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Retirement in a Life Cycle Model of Labor Supply with Home Production

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Retirement in a Life Cycle Model of Labor Supply with Home Production

Abstract

We analyze the forces that can generate retirement in different versions of standard life cycle models of labor supply. While nonconvexities in production can generate retirement, we show that the size of nonconvexities needed increases sharply as the intertemporal elasticity of substitution for labor decreases. In a model with home production, we show that these models imply a large increase in time devoted to home production at retirement. This is contrary to what is found in the ATUS data. We suggest that nonconvexities in the enjoyment of leisure time may be a promising alternative feature to generate retirement.

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1. Introduction

Understanding how changes in programs such as social security and medicare affect life cycle labor supply in general, and retirement behavior in particular, is central to assessing the effects of these changes on allocations, welfare, and government finances. Accurate assessments require a model of retirement that captures the key economic forces that lead to retirement. To date there is relatively little work that aims to isolate these key forces. Much recent work that models retirement as an optimal solution to an individual's lifetime labor supply problem assumes that workers face a discrete choice between working full time or not working at all. If the hours associated with full-time work are sufficiently great, these models can generate retirement.[1] The question as to why workers must choose between a fixed number of hours versus zero hours is often left unanswered in these models. Typically the authors appeal to the presence of nonconvexities in the production process, associated with workers getting to work and getting set up in a job, coordination issues among workers, etc..... The objective of this paper is to provide a more careful assessment of the extent to which the class of models featuring production based nonconvexities provide a sufficient basis for understanding retirement as part of an optimal labor supply choice. Assessing the empirical adequacy of these models has important implications for policy analysis, since these models give rise to large changes in retirement behavior in response to changes in effective marginal tax rates.

[1] Examples of retirement analyses that adopt this approach are Rust and Phelan (1997), Laitner and Silverman (2005) and Ljungqvist and Sargent (2006, 2009). See also Hurd (1996).

We carry out two exercises. The first exercise considers a standard life cycle model of labor supply extended to allow for nonconvexities in production. These nonconvexities are assumed to take two forms, either as a fixed time cost or as a mapping from hours of work to output (or earnings) per hour that is increasing instead of constant. We compute how large the required nonconvexities must be in order to generate retirement. An important finding is that the degree of nonconvexity required increases sharply as the intertemporal elasticity of substitution for labor decreases. Our calculations suggest a tension between understanding retirement and understanding changes in hours worked and wages for prime aged males. In particular, if the elasticity of hours worked with respect to wage changes is .50 or lower, the required nonconvexities seem unreasonably large given current estimates. Either the production based nonconvexities are not sufficient as a model of retirement, or previous estimates of labor supply elasticities for prime aged males are far too low. More definitive conclusions would require better measurement.

In the second exercise we extend our model to allow for home production. Recent work by Aguiar and Hurst (2005) emphasizes the potential importance of home production in the context of retirement. We find that allowing for home production reduces the degree of nonconvexities required to generate retirement. A closely related and robust implication of this model is that time devoted to home production should increase when a worker retires. Moreover, the magnitude of this increase is very large, with time devoted to home production at least doubling.

We then examine the ATUS data to assess the extent of the increase in time devoted to home production when workers retire. Although this data set does not allow us to observe what happens to the home production time of specific individuals as they retire, we can examine what happens to the home production time of a group of workers as more of them become retired. This data reveals no evidence of such large changes in home production time.

While our analysis does not rule out nonconvexities in production as being an important element in modeling retirement, we believe that it does suggest that this feature alone is not adequate. Uncovering the quantitatively important additional elements is an important topic for future research. Our analysis suggests that nonconvexities in preferences for leisure that take the form of higher marginal utility of leisure when not working in the market is a promising candidate to pursue.

An outline of the paper follows. In Section 2 we describe the standard life cycle model without any nonconvexities and discuss why this model does not provide a very satisfactory foundation for understanding the general phenomenon of retirement. Section 3 examines the standard model extended to include nonconvexities in production and characterizes the degree of nonconvexities required as a function of the intertemporal elasticity of substitution for labor. Section 4 extends this analysis by allowing for home production, and Section 5 analyzes time use data from the ATUS. Section 6 concludes and discusses avenues for future research.

2. Retirement in the Standard Life Cycle Model

In this section we consider a standard life cycle model of labor supply, by which we mean an individual who maximizes a time separable and strictly concave utility function subject to a convex budget set. We show why it is very difficult for this model to generate retirement when standard functional forms are used. While this result is probably not surprising to anyone familiar with these models, it is useful to consider the issue of retirement in this model since it allows us to focus on a key tension that will also be present in more complex settings considered later. Before proceeding with the analysis it is important to clarify what we mean by retirement. We use this term to describe the situation in which an individual chooses a large, abrupt and persistent decrease in their hours of work following a lengthy period of full time work. The most extreme form of this phenomenon is the case in which a worker who has worked full time for thirty or more years chooses to move from full time market work in one period to no market work in all subsequent periods. In reality retirement may be more nuanced than this, with an individual first moving from full time to part time or occasional work, or moving back and forth between no work and part time work. The key feature for our purposes is that for almost all workers who eventually withdraw from the labor force, retirement does not represent a smooth and gradual reduction of working time from full time work to withdrawal from the labor market. For ease of exposition, in what follows we will focus almost exclusively on the extreme form of retirement, where a worker moves from full time work to no work. As we will see, it is this type of transition that will be the most difficult to account for, making it an appropriate focus of our analysis.

The standard life cycle model necessarily generates a motive to smooth consumption and leisure over time. Movements in relative prices can induce individuals to choose profiles in which consumption and leisure (and hence work hours) change over time, but in the face of constant prices, and assuming that the interest rate exactly offsets the agent's discounting, the individual will choose constant sequences for consumption, leisure and work hours. Viewed from the perspective of this standard model, retirement is a puzzling phenomenon, since it represents anything but a smooth profile for leisure and work.

To facilitate discussion and make the analysis more precise, it is useful to consider a specific model, purposefully simplified so as to make the main points more transparent. We consider the utility maximization problem of a finitely lived individual who faces markets for labor and consumption, and is allowed to borrow and lend freely. For now we assume that there are no policies in place that involve taxes or transfers, either explicitly or implicitly. That is, there is no social security and there is no private pension plan. Let c_t and h_t denote consumption and hours of work at age t, and normalize the total time endowment to equal one each period, so that leisure at age t is given by $1 - h_t$. We assume that the individual has preferences described by:

$$\sum_{t=0}^{T} [\log(c_t) + \alpha_t v(1 - h_t)] \tag{2.1}$$

where T is the length of life, assumed to be known with certainty. The utility function is separable both across time and across consumption and leisure at a point in time. The choice of $\log c_t$ as the utility from consumption implies offsetting income and substitution effects, and allows the model to be consistent with the fact that hours worked have changed relatively little over time despite large changes in the real wage. The function v is assumed to be twice continuously differentiable, strictly increasing, strictly concave and have infinite derivative at 0. The parameter α_t is included to allow for the possibility that the marginal rate of substitution between consumption and leisure changes with age. To simplify exposition we have assumed that the individual does not discount the future, but will also assume that the interest rate is zero.[2] The present value budget equation faced by this individual is given by:

$$\sum_{t=0}^{T} c_t = \sum_{t=0}^{T} w_t h_t + Y \tag{2.2}$$

where Y is the present value of non-labor income for the individual.

Letting μ be the Lagrange multiplier on the budget equation and assuming an interior solution, the first order condition for h_t is:

$$\alpha_t v'(1 - h_t) = \mu w_t \tag{2.3}$$

Assuming that the solution for h_t is interior, the optimal solution for h_{t+1} is also interior if the following inequality holds:

$$v'(1) < v'(1 - h_t) \frac{\alpha_t}{\alpha_{t+1}} \frac{w_{t+1}}{w_t} \tag{2.4}$$

[2] Alternatively, we could assume that the individual discounts at a positive rate but that the interest rate is positive and perfectly offsets this discounting. All of our analysis would carry over to this case, but the algebra is somewhat simpler in the zero discounting case. More generally, we could assume that the interest rate and discount factor are not perfectly offsetting. This induces slopes to the life cycle profiles for hours of work and consumption. While there is some empirical support for the presence of these effects they are not central to the issues we focus on here, and so in the interest of simplicity we abstract from them.

Otherwise, the optimal solution is $h_{t+1} = 0$. Our focus is to understand how to account for retirement, as defined above, in this framework. That is, assuming that h_t is a number corresponding to full time work, how would this framework rationalize that h_{t+1} equals zero. As just noted, the solution for h_{t+1} is zero if:

$$v'(1) \geq v'(1 - h_t) \frac{\alpha_t}{\alpha_{t+1}} \frac{w_{t+1}}{w_t} \tag{2.5}$$

If $v'(1) = 0$ then $h_{t+1} = 0$ if and only if $w_{t+1} = 0$. But if $v'(1) > 0$ it is possible that $h_t > 0$, and $h_{t+1} = 0$ even with $w_{t+1} > 0$.

A simple calculation is informative to provide a quantitative perspective on this issue. Consider a standard choice for the function v:

$$v(1 - h) = \frac{A}{1 - \frac{1}{\gamma}} (1 - h)^{1 - \frac{1}{\gamma}} \tag{2.6}$$

The parameter γ denotes the intertemporal elasticity of substitution for leisure. There is of course an extensive literature that has estimated this value, largely focusing on the labor supply behavior of prime aged males. In connecting with the empirical literature it is preferable to consider the intertemporal substitution elasticity for labor rather than for leisure. The implied intertemporal elasticity of substitution for labor is different by a factor of $(1 - h)/h$. In what follows we will use the abbreviation IES to *always* refer to the elasticity for *labor*, which as just noted is in general not equal to γ. There is a voluminous literature that has estimated the IES using variation in hours and wages over the life cycle. Early examples include Ghez and Becker (1975), MaCurdy (1981), and Heckman and MaCurdy (1980). The early literature found relatively small estimates for males, on the order of .3 or less, but much larger values for women. (See Pencavel (1986) for a survey of early work.) Subsequent work, including recent papers by Kimball and Shapiro (2003), Pistaferri (2003) and Domeij and Floden (2006) have refined these estimates in various ways, and found larger estimates, in the range of $.7 - 1.0$ for males. (See Hall (2007) for a critical survey of the recent literature.) In a model that assumed human capital accumulation, Imai and Keane (2004) found an IES that exceeds 3. Wallenius (2007) argues that this estimate is likely to be biased upward, and that adding human capital accumulation does not lead to estimates of the IES that are much greater than 1.0.[3]

The first property that we highlight is a connection between the value of the IES and the difficulty in accounting for retirement. This connection is intuitive. As noted above, the concavity of $v(1-h)$ generates a motive to smooth leisure over time, and the lower the value of the IES, the greater is this motive. Retirement constitutes a dramatic departure from smoothness in the leisure profile, and the greater the desire for smoothness, the more difficult it is to account for this departure from smoothness. To quantify this we proceed as follows. Denote the ratio $\frac{\alpha_t}{\alpha_{t+1}}\frac{w_{t+1}}{w_t}$ in equation (2.5) by R_{t+1}, which can be interpreted as the return to work in period $t+1$ relative to period t. Equation (2.5) tells us the highest value of R_{t+1} consistent with $h_{t+1} = 0$ given a value of γ and a value of h_t. (The parameter A does not appear in this expression.) Assuming a weekly endowment of discretionary time equal to 100 hours, and that a full time worker devotes 45

[3]Although we do not consider human capital accumulation in our analysis, Wallenius (2009) contains some results about the degree of nonconvexities required in that setting.

hours to market work (including commuting), we have $h_t = .45$. Table 1 gives the maximum value of R_{t+1} that is consistent with inducing pure retirement at age $t+1$.

Table 1

Value of R_{t+1} to Induce Retirement

IES=2.0	IES=1.0	IES=.75	IES=.50	IES=.25	IES=.10	IES=.05
.61	.48	.38	.23	.05	.001	.000

The values in the table are revealing. Even with a very large value of the IES, say equal to 2, one would still require a drop of almost 40% in the return to work between consecutive years to generate retirement. If one focuses on values of the IES that are commonly used in the literature, such as those that are .25 and below, the message is that one needs almost a 100% drop in the return to work in order to generate retirement. Even for values that are at the upper end of estimates for males, such as .75 and 1.00, one still needs the return to work to drop by more than 50% between consecutive years. The basic message is clear–in this framework, the only way that one can generate retirement is by assuming dramatic decreases in wages or dramatic increases in the disutility of working precisely at the time of retirement. It is important to note that the above analysis has abstracted from uncertainty. While allowing shocks to w, α, or Y changes the analysis somewhat, the basic message is that only large shocks to these values can induce retirement. While it is undoubtedly true that some individuals experience shocks to market opportunities, wealth, and/or health that might rationalize retirement in this context, the available evidence does not support this as the prime cause of retirement (see, e.g., Blau and Shvydko (2007)). Put somewhat differently, although shocks may alter an individual's plans for retirement, they do not seem to be the underlying explanation for why individuals plan to retire in the first place.

Although we abstracted from social security and pension programs, the previous analysis can also be used to gauge how large the change in effective tax rates associated with these features would need to be in order to induce retirement. In particular, assuming no change in the return to work, i.e., constant wages and disutility of working, Table 1 tells us the required magnitude of the increase in either the implicit or explicit effective tax rate on earnings to induce retirement. Once again, the message is that these values are extremely large.[4] One issue to note regarding implicit tax rates associated with private pension plans is that these rates are specific to the job and so are typically not relevant if the individual considers working for a different employer. In this case the relevant calculation would be the value of R_{t+1} based on the other job opportunities for this individual.

The above calculations were based on the assumption that the individual moves from full-time work to no work at all. How are these values affected by someone who moves from full-time work to part time work? Or by assuming that someone moves from part-time work to pure retirement? Tables 2 and 3 show the results, using $h = .20$ to reflect part time work.

[4] While typically not the case in the US, in other countries individuals can find themselves in a situation where they face dramatic increases in effective tax rates from one year to the next. In particular, systems in which one must retire in order to collect social security benefits can induce large changes in effective tax rates at the normal retirement age.

Table 2

Value of R_{t+1} to Induce Transition from Full-Time to Part-Time

IES=2.0	IES=1.0	IES=.75	IES=.50	IES=.25	IES=.10	IES=.05
.80	.63	.54	.40	.16	.01	.000

Table 3

Value of R_{t+1} to Induce Retirement from Part-Time

IES=2.0	IES=1.0	IES=.75	IES=.50	IES=.25	IES=.10	IES=.05
.88	.76	.70	.58	.34	.07	.01

Comparing Table 2 with Table 1 we see that the required values of R_{t+1} are not as small, but the fact remains that these represent dramatic changes in either preferences or opportunities between consecutive years, even for values of the IES on the high end. A similar message applies to the results in Table 3. That is, inducing pure retirement even for an individual who is currently working part-time still requires a dramatic change in the economic returns to work.

The simple conclusion that we want to emphasize from the above analysis is that it is very difficult to reconcile retirement with the standard model of life cycle labor supply. While this statement seems to apply to all reasonable values of the IES, we also want to note that the lower the IES the more difficult it becomes to account for retirement in this framework.

One of the key motivations for developing a model of retirement is to understand how changes in effective tax rates associated with changes in social security or medicare will influence retirement behavior. While the standard life cycle model does not seem to offer a promising theory of retirement, it is interesting to note one implication of this framework. Loosely speaking, retirement in the above model occurs only as a result of a very large change in the return to work at the individual level. In such a setting it is likely that small changes in the effective return to work associated with small changes in the provisions of social security will have no effect on retirement. Put somewhat differently, the calculations described above suggest that marginal changes in the return to work have virtually no impact on the decision to retire.

3. Nonconvexities as a Source of Retirement

The analysis in the previous section illustrated the difficulty in generating retirement in a model with a strictly concave time separable utility function and a convex budget set. An obvious alternative is to relax one of these two assumptions so as to generate a nonconvexity in the consumer's optimization problem. Because nonconvexities can lead to discontinuities in the decision rule for hours, this alternative would seem to overcome the key problem encountered in the previous section.[5] As noted in the introduction, much recent work on retirement assumes that budget sets are nonconvex, implicitly because of some underlying nonconvexities in production. In this section we describe two variations of a model that features nonconvexities in production. While it is true that this model diminishes the tension that we found in the standard life cycle model, we show that the same

[5] Cogan (1981) is a classic reference for empirical work on the implications of fixed costs, though he did not focus on retirement.

tension is very much present. Specifically, it remains true that the smaller the IES, the more difficult it is to generate retirement, in the sense that the required degree of underlying nonconvexity is larger.

3.1. Fixed Time Costs

We begin with a version of the model recently put forth by Prescott et al (2009). The analysis is greatly simplified by assuming a continuous time framework. Normalizing the length of life to 1, preferences are now given by:

$$\int_0^1 [\log(c(t)) + \alpha(t)v(1 - h(t))]dt \tag{3.1}$$

We consider a nonconvexity that takes the form of a fixed time cost associated with work, which we denote by $\bar{h}$. If an individual gives up $h(t)$ units of leisure at time t this will lead to $\max\{0, h(t) - \bar{h}\}$ units of labor that can be sold in the labor market. Letting $w(t)$ denote the wage at time t, the present value budget equation for this individual now reads:

$$\int_0^1 c(t)dt = \int_0^1 w(t) \max[0, h(t) - \bar{h}]dt + Y \tag{3.2}$$

Whereas in the previous section one required changes in at least one of $\alpha(t)$ or $w(t)$ to generate retirement, with fixed costs one can generate retirement without any variation in these factors. In order to focus on the forces associated with the nonconvexity it is convenient to initially assume that $\alpha(t) = \alpha$ and $w(t) = w$ for all t. We will return to consider the more general case later in this section. With

w and α constant over time, and the interest rate and discount factor perfectly offsetting each other, the optimal timing of work for the individual is indeterminate. That is, the individual could choose to do all of the work at the beginning of life, all at the end of life, or all in the middle, etc..... As such, the model may not appear to be a good model of retirement per se. However, this is an artifact of the extreme but useful assumption that there is no change in the return to work over time. If, for example, there is even an arbitrarily small positive slope to $\alpha(t)$, or negative slope to $w(t)$ (even if only at later ages), then the indeterminacy would vanish. So while we will work with a specification in which the timing of work is indeterminate, we will focus on the solution in which work occurs at the beginning of life, followed by retirement.

Independently of the optimal labor supply decision, the optimal consumption decision for this individual is to smooth consumption perfectly.[6] The optimal solution for the hours profile can take one of two forms. The first corresponds to a solution in which it is optimal for the individual to have positive hours in all periods. By symmetry, the solution in this case will entail a constant amount of work at each time t. This case applies if the nonconvexity is not sufficiently large to overcome the forces that favor smooth leisure. The second and more interesting case is one in which the individual chooses to work at some but not all dates.[7]

[6] Later in the paper we discuss how the model can address the documented drop in consumption at retirement. The model could also be extended in different ways to generate a hump-shape in consumption during working life, but because this does not appear to be central to the issue of generating retirement, we do our analysis in the simpler specification.

[7] To be more precise we are interested in the case where the individual chooses positive hours for a positive measure of time but strictly less than measure 1. In our discussion we will ignore the issues associated with deviations on sets of measure zero.

Once again by symmetry, the individual will work the same amount of time in all periods with positive hours. If an individual works for measure e periods and gives up $h > \bar{h}$ units of leisure at each date, he or she will have a present value of income equal to:

$$e(h - \bar{h}) + Y \tag{3.3}$$

We can thus write the utility maximization problem as:

$$\max_{e,h} \log[e(h - \bar{h})w + Y] + ev(1 - h) + (1 - e)v(1) \tag{3.4}$$

Our main interest is to determine the conditions necessary for an interior solution for e, since this corresponds to there being retirement. Assuming interior solutions for both e and h we obtain the following two first order conditions:

$$\frac{(h - \bar{h})w}{e(h - \bar{h})w + Y} = v(1) - v(1 - h) \tag{3.5}$$

$$\frac{w}{e(h - \bar{h})w + Y} = v'(1 - h) \tag{3.6}$$

Divide these two equations by each other to obtain:

$$h - \bar{h} = \frac{v(1) - v(1 - h)}{v'(1 - h)} \tag{3.7}$$

This equation is similar in spirit to equation (2.5) that we derived in the previous section to characterize the conditions necessary to generate retirement at a given age. Assuming the same functional form as in the last section:

$$v(1-h) = \frac{A}{1-\frac{1}{\gamma}}(1-h)^{1-\frac{1}{\gamma}} \tag{3.8}$$

equation (3.7) becomes:

$$h - \bar{h} = \frac{1}{1-\frac{1}{\gamma}}[1-(1-h)^{1-\frac{1}{\gamma}}](1-h)^{\frac{1}{\gamma}} \tag{3.9}$$

If we choose a value of γ and specify the level of work during working years, h, this expression gives the value of $\bar{h}$ that is required for the optimal solution to display both the level of work h while working and retirement. Note that the solution for $\bar{h}$ is independent of the age at which retirement occurs. That is, subject to requiring that the optimal solution entails working hours of h when working, the size of the fixed cost that is required to generate retirement is independent of whether one wants the worker to retire at age 40 or age 65. Conditional upon an interior solution for e and the level of h, the length of working life is determined by the values of A, and Y/w.

Proceeding as before, we consider the same range of values for the IES as in the previous section, and once again consider the case where h is equal to .45 while working. Table 4 provides the results.

Table 4

Value of $\bar{h}$ Required for Retirement

IES=2.0	IES=1.0	IES=.75	IES=.50	IES=.25	IES=.10	IES=.05
.08	.14	.18	.23	.32	.40	.43

One interpretation of the fixed cost is that it represents commuting costs. Estimates of average commuting times would suggest a value of $\bar{h}$ equal to around .05. From this perspective all of the above values would seem high, with the possible exception of the $IES = 2.0$ case. However, there are several issues that should be noted in the context of interpreting $\bar{h}$ as commuting costs. First, it is not clear that average commuting costs are the appropriate measure to use in this calculation. We are not aware of any evidence to suggest that retirement does not occur for individual with relatively low commuting costs. For example, commuting times tend to be much less in smaller urban areas, but it seems that retirement remains a prominent feature of life cycle labor supply in these settings as well as larger urban areas. This being the case, we need to understand why retirement occurs not only for an individual with a high value of $\bar{h}$ but also for those individuals with low values of $\bar{h}$. From this perspective we may want to evaluate equation (3.7) using a value of $\bar{h}$ much lower than the average value of time spent commuting.

Second, although commuting costs are often mentioned as the classic example of a fixed cost, how they should be interpreted here depends very much on how one interprets a period. If one interprets a period to be a day, then commuting costs are clearly a fixed cost. If one interprets a period as a week, one might think that commuting costs are a step function of weekly hours, with the potential length of working days determining the length of the steps. In this case, marginal adjustments in working hours, say by 1%, would typically represent adjustments inside one of the steps, so that it again seems reasonable to think of commuting costs as fixed costs. But if one thinks of the period as a year, then an individual could adjust annual hours by 1% or even less by changing the number of days worked. If this is the margin of adjustment then commuting costs would be a proportional cost not a fixed cost.

Prescott et al (2009) argued that the fixed costs in this specification were intended to capture set-up costs that a worker experiences at work. Even without taking a strong stand on the appropriate level of setup costs, the results in Table 4 clearly suggest a tension. If one considers values of the IES that are .25 and below, it seems very hard to rationalize the level of fixed time costs required to generate retirement since the implication is that over two thirds of the individual's time is devoted to set-up costs. This tension has implicitly appeared in empirical work on labor supply. French (2005) estimates a life cycle model of labor supply that includes accounting for retirement, and assumes fixed time costs. When he estimates this specification he finds a relatively small intertemporal elasticity of substitution for labor, but a very large value of the fixed time costs. In fact, his estimate of the fixed time cost using annual data is more than 1200 hours per year. While this is the value that is required to make the model consistent with the data, this value seems hard to justify. In the next subsection we consider an alternative formulation which provides a somewhat sharper comparison with the data.

3.2. An Alternative Formulation

The previous subsection assumed that the nonconvexity took the form of a fixed time cost. Whether this time cost reflects time getting to and from work or time getting set up at work, it necessarily induces a nonconvex relationship between the hours of leisure that the individual gives up and the earnings per hour of leisure sacrificed. In this section we consider another form for this relationship. Specifically, we assume that the individual faces a nonlinear wage schedule for the wage per unit of time as a function of time spent working in a particular period. In particular we assume that the wage schedule $w(h)$ is given by:

$$w(h) = w_0 h^{\theta} \tag{3.10}$$

where $\theta \geq 0$. If $\theta = 0$ this reduces to the standard case in which the wage per unit of time worked is independent of the number of hours worked, and implies a convex budget set. The advantage of this specification is that there has been some empirical work to guide us in thinking about reasonable values of the parameter θ. (See, for example, Moffitt (1984), Keane and Wolpin (2001) and Aaronson and French (2004).) While there are some important issues involved in estimating this parameter and there is by no means a definitive estimate, the value suggested by this work is $\theta = .4$. This is the value that French (2005) assumed when considering this specification. In this section we consider this alternative formulation and solve for the value of θ that is required to generate retirement as a function of the preference parameter γ and the time devoted to work when working.

The relevant maximization problem now becomes:

$$\max_{e,h} \log(ew_0 h^{1+\theta} + Y) + ev(1-h) + (1-e)v(1) \tag{3.11}$$

Repeating the same steps as before, we arrive at the expression:

$$\frac{h}{1+\theta} = \frac{1}{1-\frac{1}{\gamma}}[1-(1-h)^{1-\frac{1}{\gamma}}](1-h)^{\frac{1}{\gamma}} \tag{3.12}$$

which now gives us a required value of θ given values for γ and h. The results are contained in Table 5.

Table 5

Value of θ Required for Retirement

IES=2.0	IES=1.0	IES=.75	IES=.50	IES=.25	IES=.10	IES=.05
.22	.46	.64	1.04	2.53	8.19	18.2

The qualitative pattern is the same as that found in Table 4: the smaller the IES the larger the nonconvexity needs to be in order to generate retirement as part of an optimal choice for the individual. If we take the value of $\theta = .4$ as a guideline for a reasonable magnitude, we see that any values of the IES that are .75 or below are not consistent with retirement given this degree of nonconvexity.

Subject to the issues raised earlier about commuting time, one might want to allow for a fixed time cost associated with getting to and from work in addition to the nonlinear wage schedule that applies to hours at work. It is easy to assess how this affects the numbers in Table 5. If one assumes a fixed time cost of $\bar{h}$ in addition to the nonlinear wage schedule, so that only $h - \bar{h}$ hours are productive, one obtains the following expression:

$$\frac{h-\bar{h}}{1+\theta} = \frac{1}{1-\frac{1}{\gamma}}[1-(1-h)^{1-\frac{1}{\gamma}}](1-h)^{\frac{1}{\gamma}} \tag{3.13}$$

Assuming that commuting costs represent 10% of total working time, the new values of θ are given in Table 6.

Table 6

Value of θ Required for Retirement When $\bar{h} = .1h$

IES=2.0	IES=1.0	IES=.75	IES=.50	IES=.25	IES=.10	IES=.05
.09	.31	.48	.84	2.17	7.27	16.3

While this adjustment does influence the magnitude of the required values of θ, the overall picture does not change much. There are three very different potential interpretations of the tradeoff that these tables describe. If one wants to take as given that the IES is .50 or smaller, and that θ is around .40, then Tables 5 and 6 suggest that this model is not a good model of retirement, since it cannot generate retirement for empirically reasonable parameters. Alternatively, if one accepts this model as correct and assumes that $\theta = .4$ is reasonable, we would take this as evidence in support of a value for the IES that is either near to 1.0 or even slightly above 1.0. This value is much larger than many of the traditional estimates derived from looking at changes in hours and wages for prime age males, and at the upper end of more recent estimates that challenge the earlier estimates. Lastly, if we accept the model as correct and take a stand on the value of IES, then the above table could be used to derive information about the value of θ. Given these very different potential interpretations, it is not possible to draw any firm conclusions at this point as to whether this model can be viewed as an empirically reasonable model of retirement. In the next section we show that more definitive conclusions emerge when we extend the model to allow for home production. But before considering that extension we first note the implications of the current model for the response of retirement to changes in the marginal effective tax rate.

3.3. Tax Effects

One of the reasons for wanting to assess the extent to which models featuring non-convexities in production represent empirically reasonable models of retirement is that they have a very sharp prediction for the response of retirement to changes in marginal tax rates. In this section we derive this implication. The main result derived here is essentially contained in Prescott et al (2009), but we include it here for completeness, since it will serve as a benchmark for extensions in the next section. Consider a very simple and stylized social security system with the following features. An individual pays a flat tax of τ on his or her labor earnings. The individual receives benefits later in life that are a function of his or her total tax paid into the system. Total tax paid into the system will equal $\tau e(h-\bar{h})^{1+\theta}w_0$. (Recall that we have assumed zero interest rates to simplify the analysis, so the aggregate tax paid is also the present value of taxes paid.) Because we assume perfect capital markets, all that matters to the individual is the present value of the total transfer, and not the timing of the transfer. Consider the following simple specification of the formula that determines the transfer payment:

$$T(\tau e(h-\bar{h})^{1+\theta}w_0) = B_0 + B_1 \tau e(h-\bar{h})^{1+\theta}w_0 \tag{3.14}$$

where $B_0 \geq 0$ and $0 \leq B_1 \leq 1$. The individual will take the parameters B_0 and B_1 as fixed when making his or her labor supply decision. If $B_0 = 0$ and $B_1 = 1$ then social security is a pure forced saving program, and given that we have assumed that there are perfect markets for borrowing and lending, this system will have no impact on individual choices. The lost income on the left-hand side of the budget equation is exactly offset by the increased income on the right hand side of the budget equation, and the individual understands that each dollar paid in taxes will be returned to them. But if $B_1 < 1$ then the marginal increase in benefits is less than the marginal increase in taxes from the perspective of the individual, independently of the value of B_0. To simplify analysis we will assume that in equilibrium the system is individually fair, in the sense that in equilibrium the present value of the transfer received by this individual is the same as the present value of taxes paid. This requires that the value of B_0 is set just right.[8] In this case the effect of the system is equivalent to what would be generated by having a flat tax on labor income of $(1 - B_1)\tau$ with these funds being used to fund a

[8] If there is heterogeneity among individuals this condition could only hold on average, so this analysis should be understood as reflecting the effects on such an average person.

lump-sum transfer that from the individual's perspective is independent of his or her choices. Loosely speaking, the distortion implied by the system is what is left after we take out the forced saving component. We will call $(1-B_1)\tau$ the effective marginal tax rate for this individual and denote it by τ^e

In what follows we consider changes in the social security system that change the effective marginal tax rate. This change may come from changes in the benefit formula or changes in the tax rate, or some combination thereof.[9] For convenience set nonlabor income equal to zero. The optimization problem that an individual solves is given by:

$$\max_{e,h} \log((1-\tau^e)e(h-\bar{h})^{1+\theta}w_0 + B_0) + ev(1-h) + (1-e)v(1) \tag{3.15}$$

where we assume that B_0 is set so that at the optimal choices of the individual, $B_0 = \tau^e e(h-\bar{h})^{1+\theta}w_0$. Proceeding exactly as before, and assuming an interior solution we obtain first order conditions for e and h given by:

$$\frac{(1-\tau^e)(h-\bar{h})^{1+\theta}w_0}{(1-\tau)^e e(h-\bar{h})^{1+\theta}w_0 + B_0} = v(1) - v(1-h) \tag{3.16}$$

$$\frac{(1-\tau^e)(1+\theta)(h-\bar{h})^{\theta}w_0}{(1-\tau^e)e(h-\bar{h})^{1+\theta}w_0 + B_0} = v'(1-h) \tag{3.17}$$

Dividing these two equations by each other we obtain:

$$h-\bar{h} = (1+\theta)\frac{v(1)-v(1-h)}{v'(1-h)} \tag{3.18}$$

[9] When we do this we are implicitly assuming that the value of a is changed simultaneously to eliminate any income effect associated with the change in the system.

which is the same expression that we derived earlier, implying that changes in the effective tax rate have no effect on how much people work while employed. But making use of the balanced budget condition, we can rewrite the first order condition for h as :

$$e = (1 - \tau^e)\frac{1 + \theta}{(h - \bar{h})v'(1 - h)} \tag{3.19}$$

Given that h is independent of τ^e, it follows that e, and hence total labor supply, responds with a unitary elasticity to changes in $(1 - \tau^e)$. In particular, independently of the curvature in the function $v(1-h)$ and independently of the magnitude of the fixed cost, this model features a large change in retirement in response to changes in effective tax rates. So while we have raised issues concerning the relationship between the required nonconvexity and the degree of curvature in $v(1-h)$, conditional on the model generating retirement, the responsiveness of retirement to changes in effective tax rates is independent of these details.

3.4. Sensitivity Analysis

Before moving on to the extension including home production, we assess the extent to which some of the special features assumed above are influencing the quantitative results that we derived. The first issue we consider has to do with the assumption that there are no age effects on either wages or utility from leisure. The second issue that we consider has to do with separability between consumption and leisure. We deal with each in turn.

Adding age varying wages or utility does not matter at all for the results derived above if we assume that these profiles are continuous. We demonstrate this in the context of an age varying utility from leisure, given by $\alpha(t)$. Consistent with our desire to focus on retirement, i.e., that the period of not working in the market occurs at the end of life, we assume that the $\alpha(t)$ profile is increasing.[10] It is no longer the case that hours of work when working are constant, so we will now have an hours of work profile $h(t)$. The maximization problem is now:

$$\max_{e,h(t)} \log\left(\int_0^e w(h(t)-\bar{h})dt + Y\right) + \int_0^e \alpha(t)v(1-h(t))dt + \int_e^1 \alpha(t)v(1)dt \tag{3.20}$$

Assuming an interior solution the first order condition for e is:

$$\frac{w(h(e)-\bar{h})}{\int_0^e w(h(t)-\bar{h})dt + Y} = \alpha(e)v(1) - \alpha(e)v(1-h(e)) \tag{3.21}$$

Of particular interest is the first order condition for the optimal level of hours at the time of retirement, $h(e)$. The first order condition for this value is:

$$\frac{w}{\int_0^e w(h(t)-\bar{h})dt + Y} = \alpha(e)v'(1-h(e)) \tag{3.22}$$

Dividing these two expressions gives:

$$h(e) - \bar{h} = \frac{v(1) - v(1-h(e))}{v'(1-h(e))} \tag{3.23}$$

It follows that our previous calculations all go through exactly, as long as we understand that the level of hours that we use in the calculation refers to the

[10] In fact, our analysis would go through unchanged if we instead assumed that this profile were u-shaped, thereby potentially generating a period of nonwork at the beginning of life as well.

level of hours worked at the time of retirement. But with this one proviso, the calculation is entirely unchanged.[11]

We can also extend this analysis to handle the case of a single discontinuity in the α profile. If retirement occurs at the point of the discontinuity then what matters is not what the hours were just prior to retirement, but rather what the hours worked would have been at e had the individual not retired. The first order condition for optimal hours tells us that this value must satisfy

$$\alpha(e)v'(1-h(e)) = \lim_{t\to e}\alpha(t)v'(1-h(t)) \tag{3.24}$$

The discontinuity in the α profile can reduce the needed nonconvexity through lowering the appropriate h to feed into the calculations. Basically, the implied level of hours is the value that equates marginal disutility of work at the margin with those periods in which the individual chose to work. However, as we know from our earlier calculations, for relatively small elasticities the effect of even moderate discontinuities on h is somewhat small, and moreover the effect of a small change in h on the required nonconvexity is also small. We conclude that the previous calculations are not much affected by allowing for time changing α or w, unless we allow for very large discontinuities.

The second sensitivity analysis that we consider is whether separability between consumption and leisure matters. To explore this we consider a period

[11] Rogerson and Wallenius (2009) assume that productivity varies with age, but do not target working time at retirement in their calibration, so this result does not apply to their calculations.

utility function of the form:

$$\frac{1}{1-\sigma}[c(t)^{\phi}(1-h(t))^{1-\phi}]^{1-\sigma} \tag{3.25}$$

In the limit as σ goes to one this period utility function converges to the additive log specification, which would correspond to an IES for labor of 1.22 in our earlier analysis. We consider the results for the case where the only nonconvexity is due to the presence of fixed time costs, denoted by $\bar{h}$. The results for the other cases are similar, so we do not report them. Although we did not report results for this particular value of γ, the value of $\bar{h}$ required to induce retirement assuming $h = .45$ when working is equal to .13.

With non-separable preferences consumption will be different when working and when retired, so an optimal solution to the individual's problem will now be described by four values: e, h, c_w, and c_r, where e and h are as before, and the two values c_w and c_r are consumption when working and when retired, respectively. Unfortunately, with nonseparable preferences it is not possible to derive a simple analytic expression that tells us the required degree of nonconvexity in order to generate retirement. Instead we must resort to a numerical analysis. In what follows we consider different values of σ, and then for each value of σ we find values of ϕ and $\bar{h}$ that lead to optimal solutions for h and e equal to .45 and 2/3 respectively. In the separable case we found that the degree of nonconvexity required was independent of what fraction of life was spent in employment as long as the fraction was less than one. While that result no longer holds exactly, it remains true approximately, so the choice of a target for e turns out not to be very important. Nonetheless, we note that $e = 2/3$ is a reasonable target in that it corresponds to an individual having 60 years as an adult and spending 40 of them in employment, and this is the value that we use in the calculations.

Results are reported in Table 6. In addition to reporting the required level of $\bar{h}$, we also report the value of ϕ and the ratio of consumption when retired to consumption when working.

Table 6

Results with Nonseparable Utility

σ	$\bar{h}$	ϕ	c_r/c_w
2.00	.17	.270	.71
1.50	.15	.280	.83
1.25	.14	.285	.90

The results show that assuming nonseparability between leisure and consumption tends to somewhat increase the degree of nonconvexity required to generate retirement. We have only presented results for the case in which σ is larger than one, since this leads to higher consumption when working than when retired, which is consistent with what is observed in the data. Given that the magnitude of the drop in consumption at retirement is on the order of 15% (see Laitner and Silverman (2005)), this suggests that a value of σ between 1.25 and 1.50 would be most reasonable. This increases the needed value of $\bar{h}$ by roughly 10%. We conclude that nonseparabilities do not have a very large impact on the previous calculations, and that to the extent that they do, their effect is to increase the needed degree of nonconvexity.

4. Home Production and Retirement

In this section we extend the previous analysis to include home production. There are several motivations for this extension. First, work by Aguiar and Hurst (2005) has emphasized that home production may play an important role in understanding the behavior of consumption at retirement. Second, if it is the case that households substitute at least partially from market work into home production at retirement, it follows that the change in leisure will not be as large, and this will potentially influence the nonconvexities that are needed to induce retirement. Third, broadening the analysis to include home production will allow us to assess the basic model's implications for a wider range of predictions, and therefore potentially shed more light on the empirical importance of the mechanisms that this model stresses. In particular, it may help us distinguish between the three different conclusions noted in the previous section.

4.1. Model

To maintain transparency we will return to the assumption of separability between leisure and consumption and no age effects. In the spirit of Becker (1965) we model home production by assuming that the consumption that individuals care about is an aggregate of market purchased goods and time. We assume that goods and home production time are aggregated according to a CES aggregator:

$$c(t) = [ag(t)^{\varepsilon} + (1-a)h_n(t)^{\varepsilon}]^{1/\varepsilon}. \tag{4.1}$$

In the spirit of Gronau (1977) we distinguish between leisure and working time and assume that lifetime utility is given by:

$$\int_0^1 [\log c(t) + v(1 - h_m(t) - h_n(t))]dt \tag{4.2}$$

The present value budget equation continues to be given by:

$$\int_0^1 g(t)dt = \int_0^1 w_0[h_m(t) - \bar{h}]^{1+\theta} dt \tag{4.3}$$

where for simplicity we have dropped the non-labor income term. The optimal solution can now be described by 6 numbers: e, h_m, h_w, h_r, g_w, and g_r. As before, e is the fraction of life spent in employment, and h_m is the time spent working in the market when working. The values g_w and g_r represent the consumption of market goods when working and retired, respectively, and the values h_w and h_r represent time spent in home production when working and retired, respectively.

4.2. Solution

Let μ be the multiplier on the budget equation, and define c_w and c_r as follows:

$$c_w = ag_w^{\varepsilon} + (1-a)h_w^{\varepsilon} \tag{4.4}$$

$$c_r = ag_r^{\varepsilon} + (1-a)h_r^{\varepsilon} \tag{4.5}$$

Assuming interior solutions for all variables we get the following first order conditions.

$$g_w: \quad \frac{a g_w^{\varepsilon-1}}{c_w} = \mu \tag{4.6}$$

$$g_r: \quad \frac{a g_r^{\varepsilon-1}}{c_r} = \mu \tag{4.7}$$

$$h_w: \frac{(1-a) h_w^{\varepsilon-1}}{c_w} = A(1-h_m-h_w)^{-\frac{1}{\gamma}} \tag{4.8}$$

$$h_r: \frac{(1-a) h_r^{\varepsilon-1}}{c_r} = A(1-h_r)^{-\frac{1}{\gamma}} \tag{4.9}$$

$$h_m: \quad A(1-h_m-h_w)^{-\frac{1}{\gamma}} = \mu(1+\theta)(h_m-\bar{h})^{\theta} \tag{4.10}$$

$$e: \quad \frac{1}{\varepsilon}\log(\frac{c_w}{c_r}) + \frac{A}{1-\frac{1}{\gamma}}[(1-h_m-h_w)^{1-\frac{1}{\gamma}} - (1-h_r)^{1-\frac{1}{\gamma}} = \mu[g_r - g_w - (h_m-\bar{h})^{1+\theta}] \tag{4.11}$$

plus the budget equation:

$$e g_w + (1-e) g_r = e(h-\bar{h})^{1+\theta} \tag{4.12}$$

Unfortunately, one cannot obtain simple analytic expressions like those we derived in the earlier models to describe the extent of nonconvexities required to generate retirement given targets for hours of work at the time of retirement. As a result, we proceed in the next subsection to obtain results from numerical analysis.

4.3. Results

The above model has six parameters: a, ε, A, γ, θ, and $\bar{h}$. We set $\bar{h} = .045$ and will assess the additional required nonconvexity in terms of the value of θ.

There are two elasticity parameters: ε and γ. As before, we will be interested in exploring the relationship between elasticity parameters and the level of nonconvexities required to generate retirement, so we will consider various values for each of these parameters. Given choices for $\bar{h}$, γ, and ε, we solve for values of the other three parameters that are consistent with three targets that describe labor allocations: the level of market work when working (h_m), the length of life spent in employment (e), and the amount of time devoted to home production when working (h_w). The three target values are $h_m = .45$, $h_w = .10$, and $e = 2/3$.[12] In the earlier analysis without home production we noted that the length of time spent in employment prior to retirement had no impact on the level of nonconvexity needed to generate retirement, given a level of work at the time of retirement. This sharp result does not hold in the model with home production. However, it is approximately true, so that our results about the degree of nonconvexity required is not sensitive to our target for e.

Before presenting the results it is important to note that the implication of a given value of γ for the implied intertemporal elasticity of substitution of labor is not constant across models with and without home production. As previously noted, the mapping from the intertemporal elasticity of substitution for leisure to the intertemporal elasticity of substitution for labor is determined by the ratio of working time to leisure time. Holding time devoted to market work constant

[12] The choice of $h_w = .10$ is perhaps a bit on the low side. According to the ATUS, home production time for working male head of households in their late fifties would correspond to .14 of their total discretionary time endowment. We choose a somewhat lower value to allow for the possibility that this may include some part-time workers with higher values for home production time. The results that we emphasize become even starker if we assume a higher value of home production time, so in this sense our choice is conservative.

but increasing the time devoted to home production necessarily decreases the time devoted to leisure and so changes this mapping. Given that we target home production time of .10 when the individual is working, it follows that the ratio of working time to leisure time is now .55/.45, as opposed to .45/.55 in the earlier analysis. We use this new ratio to determine the appropriate value of γ to represent a given value of the IES for labor. Table 7 provides the mapping from the IES for labor into the values of γ that we use.

Table 7

Implied Values of γ for Given IES for Labor

IES=2.0	IES=1.0	IES=.75	IES=.50	IES=.25	IES=.10	IES=.05
2.44	1.22	.92	.61	.31	.12	.06

For each of the values of γ given above, we consider values of ε ranging from 0 to .40, implying an elasticity of substitution between goods and time varying from 1 to 1.67. The existing empirical literature suggests that values as high as 2.5 are reasonable, but as we will see below, values this high do not seem reasonable in the context of this model, so we only report results for an elasticity as high as 1.67.[13]

Table 8 reports the implied values for the parameter θ. For ease of comparison the final column of this table repeats the values from the earlier calculations that

[13] Using aggregate data, McGrattan, Rogerson and Wright (1997) find a value of ε in the range of $.40 - .45$, while Chang and Schorfheide (2003) find a value in the range of $.55 - .60$. Using micro data, Rupert, Rogerson and Wright (1995) find an estimate in the range $.40 - .45$, while Aguiar and Hurst (2007a) report an estimate for their benchmark specification in the range of $.50 - .60$.

did not include home production.

Table 8

Values of θ for Home Production Model

	$\varepsilon = 0$	$\varepsilon = .20$	$\varepsilon = .40$	w/o HP
$IES = 1.00$	.18	.17	.16	.31
$IES = .50$	.41	.38	.34	.84
$IES = .25$	.70	.63	.52	2.17
$IES = .10$	1.03	.86	.70	7.27

Extending the analysis to allow for home production has quite substantial implications for the degree of nonconvexity that is required in order to generate retirement. In particular, consider the case where the IES is equal to .10. In the model without home production, the required value of θ was over 7. With $\varepsilon = .40$ this value is now reduced to .70, which is an order of magnitude smaller. Similarly, with the IES of .25 and $\varepsilon = .40$ one can generate retirement with values of θ that are not too much larger than current estimates, whereas previously this case required a value of θ greater than 2.

Two additional statistics are also of interest. Our procedure does not target the amount of time devoted to home production during retirement. This value is implicitly determined as part of the calibration procedure that chooses values of a, A, and θ to match the three targets. And although preferences are separable between consumption and leisure, the presence of a home production margin implies that consumption of goods need not be the same during working years as during retirement. Table 9 displays the results for h_r and g_r/g_w.

Table 9

Values of h_r and g_r/g_w in the Home Production Model

	h_r			g_r/g_w		
	$\varepsilon = 0$	$\varepsilon = .2$	$\varepsilon = .4$	$\varepsilon = 0$	$\varepsilon = .2$	$\varepsilon = .4$
$IES = 1.00$	.23	.23	.24	1.00	.96	.90
$IES = .50$	.24	.26	.29	1.00	.93	.83
$IES = .25$	.35	.37	.39	1.00	.89	.76
$IES = .10$	.46	.47	.48	1.00	.86	.70

We begin by discussing the results for the drop in consumption of goods at retirement. As expected, the drop in consumption of goods at retirement is increasing in the value of ε. Intuitively, since time becomes relatively more plentiful during retirement, the individual substitutes time for goods when moving from working to retirement. The greater the degree of substitutability, the greater is this effect. But while the qualitative effect is intuitive and expected, Table 9 shows that its magnitude can be quite large. In particular, for low values of the IES and high values of ε, the drop can be as much as 30%. As noted earlier, the drop in consumption of goods at retirement is estimated to be in the neighborhood of 15%. Allowing for nonseparability would induce an additional channel though which consumption of goods would drop at retirement. It therefore seems that any specification yielding a value of g_r/g_w less than .85 should be viewed as empirically unreasonable. Given the model, this would imply that either ε is no larger than .2 or that the IES must be above .5. In particular, although this model can reconcile low values of the IES with reasonable degrees of nonconvexity, these specifications have counterfactual implications along other dimensions.

Next we consider the model's implications for time devoted to home production in retirement. At the beginning of this section we argued that it may be easier to generate retirement in a model with home production if retirement is associated with a substitution from market work to both home production and leisure. Intuitively, this substitution into home production decreases the extent of the jump in leisure associated with retirement, thereby decreasing the extent of the nonconvexity required to generate retirement. This intuition suggests that the assumed increase in home production time at retirement plays a key role. However, as just noted, our procedure does not assume a level of home production time in retirement. Instead, it determines the level of home production time in retirement that is consistent with the time allocation during employment. And what our results imply is that if one is able to generate retirement in this model, which necessarily implies a large change in market work, then it is necessarily the case that there must be a large increase in time devoted to home production. The intuition for this result is as follows. If it is optimal for an individual to devote a particular amount of time to home production when they are doing a lot of market work and have little leisure, then the incentive to do home production when market work drops to zero necessarily becomes very large. As our calculations show, time devoted to home production more than doubles, even if we assume that $\varepsilon = 0$ and the IES is equal to 1. Although we did not report it in the table, this continues to be the case even for the case when the IES is equal to 2. At the other extreme, if the IES is equal to .10 the model requires that retirement basically constitutes a switch from full time market work to full time home production.

Another way to summarize the change in time allocation associated with retirement is to ask what fraction of the drop in market work is allocated to leisure. When the IES is equal to 1.0, roughly three quarters of the drop in market work goes into leisure. When the IES is equal to .50 this fraction drops to around two-thirds, and when the IES is .25 this value is about two-fifths. When the IES drops to .10 the increase in leisure is only about one fifth of the decrease in market work. When the IES is small, the individual has a strong preference for smooth leisure, and the result is that an optimal allocation involves a profile for home production that yields a relatively smooth path for leisure in the face of large changes in the amount of time devoted to market work. In the next section we will present data on the extent of changes in time allocation associated with retirement.

4.4. Tax Effects

In this subsection we reconsider the same tax policies as were considered in the model without home production to see whether the extension to include home production has significant effects on the previous results. For this model it is necessary to resort to numerical analysis. Table 10 presents results of an increase in the effective marginal tax rate of 5% for a few different specifications.

Table 10

Effect of 5% Increase in τ^e

	$IES = .25$		$IES = 1.00$	
	$\varepsilon = 0$	$\varepsilon = .40$	$\varepsilon = 0$	$\varepsilon = .40$
$\Delta \log e$	$-.05$	$-.09$	$-.05$	$-.07$
$\Delta \log h$	.00	.00	.00	.00
$\Delta \log h_w$	.00	.01	.00	.02
$\Delta \log h_r$	.00	.01	.00	.02

We begin by noting a couple of patterns that hold for both values of the IES. First, if $\varepsilon = 0$ the table shows that the results for market work are effectively identical to those from the model without home production: all labor supply adjustment occurs along the employment margin, and the response in log hours is basically one for one with changes in the tax rate. The fact that home production effectively does not matter when $\varepsilon = 0$ has previously been obtained in home production models without nonconvexities in production (see, e.g., Benhabib et al (1991)), so this result is perhaps not surprising. But perhaps somewhat surprising is the fact that even when $\varepsilon = .40$, indicating quite a lot of substitutability between goods and time, it remains true that there is no adjustment of market hours while employed. Perhaps somewhat surprisingly, this occurs even though there is an increase in time devoted to home production when working. Although the overall effect of the tax increase is to increase the amount of leisure enjoyed over the lifetime, the amount of leisure enjoyed decreases both when working and when retired. The increase in leisure occurs because a greater fraction of life is spent in retirement, and the individual has greater leisure when retired than when working in the market. Similarly, there is a substitution from market work to home production over the lifetime of the individual, but not at each point in life, since there is no reduction of time devoted to market work while employed. Finally, note that when $\varepsilon = .40$ the response of retirement is significantly larger than when $\varepsilon = 0$.

Next we compare the responses across the two different values of the IES. Given the earlier comments, it is not surprising that when $\varepsilon = 0$ there is no difference across the cases, since this is what we found analytically for the model without home production. But when $\varepsilon = .40$ the response in market work is greater when the IES is smaller. While at first pass this seems surprising, a closer look reveals that this result needs to be interpreted carefully. As one might expect, the model with a higher IES has greater adjustment along the intensive margin, i.e., home production time increases more both when working and not working. It follows that the change in employment is misleading in terms of telling us about the change in leisure. A closer analysis reveals that leisure increases by more in the case with a higher IES.

To summarize, the model with home production also predicts that all adjustment of market work to increases in the effective marginal tax rate will take place along the employment margin. If the value of ε is greater than zero, then this response is greater than that in the model without home production, implying that the response in log employment responds more than one for one with changes in tax rates.

5. A Look at the Time Use Data

In this section we analyze data from the ATUS to assess how time allocations change when individuals retire. Ideally we would like to have panel data in order to see how a given individual or household changes their time use following retirement. Unfortunately, such data does not exist. Absent panel data, one alternative would be to compare the time use allocations between workers and non-workers of a given age. But looking at a given cross section of workers and non-workers has the potential to be very misleading. For example, it turns out that among individuals aged 55-60, non-workers devote much more time to home production than do workers. However, we cannot tell if the nonworkers have always spent more time in home production, or if they increased the amount of time spent in home production because they are not working. Or put somewhat differently, it may well be that the reason they do not work in the market is because they spend a lot of time in home production, rather than the reverse. The same issue would also naturally arise in considering differences in leisure time: observing that a person who is not working enjoys more leisure does not allow us to determine the extent to which his or her leisure increased on account of moving from working to not working.

Instead, we take advantage of the fact that there is a dramatic decrease in average market work between the ages of 55 and 70, with most of this drop associated with retirement. By examining the change in average time use allocations over this age range we can hopefully isolate the average response in time devoted to various uses associated with retirement. Looking at what happens to changes in average time allocations across age avoids the selection problems noted above if it is reasonable to assume that retirement is the dominant source of changes over the age range considered. While it is certainly reasonable to think that there are age effects in addition to those associated with retirement, the change in market work is sufficiently large that we believe it is reasonable to think that retirement is the dominant force. We compute average time allocations by age using data from the ATUS for the years 2003-2006, pooling data across the four samples. Tables A1 and A2 in the appendix provide the data on average time allocations by age for both the total sample and for men.

We summarize the relationships in the data in two different ways, both of which basically tell the same story. The first method regresses time in each category on a constant and age to measure the average change with age. The second method regresses time devoted to each activity other than market work on a constant and time devoted to market work, to isolate the change in each time use that is correlated with changes in market work by age. The first method imposes that the effects are linear in age, whereas the second method imposes that the effects are linear in the change in time devoted to market work. Results of both exercises are reported in Table 11, based on time use data for all individuals aged 55-70. The headings in the table have the following meaning: MW is market work, HP is time spent in home production, SH is shopping time, LE is leisure time, ED is time spent eating and drinking, and PC is time devoted to personal care.[14]

[14] We aggregate activities in the ATUS into these categories as follows. Personal care consists of sleeping, grooming, health related self-care, personal activities and personal care emergencies. Home production includes housework, food and drink prep, presentation and cleanup, interior maintenance, repair and decoration, exterior repair, maintenance and decoration, lawn,

Table 11

Estimated Time Use Effects–Total (standard errors in parentheses)

Dep. Var.	MW	HP	SH	ED	LE	PC
Age	-1.51(.09)	.16(.04)	.02(.02)	.09(.02)	1.00(.06)	.17(.04)
Market Work	–	-.12(.02)	-.01(.01)	-.06(.01)	-.65(.04)	-.12(.02)

The first row documents the extent to which time use in the various categories is either increasing or decreasing with age. It shows that market work is decreasing at a strong rate, while everything else is tending to increase, with the exception of shopping, which displays no trend. The regression coefficient in the second row gives the change in time allocated to each activity in response to a change in market work of one hour. The key message of interest from this row is that for each hour decrease in market work, leisure time increases by almost two-thirds of an hour. Including time spent eating and drinking in the leisure category does not change this estimate much. In contrast, this same one hour decrease in market work leads to an increase in home production of slightly more than one tenth of an hour. Including shopping time has effectively no impact on this estimate.

These results are clearly of interest in terms of interpreting the results of the previous section. None of the specifications that we examined there generate such a low response of home hours to a decrease in market work. As we previously

garden and houseplants, animals and pets, vehicles, appliances, tools and toys, and household management. Shopping includes consumer purchases, professional and personal care purchases, purchasing household services, and purchasing government services. Leisure includes socializing and communicating, attending and hosting social events, relaxing and leisure, arts and entertainment, and waiting associated with the above.

discussed, any model that posits a desire for smooth leisure over time and even very weak substitution between time and goods in home production produces a sizable increase in time devoted to home production when there is a large drop in hours devoted to market work. The previous section provided some guidance as to the magnitude of these effects. Viewed against the effects that we have measured in the ATUS, even the models with a large IES and a small value of ε generate changes in time devoted to home production that are too large by almost an order of magnitude.

The above results were for the total population. If we examine the data for men only, then the picture looks even worse in terms of the model and the data. Table 12 presents the results of the simple regression analysis for men.

Table 12

Estimated Time Use Effects–Men (standard errors in parentheses)

Dep. Var.	MW	HP	SH	ED	LE	PC
Age	-1.68(.15)	.01(.01)	.08(.03)	.12(.03)	1.19(.09)	.17(.04)
Market Work	–	-.03(.03)	-.05(.02)	-.07(.02)	-.68(.04)	-.11(.02)

The overall results for men are quite similar to those for the total population, but note that the change in home production time is small and not significantly different from zero. There is however, a small and statistically significant increase in shopping time. If one considers the point estimates, then the increase in home production and shopping time amounts to less than one tenth of an hour for each hour change in market work. The increase in leisure in response to a decrease in market work is once again around two-thirds.

To summarize, the key finding from the data is that we find very little evidence for substitution between market work and home production as part of retirement. While the data we examine is not panel data and therefore is not ideally suited to shedding light on this margin, the fact that home production time changes so little as market work decreases so dramatically suggests that this finding is likely to be robust. We believe that this finding casts doubt on any model which predicts a large increase in home production associated with retirement.

5.1. Extending the Home Production Model

Having documented a very significant discrepancy between data and theory it is natural to ask if there are alterations to the basic model that would change this. We believe the answer to this is no, but will postpone discussion of this point until later in this section. Simply put, one might characterize the basic problem from the previous analysis to be that time devoted to home production increases by too much during retirement. It follows that adding features that would work to decrease the extent of this increase would presumably decrease the discrepancy between theory and data.

One simple and intuitive way to achieve this is to assume that the marginal value of home production time diminishes very rapidly, thereby creating very little incentive for high values of home production time. To capture this we assume that the home production function is now written as:

$$c(t) = [ag(t)^{\varepsilon} + (1-a)h_n(t)^{\eta\varepsilon}]^{1/\varepsilon} \tag{5.1}$$

where $0 < \eta \leq 1$. If $\eta = 1$ then this is simply the original model. One can interpret h_n^η as the efficiency units of home production, so that if $\eta < 1$ the implication is that the mapping from time devoted to home production into efficiency units of home production time displays diminishing returns, and the smaller the value of η the greater is the degree of diminishing returns. Consider repeating our earlier procedure. We now have one additional parameter and so in principle could target one more value, which could be h_r. However, it turns out that one cannot target an arbitrary value for h_r. For example, even in the most favorable of our previous cases, with the $IES = 1.0$ and $\varepsilon = 0$, setting $\eta = .1$ still yields a value of $h_r = .16$. While certainly diminished, the discrepancy is still quite large. On the other hand, if we pick a small value of the IES, say, .25, then η has very little ability to affect the value of h_r. In this case even when $\eta = .20$, the value of h_r is still as high as .34.

Of course, if one were to consider flexible specifications that allow for diminishing marginal productivity of home production time then one could easily engineer whatever result is desired. All that one would need to do is to assume that the marginal value of time spent in home production drops to zero at whatever the target value is. So the failure of the above specification to resolve the discrepancy can be interpreted as a failure to allow for sufficient flexibility in functional forms.

However, we argue that this type of solution to the discrepancy does not seem promising. The reason is that a solution of this form will imply that there is very little scope for substitution between time and goods beyond some relatively low threshold. While this may do a good job of fitting the facts in terms of what happens to home production time during retirement, this would seem to be sharply at odds with the evidence about time devoted to home production during other parts of the life cycle. We know from Aguiar and Hurst (2007b) and Francis and Ramey (2009) that as women entered the workforce there was a large drop in time devoted to home production. One still sees large differences in time devoted to home production between households with two working members as opposed to one working member. And cross country analysis has revealed quite large substitution between home production and goods. See, for example, Freeman and Schettkat (2001,2005), Davis and Henrekson (2004), Ragan (2005), and Burda, Hamermesh and Weil (2008). This evidence does not seem to be consistent with a theory that says there is very little scope for substitution. Reconciling these observations would require a theory in which the home production function in retirement is dramatically different than it is prior to retirement. To us this does not seem to be a promising solution.

6. Conclusion and Directions for Future Research

In order to understand the effects of changes in retirement programs on retirement, one needs to have a model of retirement that captures the key forces relevant for this decision. Generating retirement seems to call for alterations to the standard life cycle model in which preferences are time separable and strictly concave, and budget sets are convex. The reason is that these models have strong motives for individuals to smooth leisure over time, and this makes retirement an unattractive option. Uncovering the empirically relevant departures from this framework that are important in generating retirement, and their implications for how individuals respond to changes in programs such as social security and medicare is an important research objective.

This paper has taken a first step in this direction by examining in some detail one particular class of models that have been suggested as offering a likely explanation for retirement. This class of models maintains the assumption of strictly concave and time separable utility, but posits nonconvexities in production that lead to nonconvex budget sets. This class of models has very sharp implications for the response of retirement to marginal changes in effective tax rates. In particular, it predicts that the length of working life responds one-for-one with decreases in the after tax return, which is a big effect. It is thus quite important to assess the empirical foundations of this model.

In a version of this model without home production we show that the model's ability to account for retirement depends upon the relationship between two key parameters: the degree of nonconvexity and the IES. Better measurement is important to obtain sharper conclusions from this exercise. When we extend the analysis to allow for home production, we find a major discrepancy between this theory and the data from the ATUS.

The main message that we take away from the preceding analysis is that models in which individuals have a desire for smooth profiles of leisure and retirement is the result of nonconvexities associated with production of output would do not seem to present an adequate theory of retirement. Especially important is the fact that if individuals have a desire for smooth profiles of leisure it is hard to understand why the time devoted to home production does not increase more substantially at retirement. One possibility is that the returns to home production display drop off very sharply as home production time is increased. But this seems hard to reconcile with the fact that in different circumstances we observe lots of substitution between home and market consumption.

In this concluding section we would like to suggest that researchers (including us) may have been looking in the wrong place for the nonconvexities that are central to understanding the motives for retirement. In particular, it may be that there are important nonconvexities associated with the enjoyment of leisure in addition to those that are present in production. For example, certain leisure activities are more difficult to engage in when one is working, even if not full time. An obvious example is travel. More subtle is the general fact that work reduces one's flexibility in terms of scheduling activities. We leave a rigorous examination of this possibility for future work, but here we want to describe one simple specification and discuss why we think it is a promising candidate to address the problems noted earlier.

In the previous analysis we assumed that the utility from leisure in any given period was given by $v(1-h)$ where h was total time devoted to work. What we want to consider now is that there is one function, call it $v^W(1-h)$ that describes the utility from leisure during a period when market work is positive, and there is another function, call it $v^R(1-h)$, that describes the utility from leisure during a period when market work is zero. One simple specification of this sort would be:

$$v^N(1-h) = D + v^W(1-h)$$

where $D > 0$. This specification says that there is some additional utility associated with doing zero market work relative to doing an infinitesimal amount of market work, but that the marginal utility of leisure is unaffected. While such a specification may seem reasonable, and for sufficiently large values of D could generate retirement in an otherwise standard model, it seems unlikely to resolve the key discrepancy noted previously. In particular, this specification would still give rise to the incentives for substitution into home production following a large decrease in work. However, an alternative specification would be:

$$v^N(1-h) = Dv^W(1-h) \tag{6.1}$$

where $D > 1$. A simple implication of this specification is that it is no longer the case that the individual has a preference for smooth leisure over time. In particular, if the individual has leisure equal to .5 in half the periods and leisure equal to 1 in the other half of the periods, it is not the case that the individual would be better off having leisure equal to .75 in all periods.

This specification may also help to explain why it is that time devoted to home production does not increase more during retirement. The reason is that with D greater than one, the marginal value of leisure time increases at retirement, thereby creating an incentive to take more leisure. Loosely speaking, this increased marginal value of leisure has the potential to squeeze out some time that would have been allocated to home production. Whether such a model will be able to quantitatively account for the patterns we see in the time use data remains an open question and one that we leave to future work.

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Appendix

Table A1

Time Use By Age: Men and Women

Age	MW	HP	SH	L	E	PC
55	23.5	16.5	6.0	35.3	9.1	65.1
56	23.4	16.7	5.9	35.5	9.1	65.5
57	24.3	15.7	6.6	36.0	9.1	64.2
58	23.6	15.5	5.6	37.3	8.5	65.2
59	20.6	16.4	5.5	39.0	9.4	65.3
60	19.6	16.0	6.0	36.6	10.2	66.3
61	18.6	16.5	6.2	39.1	9.5	66.4
62	16.3	16.6	5.9	39.5	9.8	66.3
63	13.4	17.5	6.1	42.3	9.7	66.4
64	13.3	18.4	6.2	43.9	9.4	64.9
65	8.00	18.9	5.7	45.5	9.7	66.5
66	7.55	18.2	6.2	44.0	10.1	68.2
67	7.64	18.1	6.5	46.8	10.5	66.5
68	8.25	17.4	5.4	46.8	9.9	66.6
69	4.57	18.3	6.8	48.2	10.0	67.5
70	3.78	17.4	6.3	49.7	10.5	67.5
71	4.36	17.4	5.3	47.4	11.0	69.0
72	5.67	18.5	6.0	47.6	10.4	67.1

TableA2

Time Use By Age: Men

Age	MW	HP	SH	L	E	PC
55	26.4	14.1	4.7	37.3	10.0	64.2
56	28.4	13.4	4.7	38.0	9.3	63.0
57	26.1	13.0	5.7	38.9	9.4	63.8
58	30.0	11.4	5.1	37.8	8.9	63.1
59	26.1	15.1	3.9	40.0	9.4	63.2
60	25.4	12.4	5.4	37.9	11.3	64.6
61	21.4	13.4	5.0	42.6	9.9	65.2
62	19.6	12.7	4.8	41.9	10.8	65.2
63	14.8	14.8	5.1	44.7	10.4	65.4
64	17.8	13.1	5.1	45.7	10.7	64.6
65	11.1	13.5	5.2	50.0	11.0	65.3
66	7.9	15.3	5.9	49.0	10.9	66.5
67	9.2	13.4	6.6	52.1	10.3	64.8
68	11.5	13.2	5.3	50.3	10.6	64.4
69	6.0	13.5	6.9	52.3	11.3	65.9
70	7.0	12.4	5.0	52.7	11.2	66.3
71	6.2	13.2	4.3	50.8	11.7	67.4
72	7.4	15.2	6.1	48.9	11.3	65.7