes it is negligible. Thus if two specific cycles occupy approximately the same period as a business cycle, the trough between the two specific cycles and at least one peak will not correspond to any reference turn. Again, if one specific cycle spans two business cycles, its initial and terminal troughs will match reference turns and the peak is likely to do so, but at least two reference turns will not match any specific-cycle turn. In series where the relationship between the turning dates of specific and business cycles is still looser, matching of the two sets of dates becomes very hazardous unless the entire operation can be reduced to a tolerably objective basis.

The first step in the measurement of the cyclical timing of a series is to decide whether to match the specific-cycle turns with like or opposite reference turns. The rules recited in the preceding section [1] must be used discriminatingly. For example, series on new orders usually lead similar turns in general business; most of them are clearly 'positive' according to our criterion, but a few with longer leads are on the margin, while one or two with still longer leads are 'inverted'. To avoid confusion we analyze all series in this group on a positive basis. This seems the more logical treatment, although it may prove desirable later to add inverted measures for the entire group. In analyzing activities such as bond yields or bond sales, which usually have cyclical turns near the middle of reference expansions and contractions, we may make a double analysis from the start. From certain points of view it is desirable to think of upturns and downturns in long-term interest rates as lagging behind the revivals and recessions of general business; from other points of view it is important to emphasize the tendency of upturns in long-term interest rates to lead general recessions and of downturns to lead general revivals. A similar problem arises in handling short-term interest rates in the principal money markets, despite the fact that series of this type are clearly 'positive' according to our rules and have been uniformly handled on this basis. In tracing the developments that generate revivals and recessions in business activity, we are sure to add inverted measures; for a decline in interest rates is a favorable development and a rise an unfavorable development

1 When the relationship in time between specific and business cycles is rather loose, it may be difficult to decide whether the relation is positive, inverted, neutral, or irregular. However, errors in classifying such series cannot be serious: irregular timing is their most prominent feature and that will be demonstrated whether the analysis is carried through on a positive or on an inverted basis.

with respect to the business situation at large. The subtle and shifting demands of analytic work can be anticipated only in part at the stage where series are taken up for examination, one by one.

Once a decision has been reached on the issue of positive versus inverted treatment, the next step is to match individual turns of the specific cycles of a series with individual turns of business cycles. This operation calls for considerable self-restraint, lest the data be forced to suit preconceived notions. We follow fairly rigid rules in making timing comparisons. The method is not 'purely objective', for we have found no mechanical device that insures sensible results; but it is sufficiently objective to minimize if not eliminate bias.

Our basic rule is that a specific-cycle turn (S) must meet two criteria in order to be considered as corresponding to a *like* reference turn (R):

(1) there is no other reference turn in the interval between S and R (including the month of S)

(2) there is no other specific-cycle turn in the interval between S and R (including the month of R).

These criteria help to weed out 'extra' specific cycles, that is, specific-cycle movements unrelated to business cycles. It may happen, however, that two specific-cycle peaks, coming on opposite sides of a given reference peak, deviate by approximately the same number of months from the reference peak, but that the earlier specific-cycle peak meets both criteria while the later peak meets only the first criterion (since the intervening specific-cycle trough happens to follow the reference peak). To say in that event that the earlier peak 'corresponds' to a reference turn and that the later one does not is to settle a problem of causation by mechanical rule. Difficulties of this type are reduced by our third criterion: if S meets criteria (1) and (2) and another *like* specific-cycle turn on the opposite side of R meets just criterion (1), S will be treated as a corresponding turn only in the event that it deviates no more than three months from R. In applying criterion (3) we avoid guessing about the behavior of the series prior to the first or after the last specific-cycle turn; that is, a turn is treated as noncorresponding so long as it might fail to meet this criterion if the series were extended.[2]

If all three criteria are met, S is treated as corresponding to R. If not, S is treated as noncorresponding; that is, no attempt is made to compare its timing with that of business cycles. The three criteria have been phrased for series that are treated on a positive plan. To apply the criteria

[2] Criteria (1) and (2) would yield identical comparisons if the turning dates of the series were the standard against which the reference dates were compared—instead of the other way around, which of course is our practice. Criterion (3) does not satisfy this principle of reversibility. If that criterion were dropped we might gain a formal advantage, but at the cost of a less stringent weeding of 'extra' specific cycles in relation to business cycles.

Criteria (1) and (2) may be combined in the following rule: a specific-cycle turn (S) corresponds to a like reference turn (R) if no other specific or reference turn falls in the interval between S and R. Criterion (3) places a restriction on this rule.

to series treated on an inverted plan, it is necessary merely to substitute *opposite* for *like* in the wording of the rules. A simpler expedient is to think of inverted relations as being transformed into positive ones by plotting figures invertedly, and we therefore continue the exposition as if all series were treated on a positive basis.

Chart 11 illustrates the application of the timing rules. The angular line at the top identifies the American reference cycles from 1905 to 1933 and serves no other purpose.[3] All series are seasonally adjusted. The turns of the specific cycles are marked on the chart, and the dates are listed in Table 28. To facilitate comparisons between the turning dates of the

TABLE 28

Turning Dates of Specific Cycles in Six American Series

COKE PRODUCTION		BITUMINOUS COAL PRODUCTION		CALVES SLAUGHTERED UNDER FEDERAL INSPECTION	
Peak	Trough	Peak	Trough	Peak	Trough
Jan. 1913	Nov. 1914	May 1907	Jan. 1908		Feb. 1908
July 1918	May 1919	June 1910	Feb. 1911	Dec. 1909	Dec. 1911
Aug. 1920	July 1921	Oct. 1913	Nov. 1914	Aug. 1912	Nov. 1914
May 1923	July 1924	July 1918	Mar. 1919	June 1920	Dec. 1920
Feb. 1926	Nov. 1927	Dec. 1920	July 1922	July 1925	June 1929
July 1929	Aug. 1932	May 1923	June 1924	June 1931	Dec. 1932
		Mar. 1927	Dec. 1927		
		May 1929	July 1932		

PETROLEUM OUTPUT APPALACHIAN FIELD		COTTON STOCKS AT MILLS		STRUCTURAL STEEL ORDERS	
Peak	Trough	Peak	Trough	Peak	Trough
	Aug. 1908		Oct. 1914	May 1909	Nov. 1910
Nov. 1909	Jan. 1912	Nov. 1916	Oct. 1917	Oct. 1912	Nov. 1914
May 1912	Oct. 1915	Oct. 1918	Apr. 1919	Dec. 1915	Sep. 1917
July 1919	Feb. 1920	Jan. 1920	Nov. 1920	July 1918	Jan. 1919
Feb. 1921	Nov. 1924	Nov. 1922	Oct. 1924	Feb. 1920	Feb. 1921
Feb. 1930	Aug. 1931	June 1927	Sep. 1928	Mar. 1923	June 1923
Oct. 1931	May 1933	Mar. 1930	Sep. 1931	Oct. 1925	Sep. 1926
		May 1932	Mar. 1933	Mar. 1929	Jan. 1932

See Chart 11.

specific and business cycles, vertical lines are erected at the reference turns and the reference contractions are shaded. In each case where a specific-cycle turn satisfies our three criteria, an arrow is drawn from it to the reference turn with which it is compared. The absence of an arrow at a specific-cycle turn means that one or more of the criteria are not met, and accordingly that no timing comparison is made.

In the series on calves slaughtered, the specific-cycle trough of June 1929 fails to satisfy criterion (1); hence no arrow appears at this point. There is also no arrow at June 1931, for this peak fails to meet criterion (2). In cotton stocks held by mills, every specific-cycle turn can be matched with a reference turn so as to satisfy criterion (1), but four turns

[3] The straight lines joining peaks and troughs are arbitrary; so also are the uniform amplitudes.

(November 1916, October 1917, September 1931 and May 1932) fail to satisfy criterion (2). A failure to meet criterion (3) is illustrated by the peak of July 1919 in Appalachian petroleum output. Both this peak and that of February 1921 are linked by criterion (1) to the reference peak of January 1920; the first specific-cycle peak meets criterion (2) while the second specific-cycle peak does not; but the first specific-cycle peak succumbs to criterion (3) since it deviates more than three months from the reference peak in the face of a competing specific-cycle peak on the opposite side of the same reference peak. There is no arrow at the last turning points of coke production, bituminous coal production, and structural steel orders. These turns satisfy criteria (1) and (2); but since there is no assurance that they would satisfy criterion (3) if the data were carried forward, they are not classed as corresponding turns. These cases illustrate how we handle series when the records cannot be carried forward or backward. Of course, it is now possible to carry all series sufficiently beyond 1933 to decide whether or not criterion (3) is actually met.

The mechanical rules for timing comparisons separate out the cyclical movements in a series that are closely related to business cycles from others dominated by factors peculiar to the series. They thus serve to restrict our averages to cyclical turns that have some presumptive claim to being connected with the revivals and recessions of general business. But the separation is very rough and we do not consider it final. Obviously, the rules treat with excessive liberality series that move in virtual independence of business cycles. That is not especially disturbing, since timing averages are in any event of slight value in series that conform badly to business cycles. On the other hand, the rules may reduce unduly the number of timing comparisons in series that conform well to business cycles. This difficulty is serious: first, because even our longest series cover relatively few cycles, second, because the rules may weed out turns in a way that biases the averages. Thus criterion (1) weeds out leads or lags equal to, or longer than, a full reference phase. Leads or lags of this length may be of slight value in describing the sequence of cyclical turns in a particular revival or recession of general business, but they should not therefore be ignored in estimating the typical timing of an activity at business-cycle revivals or recessions. When the turns of the specific cycles of a series usually precede reference turns by half a phase, random factors are as likely to produce a lead that is slightly longer than a full reference phase as they are to produce a short lag. To give random movements an opportunity to cancel out in the averages, it seems no less necessary to include the one than to include the other.[4]

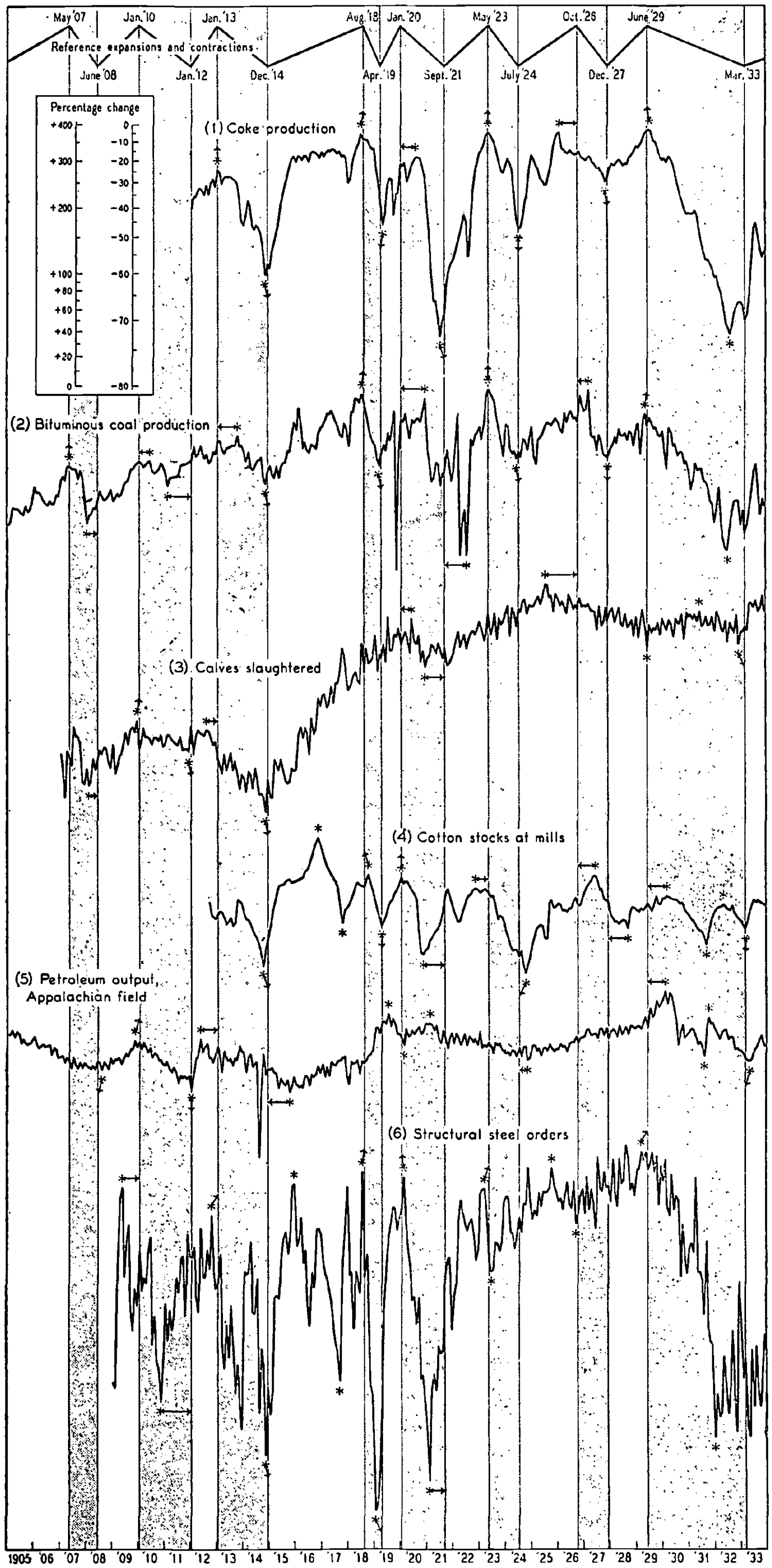
We therefore relax the mechanical rules in series that *conform well* to business cycles. If each of two specific-cycle turns deviates less than a full

[4] See the analysis in Ch. 8, Sec. III of the influence of erratic movements on our timing measures.

CHART 11

Illustrations of Mechanical Rules for Comparing the Timing of Specific Cycles with the Reference Dates

Six American Series

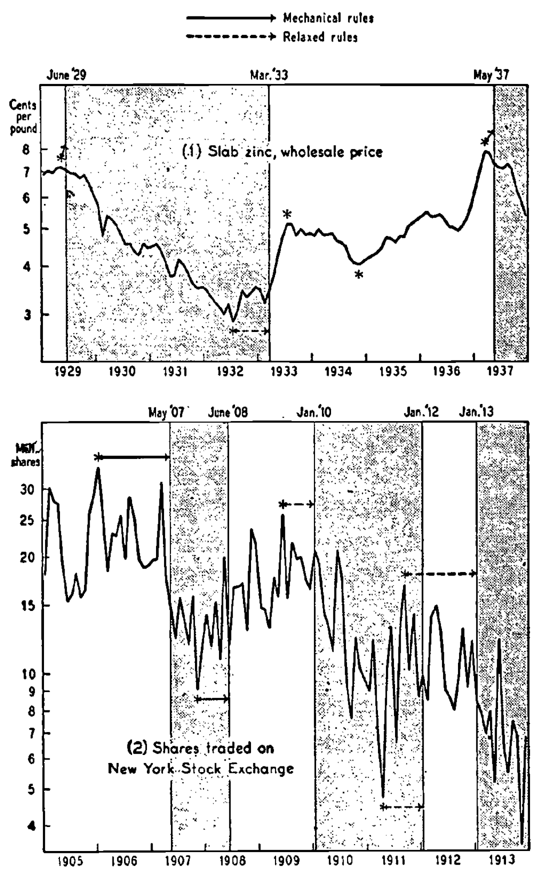


Shaded areas represent reference contractions; white areas, reference expansions. Asterisks identify peaks and troughs of specific cycles. See text for explanation of arrows and other features. Sources are listed in Appendix C.

phase from the same reference turn, we may treat one of them as corresponding regardless of the mechanical rules. When cases of this sort are encountered, we scrutinize the amplitudes and patterns of the competing movements, and draw on whatever knowledge we may have concerning the particular fluctuations. In the absence of special knowledge, if any cyclical movement in the neighborhood of the reference turn is so mild that it might have been disregarded in the first place, we ignore it in making timing comparisons; while if none of the cyclical movements is dubious, we give preference to the turn that represents the culmination of a larger specific-cycle movement. Chart 12 gives an illustration of the procedure. If we followed the mechanical rules, we would disregard the troughs in zinc prices in 1932 and 1934. But no one could reasonably

CHART 12

Illustrations of Relaxed Rules for Comparing the Timing of Specific Cycles with the Reference Dates
Two American Series



See note to Chart 11.

doubt that zinc prices were at a trough in 1932, while the contraction from 1933 to 1934 might be considered a pause in the sharp recovery that started in 1932 and culminated in 1937. We therefore disregard the trough in 1934 and match the trough in 1932 with the reference trough.

We relax the mechanical rules in two additional respects when handling well conforming series. First, in applying criterion (3) we proceed on the assumption that there is no competing turn just before the first or just after the last turn of the specific cycles that can be dated from existing records.[5] Second, we sometimes admit timing measures as long as a full reference phase or even longer. For example, the trough of June 1923, peak of October 1925, and trough of September 1926 in structural steel orders are noncorresponding according to the mechanical rules (Chart 11). But since the series on the whole conforms well to business cycles, we might compare the trough of September 1926 with the reference trough in December 1927, in spite of the fact that this comparison yields a lead longer than a full phase. Once that decision is made, the trough in June 1923 is readily matched with the reference trough in July 1924 and the peak in October 1925 with the reference peak in October 1926. Chart 12 illustrates a similar situation in share trading on the New York Stock Exchange. Here we recognize a lead longer than a phase at the reference peak of January 1913; two additional timing comparisons naturally follow from this decision.

These relaxed procedures make possible a fuller use of the statistical information than could be attained by relying exclusively on the mechanical rules. In practice we begin the analysis of every series by applying the mechanical rules. If a series does not conform well to business cycles, nothing further is done regardless of the number of specific-cycle turns that are classed as noncorresponding. But if a series conforms closely to business cycles, we try to reduce the number of noncorresponding turns by relaxing the mechanical rules. The main difficulty is to draw a reasonable dividing line between series that conform 'closely' to business cycles and those that do not. In general, we accept an index of full-cycle conformity of 50 or higher as indicative of close conformity. But this index is not an infallible indicator; and we therefore take account as well as may be of other evidence before making a decision, such as the patterns of successive reference cycles, the number of reference cycles covered, the number of specific cycles relatively to the number of reference cycles, and the behavior of intimately related series.[6]

Once decisions have been reached, by studying charts like those produced here, concerning what turns of the specific cycles of a series correspond to what reference turns, the timing measures are entered in our

[5] However, to avoid absurdities, no timing comparison longer than 12 months is made unless such timing is typical of the series.

[6] See below, Sec. X.

standard Table S1. Table 29 presents a few samples of the first five columns in Table S1, the part devoted to timing measures. Column (1) shows the turning points of each full specific cycle: the month of the initial trough, of the cyclical peak, and of the low point that marks the end of the con-

TABLE 29

Samples of a Section of Table S1: Timing of Specific Cycles
Three American Series

Dates of specific cycles Trough–Peak–Trough (1)	Timing at reference peak: Number of months lead (−) or lag (+) (2)	Timing at reference peak: Date of reference peak (3)	Timing at reference trough: Number of months lead (−) or lag (+) (4)	Timing at reference trough: Date of reference trough (5)
Bituminous coal production				
May 07–Jan. 08	0	5/07	−5	6/08
Jan. 08–June 10–Feb. 11	+5	1/10	−11	1/12
Feb. 11–Oct. 13–Nov. 14	+9	1/13	−1	12/14
Nov. 14–July 18–Mar. 19	−1	8/18	−1	4/19
Mar. 19–Dec. 20–July 22	+11*	1/20	+10	9/21
July 22–May 23–June 24	0	5/23	−1	7/24
June 24–Mar. 27–Dec. 27	+5	10/26	0	12/27
Dec. 27–May 29–July 32	−1	6/29	(−8)	3/33
Average	+3.5		−2.1	
Average deviation	4.0		4.4	
Structural steel orders				
May 09–Nov. 10	−8	1/10	−14*	1/12
Nov. 10–Oct. 12–Nov. 14	−3	1/13	−1	12/14
Nov. 14–Dec. 15–Sep. 17	...	...	...	...
Sep. 17–July 18–Jan. 19	−1	8/18	−3	4/19
Jan. 19–Feb. 20–Feb. 21	+1	1/20	−7	9/21
Feb. 21–Mar. 23–June 23	−2	5/23	(−13)*	7/24
June 23–Oct. 25–Sep. 26	(−12)	10/26	(−15)**	12/27
Sep. 26–Mar. 29–Jan. 32	−3	6/29	(−14)	3/33
Average	−4.0		−9.6	
Average deviation	3.4		5.1	
Petroleum output, Appalachian field				
Aug. 08			+2	6/08
Aug. 08–Nov. 09–Jan. 12	−2	1/10	0	1/12
Jan. 12–May 12–Oct. 15	−8*	1/13	+10	12/14
Oct. 15–July 19–Feb. 20	...	...	...	...
Feb. 20–Feb. 21–Nov. 24	...	...	+4	7/24
Nov. 24–Feb. 30–Aug. 31	+8	6/29	...	...
Aug. 31–Oct. 31–May 33	...	...	+2	3/33
Average	−0.7		+3.6	
Average deviation	5.8		2.7	

Average is the arithmetic mean; the average deviation is measured from the mean. The entries in parentheses identify the timing measures made under the 'relaxed rules'.

* Indicates that the specific-cycle turn deviates more than a half but less than a full reference phase from the corresponding reference turn.

** Indicates that the specific-cycle turn deviates a full reference phase or longer from the corresponding reference turn.

traction. If a specific-cycle peak comes before the first or after the last complete specific cycle, its date is also entered since we wish to utilize fully what information we have on timing. Columns (3) and (5) indicate the reference dates with which the specific-cycle turns are compared. Column (2) states the number of months by which the peak of a specific cycle precedes or follows the corresponding reference peak. Similarly, column (4) states the number of months by which the trough of a specific cycle, as shown by the last set of entries in column (1), precedes or follows the corresponding reference trough. A minus sign indicates a lead at a reference turn, a plus a lag, zero a coincidence; a blank indicates noncorrespondence. Thus the entries for coal production of '−5' in column (4) and '6/08' in column (5) mean that the specific-cycle trough (January 1908) precedes the reference trough (June 1908) by 5 months. To facilitate use of the timing measures, several symbols are used. Parenthetic entries represent timing comparisons in addition to those permitted by the mechanical rules. Thus Appalachian petroleum output is treated throughout according to the mechanical rules; but these rules are relaxed in bituminous coal production and structural steel orders, which conform well to business cycles.[7]

When specific cycles are treated on an inverted plan, the general procedure is the same. But now we match the specific-cycle troughs with the reference peaks and the specific-cycle peaks with the reference troughs. Column (1) gives the month of the initial specific-cycle peak, of the trough, and of the terminal peak. Column (2) indicates the number of months by which the cyclical troughs in the series precede or follow the reference peaks. Similarly, column (4) indicates the number of months by which its cyclical peaks precede or follow the reference troughs.

As explained in Chapter 4, some business-cycle turns are more difficult to date than others. Hence we do not have implicit confidence in the accuracy of the leads and lags, even when the turning points of specific cycles are perfectly clear. Moreover, our practice of setting reference dates toward the close of transition periods whenever cyclical waves seemed to culminate in a flat or double peak or trough produces a bias—leads are more numerous than lags.[8] But since the leads and lags of all series for a given country are measured from a standard set of benchmarks, they are dependable with respect to one another, within the variable margins of

[7] There is no timing entry in Chart 11 for the specific-cycle trough of July 1932 in bituminous coal production. There is an entry in Tables 29 and 30, parenthetic in the former table but not in the latter. The reasons for these differences are as follows. The chart shows what timing comparisons would be made if bituminous coal output were treated according to the mechanical rules; the tables show the comparisons we actually make in this well conforming series. But whereas Table 29 is made on the assumption that the available data stop in 1933 (the period covered by the chart), Table 30 draws upon later data.

[8] Dating specific cycles in a similar manner does not prevent a bias. For example, an expansion in general business might culminate in a plateau maintained for several months, while the peaks in the specific cycles of most individual activities are sharply angular.

error involved in dating the turning points of specific cycles. Any error we may make in fixing the reference dates affects the absolute timing measures, but does not misrepresent the sequence of the cyclical turns in individual series during a business-cycle revival or recession. Nor do errors in reference dates affect the average sequence of individual series so long as they are compared for identical periods; for the average error of the timing measures, if any, is then necessarily the same for all series. If the periods are not identical, the average timing measures may contain an error of varying size, since the average error of one set of reference dates may differ from the average error of another set of reference dates.

Further difficulties arise if a series shows 'extra' cycles or if its timing differs widely in other respects from that of business cycles—complications which we ignore in the preceding paragraph. In such instances slight shifts in reference dates may alter decisions as to what turns of the specific cycles are corresponding, and may even result in a different matching of specific and business cycles. These difficulties are inherent in any set of rules involving rigid boundary lines. There are always some cases near the margin, so that if a reference date is changed, however slightly, a turn that is barely corresponding according to the rules may become noncorresponding or vice versa. Another aspect of the same difficulty appears when the cyclical turns of a group of closely related series are nearly coincident, yet differ materially from the reference dates in some portion of the period covered. In a case of this sort a difference of one or two months in the turning points of individual series may result in a different matching of their cyclical turns with the reference dates. Moreover, the timing of 'extra' cycles will go unrecorded; in other words, part of the information concerning the timing of the specific cycles common to the group will not be used.[9] These difficulties flow from the fact that the measures in Table S1 are designed chiefly to determine what timing is characteristic of a series in regard to general business activity. If another problem is faced, such as comparing the turns of a closely related group of series, our standard measures cannot be trusted implicitly. The safest method is to check timing measures made on the standard plan by direct comparisons among the series.[10]

[9] It is proper to ignore this information if the problem is simply to determine the timing relations among the series in the vicinity of business-cycle revivals and recessions. The text refers to an effort to determine the timing relations among the specific cycles of a closely related group of series, regardless of their conformity to business cycles.

[10] It is always desirable to check timing comparisons made from the dates assigned to the turning points of specific cycles by going back to the data charts. If all series were sine curves of equal period and amplitude, the sequence of their cyclical movements could be determined completely by dating peaks and troughs alone; indeed, the dates of a single cluster of turns including all series would suffice. But in actual life the shapes of cyclical movements are so varied that a mere record of the months in which different series turned up or down may be misleading. Data charts must be studied closely before drawing conclusions about sequences even of series with clearly defined cyclical movements. The charts should show the original as well as seasonally adjusted figures so that faulty seasonal adjustments may be detected. See also below, Sec. VII.

At a later stage of the investigation we shall make tables showing the order in which different series turned up at successive reference troughs and turned down at successive reference peaks, thus establishing a record of the sequence in which various economic activities have followed one another during cyclical reversals of general business activity. In drawing up these tables we shall need to supplement the measures in Table S1. The distinction between 'corresponding' and 'noncorresponding' turns, although vital for the purpose of determining what timing is characteristic of a given activity in regard to business cycles, is of little use when interest shifts to particular business-cycle revivals or recessions. Specific-cycle movements now classed as 'noncorresponding' may be no less important for an understanding of individual business cycles than those classed as 'corresponding'. On the other hand, 'leads' or 'lags' lasting longer than a reference phase may be misleading for this purpose.

As these remarks indicate, the method of measuring sequences must be adapted to the aims pursued in different parts of this investigation. And in this connection, it is well to note briefly our reasons for choosing the particular method that we use to determine what cyclical timing is characteristic of a series instead of the traditional method, which takes as a measure of the typical lead or lag the pairing of months that yields the highest coefficient of correlation between the series and some index of business activity. Our method involves judgment at every step of the calculation: in dating business cycles, dating specific cycles, deciding whether to analyze a series on a positive or inverted basis, and deciding how to match the specific and business cycles. The traditional method, while by no means free from personal factors, is more objective and more elegant. But it has three grave shortcomings: (1) it fails to distinguish between the months of cyclical turn and other months and therefore also between troughs and peaks, (2) it tells nothing about the variation in timing from cycle to cycle, and (3) it reports at best the sequence of trend-adjusted data. For our purposes, it is highly important to distinguish between the timing at upturns and downturns of business cycles, to note the variations in timing from one business cycle to the next, and to follow the sequence of cyclical movements as they occur rather than as they appear after trends are removed. Whatever its defects, our method at least gets directly at matters that, from our viewpoint, are significant for the understanding of business cycles.

Moreover, in trying to determine the timing of cyclical movements characterizing different economic processes, we do not rely exclusively upon the leads and lags of Table S1. The reference-cycle patterns, described in Section VII, afford a check on these measures. Although the check is insensitive to minor variations in timing, it has several valuable features. First, the reference-cycle patterns indicate roughly the timing of the rate of change of a series during the expansions and contractions of business cycles, as well as the timing of its peaks and troughs. Second, though these measures have uncertainties of their own, they are free from practically every uncertainty that surrounds the calculation of leads and lags in Table S1, except the dating of business cycles. Third, the reference-cycle patterns automatically show timing in fractions of business-cycle expansions and contractions.

The last feature raises an important question about the timing measures in Table S1, which are expressed in months. Since a business cycle in a given country is the fundamental unit in our scheme of analysis, timing measures expressed as fractions of the duration of a business cycle or one of its phases may seem more defensible logically than timing measures expressed in months. Yet the issue is full of complexities. For some practical purposes the simple measures in months are best: for example, when one is trying to anticipate business-cycle recessions or revivals from the average leads of related series in the past and their current movements.[11] The critical question from an analytic viewpoint is whether timing measures are more stable in months or in fractions of cyclical units, but that question we are as yet in no position to answer. And if the answer should turn out to be that most economic groups show greater stability on one plan but that a few show greater stability on the other, the question will remain whether the method that is best for most series is best for all.

At a later stage we may want to convert the leads and lags given by months in Table S1 to other forms, but it should be noted that the conversion into cyclical units is not a simple matter. Perhaps the best plan is that implicit in our reference-cycle measures of timing, where the lead or lag of a series at a reference turn takes the form of a fraction of the reference phase within which the turn of the series falls.[12] But there are other methods that cannot be dismissed lightly. To mention just one, the lead or lag at a reference peak might be expressed as a percentage of the cycle running from the preceding to the following reference trough, and the lead or lag at a reference trough as a percentage of the cycle running from peak to peak. Preference for one base or another may shift with the purpose the investigator has at the moment. In any case the measures in months in Table S1 are of direct value as they stand, and they may be converted into other forms more readily than highly fabricated measures.

III Duration of Specific Cycles

Table 30 presents a sample of Table S1 in full. The measures of cyclical duration in columns (6) to (8) are obvious and raise no difficulties. The

[11] For a preliminary study of this problem, see our paper, Statistical Indicators of Cyclical Revivals (*Bulletin 69*).

[12] See below, note 70.

expansion is the interval from the midpoint of the date of the initial trough to the midpoint of the date of the peak; the contraction is the interval from the midpoint of the peak to the midpoint of the terminal trough. The duration of a full specific cycle is obtained by summing the lengths of the expansion and contraction. If the duration of a contraction or expansion of an incomplete specific cycle at the beginning or end of a series can be ascertained, it is recorded but excluded from the averages; e.g., the contraction from May 1907 to January 1908 in coal production.

TABLE 30

Sample of Table S1: Timing and Duration of Specific Cycles
Bituminous Coal Production, United States, 1907–1938

Dates of specific cycles	Timing at reference peak		Timing at reference trough		Duration of cyclical movements (mos.)						Per cent of duration of specific cycles	
					Specific cycles			Excess over reference cycles				
Trough – Peak – Trough (1)	No. of mos. lead (−) or lag (+) (2)	Date of reference peak (3)	No. of mos. lead (−) or lag (+) (4)	Date of reference trough (5)	Expansion (6)	Contraction (7)	Full cycle (8)	Expansion (9)	Contraction (10)	Full cycle (11)	Expansion (12)	Contraction (13)
May 07–Jan. 08	0	5/07	−5	6/08	..	8[a]	..	...	−5[a]	...	..	..
Jan. 08–June 10–Feb. 11	+5	1/10	−11	1/12	29	8	37	+10	−16	−6	78	22
Feb. 11–Oct. 13–Nov. 14	+9	1/13	−1	12/14	32	13	45	+20	−10	+10	71	29
Nov. 14–July 18–Mar. 19	−1	8/18	−1	4/19	44	8	52	0	0	0	85	15
Mar. 19–Dec. 20–July 22	+11 *	1/20	+10	9/21	21	19	40	+12	−1	+11	52	48
July 22–May 23–June 24	0	5/23	−1	7/24	10	13	23	−10	−1	−11	43	57
June 24–Mar. 27–Dec. 27	+5	10/26	0	12/27	33	9	42	+6	−5	+1	79	21
Dec. 27–May 29–July 32	−1	6/29	−8	3/33	17	38	55	−1	−7	−8	31	69
July 32–Mar. 37–Mar. 38	−2	5/37	−2	5/38	56	12	68	+6	0	+6	82	18
Average[b]	+2.9		−2.1		30.2	15.0	45.2	+5.4	−5.0	+0.4	65	35
Average deviation[c]	4.1		3.9		11.0	6.8	9.8	6.8	4.5	6.6	17	17

[a] Excluded from the average and the average deviation.

[b] Arithmetic mean determined separately for each column. Hence (6)+(7) *may* differ from (8) in the last place; likewise (9)+(10) from (11).

[c] Measured from the mean.

* Indicates that the specific-cycle turn deviates more than a half but less than a full reference phase from the corresponding reference turn.

In columns (9) to (11) the durations of specific cycles are compared with the durations of corresponding business cycles derived from the monthly reference dates. A plus sign in column (9) means that the expansion of the specific cycles is longer than the corresponding reference expansion; a minus sign means that it is shorter. Similar rules govern columns (10) and (11). Columns (9) to (11) are restricted as a rule to well conforming series, in the sense defined in the preceding section on timing measures. In series of this type we compare the durations of all corresponding phases, but base the averages on specific cycles that correspond in full to business cycles. In other words, if an expansion in the series corresponds to a reference phase while the succeeding contraction does not, the former is compared with the corresponding reference expansion but is excluded from the average at the bottom of the table. Similarly, if there is a full phase of an incomplete specific cycle at the beginning or end of a series, its duration is compared with the corresponding duration of business cycles but is excluded from the averages; the first entry in column (10) is a case in point.[13]

Finally, in columns (12) and (13) we transform the entries in columns (6) and (7) into relatives of the entries in column (8). This step facilitates comparisons of the expansion and contraction phases of specific cycles of different durations. In the averages of columns (12) and (13), each cycle receives the same weight regardless of its duration. Weighted averages, if desired, may be obtained by expressing the averages in columns (6) and (7) as percentages of the average in column (8); these percentages would show the proportion of the period covered by the full cycles that consisted of expansion and the proportion that consisted of contraction. In coal production the unweighted average for expansions is 65 per cent and the weighted average is 67 per cent; the difference is negligible because the correlation between the absolute duration of specific cycles and the percentage consisting of expansions is slight.

If a series is analyzed on an inverted plan the procedure is essentially the same. But now the contraction is entered in columns (6) and (12), the expansion is entered in columns (7) and (13), the entries in column (9) compare the contraction of specific cycles with the corresponding reference expansion, and the entries in column (10) compare the expansion of specific cycles with the corresponding reference contraction.

The measures of cyclical duration are peculiarly sensitive to uncertainties in identifying specific cycles and must be used with discrimination. If a contraction or expansion is just on the margin of being counted as a specific-cycle movement, its inclusion or omission may have considerable effect on the average duration of specific cycles and their phases, especially if the series is short. The average durations must be interpreted with care even when the specific cycles are perfectly clear. For example, the specific cycles in mill consumption of cotton in the United States averaged 36 months during 1914–32, or 8 months less than business cycles during a comparable period. Cattle slaughter, on the other hand, shows specific cycles averaging 100 months from 1908 to 1932, in contrast to an average of 42 months for business cycles. The first difference arises entirely from an 'extra' cycle during the great contraction of 1929–33, while the second arises from individual observations that are characteristically longer than those for business cycles.

[13] The entries in col. (2) to (11) check one another. The figure in col. (4) minus that in col. (2) should equal the figure on the same line in col. (10). The figure in col. (2) on a given line minus the figure in col. (4) on the preceding line should equal the figure in col. (9) on the given line, while the figure in col. (4) on a given line minus the figure in the same column on the preceding line should equal the figure in col. (11) on the given line. If these columns check, the entries in col. (6) to (8) are in all likelihood also correct.

IV Amplitude of Specific Cycles

The original data of economic time series are expressed in diverse units: tons, dollars, persons, bushels, miles, square feet, percentages, and so on. Variations of the unit do not affect the comparability of duration measures, but bar direct comparisons of the amplitude of cyclical swings. Our solution of this difficulty is to express the rise from the trough to the peak and the fall from the peak to the trough of each specific cycle as a percentage of the average monthly value of the series during the cycle. This method puts amplitudes of different types of series in a similar unit and thereby facilitates comparisons among them. Also, it eliminates the secular trend in step-wise fashion and thus facilitates comparisons of the cyclical amplitudes at different stages of the development of a given activity.

The first step in measuring the amplitude of a specific cycle is, therefore, to compute the 'cycle base', that is, the average value of the series during the cycle. Since specific cycles run into one another, a trough is as much a part of a given cycle as of the one adjacent to it. Hence we include both the initial and terminal troughs in computing the average value during a specific cycle; but to avoid a downward bias the trough values receive a weight of one-half each.[14] For example, the initial trough of the first complete specific cycle in bituminous coal production comes in January 1908, the terminal trough in February 1911, and the duration of the full cycle is 37 months. To obtain the average monthly standing during the cycle, we sum the values of the seasonally adjusted figures from February 1908 to January 1911 inclusive, add to this sum one-half the value in January 1908 and one-half the value in February 1911, and divide the grand total by 37, the number of months in the cycle. When a series is analyzed on an inverted instead of a positive basis, the procedure is the same except that the peak values are weighted one-half each. The 'cycle bases' yielded by these calculations are recorded in column (4) of Table S3, of which Table 33 gives a sample.

The second step is to compute specific-cycle relatives, which involves expressing the seasonally adjusted figure for each month during a specific cycle as a percentage of the average monthly value for the cycle. In practice it is not necessary to compute specific-cycle relatives for every month. As explained below, nearly all measures based upon these relatives cover two or more months. It is quicker to average the seasonally adjusted data for the months covered by a measure and to reduce this average to a relative than to compute relatives for the individual months and average them.[15] But our explanation can be kept simpler by assuming that the roundabout calculation is made; that is to say, that the monthly values

14 Here we follow a suggestion by Milton Friedman.

15 Also, errors from rounding numbers are reduced.

in each specific cycle are converted to specific-cycle relatives, and that subsequent computations are made from these relatives.

The next step is to determine the standing of the series at the successive troughs and peaks of the specific cycles. Since the cycle relatives at the troughs are likely to be lower and at the peaks higher than they would be if the data were adjusted for erratic movements, we represent the limits of the cyclical swings by three-month averages centered on the peaks and troughs.[16] This is the general rule, but now and then exceptions are necessary. If there is an extremely low value in the month following or preceding a cyclical peak, or an extremely high value in the month following or preceding a cyclical trough, we omit it in computing the peak or trough standing. In these instances a two-month average represents better the limit of a cyclical swing than does a three-month average. On the other hand, some series are free from erratic movements; for example, the specific cycles in series on wholesale prices and bank discount rates at times have horizontal tops and bottoms. In such cases the cyclical peaks and troughs are best represented by the highest and lowest values.

Short phases also require special treatment. When the phase of a specific cycle is extremely short, three-month averages centered on the peak and trough may use up all or most of the observations on the phase, and therefore dampen the cyclical movement as well as eliminate erratic flutterings. To reduce this difficulty we use two-month averages in handling cyclical phases of less than four months. Thus, if the short phase is an expansion, the standing at the trough is represented by an average of the values at the trough and in the preceding month, while the standing at the peak is represented by an average of the values at the peak and in the following month. Similarly, if the short phase is a contraction, the standing at the peak is represented by the peak month and the one preceding it, and the standing at the trough by the trough month and the one following. But in some short phases a two-month average fails to reach as high up into the peak or as far down into the trough as would a three-month average, and therefore is even less satisfactory than a three-month average. To provide for such cases we follow the rule that if the average value in the two months specified to represent the peak is equal to or lower than the value in the additional month that would be included in a three-month average centered on the peak, the standing at the peak should be determined from the value in the peak month alone. With obvious changes in wording, this rule applies also to troughs of short phases.

The amplitude measures are shown in three forms in Table S2, of which Table 31 gives a sample. Columns (2), (3) and (4) show the standings at the initial low point of a specific cycle, at the high point, and at the terminal low point. As stated, these standings are usually three-month

[16] In this connection, see pp. 326, 334-5.

TABLE 31
Sample of Table S2: Amplitude of Specific Cycles
Bituminous Coal Production, United States, 1907–1938

Dates of specific cycles	3-month average in specific-cycle relatives centered on			Amplitude of			Per month amplitude of		
Trough–Peak–Trough (1)	Initial trough (2)	Peak (3)	Terminal trough (4)	Rise (5)	Fall (6)	Rise & fall (7)	Rise (8)	Fall (9)	Rise & fall (10)
May 07–Jan. 08	...	111.1[a]	83.6[a]	...	27.5[a]	...	...	3.4[a]	...
Jan. 08–June 10–Feb. 11	82.7	111.6	102.2	28.9	9.4	38.3	1.0	1.2	1.0
Feb. 11–Oct. 13–Nov. 14	87.3	110.2	88.0	22.9	22.2	45.1	0.7	1.7	1.0
Nov. 14–July 18–Mar. 19	76.0	124.3	85.2	48.3	39.1	87.4	1.1	4.9	1.7
Mar. 19–Dec. 20–July 22	93.4	122.7	64.0	29.3	58.7	88.0	1.4	3.1	2.2
July 22–May 23–June 24	57.2	127.0	86.8	69.8	40.2	110.0	7.0	3.1	4.8
June 24–Mar. 27–Dec. 27	85.9	113.1	85.8	27.2	27.3	54.5	0.8	3.0	1.3
Dec. 27–May 29–July 32	100.1	124.0	57.7	23.9	66.3	90.2	1.4	1.7	1.6
July 32–Mar. 37–Mar. 38	68.4	128.0	83.2	59.6	44.8	104.4	1.1	3.7	1.5
Average[b]	81.4	120.1	81.6	38.7	38.5	77.2	1.8	2.8	1.9
Average deviation[c]	10.6	6.4	10.4	15.4	14.2	23.5	1.3	1.0	0.8
Weighted average	...	...	...	...	...	...	1.3	2.6	1.7

Average rise & fall of seasonal: 68.4 per cent

[a] Computed on base of inverted cycle, May 1907–June 1910. Excluded from the average and the average deviation.
[b] Arithmetic mean determined separately for each column. Hence (3)−(2) *may* differ from (5) in the last place; likewise (3)−(4) from (6), and (5)+(6) from (7).
[c] Measured from the mean.

averages of the specific-cycle relatives centered on the turning points of the specific cycles. If a standing is based on fewer than three months, the exceptional treatment is noted in the table. The 'terminal trough' of each cycle interlocks with the 'initial trough' of the next cycle; that is, the same item is presented as the terminal trough of one cycle and as the initial trough of the next. But the three-month averages of cycle relatives made from the same items differ almost always, sometimes widely, because the average value used as the base of the cycle relatives changes from one cycle to the next. Thus, in bituminous coal production the average of the three values centered on the trough of November 1914 is 88 per cent of the average monthly value during the cycle of February 1911–November 1914, but 76 per cent of the average monthly value during the cycle of November 1914–March 1919.

By scanning the entries on successive lines in columns (2) to (4) one gets a mental image of the specific cycles in the series. This image is severely simple, for it presents only the trough, peak, and trough of each cyclical wave measured from its own mean level. The varying shapes of the cycles from one turning point to the next and from series to series are brought out in the measures of cyclical patterns, described in Section VI. In the meantime columns (5) to (7) show the rise from trough to peak, the fall from peak to trough, and the total rise and fall. These measures are obtained from the entries in the preceding columns. Column (5) shows the absolute differences between columns (2) and (3); column (6) shows the absolute differences between columns (3) and (4); column (7) is the sum of columns (5) and (6). These figures add no information, but they make it easier to compare the amplitudes of different specific cycles in one process and of corresponding specific cycles in different processes. For convenience, the average rise and fall of the seasonal fluctuations is noted at the bottom of the table. Coal production is a highly seasonal industry, but the range of the seasonal fluctuations has been narrower on the average than the range of the cyclical fluctuations.

The third set of amplitude measurements, given in columns (8) to (10), is obtained by dividing the rise and fall figures in columns (5) to (7) of Table S2 by the corresponding duration figures in columns (6) to (8) of Table S1. This step brings out characteristics of the cyclical waves that are hidden by the preceding measurements of rise and fall. In Table S2 for bituminous coal production, the largest cyclical decline occurred in the cycle of 1927–32. But this contraction was uncommonly long, and when we divide the amplitudes of fall by their respective durations, we find that the average rate of fall was smaller in this cycle than in any other cycle except the first and second. Again, the rise exceeds the decline in five cycles out of the eight, but the *rate* of decline exceeds the *rate* of rise in every cycle except one. That contraction is a more violent change than expansion is a common finding, though it is not characteristic of all series. The per month figures are useful not only in comparing relatively long with relatively short cycles in the same country, but also cycles of unlike duration in different countries. For example, the specific cycles in British and American exports have almost the same average amplitudes. But the American cycles are on the average decidedly shorter than the British, and their per month amplitudes are therefore much greater.

The averages of columns (8) to (10) are given in two forms. In the simple averages each entry receives the same weight regardless of the length of the period to which it applies. In the weighted averages successive entries are assigned weights proportionate to the durations. Our plan of treating each cycle as a unit of analysis implies that the average should be unweighted. But we also wish to protect the averages against the distorting influence of extreme items. One way of approximating this objective is to compute weighted average rates of change.[17] Weighting reduces the influence of violently large values, for they are much more likely to come during brief than during long phases. In practice the weighted averages are obtained as a by-product of other calculations; for example, the weighted average of column (8) in Table S2 is derived by dividing the sum of column (5) in Table S2 by the sum of column (6) in Table S1. The differences between the simple and weighted averages are instructive. Thus the unweighted average rate of rise in coal production

[17] See pp. 364-5 and Tables 186-187.

is 1.8 per cent per month, the weighted average 1.3 per cent. The unweighted average rate of fall is 2.8 per cent, the weighted average 2.6 per cent. The weighted figure is commonly lower than the unweighted one, since there is a tendency for the rates of change during cyclical phases to be correlated inversely with their duration.

When a series is analyzed on an inverted basis, columns (2) to (4) show successively the three-month averages of cycle relatives centered on the initial peak of the specific cycle, the trough, and the terminal peak; columns (5) and (8) show the fall instead of the rise, and columns (6) and (9) show the rise instead of the fall. Although the method of calculation is essentially the same, the measure of average amplitude obtained by analyzing a series on an inverted basis is likely to differ from the measure that would be obtained by positive analysis. One reason for the difference is that the period covered by a group of positive cycles is slightly different from the period covered by inverted cycles: the former start and end with a trough, the latter start and end with a peak. But there is also another reason of a more technical character.

This technical reason arises from our practice of eliminating secular trends by steps covering a full cycle, not continuously as is the usual practice of statisticians. When the trend is upward a fall of specific cycles will usually appear larger in units of cycle relatives if the fall is made a part of a positive cycle than if it is made a part of an inverted cycle; for the base of the positive cycle is apt to be smaller than the base of the inverted cycle. But an upward trend will also make a rise of specific cycles smaller when the rise is included in a positive cycle than when it is included in an inverted cycle; for the base of the former is now larger than the base of the latter. If we take positive analysis as a standard, inverted analysis tends to reduce the amplitude of fall and to increase the amplitude of rise when the trend is upward; it tends to produce opposite effects when the trend is downward; but the effects on the rise are set against the effects on the fall when the amplitudes of rise and fall are combined. When the trend is horizontal, the positive and inverted measures should be identical.

These effects and their order of magnitude are illustrated by Table 32, which includes two samples of a rising trend, one of a falling trend, and three of a horizontal trend.[18] The three samples of a horizontal trend are the trend-adjusted forms of the actual data; the latter are designated in the table as 'unadjusted', being corrected merely for seasonal variations. To isolate the difference between the positive and inverted measures that is attributable to differences in the periods covered, the averages for the positive cycles are shown in two forms. One set covers a period beginning and ending with a trough, as is our usual practice. The second set omits the expansion of the first and the contraction of the last

[18] For the rate of secular advance or decline in these series, see Table 84. The series themselves are identified more fully in Ch. 7, Sec. I.

TABLE 32

Average Amplitude of Specific Cycles on Positive and Inverted Plans
Three American Series, Unadjusted and Trend-adjusted

Series and type of analysis	Period covered	No. of cycles	Direction of trend[a]	Average amplitude in specific-cycle relatives		
				Rise	Fall	Rise & fall
Unadjusted data						
A.T.&T. INDEX						
Positive	1900–1933	9	U	30.5	26.1	56.5
Positive	1903–1929	8	U	30.7	20.2	51.0
Inverted	1903–1929	8	U	31.4	19.1	50.6
PIG IRON PRODUCTION						
Positive	1879–1933	15	U	62.1	54.8	116.8
Positive	1883–1929	14	U	61.9	48.0	110.0
Inverted	1883–1929	14	U	64.5	43.9	108.4
RAILROAD BOND YIELDS						
Positive	1868–1899	8	D	6.3	14.6	20.9
Positive	1869–1896	7	D	6.0	14.4	20.4
Inverted	1869–1896	7	D	5.8	15.0	20.8
Trend-adjusted data						
A.T.&T. INDEX						
Positive	1900–1933	9	H	23.9	29.6	53.6
Positive	1903–1929	8	H	25.1	23.5	48.6
Inverted	1903–1929	8	H	24.8	23.8	48.6
PIG IRON PRODUCTION						
Positive	1879–1933	15	H	52.2	57.7	109.9
Positive	1882–1929	14	H	53.1	51.2	104.3
Inverted	1882–1929	14	H	52.1	51.9	104.1
RAILROAD BOND YIELDS						
Positive	1868–1899	8	H	8.6	8.4	17.0
Positive	1869–1896	7	H	8.6	8.4	17.0
Inverted	1869–1896	7	H	8.6	8.4	17.0

[a] U stands for upward, D for downward, H for horizontal.

specific cycle, and thus includes exactly the same cycles as do the inverted averages.[19] We find a systematic difference between the positive and inverted measures in the samples in which the secular trend has a definite direction; but the differences are small in relation to the size of the figures, and are practically confined to the rise and fall taken separately. In general, as the table suggests, the inclusion or omission of an additional cycle may well exercise a greater effect on the averages than a shift from positive to inverted analysis or vice versa for the same group of cycles.[20]

It follows, if the analysis is positive, that the amplitude of a declining

19 That applies to the rise and fall in specific-cycle relatives; the bases, on which the relatives are computed, unavoidably cover the maximum numbers of cycles shown for the positive analysis in the table.

20 The trend-adjusted forms of pig iron production and the A.T.&T. index behave a little like series with downward trends (Table 32). This or the opposite result can happen, since the cycle bases of trend-adjusted data are almost certain to vary somewhat. See pp. 287-9 for further analysis of the effect of positive versus inverted treatment on measures of amplitude.

phase of an incomplete specific cycle at the beginning of a series, or of a rising phase at the end, can be approximated by treating it as part of an inverted cycle. Similarly, if a series is analyzed on an inverted basis, the amplitude of an extra phase at the beginning or end of the series can be approximated by treating it as part of a positive cycle. We follow this practice in making Table S2, since we wish to record as fully as we can the amplitudes of the cyclical movements recognized in the series. For example, the relatives on the first line in columns (3) and (4) of Table 31 are computed on the base of the inverted cycle from May 1907 to June 1910; the remaining entries on this line are derived in the usual fashion. But we do not include the entries for the incomplete cycle in the averages, mainly because comparisons of the averages for the phases of rise and fall are as a rule facilitated if based on the same number of observations.

A more serious difficulty is that our measure of amplitude is not entirely independent of the unit in which the original data are expressed.[21] For example, the measure is the same whether a series is expressed in dollars or cents, in long tons or short tons, in miles or yards. But we shall get one measure of amplitude if the price of eggs is expressed as so many cents per dozen, and another measure if the price is expressed as so many dozens per dollar; or if London exchange is expressed in dollars per pound, instead of shillings per dollar. Again, a series showing the percentage of workers unemployed is apt to yield very different measures of amplitude than the same data converted into employment percentages. For example, the percentage of trade union members unemployed was exceptionally low in Great Britain during 1914–17, the average for this cycle being 1.091 per cent. At the peak of the cycle in August 1914 unemployment was 5.20 per cent, then fell to a trough of .33 per cent in April 1917, and rose to another peak of 1.17 per cent in December 1917; the total swing is thus 5.71 per cent.[22] When 5.71 is expressed as a relative of 1.091, the small unemployment average for the cycle, we get an impression of an enormous fluctuation—a total swing of 523 per cent. But we get the opposite impression when the unemployment percentages are converted into employment percentages; for now 5.71, expressed as a percentage of 98.909, comes out 6 per cent.

A shortcoming of the amplitude measure is that it cannot be applied to series of figures that are sometimes plus, sometimes minus. Familiar

21 Assume that the original figures of a time series are represented by a_1, a_2, a_3, a_4, etc. Then our measure of amplitude will be unchanged if each term is multiplied by a constant; that is, the measure of amplitude for the series a_1k, a_2k, a_3k, etc. is the same as for the original series. But the measure will be different if the original data are converted to the form $\frac{k}{a_1}, \frac{k}{a_2}, \frac{k}{a_3}$, etc., or if they are converted to the form $k - a_1$, $k - a_2$, $k - a_3$, etc.

22 The figures cited are three-month averages centered on the turns. The figure for 1914 is seasonally adjusted; the others are not, no seasonal calculation being made for the few disturbed years that followed. The original figures come from the Ministry of Labour, *Twenty-first Abstract of Labour Statistics of the United Kingdom (1919–33)*, p. 69.

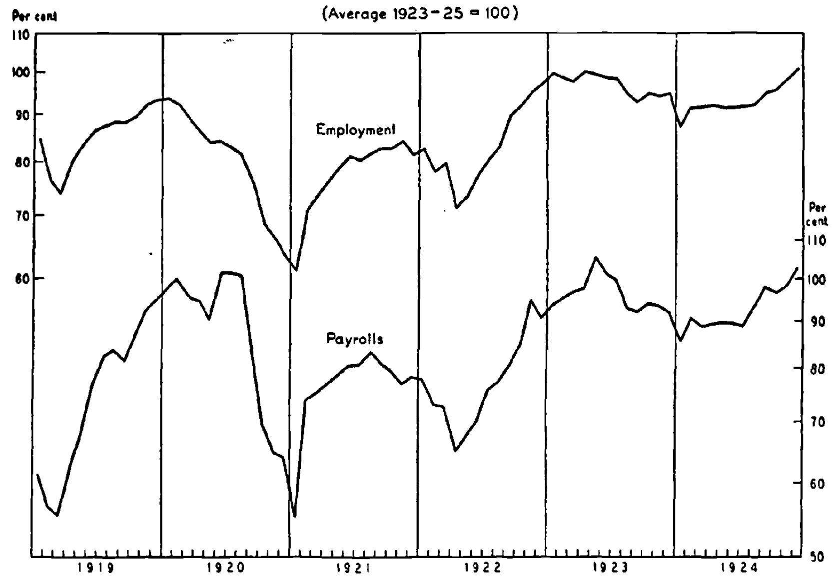
examples are series on profits or losses, net movements of gold from one country to others, differentials between interest rates in the same or different markets, net changes in inventories or plant equipment. In such series the cycle base may approach zero or even be negative, and thus produce preposterous amplitudes. This difficulty is not peculiar to the amplitude measures; it is common to all measures in our scheme of analysis that involve percentages, which means all the measures we make except those in Table S1 and the conformity indexes in Tables R3 and R4. The alternative we have adopted for handling series with plus or minus figures is to work with absolute deviations from the average for each cycle, instead of percentage deviations. Amplitudes measured in this way can be compared for series expressed in the same unit, but not for series in diverse units.

Another troublesome point is that our rules for identifying specific cycles sometimes fail to yield clear-cut results. This difficulty too is not peculiar to the amplitude measures, but applies to all measures of specific cycles. To minimize the difficulty we check carefully the specific-cycle decisions for related series, and thus attempt to prevent inconsistent treatment of marginal cases. Sometimes we leave a period beclouded by violent erratic movements out of the averages, or else use a range instead of a single figure to indicate the central tendency, one average including and the other excluding the doubtful movements.[23] But even when there are no marginal cases in the sense of our rules, the results may be unsatisfactory, for the rules themselves suffer from discontinuity. An extreme instance is illustrated by Chart 13, showing the employment and payrolls in dyeing and finishing textile plants from 1919 to 1924. The cyclical movements of the two series are similar except that payrolls have wider fluctuations than employment, which is to be expected. Yet, so long as we adhere rigidly to the rules, we must recognize one cycle less in payrolls than in employment; for the interval between the peaks of payrolls in 1920 and 1921 is only 13 months, while the minimum interval that can qualify as a specific cycle under our rules is 15 months. Since we must choose between the peaks in 1920 and 1921, it seems better to select the former. If we now recognize a trough in January 1921, which seems unavoidable, we are bound to ignore the contraction of 1921 to 1922, although it is both longer and larger than the corresponding contraction in employment. Decisions of this character are obviously absurd and can only lead to confused results and interpretations. They can be avoided by any of three devices: by omitting the period causing trouble from the averages of both series, by recognizing peaks in both 1920 and 1921 in payrolls despite the rules, or by ignoring the contraction from 1921 to 1922 in employment as well as in payrolls. It is wiser to set the rules aside

[23] Cf. Ch. 8, Sec. X.

CHART 13

Employment and Payrolls in Dyeing and Finishing Textile Plants
United States, 1919 – 1924
(Average 1923–25 = 100)



Adjusted for seasonal variations. Source: Bureau of Labor Statistics, Revised Indexes of Factory Employment and Payrolls, 1919 to 1933 *(Bulletin 610)*, Feb. 1935, pp. 60, 102. Logarithmic vertical scales

momentarily when they lead to absurd results than to stand by and permit them make a mockery of common sense.

Some modification of the standard procedure of measuring amplitudes may be desirable in other situations. Suppose, for example, that we wish to compare the amplitude of the cyclical declines in different branches of production during the contraction of 1929–33 in the United States. It would not do to measure the change during the same interval for each industry, since the timing of the peaks and troughs varies considerably from activity to activity. If the amplitude of the full cyclical declines is to be determined, the measures must be made from the specific cycles of the individual series. But work with specific cycles in our usual fashion is awkward for two reasons. In the first place, some industries skipped the contraction of 1926–27, while a few skipped both this and the 1923–24 contraction, so that the bases on which specific-cycle relatives are computed cover widely varying periods in different series. To meet this difficulty the decline of each series may be expressed as a percentage of its standing at the peak from which the decline started; or better, as a percentage of its average value during the reference cycle 1927–33. The second difficulty is that although the contraction of 1929–33 engulfed virtually the entire industrial system, the long decline was interrupted by several abortive revivals, one of which lasted long enough to produce an 'extra' specific-cycle expansion in the production of many consumer goods industries.[24] The peak of the extra expansion was as a rule lower, sometimes much lower, than the peak in the vicinity of the reference turn in 1929. For the special purpose of comparing the amplitude of the cyclical declines in different branches of industry during the business-cycle contraction of 1929–33, it is probably best therefore to ignore entirely the extra cyclical movements. But the extra cycles cannot be ignored in other connections: they are a conspicuous feature of the Great Depression, and they raise the question whether similar developments occurred in other protracted depressions of general business.

It must always be remembered that the large amplitude of one activity in units of cycle relatives and the small amplitude of another activity tell us little about the relative importance of the two activities in business cycles. For example, industrial building contracts in the United States have larger cyclical amplitudes than residential building contracts; this relation is reversed if the amplitudes are measured in dollars instead of cycle relatives. Again, since repair work has smaller amplitudes in cycle relatives than new building, it may be said to 'moderate' the cyclical fluctuations of total building construction inclusive of repair work. But this observation means merely that the cyclical swing of total building is a smaller percentage of its average level than the swing of new building is of its average. So far as the cyclical timing of repair work is similar to that of new building, repair work is obviously an intensifying factor in building fluctuations; that is, it tends to increase the fluctuations of the industry whether measured in man-hours of employment or in dollars expended. One more example may be cited. In recent business cycles the construction of new railroad lines in this country appears to be nearly as volatile, on a percentage basis, as in the eighteen-seventies or eighties, when thousands of miles of road were added each year to the railroad system. But the enormous percentage fluctuations of recent times excite little interest, since the construction of new railroads has shrunk to insignificance both absolutely and in relation to total industrial activity.

These illustrations point a moral, and one that applies to all the measures we use to describe cyclical behavior, not only to the amplitudes. These measures cannot be interpreted properly unless the relation of each process to the economy at large is brought out. For example, we must know the relative importance of agriculture, of exports and imports, of railroads and other transport agencies, of commercial banks, of construction work, of the steel industry, and so on, in each of the four countries represented in our statistical collection. In these instances, useful criteria of importance are the contribution of a factor to the national income, the number who get their living from the activity in question, and the capital invested. Such indicators of the importance of an activity

[24] See our paper, Production during the American Business Cycle of 1927–1933 (*Bulletin 61*), especially pp. 18-19.

must be supplemented by knowledge of its industrial and financial relation to other activities. It is well to know that railroads rate much higher as employers of labor than banks; yet it is no less essential to recognize that railroads and banks are alike in that all modern business depends upon their continuous functioning, that business cycles existed before railroading though not before commercial banking, that when banks provide more credit the volume of a nation's circulating medium is increased, but that when railroads move more traffic the direct monetary effect is merely an increase in the exchange velocity of 'money'.

V Measures of Secular Movements

The 'intra-cycle' trend of a series is that portion of its secular movement which occurs within the period of a single cycle. The 'inter-cycle' trend is that portion of the secular movement which cumulates from cycle to cycle. The former we retain in our measures of cyclical behavior; the latter we eliminate in computing specific-cycle and reference-cycle relatives.

The element of trend retained in our measures can be judged from the difference between the average rise and the average fall of the specific cycles, or from the tilt of the curves on the charts of cyclical patterns. But these approximations are rough and may be misleading, especially when the series is short. Hence we set out in Table S3 some of the chief facts concerning the secular movements of each series we study. A sample for bituminous coal production appears in Table 33.

Columns (2) and (3) of Table S3 show the average monthly standing of the seasonally adjusted data during the expansions and contractions of specific cycles. If there is a full phase of an incomplete specific cycle at the beginning or end of a series, its average value is also recorded in these columns. In computing the average standing during an expansion or contraction, the values at the peak and trough dates receive a weight of one-half each and the intervening monthly values a weight of one each. Column (4) shows the average standing of the seasonally adjusted data during each specific cycle. This figure may be obtained by striking a mean of the average standings in the expansion and contraction phases, each weighted by its duration; which is equivalent to assigning a weight of one-half each to the values at the initial and terminal troughs and a weight of one to each intervening value. As previously explained, the entries in column (4) are the bases on which the specific-cycle relatives are computed.

The entries in later columns are elaborations of the figures in columns (2) to (4). Column (5) states by what percentage the average standing during an expansion is higher or lower than the average during the preceding contraction. Similarly, column (6) states by what percentage the level of

TABLE 33
Sample of Table S3: Secular Movements
Bituminous Coal Production, United States, 1907–1938

Dates of specific cycles	Average monthly standing (million short tons)			Per cent change from preceding phase		Per cent change from preceding cycle on base of			
						Preceding cycle		Average of given and preceding cycle	
Trough – Peak – Trough (1)	Expansion (2)	Contraction (3)	Full cycle (4)	Contraction to expansion (5)	Expansion to contraction (6)	Total (7)	Per month (8)	Total (9)	Per month (10)
May 07–Jan. 08	...	32.3	...	...	...	...	...	...	...
Jan. 08–June 10–Feb. 11	30.8	34.0	31.5	−5	+10	...	...	...	...
Feb. 11–Oct. 13–Nov. 14	37.0	36.5	36.9	+9	−1	+17	+0.41	+16	+0.39
Nov. 14–July 18–Mar. 19	42.5	43.7	42.7	+16	+3	+16	+0.33	+15	+0.31
Mar. 19–Dec. 20–July 22	43.3	34.0	38.9	−1	−21	−9	−0.20	−9	−0.20
July 22–May 23–June 24	42.1	44.7	43.6	+24	+6	+12	+0.38	+11	+0.35
June 24–Mar. 27–Dec. 27	44.8	41.2	44.0	0	−8	+1	+0.03	+1	+0.03
Dec. 27–May 29–July 32	42.3	35.6	37.7	+3	−16	−14	−0.29	−15	−0.31
July 32–Mar. 37–Mar. 38	31.3	34.1	31.8	−12	+9	−16	−0.26	−17	−0.28
Average	...	...	...	...	...	...	...	+0.3	+0.04
Average deviation	...	...	...	...	...	...	...	12.0	0.26
Weighted average	...	...	...	...	...	...	...	...	+0.01

Average is the arithmetic mean; the average deviation is measured from the mean.

a given contraction is above or below that of the preceding expansion. In both columns the percentages are computed on the base of the preceding phase. Together they show the height of the steps from one phase of specific cycles to the next, the width of the steps being already entered in columns (6) and (7) of Table S1. When the secular trend rises, as in the first few cycles of coal output, the average standing during contraction is often higher than during the preceding expansion. The mean levels of successive phases thus bring out important features of cyclical movements in an economic system characterized by secular changes that differ in pace and direction within given processes during different periods and among different processes during the same period.

For our purposes the magnitude of the 'intra-cycle' trend can usually be judged sufficiently well from the line of 'inter-cycle' trend, that is, the step-line formed by the mean levels of successive specific cycles. In column (7) we therefore give the percentage change from the mean level of one specific cycle to the mean level of the next. We take account of the width of the steps between successive cycles in column (8), which shows the percentage change per month between the mean levels of successive cycles. This column is obtained by dividing the number of months from the midpoint of one cycle to the midpoint of the next into the entries in column (7). Thus the duration of the first specific cycle in coal output is 37 months and of the second cycle 45 months; the interval between their midpoints is 41 months; the mean level of the second cycle is 17 per cent higher than the mean level of the first; therefore the percentage change per month between the two cycles is +0.41.

While the measures in columns (5) to (8) show in a simple manner the percentage changes between successive phases or cycles, we would get biased results if we averaged these measures for a number of cycles. Assume, for example, that the average monthly standings of successive cycles are 80, 40, 80, 40, 80. There is, obviously, no secular trend in this series. Yet the percentage changes between successive cycles are −50, +100, −50, +100; hence the average percentage change from cycle to cycle is +25. This result exemplifies the familiar statement that percentage changes have an 'upward bias'. But the bias disappears if the average of the two values being compared is taken as the base of the percentage, instead of the first value. On this plan the bases are uniformly 60 in our example; the percentage changes are alternately −67 and +67, and their average is 0. We enter in column (9) percentages computed in this manner; that is, instead of using the average standing in the preceding cycle as the base, as we do in column (7), we use a simple average of the average standings in the given and preceding cycles as the base.[25] Column (10) differs from (8) in the same way that column (9) differs from (7). Averages for all cycles are restricted in Table S3 to columns (9) and (10), because these alone are free from 'secular bias'.

At the foot of column (10) both simple and weighted averages are entered. The latter weights the individual entries by the intervals to which they relate, that is, the intervals between midpoints of successive cycles. In practice the weighted average is most readily obtained by dividing the sum of the entries in column (9) by the number of months from the midpoint of the first to the midpoint of the last full cycle in the series. Whereas the unweighted average is likely to vary with specific-cycle decisions, the weighted average is virtually independent of those decisions. We use this weighted figure more commonly than any other in the table. We have found empirically that it is a good approximation to the average percentage change per month determined from a 'least squares' exponential.

When a series is analyzed on an inverted basis, the procedure is the same except that the average standing during contraction is entered in column (2), the average standing during expansion in column (3), the

[25] Let A be the average standing of a series during a given specific cycle, KA the average during the next cycle, F the customary measure of percentage change, and J the measure described in the text as free from secular bias. Then

$$F = 100\left(\frac{KA - A}{A}\right) = 100\,(K-1), \text{ and } J = 100\left(\frac{KA - A}{\frac{KA+A}{2}}\right) = 200\,\frac{K-1}{K+1}.$$

The theoretical limits of F are −100 and ∞, while the limits of J are −200 and +200. Since $J = \left(\frac{2}{K+1}\right)F = \frac{200F}{200+F}$, in practice we first compute F, then find the equivalent J from a table relating F and J.

percentage change from expansion to contraction in column (5), and the percentage change from contraction to expansion in column (6). In handling series such as net changes in inventories, the successive values of which are sometimes plus and sometimes minus, we show absolute instead of percentage changes in the table.

Table S3 supplies what we wish to know: the shifts in the average level of a series from one phase of specific cycles to the next and from one full cycle to the next. We should use different procedures were we concerned primarily with secular trends instead of cyclical fluctuations.[26] Of course, if a series is free from specific cycles, Table S3 cannot be made. In such series we use measures like those in Table S3, but made from reference cycles instead of specific cycles. Since reference cycles cover uniform periods in all series, this plan facilitates comparisons of different series for some purposes, and it may prove profitable at a later stage to extend this computation to all series. However, the weighted average in column (10) is virtually bound to be the same when computed from reference cycles as when computed from specific cycles, unless a series is short and the specific and reference cycles match very badly at the ends.

VI Specific-cycle Patterns

While Table S1 shows the timing and duration of the specific cycles of economic activities, Table S2 their amplitudes, and Table S3 the changes in their average levels, these tables do not give a clear idea of the form of the mounting wave from trough to crest or of the subsiding wave from crest to trough. Supplemental measures are needed to show in some detail the progress of the cyclical fluctuations. These measures are supplied by Table S4, which records the average standing of a series during nine segments of each specific cycle. A sample for bituminous coal production is shown in Table 34.

The nine-point 'pattern' of a specific cycle is made on the following plan. First, the average of the cycle relatives in the three months centered on the initial trough of the specific cycle is entered in column (2). Next, the expansion phase—strictly speaking, the interval beginning with the month after the trough and ending with the month before the peak—is subdivided into three parts as nearly equal as may be without using fractions of a month. The average standing of the cycle relatives in each third of expansion is computed, and the results entered in columns (3) to (5). The standing at the specific-cycle peak is represented by a three-month average centered on that date; this average is entered in column (6). The contraction phase, that is, the interval beginning with the month after the peak and ending with the month before the terminal trough, is

26 Indeed, as our work on business cycles deepens, we find at times a need for mathematical representations of secular movements or for measures other than those recorded in Table S3.

divided into thirds as is the expansion phase, and the average standings are entered in columns (7) to (9). Finally, the average standing in the three months centered on the trough that links the given cycle to the next is presented in column (10). For convenience, the successive stages into which each specific cycle is broken are designated by Roman numerals from I to IX: the stage of the initial trough is represented by I, of the peak by V, and of the terminal trough by IX.

TABLE 34

Sample of Table S4: Specific-cycle Patterns

Bituminous Coal Production, United States, 1907–1938

Dates of specific cycles	Average in specific-cycle relatives at stage								
	I	II	III	IV	V	VI	VII	VIII	IX
	3 mos. centered on initial trough	Expansion			3 mos. centered on peak	Contraction			3 mos. centered on terminal trough
Trough – Peak – Trough		First third	Middle third	Last third		First third	Middle third	Last third	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
May 07–Jan. 08	...	...	...	...	111.1[a]	109.7[a]	107.3[a]	97.0[a]	83.6[a]
Jan. 08–June 10–Feb. 11	82.7	87.8	94.9	111.4	111.6	106.4	109.5	107.8	102.2
Feb. 11–Oct. 13–Nov. 14	87.3	92.5	101.9	106.9	110.2	104.3	95.9	96.3	88.0
Nov. 14–July 18–Mar. 19	76.0	86.7	101.3	110.5	124.3	117.5	100.2	88.9	85.2
Mar. 19–Dec. 20–July 22	93.4	108.8	104.1	119.8	122.7	89.9	87.5	83.6	64.0
July 22–May 23–June 24	57.2	83.6	100.5	108.3	127.0	119.1	95.8	91.8	86.8
June 24–Mar. 27–Dec. 27	85.9	90.7	102.8	111.5	113.1	96.2	92.4	87.6	85.8
Dec. 27–May 29–July 32	100.1	109.3	113.1	114.3	124.0	114.3	95.1	74.2	57.7
July 32–Mar. 37–Mar. 38	68.4	87.3	96.3	111.1	128.0	112.2	114.4	94.8	83.2
Average	81.4	93.3	101.9	111.7	120.1	107.5	98.8	90.6	81.6
Average deviation	10.6	7.8	3.6	2.6	6.4	8.3	6.9	7.0	10.4

Average is the arithmetic mean; the average deviation is measured from the mean.

[a] Computed on base of inverted cycle, May 1907–June 1910. Excluded from the average and the average deviation.

Thus the nine-point pattern is an elaboration upon the skeleton framework in columns (2) to (4) of Table S2. The standings in stages I, V and IX are taken directly from that table. When one or two months instead of the usual three are used in Table S2 to represent the standing at a peak or trough, that is done also in Table S4. Stage V overlaps stages IV and VI; that is, of the three months usually included in stage V, the first is included also in IV and the last in VI. In a similar manner stage I (or IX) overlaps stages VIII and II. Since stages IV and VI omit the peak month, and stages VIII and II omit the trough month, the intervals designated as 'expansion' and 'contraction' in Table S4 cover one month less than the full phase. If the duration of the expansion or contraction, thus defined, is exactly divisible by three, the distribution of months into thirds is straightforward. If the division yields a remainder of one, the extra month is placed in the middle third. If the remainder is two, an extra month is assigned to the first and last thirds.[27] Thus the successive stages of expansion or contraction may cover 6, 6, 6 months, or 6, 7, 6, or 7, 6, 7. This plan ensures that the midpoint of stage III is also the midpoint of the expansion, and that the midpoint of stage VII is also the midpoint of the contraction. Further, it tends to make the average number of months in each third of a run of expansions or contractions equal, since the probability that the duration of a phase when divided by three will leave a remainder of zero is presumably the same as the probability that the remainder will be one or two.

TABLE 35
The Computation of Specific-cycle Patterns Illustrated
Bituminous Coal Production, United States, 1908–1919

Cycle	Stage	Period covered	No. of months	Average monthly standing in: Millions of short tons	Average monthly standing in: Cycle relatives	Interval from stage to stage (mos.)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
	I	Dec. 1907 – Feb. 1908	3	26.033	82.7	...
	II	Feb. 1908 – Oct. 1908	9	27.644	87.8	5.0
Jan. 1908–Feb. 1911 (Av. monthly value = 31.482 million short tons)	III	Nov. 1908 – Aug. 1909	10	29.870	94.9	9.5
	IV	Sep. 1909 – May 1910	9	35.056	111.4	9.5
	V	May 1910 – July 1910	3	35.133	111.6	5.0
	VI	July 1910 – Aug. 1910	2	33.500	106.4	1.5
	VII	Sep. 1910 – Nov. 1910	3	34.467	109.5	2.5
	VIII	Dec. 1910 – Jan. 1911	2	33.950	107.8	2.5
	IX	Jan. 1911 – Mar. 1911	3	32.167	102.2	1.5
	I	Jan. 1911 – Mar. 1911	3	32.167	87.3	...
	II	Mar. 1911 – Dec. 1911	10	34.090	92.5	5.5
Feb. 1911–Nov. 1914 (Av. monthly value = 36.853 million short tons)	III	Jan. 1912 – Nov. 1912	11	37.545	101.9	10.5
	IV	Dec. 1912 – Sep. 1913	10	39.400	106.9	10.5
	V	Sep. 1913 – Nov. 1913	3	40.600	110.2	5.5
	VI	Nov. 1913 – Feb. 1914	4	38.425	104.3	2.5
	VII	Mar. 1914 – June 1914	4	35.350	95.9	4.0
	VIII	July 1914 – Oct. 1914	4	35.500	96.3	4.0
	IX	Oct. 1914 – Dec. 1914	3	32.433	88.0	2.5
	I	Oct. 1914 – Dec. 1914	3	32.433	76.0	...
	II	Dec. 1914 – Jan. 1916	14	37.000	86.7	7.5
Nov. 1914–Mar. 1919 (Av. monthly value = 42.667 million short tons)	III	Feb. 1916 – Apr. 1917	15	43.207	101.3	14.5
	IV	May 1917 – June 1918	14	47.143	110.5	14.5
	V	June 1918 – Aug. 1918	3	53.033	124.3	7.5
	VI	Aug. 1918 – Sep. 1918	2	50.150	117.5	1.5
	VII	Oct. 1918 – Dec. 1918	3	42.767	100.2	2.5
	VIII	Jan. 1919 – Feb. 1919	2	37.950	88.9	2.5
	IX	Feb. 1919 – Apr. 1919	3	36.367	85.2	1.5

The illustration covers the first three full specific cycles in coal production; see Table 34.

The illustration worked out in Table 35 may clarify the details. The initial trough of the first complete specific cycle recorded for bituminous coal production comes in January 1908. Hence the standing in stage I is an average of the specific-cycle relatives for December 1907, January

[27] We are indebted to W. Allen Wallis for suggesting this arrangement.

1908, and February 1908, each computed on the base January 1908–February 1911. As stated before, this result is obtained more easily by averaging the original values in the three months and then converting the average into a relative, and the calculations in Table 35 are shown in this form. The 'expansion' phase covers the period from February 1908 through May 1910, or 28 months in all. Hence stages II, III and IV cover successively 9, 10 and 9 months. Stage V includes the three months May–July 1910, but May is included also in stage IV and July in stage VI. Stage IX includes the three months January–March 1911, and thus laps over stage VIII of the given cycle and stage II of the next cycle. The three months entering into stage IX of the cycle from January 1908 to February 1911 constitute also stage I of the cycle from February 1911 to November 1914. But the standing in stage IX of the first cycle is 102 and the standing in stage I of the next cycle is only 87; the difference means that the monthly average of the data for January–March 1911 is 102 per cent of the monthly average for January 1908–February 1911, and 87 per cent of the average for February 1911–November 1914.

When the specific cycles of a series are treated as inverted, the procedure is the same but now the sequence of stages runs from the initial peak (stage I) to the terminal peak (stage IX). A sample of an inverted analysis appears in Table 36. If there is a full phase of an incomplete specific cycle at the beginning or end of a positive analysis, as in coal production, this phase is subdivided in the usual fashion, but the standings are expressed as relatives on the base of an inverted cycle and excluded from the averages of all cycles. If a series is treated on an inverted plan, the standings of a full phase of an incomplete specific cycle are

TABLE 36

Sample of Table S4: Specific-cycle Patterns on Inverted Plan
Slab Zinc Stocks at Refineries, United States, 1921–1938

Dates of specific cycles	Average in specific-cycle relatives at stage								
	I	II	III	IV	V	VI	VII	VIII	IX
	3 mos. centered on initial peak	Contraction			3 mos. centered on trough	Expansion			3 mos. centered on terminal peak
Peak – Trough – Peak		First third	Middle third	Last third		First third	Middle third	Last third	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
July 21–Apr. 23–July 24	237.8	190.2	102.7	42.5	28.3	53.5	88.9	103.1	135.6
July 24–Nov. 25–Sep. 28	174.4	125.1	63.1	54.6	27.0	67.9	115.0	143.5	157.4
Sep. 28–May 29–Jan. 31	58.4	58.6	55.4	45.9	44.4	64.7	120.7	171.1	182.7
Jan. 31–Oct. 33–Dec. 34	117.7	110.7	106.3	100.3	80.1	87.4	85.1	88.7	95.6
Dec. 34–Aug. 37–June 38	153.8	145.0	107.5	39.8	16.8	35.8	114.8	175.5	194.2
Average	148.4	125.9	87.0	56.6	39.3	61.9	104.9	136.4	153.1
Average deviation	48.3	33.3	22.2	17.5	18.3	13.8	14.3	32.4	30.0

Average is the arithmetic mean; the average deviation is measured from the mean.

expressed as relatives on the base of a positive cycle. In the few series consisting of plus or minus values, whether analyzed on a positive or inverted basis, the standings in successive stages of the cycles are shown as absolute deviations from the cycle base, instead of in percentages.[28]

Our method of tracing cyclical patterns makes implicit assumptions about the general character of cyclical movements. The principal assumption is that the turns of specific cycles come in single months; that is to say, the tops and bottoms of specific cycles are rounded or angular, not flat. On the whole this assumption is amply justified by experience. Specific cycles consisting of a horizontal low level followed by a stationary high level, which in turn is followed by a stationary low level and so on, appear very rarely in practice. But intermediate types are troublesome, as when a specific cycle consists of an expansion, plateau, and contraction; or of an expansion, contraction, and flat bottom; or of an expansion, flat top, contraction, and flat bottom. Our standard practice has been to date the peak towards the end of a flat top and the trough towards the end of a flat bottom—a method that obviously biases the cyclical patterns. But such cases are few in proportion to the number of cycles we have analyzed; they can be readily spotted on the data charts, and qualitative amendments made where needed. A simple quantitative adjustment could be made by including all months within the flat top in the standing at stage V, including all months within the flat bottom in stages I and IX, assigning the months between the end of the flat bottom and the beginning of the flat top to stages II, III and IV, and the months between the end of the flat top and the beginning of the flat bottom to stages VI, VII and VIII. But this method also will not work perfectly in practice, for so-called 'flat' tops or bottoms may be jagged and therefore not easy to delimit. And when the tops and bottoms do happen to be perfectly flat, the expansions and contractions are likely to be staircase movements, and this feature will be concealed by the adjustment just as it is concealed by the standard plan of making patterns.

Another assumption that underlies our method of tracing cyclical patterns is that the phases of specific cycles cover fairly large numbers of months, so that there is reasonable opportunity for erratic flutterings to disappear. The practice of breaking expansions and contractions into thirds irrespective of their duration becomes a thin formality when the phases are very short. When an expansion lasts four months, three months are covered by the interval exclusive of the peak and trough and hence only one month each is left for stages II, III, and IV. When a phase lasts less than four months, as happens occasionally, we must resort to interpolation to get the standings needed for the successive thirds of the phase.

28 To keep the exposition simple, the remainder of this section, except for the closing paragraphs, is written from the viewpoint of positive analysis. With obvious changes in phrasing, the description applies also to inverted analysis.

Interpolated figures, if any, are enclosed in parentheses in Table S4.[29]

Where needed, the interpolations are made on the following plan. If the short phase is an expansion, the standing at stage III is determined from the cycle relatives for the one or two months in the expansion exclusive of the trough and peak months. The standing at stage II is then determined by interpolating along a straight line between the standings at stages I and III, and the standing at stage IV by interpolating between the standings at stages III and V. In these computations each standing is conceived as placed in the middle of the stage. The midpoint of stage V is treated as coming in the middle of the peak month and the midpoint of stage I as coming in the middle of the trough month, although these stages are commonly represented by two months in very brief phases. This procedure is necessary to ensure that the sum of the intervals from midpoint to midpoint of the stages equals the duration of the cycle; and may be rationalized by saying that stages I, V and IX include in principle only the trough or peak months, but that the cyclical standing at these stages is estimated in practice from one, two or three months.[30]

The disregard of differences in cycle durations in Table S4, and in its congener Table R1, is the most considerable distortion we practise upon the original data at any point in the analysis. At a later stage, we plan to make a special study of long, medium, and short cycles to see whether differences in duration are regularly associated with differences in other aspects of cyclical behavior. Meanwhile we express the relations between the amplitude and duration of cyclical fluctuations in the per month figures of Table S5,[31] which show the average change per month, in units of cycle relatives, from stage to stage of the specific cycles. As the sample in Table 37 indicates, the rate of change of coal production has been far from steady during successive intervals of expansion and contraction. As usual, within its briefer span contraction is somewhat more violent than expansion. But in coal production at least, the most violent declines tend to occur at the beginning and close of contraction.

29 For some examples, see Appendix Table B1.

30 In a three-month expansion, stages II to IV cover two months in all, or two-thirds of a month each; the middle of stage II is five-sixths and the middle of stage III is nine-sixths of a month from the middle of stage I. Hence if we interpolate along a straight line between stages I and III, the standing in stage II is equal to the standing in stage I plus five-ninths of the excess of the standing in stage III over the standing in stage I. Again, the middle of stage IV is four-sixths and the middle of stage V nine-sixths of a month from the middle of stage III; hence the standing in stage IV is equal to the standing in stage III plus four-ninths of the excess of the standing in stage V over that in stage III.

By similar reasoning, if the expansion lasts two months, the standing in stage II equals that in stage I plus two-thirds of the excess of stage III over stage I, and the standing in stage IV equals that in stage III plus one-third of the excess of stage V over stage III.

With obvious changes in stage numerals, the procedure is the same in handling short contractions.

31 Also in Tables S2, R2, R3, and R4 the amplitudes are reduced to a per month basis. In Table S3 per month figures are used to express the relation between changes in the average level of successive cycles and the intervals between their midpoints.

TABLE 37

Sample of Table S5: Rate of Change from Stage to Stage of Specific Cycles
Bituminous Coal Production, United States, 1907–1938

Dates of specific cycles	Average change per month in specific-cycle relatives between stages							
	I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
	Expansion				Contraction			
Trough–Peak–Trough (1)	Trough to first third (2)	First to middle third (3)	Middle to last third (4)	Last third to peak (5)	Peak to first third (6)	First to middle third (7)	Middle to last third (8)	Last third to trough (9)
May 07 – Jan. 08	. . .	. . .	. . .	. . .	−0.9[a]	−1.0[a]	−4.1[a]	−8.9[a]
Jan. 08 – June 10 – Feb. 11	+1.0	+0.7	+1.7	0.0	−3.5	+1.2	−0.7	−3.7
Feb. 11 – Oct. 13 – Nov. 14	+0.9	+0.9	+0.5	+0.6	−2.4	−2.1	+0.1	−3.3
Nov. 14 – July 18 – Mar. 19	+1.4	+1.0	+0.6	+1.8	−4.5	−6.9	−4.5	−2.5
Mar. 19 – Dec. 20 – July 22	+3.8	−0.7	+2.4	+0.7	−9.4	−0.4	−0.6	−5.6
July 22 – May 23 – June 24	+13.2	+5.6	+2.6	+9.4	−3.2	−5.8	−1.0	−2.0
June 24 – Mar. 27 – Dec. 27	+0.8	+1.2	+0.8	+0.3	−8.4	−1.5	−1.9	−0.9
Dec. 27 – May 29 – July 32	+3.1	+0.7	+0.2	+3.2	−1.5	−1.5	−1.7	−2.5
July 32 – Mar. 37 – Mar. 38	+2.0	+0.5	+0.8	+1.8	−6.3	+0.6	−5.6	−4.6
Average	+3.3	+1.2	+1.2	+2.2	−4.9	−2.0	−2.0	−3.1
Average deviation	2.6	1.1	0.8	2.0	2.4	2.2	1.5	1.2
Weighted average	+2.3	+0.9	+1.0	+1.6	−4.5	−1.8	−1.8	−3.2
Average interval (mos.)	*5.3*	*9.8*	*9.8*	*5.3*	*2.8*	*4.7*	*4.7*	*2.8*

$$\frac{\text{Duration of irregular movements}}{\text{Duration of specific cycles}} = \frac{16.5}{370} = 4.5 \text{ per cent}$$

Average is the arithmetic mean; the average deviation is measured from the mean.

[a] Computed on base of inverted cycle, May 1907–June 1910. Excluded from the average, the average deviation, and the average interval.

The figures in Table S5 are obtained by dividing the differences between successive entries in Table S4 by the number of months from the middle of one stage to the middle of the next stage. The calculation of these intervals is illustrated in the last column of Table 35. Whether one, two or three months are used to represent the standing in stage V, the intervals between stages IV and V and between stages V and VI are computed as if the standing in stage V were in the center of the peak month. A like remark applies to stage I (or IX) in relation to adjacent stages. Where the standings of an expansion are interpolated, the average rate of change is computed from stage I to III and from stage III to V; the first represents the change from stage I to II and from stage II to III; the second represents the change from stage III to IV and from IV to V. Contractions are handled on the same principle. The rates of change computed from interpolated values are placed in parentheses, so that they may be easily 'spotted'. There are no instances of this type in coal production.

The average rates of change at the bottom of Table S5 are first shown in unweighted form; that is, the rate of change from stage to stage of every cycle is allowed to count the same regardless of the interval to which it applies. Since the rates of change computed from interpolated values are apt to be of extreme size relatively to corresponding figures in longer cycles, we exclude them as a rule from the averages. But the averages may be distorted also by rates of change during phases that are sufficiently long not to require interpolation, yet too short to reduce erratic movements effectively. One method of handling this difficulty is to omit the 'extreme' items from the averages. Another and perhaps less arbitrary method is to weight the rates of change between successive standings by the intervals to which they apply. Weighted averages are therefore included in Table S5. They are derived, in effect, from the average standings at the bottom of Table S4 and the average intervals at the bottom of Table S5. Thus the excess of the average standing in stage II over the average in stage I, divided by the average interval between these stages, is equivalent to an arithmetic mean of the rates of change in successive cycles weighted by the intervals to which the rates apply; and so on from stage to stage.[32] These weighted averages correspond precisely to the slopes of our graphic pictures of average specific-cycle patterns, described later in this section.

Since stages I, V and IX overlap adjacent stages, the differences between the standings at the cyclical turns and adjacent standings involve a peculiar weighting of individual months. This difficulty could be avoided by not assigning to the 'expansion' any month included in stages I and V, or to the 'contraction' any month included in stages V and IX. However, the use of overlaps adds two months to each phase; and consequently, the chance of wiping out erratic flutterings is improved, the occasions on which interpolating is needed to get cyclical standings are fewer, and the inaccuracy of describing stages II to IV as successive thirds of expansion and stages VI to VIII as successive thirds of contraction is reduced. The advantages of the overlaps seem to outweigh the disadvantages.[33] In practice the two plans yield closely similar cyclical patterns.

The manner in which our method of replacing the full set of monthly values for a cycle by nine average standings irons out erratic movements and discloses the form of the successive cycles is brought out vividly by Chart 14. Curve A shows the original monthly figures of coal production in the United States from 1905 to 1939, and curve B shows these

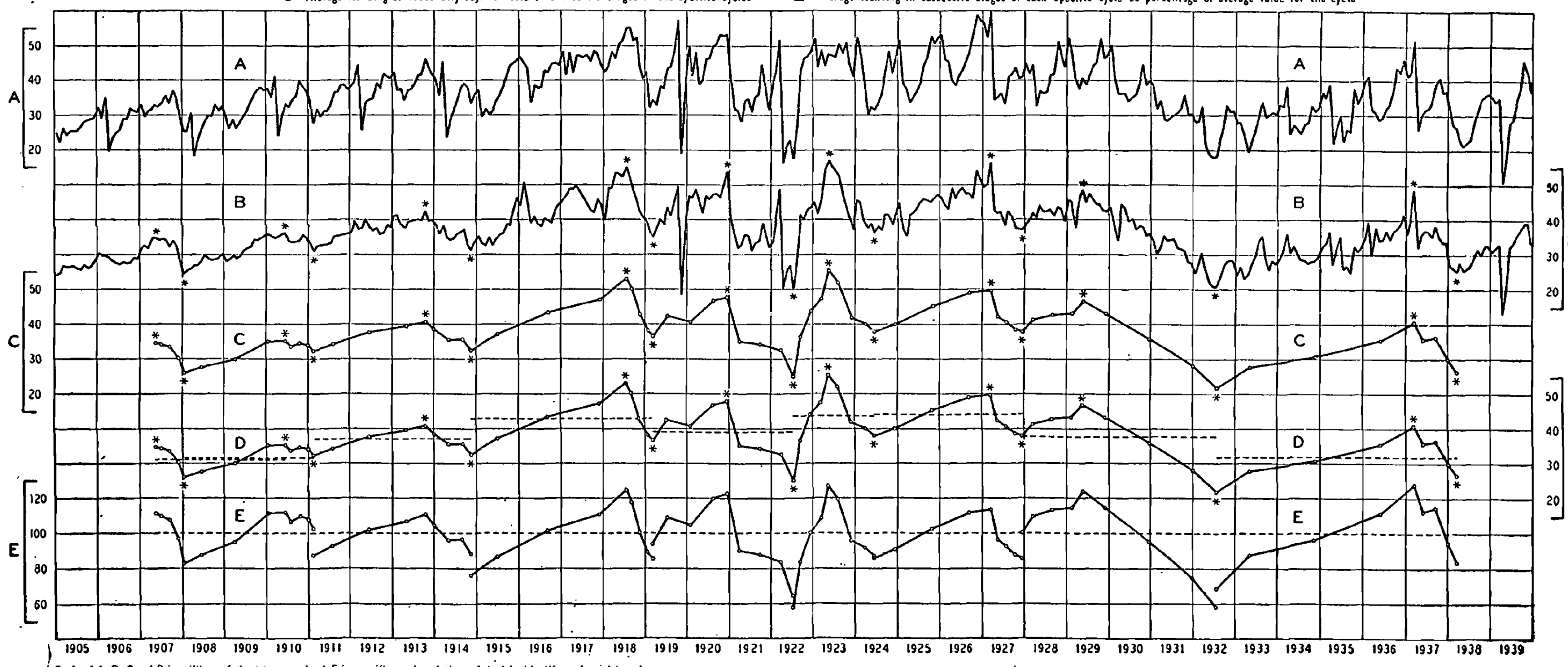
[32] Let the standings of successive cycles in a given stage be $a_1, a_2, \ldots a_N$, the standings in the next stage $b_1, b_2, \ldots b_N$, and the intervals between the midpoints of the adjacent stages $d_1, d_2, \ldots d_N$. Then the rates of change from one stage to the next in successive cycles are $\frac{b_1 - a_1}{d_1}, \frac{b_2 - a_2}{d_2}$, and so on. If these rates are weighted by the corresponding durations, the weighted average is $\frac{\Sigma b - \Sigma a}{\Sigma d}$, or $\frac{\frac{\Sigma b}{N} - \frac{\Sigma a}{N}}{\frac{\Sigma d}{N}}$. In practice, we derive the weighted averages from sums instead of means.

[33] If the overlaps were eliminated, cyclical patterns made from monthly data would be more nearly comparable with patterns made from quarterly data than at present. But our statistical analysis is designed primarily for monthly data, and the analysis of quarterly and anual data adapted as well as may be to the monthly model. See below, pp. 199, 249-50.

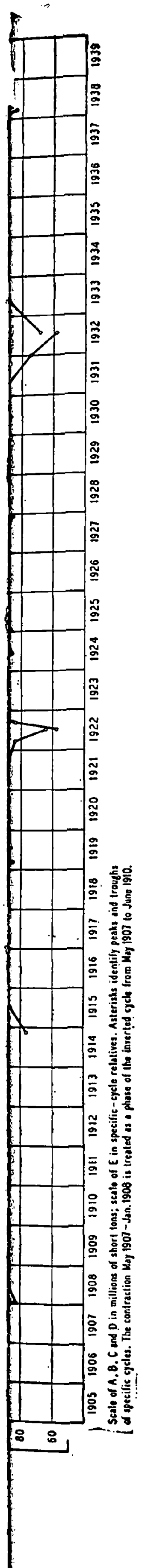
CHART 14

Derivation of Specific-cycle Patterns
Bituminous Coal Production, United States, 1905 – 1939

A Original monthly data
B Seasonally adjusted monthly data
C Average standing of seasonally adjusted data in successive stages of the specific cycles
D Step-line of trend and average standing in successive stages of specific cycles
E Average standing in successive stages of each specific cycle as percentage of average value for the cycle



Scale of A, B, C and D in millions of short tons; scale of E in specific-cycle relatives. Asterisks identify peaks and troughs of specific cycles. The contraction May 1907 – Jan. 1908 is treated as a phase of the inverted cycle from May 1907 to June 1910.



figures adjusted for seasonal variations. Curve C, derived from the seasonally adjusted figures, shows the patterns of successive specific cycles expressed in units of the original data, that is, before adjustment for the changing level of the specific cycles. Curve D is the same as C but is cut across by horizontal lines representing the mean levels of successive specific cycles, and curve E shows the cyclical patterns after elimination of the inter-cycle trend—the standard form in which we put the results for practically all series.

When the amplitudes of erratic movements are modest compared with those of specific cycles, they tend to disappear from view when the full set of monthly values within a cycle is replaced by nine average standings. But when the erratic movements are relatively large, they are likely to leave their stamp visibly on the patterns of individual cycles, sometimes even on the average patterns. Chart 14 indicates that the erratic movements in bituminous coal production are removed in large part, but not entirely. In four specific cycles the rise is continuous from stage I to V and the decline is continuous from stage V to IX; each of the other four cycles shows one irregularity. Any reversal of direction within a phase—that is, a decline between stage I and V, or rise between stage V and IX—is an irregular movement. Such reversals of direction may be due to the failure of erratic movements to cancel out, or to recognition of one specific cycle when two should have been taken, which can happen when erratic movements obscure the specific cycles. Therefore a simple though indirect method of judging the amplitude of the erratic movements in a series relatively to the amplitude of its specific cycles is to compare the number of irregularities in its stage-to-stage movements with the total number of stage-to-stage intervals; or better, the total duration of the intervals having irregular signs with the total duration of all the specific cycles. We do the latter and enter the result for each series at the bottom of Table S5.[34]

The qualitative descriptions of erratic movements noted in Section I of Chapter 4 are derived largely from these percentages. Four classes of percentages are distinguished: 0 to 2.0 is taken to indicate 'mild' erratic movements, 2.1 to 6.0 'moderate', 6.1 to 10.0 'pronounced', and over 10.0

[34] The result for coal production is obtained as follows:

List of irregular movements	*Interval in months*
Stage VI to VII, cycle 1908–11	2.5
Stage VII to VIII, cycle 1911–14	4.0
Stage II to III, cycle 1919–22	6.5
Stage VI to VII, cycle 1932–38	3.5

The total duration of intervals showing irregular signs is 16.5 months. The total duration of specific cycles from May 1907 to March 1938 is 370 months. The first figure is 4.5 per cent of the second.

A slight change between consecutive stages in Table S4 may be lost by rounding in Table S5. Hence, if a zero appears in Table S5, it is necessary to refer to Table S4 to make sure whether a sign cannot be determined.

'very pronounced'. Descriptions carried through on this plan are then checked against independent judgments made from the data charts, and frequently revised in the light of the latter. Although the method is very rough,[35] it helps us appraise the analysis of specific cycles. More confidence can be placed in the measures for specific cycles when the erratic movements are set down as mild than when they are set down as pronounced or very pronounced. These descriptions are included in our worksheets for each series, which list in detail any peculiarities of the data or doubtful features of the analysis.

The patterns developed in Chart 14 are rearranged in Chart 15 to bring out the significance of the average pattern of all the cycles in a series. The vertical scale runs in units of cycle relatives, the horizontal scale in units of time, as in Chart 14. The patterns of the single cycles are plotted so that each peak (standing in stage V) is directly under the preceding one. Since the durations of the cycles vary, the standings at other stages are out of alignment; but they can be readily identified from the dots representing the successive standings. The average specific-cycle pattern at the bottom of the chart is drawn from the average standings in Table S4 and the average intervals between successive stages in Table S5. This pattern gives a composite photograph of the durations, amplitudes and stage-to-stage changes of the individual specific cycles. As is to be expected, it is considerably smoother than the patterns of most individual cycles. Its slope represents the average rate of change from stage to stage of the cycles weighted by the intervals between the stages, and thus corresponds to the weighted averages in Table S5.[36] But the average pattern gives no heed to the sequence in which the cycles actually occurred. This disregard of the historical succession of the cycles is implicit in all the averages we strike of cyclical measures: it is a crucial feature of our technique and raises questions of great importance to which Chapters 9-12 are devoted.

Chart 16 shows our standard method of summarizing graphically some leading features of the specific cycles of a series. The diagram for coal production is the same as that at the bottom of Chart 15 except for additional detail. Several other series are included in the chart to give a glimpse of the variety of cyclical patterns found in economic activities.[37]

35 Not all irregularities of sign are due to erratic movements. If a series flattens out some months before it reaches a specific-cycle peak and falls sharply in the month following the turn, the standing in stage V may be lower than in stage IV; for similar reasons the standing in stage IX may be higher than in stage VIII. On the other hand, absence of irregular signs need not mean that erratic movements are slight. For example, if a declining segment of a highly choppy series is mistakenly treated as a specific-cycle contraction, there may be a continuous decline from stage to stage of this false contraction. In general, the longer the phase of specific cycles the more effectively will the cyclical patterns wipe out erratic movements; our method takes this principle into account, though very imperfectly.

36 See note 32.

37 For several of these series, the patterns of individual cycles are shown in Appendix Table B1.

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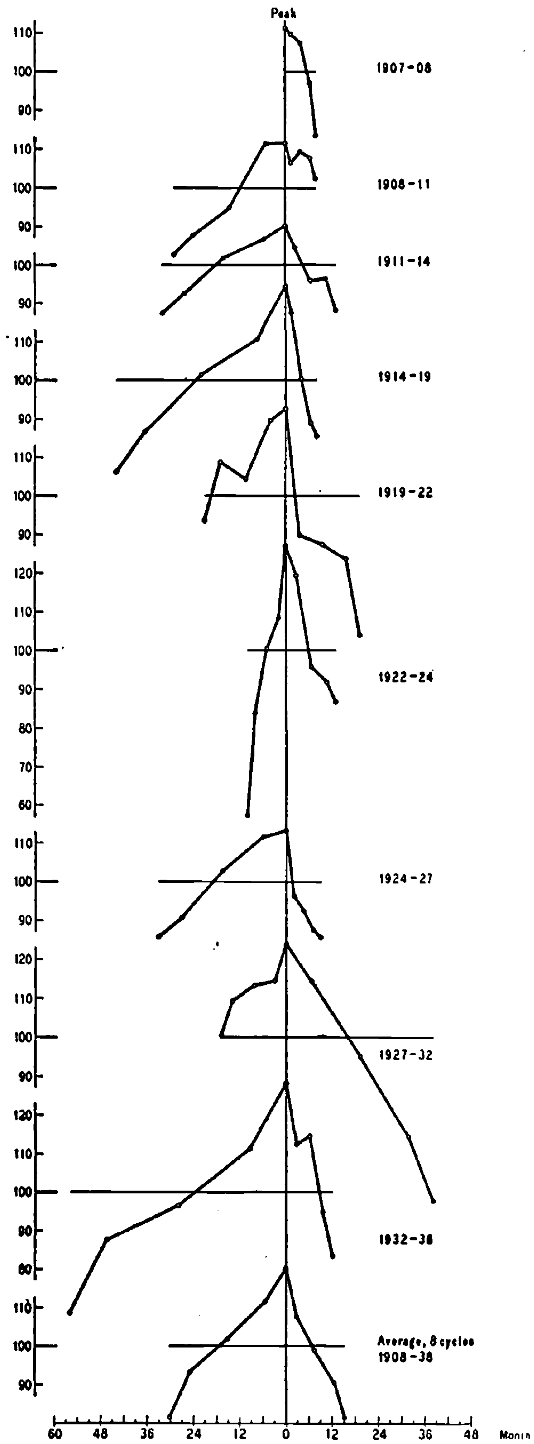
CHART 15

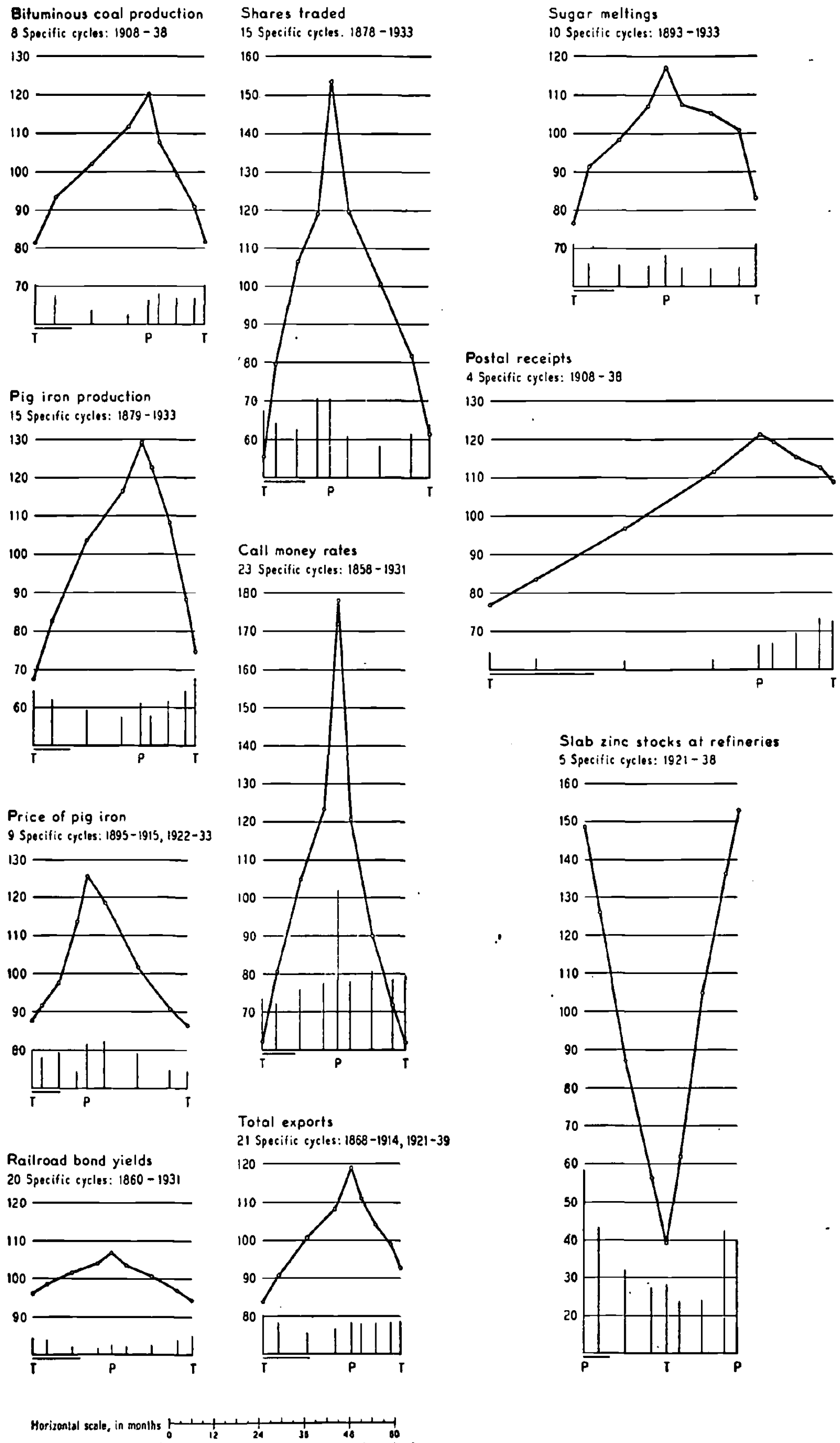
Patterns of Successive Specific Cycles and Their Average Pattern
Bituminous Coal Production, United States, 1907 – 1938



See Chart 14 and explanations in text.

CHART 16

Average Specific-cycle Patterns of Ten American Series



See explanations in text. For sources of data, and other notes, see Appendix C.

The long horizontal line at the bottom of each pattern represents the average duration of the specific cycles; the short horizontal line below it represents the average deviation of the cycle durations. The distances we take to represent time are shown by a ruler, which furnishes a scale for approximating all measurements of duration. The vertical scale at the left runs in units of cycle relatives; with its aid the successive standings of a cyclical pattern and its amplitude can be read. The horizontal line indicating the average duration of the specific cycles is broken by vertical lines erected at the midpoints of the successive cycle stages. The midpoint of the trough stage is marked by 'T' and of the peak stage by 'P' in order that the eye may quickly find these stages. The lengths of the nine vertical lines represent average deviations from the average standings at successive stages of the specific cycles.[38]

When numerous patterns have been worked out, they give lively impressions of the variety of cyclical behavior characteristic of economic processes. Among these patterns we can distinguish various types, and classify the series according to the duration and amplitude of their cyclical movements, the relative magnitude of the average deviations, the rate of change during successive stages of their cycles, the peakedness of the patterns, the presence or absence of 'saw teeth' in the patterns, as well as their positive or inverted shapes.[39] Chart 16 suggests, and our later monographs will demonstrate in detail, that there is little justification for the common notion that cyclical patterns of different activities vary merely in amplitude, or that a sine curve is a satisfactory 'approximation' or 'model' of the specific cycles found in experience.

The patterns in Chart 16 show besides the changing pace of the average rise and fall of the specific cycles, the durations of the expansions and contractions, their amplitude, and the intra-cycle trend. The latter features are standardized in Chart 17, which isolates the information that the cyclical patterns add to the average measures described in preceding sections. The new chart shows merely the rates of change from stage to stage of an average expansion relatively to one another, as well as the rates of change from stage to stage of an average contraction relatively to one another. The computations necessary to make the chart are illustrated in Table 38. The adjusted durations in column (8) are plotted along the horizontal axis, and the adjusted standings in column (4) are plotted along the vertical axis. A similar procedure is used when the analysis is inverted. Here also the expansion is treated separately from the contraction; but in making the adjustments the average standing in stage V is subtracted from the standings in other stages, since the trough now comes in stage V and the peak in stages I and IX.[40]

38 See in this connection the description of specific-cycle patterns in Chart 2.

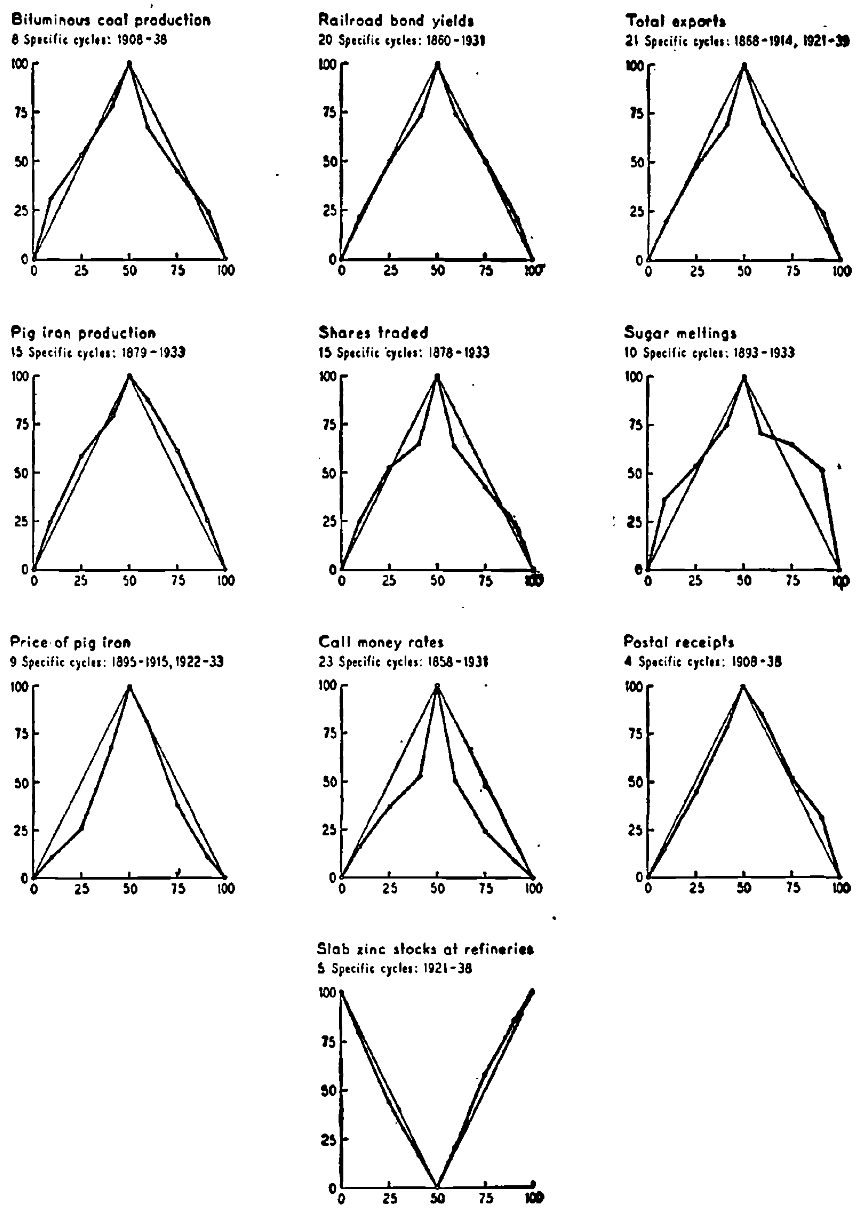
39 Note carefully, however, the last two paragraphs in Sec. I.

40 The principal features of this chart were suggested by Geoffrey H. Moore.

CHART 17

Variation of Average Rates of Change from Stage to Stage of Expansions and Contractions of Specific Cycles

Ten American Series



To facilitate study, the peaks and troughs are joined by straight lines representing uniform rates of expansion and contraction. See explanations in text and Table 38. For sources of data and other notes, see Appendix C.

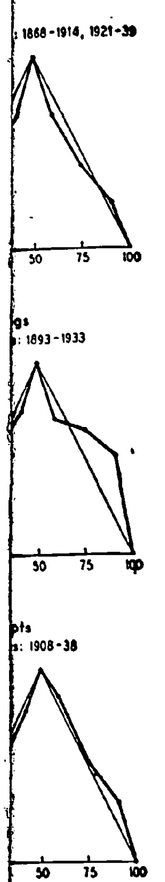


TABLE 38

Adjustment of Average Specific-cycle Pattern to Show Relative Variation of Rates of Change from Stage to Stage of Expansion and Contraction

Bituminous Coal Production, United States, 1908–1938

Expansion of specific cycles							
		Excess over stage I		Average interval in months from			
Stage	Average standing (see Table 34)	Absolute amount	As % of excess for stage V	Preceding stage (see Table 37)	Stage I as origin (cumulatives of col. 5)	Average interval from stage I as origin, as % of twice the duration of expansion	Adjusted interval from stage I as origin (same as col. 7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
I	81.4	0.0	0.0	...	0.0	0.0	0.0
II	93.3	11.9	30.7	5.3	5.3	8.8	8.8
III	101.9	20.5	53.0	9.8	15.1	25.0	25.0
IV	111.7	30.3	78.3	9.8	24.9	41.2	41.2
V	120.1	38.7	100.0	5.3	30.2	50.0	50.0

Contraction of specific cycles							
		Excess over stage IX		Average interval in months from			
Stage	Average standing (see Table 34)	Absolute amount	As % of excess for stage V	Preceding stage (see Table 37)	Stage V as origin (cumulatives of col. 5)	Average interval from stage V as origin, as % of twice the duration of contraction	Adjusted interval from stage I as origin (add 50 to col. 7)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
V	120.1	38.5	100.0	...	0.0	0.0	50.0
VI	107.5	25.9	67.3	2.8	2.8	9.3	59.3
VII	98.8	17.2	44.7	4.7	7.5	25.0	75.0
VIII	90.6	9.0	23.4	4.7	12.2	40.7	90.7
IX	81.6	0.0	0.0	2.8	15.0	50.0	100.0

The adjusted patterns are of uniform amplitude and have no tilt. In series treated on a positive plan the pattern starts at 0 in stage I, rises to 100 in stage V, and falls to 0 in stage IX; in inverted analyses the movement is from 100 to 0 to 100. The durations are also uniform: the expansion runs from 0 to 50 and the contraction from 50 to 100. The adjusted patterns can therefore differ only in their slopes. These correspond precisely to the slopes in Chart 16, in the sense that whatever set of numbers is formed by the slopes of the latter during an average expansion or contraction, taken separately, precisely the same set of numbers multiplied by some constant is formed by the slopes of Chart 17; so that the *relative* variation of the slopes during expansion or contraction is preserved. A glance at Chart 17 reveals whether the average expansion of a series has been characterized by a steady, increasing or decreasing rate of rise, how the rate of decline has behaved on the average during contractions, and the manner and degree in which the pace of expansions or contractions has varied in different series. It is clear, for example, that neither the production nor price of pig iron has moved at a steady average pace during the expansions and contractions of specific cycles; but whereas three-fifths of the rise in production came in the first half of the expansion and three-fifths of the decline in the last half of contraction, in prices only a fourth of the rise was accomplished from the trough to mid-expansion and less than two-fifths of the decline was accomplished from mid-contraction to the trough.

Our methods of analyzing cyclical patterns are far from perfect. We hope that they may excite the reader sufficiently to undertake fresh experiments. One troublesome question is whether the patterns give sufficient detail, particularly in the neighborhood of the turning points. This question is briefly explored in a later chapter. Two related issues are also considered there: whether three-month averages are satisfactory representatives of cyclical peaks and troughs, and whether the results could be improved by preliminary smoothing of the data.[41]

VII Reference-cycle Patterns

The 'S' tables do not show how different economic activities behave at one and the same time. Were our analysis confined to these measures, we would be left with vague notions concerning the relations in time of the fluctuations of different series. Study of the leads and lags in Table S1 would demonstrate that certain series agree more or less closely in timing with the reference turns, and differ by more or less regular intervals from certain other series. But if business cycles really are units of concurrent fluctuations in many activities, the critical point to establish is how different activities fluctuate during fixed periods. To that end, we transfer measurements of cyclical behavior from specific cycles, which vary in timing from series to series, to reference cycles, which occupy uniform periods in all series for a given country.

The behavior of each series during the periods occupied by successive business cycles is shown in a set of 'R' tables. These are a slighter affair than the 'S' tables, because some of the essential comparisons between specific and business cycles have already been made, and because certain measurements of specific cycles need no counterparts in reference cycles. Thus the leads and lags and the duration measurements of Table S1 have compared the timing of specific with that of business cycles. To measure the full amplitudes of cyclical waves, we must have the cyclical peaks and troughs touched by each series at whatever times these turning points are reached—the procedure followed in Table S2. Nor can we measure secular changes for our purpose any better than in Table S3, so long as we center attention on the cyclical behavior of different activities, one by one. Hence we make the 'R' tables in forms corresponding only to Tables S4 and S5, and add two sections measuring the conformity of the series to business cycles.

[41] See Ch. 8, Sec. IV and VI.

TABLE 39

Sample of Table R1: Reference-cycle Patterns
Bituminous Coal Production, United States, 1905–1938

Dates of reference cycles	Average in reference-cycle relatives at stage									Average monthly standing during cycle (million short tons)
	I	II	III	IV	V	VI	VII	VIII	IX	
	3 mos. centered on initial trough	Expansion			3 mos. centered on peak	Contraction			3 mos. centered on terminal trough	
Trough – Peak – Trough		First third	Middle third	Last third		First third	Middle third	Last third		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Aug. 04–May 07–June 08	...	...	97.2[a]	103.2[a]	119.9[a]	116.6[a]	102.9[a]	91.2[a]	98.4[a]	28.8[b]
June 08–Jan. 10–Jan. 12	86.6	88.9	89.4	103.2	108.7	105.9	101.8	105.4	113.9	32.7
Jan. 12–Jan. 13–Dec. 14	99.2	102.7	99.3	98.4	106.4	104.6	102.6	91.8	88.5	37.6
Dec. 14–Aug. 18–Apr. 19	77.9	89.4	100.9	111.2	121.1	109.7	95.6	83.2	88.3	42.7
Apr. 19–Jan. 20–Sep. 21	91.8	99.0	105.4	85.7	106.0	111.6	111.1	81.1	86.1	41.1
Sep. 21–May 23–July 24	87.9	95.0	71.5	113.1	137.5	129.0	105.4	94.2	92.7	40.2
July 24–Oct. 26–Dec. 27	84.5	90.3	102.1	106.0	111.8	116.2	100.5	89.7	85.4	44.2
Dec. 27–June 29–Mar. 33	104.2	114.3	117.9	121.6	130.2	115.6	92.2	70.9	68.1	36.2
Mar. 33–May 37–May 38	76.3	92.7	93.2	115.2	108.8	114.5	105.6	82.2	82.0	32.3
Average	88.6	96.5	97.5	106.8	116.3	113.4	101.8	87.3	88.1	...
Average deviation	7.4	6.6	9.6	8.5	10.0	5.4	4.3	8.0	7.7	...

$$\frac{\text{Average rise \& fall of reference cycles}}{\text{Average rise \& fall of specific cycles}} = \frac{56.0}{77.2} = 73 \text{ per cent}$$

Average is the arithmetic mean; average deviation is measured from the mean.

[a] Computed on base of incomplete cycle, starting Jan. 1905. Excluded from the average and the average deviation.

[b] Incomplete cycle; data start in Jan. 1905.

Table 39 presents a sample of Table R1, which parallels Table S4 in every respect except for the insertion of cycle bases.[42] But now the dates of troughs and peaks are taken from the standard list of reference dates instead of from the turning points of specific cycles, and the standings during successive stages are expressed in reference-cycle relatives instead of specific-cycle relatives. The reference cycles are divided into nine stages in exactly the same way as are the specific cycles.[43] As in the specific-cycle patterns, the standing at the terminal trough in one reference cycle is rarely the same as the standing at the initial trough in the next cycle; for although the standings are made from the same items, the base of the former is the monthly average during that reference cycle, while the base of the latter is the monthly average during the following reference cycle. The successive reference-cycle stages in Table R1 also are designated by Roman numerals from I to IX; V refers to the reference peak. But the standings in stages I, V and IX invariably cover three months, whereas they are sometimes based upon one or two months in the specific cycles. The nine standings within a reference cycle are called the 'reference-cycle pattern'.

[42] The specific-cycle bases are given in Table S3.

[43] Appendix Table A1 shows in full the division of successive reference cycles in the United States, Great Britain, Germany and France, when the analysis is based upon monthly data.

Table 40 illustrates the calculation in detail, and Charts 18-19 show the process graphically. The table and charts are precise analogues of Table 35 and Charts 14-15 for specific cycles. As said before, it is easier to compute the average standing of relatives in a cycle-stage by taking an average of the original figures and converting it to a relative than by computing relatives for individual months and then averaging them; hence the computations in Table 40 are shown on the former plan. Indeed, there is a slightly better method of calculation, though its meaning may be less obvious at first sight. Let T be the total of the monthly values included in a stage, M the number of months in the stage, S the total of the monthly values in the cycle,[44] and N the number of months in the cycle. Then the standing in a reference-cycle stage is given by $\frac{TN}{MS}$. Of course, $\frac{N}{S}$, the reciprocal of the cycle base, is a constant for a given cycle; hence the most convenient formula is $\frac{TV}{M}$, where $V = \frac{N}{S}$.

Since most of our time series do not start at the very trough of a reference cycle, they cover only part of a reference cycle at the beginning. It is desirable to utilize information on even a fraction of a cycle, especially in brief series. We handle fractional cycles as follows. If only one or two stages are missing, as in the reference cycle from 1904 to 1908 in coal production, the standings in the stages covered by the data are expressed as relatives on the base of the incomplete cycle. If more than two stages are missing, the standings in the stages covered by the data are computed on the base of the nearest complete reference cycle, whether marked off by peaks or by troughs. These standings are recorded in Table R1, but are not included in the averages.

Since the aim of Table R1 is simply to show how different series behave during the same periods, it is not necessary to distinguish between series that trace out specific cycles and those that do not. A series that appears to have no specific cycles, and for which therefore no 'S' Tables are made, can be chopped into reference-cycle segments as readily as any other. These segments can be presented in Table R1 and scrutinized to see what 'response' if any the series makes, or what relation it bears, to business cycles. Nor is it necessary to distinguish between positive and inverted analyses in making Table R1; although the decision to analyze *specific cycles* on a positive or inverted basis is actually based on a study of Table R1, which comes first in the order of computation.

Table R1 is therefore computed for every series and in every series the computation is made on a positive plan; that is, every series is broken into segments running from one reference trough to the next. Business-cycle units could be delimited in other ways: for example, from one

[44] As in the specific cycles, this includes the values from the initial to the terminal trough, but only half the value at the troughs.

TABLE 40

The Computation of Reference-cycle Patterns Illustrated

Bituminous Coal Production, United States, 1908–1919

Cycle	Stage	Period covered	No. of months	Average monthly standing in		Interval from stage to stage (mos.)
				Millions of short tons	Cycle relatives	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
June 1908–Jan. 1912 (Av. monthly value = 32.740 million short tons)	I	May 1908 – July 1908	3	28.367	86.6	...
	II	July 1908 – Dec. 1908	6	29.100	88.9	3.5
	III	Jan. 1909 – June 1909	6	29.283	89.4	6.0
	IV	July 1909 – Dec. 1909	6	33.800	103.2	6.0
	V	Dec. 1909 – Feb. 1910	3	35.600	108.7	3.5
	VI	Feb. 1910 – Sep. 1910	8	34.675	105.9	4.5
	VII	Oct. 1910 – Apr. 1911	7	33.329	101.8	7.5
	VIII	May 1911 – Dec. 1911	8	34.500	105.4	7.5
	IX	Dec. 1911 – Feb. 1912	3	37.300	113.9	4.5
Jan. 1912–Dec. 1914 (Av. monthly value = 37.611 million short tons)	I	Dec. 1911 – Feb. 1912	3	37.300	99.2	...
	II	Feb. 1912 – May 1912	4	38.625	102.7	2.5
	III	June 1912 – Aug. 1912	3	37.333	99.3	3.5
	IV	Sep. 1912 – Dec. 1912	4	37.000	98.4	3.5
	V	Dec. 1912 – Feb. 1913	3	40.000	106.4	2.5
	VI	Feb. 1913 – Aug. 1913	7	39.343	104.6	4.0
	VII	Sep. 1913 – Apr. 1914	8	38.600	102.6	7.5
	VIII	May 1914 – Nov. 1914	7	34.529	91.8	7.5
	IX	Nov. 1914 – Jan. 1915	3	33.300	88.5	4.0
Dec. 1914–Apr. 1919 (Av. monthly value = 42.746 million short tons)	I	Nov. 1914 – Jan. 1915	3	33.300	77.9	...
	II	Jan. 1915 – Feb. 1916	14	38.229	89.4	7.5
	III	Mar. 1916 – May 1917	15	43.127	100.9	14.5
	IV	June 1917 – July 1918	14	47.529	111.2	14.5
	V	July 1918 – Sep. 1918	3	51.767	121.1	7.5
	VI	Sep. 1918 – Oct. 1918	2	46.900	109.7	1.5
	VII	Nov. 1918 – Jan. 1919	3	40.867	95.6	2.5
	VIII	Feb. 1919 – Mar. 1919	2	35.550	83.2	2.5
	IX	Mar. 1919 – May 1919	3	37.733	88.3	1.5

The illustration covers the first three full reference cycles in coal production; see Table 39.

reference peak to the next, or from the midpoint of one reference expansion to the midpoint of the next. Our reason for making Table R1 on a positive plan is merely that this method is more familiar than any other; it involves no theoretical assumptions such as may have influenced other investigators who have also considered business cycles as units running from trough to trough. True, the developments during the transition from expansion to contraction of business cycles can be traced more easily from patterns made on a positive basis than from patterns made on an inverted basis. But just as clearly patterns on an inverted basis facilitate analysis of business-cycle revivals, and that is one reason among others why it is sometimes desirable to put the patterns in this form.[45]

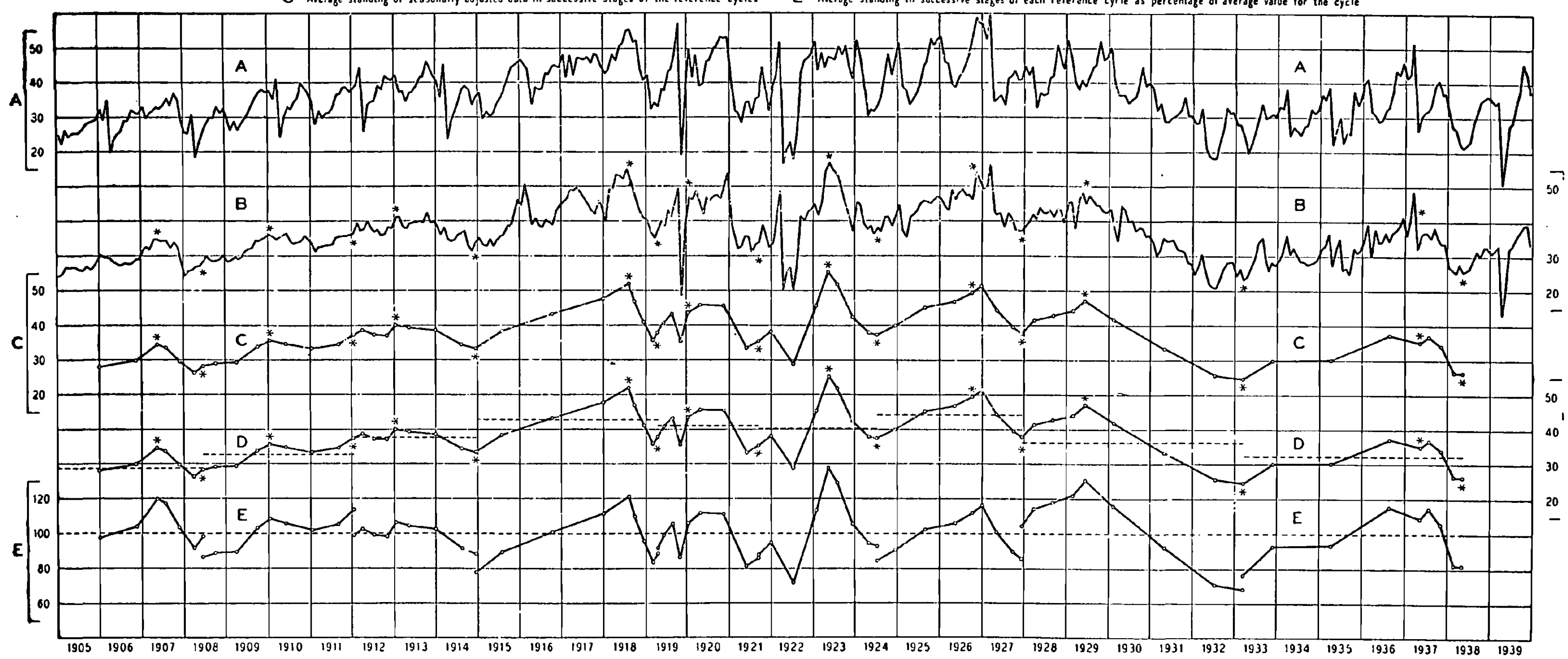
[45] See below, Ch. 11, Sec. V and Ch. 12, Sec. I. In Appendix B we show individual reference-cycle patterns on both a positive and inverted basis for several series; see Tables B3 and B4.

CHART 18

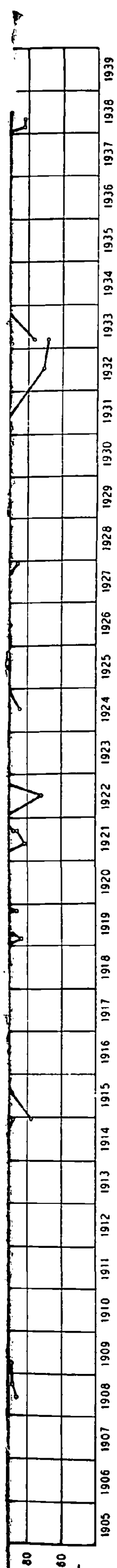
Derivation of Reference-cycle Patterns

Bituminous Coal Production, United States, 1905 – 1939

A Original monthly data
B Seasonally adjusted monthly data
C Average standing of seasonally adjusted data in successive stages of the reference cycles
D Step-line of trend and average standing in successive stages of reference cycles
E Average standing in successive stages of each reference cycle as percentage of average value for the cycle



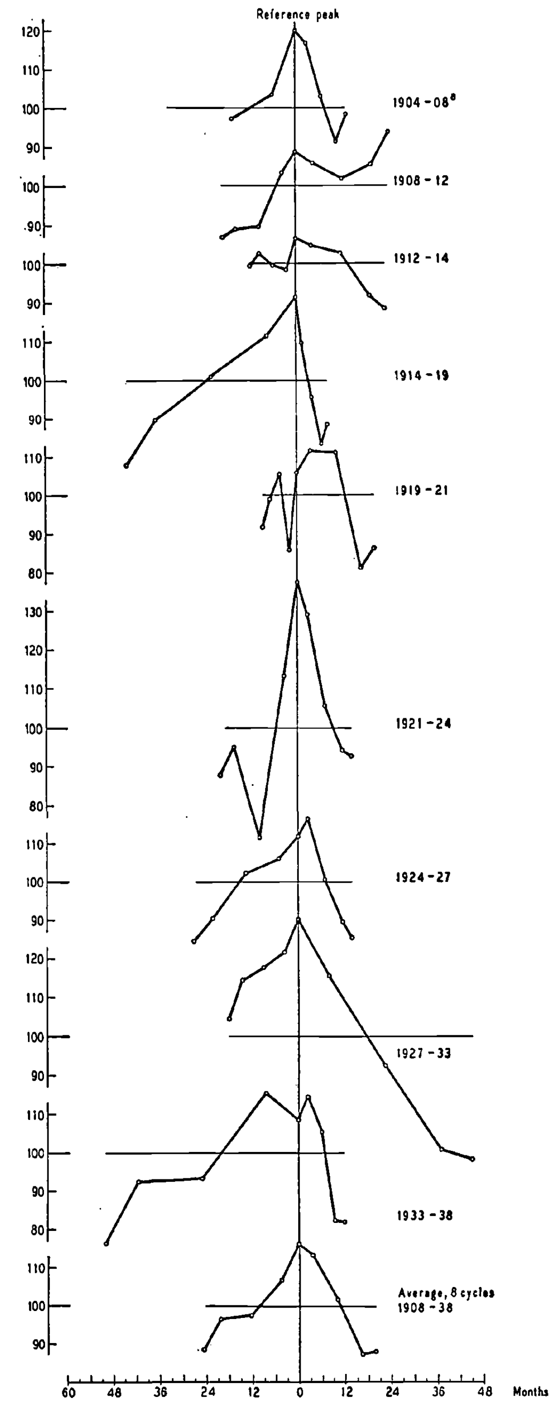
Scale of A, B, C and D in millions of short tons; scale of E in reference-cycle relatives. Asterisks identify peaks and troughs of reference cycles. The pattern of the 1904 – 08 cycle is incomplete, data not being available.



Scale of A, B, C and D in millions of short tons; scale of E in reference-cycle relatives. Asterisks identify peaks and troughs of reference cycles. The pattern of the 1904-08 cycle is incomplete, data not being available.

CHART 19

Patterns of Successive Reference Cycles and Their Average Pattern
Bituminous Coal Production, United States, 1905 – 1938



See Chart 18.

* The pattern is incomplete, data not being available.

Inverted average patterns may be derived by a fresh computation along the lines described for positive patterns, or approximated by a simple transformation of the positive patterns.[46] If the average standings in stages I and IX are the same, the inverted pattern can be estimated by accepting the values of the positive pattern, beginning in stage V and going the full round to stage V again. If the average standings in stages I and IX differ, as they usually do, a tolerable approximation can be achieved by using the average of the standings in stages I and IX to represent the trough, then adjusting the standings in stages V-IX by the ratio of this average to the standing in stage IX, and adjusting the standings in stages I-V by the ratio of this average to the standing in stage I.[47]

In the few series having plus and minus values, it is not feasible to work with reference-cycle relatives. But series of this type can still be broken into reference-cycle segments and presented in Table R1. The standings in successive stages of each cycle may then be expressed as simple averages of the original data, or as plus or minus deviations from the average value during the cycle.[48] The latter plan resembles our standard method, since it involves an adjustment for the changing level of successive reference cycles. But the absolute level of a series is frequently of considerable interest.[49] For example, in analyzing the relations among different types of interest rates during a business cycle, it is desirable to know not only the percentage variations of each about its average during the cycle, but also whether and how much one type of interest rate is above or below another. Hence we sometimes find it profitable to express the standings of a series in successive stages of business

46 The term 'inverted pattern', when applied to reference cycles, is slippery; it may refer to the shape of the pattern or to the plan on which Table R1 is computed. Thus the pattern of zinc stocks (Chart 20) is inverted in shape, though made on a positive plan. The statement in the text refers to the plan of computation solely.

47 This transformation will rarely give exactly the same results as direct calculation, for the following reasons. (1) The period covered by inverted cycles must differ somewhat from that covered by positive cycles, since the former begin and end with a reference peak while the latter begin and end with a trough. (2) The cycle bases and therefore also the cycle relatives vary according as the cycles are marked off by troughs or peaks; see Ch. 12, note 3.

It might seem that the estimated pattern should be adjusted so that the average of its nine average standings equals 100. But this average may differ from 100 even when a pattern is computed directly. To explain: In a single cycle the average of the nine average standings would necessarily equal 100 if (a) the standings at reference peaks and troughs were represented by single months, (b) the standings at the troughs (assuming positive analysis) were weighted one-half each, (c) the other standings were weighted by the number of months they cover. For a group of cycles an additional condition would be necessary in order that the average of the nine average standings equal 100: the average for each stage must be a weighted average, where the weights are the periods covered by the stage in successive cycles.

48 The difficulty of plus and minus values may be circumvented by ranking the nine standings within each reference cycle, then averaging the ranks for each stage to obtain the average pattern. This plan of computation is unsatisfactory for most purposes, because it brushes aside differences in amplitude.

49 That is our reason for showing the reference-cycle bases in Table R1.

cycles in the form of simple averages of the original data, not only when the series contains plus and minus values, but also when it consists of positive values throughout.

Table R2 elaborates upon Table R1: that is, it records the figures obtained by dividing the differences between successive stages in Table R1 by the number of months between the middle of one stage and the middle of the next. All that has been said about the relations between Tables R1 and S4 might be rephrased to apply to Tables R2 and S5. Also, Table R2 supplements R1 in the same way that Table S5 supplements S4. The rates of change per month in Table R2 are a highly valuable device for examining the influence of business cycles and random factors upon different business processes. For example, coal production rose in every business cycle from the trough to the first third of expansion; in the middle stages of expansion the rise tended to slacken; there was a fresh burst of activity from the last third of expansion to the peak, and sometimes the rise extended into the first third of contraction; a fairly continuous and vigorous decline followed; but the rate of decline tapered off from the last third of contraction in general business to the trough and in a few of the earlier cycles coal production actually rose during this interval (Table 41).

TABLE 41

Sample of Table R2: Rate of Change from Stage to Stage of Reference Cycles
Bituminous Coal Production, United States, 1905–1938

Dates of reference cycles	Average change per month in reference-cycle relatives between stages							
	I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
	Expansion				Contraction			
Trough – Peak – Trough (1)	Trough to first third (2)	First to middle third (3)	Middle to last third (4)	Last third to peak (5)	Peak to first third (6)	First to middle third (7)	Middle to last third (8)	Last third to trough (9)
Aug. 04 – May 07 – June 08	. . .	. . .	+0.6[a]	+2.8[a]	−1.3[a]	−3.4[a]	−2.9[a]	+2.9[a]
June 08 – Jan. 10 – Jan. 12	+0.7	+0.1	+2.3	+1.6	−0.6	−0.5	+0.5	+1.9
Jan. 12 – Jan. 13 – Dec. 14	+1.4	−1.0	−0.3	+3.2	−0.4	−0.3	−1.4	−0.8
Dec. 14 – Aug. 18 – Apr. 19	+1.5	+0.8	+0.7	+1.3	−7.6	−5.6	−5.0	+3.4
Apr. 19 – Jan. 20 – Sep. 21	+3.6	+2.6	−7.9	+10.2	+1.6	−0.1	−4.6	+1.4
Sep. 21 – May 23 – July 24	+2.0	−3.6	+6.4	+7.0	−3.4	−5.2	−2.5	−0.6
July 24 – Oct. 26 – Dec. 27	+1.2	+1.4	+0.5	+1.2	+1.8	−3.5	−2.4	−1.7
Dec. 27 – June 29 – Mar. 33	+2.9	+0.7	+0.7	+2.5	−1.8	−1.6	−1.5	−0.4
Mar. 33 – May 37 – May 38	+1.9	0.0	+1.3	−0.8	+2.3	−2.5	−6.7	−0.1
Average	+1.9	+0.1	+0.5	+3.3	−1.0	−2.4	−3.0	+0.4
Average deviation	0.7	1.2	2.3	2.7	2.4	1.8	1.9	1.4
Weighted average	+1.8	+0.1	+1.2	+2.1	−0.8	−1.8	−2.3	+0.2
Average interval (mos.)	*4.5*	*7.9*	*7.9*	*4.5*	*3.6*	*6.4*	*6.4*	*3.6*

Average is the arithmetic mean; the average deviation is measured from the mean.

[a] Computed on base of incomplete cycle, starting Jan. 1905. Excluded from the average, the average deviation, and the average interval.

Chart 20 shows the average reference-cycle pattern of coal production and of several other American series.[50] The chart is constructed on the same principles as Chart 16 showing specific-cycle patterns,[51] but now the graphs are drawn from the average standings in Table R1 and the average intervals in Table R2. The long horizontal line below each pattern represents the average duration of the business cycles covered by the series, and the shorter horizontal line represents the average deviation of the business-cycle durations. The other features correspond to similar features of Chart 16; for example, the slopes of the average reference-cycle pattern represent weighted averages of the rates of change from one stage of the reference cycles to the next, just as in the specific cycles. The average patterns of most series we have analyzed rise during reference expansions and decline during reference contractions, attesting the existence of business cycles in the sense of our definition. But the patterns of some series rise during contractions though at a retarded rate (e.g., postal receipts), while others move invertedly (zinc stocks) or quasi-invertedly (bond yields), and still others move in haphazard fashion (sugar meltings).

By studying Tables R1 and R2 we can learn in some detail what the behavior of a series has been during successive reference expansions and contractions, and thus supplement the knowledge derived from the timing measures of Table S1, which show merely the timing of the series at reference turns. For example, Table S1 tells us that if we count all turns in the nine specific cycles from 1893 in pig iron production and from 1895 in pig iron prices to World War I and from 1921 or 1922 to 1933, production led the reference troughs by 4.3 months on the average while prices lagged 4.4 months. At reference peaks, production lagged 1.6 months while prices led by 1.7 months. The following summary of Tables R1 and R2, based on the nine reference cycles in 1894–1914 and

	Average in reference-cycle relatives at stage								
	I	II	III	IV	V	VI	VII	VIII	IX
Production	73.1	90.5	102.9	116.9	126.1	119.0	96.4	84.5	81.5
Prices	89.5	91.2	94.2	108.9	117.4	114.7	103.2	91.6	86.9

	Average change per month in reference-cycle relatives between stages							
	I-II	II-III	III-IV	IV-V	V-VI	VI-VII	VII-VIII	VIII-IX
Production	+4.4	+1.9	+2.3	+2.6	−1.6	−3.4	−1.8	−1.0
Prices	+0.2	+0.4	+2.4	+2.0	−0.9	−1.9	−2.0	−1.5

1921–1933, adds significant information about the timing of the advances and declines.[52] Production increased at an extremely fast rate between the trough and the first third of reference expansion, much faster than during

50 For several of these series the patterns of individual cycles are shown in Appendix Table B3.

51 In this connection, see the description of reference-cycle patterns in Chart 2.

52 The rates of change shown are unweighted averages. Weighted averages would demonstrate similar results.

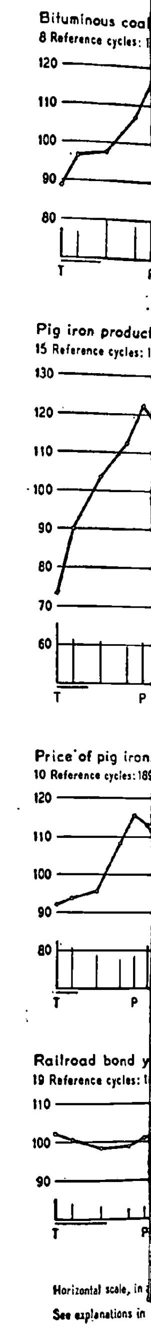
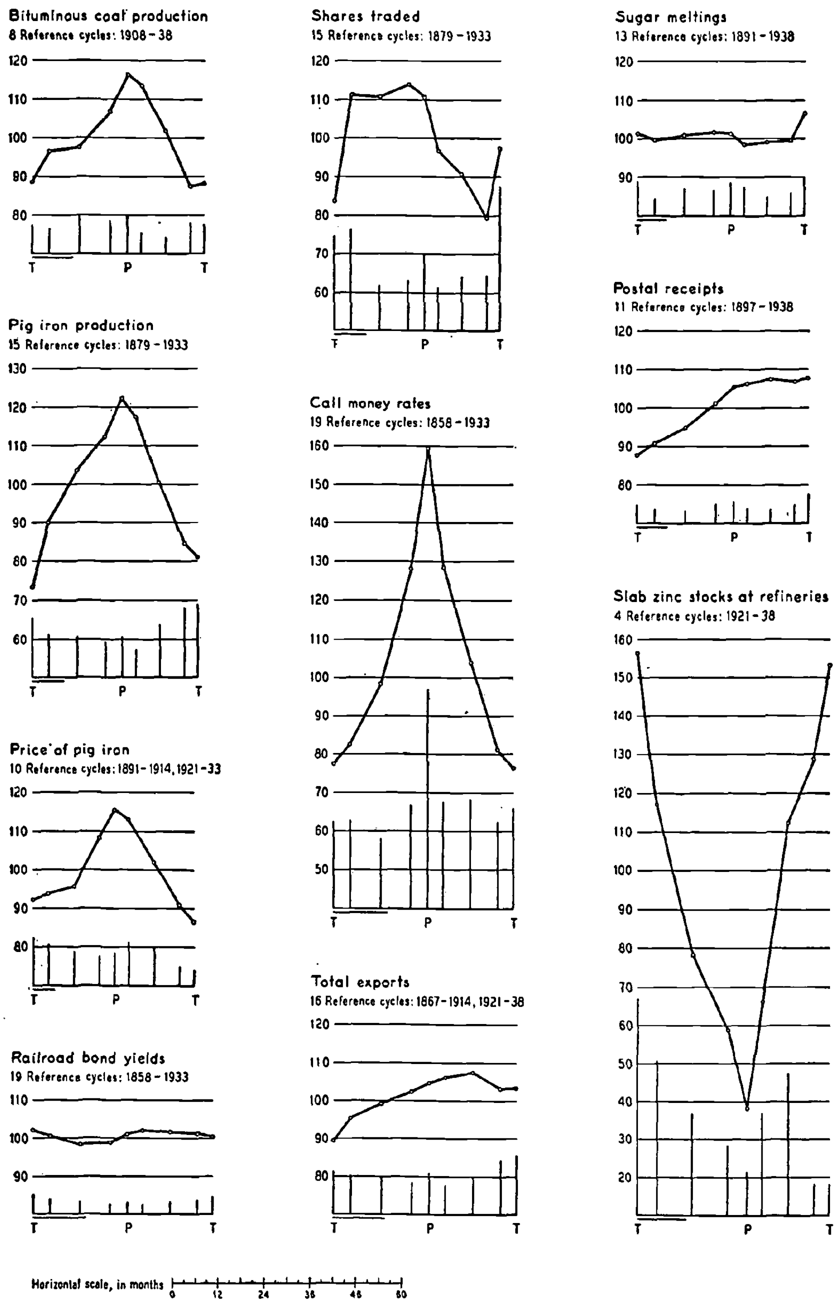


CHART 20

Average Reference-cycle Patterns of Ten American Series



See explanations in text. For sources of data and other notes, see Appendix C.

the rest of the expansion. Prices, on the other hand, did not begin to rise appreciably until the middle of the expansion. Both series fell sharply between the first and middle thirds of reference contraction, production much more than prices. After the middle of the reference contraction, the rate of decline abated considerably in production but not in prices. Comparisons of this character for numerous series should give fairly clear pictures of the timing relations among the movements of different processes during the periods when general business activity is experiencing cyclical expansions and contractions.

Tables R1 and R2 provide also a partial check on the timing measures of Table S1. For example, the average reference-cycle pattern of railroad bond yields reaches a trough in stage III and a peak in stage VI. This result is broadly confirmed by Table S1, which shows that bond yields lagged on the average 8 months at reference peaks and 12 months at reference troughs. The average reference-cycle pattern of share trading reveals a pronounced tendency to lead, and this result too is confirmed by Table S1. On the other hand, the patterns of production and prices of pig iron obscure the much smaller departures of these series from the reference dates. Both reach a peak in stage V and a trough in stage IX, although it seems clear from Table S1 that their timing differed significantly: from 1893 to 1933 the cyclical peaks in prices led the peaks in production in eight instances, coincided in three, but lagged in none, while the cyclical troughs in prices lagged behind the troughs in production in ten instances, and coincided in the two remaining instances.[53]

To extract reliable information on cyclical timing, Tables S1, R1 and R2 should be used together. As said before, the measures in Table S1 show the timing of cyclical turns in months, but are surrounded by uncertainty because of the difficulties of dating specific cycles and matching them with business cycles. The measures in Tables R1 and R2 are free from these difficulties, show timing of the rates of change of cyclical movements as well as of their reversals in direction, but indicate timing in coarse fractions of a cyclical phase. The limitations of the two sets of measures to some extent offset one another. The knowledge that can be wrung from time series by checking one set of measures carefully against the other is both more detailed and more reliable than what can be learned by relying on either set exclusively.

VIII Relation between Reference- and Specific-cycle Patterns

To facilitate study of the cyclical measures described so far, we display in a single chart the averages and average deviations of those features of the

[53] For fuller details, see Table 176. That table stops in 1933. It may be well to add here that at the downturn of business activity in 1937, the peak in pig iron prices came 9 months after the downturn in pig iron production. This is the only lag in the record, and may well excite some speculation.

cyclical behavior of a series that lend themselves readily to graphic presentation. Several samples appear in Chart 21. The purpose of this chart is to picture average behavior during the cycles peculiar to a series and during the group of contemporaneous business cycles. Hence we need merely combine the graphs of specific-cycle and reference-cycle patterns already illustrated. But we must ensure that the averages of the specific and reference cycles cover comparable periods, a matter that was of no concern so long as specific and reference cycles were observed separately. And we must also decide how to place the two patterns in relation to each other.

Our method is as follows: If each turning point of the specific cycles is treated in Table S1 as corresponding to a reference turn and every reference turn within the period covered is thus accounted for, the 'duration lines' of the two patterns—that is, the long horizontal lines representing average cyclical duration—are so placed that they show the average lead or lag of the turning dates of the specific cycles compared with the turning dates of business cycles.[54] Arrows are drawn to call attention to the average leads or lags. If the specific cycles have been analyzed on a positive basis, an arrow is drawn from the trough of the average specific-cycle pattern to the trough of the average reference-cycle pattern, and from the peak of the former to the peak of the latter. If the average lead or lag is one month or less, we draw a vertical arrow, which signifies that the timing of specific and business cycles is roughly coincident on the average at that turn. Similarly, if the specific cycles have been analyzed on an inverted basis, an arrow is drawn from the peak of the specific-cycle pattern to the reference trough and from the trough of the specific-cycle pattern to the reference peak. When the specific cycles do not bear a one-to-one correspondence to business cycles, we do not attempt to represent leads or lags on the chart. In these instances, the 'duration lines' of the two patterns are placed so that their average standings in stage V are aligned vertically. In other words, if the specific cycles have been analyzed positively, as in call money rates, the average standings at the peak stage of specific and business cycles are plotted on the same vertical; but if the specific cycles have been analyzed invertedly, as in zinc stocks, the standings at the trough stage of the specific cycles and at the peak stage of business cycles are plotted on the same vertical.

The two patterns are plotted to a common set of vertical scales; hence the average standings of the patterns can be read directly from the scale on the left. The average deviations of the patterns are drawn as vertical lines above and below the curves representing the average patterns. The

[54] The average lead or lag at the initial reference trough need not be the same as the average at the terminal reference trough. For the former is based on the leads or lags at all reference troughs except the last, while the latter includes the leads or lags at all reference troughs except the first. The average lead or lag shown on the chart at the reference peak may also differ from that in Table S1, since the former includes timing comparisons for full specific cycles only.

average deviations of the specific-cycle standings must be read down from the 'duration line' of the specific cycles, which serves as a zero line for this purpose. The average deviations of the reference-cycle standings must be read up from the 'duration line' representing the average duration of the comparable set of business cycles. The short horizontal lines above and below the 'duration lines' represent average deviations of the cyclical durations. The ruler on the chart affords a scale from which all duration measurements can be approximated.

The figures in Chart 21 may at first seem puzzling even to a person who has studied with care the manner in which they are constructed.[55] But the reader will soon learn how to use them if he bears in mind the following points: (1) The patterns indicate average leads or lags at reference turns only when there is a one-to-one correspondence of specific and business cycles. (2) The specific- and reference-cycle patterns cover periods that are as nearly the same as the turning dates of specific and business cycles allow. (3) The representative value of the patterns is indicated by the lines showing average deviations and by the captions stating the number of cycles covered. (4) Even when the number of cycles is 'large' and the average deviations are 'small', the full significance of the patterns cannot be determined without close study of the original data, the arrays from which the patterns are made, and the many measures we compute but do not record on the charts. (5) Differences among cyclical patterns of individual series may reflect persistent differences, or merely special features of the varying periods covered by the series. (6) The amplitude of the reference-cycle pattern relative to that of the specific-cycle pattern provides a quick clue to the relationship in time between a given series and business cycles. If the turning dates of the specific cycles are close to the reference dates, the two patterns will be nearly the same. But if the timing of the specific cycles is fairly independent of that of business cycles, then what may have been a large amplitude in the specific-cycle pattern will be obscured or disappear in the reference-cycle pattern.

The last remark is vital and must be understood by anyone wishing to follow closely the results yielded by our technical apparatus. When we break a series into specific-cycle segments, divide each segment into stages, and average the standings at each stage for all cycles, the cyclical movement of the series is exhibited in full. When this procedure is applied to reference-cycle segments, some portion of the cyclical movement is virtually bound to be erased, for the trough-peak-trough reference dates rarely, if ever, match precisely the trough-peak-trough points in the specific cycles. To be sure, if the specific cycles conform approximately in

[55] See also above, pp. 154-7, 168; and especially the description in Chart 2. The scales in the charts of cyclical patterns in this volume have been kept strictly uniform, so that the features of one chart may be readily compared with the features of any other. For obvious reasons Charts 2 and 50 are exceptions.

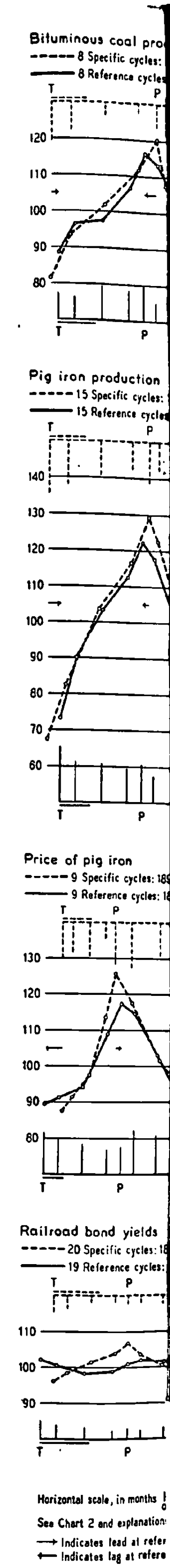
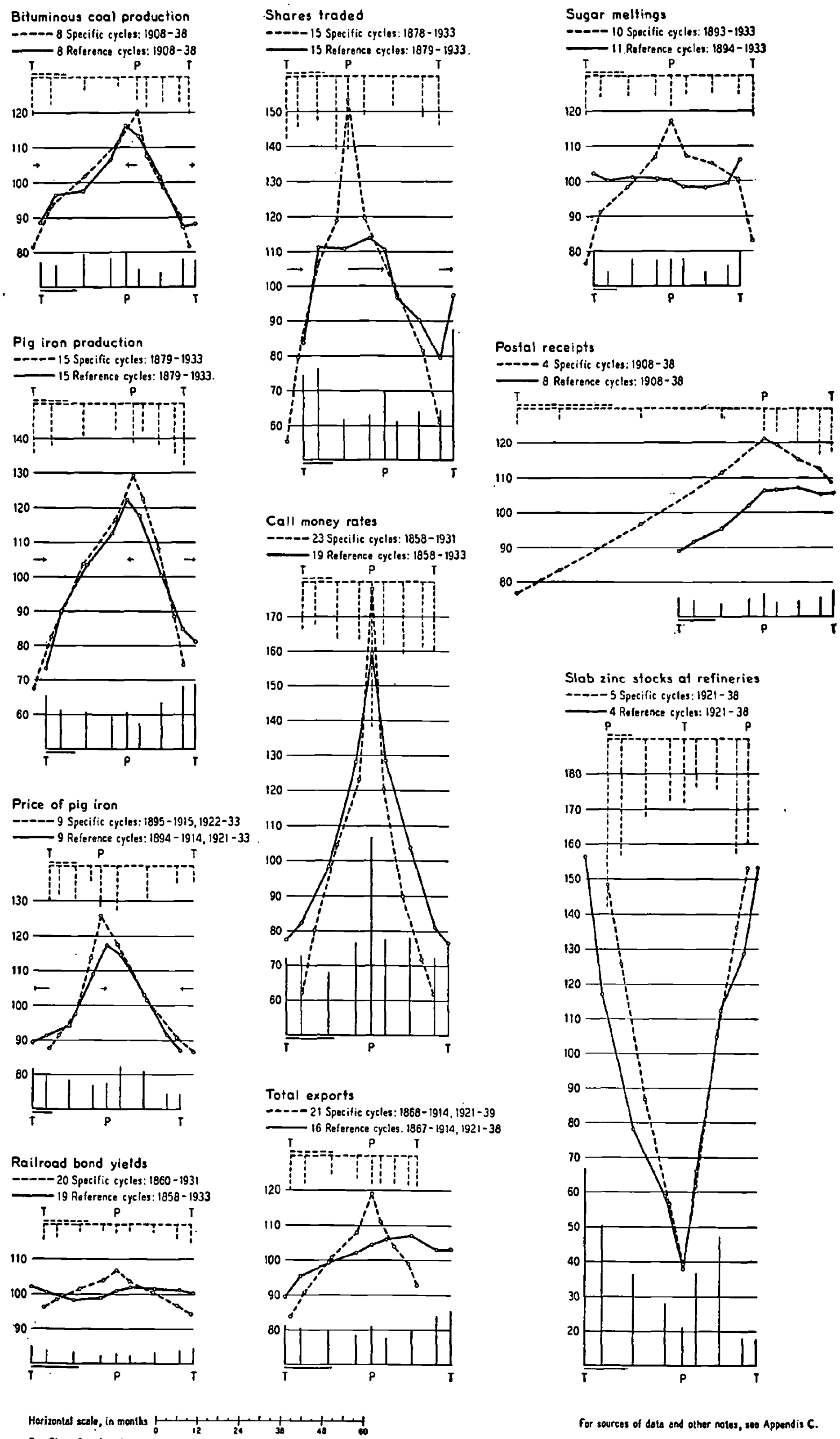


CHART 21

Average Cyclical Patterns of Ten American Series



See Chart 2 and explanations in text.

→ Indicates lead at reference peaks, or troughs.

← Indicates lag at reference peaks, or troughs.

For sources of data and other notes, see Appendix C.

timing to business cycles, as in the production of bituminous coal or pig iron, the average patterns of Tables S4 and R1 resemble each other closely. Further, if the turning dates of the specific cycles differ from the reference dates by considerable but tolerably regular intervals, the two patterns will be much alike when the difference in timing is allowed for. But if the specific cycles have no systematic relationship in time to business cycles, then what may have been a clearcut cyclical pattern in Table S4 will disappear or be obscured in the averages of Table R1. In other words, when the movements of a series are independent of the direction of business cycles, and the series covers a fairly large number of business cycles, the average reference-cycle pattern is apt to reveal merely the secular trend. Comparison of the averages in Tables R1 and S4 thus gives a snapshot of the relationship in time between the specific and business cycles.[56]

We have no satisfactory method of measuring the degree of resemblance between the specific- and reference-cycle patterns. But the feature of the patterns that concerns us most is the degree to which the amplitudes of the specific cycles are preserved in the reference-cycle patterns, and this can be readily ascertained. We could get reference-cycle amplitudes by taking the maximum rise and fall in the averages of Table R1. But the figures will be less exposed to random perturbations and more influenced by business cycles if we use the rise during those stages of reference cycles that are the characteristic period of expansion in the series and the fall during the stages that are characteristic of contraction; that is to say, if we determine the rise during the stages assigned to 'expansion' and the fall during the stages assigned to 'contraction' in making Table R4, as explained in Section X. In coal production, for example, the typical period of expansion is from stage I to V, and the typical period of contraction from stage V to IX. The average change from stage I to V of the reference cycles is +27.8, the average change from stage V to IX is −28.2.[57] The average rise and fall of the reference cycles is therefore 56.0 points, that is, +27.8 − (−28.2). According to Table S2 the average rise and fall of the specific cycles is 77.2 points. Hence the average amplitude of the reference-cycle pattern is 73 per cent of the average amplitude of the specific cycles; this figure is placed at the bottom of Table R1.

The percentage ratio of the average reference-cycle amplitude to the average specific-cycle amplitude can be used not only to indicate the relation in time between specific and business cycles, but also the relation in time of any pair of economic series. For example, a price series may be broken into segments on the basis of the turning points in the specific cycles of a corresponding production series instead of on the basis of the

[56] Cf. Ch. 12, Sec. IV.

[57] These figures are taken from col. (2) and (5) of our sample Table R3 (see Table 42). Since Tables R3 and R4 are the same for coal production, no Table R4 is explicitly presented for this series.

turning points of business cycles. In that case the percentage ratio of the amplitude computed on the basis of the special reference dates to the specific-cycle amplitude would indicate the degree of correspondence between the specific cycles in prices and output, since the output series yields reference cycles, so to speak, for the price series. Or if a series on wage rates is broken into segments on the basis of the turning points in the specific cycles of employment, the ratio of the amplitude computed on the basis of these 'reference dates' to the specific-cycle amplitude would indicate the degree of correspondence between the specific cycles in wage rates and employment.

The percentage ratio of the reference-cycle amplitude to the specific-cycle amplitude is a rough indicator of similarity, and must be used cautiously. In the first place, leads or lags do not stand on the same footing as coincidences. If the timing of a series coincides with that of business cycles, the reference-cycle amplitude must be the same as the specific-cycle amplitude; but the latter is nearly certain to exceed the former if the series leads or lags behind the reference dates by a substantial interval, even though this interval be perfectly uniform, whether in months or in fractions of a cycle phase. For example, if a series led the reference turns uniformly by half a reference phase, the peaks would be in the center of stage III and the troughs in the center of stage VII of the reference cycles. But since the values at the peaks and troughs would be averaged with all the months falling in these stages, usually more than three, the reference-cycle amplitude would be smaller for this purely technical reason than the specific-cycle amplitude. Another shortcoming of the measure is that it may be influenced unduly by extreme observations. For example, if the turning points of some specific cycle coincide with the turning points of a business cycle, and this specific cycle happens to be of enormous amplitude relatively to other cycles in the series, the average reference-cycle amplitude may approach in size the average specific-cycle amplitude regardless of the relations in time between the other cycles in the series and business cycles. Finally, the measure may defy logic by falling below 0 or rising above 100. Thus the typical period of expansion may run from stage I to V of the reference cycles, and yet the average reference-cycle pattern may not only rise from stage V to IX, but this rise may exceed that between stages I and V; in that case the reference-cycle amplitude, as we measure it, would be minus. Again, if the specific cycles in a series correspond closely to all business cycles but one, and two specific cycles occur during this cycle, the first marked by a slight decline and the second by a slight rise, the average amplitude of the reference-cycle pattern may exceed that of the specific-cycle pattern. Such absurd results are rare, though the series on zinc stocks (Chart 21) illustrates the difficulty: the average amplitude of its four reference cycles in 1921–38 is a little larger than the average of its five specific cycles within this period.

In presenting the results of our analysis of time series, we rely heavily on charts of cyclical patterns, of the type illustrated in Chart 21. The charts are drawn to a uniform set of scales, so that the features of one series can be compared directly with the corresponding features of any other series. Once a person has become expert in interpreting the charts, he will find that a glance gives a fairly comprehensive notion of the behavior of a series with respect to business cycles. But he will also find that the knowledge so quickly won must be tested by close and patient study of the full evidence. This evidence includes, besides the measures already described, Tables R3 and R4, to which we now turn.

IX Conformity to Business Cycles: Behavior during Fixed Periods

Several of our measures throw light upon the manner and degree in which various economic processes conform to business cycles. Table S1 shows the differences between the timing and the duration of specific and business cycles; Tables R1 and R2 show the movements of a series from stage to stage of business cycles. Whether these movements match business cycles closely or not is indicated by a comparison of the average amplitudes in Tables S4 and R1. But we need a numerical expression of these relationships that takes account of the type of conformity, its regularity, and that shows separately the conformity to the expansion phase of business cycles, to the contraction phase, and to full business cycles.[58]

A simple method of measuring conformity is to observe the direction and rate of movement of a series during successive reference expansions and contractions. Conformity measures on this plan are set out in Table R3, of which Table 42 provides several samples. The first step is to subtract the standing of the series in stage I of each reference cycle from its standing in stage V; this difference is recorded in column (2). Next the durations of the reference expansions covered by the series are entered in column (3). Each of these entries is then divided into the total change during a reference expansion, and the resulting average change per month entered in column (4). The average change per month during reference contractions is determined similarly. The interval from the midpoint of stage V to the midpoint of stage IX is entered in column (6). The standing in stage V of each reference cycle is subtracted from the standing in stage IX, and the difference entered in column (5). This figure is then divided by the duration of the contraction, and the quotient recorded in column (7). In column (8) we subtract the entry in column (4) from the entry in column (7), in order to show whether there is any 'response' to business cycles in the numerous instances where a series rises

58 The methods that follow may be used to measure the conformity of one series to the specific cycles of another, just as conformity to business cycles. In that case, the specific cycles of either series will yield 'reference dates' for the analysis of the other. See pp. 174-5.

TABLE 42
Samples of Table R3: Conformity to Business Cycles
Three American Series

Dates of reference cycles	Change in reference-cycle relatives during						Av. change per month during reference contraction minus that during	
	Reference expansion			Reference contraction				
Trough – Peak – Trough	Total change	Interval in months	Average change per month	Total change	Interval in months	Average change per month	Preceding reference expansion	Succeeding reference expansion[a]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Bituminous coal production								
Aug. 04–May07–June 08	...	...	...	−21.5[b]	13.0	−1.65[b]	...	−
June 08–Jan. 10–Jan. 12	+22.1	19.0	+1.16	+5.2	24.0	+0.22	−0.94	−
Jan. 12–Jan. 13–Dec. 14	+7.2	12.0	+0.60	−17.9	23.0	−0.78	−1.38	−
Dec. 14–Aug. 18–Apr. 19	+43.2	44.0	+0.98	−32.8	8.0	−4.10	−5.08	−
Apr. 19–Jan. 20–Sep. 21	+14.2	9.0	+1.58	−19.9	20.0	−1.00	−2.58	−
Sep. 21–May23–July 24	+49.6	20.0	+2.48	−44.8	14.0	−3.20	−5.68	−
July 24–Oct. 26–Dec. 27	+27.3	27.0	+1.01	−26.4	14.0	−1.89	−2.90	−
Dec. 27–June29–Mar.33	+26.0	18.0	+1.44	−62.1	45.0	−1.38	−2.82	−
Mar.33–May37–May 38	+32.5	50.0	+0.65	−26.8	12.0	−2.23	−2.88	...
Average[c]	+27.8	...	+1.24	−28.2	...	−1.80	−3.03	...
Average deviation[d]	...	...	0.45	...	...	1.06	1.17	...

Index of conformity to reference
Expansions .... +100
Contractions .... +78
Cycles, trough to trough .... +100
Cycles, peak to peak .... +100
Cycles, both ways .... +100

Dates of reference cycles	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Postal receipts in 50 large cities								
June 97–June99–Dec. 00	+12.9	24.0	+0.54	+12.6	18.0	+0.70	+0.16	−
Dec. 00–Sep. 02–Aug. 04	+18.8	21.0	+0.90	+17.1	23.0	+0.74	−0.16	−
Aug. 04–May07–June 08	+23.1	33.0	+0.70	+0.5	13.0	+0.04	−0.66	−
June 08–Jan. 10–Jan. 12	+15.7	19.0	+0.83	+14.0	24.0	+0.58	−0.25	−
Jan. 12–Jan. 13–Dec. 14	+10.7	12.0	+0.89	+0.8	23.0	+0.03	−0.86	−
Dec. 14–Aug. 18–Apr. 19	+43.3	44.0	+0.98	+4.9	8.0	+0.61	−0.37	+
Apr. 19–Jan. 20–Sep. 21	+3.7	9.0	+0.41	+8.5	20.0	+0.42	+0.01	−
Sep. 21–May23–July 24	+16.9	20.0	+0.84	+1.8	14.0	+0.13	−0.71	−
July 24–Oct. 26–Dec. 27	+19.1	27.0	+0.71	+1.5	14.0	+0.11	−0.60	−
Dec. 27–June29–Mar.33	+3.2	18.0	+0.18	−30.2	45.0	−0.67	−0.85	−
Mar.33–May37–May 38	+26.6	50.0	+0.53	−5.9	12.0	−0.49	−1.02	...
Average[c]	+17.6	...	+0.68	+2.3	...	+0.20	−0.48	...
Average deviation[d]	...	...	0.20	...	...	0.37	0.33	...

Index of conformity to reference
Expansions .... +100
Contractions .... −64
Cycles, trough to trough .... +64
Cycles, peak to peak .... +80
Cycles, both ways .... +71

(Table continued on next page)

TABLE 42—*Continued*

Samples of Table R3: Conformity to Business Cycles

Three American Series

Dates of reference cycles	Change in reference-cycle relatives during						Av. change per month during reference contraction minus that during	
	Reference expansion			Reference contraction				
Trough – Peak – Trough	Total change	Interval in months	Average change per month	Total change	Interval in months	Average change per month	Preceding reference expansion	Succeeding reference expansion[a]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Slab zinc stocks at refineries								
Apr. 19–Jan. 20–Sep. 21	...	...	...	+86.4[e]	20.0	+4.32[e]	...	+
Sep. 21–May 23–July 24	−190.8	20.0	−9.54	+109.3	14.0	+7.81	+17.35	+
July 24–Oct. 26–Dec. 27	−136.4	27.0	−5.05	+96.9	14.0	+6.92	+11.97	+
Dec. 27–June 29–Mar. 33	−2.6	18.0	−0.14	+103.9	45.0	+2.31	+2.45	+
Mar. 33–May 37–May 38	−143.3	50.0	−2.87	+150.6	12.0	+12.55	+15.42	...
Average[c]	−118.3	...	−4.40	+115.2	...	+7.40	+11.80	...
Average deviation[d]	...	...	2.90	...	...	2.78	4.67	...

Index of conformity to reference

- Expansions . . . . . −100
- Contractions . . . . . −100
- Cycles, trough to trough . . . . . −100
- Cycles, peak to peak . . . . . −100
- Cycles, both ways . . . . . −100

[a] Only the sign of the difference is entered.
[b] Computed on base of incomplete cycle, starting Jan. 1905. Excluded from the average and the average deviation.
[c] Arithmetic mean determined separately for each column. Hence (7)−(4) *may* differ from (8) in the last place.
[d] Measured from the mean.
[e] Computed on base of inverted cycle, 1920–23. Excluded from the average and the average deviation.

or falls throughout a reference cycle. The explanation of column (9) is best postponed to a later point.

Once entries have been made for each reference cycle in Table R3, the results are summarized in two sets of conformity measures. The first set shows arithmetic means, for all business cycles covered by the series, of the rates of change during reference expansions and contractions, and of the differences between the two rates. The averages are given below the entries for individual cycles in columns (4), (7), and (8). For the eight business cycles from 1908 to 1938 covered by bituminous coal, the averages come out +1.2, −1.8, and −3.0 per cent. The first figure indicates that coal output conformed positively to cyclical expansions in business, for output rose on the average during reference expansions. The second figure indicates that coal output also conformed positively to contractions in general business, for it declined on the average during reference contractions. The last figure indicates that coal output conformed positively to full business cycles, for the average rate of change was lower during reference contractions than during reference expansions.

A different type of behavior is illustrated by zinc stocks. For the four business cycles from 1921 to 1938, the averages are −4.4, +7.4, and +11.8; in other words, zinc stocks conformed inversely to reference expansions, contractions, and full cycles. In the series on postal receipts in 50 large American cities, the averages are +0.7, +0.2, −0.5 for eleven business cycles. These results indicate that on the average postal receipts 'responded' to contractions in general business by slackening their rate of increase; the conformity may therefore be said to be positive to expansions, inverted to contractions, but positive to business cycles taken as wholes.

While the average rates of change are significant for many purposes, they do not reveal the degree of consistency in the movements that occurred in successive business cycles. For example, the illustrations just given tell us that the reaction of coal production to business cycles differed, on the average, in direction and degree from the reaction of zinc stocks. They do not tell us whether the reaction of either or both series was uniform or variable from cycle to cycle. Average deviations from the average rates of change during the cycles covered by a series are an important supplement to the averages; so also is the ratio of the average amplitude of the reference-cycle pattern to the average amplitude of the specific-cycle pattern. But these measures still do not show how regularly a series has conformed positively or inversely to business cycles. In the period covered by our illustration, coal production rose during every reference expansion, and declined during every reference contraction but one. Zinc stocks declined invariably during reference expansions, and rose invariably during reference contractions. Postal receipts rose in every expansion, also in nine out of eleven contractions. To measure such differences in the type and regularity of reactions to cyclical changes in general business conditions, we make a second set of conformity measures, namely, 'indexes of conformity'.

We obtain an index of the conformity of a series to reference expansions—more briefly, an 'expansion index'—from column (4) of Table R3 by crediting a series with 100 for every rise, debiting it with 100 for every fall, writing 0 when there is no change, and taking an arithmetic mean of all the entries. In similar fashion column (7) yields a 'contraction index'. But now we weight every decline with +100 and every rise with −100; for a decline means positive conformity to reference contractions and a rise means inverse conformity. Indexes made on this plan range from +100, which means invariable positive conformity, to −100, which means invariable inverse conformity. An index of zero means an equal number of positive and inverse movements, combined perhaps with one or more instances of no change. The magnitude of an expansion index indicates the degree of consistency in the direction of movement of a series from cycle to cycle during reference expansions; its sign indicates whether the process is related in the main positively or inversely to reference expansions. Similarly, the contraction index shows the regularity of movements during reference contractions, and whether the series is characterized by a positive or an inverted relation to reference contractions.[59]

Since these indexes are based simply on the sign of the net difference between three-month averages centered on reference troughs and peaks, they could be computed directly from the original data instead of from reference-cycle relatives. The former method would be easier if the conformity indexes were the sole measure of conformity. But we need the relatives in any case to determine the average rates of change during business cycles—the measure of conformity previously described. Once the relatives are computed and the entries for successive cycles set out in this form in Table R3, it is simpler to work from the relatives. If an entry for a reference cycle appears in column (7) but not in column (4), or vice versa, it is excluded from the average rates of change for the group of cycles covered by the series but not from the conformity indexes. The main reason for the difference in treatment is that the average rates of change bear a simple additive relation to each other, but not the conformity indexes.

A need remains for an index of the conformity of a series to full business cycles, that is, for a 'full-cycle index'. Since the intra-cycle trend is retained in the reference-cycle relatives, it is desirable that the full-cycle index count a decline in the *rate of increase* during reference contractions as equivalent to an actual decline. Hence we consider a series as conforming positively to a business cycle when the sign for that cycle in column (8) of Table R3 is minus, as conforming inversely when the sign is plus, as showing zero conformity when the entry is 0. By counting every minus in column (8) as +100, every plus as −100, and every zero as 0, casting an algebraic sum of these entries, and dividing by the number of business cycles covered, we obtain a preliminary index of conformity to business cycles. The index is entered in the table on the line reading 'Index of conformity to reference cycles, trough to trough'.

This index is based on comparisons of rates of change during reference contractions with the rates of change during the preceding reference expansions. But it is equally important to see how the rate of change during a contraction compares with that during the following expansion, and this requires that business cycles be marked off by successive peaks.

[59] It is sometimes convenient to transform conformity indexes so as to show the number of instances of positive conformity as a percentage of the total number of cycles. This may be done by dividing the conformity index by 2 and adding 50 to the quotient. That is, let P be the number of instances of positive conformity and I the number of instances of inverted conformity; then

$$\frac{100\left(\frac{P-I}{P+I}\right)}{2}+50=100\left(\frac{P}{P+I}\right).$$

The transformation assumes that there are no instances of zero conformity, or that one-half of such instances are allotted to the positive group and one-half to the inverted group.

If the expansion and contraction indexes of a series are both +100, its conformity to full business cycles will be the same whether the cycles are considered as running from peak to peak or from trough to trough. In other instances a difference may arise between indexes made on the two plans, and the difference may be substantial if a series shows no specific cycles. Suppose, for example, that the successive terms of a series fall exactly on a parabola whose second derivative with respect to time is negative; then the full-cycle index will be +100 if the reference cycles are taken as units from trough to trough, but −100 if they are taken as units from peak to peak. If we are not to be misled by the first figure, we should have the second.[60] Hence we supplement our first preliminary full-cycle index by a second, which is entered in Table R3 on the line reading 'Index of conformity to reference cycles, peak to peak'.

The second index is computed from the entries in column (9) in exactly the same manner as the first index is computed from the entries in column (8). Column (9) contains only signs; that is, it shows merely whether the average rate of change during each contraction specified in column (1) is algebraically larger or smaller than the average rate of change during the next reference expansion. In most instances the sign in column (9) can be inferred from the signs in columns (4) and (7). If the entry is minus on a given line in column (7) and plus on the next line in column (4), the entry on the given line in column (9) must be minus. Or if the entry is plus on a given line in column (7) and minus on the next line in column (4), the entry on the given line in column (9) must be plus. A difficulty arises, however, when an entry on one line in column (7) has the same sign as the entry on the next line in column (4), for the two rates are computed on different bases and therefore are incomparable. They can be made comparable by measuring the average rates of change from the original data or from cycle relatives expressed on the same base, which may be any convenient figure. The method we find simplest in practice is to shift the standing in stage V of a given cycle to the base of the next cycle, and then compare the change per month from stage V to IX of the given cycle with the change per month from stage I to V of the next cycle.

We obtain the final full-cycle index by striking an arithmetic mean of the two preliminary indexes, each weighted by the number of cycles it covers, and record the result on the line reading 'Index of conformity to reference cycles, both ways'.[61] The final index must be +100 if a series

[60] Cf. pp. 298-9.

[61] The description in the text follows the method of calculation we have found most convenient in practice. But the computations can be organized on a plan that dispenses with the preliminary indexes.

The final full-cycle index involves a comparison of each phase with the preceding and following phase. Let the reference phase therefore be the unit of observation. If the rate of change during a contraction is algebraically smaller than during the preceding expansion, credit the contraction always rises during reference expansions and declines during reference contractions, or if the average rise per month during contractions is always less than the rise during the next preceding and following expansions, or if the decline during contractions is always more rapid than the decline during the next preceding and following expansions; or, in general, if the average change per month during reference contractions is in every instance algebraically smaller than the average change per month during the next preceding and following reference expansions. The full-cycle index will be -100 if the average change per month during reference contractions is in every instance algebraically larger than the average change per month during the next preceding and following reference expansions.

To illustrate: the conformity indexes of coke production (Table 12) are all $+100$; that is, coke output conformed positively to all the cyclical tides in general business covered by the data, rising and falling in harmony with them. The conformity indexes of zinc stocks are all -100; in other words, this process consistently moved inversely to the few cycles in general business covered by the series. Coal production has an expansion index of $+100$ and a full-cycle index of $+100$; but the contraction index is $+78$ because the series moved counter to the business tide in one of the nine contractions covered $\left(\frac{+800-100}{9}=+78\right)$. Still another result is illustrated by postal receipts, which have conformity indexes of $+100$, -64, $+71$. This series conformed positively to every reference expansion $\left(\frac{+1100}{11}=+100\right)$; it conformed inversely to nine reference contractions and positively to only two $\left(\frac{+200-900}{11}=-64\right)$; yet its conformity was preponderantly positive to full reference cycles, in the sense that its rate of rise was usually lower during contractions than during the next preceding and following expansions $\left(\frac{+1800-300}{21}=+71\right)$. The last result may be clearer if stated another way: the 'index of conformity to reference cycles, trough to trough' is $\frac{+900-200}{11}=+63.6$; the 'index of conformity to reference cycles, peak to peak' is

with $+50$; if it is also smaller than the rate of change during the following expansion, credit the contraction with another $+50$, which makes $+100$ in all; but if the rate of change is larger than during the following expansion, the additional credit is -50 and the total credit is 0; while if the rate of change is larger than during both the preceding and following expansions, the contraction gets a total credit of -100. The accounting is similar for expansions, except that the expansion is credited with $+50$ or -50 according as the rate of change is algebraically larger or smaller than that in a contiguous contraction. Obviously, the first and last phases cannot receive a credit of $+100$ or -100, since they can be compared with only one adjacent phase. Once a credit has been assigned to each reference phase covered by the analysis, the total is cast up and divided by $(N-1)$, where N is the number of reference phases. The quotient is the final index of full-cycle conformity.

$\frac{+900-100}{10}=+80$; hence the 'index of conformity to reference cycles, both ways' is $\frac{11(+63.6)+10(+80)}{21}=+71$.

The conformity indexes resemble coefficients of correlation. Except for the use of percentages, the limits of both are -1 and $+1$. The coefficient of correlation (squared) states the degree to which estimates of a given variable may be improved by taking account of its correlation with other variables. The index of conformity may be interpreted in a similar manner: it states the degree to which errors can be reduced in estimating the direction of movement of a series by taking account of its conformity to business cycles instead of guessing.[62] If in eight reference expansions or contractions there are seven movements in the same direction, that is 75 per cent better than chance, and the conformity index is precisely this figure, plus or minus. The greater the number of observations and the firmer the rational analysis, the greater is our confidence in the significance of a coefficient of correlation, and so it is also with indexes of conformity. An index of conformity as low as +33 indicates that instances of positive conformity preponderate over instances of inverted conformity in the ratio of 2 to 1; but the index commands more serious attention when the cycles number ten than when they number three, and when they number thirty than when they number ten.

Conformity indexes are highly unstable when few cycles are covered (Table 43). If there are only four cycles and no zero movements, the index of conformity to reference expansions or contractions may be -100, -50, 0, $+50$, or $+100$. The gaps between the possible values of the index diminish as the cycles become more numerous, but the gaps are appreciable even for 15 cycles. If the conformity index for four cycles is $+100$, the index will drop to $+60$ if a fifth observation happens to be inverted, and to $+33$ if a sixth observation also is inverted. If the index is $+100$ for thirteen cycles, it will drop to $+86$ if the fourteenth cycle happens to be inverted and to $+73$ if the fifteenth also is inverted. A conformity index of $+100$ or -100 for four expansions (or contractions) may seem significant at first glance; but is unimpressive when we remember that a random series would yield such a result one time in eight. Probability tests are therefore helpful in judging the statistical significance of conformity indexes, just as they help to judge coefficients of correlation. Such tests, however, must be used with fine discrimination, because as a rule the values of economic time series are serially correlated and different series are interdependent. In the case of the full-cycle index there is the additional difficulty that the same phase is used twice.[63]

62 We are indebted to W. Allen Wallis for suggesting these remarks.

63 See in this connection Geoffrey H. Moore and W. Allen Wallis, Time Series Significance Tests Based on Signs of Differences, *Journal of the American Statistical Association*, June 1943. Methods of testing the statistical significance of reference-cycle patterns are developed in this paper.

TABLE 43

Conformity Indexes for P Instances of Positive Conformity in N Cycles

No. of observations on cycles (N)	Number of instances of positive conformity (P)																								
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	−100	+100																								
2	−100	0	+100																							
3	−100	−33	+33	+100																						
4	−100	−50	0	+50	+100																					
5	−100	−60	−20	+20	+60	+100																				
6	−100	−67	−33	0	+33	+67	+100																			
7	−100	−71	−43	−14	+14	+43	+71	+100																		
8	−100	−75	−50	−25	0	+25	+50	+75	+100																	
9	−100	−78	−56	−33	−11	+11	+33	+56	+78	+100																
10	−100	−80	−60	−40	−20	0	+20	+40	+60	+80	+100															
11	−100	−82	−64	−45	−27	−9	+9	+27	+45	+64	+82	+100														
12	−100	−83	−67	−50	−33	−17	0	+17	+33	+50	+67	+83	+100													
13	−100	−85	−69	−54	−38	−23	−8	+8	+23	+38	+54	+69	+85	+100												
14	−100	−86	−71	−57	−43	−29	−14	0	+14	+29	+43	+57	+71	+86	+100											
15	−100	−87	−73	−60	−47	−33	−20	−7	+7	+20	+33	+47	+60	+73	+87	+100										
16	−100	−88	−75	−62	−50	−38	−25	−12	0	+12	+25	+38	+50	+62	+75	+88	+100									
17	−100	−88	−76	−65	−53	−41	−29	−18	−6	+6	+18	+29	+41	+53	+65	+76	+88	+100								
18	−100	−89	−78	−67	−56	−44	−33	−22	−11	0	+11	+22	+33	+44	+56	+67	+78	+89	+100							
19	−100	−89	−79	−68	−58	−47	−37	−26	−16	−5	+5	+16	+26	+37	+47	+58	+68	+79	+89	+100						
20	−100	−90	−80	−70	−60	−50	−40	−30	−20	−10	0	+10	+20	+30	+40	+50	+60	+70	+80	+90	+100					
21	−100	−90	−81	−71	−62	−52	−43	−33	−24	−14	−5	+5	+14	+24	+33	+43	+52	+62	+71	+81	+90	+100				
22	−100	−91	−82	−73	−64	−55	−45	−36	−27	−18	−9	0	+9	+18	+27	+36	+45	+55	+64	+73	+82	+91	+100			
23	−100	−91	−83	−74	−65	−57	−48	−39	−30	−22	−13	−4	+4	+13	+22	+30	+39	+48	+57	+65	+74	+83	+91	+100		
24	−100	−92	−83	−75	−67	−58	−50	−42	−33	−25	−17	−8	0	+8	+17	+25	+33	+42	+50	+58	+67	+75	+83	+92	+100	
25	−100	−92	−84	−76	−68	−60	−52	−44	−36	−28	−20	−12	−4	+4	+12	+20	+28	+36	+44	+52	+60	+68	+76	+84	+92	+100

This table is made on the assumption that there are no instances of zero conformity. To take account of the zeroes, it would be necessary to insert columns for 0.5, 1.5, 2.5, etc. instances of positive conformity. In computing conformity indexes a series is credited with 0 for every zero rate of change; which is equivalent to counting the zero change as a half instance of positive conformity and a half instance of inverted conformity. Thus if a series rises during four expansions (or falls during four contractions) and shows no change in a fifth, the conformity index is $\frac{+400 + 0}{5}$ or $\frac{+450 - 50}{5}$.

In using the table, it must be remembered that, in the case of the index of full-cycle conformity, the count includes business cycles marked off both by peaks and by troughs.

Strictly speaking, an expansion index of +100 does not mean that a series rose consistently *during* the reference expansions covered; nor does a contraction index of +100 mean that a series declined consistently *during* reference contractions. For these indexes as well as the full-cycle index are based simply on the standings at the initial trough, peak, and terminal trough of each reference cycle, and take no account of what happens *within* the phases of expansion and contraction. If the standing in stage V of a reference cycle is higher than the standing in stage I, the series is considered to conform positively to the given expansion, although the movement from stage I to IV may be sharply downward. On the other hand, if the series rises steadily from stage I to IV, but falls to a lower level in stage V than in stage I, the movement as a whole is counted as inverted.[64] Nor does the size of the difference between the standings at peaks and troughs have any influence; a tiny rise and a huge rise count the same in the expansion and contraction indexes. These considerations reinforce what has already been said, namely, that variations in conformity indexes may have little meaning, especially when cycles are few. They suggest also the wisdom of using the average rates of change in conjunction with the indexes.

X Conformity to Business Cycles: Timing Differences Recognized

The measures of conformity just described are valuable because they reveal the consistency with which a given process and general business activity have moved concurrently in the same or opposite directions, but they may obscure regular responses to the cyclical rhythm in business when a process leads or lags by considerable intervals. Imagine a series with no definite secular trend, lagging consistently a quarter of a cycle (strictly, by half a phase) behind the turns in general business. The conformity of such a series to business cycles is in a sense perfect; yet the measures described in the preceding section would probably show zero conformity, for within each reference cycle the direction of movement will be opposite to that of general business over as many stages as it will be similar, and the standing of the series at reference peaks will be about the same as its standing at reference troughs. To avoid such pitfalls, we supplement the measures of conformity based on rigid periods with another set that takes differences in timing into account. The additional measures

[64] During the reference contraction from 1910 to 1912, coal production declined from stage V to VII, then rose to stage IX. This rise carried the figure in stage IX above that in stage V, and thus accounts for the lapse of the contraction index from perfect conformity. Indexes of conformity that take account of every movement from stage to stage of the reference cycles are considered in Ch. 8, Sec. VIII.

are entered in Table R4, a sample of which is shown in Table 47.[65]

The preparation of this supplement follows the lines of Table R3, but requires several preliminary steps. The successive reference-cycle patterns in Table R1 must first be charted, these nine-point patterns studied cycle by cycle, and a decision reached how they can best be divided into two uniform segments—one representing the stages during which the series most regularly advances and the other the stages during which it most regularly declines. For this purpose we frequently draw charts on the plan illustrated by Chart 22. The midpoint of stage I of each reference cycle is taken as the origin. The midpoint of stage II is then placed one-half of a standard unit (which may be any convenient figure) to the right of the origin; in other words, the abscissa of the midpoint of stage II is .5. The abscissas of the midpoints of the later stages are as follows: 1.5 for stage III, 2.5 for stage IV, 3 for stage V, 3.5 for stage VI, 4.5 for stage VII, 5.5 for stage VIII, and 6 for stage IX. This method of plotting reference-cycle patterns blinks the varying durations of reference phases; also the fact that the successive thirds of a reference phase, as we identify them, do not add up to the full duration of the phase, but to this figure minus one month. The advantage of standardizing the durations of all reference phases is that corresponding standings of successive reference cycles, when plotted on a chart, are aligned vertically. The movement of successive reference cycles from one stage to the next can therefore be seen at a glance—in sharp contrast to Chart 19, which represents the varying intervals between successive stages with fidelity. Of course, disregard of the varying durations of reference expansions and contractions may give a very erroneous impression of the behavior of a series that usually or frequently rises or falls throughout a reference cycle; in such cases there is no satisfactory substitute for a chart of the type exemplified by Chart 19.

In general, the task of determining what stages are characteristic of expansion and contraction in a series is easy or difficult according as its timing with respect to business cycles is regular or irregular. In a series like coke production (Table 10) a decision is quickly reached. The series typically rises from stage I to V and declines from stage V to IX; no Table R4 is therefore necessary, or more precisely, Table R3 is the equivalent of Table R4. In bituminous coal production (Table 39 and Chart 22) the timing is less regular, especially at reference troughs. In railroad bond yields (Table 45) the timing varies so much from cycle to cycle that it is not easy to decide upon the typical stages of expansion and contraction, although it is plain that the trough was typically reached later than stage I and that the peak was typically reached later than stage V.

To expose and test what parallelism there may be in the successive reference-cycle patterns of a series, we use a variety of devices. For

65 This step is analogous to that traditionally followed in correlation analysis of time series when one series is 'lagged' after the other, except that our procedure permits differentiation between the timing at peaks and troughs.

Table 47.[65]
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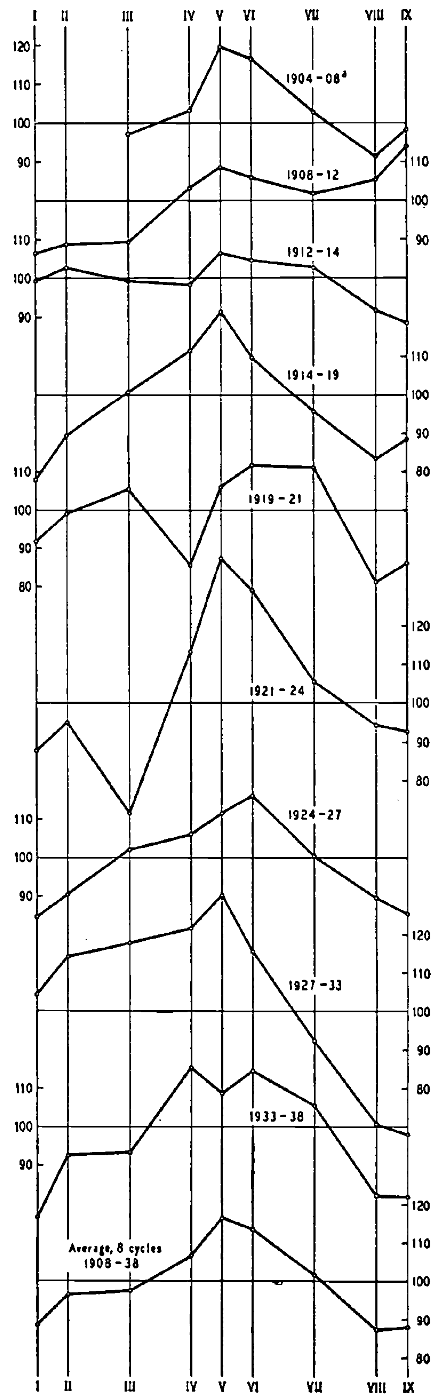
CHART 22

Patterns of Successive Reference Cycles and Their Average Pattern
Bituminous Coal Production, United States, 1905 – 1938
(Drawn to a Schematized Time Scale)



See explanations in text; cf. Chart 19.
[a] The pattern is incomplete, data not being available.

example, we may first examine the timing of the average reference-cycle pattern, then see whether this timing is confirmed by averages of subgroups of cycles, by a count of the rises and falls between successive stages of the reference cycles, by patterns formed by rankings of the reference-cycle standings, by averages of the leads or lags expressed in fractions of the duration of a cycle phase ($\frac{1}{6}$, $\frac{1}{2}$, $\frac{5}{6}$), and so on. When a decision is finally reached, we usually feel reasonably certain that we have made that division into 'expansion' and 'contraction' which best represents the behavior of the series during successive reference cycles. If the behavior of a series is so irregular from one business cycle to the next that there is serious doubt whether it is correlated at all with business cycles, we dispense with Table R4.

We rarely make divisions more unequal than five stages for expansion and three for contraction, or three for the former and five for the latter. The terms 'expansion' and 'contraction' are used in a relative sense. Thus, if a series shows no specific cycles, we mean by 'expansion' the stages during which the advance is more rapid or the decline less rapid, according as the secular trend slopes upward or downward. Table 44 indicates that twenty-four divisions of reference cycles into expansion and contraction are possible when no division more unequal than five and three stages is allowed. Of these, six are coterminous with the reference cycles (lines 1-3, 12, 14, and 16), two are coterminous with the reference phases (lines 2 and 14), and one is coincident with both reference troughs and peaks (line 2).

Once the division of reference cycles in a series has been decided upon, the next step is to class it as 'positive', 'inverted' or 'neutral'. This classification determines whether the *specific* cycles are to be marked off from trough to trough or from peak to peak,[66] and also plays a part in the computation of Table R4. We class a division of reference cycles as positive when the selected expansion segment contains more stages in reference expansion than in reference contraction; or, what comes to the same thing, when the selected contraction contains more stages in reference contraction than in reference expansion. We class the division of reference cycles as inverted when the selected expansion covers more stages in reference contraction than in reference expansion. When the selected expansion overlaps equally reference expansions and contractions, the division is classed as neutral.

Whatever the division, it is applied uniformly in subsequent operations to all the reference cycles covered by the series.[67] When the division is positive, the expansion segments of the series are matched with reference expansions and the contraction segments with reference contractions. When the division is inverted, the contraction segments are

66 See Sec. I of this chapter.
67 See, however, Ch. 10, Sec. VIII.

TABLE 44

Possible Divisions of Reference Cycles when Three to Five Stages Are Assigned to Expansion

Line	Division of the reference cycles of a series		Classification of the division	In computing Table R4 the following stages are matched with	
	Expansion covers stages	Contraction covers stages		Reference expansion	Reference contraction
1	I-IV	IV-IX	Positive	I-IV	IV-IX
2	I-V	V-IX	Positive	I-V	V-IX
3	I-VI	VI-IX	Positive	I-VI	VI-IX
4	II-V	V-II	Positive	II-V	V-II
5	II-VI	VI-II	Positive	II-VI	VI-II
6	II-VII	VII-II	Positive	II-VII	VII-II
7	III-VI	VI-III	Positive	III-VI	VI-III
8	III-VII	VII-III	Neutral	III-VII	VII-III
9	III-VIII	VIII-III	Inverted	VIII-III	III-VIII
10	IV-VII	VII-IV	Inverted	VII-IV	IV-VII
11	IV-VIII	VIII-IV	Inverted	VIII-IV	IV-VIII
12	IV-IX	I-IV	Inverted	I-IV	IV-IX
13	V-VIII	VIII-V	Inverted	VIII-V	V-VIII
14	V-IX	I-V	Inverted	I-V	V-IX
15	V-II	II-V	Inverted	II-V	V-II
16	VI-IX	I-VI	Inverted	I-VI	VI-IX
17	VI-II	II-VI	Inverted	II-VI	VI-II
18	VI-III	III-VI	Inverted	III-VI	VI-III
19	VII-II	II-VII	Inverted	II-VII	VII-II
20	VII-III	III-VII	Neutral	VII-III	III-VII
21	VII-IV	IV-VII	Positive	VII-IV	IV-VII
22	VIII-III	III-VIII	Positive	VIII-III	III-VIII
23	VIII-IV	IV-VIII	Positive	VIII-IV	V-VIII
24	VIII-V	V-VIII	Positive	VIII-V	V-VIII

matched with reference expansions and the expansion segments with reference contractions. When the division is neutral, the division is treated as if it were positive; that is, the expansion segments are matched arbitrarily with reference expansions and the contraction segments with reference contractions. For each series we indicate the procedure by writing at the top of Table R4 what stages are considered characteristic of expansions, and whether expansions are matched with reference expansions or reference contractions, implying that contractions are matched with the other reference phase (see Table 47).

We then compute the average rate of change per month during the stages matched with reference expansions and during the stages matched with reference contractions, and enter the results in Table R4. The computations correspond in every respect to those in Table R3. For example, if the selected expansion runs from stage I to IV of the reference cycles, the duration of this segment, that is, the number of months between the midpoints of stages I and IV, is entered in column (3), for the segment is matched with reference expansion. The excess of the standing in stage IV over that in stage I is entered in column (2), and this difference divided by the interval between the midpoints of the two stages is entered in column (4). Similarly, column (6) shows the number of months in the segment matched with reference contraction, column (5) the total (net) change during this interval, and column (7) the rate of change during this interval. Column (8) is determined from columns (4) and (7) just as in Table R3; so also is column (9), though here, as previously explained, special calculations may be necessary.

A special calculation is also necessary when the segment matched with reference expansion does not begin in stage I; for in that case the standings necessary to determine the average rates of change are expressed on different bases in Table R1. The series on railroad bond yields (see Tables 45 and 47), in which stages III-VI are matched with reference expansion, may serve as an example. There is no difficulty in determining from Table R1 the average change per month from stage III to VI, since the stand-

TABLE 45

Sample of Table R1: Reference-cycle Patterns
Railroad Bond Yields, United States, 1857–1933

Dates of reference cycles	Average in reference-cycle relatives at stage									Av. monthly standing during cycle (per cent)
	I	II	III	IV	V	VI	VII	VIII	IX	
	3 mos. centered on initial trough	Expansion			3 mos. centered on peak	Contraction			3 mos. centered on terminal trough	
Trough – Peak – Trough		First third	Mid. third	Last third		First third	Mid. third	Last third		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Dec. 54–June 57–Dec. 58	...	...	...	...	105.8[a]	115.9[a]	104.6[a]	98.4[a]	95.0[a]	6.438[b]
Dec. 58–Oct. 60–June 61	99.4	100.7	101.6	96.5	96.8	102.5	100.1	102.9	104.2	6.150
June 61–Apr. 65–Dec. 67	112.5	104.8	84.4	87.6	103.9	109.8	111.2	111.0	111.9	5.696
Dec. 67–June 69–Dec. 70	99.6	97.4	97.8	101.0	100.7	103.0	100.8	99.6	100.7	6.396
Dec. 70–Oct. 73–Mar. 79	113.4	111.7	109.0	108.6	112.4	102.6	92.0	89.8	85.3	5.677
Mar. 79–Mar. 82–May 85	112.8	109.5	102.0	96.6	98.2	97.8	97.5	96.7	93.3	4.292
May 85–Mar. 87–Apr. 88	106.3	103.6	98.1	98.2	98.8	98.9	101.4	99.5	98.9	3.767
Apr. 88–July 90–May 91	102.3	100.4	96.7	98.7	100.3	101.8	104.0	104.0	105.8	3.642
May 91–Jan. 93–June 94	102.3	102.8	98.9	98.6	98.8	99.5	103.2	96.9	95.9	3.770
June 94–Dec. 95–June 97	103.6	102.1	101.6	98.0	99.4	99.6	102.7	96.9	94.8	3.488
June 97–June 99–Dec. 00	103.1	102.1	102.9	98.1	96.0	98.3	98.9	99.6	98.7	3.207
Dec. 00–Sep. 02–Aug. 04	94.6	94.5	95.9	96.4	98.5	101.0	105.6	105.7	104.2	3.346
Aug. 04–May 07–June 08	96.6	95.9	96.8	99.8	102.7	104.7	109.0	105.5	104.4	3.612
June 08–Jan. 10–Jan. 12	100.3	98.2	96.4	98.1	99.3	101.4	101.8	102.4	102.2	3.761
Jan. 12–Jan. 13–Dec. 14	96.4	96.6	97.5	98.3	98.4	101.7	101.8	100.8[c]	105.2[d]	3.988[e]
Dec. 14–Aug. 18–Apr. 19	97.3[d]	96.3	93.9	106.1	111.1	110.5	103.5	106.6	107.6	4.312
Apr. 19–Jan. 20–Sep. 21	92.4	92.4	96.7	96.6	99.0	105.4	100.5	101.7	99.0	5.020
Sep. 21–May 23–July 24	109.8	102.2	95.6	99.4	101.3	101.3	101.9	100.6	98.4	4.525
July 24–Oct. 26–Dec. 27	102.7	102.8	102.5	99.9	99.5	98.4	96.7	95.0	93.4	4.336
Dec. 27–June 29–Mar. 33	93.1	94.7	99.5	101.9	103.8	100.5	95.0	105.8	99.8	4.349
Average	102.0	100.5	98.3	98.9	101.0	102.0	101.5	101.1	100.2	...
Average deviation	5.1	4.0	3.3	2.6	3.1	2.5	3.2	3.8	4.7	...

Average is the arithmetic mean; the average deviation is measured from the mean.

[a] Computed on base of inverted cycle, June 1857–Oct. 1860. Excluded from the average and the average deviation.

[b] Base of inverted cycle, June 1857–Oct. 1860.

[c] Includes 3 months, instead of 7; see note 'e'.

[d] Includes 2 months, instead of 3; see note 'e'.

[e] Omits Aug.–Nov. 1914. No data (N. Y. Stock Exchange closed).

ings in both stages are expressed as relatives on the base of the same reference cycle. Thus the standing of bond yields in stage III of the reference cycle of 1858–61 is 101.6; the standing in stage VI is 102.5; the total change is +0.9 over an interval of 12.5 months, or an average of +0.07 per cent per month. But we cannot determine directly the average change per month during the segment matched with the contraction of the given reference cycle—that is, from stage VI of the given cycle to stage III of the next cycle, because the required standing in stage III is expressed in Table R1 as a relative on the base of the next reference cycle. We meet the difficulty by shifting the standing in stage III to the base of the given reference cycle. Thus the standing of bond yields in stage VI of the reference cycle of 1858–61 is 102.5; the standing in stage III of the reference cycle of 1861–67 is 84.4; this standing becomes 78.1 when shifted to the base of the reference cycle of 1858–61; [68] the change from stages VI to III is therefore −24.4 (in relatives of the base of the 1858–61 cycle); and since the interval between these stages is 29.5 months, the average change per month is −0.83 per cent.

The procedure is similar whenever shifts of base are required. For example, if the stages matched with reference expansion were VII-IV, we would need to shift the standing in stage VII of the preceding reference cycle to the base of the given reference cycle in order to compute the average rate of change during the segment matched with the reference expansion of the given cycle. On the other hand, if the stages matched with reference expansion were II-V, we would need to shift the standing in stage II of the following reference cycle to the base of the given reference cycle to compute the average rate of change during the segment matched with the reference contraction of the given cycle.

Once the entries in Table R4 have been made, they are summarized by two sets of conformity measures, in exactly the same way as in Table R3. How useful the new measures can be appears from a comparison of Tables 46 and 47. From Table 46 we learn that the synchronous movements of bond yields and general business have been almost as often in the opposite as in the same direction. We cannot infer, however, that there is no relation between the movements of bond yields and general business, since the table ignores not only haphazard but also systematic differences in timing. Table 47 indicates that when the lagging tendency of bond yields is allowed for, they move preponderantly in the same direction as business cycles.

The conformity measures in Tables R3 and R4 thus serve different ends and we compute both for each series.[69] The measures in the former

[68] The base of the 1861–67 cycle is lower than the base of the 1858–61 cycle in the ratio of 5.696 to 6.150, as is indicated by the entries in col. (10) of Table 45.

[69] Except when no typical expansion or contraction segments can be marked off; in such instances Table R4 is omitted. Of course, when the typical expansion of a series runs from stage I to V or from stage V to IX, Table R4 is the same as Table R3.

tell the behavior of different series during fixed periods, and thereby help to characterize these periods. In particular, they impose a severe test of whether the fixed periods are or are not business cycles in the sense of our definition. But when the problem is to ferret out the 'response' or 'relation' of a series to business cycles, it is essential to take account of the adjustment in time between the given activity and business as a whole.

TABLE 46
Sample of Table R3: Conformity to Business Cycles
Timing Differences Ignored
Railroad Bond Yields, United States, 1857–1933

Dates of reference cycles	Change in reference-cycle relatives during						Av. change per month during reference contraction minus that during	
	Reference expansion			Reference contraction				
Trough – Peak – Trough	Total change	Interval in months	Average change per month	Total change	Interval in months	Average change per month	Preceding reference expansion	Succeeding reference expansion[g]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dec. 54–June 57–Dec. 58	...	...	...	−10.8[a]	18.0	−0.60[a]	...	−
Dec. 58–Oct. 60–June 61	−2.6	22.0	−0.12	+7.4	8.0	+0.92	+1.04	+
June 61–Apr. 65–Dec. 67	−8.6	46.0	−0.19	+8.0	32.0	+0.25	+0.44	+
Dec. 67–June 69–Dec. 70	+1.1	18.0	+0.06	0.0	18.0	0.00	−0.06	+
Dec. 70–Oct. 73–Mar. 79	−1.0	34.0	−0.03	−27.1	65.0	−0.42	−0.39	−
Mar. 79–Mar. 82–May 85	−14.6	36.0	−0.41	−4.9	38.0	−0.13	+0.28	+
May 85–Mar. 87–Apr. 88	−7.5	22.0	−0.34	+0.1	13.0	+0.01	+0.35	+
Apr. 88–July 90–May 91	−2.0	27.0	−0.07	+5.5	10.0	+0.55	+0.62	+
May 91–Jan. 93–June 94	−3.5	20.0	−0.18	−2.9	17.0	−0.17	+0.01	+
June 94–Dec. 95–June 97	−4.2	18.0	−0.23	−4.6	18.0	−0.26	−0.03	+
June 97–June 99–Dec. 00	−7.1	24.0	−0.30	+2.7	18.0	+0.15	+0.45	−
Dec. 00–Sep. 02–Aug. 04	+3.9	21.0	+0.19	+5.7	23.0	+0.25	+0.06	+
Aug. 04–May 07–June 08	+6.1	33.0	+0.18	+1.7	13.0	+0.13	−0.05	+
June 08–Jan. 10–Jan. 12	−1.0	19.0	−0.05	+2.9	24.0	+0.12	+0.17	−
Jan. 12–Jan. 13–Dec. 14	+2.0[b]	12.0	+0.17[b]	+6.8[c]	23.0	+0.30[c]	+0.13[c]	−
Dec. 14–Aug. 18–Apr. 19	+13.8[d]	44.0	+0.31[d]	−3.5	8.0	−0.44	−0.75[d]	−
Apr. 19–Jan. 20–Sep. 21	+6.6	9.0	+0.73	0.0	20.0	0.00	−0.73	+
Sep. 21–May 23–July 24	−8.5	20.0	−0.42	−2.9	14.0	−0.21	+0.21	−
July 24–Oct. 26–Dec. 27	−3.2	27.0	−0.12	−6.1	14.0	−0.44	−0.32	−
Dec. 27–June 29–Mar. 33	+10.7	18.0	+0.59	−4.0	45.0	−0.09	−0.68	...
Average[e]	−1.0	...	−0.01	−0.8	...	+0.03	+0.04	...
Average deviation[f]	...	...	0.24	...	...	0.26	0.35	...

Index of conformity to reference
- Expansions .... −26
- Contractions 0
- Cycles, trough to trough .... −16
- Cycles, peak to peak .... −16
- Cycles, both ways .... −16

[a] Computed on base of inverted cycle, June 1857–Oct. 1860. Excluded from the average and the average deviation.
[b] Computed on base omitting Aug.–Nov. 1914.
[c] Computed on base omitting Aug.–Nov. 1914; standing in stage IX includes 2 months.
[d] Standing in stage I includes 2 months.
[e] Arithmetic mean determined separately for each column. Hence (7)−(4) *may* differ from (8) in the last place.
[f] Measured from the mean.
[g] Only the sign of the difference is entered.

The measures in Table R4 are therefore more useful for analytical purposes than the measures in Table R3, and we use the former more extensively than the latter. Whenever measures of 'conformity' are referred to without qualification, it is to be understood that Table R4 is meant.

Table R4 shares the defects of Table R3 noted in the preceding section, and suffers besides from the uncertainty inherent in any effort to

TABLE 47

Sample of Table R4: Conformity to Business Cycles
Timing Differences Recognized
Railroad Bond Yields, United States, 1857–1933

(Expansion covers stages III-VI. Expansions are matched with reference expansions.)

Dates of reference cycles	Change in reference-cycle relatives during stages matched with						Av. change per month during stages matched with reference contraction minus that during stages matched with	
	Reference expansion			Reference contraction				
Trough–Peak–Trough	Total change	Interval in months	Average change per month	Total change	Interval in months	Average change per month	Preceding reference expansion	Succeeding reference expansion[e]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dec. 54–June 57–Dec. 58	...	...	...	–18.8[a]	25.5	–0.74[a]	...	–
Dec. 58–Oct. 60–June 61	+0.9	12.5	+0.07	–24.4	29.5	–0.83	–0.90	–
June 61–Apr. 65–Dec. 67	+25.4	28.5	+0.89	0.0	35.5	0.00	–0.89	–
Dec. 67–June 69–Dec. 70	+5.2	12.5	+0.42	–6.3	31.5	–0.20	–0.62	+
Dec. 70–Oct. 73–Mar. 79	–6.4	28.0	–0.23	–25.5	72.0	–0.35	–0.12	–
Mar. 79–Mar. 82–May 85	–4.2	24.5	–0.17	–11.7	42.5	–0.28	–0.11	–
May 85–Mar. 87–Apr. 88	+0.8	13.5	+0.06	–5.4	24.0	–0.22	–0.28	–
Apr. 88–July 90–May 91	+5.1	15.5	+0.33	+0.6	18.0	+0.03	–0.30	–
May 91–Jan. 93–June 94	+0.6	13.0	+0.05	–5.5	23.0	–0.24	–0.29	–
June 94–Dec. 95–June 97	–2.0	12.5	–0.16	–5.0	26.5	–0.19	–0.03	+
June 97–June 99–Dec. 00	–4.6	15.5	–0.30	+1.7	25.0	+0.07	+0.37	–
Dec. 00–Sep. 02–Aug. 04	+5.1	14.5	+0.35	+3.5	35.5	+0.10	–0.25	–
Aug. 04–May 07–June 08	+7.9	19.0	+0.42	–4.3	20.0	–0.22	–0.64	–
June 08–Jan. 10–Jan. 12	+5.0	14.0	+0.36	+2.0	25.5	+0.08	–0.28	–
Jan. 12–Jan. 13–Dec. 14	+4.2[b]	10.0	+0.42[b]	–0.2[b]	41.0	0.00[b]	–0.42[b]	–
Dec. 14–Aug. 18–Apr. 19	+16.6	23.5	+0.71	+2.1	11.0	+0.19	–0.52	–
Apr. 19–Jan. 20–Sep. 21	+8.7	8.0	+1.09	–19.2	26.5	–0.72	–1.81	–
Sep. 21–May 23–July 24	+5.7	12.5	+0.46	–3.0	25.0	–0.12	–0.58	+
July 24–Oct. 26–Dec. 27	–4.1	16.0	–0.26	+1.4	20.5	+0.07	+0.33	+
Dec. 27–June 29–Mar. 33	+1.0	17.0	+0.06	–19.4	62.0	–0.31	–0.37	...
Average[c]	+3.7	...	+0.24	–6.2	...	–0.17	–0.41	...
Average deviation[d]	...	...	0.32	...	...	0.20	0.33	...

Index of conformity to reference
- Expansions ... +47
- Contractions ... +30
- Cycles, trough to trough ... +79
- Cycles, peak to peak ... +58
- Cycles, both ways ... +68

[a] Computed on base of inverted cycle, June 1857–Oct. 1860. Excluded from the average and the average deviation.
[b] Computed on base omitting Aug.–Nov. 1914.
[c] Arithmetic mean determined separately for each column. Hence (7)–(4) *may* differ from (8) in the last place.
[d] Measured from the mean.
[e] Only the sign of the difference is entered.

determine leads or lags. Our leads or lags are expressed in cycle-stage units, since they are derived from Table R1. This method confines the leads or lags to coarse fractions of a cycle. Cycle stages sometimes cover many months, and summation over these intervals may conceal or obscure the true timing. To meet this difficulty we might follow another plan, namely: (1) convert the leads or lags in Table S1 into percentages of the duration of the reference phase within which the turn occurs,[70] (2) determine the typical lead or lag from these percentages, (3) divide the reference cycles into expansion and contraction segments on the basis of the typical percentages, (4) represent the beginnings of the expansion and contraction segments by centered three-month averages of cycle relatives, (5) compute the average rates of change during the expansion and contraction segments from these standings, (6) then proceed as in Table R4. Measures made on this plan might be more sensitive to cyclical influences than our present measures,[71] but they would be more laborious to compute, and it is unlikely that they would yield results materially different from those we get by our simpler method.

Another plan we have considered is to divide reference cycles into segments of expansion and contraction on the basis of typical leads or lags measured in months, instead of in percentages of a cycle phase.[72] If the timing of economic series were more uniform in months than in cyclical percentages,[73] a matter of which we as yet have little knowledge, this plan would be preferable in principle to the method we are using. But this variant upon our current practice may break down when the typical lead or lag is long relatively to a particular reference phase. Thus, if a series typically lagged 5 months at reference peaks but led by 5 months at reference troughs, its conformity to a reference contraction lasting just 10 months could not be determined. If the typical lead were 12 months at both reference peaks and troughs, conformity to a reference phase lasting less than 12 months could be determined arithmetically, but it might be difficult to interpret or justify the computation. On the other hand, a typical lag of 12 months under the assumed circumstances would seem plausible, if it reflected the period required by technical or administrative adjustments. These observations are inconclusive; they mean merely that the division of reference cycles on the basis of monthly leads or lags

70 This statement applies only when the lead or lag is shorter than a reference phase. The following statement covers also longer leads or lags: a lead would be expressed as a percentage of the duration of the reference phase just preceding the reference turn with which the specific-cycle turn is compared, and a lag would be expressed as a percentage of the reference phase just following the reference turn with which the specific-cycle turn is compared.

71 The uncertainties of measuring leads or lags should be recalled in this connection; see Sec. II. Also, the 'typical' lead or lag is not necessarily the arithmetic mean.

72 See above, p. 128.

73 In the course of an intensive study of inventory cycles, Moses Abramovitz found this to be true of the behavior of inventories of finished goods held by manufacturers, and certain other classes of inventories.

would work badly under conditions we are sure to encounter occasionally. For the time being at least, our method has the advantage that it avoids these embarrassments.

Our method of making conformity indexes has been criticized on the ground that the cyclical responses of a series may vary in timing from cycle to cycle, and that a uniform division of reference cycles, whether on the basis of leads or lags expressed in months or in percentages, fails to take this variation into account.[74] This criticism may well be justified, but it raises a host of questions in regard to long economic waves, also cyclical versus random movements, with which we cannot cope adequately at present. At this stage of our work there seems to be no satisfactory alternative to uniform divisions; for unless the same division is applied to each reference cycle of a series there is danger that the results will be indistinguishable from subjective judgments of conformity.[75] The method has been criticized also on the ground that it maximizes the conformity indexes, but this criticism involves a misinterpretation. The conformity measures are based on the division of reference cycles that seems most representative, not on the division that yields the highest conformity indexes. For example, our series on sales per store of 'variety chains' in the United States rises typically from stage I to VI, but in one of the five cycles covered there is an abrupt drop between stage V and VI. The standing in stage V of this cycle is above while that in stage VI is below the standing in stage I. In Table R3 the expansion index is +100, and the contraction index +20. But in Table R4 we take stages I-VI to represent the expansion and VI-IX the contraction, so that the expansion index drops to +60, while the contraction index remains +20. The conformity indexes in Table R4 not only need not be maximum indexes, but—as this example illustrates—may actually be lower than those in Table R3, though as a rule they are of the same size or higher.

There is, however, a difficulty in the fact that the margin of preference for the division we make is sometimes slight. In bituminous coal production there is little to choose between stages VIII-V and I-V as the typical period of expansion. In railroad bond yields we take stages III-VI to represent expansion, and on this basis the indexes of conformity to reference expansions, contractions, and full cycles come out +47, +30, +68. But there is some uncertainty whether the expansion culminates typically in stage VI or stage VII. If we took stages III-VII to represent expansion, we would get another set of conformity indexes: +37, +60, +58. The dependence of conformity indexes on the particular division that is made of the reference cycles is frequently much greater than in bond yields, especially in short series. The difficulty in question must be met by exercising care and caution. If there is little to choose between two

74 See Ch. 8, Sec. VIII.

75 See, however, method C in Table 25, and the accompanying text.

divisions of reference cycles, the sensible procedure is to compute Table R4 in a form corresponding to each division and to express each result by a range instead of a single figure.

There is no logical justification of our practice of treating neutral divisions, which overlap equally upon reference expansions and contractions, in the same way as positive divisions. Being marginal (neutral), such divisions could just as well be treated as inverted. However, the consequences of our arbitrary practice are less disturbing than may seem at first glance. If a series consistently rises from stage III to VII and declines from stage VII to III, positive treatment must yield conformity indexes of +100, +100, +100; for stages III-VII, the expansion segments, are matched with reference expansions and stages VII-III, the contraction segments, are matched with reference contractions. Inverted treatment, on the other hand, would yield indexes of −100, −100, −100; since the expansion segments would be matched with reference contractions, and the contraction segments with reference expansions.[76] The signs of the conformity indexes are uncertain, since they reflect the arbitrariness of our procedure. No one who notes that the timing is neutral should be misled. In any case, the absolute magnitudes of the indexes are unequivocal: they tell with what consistency the series rose or fell during the two segments into which the reference cycles are broken. Whether these movements are related positively or inversely to business cycles must be determined on independent grounds.

The marginal divisions of reference cycles merely focus a difficulty that applies also to other, if not to all, divisions. Suppose that stages III-VIII are selected to represent expansions; then we are bound by our rules to match the expansion segments in the series with reference contractions and the contraction segments with reference expansions, and so will get negative conformity indexes.[77] The procedure in this instance is more defensible than in the marginal case, but it rests upon rules that have slight relevance to a theoretical understanding of economic processes. If we can attach no importance to signs of conformity indexes when the expansion runs from stage III to VII, or from stage VII to III, we cannot attach much importance to the signs *per se* when the selected expansion covers stages III-VIII or stages VI-III. If an occasional series on business orders rises typically between stages VI and III, we should probably regard it as conforming positively, although the signs of the conformity indexes were all negative; at any rate we would do so if our

76 Note that if (1) the selected expansion runs from stage III to VII, (2) the conformity analysis is positive, and (3) the expansion, contraction, and full-cycle indexes come out, respectively, +80, +20, +90: then, a shift to inverted treatment would produce conformity indexes of −20, −80, −90, respectively, not −80, −20, −90.

77 They will not be negative necessarily. In the first place, the division may be improper. Even if proper, the segment from stage VIII to III may represent a retardation of growth rather than actual decline, so that the expansion index at least will be positive.

immediate task was to trace the transition of general business from contraction to expansion. Of course, the formal inconsistency can be removed by matching stages VI-III with reference expansions, in defiance of the rules, and we have done that in several instances to facilitate comparisons of closely related series. But the thing to note is that even rigid observance of the rules will never lead an investigator astray if he bears in mind that the signs of conformity measures cannot be taken at face value and must submit to the test of reason.[78]

The list of limitations of the conformity measures might be extended, but enough has been said to demonstrate that judgment of the way a process conforms to business cycles cannot be based solely upon the conformity indexes. In addition one should note carefully how the reference cycles are divided into segments of expansion and contraction, remember that the conformity indexes report merely the consistency of rise or fall between the initial and terminal stages of these segments, study the average rates of change, the average deviations about these averages, the full array from which both averages are made, the number of business cycles and the periods they cover, the relation of the average reference-cycle pattern to the average specific-cycle pattern, the behavior in these several respects of related series in the same country and in different countries, and whatever historical and theoretical analysis can teach concerning the causal connections between the given process and business at large. To determine just how much the available time series can be made to reveal about the business-cycle conformity of different economic processes is an elaborate undertaking that requires skillful use of most of the tools in our kit.

XI Analysis of Quarterly and Annual Data

The methods described in this chapter were originally designed to handle monthly records, but they can be adapted to series that run by quarters and even by years. Table 48 shows, measure by measure, how we analyze quarterly and annual data. The description of some of the operations is limited to positive analysis, but with obvious changes in phrasing the descriptions apply also to series treated on an inverted plan.

The analysis of quarterly series is modeled closely upon the analysis of monthly series. But nothing can overcome the greater coarseness of the time unit in measuring leads and lags. Interpolations are more often necessary in making cyclical patterns than when monthly data are used. Nor does our practice of representing peaks and troughs of specific cycles by single quarterly values ensure that the amplitude measurements will be the same as those derived from three-month averages centered on the peaks and troughs of monthly series; for in quarterly series the bound-

[78] See above, p. 116.

TABLE 48

Methods Used in Analyzing Quarterly and Annual Series

Measure	*Quarterly series*	*Annual series*
Table S1, col. (2)-(5)	If the series expresses totals or averages for the full quarter, the middle month of the quarter that marks the turn of the specific cycles is compared with the corresponding monthly reference turn. When the series relates to the first or third month of the quarter instead of the full quarter, the leads or lags are computed from that month. In other respects, the method is the same as in monthly series. See Ch. 6, Sec. VI for contrast with another method of measuring leads or lags; namely, comparison of quarterly specific-cycle turns with quarterly reference turns.	Leads or lags are entered only if the series conforms very well to business cycles, and permits at least a dozen comparisons of specific-cycle and reference turns. In such series the lead or lag is obtained by comparing the midpoint of the year of the specific-cycle turn with the midpoint of the corresponding monthly reference turn. If the series relates to a single month in the year, that month is compared with the monthly reference turn. If the series starts before the monthly reference dates, the specific-cycle turns are compared with the annual reference turns. In other respects, the method is the same as in monthly series. The timing measures for individual annual turns have little value of themselves; their main use is in estimating the average timing. See Ch. 6, Sec. VI for further discussion of the method and its limitations.
Table S1, col. (6)-(13)	Same as monthly.	Col. (6)-(8) and (12)-(13) same as monthly. Entries in col. (9)-(11) are omitted.
Table S2, col. (2)-(4)	The standings are based on single quarters marking the cyclical turns.	The standings are based on years marking the cyclical turns.
Table S2, col. (5)-(10)	Same as monthly.	Same as monthly.
Table S3, col. (2)-(3)	The average includes all quarters from one turn to the next; the values at the turns are weighted one-half each and the intervening quarters one each.	The average includes all years from one turn to the next. The values at the turns are weighted one-half each; the intervening values, if any, are weighted one each.

Measure	*Quarterly series*	*Annual series*
Table S3, col. (4)	The average includes all quarters from trough to trough; the troughs are weighted one-half each and the intervening values one each.	The average includes all years from trough to trough; the troughs are weighted one-half each and the intervening values one each.

Table S2, col. (5)-(10)	Same as monthly.	Same as monthly.
Table S3, col. (2)-(3)	The average includes all quarters from one turn to the next; the values at the turns are weighted one-half each and the intervening quarters one each.	The average includes all years from one turn to the next. The values at the turns are weighted one-half each; the intervening values, if any, are weighted one each.

Measure	*Quarterly series*	*Annual series*
Table S3, col. (4)	The average includes all quarters from trough to trough; the troughs are weighted one-half each and the intervening values one each.	The average includes all years from trough to trough; the troughs are weighted one-half each and the intervening values one each.
Table S3, col. (5)-(10)	Same as monthly.	Same as monthly.
Table S4, col. (2), (6), (10)	Same as Table S2, col. (2)-(4).	Same as Table S2, col. (2)-(4).
Table S4, col. (3)-(5)	The period from the quarter following the initial trough through the quarter preceding the peak is divided into three parts as nearly equal as possible. The period called 'expansion' in this table is therefore 3 months shorter than the true expansion. If exact division of the 'expansion' into thirds is impossible, the remainder, plus or minus one, is placed in the middle third. The operations thus far are, in principle, the same as in monthly series. But interpolations of standings in short phases are made on a different principle: along a straight line connecting the quarterly relatives. For example, if two quarters in all are available for stages II, III and IV, the standing at (the midpoint of) stage II equals the relative at the trough (standing in stage I) plus five-sixths of the excess of the relative in the quarter following the trough over the relative at the trough. Interpolations from quarterly relatives are sometimes identical with interpolations from cycle-stage standings (the method applied in monthly series); that is true of stages II and IV when only one quarter is available for the 'expansion'. If no quarter is available for stages II, III and IV (in other words, if the full length of the expansion is just one quarter) the standings in these stages are interpolated from the standings at stages I and V, being placed three-quarters of a month apart. The slight difference in the method of interpolating for monthly and quarterly series is explained by the fact that single monthly values are more erratic than single quarterly values.	Only one standing is computed instead of three; namely, at mid-expansion. It is entered in col. (4), which refers to stage III. Col. (3) and (5) are left blank; in other words, there is no stage II or IV. The standing in stage III of each cycle is an arithmetic mean computed according to the following schedule of weights:

No. of years from turn to turn, counting both	Weights of successive values in obtaining mid-phase standing
2	1, 1
3	0, 1, 0
4	0, 1, 1, 0
5	0, 1, 2, 1, 0
6	0, 0, 1, 1, 0, 0
7	0, 0, 1, 2, 1, 0, 0
8	0, 0, 1, 2, 2, 1, 0, 0
9	0, 0, 1, 2, 2, 2, 1, 0, 0
10	0, 0, 1, 1, 2, 2, 1, 1, 0, 0
11	0, 0, 1, 1, 2, 2, 2, 1, 1, 0, 0

For longer periods the weight scheme is the same as for ten years if the number of years is even, and the same as for eleven years if the number is odd. The long intervals in this schedule occur rarely.

TABLE 48 – *Continued*

Methods Used in Analyzing Quarterly and Annual Series

Measure	*Quarterly series*	*Annual series*
Table S4, col. (7)-(9)	Same as Table S4, col. (3)-(5), except for obvious changes in wording.	Only one standing is computed instead of three; namely, at mid-contraction. It is entered in col. (8), which refers to stage VII. For the rest, see comments on col. (3)-(5) of this table.
Table S5, col. (2)-(9)	Same as monthly. Note, however, that when the standings in Table S4 are interpolated from quarterly relatives instead of cycle-stage standings, the calculation of the rate of change is made independently for each stage-to-stage interval.	In principle, same as monthly. But now the entries are confined to the rate of change from the trough to the mid-expansion (I-III), the mid-expansion to the peak (III-V), the peak to mid-contraction (V-VII), and mid-contraction to the trough (VII-IX). When the standing in stage III (or VII) is computed by interpolating between the standings in stages I and V (or V and IX), the rate of change from stage I to V represents the rate of change both from stage I to III and from stage III to V (or V to VII and VII to IX). The measure (at the bottom of the table) of the relative amplitude of erratic movements is omitted.
Table R1, col. (2)-(11)	The reference-cycle pattern is made on the basis of quarterly reference dates. The base of the reference cycles is computed on the same principle as the base of the specific cycles. For the rest, see comments on Table S4.	The reference-cycle pattern is made on the basis of annual reference dates. Note that two sets of annual reference dates have been prepared for the United States: by calendar years, and years ending June 30. The base of the reference cycles is computed on the same principle as the base of the specific cycles. For the rest, see comments on Table S4. The comparison of the amplitudes of reference and specific cycles is omitted.
Table R2, col. (2)-(9)	See comments on Table S5.	See comments on Table S5.
Table R3, col. (2)-(9)	Same as monthly.	Same as monthly.

Measure	*Quarterly series*	*Annual series*
Table R4, col. (2)-(9)	Same as monthly.	Stages matched with reference expansion may be I-III, I-V, I-VII, III-V, III-VII, III-IX, V-VII, V-IX, [illegible]

and specific cycles is omitted.

Table R2, col. (2)-(9)	See comments on Table S5.	See comments on Table S5.
Table R3, col. (2)-(9)	Same as monthly.	Same as monthly.

Measure	*Quarterly series*	*Annual series*
Table R4, col. (2)-(9)	Same as monthly.	Stages matched with reference expansion may be I-III, I-V, I-VII, III-V, III-VII, III-IX, V-VII, V-IX, V-III, VII-IX, VII-III, VII-V. In practice, the choice is more restricted, since the duration of reference phases (notably American contractions) is frequently one year. In such cases the standings in stages III and VII are entirely artificial. In view of the difficulty of recognizing leads or lags in cycle-stage units when handling annual data, we often follow another plan. Thus, if an annual series shows a strong tendency to lead the annual reference dates by one year, we postdate the series one year and then compute the conformity on the basis of a typical expansion running from stage I to V, as in Table R3. If the series lags with considerable regularity by one year, we predate the figures one year and then proceed as in Table R3. In handling American series, two sets of annual reference dates are at our disposal. Hence if a calendar-year series shows cyclical turns that lag regularly 6 months behind the fiscal-year reference dates (years ending June 30), we may predate the figures 6 months (the figures for the calendar years 1929, 1930, etc. are treated, respectively, like figures for the fiscal years 1929, 1930, etc.) and then proceed as in Table R3. But we never postdate or predate a series by more than one year.
Chart of cyclical patterns	Same as monthly. Note, however, that leads or lags shown on the chart are obtained by comparing the quarter in which the series makes a cyclical turn with the corresponding quarterly reference date, not by comparing the midmonth of the quarter (or the first or third month if the series refers to the beginning or end of the quarter) with the monthly reference date, as we do in Table S1. The former procedure is necessary because the duration lines of the reference-cycle patterns, as well as the intervals into which they are broken, are derived from the quarterly reference dates.	Same as monthly, except that leads or lags of specific cycles are in no case represented.

aries of the quarters are fixed by conventions of the calendar while in monthly series the boundaries of the quarters are flexible. Nevertheless, as the following chapter demonstrates, the results obtained by applying our analysis to quarterly series are very similar to those we get from monthly series.

When forced to use annual data we simplify our procedure. The observations are not numerous enough to allow the cyclical patterns to be made on the same plan as when the data run by months or quarters. Instead, we take the standing in the year of the initial trough, of the peak, and of the terminal trough as a framework, and complete the pattern by computing the average standing in mid-expansion and in mid-contraction. This procedure gives five-point patterns; but to avoid confusion the numerals I, III, V, VII, and IX are used to indicate the five stages. In Table S1 we usually omit timing comparisons; that is, measures of leads or lags are restricted to fairly long series that conform exceptionally well to business cycles. In Table R4 we modify the procedure in another respect. Since the small number of observations per reference phase makes it difficult to take account of leads or lags in cycle units, we mark off expansion and contraction segments in some series by years instead of by cycle-stages.[79] The brevity of reference phases limits the extent to which we can prudently go in this direction: in making conformity measures we never assume a systematic lead or lag greater than one year.[80]

The results obtained from annual series are less trustworthy than those from monthly or quarterly. Cycle durations become crude approximations, even when the series undergoes cyclical fluctuations that are large in relation to its secular movements. Summation by years tends to reduce or obliterate cyclical swings. Hence cycle durations obtained by our methods from annual series tend to exceed those obtained from monthly or quarterly series. Amplitude measurements, on the other hand, tend to be reduced; so also are the indexes of conformity. Even our simplified patterns involve an over-elaboration of the data when a cycle lasts less than four years, which is fairly common, particularly in American records.[81] These and other shortcomings of annual data for the study of cyclical behavior are analyzed in detail in the next chapter.

[79] We are indebted to Moses Abramovitz for this procedure, which is described in Table 48.

[80] See above, p. 194.

[81] A cycle with a duration of four years gives us five observations; a shorter cycle therefore requires interpolating.