

The Macroeconomic Effects of Government Spending

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I. INTRODUCTION

Peacetime government spending has risen steadily from less than 10 percent of GNP in the 1920s to about 30 percent of GNP today.¹ The larger role of government has generated increasing interest in the macroeconomic effects of government spending. This paper examines the effects of government spending in a simple macromodel. A small-scale neoclassical model is used for analyzing a classical problem in the literature, namely, the effects of temporary and persistent changes in government spending under a balanced budget. It is found that under a simple lump-sum tax financing scheme, persistent changes in government spending have much larger effects on economic aggregates (such as consumption, output, labor, and investment) than do temporary changes. This result replicates the findings of recent studies by King (1989) and Aiyagari, Christiano, and Eichenbaum (1990).

The second purpose of this paper is to analyze the effects of government spending under different tax financing regimes. For simplicity or technical reasons, the above studies assume that government purchases are financed by lump-sum taxes.² This assumption severely limits the applicability of the theory because most taxes are distortionary. The current paper extends the existing literature to the important case of income tax financing. The results stemming from this extension are fundamentally different from those of lump-sum tax financing. For example, an increase in government spending that is financed by a lump-sum tax under a balanced budget will increase labor effort and real output because of the dominating income effect. Under income tax financing, however, both labor effort and output decline instead of rise in response to an increase in government spending.

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¹ For a statistical review of government spending, see Barro (1984).

² A notable exception is Baxter and King (1990) who considered the case of a proportional tax. Barro (1984) also discussed the implications of income tax financing.

This paper is organized as follows: Section II describes a model economy that will be used for analyzing the effects of government spending. Section III analyzes the consumer's problem. Section IV then calibrates the model economy and considers a specific example. The effects of temporary and persistent changes in government spending, under both the lump-sum tax and the income tax regime, are discussed in Section V. Section VI concludes the paper and points out possible extensions for future studies.

II. THE ECONOMY

The hypothetical economy is assumed to be populated by a large number of identical and infinitely lived consumers. Since consumers are all alike by assumption, their behavior can be represented in terms of a single representative agent. At each date t , the representative agent values services from consumption of a single commodity c_t and leisure l_t . It is assumed that both leisure and the consumption goods are normal in the sense that more is always desired to less and that the utility function $u(c_t, l_t)$ satisfies the usual restrictions, namely, it is strictly increasing, concave, and twice differentiable.

The consumer derives his income from three different sources. First, at time t the consumer provides labor services n_t (hours of work) to the market and earns wage income $w_t n_t$, where w_t is the market-determined hourly real wage rate expressed in consumption units. Labor hours are constrained by the total time endowment, which is normalized to one. Thus, $l_t + n_t = 1$. The second source of income derives from the holding of a single asset called capital. At the beginning of each period, the consumer rents to firms the amount of capital k_t carried from the previous period and collects capital income $r_t k_t$, where r_t is the market-determined rental rate expressed in consumption units. In each period, the government imposes a uniform tax rate τ_t on wage income and capital income so that the net-of-tax earned income for the consumer is $(1 - \tau_t)(w_t n_t + r_t k_t)$.³ The final source of income is the lump-sum

³ For simplicity, wage income and capital income are assumed to be taxed at the same rate. This assumption may not represent the actual tax scheme in the U.S. where capital income (e.g., capital gains) is usually taxed at a lower rate than is wage income.

transfer v_t from the government. Depending upon the budget constraint of the government, the lump-sum payment may be negative, in which case there is a lump-sum tax imposed on the consumer. The total disposable income for the consumer at time t is therefore $(1 - \tau_t)w_t n_t + (1 - \tau_t)r_t k_t + v_t$, which will be allocated between consumption and investment. In short, the budget constraint for the consumer at time t is:

$$c_t + i_t = (1 - \tau_t)w_t n_t + (1 - \tau_t)r_t k_t + v_t, \quad (1)$$

where $i_t = k_{t+1} - (1 - \delta)k_t$ is gross investment⁴ and δ is the depreciation rate of capital ($0 < \delta < 1$). While the capital stock will always be positive, gross investment is allowed to become negative. That is, investment is reversible in the sense that the consumer may actually eat some existing capital stock if he decides to do so.⁵

The consumer's problem is to choose a sequence of contingent plans for consumption and labor supply, taking prices as given, so as to maximize his discounted expected lifetime utility subject to the budget constraint. Formally, the consumer solves the following maximization problem:

$$\max E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \right], \quad 0 < \beta < 1,$$

$$\text{subject to } c_t + i_t = (1 - \tau_t)(w_t n_t + r_t k_t) + v_t,$$

and

$$l_t + n_t = 1, \quad \text{for all } t,$$

where β is the time preference discounting factor and E_0 is the conditional expectation operator. Expectations are taken conditional on the future course of government spending, which will be discussed shortly. The optimal solution of the consumer's problem will be characterized in the next section.

As in the case of consumers, there are a large number of identical firms in the economy; each firm accesses a constant returns to scale technology represented by the production function $F(k_t, n_t)$. During each period, the firm chooses inputs in order to maximize the current profit (or output) at the market-determined wage rate and rental rate. Let y_t denote output at time t ; then the firm solves the following problem:

$$\begin{aligned} \max & [y_t - w_t n_t - r_t k_t] \\ \text{subject to } & y_t = F(k_t, n_t). \end{aligned}$$

Note that the firm's problem is much simpler than that of consumers; it does not involve any intertemporal trade-off as in the consumer's problem. Since the market is assumed to be competitive, the zero profit condition dictates that capital and labor will be employed up to the point where the rental rate r_t and the wage rate w_t equal the marginal product of capital and labor, respectively. That is:

$$w_t = F_n(k_t, n_t) \text{ and } r_t = F_k(k_t, n_t) \quad (2)$$

where F_n and F_k are the marginal product of labor and capital, respectively.⁶ To focus on government fiscal shocks, it has been assumed that there is no uncertainty in the firm's production process. Incorporating such uncertainty into the model is easy, but unnecessary. Also, for simplicity, it is assumed that the firm's income or profit is not taxed.⁷

The role of the government in this hypothetical economy is a simple one. It collects taxes and consumes portions of real output each period. It is assumed that government spending is not utility- or production-enhancing; the resources claimed by the government are simply "thrown into the ocean" and vanish. This assumption may not be the most interesting way to model the function of the government, but it serves as a useful point of departure. Thus, let g_t be the percentage of output that the government claims each period. Then government purchases at time t are $g_t y_t$. In order to finance its purchases, the government collects taxes $\tau_t(w_t n_t + r_t k_t)$, which are equal to $\tau_t y_t$ in view of the constant returns to scale technology. As noted before, the variable τ_t is the income tax rate. The budget constraint of the government at time t , expressed in per capita terms, is:

$$g_t y_t + v_t = \tau_t y_t. \quad (3)$$

In short, equation (3) states that the sum of government purchases $g_t y_t$ and lump-sum transfers v_t must equal tax revenues $\tau_t y_t$. I rule out the possibility of debt financing and money creation as alternative means to finance government purchases. That is, the

⁴ The gross investment i_t is the sum of the net investment $(k_{t+1} - k_t)$ and the replacement investment δk_t .

⁵ Later on, the shock I choose turns out to generate negative gross investment at the time of impact, but not later.

⁶ Throughout the paper, the notation $F_n(\cdot)$ will be used to denote the partial derivative of the function F with respect to the argument n_t , which is the marginal product of labor. Similar quantities are defined accordingly.

⁷ It should be mentioned, however, that the personal income tax in the hypothetical economy is equivalent to a production tax.

government fiscal policy will be conducted under a balanced budget constraint.

The variable g_t is an exogenous policy instrument that is assumed to be a random variable. Ideally, the government would make g_t contingent on certain variables in the economy such as output and labor hours. However, a simplistic view will be taken regarding the policy process $\{g_t\}$. Specifically, g_t is assumed to follow a first-order Markov process with a given transition probability that is known to all agents in the economy. For the bulk of the analysis, the transition probability will be further structured so that it gives rise to the following autoregressive representation:

$$E[g_{t+1}|g_t] = (1 - \rho)g^* + \rho g_t, \quad 0 \leq \rho < 1. \quad (4)$$

In this representation, the conditional mean of g_{t+1} depends only on its immediate past plus a constant term $(1 - \rho)g^*$. The quantity g^* is the steady state or long-run level of the government share of GNP. The autoregressive parameter ρ , assumed to be non-negative and less than one, will determine the persistence of government spending. The larger ρ is, the more lasting will be the displacement of g_t . If $\rho = 0$, then changes in government spending will be completely temporary.

Although the government is not allowed to print money or issue debts to finance its purchases, it still has some latitude in choosing different tax schemes. Two idealized tax systems will be considered in this paper: (1) $\tau_t = 0$ and (2) $\tau_t = g_t$. In the first case, the government finances all its purchases by a lump-sum tax. That is, the transfer v_t is negative and equals g_t in absolute value. In the second case, all government purchases are financed by an income tax and the lump-sum transfer will be zero (i.e., $v_t = 0$). This policy exerts the greatest distortion on the behavior of consumers.

It is not difficult to conceive that the effects of government spending will depend upon the way it is financed. For instance, if the spending is financed by an income tax, there will be substitution effects that will distort market outcomes. Even in the case of a lump-sum tax, market prices will still have to adjust in response to changes in quantities that are induced by income effects. It is impossible to assess the impact of government spending without explicitly considering the market equilibrium.

III. THE EQUILIBRIUM

The equilibrium of the model economy requires that the commodity market clear at each date and that consumers and firms solve their maximization problems at the given market prices. A formal definition of the equilibrium is discussed in the appendix. Here we focus on characterizing the firm's and consumer's equilibrium.

As noted before, the firm's problem is straightforward. It requires, as stated in equation (2), that the rental rate and the real wage rate equal the marginal product of capital and labor, respectively. The consumer's problem requires that the following two first-order necessary conditions be satisfied in equilibrium:

$$u_l(c_t, l_t) / u_c(c_t, l_t) = (1 - \tau_t)w_t. \quad (5)$$

$$u_c(c_t, l_t) = \beta E_t[u_c(c_{t+1}, l_{t+1}) [1 + (1 - \tau_{t+1})r_{t+1} - \delta]]. \quad (6)$$

Equation (5) states that the rate of substitution of consumption for leisure (i.e., the ratio of their marginal utilities) should equal the opportunity cost of leisure, which is the after-tax wage rate. Equation (6) states that the utility-denominated price of current consumption (i.e., marginal utility of consumption) should equal the discounted expected return on saving, which is the expected value of the product of the after-tax return to investment $[1 + (1 - \tau_{t+1})r_{t+1} - \delta]$ and the next period's marginal utility of consumption discounted at the rate β .⁸ This condition implies that in equilibrium the consumer is indifferent between consuming one extra unit of output today and investing it in the form of capital and consuming tomorrow. Equations (5) and (6) together with the budget constraint (1) and the time constraint $l_t + n_t = 1$ completely characterize the consumer's equilibrium.

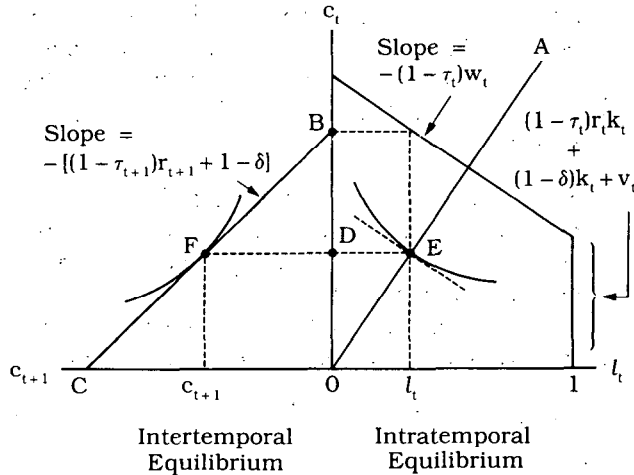
Figure 1 presents a two-quadrant diagram to illustrate the determination of the consumer's equilibrium. For this purpose, we assume that there is no uncertainty in the economy and that the utility function is homothetic.⁹ The right-hand quadrant depicts the

⁸ Note that in a deterministic context, the gross return to investment will be equal to one plus the real interest rate, which is the ratio of marginal utilities of consumption between time t and time $t+1$.

⁹ A utility function is called homothetic if the marginal rate of substitution depends only on the consumption-leisure ratio. A homothetic utility function has the property that the slope of the indifference curve is constant along a given ray from the origin.

Figure 1
CONSUMER'S EQUILIBRIUM

Taking the real interest rate, the after-tax wage rate, and the after-tax rental rate as given



trade-off between consumption (measured along the vertical axis) and leisure (measured along the horizontal axis) for a given wage rate and tax rate at time t . The budget line in the right-hand quadrant for the consumer at time t has two components: the vertical segment corresponds to the nonwage income which is fixed at the beginning of the period and equals $[1 + (1 - \tau_t)r_t - \delta]k_t + v_t$; the sloping segment corresponds to labor income $(1 - \tau_t)w_t n_t$ and has the slope $-(1 - \tau_t)w_t$. From equation (5), the slope of the indifference curve must equal the after-tax wage rate in equilibrium. Since the utility function is homothetic, this condition determines an equilibrium consumption-leisure ratio that is represented by the ray OA extended from the origin.

Equation (5) alone cannot pin down the equilibrium point, however. To locate the equilibrium, one must determine saving from equation (6). Consider the point E along the ray OA. There is an indifference curve tangent at E with slope equal to $-(1 - \tau_t)w_t$. The total income associated with this point, OB, is divided between consumption and investment. If invested, the income available at time $t + 1$ is OC, which is measured from right to left along the horizontal axis in quadrant 2. The absolute value of the slope of the budget line BC is the after-tax rate of return to capital [i.e., $1 + (1 - \tau_{t+1})r_{t+1} - \delta$]. According to equation (6), the intertemporal equilibrium will be achieved at point F, where the indifference curve is tangent to the budget line BC. The point F determines the optimal saving (i.e., k_{t+1}) BD and time t consumption OD which coincide with

those implied by the intratemporal equilibrium point E.¹⁰ Points E and F jointly characterize the consumer's equilibrium. Other quantities such as leisure (labor hours) and time $t + 1$ consumption can be easily derived once the equilibrium point is determined.

The appendix sketches a numerical procedure which permits computation of the equilibrium and quantitative assessment of the effects of government spending. This approach requires one to take an explicit stand on the parameter structure of the economy. The rest of the paper therefore focuses on a specific example and works out the equilibrium implications of changes in government spending.

IV. CALIBRATING THE MODEL

The example considered here involves a logarithmic utility function:

$$u(c_t, l_t) = \theta \ln c_t + (1 - \theta) \ln l_t, \quad 0 < \theta < 1,$$

and a Cobb-Douglas production function:

$$F(k_t, n_t) = k_t^\alpha n_t^{1-\alpha}, \quad 0 < \alpha < 1.$$

This specification is widely used in the literature because it generates dynamics that roughly match several important features of business cycles in the U.S. (see, for example, King, Plosser, and Rebelo (1988)). Our experiment assumes the following values: $\alpha = 0.3$, $\theta = 0.3$, $\beta = 0.96$, and $\delta = 0.05$.

In addition to preferences and technology, one needs explicitly to spell out the process of government spending. As mentioned before, the variable g_t , i.e., the ratio of government spending to real output, is assumed to follow a first-order Markov process. In what follows, the autoregressive parameter ρ is assigned either a value of 0 in the case of a temporary government spending or a value of 0.9 in the case of a more persistent government spending. Further, the random variable g_t is assumed to possess a binomial distribution with probabilities concentrated on five distinct points over a bounded interval. The mean and variance of g_t are taken to be 0.3 and 0.005, respectively. These figures imply that g_t will fluctuate around 30 percent (i.e., $g^* = 0.3$), ranging approximately from 15 percent to 45

¹⁰If the intratemporal equilibrium and the intertemporal equilibrium do not imply the same consumption and saving decisions, then another point along the ray OA must be chosen until the two equilibria are consistent.

percent. Given this specification, the transition probability of g_t , needed to numerically solve the model, is constructed using the method proposed by Rebelo and Rouwenhorst (1989).

V. DYNAMIC EFFECTS OF GOVERNMENT SPENDING

Consider the following scenario: Suppose, initially, that the economy has settled at its steady state equilibrium, and that government spending has reached its long-run level relative to the economy's real output such that 30 percent of real output is claimed by the government. At date 1 the government raises taxes and increases spending. Thereafter, the ratio of government spending to real output follows a time path prescribed by the autoregressive process and gradually returns to its steady state. The left- and right-hand sides of Figure 2 plot the mean path of g_t , measured as percentage deviations from the steady state, for $\rho = 0$ and $\rho = 0.9$, respectively. These hypothetical paths are generated by taking an average of 5000 random realizations of g_t , conditional on the given change at the initial date. Notice that the case of $\rho = 0.9$ yields a more lasting displacement of g_t .

Given the displacement of government spending, what would be the dynamic response of quantities and prices in the pure lump-sum versus pure income tax regime? To answer this question, one needs to understand the forces that govern individual behavior. It is instructive to consider a simpler case in which the increase in government spending is financed by a lump-sum tax. Figure 3 shows the shift in the consumer's equilibrium for this case. As in Figure 1, the points E and F represent the initial equilibrium prior to the occurrence of shocks to government spending. As government spending rises, the budget line shifts downward by an amount equal to the increment of government spending, i.e., $-\Delta v_t = \Delta(g_t y_t)$. With lump-sum tax financing, the slope of the budget line or the after-tax wage rate remains unchanged. As a result, the new equilibrium will still lie on the rays OA and OB (recall that the utility function is homothetic). Given the new budget constraint, the intratemporal and intertemporal equilibrium will be achieved at point E' and F', respectively. Since there is only an adverse income effect, represented by the downward and parallel shift of the budget line, the new equilibrium displays less consumption for both periods and greater work effort. The individual is willing to work harder because leisure is a normal good and the individual is poorer than before due to tax

increases. Because both income and consumption are lower, the effect on saving is indeterminate. In other words, at the initial interest rate, saving or investment may rise or fall, so it appears that the equilibrium interest rate may go either way. In the simulation below, however, we will see that it rises.

How might results differ with income tax financing? Now, substitution effects of changes in the after-tax wage rate and rental rate become potentially important. A change in the income tax rate will induce not only a substitution between consumption and leisure at a given date, but also a substitution of consumption over time. In order to assess the impact of government spending, it is necessary to trace out the equilibrium paths of quantities and prices.

The dynamic responses of the system are displayed below the dotted line in Figure 2. These response functions are calculated by taking an average from 5000 random realizations of the system, conditional on the initial displacement of government spending. To contrast the effects under different tax regimes, each figure contains two transition paths of the same variable; the solid line traces out the dynamic response under lump-sum tax financing; the dotted line traces out the dynamic response under income tax financing. Since the steady state is different for the two tax regimes, these responses are expressed in terms of percentage deviations from the steady state. The following discusses the different implications under the two tax financing schemes.

Lump-Sum Tax vs. Income Tax Financing (Temporary Case)

Consider first the case of a temporary increase in government spending in which g_t jumps from 30 percent to above 40 percent at date 1. Since the shock is temporary, it lasts for about one period (see Figure 2, left-hand side). As the left-hand side of Figure 2 shows, both lump-sum tax financing and income tax financing have negative effects on capital, consumption, and investment. The magnitudes are quite different, however. In the case of lump-sum tax financing, capital falls by 3 percent on impact, while consumption and investment decrease by 2 percent and 70 percent, respectively. The negative effects are much more severe under income tax financing; capital falls by over 9 percent while consumption and investment drop by more than 5 percent and 180 percent, respectively. Two reasons are responsible for these results. First, a rise in the income tax rate decreases the after-tax marginal product of capital. In addition, a decrease in labor

Figure 2

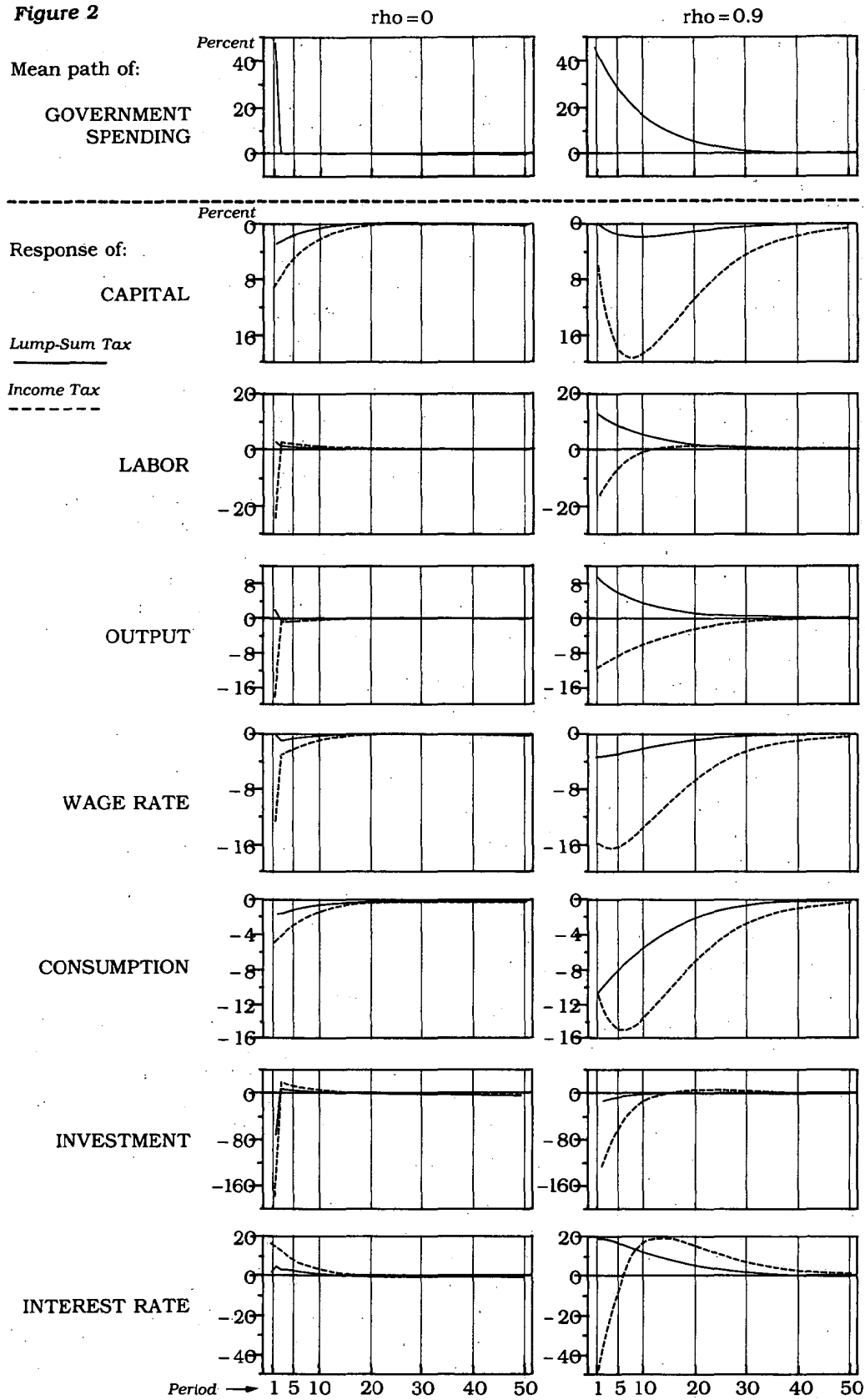
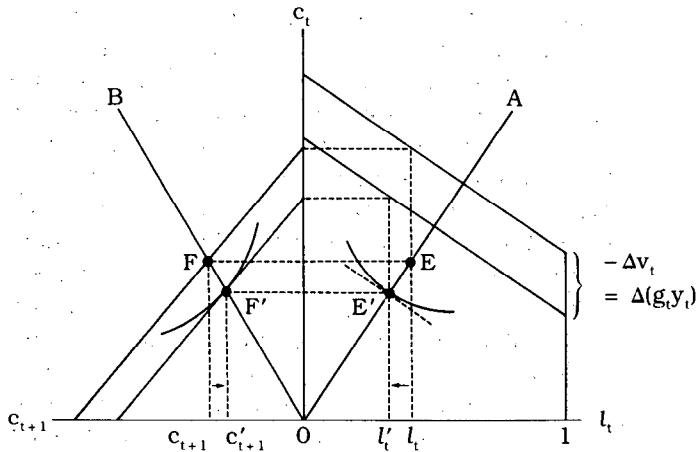


Figure 3
**CONSUMER'S EQUILIBRIUM:
 EFFECTS OF AN INCREASE
 IN LUMP-SUM TAX**



hours, which results also from a lower after-tax wage rate, pushes the marginal product of capital even lower.¹¹ As the productivity of capital falls, agents have less incentive to accumulate capital so that the decrease in investment is larger under income tax financing. Finally, the lower productivity of capital and labor represents an additional loss of income which makes agents poorer than in the lump-sum tax case. Therefore, the decrease in consumption is also larger under income tax financing. In order to induce agents to consume less, the real interest rate will go up to maintain equilibrium in the goods market.

The most visible difference between lump-sum tax financing and income tax financing shows up in their effects on labor effort and real output. In the case of lump-sum tax financing, both labor effort and real output rise on impact by about 2 percent, while income tax financing causes them to decrease by more than 24 percent and 17 percent, respectively. Three forces determine the response of labor supply. First, an increase in government spending leads to the use of real resources and makes agents poorer. This adverse income effect motivates consumers to work harder. However, since the disturbances are temporary, this effect is relatively small. Second, there is a wage effect. As Figure 2 shows, the after-tax wage rate falls by more than 13 percent in the income tax case, as opposed to a tiny 0.6 percent drop in the lump-sum tax case. The larger decrease in the wage rate tends to dampen the response of labor supply.

¹¹ Since capital and labor are complements in production, a decrease in labor input lowers the productivity of capital.

The lower wage rate implies that leisure is less expensive relative to consumption and as a result, consumers are more willing to take leisure instead of consumption. Finally, there is an interest rate effect. According to Figure 2, the real interest rate rises on impact, which largely reflects the increase of aggregate demand associated with an increase in government spending. The rise in the interest rate encourages consumers to work harder due to a higher rate of return. Under lump-sum tax financing, the wage effect is dominated by the income effect and the interest rate effect, resulting in greater labor effort. Since the capital stock is fixed at the beginning of the period, output also increases. Although the interest rate rises even higher in the case of income tax financing, this rise together with the income effect is not sufficient to outweigh the wage effect so that both labor hours and real output decrease.

The initial response of the interest rate and output can be analyzed using the traditional aggregate demand and aggregate supply paradigm. Figure 4a depicts the equilibrium shift in the goods market when a lump-sum tax is used to finance government spending. The real interest rate and output are measured on the vertical and horizontal axis, respectively. The point E is the initial equilibrium point. As government spending rises, the aggregate demand schedule shifts to the right because of the increase in goods demanded by the government. The aggregate supply schedule also shifts to the right because, as explained above, labor supply increases. However, since the increase in government spending is temporary, the shift in aggregate supply will be relatively small due to the negligible income effect. As a result, there is an excess demand at the initial interest rate r^* , which must rise in order to restore equilibrium in the goods market. As the real interest rate rises, aggregate supply (labor effort) increases while aggregate demand (consumption and investment) decreases and the new equilibrium is reached at point F. Comparing points E and F reveals that both output and the real interest rate are higher.

The case of income tax financing can be analyzed in a similar fashion (see Figure 4b). The principal difference here is that the aggregate supply schedule will now shift to the left because of the decrease in labor supply. The shift in aggregate supply will of course depend on the extent to which the marginal

**EFFECTS OF TEMPORARY INCREASES
IN GOVERNMENT SPENDING**

Figure 4a
With Lump-Sum Tax

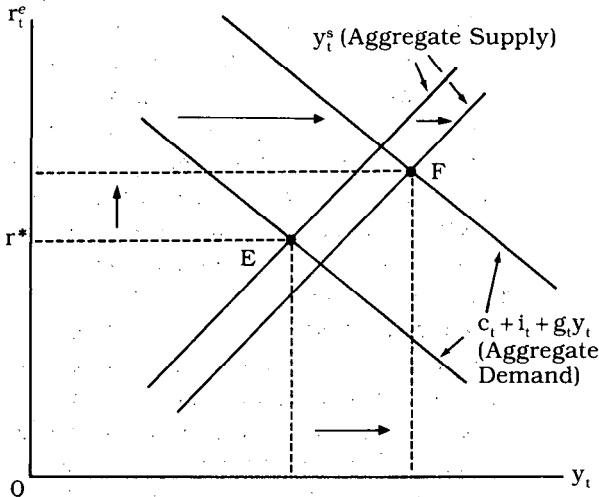
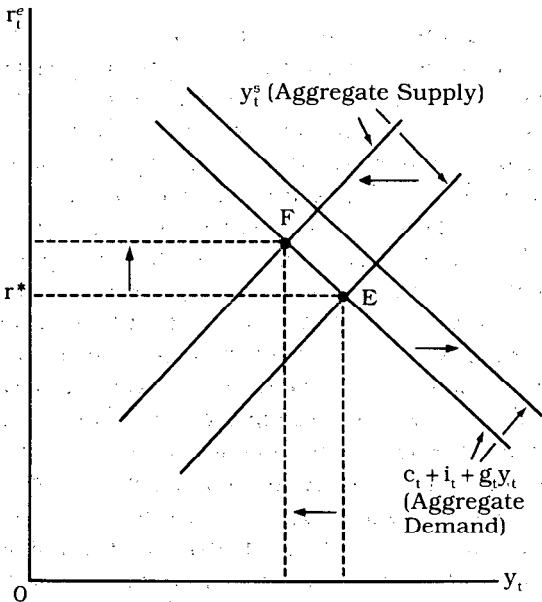


Figure 4b
With Income Tax



product of labor is reduced. It turns out that in the case under consideration the shift of aggregate supply outweighs that of aggregate demand so that output decreases while the interest rate rises.

The analysis up to this point has focused on the short-run effects of an increase in government spending. Consider now the transition dynamics of the system after the initial impact. Since the capital stock is lower at date 1, the marginal product of capital

increases.¹² As a result, agents begin to accumulate more capital after date 1. As the capital stock or investment increases, the real interest rate (or the marginal product of capital) falls and consumption begins to rise. Consumption rises over the transition period because current consumption becomes less expensive relative to future consumption as the interest rate declines over time.¹³ This response applies to the lump-sum tax financing as well as the income tax financing. Figure 2 shows, however, that the transition path of real output and labor effort will depend on the tax regimes. In the lump-sum tax case, both labor hours and real output decrease over time because the real interest rate falls (recall that a lower interest rate implies a lower labor effort). In the case of income tax, the rising wage rate, due to a decrease in the income tax rate and an increase in the capital stock, becomes an overriding force that pushes labor hours up over the transition period. As can be seen from the figure, labor supply will temporarily overshoot the steady state and then decline to the initial equilibrium. As labor supply and the capital stock rise, output also increases until the steady state is reached.

**Lump-Sum Tax vs. Income Tax Financing
(Persistent Case)**

Suppose now that the increase in government spending is more persistent (i.e., $\rho = 0.9$). The right panel shows that the responses are very similar to those of a temporary increase in government spending. The principal difference is the implied wealth effect. Because the shock is expected to persist for a longer period of time, the wealth effect will now play a more important role in the response of quantities and prices.

Consider the case of lump-sum tax financing. Figure 2 shows that labor hours rise by 13 percent and consumption falls by 10 percent on impact. These responses are more than five times the responses in the temporary case. These results occur because consumers are poorer than in the case of a temporary shock. To induce agents to consume less and work harder, the real interest rate will also

¹² Since labor hours rise under lump-sum tax financing, it pushes the marginal product of capital even higher. Under income tax financing, labor effort decreases, but the decrease outweighs that of capital (see Figure 2, left-hand side) and the capital-labor ratio is lower at date 1, implying a higher marginal product of capital.

¹³ The negative correlation between current consumption and the real interest rate is sometimes called the effect of intertemporal substitution.

increase by a larger magnitude. Again, since capital is predetermined, real output rises with labor supply. Perhaps the most interesting difference here is that investment does not go down as much as in the temporary case. The principal reason for this result is that the increase in labor hours occurs over a more extended time period and pushes up the marginal product of capital both now and in the future, thus raising the rate of return to investment. It should be noted, however, that investment will still go down on impact as consumers try to smooth out consumption by holding less capital.

The adverse income effect works in a similar fashion under the income tax regime. In particular, consumption drops by more than 10 percent, as opposed to a 5 percent decrease in the temporary case. Because of the income effect, the decrease in labor hours, which is caused by a lower after-tax wage rate, is smaller than that in the temporary case. Consequently, the decrease of real output is also smaller. Because the decrease of labor effort is smaller, the marginal product of capital does not go down as much as in the temporary case, leading to a smaller decrease in investment.

Although the initial effects of a persistent increase in the income tax rate are not as large as those in the temporary case (except consumption), major variables such as output and investment will stay below their steady state for a long period of time. In fact, the shock is so persistent that agents will eat up some existing capital for one period before consumption (and capital) begins to rise over the transition period. This is the case of a severe recession. The reason for this result is that the marginal product of capital is so low in the future that agents have very little incentive to accumulate capital.

A surprising feature of the income tax regime is that the real interest rate declines in response to a persistent increase in government spending. Again, this result can be attributed to the income effect. As noted before, output supply will decline, but the decrease will not be as much as that in the temporary case because the income effect motivates agents to work harder. On the demand side, the income effect and the lower productivity of capital in the future decrease both consumption and investment at the initial real interest rate. The decrease of consumption and investment may reach the point at which it outweighs the increase of government purchases, leading to a decrease of aggregate demand. The extent to which aggregate demand decreases will depend on how long the shock persists. It turns out

that in the case under consideration, the decrease in aggregate demand is quite sizable so that at the initial real interest rate there is an excess supply, resulting in a lower interest rate. Clearly, this argument hinges on the persistence of the shock and the intensity of the income effect. If the government spending shock is less persistent, then the interest rate will decline by a smaller amount or even increase as in the pure temporary case.

VI.

CONCLUSIONS AND EXTENSIONS

This paper examines the balanced budget effects of government spending under different tax financing schemes. The results suggest that, in the case of lump-sum tax financing, persistent changes in government spending have larger effects on prices and quantities except investment. This result, due to larger income effect and interest rate effect, is consistent with the findings of King (1989) and others. In general, an increase in government spending under lump-sum tax financing will reduce consumption and investment but raise labor effort and real output. This result is driven by the income and interest rate effects that encourage individuals to work harder. Under income tax financing, however, some of the above results are reversed. In particular, regardless of the persistence of spending shocks, both output and labor effort now decline in response to an increase in government spending. This result occurs because the decline in the wage rate dominates the income and interest rate effects.

There are several features of the model that are oversimplified and can be improved upon. Most notably, the government budget is assumed to be balanced in each period. This assumption prevents one from seriously considering the implications of deficit or debt financing. It is relatively easy to introduce such a financing scheme into the model. Extension along this line will probably yield fruitful results if government debts coexist with some types of distortionary tax such as the income tax considered in this paper. The most important implication of debt financing is that it allows the tax burden to be smoothed out over time. This mechanism reduces the distortionary effect on labor supply, particularly when the increase in government spending is temporary. In this case, real output and labor hours may no longer decline as in the case of a balanced budget.

Another extension worth undertaking concerns the function of government spending. The current paper assumes that government spending is a waste of

resources and is not utility- or production-enhancing. This assumption is inappropriate for some types of government spending that may either substitute for private consumption or increase the economy's productivity. These features could be introduced into

the model by specifying a more general utility function or production function, such as those employed by Barro (1984). Such refinements would nullify or even reverse some of the negative effects associated with income tax financing.

APPENDIX

This appendix presents a definition of the equilibrium discussed in the text and outlines a numerical method to construct the equilibrium. Formally, the general equilibrium for the model economy consists of a sequence of quantities $\{c_t, k_{t+1}, n_t, l_t\}$ and prices $\{w_t, r_t\}$ that satisfy the following two conditions: (1) the sequence $\{c_t, k_{t+1}, n_t, l_t\}$ solves the maximization problems of consumers and firms for a given sequence of prices $\{w_t, r_t\}$ and (2) the commodity market clears at each date t such that aggregate demand equals aggregate supply:

$$c_t + i_t + g_t y_t = y_t. \quad (A1)$$

Equation (A1) states that the total of consumption, investment, and government purchases must exhaust total output. The government budget constraint, which must also be satisfied in equilibrium, is implied by the market-clearing condition (A1) and the individual budget constraint (1) in the text.

To further characterize the equilibrium one must solve the maximization problems of consumers and firms. The firm's problem is straightforward. It requires, as stated in equation (2), that the rental rate and the wage rate be equal to the marginal product of capital and labor, respectively. This condition defines the equilibrium prices that will clear the labor market and the rental market for the existing capital stock. As discussed in the text, the consumer's equilibrium is characterized by the budget constraint (1) and the time constraint $l_t + n_t = 1$ together with two first-order necessary conditions, which are re-written as follows:

$$u_l(c_t, l_t)/u_c(c_t, l_t) = (1 - \tau_t)w_t. \quad (A2)$$

$$u_c(c_t, l_t) = \beta E_t[u_c(c_{t+1}, l_{t+1}) \\ [1 + (1 - \tau_{t+1})r_{t+1} - \delta]]. \quad (A3)$$

The meaning of (A2) and (A3) is discussed in the text.

The approach used to determine the equilibrium of the model economy is as follows. First, substitute the time constraint and equations (1) – (3) and (A1) into (A2) and (A3) to obtain

$$\frac{u_l[(1 - g_t)F(k_t, n_t) + (1 - \delta)k_t - k_{t+1}, 1 - n_t]}{u_c[(1 - g_t)F(k_t, n_t) + (1 - \delta)k_t - k_{t+1}, 1 - n_t]} \\ = (1 - \tau_t)F_n(k_t, n_t), \quad (A4)$$

and

$$u_c[(1 - g_t)F(k_t, n_t) + (1 - \delta)k_t - k_{t+1}, 1 - n_t] = \\ \beta E_t \{u_c[(1 - g_{t+1})F(k_{t+1}, n_{t+1}) \\ + (1 - \delta)k_{t+1} - k_{t+2}, 1 - n_{t+1}] \\ \times [(1 - \tau_{t+1})F_k(k_{t+1}, n_{t+1}) + (1 - \delta)]\}. \quad (A5)$$

Note that equations (A4) and (A5) are alternative versions of the consumer's equilibrium with quantities and prices replaced by the market-clearing condition and the firm's marginal conditions. These two equations jointly determine the equilibrium level of capital k_{t+1} and labor n_t ,¹⁴ which can be used to determine consumption, investment, output and equilibrium prices. Note that given the beginning of period capital k_t , a decision rule for k_{t+1} is equivalent for a saving decision made at time t .

In general, an analytical solution to equations (A4) and (A5) does not exist except for a very few special cases. Numerical methods are therefore required to obtain an approximate solution. The following briefly describes an iterative procedure used to solve the model. Technical details of this method can be found in Coleman (1989) and will not be presented here. Basically, the solution to equations (A4) and (A5) comprises a pair of decision rules for capital k_{t+1} and labor n_t that can be expressed as functions of k_t and

¹⁴ Note that k_{t+2} and n_{t+1} are "integrated out" when (A4) and (A5) are solved.

g_t (i.e., the state of the system). The numerical procedure involves approximation of these decision rules over a finite number of discrete points on the space of k_t and g_t . Starting from an arbitrary capital rule (usually, a zero function), the procedure first solves the labor rule from equation (A4) and then iterates on equation (A5) until the capital rule converges to a stationary point, that is, until capital as a function of k_t and g_t does not change over consecutive iterations. The resulting stationary function is the equilibrium solution for capital and labor.

By construction, the above procedure yields solutions that satisfy both (A4) and (A5) for all contingencies of government spending. These solutions imply three imputed or shadow prices that are consistent with the market equilibrium. Specifically, the equilibrium wage rate w_t and rental rate r_t can be computed from the firm's marginal condition (2), and the real interest rate r_t^e , by definition, is the ratio of the marginal utilities of consumption between time t and time $t + 1$, i.e., $u_c(c_t, l_t) / [\beta E_t u_c(c_{t+1}, l_{t+1})]$. In a deterministic equilibrium, the gross real interest rate r_t^e is equal to $(1 - \delta)$ plus the capital rental rate r_{t+1} , as can be seen from equations (A3) and (A5). This is the price that will clear the commodity market.

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