Price discrimination and price sensitivity in the car market

working paper, comment welcome

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Abstract

The model in Verboven (2002) is extended to include non-zero price elasticities and behavior in the fuel market is modelled explicitly. With the use of simulations it is shown, that this makes quite a difference and, therefore, might lead to bias in parameter estimates

1 Introduction

This paper integrates two approaches of modelling the purchase of a durable. The first approach has been used to model the purchase of a specific durable; the automobile. It assumes, that consumers have different tastes for quality. Quality can be one dimensional (Bresnahan, 1987), which means that a certain composite of product characteristics is used to indicate quality. Alternatively, quality can be multi-dimensional (Berry, 1994; Berry et.al. 1995; Feenstra et.al. 1995). A consumer with a certain taste for quality chooses the car, which best matches his preferences given the price of the car. On the supply side producers maximize profits by setting prices. The variable costs of the car, i.e. the cost of driving one kilometer, is part of the taste for quality. People with a tendency to drive more kilometers are said to have a strong taste for the component of quality indicating fuel

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cars. In the Netherlands sales tax on diesel cars is considerably higher than sales tax on gasoline cars. There is a lump-sum tax difference of 1868 euros compared to an average price difference of 2395 euros, but this still leaves a sizeable price difference of 517 euros. Verboven (2002) finds a similar difference in price, but there are no noteworthy lump-sum tax differences. A follow-up to this paper will investigate whether markups on diesel cars exceed the markups on gasoline cars in the Dutch market. Therefore I will at times point out the peculiarities of the Dutch car market.

In the presence of non-zero price elasticities of demand for kilometers, the behavior of fuel producers can also be modelled. If prices had no effect on demand, the optimal strategy for producers is to set prices as high as possible, which is not realistic and will always be rejected by the data. In this paper I will just show how fuel producer behavior could be modelled.

I will start by giving an outline of the game-theoretical framework underlying the model in Section 2, then the model will be discussed in detail in Section 3. With the aid of simulation I will show the possible effect of this extended model in Section 4. Finally, a conclusion is given in Section 5.

2 Outline of the model

In this section an outline of the model of the automobile market is given. In the next section it will be discussed in detail. An automobile is characterized by its model (indexed by j) and its engine (indexed by k). There are 3 groups of actors; consumers, producers of cars and producers of fuel (i.e. diesel and gasoline).

It is assumed that consumers have already decided, which model they want to purchase. Now, they have to choose between a diesel engine and a gasoline engine. For each model car manufacturers usually offer a version with a gasoline engine and a version with a diesel engine. Moreover, the technical performances of these cars do not differ much. The points on which they do differ, which is basically cost, are easily measurable. So, it is assumed that consumers are able to separate the choice of engine. Consumers maximize utility, which has three arguments; the amount of income spent on goods other than cars, mean intrinsic utility of owning model (j, k) and the amount consumers drive (mileage). Consumers know that the cost of driving a diesel car are lower than the cost of driving a gasoline car, but the fixed cost of a diesel car exceed the fixed cost of a gasoline car. Therefore, only consumers who are planning to drive more than a certain amount of kilometers per year will choose a diesel car (see Section 3.1).

Assume that for every car model, there is one producer. Since consumers have already decided which model to choose, producers act as a monopolist. They set the difference in price between the car equipped with a diesel engine and the one equipped with a gasoline engine, such that their profit is maximized. Producers are aware that people who have a high mileage are willing to pay more for a diesel, regardless of the actual (marginal) cost differences. This allows for (third degree) price discrimination. (see Section 3.2).

It is assumed, that all fuel is supplied by one producer, who can act as a monopolist.[1] Profit is maximized by setting the price of diesel and gasoline. (see Section 3.4).

Endogenous variables are: choices made by individual consumers (represented by market share of gasoline cars: the number of gasoline cars of model j sold divided by the number of cars of model j sold), price difference between diesel cars and gasoline cars, the price of diesel and the price of gasoline. It is assumed that all endogenous variables are set simultaneously, which leads to a one-shot pure Nash equilibrium (cf. Section 3.3).

[1]In the Netherlands a cartel might be active in this particular market. This has still not been properly investigated by the Dutch Competition Authority.

3 The model

3.1 The demand for mileage

Consider the following indirect utility function (see Appendix A for the derivation):

$$u_{jk} = y - \rho p^*_{jk} - \tau_{jk} + a_{jk} + \frac{1}{\lambda}\mathrm{e}^{-\lambda\pi_{jk}}\theta^j_0, \tag{1}$$

where j denotes the car model, $k = G, D$ engine type, gasoline or diesel, u_{jk} utility derived from owning and using car model (j, k), y is income, ρ an annualization coefficient (see Appendix B), p^*_{jk} price of car (j, k), including sales tax, τ_{jk} lump sum tax on car (j, k), a_{jk} mean intrinsic utility of consuming (j, k), λ a price sensitivity parameter and π_{jk} fuel cost of driving one kilometer or equivalent measure.

π_{jk} is the marginal cost of driving. It is the product of the efficiency of a car (in liters/km, denoted by w_{jk}) and the price per liter of fuel (denoted by $r_k(1 + t_k)$, where r_k is price minus taxes and t_k an ad valorem tax.)

Consumer heterogeneity is introduced through the θ^j_0-constant, which at $\pi_{jk} = 0$ can be interpreted as the amount of kilometers owner of model j would drive in that particular point. Let $\theta^j_0 \in \Theta_j$. It is assumed, that there exists a function $F_{\Theta,j} : \Theta_j \to [0, 1]$ and $F_{\Theta,j}$ is non-decreasing and differentiable. $F_{\Theta,j}$ is the c.d.f. of θ^j_0. Denote the p.d.f. by $f_{\Theta,j}$. The F-function describes the continuum of consumers on Θ.

From Roy's identity it follows that the demand for mileage given j equals:

$$\theta^j_k = \theta^j_0\mathrm{e}^{-\lambda\pi_{jk}}, \tag{2}$$

where θ^j_k denotes the amount owner of car (j, k) drives given π_{jk}. The price elasticity ϵ is defined as:

$$\epsilon = \frac{\mathrm{d}\theta^j_k}{\mathrm{d}\pi_{jk}}\frac{\pi_{jk}}{\theta^j_k} = -\lambda\theta^j_k\frac{\pi_{jk}}{\theta^j_k} = -\lambda\pi_{jk}. \tag{3}$$

Assume that $\lambda \in [0, 1/\pi_{jk})$. The higher the variable cost (π_{jk}) of car (j, k), the more sensitive the consumer is to small price changes. Since $\pi_{jD} < \pi_{jG}$, by definition, and thus $\theta_D^j > \theta_G^j$, a rise in the variable cost of gasoline cars would have a larger relative effect, but not necessarily a larger absolute effect.

There exists a unique θ_0^j, denoted as θ_*^j, such that $u_{jD} = u_{jG}$. Straightforward manipulation of (1) yields:

$$\theta_*^j = \frac{\Delta a_j - \rho \Delta p_j^* - \Delta \tau_j}{\frac{1}{\lambda}\left[\mathrm{e}^{-\lambda \pi_{jG}} - \mathrm{e}^{-\lambda \pi_{jD}}\right]}, \tag{4}$$

where $\Delta x_j = x_{jD} - x_{jG}$ for a variable x_j. Equation (4) is an extension of Verboven (2002, eq.2) If $\lambda \to 0$, then the denominator converges to $\Delta \pi_j$, as can be verified using the rule of L'Hôpital.

Consumers for which $\theta_0^j < \theta_*^j$ choose a gasoline car. Market share of gasoline cars for model j is the probability, that $\theta_0^j < \theta_*^j$. If this market share is denoted by $s_{G|j}$, then:

$$s_{G|j} = \mathrm{P}(\theta_0^j < \theta_*^j) = F_{\Theta,j}(\theta_*^j) \Leftrightarrow F_{\Theta,j}^{-1}(s_{G|j}) = \theta_*^j. \tag{5}$$

Note, that $s_{G|j} + s_{D|j}$=1, where $s_{D|j}$ is the market share of diesel cars. Interestingly, if $\lambda \to 0$, then (5) can be transformed into a linear relationship by multiplying both sides by $\Delta \pi_j$ (cf. Verboven, 2002, eq. 7).

Let observed mileage demand be denoted by θ^j. If $\lambda \to 0$, then $\theta^j = \theta_0^j$. However if $\lambda > 0$, then:

$$\theta^j = \begin{cases} \theta_G^j & \text{if } \theta_0^j < \theta_*^j \\ \theta_D^j & \text{else} \end{cases} \tag{6}$$

At $\theta^j = \theta_*^j$, it is clear that $\theta_G^j < \theta_D^j$ and the distribution of mileage demand is not continuous in this particular point.

3.2 The price of cars

Every firm produces one model, j, and equips this model with either a gasoline engine or a diesel engine. By setting different prices the firm can influence $s_{G|j}$ and maximize its profits, which are given by:

$$(p_{jG} - c_{jG})s_{G|j} + (p_{jD} - c_{jD})(1 - s_{G|j}), \tag{7}$$

A Derivation of the indirect utility function

Consumers receive extra utility when they drive more. This can be incorporated in two equivalent ways: assume a demand function for mileage and derive (indirect) utility from the demand function or assume a utility function dependent on the amount of kilometers consumed and then derive the demand function and the indirect utility.

A.1 Starting point: a utility function

Let (j,k) be given, then the utility of the consumer depends on the expenditure on other goods (z) as a function of θ_k^j and the utility received from consuming θ_k^j. If $y^* = y - \rho p_{jk}^* - \tau_{jk}$, then $z = y^* - \pi_{jk}\theta_k^j$. The utility received from consuming θ_k^j is $f(\theta_k^j)$, where $f > 0$, $f' > 0$ and $f'' < 0$. Utility is given by:

$$\bar{u}_{jk}(\theta_k^j) = y^* - \pi_{jk}\theta_k^j + a_{jk} + f(\theta_k^j). \tag{34}$$

Maximize with respect to θ_k^j:

$$\frac{\mathrm{d}\bar{u}_{jk}(\theta_k^j)}{\mathrm{d}\theta_k^j} = -\pi_{jk} + f'(\theta_k^j) = 0 \Longrightarrow f'(\theta_k^j) = \pi_{jk}. \tag{35}$$

From (35) follows the demand function of θ_k^j. Substituting the demand function in (34) gives the indirect utility.

A.2 Starting point: a demand function

Suppose the demand function of θ_k^j is given by:

$$\theta_k^j = \theta_0^j e^{-\lambda\pi_{jk}}, \tag{36}$$

where θ_0^j and λ are parameters. Note that if the variable cost are zero, the consumer will drive θ_0^j kilometers. So, θ_0^j is the maximum amount of θ_k^j the consumer will 'purchase'. Substituting (36) in (34) gives the indirect utility, but only if the function f is chosen in such a way that the demand function that follows from (35) is the same as the demand function given in (36).

A.3 Equivalence

From (36) follows:

$$\pi_{jk} = \frac{\log\theta_k^j - \log\theta_0^j}{-\lambda} = -\frac{1}{\lambda}[\log\theta_k^j - \log\theta_0^j]. \tag{37}$$

Substituting this into (35) gives:

$$f'(\theta_k^j) = -\frac{1}{\lambda}[\log\theta_k^j - \log\theta_0^j]. \tag{38}$$

Integrating over θ_k^j:

$$f(\theta_k^j) = -\frac{1}{\lambda}\int[\log\theta_k^j - \log\theta_0^j]\mathrm{d}\theta_k^j = -\frac{1}{\lambda}[\theta_k^j\log\theta_k^j - \theta_k^j - \theta_k^j\log\theta_0^j]. \tag{39}$$

After some rearranging the following is obtained:

$$f(\theta_k^j) = -\frac{\theta_k^j}{\lambda}[\log\left(\frac{\theta_k^j}{\theta_0^j}\right) - 1]. \tag{40}$$

From $\theta_k^j/\theta_0^j \leq 1$ it follows that $f > 0$. Note that $f'(\theta_k^j) = (-1/\lambda)[\log\theta_k^j - \log\theta_0^j] > 0$ and $f''(\theta_k^j) = -1/(\lambda\theta_k^j) < 0$. Substitute (36) and (40) in (34) to obtain the indirect utility function given in (1).

I would like to end by making some remarks about the function f:

1. $f(0) = -\infty$
2. $f(\theta_0^j\mathrm{e}) = 0$
3. $f'(\theta_0^j) = 0$ and $f(\theta_0^j) = \theta_0^j/\lambda > 0$.

This implies, that $\forall\theta \in [0, \theta_0^j]$ $f'(\theta) \geq 0$ and $\forall\theta \in (\theta_0^j, \theta_0^j\mathrm{e}]$ $f'(\theta) < 0$ Since $f'(\theta_k^j) = \pi_{jk}$ and π_{jk} is finite and non-negative, $\theta_k^j \in (0, \theta_0^j]$.

B Derivation of the annualization coefficient

One of the first papers using this concept is Hausman (1979). Instead of referring to this paper I would rather give the derivation, basically because Hausman (1979) does not.

efficiency. It should, however, be part of the price of owning and driving a car.

An example of such an approach is Verboven (2002), a study much in the vein of Berry (1994) and Berry et.al. (1995). It focuses on the choice of engine in an automobile. The main factor determining this choice is the amount of kilometers a person drives. If a person drives a lot the high price of purchasing a diesel car is offset by its lower cost of driving one kilometer. Instead of assuming that people who intensively use a car have strong taste for the fuel efficiency component of quality, the choice of engine can depend on evaluation of the cost structure difference.

Unfortunately in Verboven (2002), the amount of kilometers demanded is fixed (but varies over persons) or, put differently, the (short term) price elasticity of the demand for kilometers is equal to zero. Most empirical studies have found small but significant price elasticities. These estimates are usually between 0 and -0.5. See Goodwin (1992) for a review.

The second approach are studies, like Hausman (1979) and Dubin et.al. (1984), which provide a useful alternative for examining the purchase and use of a durable. Although both articles are about electric appliances, the basic structure of the problem is the same and in most respects quite similar to Verboven (2002). The difference is that Hausman (1979) and Dubin et.al. (1984) use a utility function, in which extra utility is derived from the usage of the durable. It is acknowledged that the decision to buy a certain brand of durable depends on the cost of use and since these cost may vary over brands, the intensity of use may differ over brands and this leads to non-zero price-elasticities. Verboven (2002) argues that these elasticities are so small that their effect is negligible. It is investigated whether this is the case.

Verboven (2002) showed that for the Belgian, French and Italian car market, observed price differences between cars with diesel engines and cars with gasoline engines are mainly due to a higher mark-up on diesel where p_{jk} is the price of car model (j,k) before taxes and c_{jk} the constant marginal cost of this model. Let $p^*_{jk} = p_{jk}(1+b) + \beta_k$, where b denotes value added taxes on car (j,k) and β_k a lump sum tax or subsidy.

It is clear from (4), that consumers care only about the difference in sales prices. Since there is a linear relationship between Δp^*_j (sales price) and Δp_j (sales price minus tax), this implies that instead of setting p_D and p_G, producers can set Δp_j. This follows also from the non-existence of an outside opportunity (i.e. not purchasing a car, but an alternative mode of transport). There is no need to model an outside opportunity, because we are only interested in the difference in price.

To find the FOC we differentiate (7) with respect to p_{jD} to obtain:

$$(p_{jG} - c_{jG})\frac{\partial s_{G|j}}{\partial p_{jD}} + (1 - s_{G|j}) - (p_{jD} - c_{jD})\frac{\partial s_{G|j}}{\partial p_{jD}} = 0. \tag{8}$$

Substituting

$$\frac{\partial s_{G|j}}{\partial p_{jD}} = \frac{\partial s_{G|j}}{\partial \theta^j_*} \times \frac{\partial \theta^j_*}{\partial p^*_{jD}} \times \frac{\partial p^*_{jD}}{\partial p_{jD}} = f_{\Theta,j}(\theta^j_*)\frac{-\rho(1+b)}{\frac{1}{\lambda}[\mathrm{e}^{-\lambda\pi_{jG}} - \mathrm{e}^{-\lambda\pi_{jD}}]}$$

in (8) yields (cf. Verboven, 2002, eq.8):

$$\Delta p_j = \Delta c_j - \frac{1 - F_{\Theta,j}(\theta^j_*)}{f_{\Theta,j}(\theta^j_*)} \times \frac{\mathrm{e}^{-\lambda\pi_{jG}} - \mathrm{e}^{-\lambda\pi_{jD}}}{\rho\lambda(1+b)}. \tag{9}$$

The difference in price can be split in two parts; difference in marginal cost and difference in mark-up. It is widely believed, that diesel cars are more costly to produce. However, Verboven (2002) shows that for the Belgian, French and Italian market just 16% of price differences can be explained from cost differences.

While the model does not exclude that $\Delta c_j < 0$, markups are positive by construction. This follows from the observed fact, that $\pi_{jG} > \pi_{jD}$. If consumers discount future cash flows heavily, then ρ is small and, therefore, markups are high. This is because consumers are less sensitive to price changes. If the value added tax (b) increases, then markups decrease. This can be seen as a typical trait of a monopolistic market, where the burden of additional taxes are worn by both consumers and producers. The roles of the p.d.f. and c.d.f of θ are less clear. For instance, if the value of F is close to one, then the value of f will be close to zero. The ratio of $(1-F)/f$ can be anything. If θ has an exponential distribution, then the ratio is equal to one. It is also hard to pinpoint the economic interpretation of $(1-F)/f$. A possible explanation is that it represent how sensitive to car price differences consumers are given the market share.

There are two obvious extensions:competition between models and determination of car model characteristics.

A car producer may acknowledge the fact that by setting his prices, he may lose or win customers to other models. Then one must drop the assumption, that model choice is given. This implies that consumer preferences must be modelled more explicitly. In this paper we have been able to mostly avoid the problem of measuring consumer preferences, because the source of heterogeneity in this paper are not preferences but the demand for mileage. See Bresnahan (1987) and Berry (1994) for examples of these extensions.

Differences in technological performance are captured by the Δa_j-variable. It is assumed, that this is a constant, but you could let producers increase or decrease the difference in performance to maximize profit. It is reasonable that Δa_j is constant in the short run, because most manufactures leave a model unchanged for two to four years.

3.3 Solution

Summarizing: if mileage demand is not constant the following model is obtained:

$$F_{X,j}^{-1}(s_{G|j}) = \theta_*^j. \tag{10}$$

$$\Delta p_j = \Delta c_j - \frac{1 - F_{\Theta,j}(\theta_*^j)}{f_{\Theta,j}(\theta_*^j)} \times \frac{\mathrm{e}^{-\lambda \pi_{jG}} - \mathrm{e}^{-\lambda \pi_{jD}}}{\rho \lambda (1+b)}. \tag{11}$$

If θ_0^j is uniformly distributed on the interval $[0, \theta_m^j]$ then the following solution is obtained analytically (see Appendix C):

$$s_{G|j} = \frac{1}{2} + \frac{1}{2}\frac{\Delta a_j - \rho(1+b)\Delta c_j - \rho\Delta\beta - \Delta\tau_j}{\frac{1}{\lambda}[\mathrm{e}^{-\lambda\pi_{jG}} - \mathrm{e}^{-\lambda\pi_{jD}}]}\frac{1}{\theta_m^j}, \tag{12}$$

$$\begin{aligned}\Delta p_j &= \Delta c_j \\ &+ \tfrac{1}{2}\tfrac{\Delta a_j - \rho(1+b)\Delta c_j - \rho\Delta\beta - \Delta\tau_j - \frac{1}{\lambda}[\mathrm{e}^{-\lambda\pi_{jG}} - \mathrm{e}^{-\lambda\pi_{jD}}]\theta_m^j}{\rho(1+b)}.\end{aligned} \tag{13}$$

Note that since $\frac{1}{\lambda}[\mathrm{e}^{-\lambda\pi_{jG}} - \mathrm{e}^{-\lambda\pi_{jD}}]$ is negative by assumption, one of the conditions for θ_*^j is in the interior of $[0, \theta_m^j]$ is that $\Delta a_j - \rho(1+b)\Delta p_j - \rho\Delta\beta - \Delta\tau_j < 0$ (cf. 4). If θ_*^j can be any point in the interior of $[0, \theta_m^j]$, then $s_{G|j} \in (0,1)$. From (11) it follows, that $\Delta p_j > \Delta c_j$. Therefore $\Delta a_j - \rho(1+b)\Delta c_j - \rho\Delta\beta - \Delta\tau_j$ can have any sign. If it is positive, then $s_{G|j} < 1/2$ and else it is larger than $1/2$.

The second part of (13) is a different way to express the markup, which is positive, because $\frac{1}{\lambda}[\mathrm{e}^{-\lambda\pi_{jG}} - \mathrm{e}^{-\lambda\pi_{jD}}]$ is negative. Since there are no prior assumptions on Δc_j, theoretically the sign of Δp_j is undetermined. However, we know from practice, that $\Delta p_j \gg 0$. Therefore, it is believed that either Δc_j and the mark-up have the same positive sign or, $\Delta c_j < 0$ and the absolute value of the marginal cost difference is much smaller than the positive mark-up.

3.4 Endogenizing fuel prices

This is a simplified example of how fuel prices could be endogenized. It is simplified not because the algebra is that difficult, but this approach yields clearer expressions. It, therefore, serves as a starting point for further analysis.

The average value of θ_0^j given that $\theta_0^j \geq \theta_*^j$ is:

$$\mathrm{E}[\theta_0^j | \theta_0^j \geq \theta_*^j] = \frac{\int_{z \geq \theta_*^j} z f_{\Theta,j}(z)\mathrm{d}z}{1 - F_{\Theta,j}(\theta_*^j)}. \tag{14}$$

The average diesel car owner will drive:

$$\mathrm{E}\theta_D^j = \mathrm{E}[\theta_0^j | \theta_0^j \geq \theta_*^j]\mathrm{e}^{-\lambda\pi_{jD}}, \tag{15}$$

and will demand $\mathrm{E}\theta^j_D w_{jk}$ liters of fuel. Multiplying this with the market share of diesel cars we obtain the total demand for diesel. The demand for gasoline can be derived in an analogous way and is given by the following equation:

$$\mathrm{E}\theta^j_G = \mathrm{E}[\theta^j_0 | \theta^j_0 \leq \theta^j_*] \mathrm{e}^{-\lambda \pi_{jG}}, \tag{16}$$

where

$$\mathrm{E}[\theta^j_0 | \theta^j_0 \leq \theta^j_*] = \frac{\int_{z \leq \theta^j_*} z f_{\Theta,j}(z) \mathrm{d}z}{F_{\Theta,j}(\theta^j_*)}. \tag{17}$$

As noted before, $\pi_{jk} = r_k(1 + t_k)w_{jk}$. Note that we split fuel price in a price component set by firms and a tax component set by the government. Also note that if the price of diesel is increased *ceteris paribus* two effects occur. The market share of diesel cars falls and those who remain drive less. These are the short term effects. For the sake of simplicity, we assume that the fuel producer takes market shares as given. In the long term w_{jk} cannot be assumed to be constant. For instance, after the oil crisis in the 1970s, more fuel efficient cars were produced, at least in Europe, but this happened with a lag of a decade.

There is only one firm in the fuel market.[2] Let the marginal cost of fuel k be d_k, then the firm maximizes:

$$\max_{r_k} \sum_{k=G,D} (r_k - d_k) \sum_j \frac{\mathrm{E}\theta^j_k}{w_{jk}} s_{k|j}, \tag{18}$$

where $s_{G|j} = F_{\Theta,j}(\theta^j_*)$ and $s_{D|j} = 1 - s_{G|j}$. Note that it is not sufficient to calculate the difference in prices, because unlike car prices consumer do care about the height of both diesel and gasoline price (cf.4). Since we assume that the firm takes market shares as given, it follows that:

$$\frac{\partial \mathrm{E}\theta^j_k}{\partial r_{k^*}} = 0 \quad k^* \neq k \tag{19}$$

[2] In view of the Dutch market for fuel, it maybe would be more appropriate to think of a Stackelberg leader in prices. Shell, the largest seller of fuel to petrol stations in the Netherlands, advices these petrol stations on sales price. Although owners are free to set prices, in practice this is the sales price. The petrol station, which are not supplied by Shell, can take this price as given and set their own price.

and

$$\frac{\partial \mathrm{E}\theta_k^j}{\partial r_k} = -\lambda w_{jk}(1+t_k)\mathrm{E}\theta_k^j. \tag{20}$$

The first order conditions are:

$$\sum_j \frac{\mathrm{E}\theta_G^j}{w_{jG}} s_{G|j} + (r_G - d_G) \sum_j \frac{\partial \mathrm{E}\theta_G^j}{\partial r_G} \frac{s_{G|j}}{w_{jG}} = 0 \tag{21}$$

$$\sum_j \frac{\mathrm{E}\theta_D^j}{w_{jD}} (1 - s_{G|j}) + (r_D - d_D) \sum_j \frac{\partial \mathrm{E}\theta_D^j}{\partial r_D} \frac{1 - s_{G|j}}{w_{jD}} = 0 \tag{22}$$

Substituting (20) for $\partial \mathrm{E}\theta_k^j / \partial r_k$ and solving for r_k this leads to:

$$r_G = d_G + \frac{\sum_j \mathrm{E}\theta_G^j \frac{s_{G|j}}{w_{jG}}}{\lambda(1+t_G)\sum_j \mathrm{E}\theta_G^j s_{G|j}} \tag{23}$$

and

$$r_D = d_D + \frac{\sum_j \mathrm{E}\theta_D^j \frac{(1-s_{G|j})}{w_{jD}}}{\lambda(1+t_D)\sum_j \mathrm{E}\theta_D^j (1-s_{G|j})}. \tag{24}$$

These have the same structure as (11). The second term on the RHS is a markup. A rise in markup can have three reasons: more efficient cars, less excise and/or VAT and less price sensitive consumers. If cars are more efficient or there is less excise, then the price of driving one kilometer decreases and this enables fuel producers to capture at least part of this effect. It is well known that price sensitivity and markups are inversely related.

If there is just one car model we get the trivial solution: $r_k = d_k + (1/\lambda(1+t_k)w_{jk})$

4 Simulation

In order to show the effects of introducing a price sensitivity parameter in the Verboven (2002) model, I have done some simulation on the model described in equations 10 and 11. In this simulation fuel producer behavior is exogenous and we treat fuel prices as given. Parameter values are taken from Verboven (2002, Table 5). Demand and pricing were estimated jointly and car producers behave monopolistically. The hypothetical average car model is sold in Belgium and is not produced in France, Germany or Italy. It has average horsepower, displacement and weight. These numbers are taken from Verboven (2002, Table 1 and 2).

Mean intrinsic utility is a linear combination of a constant, horsepower, displacement and weight. The same applies to marginal cost. The parameter values are shown in Table 1. The value of ρ implies that given an average lifetime of the car of 10 years consumers have an implicit interest rate of 6.8% and if the average lifetime of the car is 15 years an implicit interest rate of 12.2%. Note that the marginal cost difference is negative but small compared to an average price difference of 2000 euros.

Note, that we observe θ^j, which is either θ^j_D or θ^j_G. If λ, the price sensitivity parameter, equals zero, then $\theta^j = \theta^j_0$. However if $\lambda > 0$, then $\theta^j_k = \mathrm{e}^{-\lambda\pi_{jk}}\theta^j_0$. Recall that

$$\theta^j = \begin{cases} \theta^j_G & \text{if } \theta^j_0 < \theta^j_* \\ \theta^j_D & \text{else} \end{cases} \tag{25}$$

This means that the expected value of θ^j is defined as:

$$\mathrm{E}\theta^j = \mathrm{P}(\theta^j = \theta^j_G)\mathrm{E}\theta^j_G + \mathrm{P}(\theta^j = \theta^j_D)\mathrm{E}\theta^j_D. \tag{26}$$

The probability that $\theta^j = \theta^j_G$ is the probability that $\theta^j_0 \leq \theta^j_*$. The probability that $\theta^j = \theta^j_D$ is the probability that $\theta^j_0 > \theta^j_*$. This leads to:

$$\mathrm{E}\theta^j = \mathrm{P}(\theta^j_0 \leq \theta^j_*)\mathrm{E}\theta^j_G + \mathrm{P}(\theta^j_0 > \theta^j_*)\mathrm{E}\theta^j_D. \tag{27}$$

Of course the probability that $\theta^j_0 \leq \theta^j_*$ is the market share of gasoline engines ($s_{G|j}$):

$$\mathrm{E}\theta^j = s_{G|j}\mathrm{E}\theta^j_G + (1 - s_{G|j})\mathrm{E}\theta^j_D. \tag{28}$$

The expected value of θ^j_k equals $\mathrm{e}^{-\lambda\pi_{jk}}\mathrm{E}\theta^j_0$. After some rearranging this results in:

$$\mathrm{E}\theta^j_0 = \frac{\mathrm{E}\theta^j}{s_{G|j}\mathrm{e}^{-\lambda\pi_{jG}} + (1 - s_{G|j})\mathrm{e}^{-\lambda\pi_{jD}}}. \tag{29}$$

Table 1: Parameter values based on Verboven (2002)

Δa_j	$-98,722$
ρ	$0,132$
τ_D	285.9
τ_G	173.7
π_{jD}	$0,043076$
π_{jG}	$0,070384$
Δc_j	$-31,29$

Table 2: Description of the 4 scenarios

(1)	(2)	(3)	(4)
1	0	0	0
2	0.1	0.06	1.42
3	0.2	0.12	2.84
4	0.5	0.31	7.10

Note: In column (1) are the scenario numbers, in column (2) the absolute price elasticity of gasoline, in column (3) the implied absolute price elasticity of diesel and in column (4) the implied value of λ

If the expected value of θ^j is known, then the expected value of θ_0^j can be calculated from the equation above. For now assume that $\mathrm{E}\theta^j = 18257$ and $\theta_0^j \sim \mathrm{U}(0, 2\mathrm{E}\theta_0^j)$.

I consider four scenarios based on four different price elasticities of gasoline. These correspond to values of λ and the price elasticity of diesel. See Table 2 for the values. Scenario 1 is the model as estimated in Verboven (2002). Scenario's 2 and 3 are based on price elasticities as found in the literature (see Goodwin (1992) for an overview). Scenario 4 is based on a price-elasticity, which is larger than found in empirical studies. The scenarios stay the same throughout this section.

Equilibrium is calculated and shown in Table 3. The outcomes are reasonably close to observed differences in the Belgian car market. Price difference and market share of gasoline cars are both a bit too high. The effect of introducing non-zero price sensitivities are minimal. The percentage change in both market share and price relative to scenario 1 is less than one percent for all scenario's.

Table 3: Outcomes, when θ_0^j has a uniform distribution

Scenario	Market share	Price	Demand for diesel	Demand for gasoline
1	0.6037	2962	22043	22043
2	0.6033	2977	22546	21688
3	0.6029	2991	23052	21331
4	0.6020	3026	24590	20254

Note: Market share is the market share of gasoline cars, price is the price difference between diesel and gasoline cars in US dollars and the demand for diesel and gasoline is the demand at the threshold in kilometers.

Table 4: Outcomes, when θ_0^j has a double exponential distribution

Scenario	Market share	Price	Demand for diesel	Demand for gasoline
1	0.6153	2461	19617	19617
2	0.5922	2316	19288	18554
3	0.5679	2185	19003	17584
4	0.4914	1857	18371	15131

Note: Market share is the market share of gasoline cars, price is the price difference between diesel and gasoline cars in US dollars and the demand for diesel and gasoline is the demand at the threshold in kilometers.

Verboven (2002) uses the double exponential distribution:

$$F(\theta) = \exp(-\exp(-(\theta - \mu)\frac{\pi}{\sqrt{6}\sigma} - \gamma)), \tag{30}$$

where μ is the mean, σ the standard deviation and $\gamma = 0.57721566$.[3] Take $\mu = \mathrm{E}\theta_0^j$ and $\sigma = 12000$ and calculate the equilibrium for each of the scenario's. See Table 4.

As we see, the differences between scenario 1 and scenario 2 and 3 are much larger when using the double exponential distribution, probably due to a much steeper distribution. The relative changes are captured in Table 5. For the likely scenario's 2 and 3 these changes are between 4 to 11 percent and for scenario 4 they are above 20%. This indicates, that ignoring price sensitivity might have quite some implications.

There are two taxes in the model; fuel tax and annual car tax. So far, the problem has been analyzed in terms of differences between cars with

[3]This is Euler's constant, a well known constant in number theory. It is defined as $\gamma = \lim_{n\to\infty}(\sum_{i=1}^{n} 1/i - \ln(n))$.

Table 5: Percentage change relative to scenario 1

Scenario	Market share	Price
2	4	6
3	7	11
4	20	25

Table 6: Elasticities of the demand of gasoline cars

Scenario	ϵ_j^F	ϵ_j^C
1	-1.203	-0.618
2	-1.385 (15%)	-0.743 (20%)
3	-1.550 (29%)	-0.810 (31%)
4	-2.035 (69%)	-1.099 (78%)

Note: Behind the price elasticities in parentheses are the percentage difference with scenario 1.

gasoline engines and cars with diesel engines. This will also be the way in which changes in taxes will be analyzed. For both fuel tax and annual car tax, the tax on gasoline will be increased by a half percent and the tax on diesel will be decreased by a half percent. The difference will, more or less, increase by one percent. Therefore the percentage change in market share can be interpreted as the price elasticity of the demand of gasoline cars. Formally, I computed:

$$\epsilon_j^F = \frac{1}{2}\frac{\mathrm{d}s_{G|j}}{\mathrm{d}\pi_{jG}}\frac{\pi_{jG}}{s_{G|j}} - \frac{1}{2}\frac{\mathrm{d}s_{G|j}}{\mathrm{d}\pi_{jD}}\frac{\pi_{jD}}{s_{G|j}} \tag{31}$$

and

$$\epsilon_j^C = \frac{1}{2}\frac{\mathrm{d}s_{G|j}}{\mathrm{d}\tau_{jG}}\frac{\tau_{jG}}{s_{G|j}} - \frac{1}{2}\frac{\mathrm{d}s_{G|j}}{\mathrm{d}\tau_{jD}}\frac{\tau_{jD}{}'}{s_{G|j}}. \tag{32}$$

Note, that when calculating ϵ_j^F I change the cost of driving one mile (π_{jk}) by a half percent instead of changing tax by a half percent. As long as fuel producer behavior is not modelled there is no difference. The results are shown in Table 6. The difference between scenario 1 and the other scenario's are even larger in the elasticities than they were in market share and price.

Define government revenue of annual car tax as $R = s_{G|j}\tau_{jG} + (1 - s_{jG})\tau_{jD}$ and define the elasticity of government revenue with respect to

Table 7: Elasticities of government revenue

Scenario	ϵ_j^R
1	0.947
2	0.938
3	0.949
4	0.931

annual car tax as:

$$\epsilon_j^R = \frac{1}{2}\frac{\mathrm{d}R}{\mathrm{d}\tau_{jG}}\frac{\tau_{jG}}{R} - \frac{1}{2}\frac{\mathrm{d}R}{\mathrm{d}\tau_{jD}}\frac{\tau_{jD}}{R}. \tag{33}$$

The results are shown in Table 7. As can be seen almost all the effects cancel out. It is no surprise that this elasticity is almost equal to one since it is assumed, implicitly, that everybody buys a car, despite the cost attached to this.

5 Concluding remarks

In this paper the model in Verboven (2002) has been extended to include non-zero price elasticities and the fuel market has been modelled explicitly. The effects of the first extension have been examined with simulation. They have shown, that an error of 5-10% can easily occur if zero price elasticity is assumed.

The next task is to estimate the model presented in this paper using data for the Dutch and (probably) the German market. Because each market, however it is defined, just gives us 100-150 observations (i.e. j, the number of models on sale during a particular period) empirical research requires the combining of several regional or national markets.

Another possibility is to rearrange the model presented in this paper to fit existing panel data. Instead of looking at the aggregate of all choices, one could look at each individual choice. This expands the number of observations to the number of members of the panel. Since most studies focus on mobility, i.e. how, why and how much people drive, detailed information on the mode of transport, as is needed in the kind of models discussed in this paper, is unavailable and therefore it is almost impossible to include producer behavior.

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Assume that a product has a price of 1 euro and lasts for T years, therefore a product originally purchased at $t = 0$, where $t \in [0, 1, 2, \ldots]$ denotes time, has to be replaced at $t = T, 2T, 3T, \ldots$ Let $r > 0$ denote the implicit interest rate, then the present value (denoted by PV) of the payment equals:

$$\mathrm{PV} = 1 + \frac{1}{(1+r)^T} + \frac{1}{(1+r)^{2T}} + \ldots \tag{41}$$

Since $1/(1+r)^T < 1$, it follows that

$$\mathrm{PV} = \frac{1}{1-(1+r)^{-T}} \tag{42}$$

Let ρ be a payment made at $t = 0, 1, \ldots,$ such that the present value of these payments equal PV. This leads to:

$$\rho + \frac{\rho}{1+r} + \frac{\rho}{(1+r)^2} + \ldots = \frac{1}{1-(1+r)^{-T}} \Leftrightarrow \tag{43}$$

$$\rho\left[\frac{1}{1-(1+r)^{-1}}\right] = \frac{1}{1-(1+r)^{-T}} \Leftrightarrow \tag{44}$$

$$\rho = \frac{r}{1+r} \times \frac{1}{1-(1+r)^{-T}}. \tag{45}$$

A person, who has an implicit interest rate of r and knows that a product is going to last T years is indifferent between paying one euro every T years or paying ρ euro every year. Since $T \geq 1$, it follows that $\rho \leq 1$. Since $PV > 0$ and $r/(1+r) > 0$, it follows that $\rho > 0$ as well.

C Derivation of the solution of the model with a uniform distribution

Observe, that $s_{G|j} = F_{\Theta,j}(\theta_*^j)$ and $f_{\Theta,j}(\theta_*^j) = 1/\theta_m^j$. Let $\Lambda = \frac{1}{\lambda}[\mathrm{e}^{-\lambda\pi_{jG}} - \mathrm{e}^{-\lambda\pi_{jD}}]$. Substitute above in (11) to obtain:

$$\Delta p_j = \Delta c_j - (1 - s_{G|j})\frac{\Lambda\theta_m^j}{\rho(1+b)}. \tag{46}$$

Substituting above in (10) gives:

$$\theta_m^j s_{G|j} = \frac{\Delta a_j - \rho(1+b)\Delta c_j + (1-s_{G|j})\Lambda\theta_m^j - \rho\Delta\beta - \Delta\tau_j}{\Lambda}. \tag{47}$$

This is equivalent to:

$$\theta_m^j s_{G|j} = \frac{\Delta a_j - \rho(1+b)\Delta c_j - \rho\Delta\beta - \Delta\tau_j}{\Lambda} + (1 - s_{G|j})\theta_m^j. \tag{48}$$

Adding $\theta_m^j s_{G|j}$ to both sides and dividing both sides by $2\theta_m^j$ leads to (12).

Substitute (12) in (11) to obtain:

$$\Delta p_j = \Delta c_j - \left[\frac{1}{2} - \frac{1}{2} \times \frac{\Delta a_j - \rho(1+b)\Delta\beta - \Delta\tau_j}{\Lambda\theta_m^j}\right] \times \left[\frac{\Lambda\theta_m^j}{\rho(1+b)}\right]. \tag{49}$$

After some rearranging (13) is obtained.