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Accounting for Obsolescence: An Evaluation of Current NIPA Practice

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"The measurement of capital is one of the nastiest jobs that economists have set to statisticians."
[J.R. Hicks,1981, p. 204]

The Bureau of Economic Analysis (BEA) makes estimates of consumption of fixed (CFC) that are used in its estimates of the nation's net income and product. These estimates reflect expected obsolescence and are adjusted for quality change in the underlying capital assets. Recently, some papers have attempted to measure obsolescence. This work raises questions about what obsolescence is and whether it is properly accounted for in BEA's methodology. This paper addresses those questions.

The paper is organized as follows. It defines CFC, outlines BEA's basic methodology for estimating it, and then elaborates on how the methodology handles quality change. It then examines what CFC is in theory: first from an economic accounting perspective and then from a productivity perspective. It then examines the question of what obsolescence is and finds that it is due to many factors that cause a sharp drop in the demand for certain capital assets. It also finds that the effects of expected obsolescence are built into the CFC estimates and that much of the controversy in the field results from disagreements over how to account for unexpected declines in the value of capital. The paper then examines the appropriateness of making adjustments for these declines.

BEA's estimates of CFC are an integral part of the National Income and Product Accounts (NIPAs). Specifically, CFC is a component of Gross Domestic Income (GDI), which measures output as the costs incurred and the incomes earned in the production of the Gross Domestic Product (GDP) and is, therefore, the equivalent of GDP on the income side of the National Income and Product Account. As a cost incurred in the production of GDP, it reflects the use of private and government fixed assets located in the United States, and is defined as the decline in the value of the stock of assets due to wear and tear, obsolescence, accidental damage, and aging. CFC is used in the context of measuring sustainable income and product. It is

deducted from GDP and GDI to derive net domestic product and net domestic income-- rough measures of the level of consumption that can be maintained while leaving capital assets intact. Similarly, CFC is deducted from the appropriate NIPA gross investment flows to obtain net investment in fixed assets for the total economy, for private business, and for government. These measures of net investment are rough indicators of whether the corresponding capital stocks have been maintained intact.

Because the output of government and nonprofit institutions is measured in the NIPAs by adding up the costs of production including CFC, the estimates of CFC are one of the components used to measure the output of these types of institutions. Additionally, the estimates of CFC for private fixed assets are used in constructing NIPA estimates of corporate profits and the income of other types of business.

CFC is generally synonymous with the term "depreciation." In the NIPAs the two can differ because CFC can include catastrophic damage incurred in wars and natural disasters in addition to normal depreciation. To avoid confusion, in the remainder of this paper, I shall use only the term "depreciation."

The question of what unexpected obsolescence is and how it differs from depreciation is quite complex. A full answer to this question will evolve over the course of this paper. As a first approximation, we can state that depreciation generally deals with gradual declines in the value of existing capital assets due to their aging. Unexpected obsolescence on the other hand, generally reflects a sudden and sharp decline in the value of these assets that may result from events that do not affect real depreciation such as the introduction of new capital assets that are based on a superior technology.

Basic BEA Estimation Methodology

Although its value is recorded in the NIPAs, BEA's estimates of depreciation are estimated as part of its program to measure the nation's net stock of fixed assets. This stock is a "wealth" stock, i.e., it represents the monetary value of the stock. In current-prices, i.e., prices of the current year or nominal dollars, it represents an estimate of what the stock of fixed assets would sell for if there were markets for those used assets.

BEA estimates the values of virtually all fixed assets using the perpetual inventory

method.[1] With this method, both net stocks and depreciation are weighted averages of past investment in the relevant assets. The weights are directly obtained from an assumed age-price profile for each asset. Estimates are first made at constant prices of a given reference year. Real estimates of aggregates, expressed in chained-prices of the reference year, are made by chaining together annual growth rates derived from a Fisher Ideal quantity index. Estimates that are expressed in current prices are obtained by multiplying the detailed constant-price estimates by the price index for gross (new) investment in the relevant asset.

The process of deriving estimates with the perpetual inventory method for a given type of asset, say assets of type i, is as follows. First, we obtain a time series on investment in (purchases of) the asset expressed in current prices.[2] The next step is to obtain a time series on a constant-quality price index for this investment, which is normalized to yield a value of unity in the reference year. By dividing each of the elements in the current-price series by the corresponding value of this price index, the time series is converted to one of investment measured in prices of the reference year. Because the price index is measured in terms of constant quality, an increase in the price of new assets of type i that is due to an improvement in their quality is treated as an increase in the quantity of constant-price investment in assets of type i rather than as an increase in the price of this investment.

The constant-price value of the stock of assets of type i is then determined by multiplying constant-price investment in these assets in each year by the appropriate value from their age-price profile. Each point on the profile denotes the ratio of the price of an s-year old asset of type i to the price of a new asset of this type, where both prices are for assets that were produced in the same year. The profile itself shows how this ratio declines from one to zero over the service life of the asset. This is a theoretical ratio. The actual ratio cannot be observed because assets of type i that were produced in year t are s-years old in only one year, year t+s. We can obtain data on the prices of new assets of type i that are produced in year t+s. But, this is not the

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[1] An alternative is the physical inventory method, which applies independently estimated prices to a direct count of the number of physical units of each type of asset. BEA uses this method only for autos because they are the only type of asset for which detailed data on the prices and physical number of units of used assets in the stock of each vintage is available.

[2] The detailed mechanics of BEA's perpetual inventory method are described in greater detail in Appendix A of this paper. This appendix also traces out the methodology for a specific example as the details are difficult to grasp without one.

price we need because these assets may embody qualities that are different from those produced in year t. The price that we need for our comparison is the hypothetical value that would exist in year t+s if assets of type i were produced new at that time with the same quality as newly produced assets of type i that were produced in year t.

BEA's age-price or depreciation profiles are completely deterministic. The profile for a given asset is fixed forever when the asset is first purchased and it enters the net stock; i.e., the profile gives price ratios that will hold in all future years of the asset's life. The profile is dependent on t because assets that are produced in different years may have somewhat different age-price profiles. The age-price profile of every asset is based on its expected service life. Each asset is assumed to remain in the stock until this life is reached. At that point, the value of the asset is zero and it is discarded. Thus, the value of assets discarded from the stock is zero and there is no difference between an asset's actual life and its expected life. Note that the relevant lives are service lives rather than physical lives; assets may be retired from service before the end of their physical lives. Consequently, the lives reflect the purchaser's expectations regarding all of the factors that contribute to obsolescence.

The prices used to value the net stock are also used to measure depreciation. By definition, constant-price depreciation on an asset in any given year is measured by the difference between its constant-price value at the beginning of the year and its constant-price value at the end of the year. Both of these values come from different points on the same age-price profile.

Current-price estimates of depreciation and net stocks are obtained by multiplying the values of the corresponding constant-price estimates by the relevant "reflator" (that is, the price index that is used to convert the constant-price value to current prices). Recall that the constant-price series on investment was constructed by deflating nominal investment by a price index for new purchases of asset i. This index, which measures the average price of asset i in the current year relative to its average price in the reference year, is the required reflator. Because net stocks are measured as of the end of the year, the required reflator for net stocks is the end-of-year value of this price index.

A major consequence of the BEA definition of depreciation is that there is an identity between changes in stocks and the corresponding flows when all are expressed in constant

prices. Specifically, the change in the net stock of assets of type i between the beginning and of the year is identically equal to the difference between gross investment in and depreciation of that type of asset. This stock-flow identity holds for all vintages of asset i. The identity has nothing to do with the shape of the age-price profile; it holds for all shapes. It does require that all assets that leave the stock do so because they have been depreciated to a zero value.[3]

A second key property of the BEA methodology is that for any given capital asset that stays in the stock for its entire service life, constant-price depreciation charges on the asset over its lifetime will sum to the asset's purchase price when new (in constant prices). The two properties have important implication for net investment and depreciation in a steady state. Consider the example given in appendix A, where in all years prior to year B+1, constant-price investment in asset i is a constant. Then, the constant-price value of the stock of this asset is unchanged from year to year. This is no fluke. Our assumption of no quality change means that goods from each vintage of the stock are identical. Under our assumptions, the stock of asset i is physically invariant with respect to time. Both the number of assets in the stock and their age composition are constant. This is what we term a "steady state." Because the net stock is constant and all subtractions from the stock are registered as depreciation, constant-price depreciation must equal constant-price gross investment for asset i. Thus, constant-price gross investment, depreciation, and the net stock of asset i are all constant over time. (Net investment is also constant and is equal to zero.) However, the corresponding current-price values of these aggregates all increase at whatever rates we have assumed for inflation in prices of new assets of type i.[4]

Adjustments for quality change

Let us now investigate how quality change is handled within the BEA framework. Continue with the example from appendix A and assume that in year B+1, the new vintage of asset i does embody some quality change so that although the price of a new unit of asset i increases as the same constant rate as before, some of this increase is due to quality change.

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[3] Thus, for example, the requirement will not be met if some of the used assets in the stock are exported to other countries or if used assets are imported.

[4] Although in appendix A it is assumed that the rate of price inflation in asset i is a constant, none of the results mentioned here depend on this assumption.

Because of this quality change, new units of asset i are treated as embodying a greater quantity of capital than new units of asset i purchased in prior years. Consequently, the rate of price inflation for constant-quality investment in asset i is lower than before and the rate of constant-price investment in asset i is higher than before. Because the constant-price value of existing (old) units of asset i is unaffected by this change, the higher rate of investment for new units of asset i results in an increase in the constant-price value of the stock of this asset. Although the current-price value of the stock of new units of asset i increases at the same rate as before, the current-price value of the stock of existing assets increases at a lower rate reflecting the lower rate of increase of the deflator for new investment.

The effects of quality change on measured depreciation are similar. As noted above, beginning in year B+1, new investment in assets of type i is effectively larger than in prior years because of the increase in quality. This causes constant-price depreciation on new assets of type i to be larger than in prior years. Depreciation on used type i assets is unaffected by the change in quality. Consequently, constant-price depreciation on the total stock of type i assets is larger than before. The effects on current-price depreciation are similar to the corresponding effects on the stock. In year B+1, current-price depreciation on the part of the stock due to new assets increases at the same rate as in the past as the real effects resulting from a larger quantity of investment are offset by the effects of a smaller increase in the price reflator. Depreciation on the stock of existing assets increases at a rate that is smaller than in prior years because the effects of reduced growth in the investment reflator are not offset. Consequently, current-price depreciation on the total stock of type i assets increases as a rate that is lower than in prior years.

A number of economists have modeled "obsolescence" as being caused by an increase in the quality of new vintages of an asset. One has concluded that such obsolescence shows itself as being part of measured depreciation.[5] But, we have just shown that with the BEA methodology obsolescence caused by quality change in a new vintage of an asset has no effect on constant-price depreciation on used assets of earlier vintages. Further, given a fixed rate of nominal inflation in the price of new assets of type i (without respect to quality), an increase in the quality of new vintages will cause current-price depreciation to be lower than it would have been in the absence of any quality change. Thus, there appears to be some question

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[5] See Wykoff [2004].

of whether the BEA results are theoretically correct. We will examine this in the next part of the paper. Before we do this, we mention how BEA handles capital destroyed in natural disasters.

BEA deviates from its normal capital stock methodology when extraordinary amounts of capital are destroyed, i.e., when the cost of the damage in a catastrophic event exceeds about one-quarter of one percent of total private CFC. Damage of this magnitude often occurs as a result of a hurricane or an earthquake. When it does and the damage is to private fixed capital, BEA estimates the NIPA measure of CFC as the sum of the value of this damage and normal depreciation. When the damage is to government capital, its value is not included in CFC because government consumption expenditures are measured by adding up costs that include CFC. Instead, the damage to government capital is deducted directly from the end-of-period capital stock (see NIPA table 5.9)." This treatment avoids increasing the measured output of fixed government assets in GDP whenever there are catastrophic losses.

Relationship to Economic Theory

There are two main strains of economic theory that pertain to the measurement of depreciation and obsolescence. The first is found in the literature related to economic accounting. For more than 150 years this literature has focused on the proper ways to define and measure the "income" of individuals, firms, and the nation as whole. The second is found in the literature on productivity measurement that has developed during the last four decades on the foundation of the user cost of capital approach to measuring capital services as developed by Dale Jorgenson and his associates. While most of this literature has focused on measuring productivity, parts of it have also been concerned with determining empirical age-price profiles for economic depreciation and integrating such measures within national accounting frameworks. We will first examine relevant themes from the national accounting literature.

BEA's measure of depreciation reflects its solution to the problem of what is meant by "maintaining capital intact." This question has always been of central importance to the definition of the net income of individuals, firms, and nations. As noted earlier, depreciation is a charge against profits for the costs incurred in the using up of fixed capital. When we measure the profits of an industry (or firm), we know that we must subtract from the value of its sales any reduction in the value of assets held in inventory. Inventories of raw materials and other inputs

are used up in the production process and need to be replenished. Likewise, the selling of finished goods from inventories should not increase profits until those inventories are replenished. In economic jargon, the inventories must be maintained intact in order for sales to be regarded as contributing to profits. The same principle holds for the stock of fixed capital assets. This stock must be maintained intact before sales of final goods are regarded as contributing to profits.

The issue of exactly what we mean by maintaining capital intact was a source of controversy for much of the last century. The matter was hotly debated in leading journals during the 1930's and 1940's and some aspects of the controversy have never been fully resolved. A complete discussion of all of the major concepts of capital maintenance is well beyond the scope of this paper. Instead, we focus on some of the most important concepts discussed in the literature and how BEA's measure relates to them.

Two of the leading protagonists in the debate were Arthur Cecil Pigou and Frederick von Hayek. Pigou has been interpreted as maintaining that it is the quantity or physical nature of capital that must be maintained intact. Specifically, Pigou [1941, p. 273] argued that if every item in the physical inventory of the capital stock was unaltered, then (except for damage due to wars or natural disasters) the capital stock must be judged to have been maintained intact. If however, capital assets of type A are replaced by assets of type B, the relevant relative values are given by their discounted present values at the time of replacement. Thus, if an asset suddenly loses almost all of its value due to obsolescence, regardless of whether this obsolescence was foreseen or not, the loss in value due to the obsolescence is not included in depreciation and the replacement asset only has to have a discounted present value equal to that of the asset being replaced, which is virtually nil.

Hayek [1941, p. 277] argued that physical lives had little to do with the problem of what is meant by capital maintenance and that expected obsolescence should be included as part of depreciation. A machine that made items of fashion might be expected to be discarded after a year because the items it made were out of style. Although the physical life of the asset could have been much longer, the entire value of the discarded asset should be charged to depreciation because the asset reached its expected service life. Hayek [1941, p. 280] also maintained that "...while the ordinary practice of trying to keep the money value of capital constant is in most circumstances a fairly good approximation to the real purpose of capital accounting, this is not true in all circumstances." This last statement appears to argue for using historical-cost

depreciation in practical measurement.

Basing his concept of real net income on the ex-post concept developed by the Swedish economist Eric Lindahl, Sir John Hicks [1942, p. 177] argued that changes in the value of a capital asset during the course of a year that were due to changes in expectations should be excluded from measures of depreciation. Consequently, Hicks would include changes due to expected obsolescence in depreciation but would exclude changes due to unexpected obsolescence. He went on to conclude that losses in an asset's value when it remains idle are not depreciation but a capital loss; similarly, he concluded that maintenance expenditures made on an idle machine should be treated as investment because they contribute to the output of future years but do not contribute to the output of the current year.

The discussion above represents the most important ideas raised by the most important players. But, many other important questions have been raised and little agreement has been reached, especially regarding what the theoretically ideal measure is.[6] Some have stated that what has to be maintained intact is some measure of the "quantity of capital" or the "productive capacity" of the stock. Here there is a further controversy over whether it is only the capacity to produce goods in the current period that is relevant or whether the capacity to produce goods in future periods is also relevant. Others have argued that these concepts cannot be measured in practice and that practical measures can only be based on the cost or price of capital assets.

As one attempts to develop any practical measure of depreciation, a host of other issues emerges. If the value of a certain capital asset is to be maintained intact, is this a nominal value or a real one? If it is the real value, how is this to be calculated? Are nominal values to be deflated using prices for capital assets or using prices for consumption goods? Do changes in nominal capital values have to be adjusted for any changes in interest rates?[7] One line of thought concerns whether it is acceptable to use the paradigm of a "sinking fund." Here, depreciation is conceived as an amount that would be deposited into a fund that would be used to purchase the replacement asset. Issues concern whether we should conceive of the fund as earning interest, and if so, how we should account for it.

BEA's Solution

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[6] For a wide-ranging discussion of alternative concepts of capital maintenance, see Break [1954].
[7] Hicks [1946, p. 177] thought that this question was incapable of solution. It led him to state, "…we shall be well advised to eschew *income* and *saving* in economic dynamics. They are bad tools, which break in our hands."

Exactly how does the BEA measure of depreciation fit in with these controversies? The BEA methods for measuring depreciation can be viewed as embodying some of the arguments of both Pigou and Hayek. To a certain extent, Hayek's views are adopted by BEA's use of service lives rather than physical lives. We do not care whether it is physically possible for an airline to maintain and fly a certain airplane for 50 years. We only care that, given technical change and the cost of maintaining and repairing the plane, it may be profitable to fly it for only 25 years and then discard it.

On the other hand, the BEA treatment is largely consistent with the spirit of Pigou's views. Consider the example given earlier of a steady state for a single type of asset. In this state, constant-price gross investment is constant over time and, with BEA's methodology, the capital stock and its age composition are physically unchanged from year to year. The quantity of capital, however you wish to define it, is surely also constant in this state. In such a state, we want depreciation to equal gross investment so that net investment is zero. Net investment should be greater than zero only if the stock is increasing. With the BEA methods, we obtain these desired results. Therefore, we can argue that in this specific instance, the BEA methods are consistent with the principle of maintaining the quantity of capital intact.

In its use of replacement-cost depreciation, BEA implicitly rejects Hicks' suggestion that the measure of depreciation be based on expected changes in net worth. Changes in net worth due to expected changes in prices are treated as revaluation rather than as depreciation.

One of the key properties of BEA's methodology is that, over the lifetime of any capital asset, constant-price depreciation on it will sum up to the asset's initial purchase price (also measured in constant prices). It is this property that causes net investment to be zero in a steady state and makes it a meaningful measure. Conversely, other measures of depreciation that do not share this lifetime adding-up property lead to poor measures of net investment.

For example, BEA could have defined depreciation as being the amount that owners of assets would have to put into a sinking fund, which earns interest, so that when the assets are discarded from the stock there will be funds that are just sufficiently large to purchase replacements for them. Because the sinking fund earns interest, unless the nominal interest rate is less than the rate of inflation in the price of the asset, the amount that needs to be put into it must be less than the value of depreciation obtained with the replacement-cost measure. As a result, under these conditions, net investment must always be positive in a steady state when depreciation is measured using the sinking-fund measure.

Likewise, BEA could have based NIPA depreciation on historical-cost depreciation. Here over the lifetime of any capital asset, the charges to depreciation will total the original purchase price in nominal dollars. This is the concept of depreciation that is used in company reports and in tax accounting. When there is inflation, depreciation charges under this concept are less than those under the replacement-cost concept. In fact, they may even be less than those under the sinking fund concept. It should be obvious that the historical-cost depreciation would lead to measures of real net investment that would not be very meaningful. In sum, it is our desire to make real net investment a truly meaningful concept that causes us to abandon the sinking fund and historical-cost depreciation approaches to capital maintenance.

User Cost of Capital Approach

There is another framework for measuring economic depreciation that is largely due to Dale Jorgenson and his various associates. It is based on the user cost of capital approach to measuring capital services using vintage production functions as exemplified in the work of Hall [1968 and 1971]. In this framework, capital services, capital stocks, and related income flows are treated in an internally consistent manner. Although the framework has mostly been associated with studies related to measuring productivity, Jorgenson has shown how it can be used within the context of national economic accounting.[8] Charles R. Hulten and Frank C. Wykoff have conducted a number of important studies in which they measured empirical patterns of depreciation based on data on used asset prices utilizing Jorgenson's concepts. Many of the rates of depreciation that BEA utilizes for specific assets are based on these studies.[9]

The cornerstone of Jorgenson's framework is what has been termed the fundamental equation of capital theory. This equation, which is more than a century old, states that in equilibrium, the price of an asset will equal the discounted present value of the net income expected to be derived from owning it. The equation holds for both financial and nonfinancial assets; for the latter the net income is equal to the market rental price of the asset's services. When a nonfinancial asset is used by its owner, this service (or implicit rental) value is known as the user cost of capital.

The framework is implemented by assuming that the equilibrium relationship actually

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[8] See Christensen and Jorgenson [1973] and Jorgenson and Landefeld [2006].
[9] See Fraumeni [1997].

holds at all times. As is shown in Appendix B, when this is done, we can derive an expression that gives the implicit rental price of the asset's services for the year. If all of the asset's services are assumed to be received on the last day of the year, this price is given by the sum of the interest forgone on the asset, the expected decline in the asset's price during the year, and the operating costs incurred in using the asset, i.e., by equation (B4).

Jorgenson defines the relative efficiency of a capital asset, denoted by d (i,t,s) as the ratio of the quantity of services that the asset will yield in the current period to the quantity of services that the asset yields when it is new. In other words, if q(i,t,s) denotes the quantity of services produced by an asset that is of type i that was produced in year t and is s-years old, then by definition, d(i,t,s) is given by the quotient q(i,t,s) / q(i,t,0). Thus, the relative efficiency of a capital asset is normalized so that it declines from one to zero over its service life. Jorgenson assumes that the exact pattern of these declines in relative efficiency is fixed when the asset is first purchased and is known with certainty by the purchaser. Some authors have described the relative efficiency function in physical terms, so that it essentially denotes the relative marginal physical products of assets of different ages. This analogue is correct in so far as the function measures "output decay," i.e., declines in the ability of the asset to produce gross output.[10] But, Jack Faucett [1980] has shown that if we do not explicitly incorporate the impacts of non-capital operating costs on the price of capital assets, the function will also reflect the effects of what some have termed "input decay". For example, some durables experience rising maintenance and repair costs as they age. This can be handled explicitly. It can also be handled implicitly by ignoring these costs and choosing a relative efficiency schedule that ultimately gives us the same age-price profile as the one we would have obtained had we explicitly handled these costs. It is, therefore, necessary for practitioners to spell out exactly how they are treating maintenance and repair and other operating costs..

The relative efficiency function is different from the age-price profile. It is, in fact, an age-efficiency profile. If we know this profile and the time pattern of future real own interest rates, we can derive the implied age-price profile as shown in appendix B.

In Jorgenson's framework, economic depreciation is defined as the decline in the price of an asset due to age alone. From the mathematical expressions given for the concept, it is clear

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[10] For assets that are rented to consumers, the relative efficiency does not refer to a strictly physical concept of output because it ultimately depends on consumers' subjective preferences for different qualities and ages of capital assets.

that annual depreciation on an asset is measured as the difference between the asset's price and the price of a second asset that is identical to the first except that it is one year older as in equation (B5). Both prices are measured at the same point in time. But, exactly when during the year is the measurement to take place? Christensen and Jorgenson [1973] implement the measure using prices as of the end of the year. Hulten and Wykoff [1980] use prices as of the first of the year. BEA uses average annual prices, which is tantamount to using prices as of the middle of the year. Note that in the Jorgensonian framework, "capital gains" on an asset are defined to be equal to the difference between the decline in the market value of the asset during the year and depreciation on the asset. Consequently, the differences between depreciation measures that are due to differences regarding when during the year the measurement takes place cause corresponding differences in the estimated measures of capital gains.

Note that the two assets whose prices go into equation (B5) have identical age-price profiles. When the assumed ultimate service life of an asset that is s years old at the beginning of the year differs from that of an asset of the same type that is s+1 years old at the beginning of the year, BEA measures depreciation on the former using prices that come solely from its own age-price profile. In other words, the two prices involved in the calculation come from the same assumed or hypothetical age-price profile.

An important result of this is that, for most capital assets, the difference in when during the year depreciation is measured is the only difference between estimates derived using BEA's methodology and those obtained using Jorgenson's methodology.[11] In other words, BEA's methodology is completely consistent with Jorgenson's framework. The property that BEA's constant-price estimates of depreciation possess of summing up over the lifetime of an asset to the asset's purchase price is also shared by Jorgenson's measures.

The sharing of this lifetime adding-up property depends crucially on the use of identical age-price profiles for the two prices used to measure depreciation in equation (B5). Recently, Ahmad et al. [2005] have indicated that they believe that different profiles are permitted in Jorgenson's framework while identical profiles are required for work in economic accounting. We know that profiles are not identical in much of the empirical work on depreciation. For example, in Hulten and Wykoff's work the average ultimate service lives of assets in different

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[11] For most assets, BEA uses age-price profiles that are strictly geometric. For any given asset, Jorgenson advocates the use of relative efficiency function that declines at a strictly geometric rate, which implies an age-price profile that declines at the identical strictly geometric rate.

vintages of the same asset are permitted to be different. In fact, it is the difference between two estimates of depreciation, where one's prices come solely from identical age-price profiles and where the second's come from two different age-price profiles that Wykoff [2004] has labeled as "obsolescence." This obsolescence corresponds precisely to what, in the BEA framework, amounts to the effect on the price of an asset due to quality change in the succeeding vintage. Consequently, this type of obsolescence is not in BEA's measure of depreciation.

Defining Obsolescence

It is rather remarkable that we have been able to say so much about measuring obsolescence without actually defining it. But, it cannot be put off any longer. Previously, we identified obsolescence with the effect on the price of existing capital assets of the introduction of new capital assets of superior quality. But, the word has a broader connation in common parlance. We should recognize that a broader concept is necessary because there are many nuances in capital measurement that cannot be properly understood if we think that measuring the effects of obsolescence and quality change are the same.

So, exactly what do we mean by obsolescence? A number of different phenomena could be described as reflecting obsolescence. The common thread that runs through them is that the demand for the services of an existing capital asset declines sharply. This leads to one or more consequences including: (1) the asset's price on used assets' markets declines, (2) the asset is used less than earlier vintages of comparable assets of the same age, and (3) the asset is discarded before it reaches a service life attained by comparable assets of earlier vintages.

These considerations lead us to the following definition. Unexpected obsolescence is a sharp decline in the value of an asset due to factors other than physical damage, deterioration, aging, and the passage of time.[12] The relevant factors are external ones that reduce the demand for the asset's services. This definition rules out declines in value caused by many of the factors whose effects are measured in depreciation. Thus, declines in value caused by the fact that an asset has aged and become closer to the end of its expected service are ruled out. We also rule out similar declines caused by an asset's becoming less efficient (relative to when it was new) in the production of services because of physical wear and tear and physical damage. The excluded declines also include those due to an asset's requiring an increased quantity of associated inputs

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[12] The sharpness of the decline is measured relative to the past price trends for the asset.

(relative to when it was new). Thus, in the terminology of the productivity literature, declines in value due to output or input decay are not unexpected obsolescence.

This definition is broader than that used by many authors. Quite a few have written as if obsolescence was solely due to improvements in quality. But, Hulten, Robertson, and Wykoff [1989, p. 235] have stated, "Obsolescence, as conventionally defined, refers to the loss in the value of existing capital because it is no longer technologically suited to economic conditions or because technically superior alternatives become available." My definition can be viewed as clarifying some of the vagueness in the stated concept. Its broadness is entirely justified. From the viewpoint of economic accounting, no analytic purpose is served by narrowly restricting our measures to a subset of the relevant economic phenomena.

Unexpected obsolescence can be caused by many phenomena. The one that most people associate with the term is the introduction of a capital asset that embodies a superior technology such as a faster computer chip. The new asset is a direct substitute for the old, it is clearly superior, and it costs little more than the old asset. As a result, hardly anyone purchases new assets that embody the older technology. Note, however, that it may still be profitable for businesses that own assets embodying the older technology to keep those assets in service. In fact, this is generally the case. Quite often there is a long time between the invention of an asset and its becoming economic enough to be used by most of the population. Thus, the diffusion of a new technology into the economy is often quite slow.

It is not necessary for the new technology to be technologically superior. In the early 1980's, the Sony Betamax video cassette recorder was arguably technologically superior to its competitors based on the VHS format. But, it lost market share to its rival to the point where companies that rented movie cassettes stopped carrying tapes based on the beta format used by the Sony machines. This caused the Sony machines to become "obsolete," at least for people who wanted to use the machines to view rental tapes. One lesson from this example is that a technologically inferior machine can drive a technologically superior one out of the market if the former is sufficiently cheaper than the latter. Furniture made from manufactured wood and wood substitutes will generally not last as long nor look as good as furniture made solely from wood. However, the high relative cost of wood furniture has limited its use in many offices.

Unexpected obsolescence can be caused by the development of a completely different type of asset that serves a similar purpose and, therefore, is a substitute for the initial one. Thus, canal barge traffic in the U.S. was largely supplanted by railroad traffic that was, in turn, largely

supplanted by motor freight traffic.

The Betamax example also demonstrates that the continued use of an asset may be dependent on the availability of certain inputs, in this case the rental tape. If the input becomes unavailable, the asset may become obsolete. This type of phenomenon happens quite often. Old typewriters can continue to be used if one can find ribbons for them and if parts and repairmen can be found when they need repairs. Similarly, old computer printers can be used if one can find cartridges that are compatible with them.

A variant on this theme occurs when the relative price of a needed input increases sharply. This prompts a switch to assets that use relatively less of this input. Thus, the use of certain types of motor vehicles is greatly dependent on the availability of relatively cheap gasoline. Sharp increases in the relative price of gasoline induce customers to purchase other types of vehicles. Similarly, certain airplanes use less jet fuel than others. When the price of fuel increases sharply, there is a shift to other types of aircraft.

A special case of the above is when the input whose relative price increases is repairs to the original asset. When an old asset needs repairs, the owner is faced with the choice of making the repairs or purchasing a new version of the asset. Consequently, a sharp increase in the relative price of repairs will lead to a decrease in the average age of the asset as repairs that were economic to make at lower prices become non-economic at higher prices. Because repairs are generally labor intensive, an increase in wages relative to the cost of materials could result in such an effect. A similar result has occurred in the case of personal computers because technological progress has led to marked decreases in the labor required to produce new computers but not in the labor needed to make repairs to existing ones.

Interest is generally a cost of capital. It is easy to demonstrate through the use of the fundamental equation of capital theory that a rise in interest rates will have a larger impact on the prices of long-lived assets than it will on short-lived ones. A sufficiently large and prolonged increase in interest rates could, therefore, cause long-lived capital assets to become obsolete and replaced with shorter-lived ones.

Finally, there are many external factors that affect the demand for an asset's services. The elimination of a rail line can force a factory to relocate. The opening of a new highway can divert traffic and cause certain businesses on an existing road to lose their customers. For many capital assets, the demand for their services is very indirect. Buggy whips became obsolete, not because someone produced a better whip, nor because someone developed a substitute for the

whip, but because someone developed a substitute for horse-drawn transportation. Thus, the demand for tools to fashion or repair a capital asset is dependent on the demand for that asset itself. Many factories cannot be readily converted to produce products other than the ones that they were initially designed to produce. Thus, they will become obsolete if the demand for that product falters. The imposition of a new tax (or a higher tax) on used capital assets may accelerate obsolescence in these assets. Special tax breaks on new investment may have a similar effect.

A key point that needs to be clarified is that from a national accounting perspective, we are not interested in measuring expected obsolescence. Assets are depreciated over their expected economic service lives. These lives are not based solely on how long an asset can physically be used; they reflect how long it is profitable to use the assets rather than some alternative asset that can be purchased. We do not care what causes an asset to be retired. We only care how long it is in service before it is retired and whether its actual life differed significantly from its expected life. This is brought out clearly in the theoretical apparatus based on the fundamental equation of capital theory. It shows that the market prices of capital assets are based on what the purchasers of the assets expect to receive. If the effects of obsolescence are expected, then through the fundamental equation they determine the market prices of capital assets and are, consequently, are already embodied in our estimates of depreciation. We would not want to measure these effects again as that would be a form of double-counting. Moreover, there is no analytical way to separate out the effects of expected obsolescence from all of the other factors that affect depreciation.

The catalogue of obsolescence-inducing phenomena given above illustrates that there are many reasons why an age-price profile for a specific capital asset that is computed from historical data may differ significantly from the profile that was expected when the asset was purchased. The question then is whether the national income accountant should alter his/her estimation methodology to take this into account. As noted earlier, in the usual implementation of the perpetual inventory model, age-price profiles are held constant over the life of assets as if their purchasers had perfect foresight. Consequently, we must ask if the existence of unexpected obsolescence makes it desirable to change this treatment.

Analysis of Practical Treatments

BEA's measures of depreciation have been generally well regarded and stood the test of time. It is, therefore, reasonable to insist that modifications to this methodology to account for unexpected obsolescence only be made if they meet a relatively high hurdle. Specifically, any modifications need to be theoretically justified. They must not introduce a bias into the measure of net investment or too much subjectivity into the estimation procedure. Finally, as a practical consideration, they should result in sizable changes to present measures of depreciation without requiring large changes in BEA's workload.

There are a few accounting treatments for obsolescence that are feasible to implement. One treatment is to alter an asset's age-price profile after it is put in service. Another involves classifying some of the real change in the value of assets as "other changes in the volume of assets." In one variant, this treatment would be accorded any observed differences between normal depreciation and actual depreciation. In a second variant, only large scale and infrequent effects would be accorded this treatment.

Let us examine the appropriateness of these treatments. The first involves adjusting age-price profiles of assets that are thought to be experiencing obsolescence. This treatment involves examining the actual depreciation patterns of assets and replacing the *ex ante* estimates with *ex post* ones. (Modifications to service lives can also be made if data on actual retirements is available.) As BEA has a history of changing service lives and the shape of age-price profiles when there is empirical evidence for such changes, this methodology is not much of a departure from the present methodology and there is little question of its theoretical appropriateness. Experience has shown, however, that the data needed to make such estimates only exists for a few assets, is expensive to acquire, and is time consuming to implement. Thus, it would be surprising if this treatment could be implemented on more than just a limited basis.

Another possible treatment for unexpected obsolescence is to write off differences between the actual value of used assets and their expected value as an other change in the volume of assets. It would, therefore, affect measured depreciation in both current and constant prices but would not affect measures of the net stock. Such a treatment would have the advantage that it would treat such differences symmetrically, i.e., it would not solely account for negative differences caused by obsolescence. It would also recognize that there are many causes of obsolescence and account for all of them. This type of treatment is recommended in paragraph 12.43 of the *System of National Accounts 1993* (SNA93).[13]

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[13] Paragraph 12.43 identifies unforeseen obsolescence with quality change. This is unfortunate

There are some practical difficulties with the above treatment. One is that it would result in a biased measure of net investment unless positive and negative differences between expected and actual changes in prices were treated in the same manner. Thus, unexpected declines in the prices of used assets are treated as reductions in constant-price depreciation, i.e., as if they were reductions in the quantity of depreciation. To avoid bias, unexpected increases in the prices of used assets and declines that are smaller than expected would have to be treated as increases in the amount of constant-price depreciation. This would appear to require a corresponding counter entry as a change in the volume of assets. Without this symmetrical treatment, this method of treating obsolescence would result in larger measures of constant-price net investment while the measures of constant-price net stocks were unchanged.

Note that an asymmetric treatment of positive and negative deviations from expected prices and service lives cannot be justified within the framework of the user cost of capital approach to capital measurement. In practice, the fundamental equation of capital theory is treated as more than just an abstract equilibrium condition. It is assumed to hold at all times and for all assets, not just the ones that are purchased on the margin. It is generally implemented as if purchasers of capital assets had perfect foresight with respect to future prices of these assets. As a result, the capital services that they receive in the current period are always equal in value to those that they expect to get. It would be totally inconsistent with this framework to claim that recurrent obsolescence systematically reduces service lives and prices of capital assets but that purchasers of these assets never realize this and are fooled into believing that these effects will not recur.

There are other practical problems besides bias. One is that it is not clear exactly what set of expectations is relevant. Are we concerned about the prices expected by the owner of the asset at the beginning of the income period or are we concerned about the prices that the owner expected when the asset was purchased? It is the latter set of expectations that seem to be the most theoretically relevant. But, it is the former that are estimated (or at least proxied) in empirical studies.

Similar treatments can be developed using other data. For example, suppose that data on the actual retirements of a type of asset is available. If the number of actual retirements exceeds that of normal retirements, a write-off to reflect the value of prematurely retired assets could be made to the NIPA table on changes in net stock of produced assets in which other changes in the

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because quality change is but one of many possible causes of obsolescence.

volume of assets are shown.

The last accounting treatment for unexpected obsolescence that we will examine involves writing off only declines in value due to large-scale obsolescence as an other change in the volume of assets. There are a few instances that might merit such a treatment. One is the obsolescence induced by the large increase in energy prices in the 1970's. Some have suggested these effects were quite large.[14] There are a number of reasons why such a treatment could be regarded as theoretically appropriate. First, the large size and suddenness of the effects of this price shock suggest that they must be regarded as being abnormal and unexpected. The treatment is, therefore, in accord with the recommendations of SNA93. Further, BEA has gone on record as supporting such a treatment in principle. For example, BEA's 2003 volume on fixed assets states "Ideally, the estimates of net stocks should also be adjusted for declines in value caused by unusually or unexpectedly large amounts of obsolescence as the depreciation rates used to derive them reflect only the effects of normal obsolescence over time. However, the data on the impact of obsolescence that are necessary to make such adjustments are not available."[15]

Prudence suggests that we reserve obsolescence charges to a few instances where it reasonable to believe that they are very large in magnitude (so that accounting for them will make a noticeable difference in the estimates) and that the underlying event can be thought to be unexpected by the overwhelming majority of purchasers of capital assets. BEA does have a tradition of making adjustments for similar effects. The stock of military equipment constructed during World War II is written down for war damage and the large amount of equipment that was scrapped (i.e., "surplussed") at the end of the war because it was no longer needed. The stock is also written down to reflect damage due to natural disasters such as major hurricanes and earthquakes. BEA has made special adjustments for 25 natural disasters in the past 34 years.

For any given individual, these natural disasters always represent unexpected events. For the nation as a whole, however, we can make the case that it is reasonable to expect that there will be a major natural disaster more than once in every two years. If, say, these major disasters have resulted in average annual damages of $X per year, one can make the case that unexpected damage only results to the extent that the aggregate damage in any year exceeds $X.[16]

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[14] Baily [1981] has suggested this but Hulten and Wykoff [1989] failed to find any measurable effects of this obsolescence on the prices of used capital assets.

[15] See BEA [2003, p. M-11].

[16] There are a myriad of other ways of estimating expected damages for major disasters that can be defended. The point here is that the expected value of these damages is not zero.

Conclusion

When BEA estimates depreciation, it does not make any specific adjustments for obsolescence. But, the deflators for net stocks and depreciation are adjusted for quality change. As a result, when quality change takes place, constant-price depreciation on and net stocks of used assets are unaffected by the quality change but the corresponding current-price measures are reduced because some of the increase in the price of new assets is attributed to quality improvements rather than inflation. This treatment has been challenged in recent studies that have advocated reducing estimated levels of depreciation for the effects of obsolescence.

A review of BEA's methodology suggests that its key property is that for each asset in the capital stock, constant-price depreciation charges over its lifetime will sum to the asset's initial purchase price. This property appears designed to make constant-price net investment a meaningful measure and to have this measure equal zero in a steady state. BEA's treatment of depreciation was compared to the one utilized in Dale Jorgenson's user cost of capital approach to capital measurement. Jorgenson's framework definitely includes expected obsolescence in its measure of depreciation. The BEA framework was shown to be consistent with Jorgenson's when the prices of used capital assets that determine depreciation come from a single age-price profile. It was argued that the use of a single profile is necessary to give meaningful measures.

Unexpected obsolescence was defined to include all situations where there is a sharp decline in the value of an asset due to factors other than physical damage, deterioration, aging, and the passage of time. Specific examples reveal that it has probably always been widespread and a normal part of economic processes.

It is impossible to separate out the effects of expected obsolescence from the effects of all of the other factors that affect asset prices. But, there is no need to do so because the effects of expected obsolescence are already embodied in BEA's estimates of depreciation. It is only the effects of unexpected obsolescence that we need concern ourselves with.

Four possible ways to account for unexpected obsolescence were discussed. The first is to retrospectively revise service lives to reflect actual lives that may have been shortened due to unexpected obsolescence. This differs little from the current treatment. The second is to treat the difference between expected depreciation and actual depreciation as an "other change in the

volume of assets." A third treatment is essentially a modification of the second in which normal depreciation is used rather than expected depreciation in an attempt to make the measure less subjective. This paper argues that negative and positive differences must be treated in the same way. Otherwise the measures of depreciation and net investment are biased and made less meaningful. A fourth treatment is to only account for very large effects of unexpected obsolescence in the way that BEA accounts for disaster damage. Given Hulten, Robertson, and Wykoff's [1989] inability to find any measurable effects of obsolescence from the last energy crisis, it is questionable whether the implementation of any of these treatments would have a large impact on aggregate depreciation. Thus, decision-makers may question whether the large amount of resources that would have to be devoted to implementing these treatments is justified.

Appendix A - Basic BEA Estimation Methodology

BEA estimates the values of virtually all fixed assets using the perpetual inventory method. With this method, both net stocks and depreciation are weighted averages of past investment in the relevant assets. The weights are directly obtained from an assumed age-price profile for the asset. Estimates are first made at constant prices of a given reference year, denoted by B. Real estimates of aggregates, expressed in chained-prices of the reference year, are made by chaining together annual growth rates derived from a Fisher Ideal quantity index. Estimates that are expressed in current prices are obtained by multiplying the detailed constant-price estimates by the price index for gross (new) investment in the relevant asset. [17] Because the methodology is rather complicated, it is useful to explain it through the use of an example that we will build on throughout this appendix. Let us consider the stock of a single asset, i, such as truck. Assume that all assets of type i have a service life of 10 years. Consequently, there are no type i assets in the stock that are 10-years old or older because, by assumption, all type i assets expire and are discarded from the stock when they are precisely 10-years old. At that time, each of the type i assets has a value of zero and is discarded from the stock. In other words, as their age approaches their maximum service life, the price that each of these assets can be sold for approaches zero.

The first step in the estimation process is to obtain a time series on purchases of the asset expressed in current prices. This is illustrated by the vector in figure A1. Here, $I\$_{i,t}$ denotes current-price purchases of asset i in year t. The vector is shown for years B+1 through B-10. The next step is to obtain a time series on a constant-quality price index for the time series. Here, $P_{i,t}$ denotes the value of this index in year t. The values of the index are normalized so that $P_{i,B}$ is equal to 1. In other words, if the price of a new (0-year old) asset of type i in year t is given by $p_{i,0,t}$, then the value of $P_{i,t}$ is given by $p_{i,0,t}/p_{i,0,B}$. These prices are average annual values of the index. They are roughly equal to the value that the price index would have in the middle of year t. Let $I_{i,t}$ denote the constant-price value of purchases of asset i in year t. By assumption,

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[17] Values that are expressed in constant-prices are strictly additive across different goods at a single point in time as are values that are expressed in current prices. Values for different goods that are expressed in chained prices, however, are not strictly additive. For a specific good, constant-price values are additive across different time periods because, by definition, all of the values are based on the same price. This property is taken advantage of in establishing the constant-price stock/flow identity that is central to BEA's estimation methodology.

this value is obtained by deflating the current-price measure by the relevant price, i.e., by dividing $I\$_{i,t}$ by $P_{i,t}$. The vector of values for the constant-price series is shown in figure A2. The elements of this vector are essentially equivalent quantities of new assets of type i, given the quality embodied in new type i assets in year B.

Figure A1. - Current-price Investment in Asset i

Year	B-10	B-9	B-8	. . .	B-2	B-1	B	B+1
current-price investment	$I\$_{i,B-10}$	$I\$_{i,B-9}$	$I\$_{i,B-8}$		$I\$_{i,B-2}$	$I\$_{i,B-1}$	$I\$_{i,B}$	$I\$_{i,B+1}$

Figure A2. - Constant-price Investment in Asset i

Year	B-10	B-9	B-8	. . .	B-2	B-1	B	B+1
constant-price investment	$I_{i,B-10} = I\$_{i,B-10}/P_{i,t}$	$I_{i,B-9}$	$I_{i,B-8}$		$I_{i,B-2}$	$I_{i,B-1}$	$I_{i,B}$	$I_{i,B+1}$

Now, let us denote by M(i,t,s) the function that gives the age-price profile for assets of type i that are s-years old and were produced in year t. By definition, this profile shows how the quotient $p_{i,s,t} / p_{i,0,t}$ varies with the age of the asset. In other words, for any given value of s, the value of the profile is given by the ratio of the price of an s-year old asset of type i to the price of a new asset of type i, both assets being identical in all respects to assets of type i produced in year t. Note that this is a theoretical ratio. It cannot be observed because assets of this type are s-years old in only one year, year t+s. Assets of type i that are produced in year t+s generally embody different qualities from those produced in year t. The price that the used asset is being compared to is a hypothetical value that would exist in year t+s if assets of type i were produced new at that time with the same quality as newly produced assets of type i in year t. Note that by definition, constant-price depreciation on asset i is equal to the product of $p_{i,0,t}$ and the change in the value of the asset's age-price profile from when it is s-years old to when it is s+1-years old.[18]

BEA's age-price or depreciation profiles are completely deterministic; they are known

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[18] For new assets the first of these values is one while for old assets that have reached the end of their service lives, the second of the two values is zero.

with complete certainty. The profile for a given asset is fixed forever when the asset is first purchased and it enters the net stock; i.e., the profile gives price ratios that will hold in the future years of the asset's life when it is older. The function is dependent on t because assets that are produced in different years may have somewhat different age-price profiles. The age-price profile of an asset is based on its expected service life. All assets are assumed to remain in the stock until their expected service life is reached. At that point, the value of asset is zero and it is discarded. Thus, by definition, the value of assets discarded from the stock is zero and there is no difference between an asset's actual life and its expected life.[19] Note that the relevant lives are service lives rather than physical lives; assets may be retired from service before the end of their physical lives. Consequently, the lives reflect the purchaser's expectations regarding all of the factors that contribute to obsolescence.

Now, for the sake of definiteness, let us assume that all assets of type i have a service life of 10 years, that the function $M(i,t,s)$ does not depend on the value of t, and that it takes on the following form. On average, assets of type i are purchased new in the middle of year t. They are ½-year old at the end of year t, at which time, the value of $M(i,t,s)$ is given by $(1-\delta/2)$, where δ is an assumed annual depreciation rate. For the next 9 years the annual depreciation rate is constant so that the end-of-year values of $M(i,v,t)$ are given by $(1-\delta/2) \cdot (1-\delta)^v$, for all goods whose end-of-year age, v, is between ½ and 9 ½. Thus, at the end of year t+9, the assets are 9.5 years old and their value relative to a new one is given by $(1-\delta/2) \cdot (1-\delta)^9$. The value of the assets then declines to zero during their next and last year of life. (On average, this zero value is reached in the middle of year t+10).[20]

Under these assumptions, the value of the net stock of asset i in constant prices is given in figure A3. This figure shows how the stock is composed of vintages reflecting investment in 10 different years. To see this, examine the column that gives the stock at the end of year B. Looking at the element from the first row in this column, we see that it consists of assets that were purchased in year B, are ½-year old at the end of year B, and have a (cohort) value of $I_{i,B} \cdot$

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[19] BEA currently uses finite service lives for only a few types of assets. Since 1996, for most assets BEA has used age-price profiles that are strictly geometric. Here, the service life is technically infinite so that the assets are never discarded and the zero value for old assets is only reached asymptotically.

[20] This time pattern for depreciation is used for expository purposes only. It greatly simplifies the calculations but gives a relatively large amount of depreciation in the last year of the asset's life.

(1- δ/2). In the second row of this column are the assets in the stock that were purchased in year B-1. They have a value of $I_{i,B-1} (1 - \delta/2) \cdot (1 - \delta)$. Continuing like this, we see in row 10 that the oldest assets in the stock in year B were purchased in year B-9 and have a value of $I_{i,B-9} \cdot (1 - \delta/2) \cdot (1 - \delta)^9$. The value for the entire stock is shown in row 11 as the sum of the values in this column for rows 1 through 10.

Figure A3. - (End-of-year) Constant-price Net Stock of Asset i.

Row	Year	B-2	B-1	B	B+1
1	value of 0.5-yr old assets	$I_{i,B-2} * (1-\delta/2)$	$I_{i,B-1} * (1-\delta/2)$	$I_{i,B} * (1-\delta/2)$	$I_{i,B+1} * (1-\delta/2)$
2	value of 1.5-yr old assets	$I_{i,B-3} * (1-\delta/2)$ $* (1-\delta)$	$I_{i,B-2} * (1-\delta/2)$ $* (1-\delta)$	$I_{i,B-1} * (1-\delta/2)$ $* (1-\delta)$	$I_{i,B} * (1-\delta/2)$ $* (1-\delta)$
	. . .	. . .	. . .	. . .	. . .
10	value of 9.5-yr old assets	$I_{i,B-11} * (1-\delta/2)$ $* (1-\delta)^9$	$I_{i,B-10} * (1-\delta/2)$ $* (1-\delta)^9$	$I_{i,B-9} * (1-\delta/2)$ $* (1-\delta)^9$	$I_{i,B-8} * (1-\delta/2)$ $* (1-\delta)^9$
11	value of total stock	$\sum_{j=1}^{10} I_{i,B-j-1} * (1-\delta/2)$ $* (1-\delta)^{j-1}$	$\sum_{j=1}^{10} I_{i,B-j} * (1-\delta/2)$ $* (1-\delta)^{j-1}$	$\sum_{j=1}^{10} I_{i,B-j+1} * (1-\delta/2)$ $* (1-\delta)^{j-1}$	$\sum_{j=1}^{10} I_{i,B-j+2} * (1-\delta/2)$ $* (1-\delta)^{j-1}$

Constant-price depreciation on the stock of asset i is shown, by vintage, in figure A4. Examine the column for year B. The element in the first row of this column gives depreciation on assets that were purchased new in year B. Because these assets were, on average, only in the stock for half a year, depreciation on them is given by the product of one half of the annual depreciation rate and the amount of constant-price investment in the assets. The element in the second row of this column gives depreciation on assets that were purchased in year B-1. These assets were ½-year old at the end of year B-1. Depreciation on them is estimated by taking their value at that time and multiplying it by δ because these assets are in the stock for the entire year. The formulas for all of the other vintages are obtained in a similar manner. The only anomaly is for the last vintage, which was 9 ½ years old at the beginning of year B. By assumption, all of these assets are retired during year B so that constant-price depreciation on them must be given by their beginning-of-year value.

A major consequence of the BEA definition of depreciation is that there is an identity

between changes in stocks and the corresponding flows when all are expressed in constant prices. Specifically, the change in the net stock of a given type of asset between the beginning and of the year is identically equal to the difference between gross investment in and depreciation of that type of asset. This can be verified using the data from figures A3 and A4. The stock-flow identity holds for all of the vintages of asset i. The identity has nothing to do with the shape of the age-price profile; it holds for all shapes. The only requirement is that all assets that leave the stock do so because they have been depreciated to a zero value. In the example, this requirement is met and the identity holds. Consequently, depreciation on the stock can be estimated as a residual, i.e., as the difference between the constant-price values of the stock between the beginning of the year and the end of the year.

Figure A4. - Constant-price Depreciation on Stock of Asset i.

Row	Year	B-2	B-1	B	B+1
1	Depreciation on assets purchased in current year	$I_{i,B-2} * (\delta/2)$	$I_{i,B-1} * (\delta/2)$	$I_{i,B} * (\delta/2)$	$I_{i,B+1} * (\delta/2)$
2	Depreciation on assets purchased 1 year ago	$I_{i,B-3} * (1 - \delta/2) * \delta$	$I_{i,B-2} * (1 - \delta/2) * \delta$	$I_{i,B-1} * (1 - \delta/2) * \delta$	$I_{i,B} * (1- \delta/2) * \delta$
3	Depreciation on assets purchased 2 years ago	$I_{i,B-4} * (1- \delta/2) * (1- \delta) * \delta$	$I_{i,B-3} * (1- \delta/2) * (1- \delta) * \delta$	$I_{i,B-2} * (1- \delta/2) * (1- \delta) * \delta$	$I_{i,B-1} * (1- \delta/2) * (1- \delta) * \delta$
	. . .	. . .	. . .	. . .	. . .
10	Depreciation on assets purchased 9 years ago	$I_{i,B-11} * (1- \delta/2) * (1- \delta)^8 * \delta$	$I_{i,B-10} * (1- \delta/2) * (1- \delta)^8 * \delta$	$I_{i,B-9} * (1- \delta/2) * (1- \delta)^8 * \delta$	$I_{i,B-8} * (1- \delta/2) * (1- \delta)^8 * \delta$
11	Depreciation on assets purchased 10 years ago	$I_{i,B-12} * (1- \delta/2) * (1- \delta)^9$	$I_{i,B-11} * (1- \delta/2) * (1- \delta)^9$	$I_{i,B-10} * (1- \delta/2) * (1- \delta)^9$	$I_{i,B-9} * (1- \delta/2) * (1- \delta)^9$
12	Depreciation on entire stock of asset i	$I_{i,B-2}*(\delta/2)$ $+I_{i,B-12}*(1-\delta/2)$ $+\left(\sum_{j=1}^{9}(I_{i,B-2-j} - I_{i,B-12})*(1-\delta/2)*(1-\delta)^{j-1}*\delta\right)$	$I_{i,B-1}*(\delta/2)$ $+I_{i,B-11}*(1-\delta/2)$ $+\left(\sum_{j=1}^{9}(I_{i,B-1-j} - I_{i,B-11})*(1-\delta/2)*(1-\delta)^{j-1}*\delta\right)$	$I_{i,B}*(\delta/2)$ $+I_{i,B-10}*(1-\delta/2)$ $+\left(\sum_{j=1}^{9}(I_{i,B-j} - I_{i,B-10})*(1-\delta/2)*(1-\delta)^{j-1}*\delta\right)$	$I_{i,B+1}*(\delta/2)$ $+I_{i,B-9}*(1-\delta/2)$ $+\left(\sum_{j=1}^{9}(I_{i,B+1-j} - I_{i,B-9})*(1-\delta/2)*(1-\delta)^{j-1}*\delta\right)$

Current-price estimates of depreciation and net stocks are obtained by multiplying constant-price estimates by the relevant reflator. Recall that constant-price stocks and

depreciation for asset i in year t are measured using the price for new units of asset i in the reference year, $p_{i,0,B}$. Consequently, the required procedure for reflating depreciation involves multiplying the constant-price measure by $P_{i,t} = p_{i,0,t}/p_{i,0,B}$. As noted earlier, both prices in the quotient measuring $P_{i,t}$ are average annual values because investment takes place continuously over the year as does depreciation. Stocks, however, are valued as of the end of the year and, therefore, require a slightly different reflator. It is given by $\hat{P}_{i,t} = \hat{p}_{i,0,t} / \hat{p}_{i,0,B}$. Here, the symbol ^ denotes that the price is measured as of the end of year t. Figures A5 and A6 show the current-price estimates of net stocks and depreciation, respectively, which correspond to the constant-price estimates shown in figures A3 and A4. Notice that the stock/flow identities do not hold in current-prices because depreciation and investment are measured in annual average prices while stocks are measured using end-of-year prices.

Figure A5. - (End-of-year) Current-price Net Stock of Asset i.

Row	Year	B-2	B-1	B	B+1
1	value of 0.5-yr old assets	$I_{i,B-2} * (1-\delta/2)$ $* \hat{P}_{i,B-2}$	$I_{i,B-1} * (1-\delta/2)$ $* \hat{P}_{i,B-1}$	$I_{i,B} * (1-\delta/2) *$ $\hat{P}_{i,B}$	$I_{i,B+1} * (1-\delta/2)$ $* \hat{P}_{i,B+1}$
2	value of 1.5-yr old assets	$I_{i,B-3} * (1-\delta/2)$ $* (1-\delta)$ $* \hat{P}_{i,B-2}$	$I_{i,B-2} * (1-\delta/2)$ $* (1-\delta)$ $* \hat{P}_{i,B-1}$	$I_{i,B-1} * (1-\delta/2)$ $* (1-\delta)$ $* \hat{P}_{i,B}$	$I_{i,B} * (1-\delta/2)$ $* (1-\delta)$ $* \hat{P}_{i,B+1}$
	. . .	. . .	. . .	. . .	. . .
10	value of 9.5-yr old assets	$I_{i,B-11} * (1-\delta/2)$ $* (1-\delta)^9$ $* \hat{P}_{i,B-2}$	$I_{i,B-10} * (1-\delta/2)$ $* (1-\delta)^9$ $* \hat{P}_{i,B-1}$	$I_{i,B-9} * (1-\delta/2)$ $* (1-\delta)^9$ $* \hat{P}_{i,B}$	$I_{i,B-8} * (1-\delta/2)$ $* (1-\delta)^9$ $* \hat{P}_{i,B+1}$
11	value of total stock	$\sum_{j=1}^{10} I_{i,B-j-1} * (1-\delta/2) *$ $(1-\delta)^{j-1} * P\hat{}_{i,B-2}$	$\sum_{j=1}^{10} I_{i,B-j} * (1-\delta/2) *$ $(1-\delta)^{j-1} * P\hat{}_{i,B-1}$	$\sum_{j=1}^{10} I_{i,B-j+1} * (1-\delta/2) *$ $(1-\delta)^{j-1} * P\hat{}_{i,B}$	$\sum_{j=1}^{10} I_{i,B-j+2} * (1-\delta/2) *$ $(1-\delta)^{j-1} * P\hat{}_{i,B+1}$

We now investigate how quality change and obsolescence are handled within the BEA framework. Let us build on our example for asset i and make it more specific. Assume that in all years through year B+1, $I\$_{i,t}$ has increased at an annual rate of 5-percent per annum. Assume that there has not been any quality change in asset i so that the constant-quality price index for the asset has also increased at an annual rate of 5-percent per annum. As a result, constant-price investment in asset i through year B+1 is a constant, which we denote by I'. Then, the value of the constant-price stock by vintage is given by the values shown in figure A7. Here, the value of

Figure A6. - Current-price Depreciation on Stock of Asset i.

Row	Year	B-2	B-1	B	B+1
1	Depreciation on assets purchased in current year	$I_{i,B-2} * (\delta/2) * P_{i,B-2}$	$I_{i,B-1} * (\delta/2) * P_{i,B-1}$	$I_{i,B} * (\delta/2) * P_{i,B}$	$I_{i,B+1} * (\delta/2) * P_{i,B+1}$
2	Depreciation on assets purchased 1 year ago	$I_{i,B-3} * (1 - \delta/2) * \delta * P_{i,B-2}$	$I_{i,B-2} * (1 - \delta/2) * \delta * P_{i,B-1}$	$I_{i,B-1} * (1 - \delta/2) * \delta * P_{i,B}$	$I_{i,B} * (1 - \delta/2) * \delta * P_{i,B+1}$
3	Depreciation on assets purchased 2 years ago	$I_{i,B-4} * (1 - \delta/2) * (1-\delta) * \delta * P_{i,B-2}$	$I_{i,B-3} * (1 - \delta/2) * (1-\delta) * \delta * P_{i,B-1}$	$I_{i,B-2} * (1 - \delta/2) * (1-\delta) * \delta * P_{i,B}$	$I_{i,B-1} * (1 - \delta/2) * (1-\delta) * \delta * P_{i,B+1}$
	. . .	. . .	. . .	. . .	. . .
9	Depreciation on assets purchased 8 years ago	$I_{i,B-10} * (1 - \delta/2) * (1-\delta)^7 * \delta * P_{i,B-2}$	$I_{i,B-9} * (1 - \delta/2) * (1-\delta)^7 * \delta * P_{i,B-1}$	$I_{i,B-8} * (1 - \delta/2) * (1-\delta)^7 * \delta * P_{i,B}$	$I_{i,B-7} * (1 - \delta/2) * (1-\delta)^7 * \delta * P_{i,B+1}$
10	Depreciation on assets purchased 9 years ago	$I_{i,B-11} * (1 - \delta/2) * (1-\delta)^8 * P_{i,B-2}$	$I_{i,B-10} * (1 - \delta/2) * (1-\delta)^8 * P_{i,B-1}$	$I_{i,B-9} * (1 - \delta/2) * (1-\delta)^8 * P_{i,B}$	$I_{i,B-8} * (1 - \delta/2) * (1-\delta)^8 * P_{i,B+1}$
11	Depreciation on entire stock of asset i	$\left[I_{i,B-2}*(\delta/2) + I_{i,B-12}*(1-\delta/2) + \left(\sum_{j=1}^{9}(I_{i,B-2-j} - I_{i,B-12}) * (1-\delta/2)*(1-\delta)^{j-1} * \delta \right) \right]*P_{i,B-2}$	$\left[I_{i,B-1}*(\delta/2) + I_{i,B-11}*(1-\delta/2) + \left(\sum_{j=1}^{9}(I_{B-1-j} - I_{i,B-11}) * (1-\delta/2)*(1-\delta)^{j-1} * \delta \right) \right]*P_{i,B-1}$	$\left[I_{i,B}*(\delta/2) + I_{i,B-10}*(1-\delta/2) + \left(\sum_{j=1}^{9}(I_{i,B-j} - I_{i,B-10}) * (1-\delta/2)*(1-\delta)^{j-1} * \delta \right) \right]*P_{i,B}$	$\left[I_{i,B+1}*(\delta/2) + I_{i,B-9}*(1-\delta/2) + \left(\sum_{j=1}^{9}(I_{i,B+1-j} - I_{i,B-9}) * (1-\delta/2)*(1-\delta)^{j-1} * \delta \right) \right]*P_{i,B+1}$

Figure A7. - (End-of-year) Constant-price Net Stock of Asset i with constant real investment.

Row	Year	B-1	B	B+1	B+2
1	value of 0.5-yr old assets	$I' * (1 - \delta/2)$	$I' * (1 - \delta/2)$	$I' * (1 - \delta/2)$	$I' * (1 - \delta/2)$
2	value of 1.5-yr old assets	$I' * (1 - \delta/2) * (1-\delta)$	$I' * (1 - \delta/2) * (1-\delta)$	$I' * (1 - \delta/2) * (1-\delta)$	$I' * (1 - \delta/2) * (1-\delta)$
	. . .	. . .	. . .	. . .	. . .
10	value of 9.5-yr old assets	$I' * (1 - \delta/2) * (1-\delta)^9$	$I' * (1 - \delta/2) * (1-\delta)^9$	$I' * (1 - \delta/2) * (1-\delta)^9$	$I' * (1 - \delta/2) * (1-\delta)^9$
11	value of total stock	$\sum_{j=1}^{10} I' * (1-\delta/2) * (1-\delta)^{j-1}$	$\sum_{j=1}^{10} I' * (1-\delta/2) * (1-\delta)^{j-1}$	$\sum_{j=1}^{10} I' * (1-\delta/2) * (1-\delta)^{j-1}$	$\sum_{j=1}^{10} I' * (1-\delta/2) * (1-\delta)^{j-1}$

the capital stock is unchanged from year to year. This is no fluke. Our assumption of no quality
change means that each vintage of the stock is the same as the last. Under this and our earlier

assumptions, the stock of asset i is physically invariant with respect to time. Both the number of assets in the stock and their age composition are constant. This is what we term a "steady state." Because the net stock is constant, constant-price depreciation must equal constant-price gross investment for asset i. Thus, constant-price gross investment, depreciation, and the net stock of asset i are all constant over time. However, the corresponding current-price values of these aggregates all increase at 5-percent per annum as a result of inflation.

Now, suppose that in year B+1, the new vintage of asset i does embody some quality change so that although the price of asset i in this year is still 5 percent greater than its value in year B, 2 percent of the 5-percent increase is due to quality change. (That is, the constant-price index has increased by (1.05/1.02) - 1 or 2.94 percent.) Also, assume that this quality change affects all subsequent vintages, but that there are no further increases in quality and that the price of new assets in year in year B+2 is 5 percent greater than the year before. Then, the composition of the constant-price stock is given in figure A8. The stock has increased in value in year B+1 because constant-price investment in the new vintage of asset i is given by 1.02 I'

Figure A8. - (End-of-year) Constant-price Net Stock of Asset i with quality change in year B+1.

Row	Year	B-1	B	B+1	B+2
1	value of 0.5-yr old assets	$I' * (1 - \delta/2)$	$I' * (1 - \delta/2)$	$I' * 1.02 * (1 - \delta/2)$	$I' * 1.02 * (1 - \delta/2)$
2	value of 1.5-yr old assets	$I' * (1 - \delta/2) * (1- \delta)$	$I' * (1 - \delta/2) * (1- \delta)$	$I' * (1 - \delta/2) * (1- \delta)$	$I' * 1.02 *(1 - \delta/2) * (1- \delta)$
3	value of 2.5-yr old assets	$I' * (1 - \delta/2) * (1- \delta)^2$	$I' * (1 - \delta/2) * (1- \delta)^2$	$I' * (1 - \delta/2) * (1- \delta)^2$	$I' * (1 - \delta/2) * (1- \delta)^2$
	. . .	. . .	. . .	. . .	. . .
10	value of 9.5-yr old assets	$I' * (1 - \delta/2) * (1- \delta)^9$	$I' * (1 - \delta/2) * (1- \delta)^9$	$I' * (1 - \delta/2) * (1- \delta)^9$	$I' * (1 - \delta/2) * (1- \delta)^9$
11	Value of total stock	$I' * (1 - \delta/2) * \left[\sum_{j=1}^{10}(1 - \delta)^{j-1} \right]$	$I' * (1-\delta/2) * \left[\sum_{j=1}^{10}(1-\delta)^{j-1} \right]$	$I' * (1-\delta/2) * \left[\left(\sum_{j=1}^{10}(1-\delta)^{j-1} \right)+.02 \right]$	$I' * (1 - \delta/2) * \left[\left(\sum_{j=1}^{10}(1 - \delta)^{j-1} \right) + .02 * (2 - \delta) \right]$

rather than by I'. Now, as shown in figure A9, the corresponding current-price value of the stock

does not increase by 5 percent from year B to year B+1 as before. The value of the part of the stock due to new assets (the first row) does increase by 5 percent in year B+1 from its value in year B. But, the value of the other parts of the stock increases by only 2.94 percent because these older assets do not embody the quality change of the latest vintage.

Figure A9. - (End-of-year) Current-price Net Stock of Asset i with quality change in year B+1.

Row	Year	B-1	B	B+1	B+2
1	value of 0.5-yr old assets	$I' * (1 - \delta/2)$ $* (1.05)^{-0.5}$	$I' * (1 - \delta/2)$ $* (1.05)^{+0.5}$	$I' * (1 - \delta/2)$ $* (1.05)^{+1.5}$	$I'* (1 - \delta/2)$ $* (1.05)^{2.5}$
2	value of 1.5-yr old assets	$I' * (1 - \delta/2)$ $* (1- \delta)$ $* (1.05)^{-0.5}$	$I' * (1 - \delta/2)$ $* (1- \delta)$ $* (1.05)^{+0.5}$	$I' * (1 - \delta/2) *$ $(1- \delta)$ $* (1.05)^{+0.5}$ $* 1.0294$	$I' * (1 - \delta/2)$ $* (1- \delta)$ $* (1.05)^{2.5}$
3	value of 2.5-yr old assets	$I' * (1 -\delta/2)$ $* (1- \delta)^2$ $* (1.05)^{-0.5}$	$I' * (1 - \delta/2)$ $* (1- \delta)^2$ $* (1.05)^{+0.5}$	$I' * (1 - \delta/2)$ $* (1- \delta)^2$ $* (1.05)^{+0.5}$ $* 1.0294$	$I' * (1 - \delta/2)$ $* (1- \delta)^2$ $* (1.05)^{+1.5}$ $* 1.0294$
	. . .	. . .	. . .	. . .	. . .
10	value of 9.5-yr old assets	$I' * (1 - \delta/2)$ $* (1- \delta)^9$ $* (1.05)^{-0.5}$	$I' * (1 - \delta/2)$ $* (1- \delta)^9$ $* (1.05)^{+0.5}$	$I' * (1 - \delta/2)$ $* (1- \delta)^9$ $* (1.05)^{+0.5}$ $* 1.0294$	$I' * (1 - \delta/2)$ $* (1- \delta)^9$ $* (1.05)^{+1.5}$ $* 1.0294$
11	value of total stock	$I' * (1 - \delta/2)$ $* \left[\sum_{j=1}^{10} (1-\delta)^{j-1} \right]$ $* (1.05)^{-0.5}$	$I' * (1-\delta/2)$ $* \left[\sum_{j=1}^{10} (1-\delta)^{j-1} \right]$ $* (1.05)^{+0.5}$	$I' * (1-\delta/2)$ $* \left[\left(\sum_{j=1}^{10} (1-\delta)^{j-1} \right) + .02 \right]$ $* (1.05)^{+0.5} * (1.0294)$	$I' * (1 - \delta/2)$ $* \left[\left(\sum_{j=1}^{10} (1-\delta)^{j-1} \right) + .02 * (2 - \delta) \right]$ $* (1.05)^{+1.5} * (1.0294)$

The effects of quality change on measured depreciation are similar. The quality change for new investment in asset i in year B+1 causes the effective quantity of investment in new assets for that type to be effectively 2 percent larger than in past years. As shown in figure A10, this causes constant-price depreciation on new assets of type i to be 2 percent larger than it was in prior years. But, depreciation on type i assets in year B+1 that are used (old) is unaffected by the change in quality. The effects on current-price depreciation, which are shown in figure A11, are similar to the corresponding effects on the stock. In year B+1, depreciation on the part of the stock due to new assets (the first row) increases by 5 percent from its year ago value while depreciation on the other parts of the stock increases by 2.94 percent.

In sum, we have just shown that, with the BEA methodology, obsolescence caused by

quality change in a new vintage of an asset has no effect on constant-price depreciation on used assets of earlier vintages. Further, given a fixed amount of nominal inflation in the price of new assets of type i (without respect to quality), an increase in the quality of new vintages will cause current-price depreciation to be lower than it would have been in the absence of any quality change.

Figure A10. - Constant-price Depreciation on Stock of Asset i with Quality Change in Year B+1.

Row	Year	B-1	B	B+1	B+2
1	Depreciation on assets purchased in current year	$I' * (\delta/2)$	$I' * (\delta/2)$	$I' * (\delta/2) * 1.02$	$I' * (\delta/2) * 1.02$
2	Depreciation on assets purchased 1 year ago	$I' * (1 - \delta/2) * \delta$	$I' * (1 - \delta/2) * \delta$	$I' * (1- \delta/2) * \delta$	$I' * (1- \delta/2) * \delta * 1.02$
3	Depreciation on assets purchased 2 years ago	$I' * (1- \delta/2) * (1- \delta) * \delta$	$I' * (1- \delta/2) * (1- \delta) * \delta$	$I' * (1- \delta/2) * (1- \delta) * \delta$	$I' * (1- \delta/2) * (1- \delta) * \delta$
	. . .	. . .	. . .	. . .	. . .
8	Depreciation on assets purchased 7 years ago	$I' * (1- \delta/2) * (1- \delta)^6 * \delta$	$I' * (1- \delta/2) * (1- \delta)^6 * \delta$	$I' * (1- \delta/2) * (1- \delta)^6 * \delta$	$I' * (1- \delta/2) * (1- \delta)^6 * \delta$
9	Depreciation on assets purchased 8 years ago	$I' * (1- \delta/2) * (1- \delta)^7 * \delta$	$I' * (1- \delta/2) * (1- \delta)^7 * \delta$	$I' * (1- \delta/2) * (1- \delta)^{7} * \delta$	$I' * (1- \delta/2) * (1- \delta)^{7} * \delta$
10	Depreciation on assets purchased 9 years ago	$I' * (1- \delta/2) * (1- \delta)^8$	$I' * (1- \delta/2) * (1- \delta)^8$	$I' * (1- \delta/2) * (1- \delta)^8$	$I * (1- \delta/2) * (1- \delta)^8$
11	Depreciation on entire stock of asset i	I'	I'	$I' * (1 +(\delta/2) * .02)$	$I' * (1 + (.02 * (\delta/2) * (3-\delta)))$

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Figure A11. - Current-price Depreciation on Stock of Asset i with Quality Change in Year B+1.

Row	Year	B-1	B	B+1	B+2
1	Depreciation on assets purchased in current year	$I' * (\delta/2) * (1.05)^{-1.0}$	$I' * (\delta/2)$	$I' * (\delta/2) * 1.05$	$I' * (\delta/2) * (1.05)^2$
2	Depreciation on assets purchased 1 year ago	$I' * (1 - \delta/2) * \delta * (1.05)^{-1.0}$	$I' * (1 - \delta/2) * \delta$	$I' * (1- \delta/2) * \delta * 1.0294$	$I' * (1- \delta/2) * \delta * (1.05)^2$
3	Depreciation on assets purchased 2 years ago	$I' * (1- \delta/2) * (1- \delta)^* \delta * (1.05)^{-1.0}$	$I' * (1- \delta/2) * (1- \delta) * \delta$	$I' * (1- \delta/2) * (1- \delta)^* \delta * 1.0294$	$I' * (1- \delta/2) * (1- \delta)^* \delta * 1.0294 * 1.05$
	. . .	. . .	. . .	. . .	. . .
8	Depreciation on assets purchased 7 years ago	$I' * (1- \delta/2) * (1- \delta)^6 * \delta * (1.05)^{-1.0}$	$I' * (1- \delta/2) * (1- \delta)^6 * \delta$	$I' * (1- \delta/2) * (1- \delta)^6 * \delta * 1.0294$	$I' * (1- \delta/2) * (1- \delta)^6 * \delta * 1.0294 * 1.05$
9	Depreciation on assets purchased 8 years ago	$I' * (1- \delta/2) * (1- \delta)^7 * \delta * (1.05)^{-1.0}$	$I' * (1- \delta/2) * (1- \delta)^7 * \delta$	$I' * (1- \delta/2) * (1- \delta)^{7*} \delta * 1.0294$	$I' * (1- \delta/2) * (1- \delta)^{7*} \delta * 1.0294 * 1.05$
10	Depreciation on assets purchased 9 years ago	$I' * (1- \delta/2) * (1- \delta)^8 * (1.05)^{-1.0}$	$I' * (1- \delta/2) * (1- \delta)^8$	$I' * (1- \delta/2) * (1- \delta)^8 * 1.0294$	$I * (1- \delta/2) * (1- \delta)^8 * 1.0294 * 1.05$
11	Depreciation on entire stock of asset i	$I' * (1.05)^{-1.0}$	I'	$I' * (1 +(\delta/2) * .02) * 1.0294$	$I' * (1 +(.02 * (\delta/2) * .(3- \delta))) * 1.0294 * 1.05$

Appendix B - Depreciation and the Fundamental Equation of Capital Theory

In order to understand what impact obsolescence might have on measured depreciation and the prices of used capital assets, it necessary to understand the fundamental equation of capital theory. This equation, which has been known for more than a hundred years states that in equilibrium the price of a capital asset will equal the discounted present value of the net income expected to be derived from owning it over its lifetime. In the case of a durable good that is used by its owner, the net income is given by the implicit rental price or user cost of capital for the asset, i.e., its gross income, less any associated inputs such a maintenance and repairs, fuel, etc. that we can describe as being operating costs.

To spell out the equation concretely, let $P_{s,t}$ denote the purchase price of an s-year old asset at the beginning of year t; $P^e_{s+1,t+1}$ denote its expected purchase price at the beginning of year t+1 when the asset is one year older; $C^e_{s,t}$ denote the expected value of the services of this s-year old asset in year t; $O^e_{s,t}$ denote the expected operating expenses for this s-year old asset in year t; and r^e_t denote the expected nominal discount rate (i.e., the rate of return on the best alternative investment) in year t. (We assume that all of the assets are of type i so that there is no need to include a subscript for this variable.) Expected variables are measured as of the beginning of year t. Assume that the entire value of the asset's services in any year will be received at the end of the year, and that the asset is expected to have a service life of m years. From the definition of discounted present value, the fundamental equation is given by

(B1)
$$P_{s,t} = \frac{C^e_{s,t}}{1+r^e_t} + \frac{C^e_{s+1,t+1}}{(1+r^e_t)(1+r^e_{t+1})} + \ldots + \frac{C^e_{m-1,t+m-s-1}}{\Pi^{t+m-s-1}_{j=t}(1+r^e_j)}$$
$$- \frac{O^e_{s,t}}{1+r^e_t} - \frac{O^e_{s+1,t+1}}{(1+r^e_t)(1+r^e_{t+1})} - \ldots - \frac{O^e_{m-1,t+m-s-1}}{\Pi^{t+m-s-1}_{j=t}(1+r^e_j)}$$

When the asset is one year older, the services it renders in year t will have been received and the operating expenses of year t already incurred. Consequently, the expected price of the asset at the beginning of year t+1 is given by

$$P^e_{s+1,t+1} = \frac{C^e_{s+1,t+1}}{1+r^e_{t+1}} + \frac{C^e_{s+2,t+2}}{(1+r^e_{t+1})(1+r^e_{t+2})} + \ldots + \frac{C^e_{m-1,t+m-s-1}}{\Pi^{t+m-s-1}_{j=t+1}(1+r^e_j)}$$

(B2)

$$- \frac{O^e_{s+1,t+1}}{1+r^e_{t+1}} - \frac{O^e_{s+2,t+2}}{(1+r^e_{t+1})(1+r^e_{t+2})} - \ldots - \frac{O^e_{m-1,t+m-s-1}}{\Pi^{t+m-s-1}_{j=t+1}(1+r^e_j)}$$

Dividing both sides of equation (B2) by $(1+r^e_t)$ and subtracting the result from equation (B1) yields

(B3) $\qquad P_{s,t} - \dfrac{P^e_{s+1,t+1}}{1+r^e_t} = \dfrac{C^e_{s,t}}{1+r^e_t} - \dfrac{O^e_{s,t}}{1+r^e_t}$

Multiplying both sides of equation (B3) by $(1+r^e_t)$ and combining terms, one obtains the standard user cost measure:

(B4) $\qquad C^e_{s,t} = r^e_t P_{s,t} + (P_{s,t} - P^e_{s+1,t+1}) + O^e_{s,t}$

Equation (B4) expresses the expected value of the asset's services as the sum of three components: the expected nominal net operating surplus, the expected decline in the price of the asset during the year, and the expected value of operating expenses. The expected decline in the price of the asset is usually partitioned into two components: depreciation and the expected capital loss on the asset. Thus, for example, if we measured depreciation using prices as of the beginning of the year, as Jorgenson does, depreciation on an s-year old asset in year t, $D_{s,t}$, would be measured by

(B5) $\qquad D_{s,t} = P_{s,t} - P_{s+1,t}$

It is also possible to measure depreciation using end-of-year prices as Hulten and Wykoff [1980, p. 86] do or using average prices during the year as BEA does. The expected capital loss component can be summed with the nominal net operating surplus to yield an expected real net operating surplus. When this is done, the expected value of the asset's services is, consequently, expressed as the sum of the expected real net operating surplus, depreciation, and the expected value of operating expenses.

The relative efficiency profile d (k,s) is a schedule of m values, one for each possible each of the asset, that gives the ratio of the (expected) net service value of an s-year old asset that was produced in year k to that of a new one of the same type also produced in that year, i.e., the

values of the function are given by

(B6) $d(k, s) = C^e_{s,t} / C^e_{0,t}$

This equation should be interpreted as denoting that in all remaining years of the asset's life, including the current year t, the relative expected net service values for any two ages are in the same ratio as originally expected when the asset was first produced and purchased (in year k).

Faucett [1980] gives an example that shows that if expenditures on the maintenance and repair of motor vehicles increase as these vehicles age, then taking these (operating) expenditures explicitly in account and assuming a one-hoss shay pattern of no declines in relative efficiency will produce the same age-price profile as one obtained assuming hyperbolic declines in relative efficiency (produced by a beta-decay function) and ignoring the effects of these expenditures. Specifically, both methods produce an age-price profile that declines to zero in nearly a linear manner.

To show how the age-price profile can be derived from the associated relative efficiency profile, let us note that, by definition, the expected rate of inflation during year t in the price of assets of type i, $\dot{P}^e_{0,t}$, is given by

(B7) $(1 + \dot{P}^e_{0,t})$ $\equiv$ $P^e_{0,t+1} / P_{0,t}$

The definition of expected rates of inflation during years t+1 and beyond is similar except that both prices on the right hand side of (B7) are expected prices.

We define the expected real own rate of interest for assets during year t for assets of type i, ρ^e_t, by

(B8) $(1 + \rho^e_t)$ $\equiv$ $(1 + r^e_t) / (1 + \dot{P}^e_{0,t})$

Let us assume that the relative efficiency sequence does not change over time and that

there are no operating expenses.[21] Let us also assume for simplicity that the asset is new in year t. Under these assumptions, we have

(B9) $\qquad C^e_{s,j+1} = (1 + \dot{P}^e_{s,j+1}) \cdot C^e_{s,j}$

This equation holds for all values of s and for all years j from t to the end of the asset's life in year t+m-1. Substituting equations (B6), (B8), and B(9) into equation (B1), we obtain

(B10) $\qquad P_{s,t} = C^e_{0,t} \cdot \left[\dfrac{d(t,s)}{1+\rho^e_t} + \dfrac{d(t,s+1)}{(1+\rho^e_t)(1+\rho^e_{t+1})} + \ldots + \dfrac{d(t,m-1)}{\Pi^{t+m-s-1}_{j=t}(1+\rho^e_j)} \right]$

Equation (B10) is really a system of m equations, one for each possible age of the asset. To obtain the age-price profile we take the right hand side of (B10) for any given age of the asset and divide it by the comparable expression for new assets. The value of $C^e_{0,t}$ in the numerator and denominator of the resulting quotient cancel out and the values of all of the other variables are known by assumption. Consequently, we are able to compute the value of the quotient, which gives us one point on the age-price profile. We then repeat this procedure for all other possible ages of the asset. This exercise helps to point out that if the relative efficiency schedule is fixed over time, the implied age-price profile will generally be a function of the real own rate of interest. An important exception is geometric depreciation. A relative efficiency function that declines at a strictly geometric rate implies an age-price profile that declines at the identical geometric rate regardless of what the real own rate of interest is.

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[21] When there are operating expenses, the derivation is more complicated because we have to specify the future time path of these expenditures and the real own rate of interest for them.

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