Rent seeking with efforts and bids

Marco Haan and Lambert Schoonbeek*
Department of Economics
University of Groningen
The Netherlands

Abstract

We introduce elements of an auction in a rent-seeking contest. Players compete for a prize. Apart from exerting lobbying efforts, they also submit a bid which is payable only if they win the prize. First, we analyze the model if the returns-to-scale parameters of both bids and efforts are unity. In that case there exists a unique Nash equilibrium in pure strategies, in which each active player submits the same bid, while the sum of all efforts equals that bid. Second, we analyze the case in which the returns-to-scale parameters differ from unity, and derive the implications of that specification.

Keywords: rent seeking; lobbying; auctions

JEL classification codes: C72; D44; D72

---

*Corresponding author: Lambert Schoonbeek, Department of Economics, University of Groningen, PO Box 800, 9700 AV Groningen, The Netherlands. E-mail: L.Schoonbeek@eco.rug.nl. Tel. + 31 50 363 3798. Fax: + 31 50 363 7337. Financial support from The Netherlands Organization for Scientific Research (NWO) is gratefully acknowledged by Marco Haan. The E-mail address of Marco Haan is M.A.Haan@eco.rug.nl. We thank Bouwe Dijkstra, Peter Kooreman, Adriaan Soetevent and Linda Toolsema for helpful suggestions.

1 Introduction

In many economic situations, a number of contestants try to obtain some prize or rent. Several mechanisms can be used to assign a prize to one of the competitors. One obvious way to do so is through a regular auction. Then, all contestants submit a bid and as a rule the one submitting the highest bid obtains the prize, and pays an amount that depends in some pre-described way on the total vector of bids. In the simplest case, a first-price sealed bid auction, the highest bidder pays his own bid, whereas the other bidders pay nothing.[1] In the case of policy decisions, the parties involved often exert effort in an attempt to influence the decision process. This effort can take the form of lobbying, but can also consist of bribes. Such a process can be modelled as an all-pay auction or a rent-seeking contest. In an all-pay auction (see e.g. Baye, Kovenock, and de Vries, 1993), all contestants have to pay for their effort, and the one with the highest effort wins the auction. In a rent-seeking contest, all players also exert some effort, but the outcome of the process is stochastic: each contestant wins with a probability that is increasing in his own effort, but decreasing in that of his competitors. The extensive literature on such contests started with Tullock (1980).[2]

Yet, in practice, we often have situations that lie somewhere between the two extremes of rent-seeking contests and regular auctions. An example is the procedure by which major sports events, such as the Olympic Games, are assigned to cities or countries. On the one hand, this decision is influenced by lobbying or bribing. Yet, the contestants also submit bids, which come in the form of e.g. the quality or quantity of new stadiums and infrastructure, which will only be built by a city or country if it becomes the actual organiser of the event. As another example, note that often when an auction is held, the outcome is not solely determined by the height of the bid. In many cases, other aspects of the competing offers also play a role. In public procurement, the quality of the offers made is also taken into account, usually by some predefined rule that weighs different quantifiable quality criteria of the offers made. A final example is a takeover battle. Suppose two firms try to take over a third firm. Both firms submit a bid. Shareholders decide whom to tender their shares to. They will usually base their decisions not only on the bids submitted, but also on the extent to which they feel each firm contributes to the long-term prospects of the firm being taken over.[3] Thus, in practice we often see hybrid forms of rent-seeking contests and regular auctions.

In this paper, we try to model this notion. We build on the rent-seeking literature, but assume that the probability of winning not only depends on the effort exerted, but also on the bid made. A bid is payable for a player only if he wins the prize. This is the first-price sealed bid aspect of our model. In section 2, we describe our general framework, and show that it can be seen as an extension of the standard rent-seeking game. In section 3, we consider the simplest possible version of our model in which returns-to-scale parameters of both bids and efforts are equal to unity. We demonstrate for this model the existence of a unique (Nash) equilibrium in pure strategies. Denoting a player as *active* if and only if he submits positive bid and effort, it turns out that in the equilibrium one of the following two possibilities must hold: either all players are active, or there is a subset of players with a relatively high valuation of the prize who are active whereas the other players with a relatively low valuation of the prize are not active. We further show that in the equilibrium (a) each active player submits the same bid, (b) the sum of all efforts equals that bid, and (c) there is underdissipation of rent. Furthermore, we give the equilibrium solution in explicit form for the case of equal valuations, and for the case in which there are only two contestants. Section 4 uses a more general model, in which the returns-to-scale parameters of bids and efforts may differ from unity, and derives the implications of that specification. If a pure strategy equilibrium in which all players are active exists, it now has that the sum of all individual ratios of effort and bid, equals the ratio of the returns-to-scale parameters associated with efforts and bids. We further present a sufficient condition for the existence of a unique symmetric equilibrium in pure strategies of this model for the case of equal valuations. Section 5 concludes.

2 The general model

Our basic model is the following. There are $n > 1$ given players trying to obtain some prize. Player i values the prize at $v_i > 0$. We thus allow for asymmetric valuations. Different from the auction literature, but consistent with the rent-seeking literature, we assume that the valuations v_i are common knowledge. Each player i can submit a bid $b_i \geq 0$, and spend effort $e_i \geq 0$. The bid b_i only has to be paid if i wins the prize. However, effort outlays e_i are sunk. A player cannot retrieve these, regardless of whether or not he wins the prize. The probability p_i that i wins is given by the logit form contest success function

$$p_i(b_1,\dots,b_n,e_1,\dots,e_n) = \frac{f(b_i,e_i)}{\sum_{j=1}^n f(b_j,e_j)}, \qquad i=1,\dots,n, \tag{1}$$

if $b_j > 0$ and $e_j > 0$ for at least one j, and $p_i = 0$ if that is not the case. Here, $f(b_i, e_i)$ is non-negative, and $\partial f/\partial b_i$, $\partial f/\partial e_i \geq 0$. This implies $\partial p_i/\partial b_i$, $\partial p_i/\partial e_i \geq 0$, and $\partial p_i/\partial b_j$, $\partial p_i/\partial e_j \leq 0$ $(j \neq i)$. Thus, based on the bid b_i and the effort e_i, a 'score' $f(b_i, e_i)$ is computed for each player. The probability that a certain player wins this contest, is equal to the share of his score in the total sum of scores. Note that these probabilities sum to unity.[4] Given (1), player i maximizes his expected payoff, given by

$$\Pi_i = p_i\,(v_i - b_i) - e_i. \tag{2}$$

This expression reflects that the bid only has to be paid if the player wins the prize, whereas the effort outlays are non-refundable.

A natural assumption is that the score $f(b_i, e_i)$ links b_i and e_i in some multiplicative fashion. In that way, we capture the idea that there is a trade-off between bid b_i and effort e_i, and that both a positive bid and a positive effort are necessary to have a positive probability of winning. In section 3, we simply assume $f(b_i, e_i) = b_i e_i$, which we denote as a constant-returns-to-scale score. In section 4, we use a more general Cobb-Douglas score function $f(b_i, e_i) = b_i^\alpha e_i^\beta$, with $\alpha, \beta > 0$ returns-to-scale parameters of, respectively, the bids and efforts. Such a more general function, however, leads to a less tractable model.

In a standard rent-seeking model, only some effort e_i is exerted. Expected payoffs then equal

$$\pi_i = \frac{g(e_i)}{\sum_j g(e_j)} v_i - e_i. \tag{3}$$

Many papers in this literature assume $g(e_i) = e_i$. Hillman and Riley (1989) analyze this model, allowing for n contestants and asymmetric valuations. Ellingsen (1991) gives an application. Our model in section 3 can be seen as a generalization of this approach. Some papers, including Tullock (1980), use a more general contest success function $g(e_i) = e_i^r$, with $r > 0$. Nti (1999) analyzes this model, allowing for asymmetric valuations, but restricting attention to the case $n = 2$. Our model in section 4 generalizes this approach. Finally, we refer to Skaperdas (1996) and Kooreman and Schoonbeek (1997) for a general discussion of the foundations of logit form contest success functions in rent-seeking models.

3 A constant-returns-to-scale score

In this section we use the constant-returns-to-scale score $f(b_i, e_i) = b_i e_i$. The expected payoff for player i then equals

$$\Pi_i = \left(\frac{b_i e_i}{\sum_j b_j e_j} \right) (v_i - b_i) - e_i \tag{4}$$

if $b_j > 0$ and $e_j > 0$ for at least one j, and $\Pi_i = 0$ otherwise. We look for Nash equilibria in pure strategies. Without loss of generality, order valuations such that $v_1 \geq v_2 \geq \ldots \geq v_n$. To derive our main result, we need the following continuous auxiliary functions

$$h_m(b) = \sum_{j=1}^{m} \left(\frac{1}{v_j - b} \right) - \frac{m-1}{b}, \tag{5}$$

for $0 < b < v_m$, with $m = 2, \ldots, n$. For fixed m, the function $h_m(b)$ is strictly increasing in b. Moreover, $\lim_{b \downarrow 0} h_m(b) = -\infty$, and $\lim_{b \uparrow v_m} h_m(b) = \infty$. This implies that $h_m(b)$ has a unique root, $b(m)$ say, on $(0, v_m)$, i.e. $h_m(b(m)) = 0$. It is not possible to find a general closed form expression for $b(m)$.

In an equilibrium, not every player necessarily submits a positive bid and effort. There are circumstances in which a player i is better off setting $b_i = e_i = 0$, and earning zero expected profits. We will describe such a player as inactive. The following theorem now states the unique equilibrium of our model.

Theorem 1 *With $n > 1$ players, whose valuations are given by $v_1 \geq v_2 \geq \ldots \geq v_n$, there is a unique equilibrium $(\hat{b}_1, \ldots, \hat{b}_n, \hat{e}_1, \ldots, \hat{e}_n)$. There is some player k $(2 \leq k \leq n)$, such that in the equilibrium every player j with $v_j \geq v_k$ is active, whereas the other players (if any) are inactive. With $b(m)$ the unique root of the function $h_m(b)$ as defined in (5), with $m = 2, \ldots, n$, we have:*

(i) $\hat{b}_i = b(k)$, *for all* $i = 1, \ldots, k$,

(ii) $\hat{e}_i = \frac{\hat{b}_i (v_i - 2\hat{b}_i)}{(v_i - \hat{b}_i)}$, *for all* $i = 1, \ldots, k$,

(iii) $\sum_{i=1}^{k} \hat{e}_i = b(k)$,

(iv) $k = \sup_m \{m : b(m) < v_m/2\}$.

Proof. See the Appendix. ■

The unique equilibrium has a number of interesting properties. First, all active players submit the same bid $b(k)$, regardless of their valuation. This implies that, in equilibrium, differences in success probabilities of active players are solely determined by differences in the efforts $\hat{e}_i$. Second, the bid $b(k)$ every active player submits, equals the sum of total efforts. Third, that bid is strictly increasing in the size of the valuations of the active players: $\partial b(k)/\partial v_i > 0$, $\forall i = 1, \ldots, k$. This can be seen from (i) of Theorem 1 and (5). Fourth, equilibrium bid and efforts are linear homogeneous in valuations: if all valuations are multiplied with the same factor, then the equilibrium bid and efforts all are multiplied with this factor as well. Fifth, we have $\hat{e}_1 \geq \hat{e}_2 \geq \ldots \geq \hat{e}_k > 0$. Thus, the higher the valuation of an active player, the greater the effort he exerts. Sixth, $\hat{p}_1 \geq \hat{p}_2 \geq \ldots \geq \hat{p}_k > 0$, which follows from the fact that $\hat{p}_i = \hat{e}_i/b(k)$. Thus, the player with the highest valuation also has the highest probability to win the prize. Seventh, a player with a higher valuation also has a higher expected profit: $\widehat{\Pi}_1 \geq \widehat{\Pi}_2 \geq \ldots \geq \widehat{\Pi}_k > 0$. This follows from the fact that $\widehat{\Pi}_i = (v_i - 2b(k))^2/(v_i - b(k))$ for $i = 1, \ldots, k$. Eighth, if not all players are active, then only the players with the highest valuations are. Finally, all n players are active if and only if $b(n) < v_n/2$.

Now consider the extent of rent dissipation that occurs in equilibrium. To study this issue, we need a definition of rent dissipation in the context of our model. Usually, it is defined as the total sum of efforts of the contestants trying to obtain the prize. Yet, in our model, there is also a bid $b(k)$ paid by the winner. Arguably, this should not be counted as rent dissipation, since it merely consists of a transfer from the winner of the prize to the authority selling the prize. On the other hand, it is often argued that efforts $\hat{e}_i$ consist of bribes rather than efforts. Since bribes are also merely transfers, then if bribes are counted as dissipated rent, winning bids should also be. We therefore consider both possibilities. First, suppose that the winning bid is considered as dissipated rent. Total dissipation then equals $D = \sum_i \hat{e}_i + b(k)$. Using Theorem 1, it follows that $D = 2b(k) < v_k$. Thus, in this case there is always underdissipation of rent: total rent dissipation is less than the size of (even) the smallest valuation of the prize among the active players. If we do not consider the winning bid as dissipated rent, then total rent dissipation, D' say, satisfies $D' = \frac{1}{2}D < \frac{1}{2}v_k$. Again there is always underdissipation of rent.

3.1 The case of n equal valuations

Now consider the case in which all players have the same valuation. We then obtain the following result.

Corollary 1 *Take the model with $n > 1$ players. If $v_i = v, \forall i$, then the unique equilibrium bids and efforts are given by:*

(i) $\hat{b}_i = b(n) = \frac{(n-1)v}{(2n-1)}, \forall i,$

(ii) $\hat{e}_i = \frac{(n-1)v}{(2n-1)n}, \forall i.$

Proof. See the Appendix. ■

For this case, we do have explicit solutions for $\hat{b}_i$ and $\hat{e}_i$. Therefore, we can explicitly characterize the extent of rent dissipation in equilibrium. If the winning bid is considered as dissipated rent, then total rent dissipation is $\frac{2}{3}v$ with $n = 2$, and it strictly increases to v as n goes to infinity. If the winning bid is not considered as dissipated rent, then total rent dissipation is $\frac{1}{3}v$ with $n = 2$, and it strictly increases to $\frac{1}{2}v$ as n goes to infinity. In the standard rent-seeking model, total rent dissipation equals $(n-1)v/n$, see e.g. Hillman and Riley (1989). Thus, in our model, total rent dissipation is lower than in the standard rent-seeking model when the winning bid is not considered as dissipated rent, but higher when it is.

For the standard rent-seeking model, equilibrium efforts are $e_i^* = e^* = (n-1)v/n^2, \forall i$ and expected payoffs $\pi_i^* = v/n^2$, see again Hillman and Riley (1989). In our model, using Corollary 1,

$$\widehat{\Pi}_i = \frac{v}{n}\left(\frac{1}{2n-1}\right). \tag{6}$$

In a regular (first-price) auction, it is easy to see that each player would bid the common valuation of the prize (v), leaving expected payoffs equal to zero. Therefore, in our model, expected payoffs for contestants are higher than in a regular auction, but lower than in a standard rent-seeking contest.

3.2 The case of two players

Next, we return to the general model in which valuations are allowed to differ, but restrict attention to the case of two contestants, thus $n = 2$. We then have the following result.

Corollary 2 *Take the model with $n = 2$ players. The unique equilibrium bids and efforts are given by:*

(i) $\hat{b}_i = b(2) = \frac{v_1+v_2}{3} - \frac{1}{3}\sqrt{(v_1+v_2)^2 - 3v_1v_2}$,

(ii) $\hat{e}_i = \frac{\hat{b}_i(v_i-2\hat{b}_i)}{(v_i-\hat{b}_i)}$,

for $i = 1, 2$. Substituting $\hat{b}_i = b(2)$ into (ii), we have an explicit solution for $\hat{e}_i$.

Proof. See the Appendix. ■

Suppose we consider the winning bid $b(2)$ as dissipated rent. Total rent dissipation then equals $D = \hat{e}_1 + \hat{e}_2 + b(2) = 2b(2)$. In order to study the effect of asymmetry, suppose the sum of valuations of both contestants is fixed: $v_1 + v_2 = V$. Assuming that $v_1 \geq v_2$, we may write $v_1 = \rho V$ and $v_2 = (1-\rho)V$, with $\rho \in [\frac{1}{2}, 1)$. We can study the effect of decreased asymmetry as a decrease in ρ. Total rent dissipation equals

$$D = \frac{2}{3}V\left(1 - \sqrt{1 - 3\rho(1-\rho)}\right). \tag{7}$$

Observe that

$$\frac{\partial D}{\partial \rho} = \frac{1-2\rho}{\sqrt{1-3\rho(1-\rho)}}V. \tag{8}$$

Thus, rent dissipation is maximized when $\rho = \frac{1}{2}$, i.e. when the two valuations are equal. Further, $\partial D/\partial \rho < 0$ for all $\rho \in (\frac{1}{2}, 1)$. Therefore, with two players, we have that more equal valuations lead to higher total rent dissipation. [5] This result does not hinge on the definition of rent dissipation. When we also take the winning bid into account, total rent dissipation simply equals $D' = \frac{1}{2}D$.

4 A general Cobb-Douglas score

In the previous section, we analyzed a model where the returns-to-scale parameters associated with both bidding and exerting effort equal unity. In this section, we use the more general Cobb-Douglas score function $f(b_i, e_i) =$

$b_i^\alpha e_i^\beta$. The returns-to-scale parameters satisfy $\alpha, \beta > 0$. Hence, the model analyzed in the previous section is a special case of this model, with $\alpha = \beta = 1$. The expected payoff of player i now equals

$$\Pi_i = \left(\frac{b_i^\alpha e_i^\beta}{\sum_j b_j^\alpha e_j^\beta} \right) (v_i - b_i) - e_i \tag{9}$$

if $b_j > 0$ and $e_j > 0$ for at least one j, and $\Pi_i = 0$ otherwise.

For this model we have the following general result.

Theorem 2 *Take the model with $n > 1$ players. Consider an equilibrium $(\hat{b}_1, \ldots, \hat{b}_n, \hat{e}_1, \ldots, \hat{e}_n)$ in which all n players are active. We then have:*

$$\left(\sum_{j=1}^{n} \frac{\hat{e}_j}{\hat{b}_j} \right) = \frac{\beta}{\alpha}. \tag{10}$$

Proof. See the Appendix. ■

Thus, if we have an equilibrium in which all players are active, then the sum of all individual ratios of the effort and bid, equals the ratio of the returns-to-scale parameters associated with efforts and bids. This theorem has a natural interpretation. As β, the parameter that reflects returns to scale with respect to the efforts increases, then efforts become more important, in the sense that the sum of the individual ratios of the equilibrium effort and equilibrium bid increases. As α, the parameter that reflects returns to scale with respect to bids increases, then bids become more important, in the sense that the sum of the individual ratios of the equilibrium effort and equilibrium bid decreases.

To further analyze this model, we assume that all contestants have equal valuations. Even for the case of two players, it is in general not possible to find the equilibrium with both agents active in closed form.[6]

4.1 The case of n equal valuations

Suppose that all players have equal valuations. We then have the following result.

Theorem 3 *Take the model with $n > 1$ players. Suppose that $v_i = v, \forall i$, and $\beta \leq 1$. Then there exists a unique symmetric equilibrium with bids and efforts given by:*

(i) $\hat{b}_i = \hat{b} = \frac{\alpha(n-1)v}{\alpha(n-1)+n}, \forall i,$

(ii) $\hat{e}_i = \hat{e} = \frac{1}{n}\frac{\beta(n-1)v}{\alpha(n-1)+n}, \forall i.$

Proof. See the Appendix. ■

Thus, when valuations are equal, $\beta \leq 1$ is a sufficient condition for the existence of a unique symmetric equilibrium. Again, bids and efforts are linear homogeneous in the valuation v. Second, if α, the returns-to-scale parameter of bids, increases, then equilibrium bids strictly increase, whereas the equilibrium efforts strictly decrease. Third, if β, the returns-to-scale parameter of efforts, increases, then equilibrium efforts strictly increase. There is no effect on equilibrium bids. Fourth, in equilibrium the probability that player i wins, equals $\hat{p}_i = 1/n$. His expected payoff equals

$$\widehat{\Pi}_i = \frac{1}{n}(v - \hat{b}) - \hat{e} = \frac{1}{n}\left(\frac{nv - \beta(n-1)v}{\alpha(n-1)+n}\right), \tag{11}$$

which is positive, because we assumed that $\beta \leq 1$.

Using Theorem 3, we can again study the extent of rent dissipation. Suppose that the winning bid is considered as dissipated rent. We then have from Theorem 3 that

$$D = n\hat{e} + \hat{b} = \frac{(\alpha+\beta)(n-1)v}{\alpha(n-1)+n}. \tag{12}$$

Consequently, with two contestants, total rent dissipation is $(\alpha+\beta)v/(\alpha+2)$. The extent of rent dissipation strictly increases to $(\alpha+\beta)\,v/(\alpha+1)$ as n goes to infinity. Note that

$$\frac{\partial D}{\partial \alpha} = \frac{n - \beta(n-1)}{(\alpha n - \alpha + n)^2}\,(n-1)\,v, \tag{13}$$

which is positive, since by assumption $\beta \leq 1$. Thus here, rent dissipation strictly increases in α and β.

Now suppose we do not count the winning bid as dissipated rent. From Theorem 3 we obtain

$$D' = n\hat{e} = \frac{\beta(n-1)v}{\alpha(n-1)+n}. \tag{14}$$

With two contestants, total rent dissipation now equals $\beta v/(\alpha+2)$. The extent of rent dissipation strictly increases to $\beta v/(\alpha+1)$ as n goes to infinity.

Rent dissipation now strictly *decreases* in α, but strictly increases in β. It is easy to verify that there is always underdissipation of rent, regardless of the treatment of the winning bid.

5 Conclusion

In this paper, we presented a model that combines a rent-seeking contest with elements of a first-price (sealed bid) auction. The model considers a situation in which players compete for a prize. The probability that a player wins the prize depends not only on the amount of effort exerted, but also on the bid submitted. The bid only has to be paid if the player wins the prize, the effort outlays are sunk.

First, we discussed the model with constant returns to scale in both bids and efforts. We showed the existence of a unique Nash equilibrium in pure strategies for that model. Further, we found that in such an equilibrium all active players will submit the same bid, regardless of their valuations, and that total efforts equal that bid. Moreover, we found underdissipation of rent. For the two player case, we showed that the extent of total rent dissipation is strictly decreasing in the extent of asymmetry in valuations.

Second, we studied pure strategy Nash equilibria of a more general model, in which the probability of success depends on a general Cobb-Douglas function in bids and efforts. We demonstrated for that model that in an equilibrium in which all players are active, the sum of the individual ratios of the effort and bid is equal to the ratio of their respective returns-to-scale parameters. Focusing on the case of equal valuations, we showed that the model has a unique symmetric equilibrium if the returns-to-scale parameter of efforts is not greater than unity. We showed that in equilibrium there is underdissipation of rent. Total rent dissipation strictly increases in the returns-to-scale parameter of efforts. If the winning bid is considered as dissipated rent, then total rent dissipation strictly increases in the returns-to-scale parameter of bids, whereas if it is not, total rent dissipation is strictly decreasing in this parameter.

Appendix

Proofs of section 3

To prove the results in section 3, we first state the first-order conditions for an interior solution of player i's maximization problem, given bids b_j and efforts e_j $(j \neq i)$ of his rivals:

$$\frac{\partial \Pi_i}{\partial b_i} = \frac{\left(\sum_j b_j e_j\right) e_i \left(v_i - 2b_i\right) - b_i e_i^2 \left(v_i - b_i\right)}{\left(\sum_j b_j e_j\right)^2} = 0 \tag{A.1}$$

and

$$\frac{\partial \Pi_i}{\partial e_i} = \frac{\left(b_i \left(\sum_j b_j e_j\right) - b_i^2 e_i\right) \left(v_i - b_i\right)}{\left(\sum_j b_j e_j\right)^2} - 1 = 0. \tag{A.2}$$

In stating these, we implicitly assume $\sum_{j \neq i} b_j e_j > 0$.

Next, we present three lemma's that will be used in the proofs of Theorem 1 and its corollaries. Lemma 1 characterizes the optimal bid and effort of player i, given bids and efforts of the other players.

Lemma 1 *Take the model with $n > 1$ players. Consider player i. Let $c_i = \sum_{j \neq i} b_j e_j > 0$. Then the optimal bid and effort of player i, $\tilde{b}_i$ and $\tilde{e}_i$, are as follows:*
(i) If $c_i \geq \left(\frac{v_i}{2}\right)^2$, then $\tilde{b}_i = \tilde{e}_i = 0$. Hence, $\tilde{\Pi}_i = 0$.
(ii) If $c_i < \left(\frac{v_i}{2}\right)^2$, then $\tilde{b}_i = \bar{b}_i$ and $\tilde{e}_i = c_i(v_i - 2\bar{b}_i)/\bar{b}_i^2$, with $\bar{b}_i$ the unique root of the continuous auxiliary function $k_i(b) = b^3 - c_i(v_i - b)$, $b \in (0, v_i)$. In this case $0 < \tilde{b}_i < v_i/2$ and $\tilde{e}_i > 0$, and $\tilde{\Pi}_i > 0$.

Proof. Given $c_i > 0$, player i considers three possible options. If he chooses $b_i = 0$, then his corresponding best effort is 0, and his expected payoff also is. If i chooses $e_i = 0$, his expected payoff equals 0 irrespective of his bid. If i chooses positive bid and effort, these must satisfy (A.1) and (A.2). Rewriting these yields

$$c_i(v_i - b_i) = b_i(b_i e_i + c_i) \tag{A.3}$$

and

$$c_i b_i (v_i - b_i) = (b_i e_i + c_i)^2, \tag{A.4}$$

or, equivalently,

$$b_i^3 = c_i(v_i - b_i) \tag{A.5}$$

and

$$e_i = \frac{c_i(v_i - 2b_i)}{b_i^2}. \tag{A.6}$$

Defining the function $k_i(b) = b^3 - c_i(v_i - b)$ for $0 < b < v_i$, we have that $k_i(b)$ is strictly increasing in b, that $\lim_{b \downarrow 0} k_i(b) = -c_i v_i < 0$, and that $\lim_{b \uparrow v_i} k_i(b) = v_i^3 > 0$. This implies that $k_i(b)$ has a unique root $\bar{b}_i$ on $(0, v_i)$. Thus there exist $b_i > 0$ and $e_i > 0$ that satisfy (A.5) and (A.6) if and only if $\bar{b}_i < v_i/2$. In particular, if $\bar{b}_i < v_i/2$, then the relevant b_i and e_i are unique and given by $b_i = \bar{b}_i > 0$ and $e_i = c_i(v_i - 2\bar{b}_i)/\bar{b}_i^2 > 0$. Moreover, using (A.3) and (A.5), we now have $\Pi_i = \left(\frac{\bar{b}_i^2}{c_i} - 1\right) e_i = \left(\frac{v_i - 2\bar{b}_i}{\bar{b}_i}\right) e_i > 0$, i.e. the expected payoff is strictly positive. Finally, observing that $k_i(\frac{v_i}{2}) = \left(\frac{v_i}{2}\right)\left(\left(\frac{v_i}{2}\right)^2 - c_i\right)$, it easily follows that $\bar{b}_i < v_i/2$ if and only if $c_i < (v_i/2)^2$. The proof of the lemma now follows directly. ■

Lemma 2 *Consider for $n = 2$ the function $h_2(b)$ as defined in (5), with $v_1 \geq v_2$. The unique root $b(2)$ of $h_2(b)$ is given by*

$$b(2) = \frac{v_1 + v_2}{3} - \frac{1}{3}\sqrt{(v_1 + v_2)^2 - 3v_1 v_2} \tag{A.7}$$

and satisfies $b(2) < v_2/2$.

Proof. Straightforward manipulations show that $b(2)$ is given by (A.7). We then have to show that $b(2) < v_2/2$, i.e. $v_2 - 2b(2) > 0$. Now,

$$v_2 - 2b(2) = \frac{v_2 - 2v_1}{3} + \frac{2}{3}\sqrt{(v_1 + v_2)^2 - 3v_1 v_2}.$$

For this expression to be positive, we need

$$2\sqrt{(v_1 + v_2)^2 - 3v_1 v_2} > 2v_1 - v_2. \tag{A.8}$$

With $v_1 \geq v_2$, the RHS of this expression is positive. Taking squares on both sides and rearranging, (A.8) simplifies to $3v_2^2 > 0$, which is always satisfied. ■

Lemma 3 *Take the model with $n > 1$ players, whose valuations are given by $v_1 \geq v_2 \geq \ldots \geq v_n$. Let $b(n)$ be the unique root of the function $h_n(b)$ as defined in (5). Then there exists an equilibrium $(\hat{b}_1, \ldots, \hat{b}_n, \hat{e}_1, \ldots, \hat{e}_n)$ in which all n players are active if and only if $b(n) < v_n/2$. Moreover, if such an equilibrium exists, then it is the unique equilibrium in which all players participate, and the bids and efforts satisfy:*

(i) $\hat{b}_i = b(n)$, *for all* $i = 1, \ldots, n$,

(ii) $\hat{e}_i = \frac{\hat{b}_i(v_i - 2\hat{b}_i)}{(v_i - \hat{b}_i)}$, *for all* $i = 1, \ldots, n$,

(iii) $\sum_{i=1}^{n} \hat{e}_i = b(n)$.

Proof.
— First, assume that $(\hat{b}_1, \ldots, \hat{b}_n, \hat{e}_1, \ldots, \hat{e}_n)$ is an equilibrium with $\hat{b}_i > 0$ and $\hat{e}_i > 0$, $\forall i$. We then have to show that $b(n) < v_n/2$, that there cannot exist another equilibrium in which all n players are active, and that the given equilibrium bids and efforts satisfy (i), (ii) and (iii) of the lemma. Note that, using (A.1) and (A.2), in the given equilibrium we must have $\hat{b}_i < v_i/2$, $\forall i$. Evaluated in the given equilibrium, (A.1) implies

$$\left(\sum_{j \neq i} \hat{b}_j \hat{e}_j\right) \left(v_i - \hat{b}_i\right) = \hat{b}_i \left(\sum_j \hat{b}_j \hat{e}_j\right), \tag{A.9}$$

whereas (A.2) yields

$$\left(\sum_{j \neq i} \hat{b}_j \hat{e}_j\right) \hat{b}_i \left(v_i - \hat{b}_i\right) = \left(\sum_j \hat{b}_j \hat{e}_j\right)^2. \tag{A.10}$$

Substituting (A.9) into (A.10) yields $\hat{b}_i^2 \left(\sum_j \hat{b}_j \hat{e}_j\right) = \left(\sum_j \hat{b}_j \hat{e}_j\right)^2$, thus

$$\hat{b}_i^2 = \sum_j \hat{b}_j \hat{e}_j. \tag{A.11}$$

The RHS of (A.11) is a constant, independent of i. Thus we can write $\hat{b}$ for the bid of each player. Hence, the condition $\hat{b}_i < v_i/2$, $\forall i$, reduces to $\hat{b} < v_n/2$. From (A.11) we immediately have

$$\hat{b} = \sum_j \hat{e}_j. \tag{A.12}$$

Using (A.12), (A.9) implies $\left(\hat{b}^2 - \hat{b}\hat{e}_i\right)(v_i - \hat{b}) = \hat{b}^3$, so we have

$$\hat{e}_i = \frac{\hat{b}(v_i - 2\hat{b})}{(v_i - \hat{b})}. \tag{A.13}$$

Substituting (A.13) into (A.12) yields $\sum_j \left(\frac{v_j - 2\hat{b}}{v_j - \hat{b}}\right) = 1$, thus $\sum_j \left(1 - \frac{\hat{b}}{v_j - \hat{b}}\right) = 1$, or

$$\hat{b} \sum_j \left(\frac{1}{v_j - \hat{b}}\right) = n - 1. \tag{A.14}$$

From (A.14), $\hat{b}$ is a root of $h_n(b)$ of (5). Since $h_n(b)$ has a unique root, $\hat{b} = b(n)$, and we must have $b(n) < v_n/2$. Next, (i) of the lemma is now obvious, and (ii) and (iii) follow from, respectively, (A.13) and (A.12). Because $b(n)$ is the unique root of $h_n(b)$, there cannot exist another equilibrium in which all n players are active.

— Next, assume that $b(n) < v_n/2$. We then have to prove that there exists an equilibrium in which all n players are active, and that its bids and efforts are given by (i) and (ii) of the lemma ((iii) is then automatically satisfied). Note that these satisfy $\hat{b}_i > 0$ and $\hat{e}_i > 0$, $\forall i$. It remains to be shown that each player i maximizes his expected payoff by choosing $b_i = b(n)$ and $e_i = \hat{e}_i$, given the rivals' choices $\hat{b}_j$ and $\hat{e}_j$ $(j \neq i)$. Consider $c_i = \sum_{j \neq i} \hat{b}_j \hat{e}_j$. Note that $c_i > 0$. Further,

$$c_i = b(n) \sum_{j \neq i} \left(\frac{b(n)(v_j - 2b(n))}{v_j - b(n)}\right) = b(n)^2 \left(n - 1 - \sum_{j \neq i} \left(\frac{b(n)}{v_j - b(n)}\right)\right).$$

Rewriting $h_n(b(n)) = 0$, it follows that

$$\sum_{j \neq i} \left(\frac{b(n)}{v_j - b(n)}\right) = n - 1 - \left(\frac{b(n)}{v_i - b(n)}\right).$$

Combining results, we thus derive that

$$c_i = b(n)^2 \left(\frac{b(n)}{v_i - b(n)}\right).$$

Because $b(n) < v_i/2$, we see that $c_i < (v_i/2)^2$. Applying part (ii) of Lemma 1, consider the function $k_i(b)$ for this case, with $0 < b < v_i$. We have

$$k_i(b) = b^3 - \left(\frac{b(n)^3}{v_i - b(n)}\right)(v_i - b).$$

Clearly, $b = b(n)$ is the (unique) root of $k_i(b)$. Thus, indeed, the optimal bid and effort of player i are given by $b_i = b(n)$ and $e_i = \hat{e}_i$. ■

Proof of Theorem 1
We give the proof in 4 steps. Throughout the proof, $b(m)$ denotes the unique root of the function $h_m(b)$ defined in (5), with $m = 1, \ldots, n$.
— Step 1: First, we show that there is no equilibrium in which only one agent is active. Suppose there is such an equilibrium and, without loss of generality, it is given by $(b_1^*, 0, \ldots, 0, e_1^*, 0, \ldots, 0)$ with $b_1^*, e_1^* > 0$. Then $\Pi_1 = v_1 - b_1^* - e_1^*$. By not being active, player 1 can always secure zero payoff. Hence, $v_1 - b_1^* - e_1^* \geq 0$. For $\epsilon > 0$ sufficiently small, $v_1 - (b_1^* - \epsilon) - (e_1^* - \epsilon) > 0$. Thus, player 1 strictly prefers to set $(b_1^* - \epsilon,\, e_1^* - \epsilon)$ rather than (b_1^*, e_1^*), and the original situation is not an equilibrium. We have a contradiction. In the remainder, we can therefore concentrate on (candidate) equilibria with more than 1 player active.
— Step 2: Next, we show that in an equilibrium in which some player s is active, necessarily all players $1, \ldots, s-1$ are active as well. Suppose there is an equilibrium $(b_1^*, \ldots, b_n^*, e_1^*, \ldots, e_n^*)$ in which exactly t players are active, with $1 < t < n$, and in which both there is a player s who is active and a player r with $v_r \geq v_s$ who is *inactive*. We will derive a contradiction. Denote the set of active players in the given equilibrium as T. Consider the hypothetical contest that is obtained from our original contest with n players by removing the $n-t$ players with $i \notin T$. Removing these non-active players does not affect the incentives of the active players in the equilibrium. Hence, for this hypothetical contest, the bids and efforts b_i^* and e_i^* with $i \in T$ must constitute an equilibrium. Noting that all t players are active in this equilibrium, we conclude from Lemma 3 that $b_i^* = b(t)$ and $\sum_{i \in T} e_i^* = b(t)$. Also, $b(t) < v_s/2$.

Returning to the original contest with n players, consider now player r. Using the notation of Lemma 1, $c_r = \sum_{i \neq r} b_i^* e_i^* = \sum_{i \in T} b_i^* e_i^* = b(t)^2 < (v_s/2)^2 \leq (v_r/2)^2$. Applying part (ii) of Lemma 1, player r prefers to choose positive bid and effort, rather than $b_r^* = e_r^* = 0$. Thus, we have a contradiction.
— Step 3: Suppose there exists an equilibrium in which the set of active players is $S = \{1, \ldots, s\}$ for some $1 < s \leq n$. We show that there cannot exist another equilibrium in which the set of active players is a strict subset of S. Without loss of generality, take $s = n$, and assume $(b_1^*, \ldots, b_n^*, e_1^*, \ldots, e_n^*)$ is an equilibrium in which all players are active. From Lemma 3, $b(n) < v_n/2$. Suppose now there is another equilibrium $(b_1^{**}, \ldots, b_n^{**}, e_1^{**}, \ldots, e_n^{**})$ in which only t players are active, $t < n$. We will derive a contradiction. Consider the hypothetical contest among only the players $\{1, \ldots t\}$. For this hypothetical

contest, $(b_1^{**}, \ldots, b_t^{**}, e_1^{**}, \ldots, e_t^{**})$ must be an equilibrium. From Lemma 3, $b_i^{**} = b(t)$ for all $i = 1, \ldots, t$, and $\sum_{i=1}^{t} e_i^{**} = b(t)$, with $b(t) < v_t/2$. Evaluating $h_t(b)$ in $b = b(n)$, using $h_n(b(n)) = 0$ and $b(n) < v_n/2$, we have

$$h_t(b(n)) = -\sum_{j=t+1}^{n} \left(\frac{1}{v_j - b(n)} \right) + \frac{n-t}{b(n)} = -\sum_{j=t+1}^{n} \left(\frac{2b(n) - v_j}{(v_j - b(n))b(n)} \right) > 0,$$

thus $b(t) < b(n)$. This implies $b(t) < v_n/2$.

Now return to the original n-player contest. For player $t+1$, we have $c_{t+1} = \sum_{j=1}^{t} b_j^{**} e_j^{**} = (b(t))^2 < (v_n/2)^2 \leq (v_{t+1}/2)^2$. Using part (ii) of Lemma 1, player $t+1$ thus prefers positive bid and effort, rather than $b_{t+1}^{**} = e_{t+1}^{**} = 0$. We have a contradiction.
— Step 4: Using steps 1, 2 and 3, we know that if there exists an equilibrium, in this equilibrium either all players are active, or there is a player with a 'critical' valuation such that all players with a valuation larger than or equal to this critical valuation are active, whereas all players with a lower valuation are inactive. Also, if the equilibrium exists, it must be unique.

Using Lemma's 1 and 3, it now follows that the proof of Theorem 1 is completed if we demonstrate that there exists a value $k \in (1, n]$ such that (a) $b(k) < v_k/2$, and (b) if $k < n$, we also have $b(k) \geq v_{k+1}/2$. By doing so, we show the existence of an equilibrium in which, in case $k = n$, all n players are active, while in case $k < n$, the players $1, \ldots, k$ are active and the players $k+1, \ldots, n$ are not. First, note that Lemma 2 implies that $b(2) < v_2/2$. Hence, there is at least one t such that $b(t) < v_t/2$. Second, suppose now that there does not exist a value k satisfying (a) and (b), i.e. suppose that the contest has *no* equilibrium. Then both (i) we must have $b(n) \geq v_n/2$ (use Lemma 3) and (ii) for all $t < n$ such that $b(t) < v_t/2$, we must have $b(t) < v_{t+1}/2$ (use (a) and (b) mentioned above). We will derive a contradiction. Consider (ii) and take a $t^* < n$ such that $b(t^*) < v_{t^*}/2$. According to (ii), this implies $b(t^*) < v_{t^*+1}/2$, thus $h_{t^*}(v_{t^*+1}/2) > 0$. Since $h_{t^*}(v_{t^*+1}/2) = h_{t^*+1}(v_{t^*+1}/2)$, this implies $h_{t^*+1}(v_{t^*+1}/2) > 0$, thus $b(t^*+1) < v_{t^*+1}/2$. If $t^*+1 = n$, then we have a contradiction with (i). If $t^*+1 < n$, by induction, repeating the argument finally implies $b(n) < v_n/2$, which again violates (i). Thus, (i) and (ii) cannot be both satisfied, which establishes Theorem 1. ∎

Proof of Corollary 1

Using $v_i = v$, $\forall i$, it follows that the root $b(n)$ of the function $h_n(b)$ is equal to $b(n) = (n-1)v/(2n-1)$, thus $b(n) < v/2$. Part (i) of the corollary then follows from (i) of Theorem 1. Invoking symmetry, $\hat{e}_i = \hat{e}$, $\forall i$, which implies (ii). ■

Proof of Corollary 2
The proof follows directly from Lemma 2 and Theorem 1. ■

Proofs of section 4

In section 4, the first-order conditions of an interior solution of the maximization problem of player i, given the bids b_j and efforts e_j $(j \neq i)$ of his rivals, are given by

$$\frac{\partial \Pi_i}{\partial b_i} = \frac{\left(\sum_j b_j^\alpha e_j^\beta\right)\left(\alpha b_i^{\alpha-1} e_i^\beta (v_i - b_i) - b_i^\alpha e_i^\beta\right) - \alpha b_i^{2\alpha-1} e_i^{2\beta}(v_i - b_i)}{\left(\sum_j b_j^\alpha e_j^\beta\right)^2} = 0 \tag{A.15}$$

and

$$\frac{\partial \Pi_i}{\partial e_i} = \frac{\beta b_i^\alpha e_i^{\beta-1}\left(\sum_j b_j^\alpha e_j^\beta\right) - \beta b_i^{2\alpha} e_i^{2\beta-1}}{\left(\sum_j b_j^\alpha e_j^\beta\right)^2}(v_i - b_i) - 1 = 0, \tag{A.16}$$

where we assume that $\sum_{j \neq i} b_j^\alpha e_j^\beta > 0$. Note that (A.15) reduces to

$$\alpha \left(\sum_{j \neq i} b_j^\alpha e_j^\beta\right)(v_i - b_i) = b_i \left(\sum_j b_j^\alpha e_j^\beta\right), \tag{A.17}$$

whereas (A.16) reduces to

$$(v_i - b_i)\beta b_i^\alpha e_i^{\beta-1}\left(\sum_{j \neq i} b_j^\alpha e_j^\beta\right) = \left(\sum_j b_j^\alpha e_j^\beta\right)^2. \tag{A.18}$$

Using these conditions we present the proofs of Theorems 2 and 3.

Proof of Theorem 2
Suppose that $(\hat{b}_1, \ldots, \hat{b}_n, \hat{e}_1, \ldots, \hat{e}_n)$ is an equilibrium in which all n players are active. In the equilibrium each player's expected payoff must be nonnegative. This implies that we must have $\hat{b}_i < v_i$, $\forall i$. We further know that the equilibrium must satisfy (A.17) and (A.18). Using this, we obtain

$$\hat{b}_i^{\alpha+1} \hat{e}_i^{\beta-1} = \frac{\alpha}{\beta}\left(\sum_{j=1}^n \hat{b}_j^\alpha \hat{e}_j^\beta\right). \tag{A.19}$$

The RHS of this equality is a constant, independent of i. Thus, the products $\hat{b}_i^{\alpha+1}\hat{e}_i^{\beta-1}$ are a constant. As a result, (A.19) yields

$$\left(\sum_{j=1}^{n}\frac{\hat{e}_j}{\hat{b}_j}\right)=\frac{\beta}{\alpha}, \tag{A.20}$$

which completes the proof. ∎

Proof of Theorem 3

Assume that $v_i = v$, $\forall i$, and $\beta \leq 1$. We will show that there exists a unique symmetric equilibrium, which is given by parts (i) and (ii) of the theorem.

Imposing symmetry, i.e. $b_i = \hat{b}$ and $e_i = \hat{e}$, $\forall i$, we observe that $\hat{b} = 0$ and/or $\hat{e} = 0$ is not an equilibrium. Substituting $b_i = \hat{b} > 0$ and $e_i = \hat{e} > 0$, $\forall i$, into conditions (A.17) and (A.18), we have

$$\hat{b}=\frac{\alpha(n-1)v}{\alpha(n-1)+n} \tag{A.21}$$

and

$$\hat{e}=\frac{1}{n}\frac{\beta(n-1)v}{\alpha(n-1)+n}. \tag{A.22}$$

This implies that if there is a symmetric equilibrium, it must be given by $\hat{b}$ and $\hat{e}$ of (A.21) and (A.22). To show that this indeed constitutes an equilibrium, we have to prove that player i maximizes his expected payoff by choosing $\hat{b}$ and $\hat{e}$, given the same choices of his rivals.

If i chooses $b_i = 0$, then his corresponding best effort is 0, and his expected payoff also is. If i chooses $e_i = 0$, then his expected payoff is 0 as well. Now examine positive b_i and e_i that satisfy (A.17) and (A.18). These conditions then reduce to

$$\alpha d_i(v-b_i)=b_i(b_i^{\alpha}e_i^{\beta}+d_i) \tag{A.23}$$

and

$$d_i\beta b_i^{\alpha}e_i^{\beta-1}(v-b_i)=(b_i^{\alpha}e_i^{\beta}+d_i)^2, \tag{A.24}$$

where for notational convenience we have defined $d_i = \sum_{j\neq i}\hat{b}^{\alpha}\hat{e}^{\beta}$. Note that $d_i > 0$. From (A.23), we must have $b_i < \alpha v/(1+\alpha)$.

Observe that (A.23) and (A.24) are satisfied if player i chooses $b_i = \hat{b}$ and $e_i = \hat{e}$ (note that $0 < \hat{b} < \alpha v/(1+\alpha)$ and $\hat{e}_i > 0$). Also note

$$\widehat{\Pi}_i=\frac{1}{n}(v-\hat{b})-\hat{e}=\frac{1}{n}\left(\frac{nv-\beta(n-1)v}{\alpha(n-1)+n}\right), \tag{A.25}$$

which is positive, since $\beta \leq 1$. The proof is completed if we show that there exist no other $b_i \in (0, \alpha v/(1+\alpha))$ and $e_i > 0$ that satisfy (A.23) and (A.24) (which implies that player i indeed globally maximizes his payoff by choosing $b_i = \hat{b}$ and $e_i = \hat{e}$). To do so, we distinguish two cases: $\beta = 1$ and $\beta < 1$.

First, take the case $\beta < 1$. From (A.23), we obtain that $e_i = s_i(b_i)$, where the continuous auxiliary function $s_i(b)$ is defined as

$$s_i(b) = \Big(d_i(\alpha v - (\alpha+1)b)b^{-(\alpha+1)}\Big)^{\frac{1}{\beta}}, \tag{A.26}$$

for $0 < b < \alpha v/(1+\alpha)$. Observe that $s_i(b)$ is strictly decreasing in b, and that $\lim_{b\downarrow 0} s_i(b) = \infty$ and $\lim_{b\uparrow \frac{\alpha v}{(1+\alpha)}} s_i(b) = 0$. Substitution of (A.23) into (A.24) yields

$$\beta b_i^{\alpha+1} e_i^{\beta-1} = \alpha(b_i^{\alpha} e_i^{\beta} + d_i), \tag{A.27}$$

which with (A.23) implies that

$$\alpha^2 d_i(v - b_i) = \beta b_i^{\alpha+2} e_i^{\beta-1}. \tag{A.28}$$

The latter gives that $e_i = t_i(b_i)$, where the continuous auxiliary function $t_i(b)$ is defined as

$$t_i(b) = \left(\frac{\alpha^2 d_i}{\beta}(v-b)b^{-(\alpha+2)}\right)^{\frac{1}{\beta-1}}, \tag{A.29}$$

for $0 < b < \alpha v/(1+\alpha)$. Since $\beta < 1$, $t_i(b)$ is strictly increasing in b, and $\lim_{b\downarrow 0} t_i(b) = 0$. As a result, the functions $s_i(b)$ and $t_i(b)$ have a unique point of intersection. By implication, this unique point of intersection is given by $b = \hat{b}$. It follows that for player i there exist a unique bid $0 < b_i < \alpha v/(1+\alpha)$ and a unique effort $e_i > 0$ which satisfy (A.23) and (A.24), i.e. $b_i = \hat{b}$ and $e_i = \hat{e}_i$.

Second, take the case $\beta = 1$. It then follows from (A.23) and (A.24) that

$$e_i = \frac{d_i(\alpha v - (\alpha+1)b_i)}{b_i^{\alpha+1}} \tag{A.30}$$

and

$$\alpha^2 d_i(v - b_i) = b_i^{\alpha+2}. \tag{A.31}$$

It is easy to verify that $b_i = \hat{b}$ is the unique solution of (A.31). In turn, we can conclude that for player i there exist a unique $0 < b_i < \alpha v/(1+\alpha)$ and a unique effort $e_i > 0$ which satisfy (A.23) and (A.24), i.e. $b_i = \hat{b}$ and $e_i = \hat{e}_i$. ∎

Notes

[1]For recent surveys of this literature, see e.g. Wolfstetter (1996) or Klemperer (1999).

[2]See further e.g. Dixit (1987), Hillman and Riley (1989), and for a concise survey, Nitzan (1994). Lockard and Tullock (2001) contains a comprehensive collection of articles on rent seeking.

[3]In a recent hostile takeover battle, the British telephone company Vodafone bid some 132 billion euro to obtain control of its German rival Mannesmann. Reportedly, both firms set aside a total amount of 850 million euro for this fight, trying to influence the voting behavior of shareholders. From this amount, 150 million was reserved for advertising. See The Economist (2000).

[4]As long as at least one player both submits a positive bid and exerts a positive effort. We assume that the contest is cancelled, i.e. the prize is not awarded at all, if none of the players both submits a positive bid and exerts a positive effort.

[5]Nti (1999) proposes the following way to study how the extent of asymmetry in valuation influences total rent dissipation. Without loss of generality, assume again that $v_1 \geq v_2$, and write $v_2 = \lambda v_1$, with $\lambda \leq 1$. We then have

$$D = \frac{2}{3} v_1 \left(1 + \lambda - \sqrt{1 - \lambda + \lambda^2}\right).$$

Observe that $\partial D / \partial \lambda > 0$. Thus, the more equal valuations are (i.e. the higher λ is), the higher total rent dissipation. Yet, this analysis is in terms of a fixed v_1. More equal valuations then imply a higher v_2, while keeping v_1 fixed. In this analysis, increased rent dissipation is not so much due to lower asymmetry, but rather to a higher v_2. This can be seen as follows. Rather than writing $v_2 = \lambda v_1$, we can also write $v_1 = \mu v_2$, with $\mu \geq 1$. We then have

$$D = \frac{2}{3} v_2 \left(1 + \mu - \sqrt{1 - \mu + \mu^2}\right).$$

Now, $\partial D / \partial \mu > 0$. Thus, this suggests that having more equal valuations (i.e. lower μ) leads to *lower* dissipation, since we now do the analysis in terms of a fixed v_2 rather than a fixed v_1. We prefer our own approach, as it circumvents the above scale effects and leads to unambiguous results.

[6]In the special case where $\beta = 1$, bids are again equal among agents, regardless of the size of α. This follows from (A.19) of the Appendix. The results given in section 3 can easily be generalized to this special case. However, if $\beta \neq 1$, bids are no longer equal among agents.

References

Baye, M., D. Kovenock, and C. de Vries (1993), "Rigging the lobbying process: An application of all-pay auctions", *American Economic Review*, **83**, 155–172.

Dixit, A. (1987), "Strategic behavior in contests", *American Economic Review*, **77**, 891–898.

Ellingsen, T. (1991), "Strategic buyers and the social cost of monopoly", *American Economic Review*, **81**, 648–657.

Hillman, A.L. and J.G. Riley (1989), "Politically contestable rents and transfers", *Economics and Politics*, **1**, 17–37.

Klemperer, P. (1999), "Auction theory: A guide to the literature", *Journal of Economic Surveys*, **13**, 227–286.

Kooreman, P. and L. Schoonbeek (1997), "The specification of the probability functions in Tullock's rent-seeking contest", *Economics Letters*, **56**, 59–61.

Lockard, A. and G. Tullock, editors (2001), *Efficient rent-seeking: chronicle of an intellectual quagmire*, Kluwer.

Nitzan, S. (1994), "Modelling rent-seeking contests", *European Journal of Political Economy*, **10**, 41–60.

Nti, K.O. (1999), "Rent-seeking with asymmetric valuations", *Public Choice*, **98**, 415–430.

Skaperdas, S. (1996), "Contest success functions", *Economic Theory*, **7**, 283–290.

The Economist (2000), "Vodafone and Mannsmann - endgame", *January 15*.

Tullock, G. (1980), "Efficient rent seeking", in J.M. Buchanan, R. Tollison, and G. Tullock, editors, *Toward a Theory of the Rent Seeking Society*, Texas A&M Press, 224–232.

Wolfstetter, E. (1996), "Auctions: An introduction", *Journal of Economic Surveys*, **10**, 367–420.