

Having more potential raiders weakens the takeover threat

Linda A. Toolsema*

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Abstract

We argue in this paper that a more active market for corporate control may weaken the takeover threat. We show that an increase in the number of potential raiders tends to decrease the probability of a takeover. This in turn weakens managerial incentives. The lower managerial effort level that results in equilibrium negatively affects the ex ante value of the firm.

Keywords: Takeover threats; Managerial incentives; Competition.

JEL classification: G30, G34, L13.

*Department of Economics, University of Groningen, P.O.Box 800, 9700 AV Groningen, The Netherlands. E-mail: L.A.Toolsema@eco.rug.nl. The author thanks Hans van Ees and Marco Haan for useful discussion and comments.

1 Introduction

Takeover threats can function as a corporate governance mechanism, inducing managers to increase their efforts (see e.g. Scharfstein, 1988; Hart, 1995). In this paper, we study the effects of an increase in the number of potential raiders on managerial effort. The term *potential raiders* refers to firms or persons who consider whether or not to actively monitor the firm with the aim to take over if the firm is not managed well. We show that an increase in the number of potential raiders may have adverse effects and reduce managerial effort when monitoring is costly. This in turn lowers the ex ante value of the firm.

In the corporate governance literature, it is often stated that an ‘active market for corporate control’ is required for takeover threats to discipline management. In most cases, it is not explained in detail what this means. It usually refers to a somewhat vague assumption that a firm’s shares can be freely traded, and raiders are able to quickly obtain large amounts of resources and gain control over the firm (e.g. Allen and Gale, 2000, p. 97; see also Shleifer and Vishny, 1997, p. 756). In this paper we consider the effects of a ‘more active’ or larger market for corporate control, in the sense of the *number of actors* on that market. We show that having more potential raiders may attenuate managerial incentives. Note that in some situations the takeover threat itself may decrease managerial effort as compared to the situation without a takeover threat. For example, since a takeover implies that the current manager will be overruled, the takeover threat may reduce the manager’s incentives to exert effort in the first place (Haan and Riyanto, 2002). Also, a takeover may break the implicit contract between workers and managers, inducing workers to invest less in relationship-specific human capital (Shleifer and Summers, 1988). Even so, we focus on the standard case where a takeover threat disciplines management relative to the case without

the threat (i.e. with zero potential raiders).

We focus on outside raiders and do not consider the possibility that a large shareholder takes over the firm. The reason is that we require dispersed ownership (see section 2). Whenever an outside (inside) party takes over and there is a (another) large shareholder, this assumption is violated. Furthermore, a large shareholder may have an incentive not to tender but retain his shares in case of a takeover, in order to obtain his share of the capital gain involved (see also section 2) Therefore, we focus on governance structures characterized by dispersed ownership and our model applies in particular to takeovers in Anglo-Saxon countries (see Shleifer and Vishny, 1997).

Even if a firms' shares are freely traded, the number of potential raiders is likely to be limited. Despite the profit opportunity, not everybody is willing and able to take over a badly managed firm and run it himself. A successful takeover requires more than acquiring sufficient funds and buying the shares. In order to be profitable, the firm needs to be run properly after the takeover. For this reason, a takeover may well require some knowledge of the industry or technology. Empirical evidence shows that acquisitions often concern related firms, e.g. firms in the same line of business (see Bolton and Scharfstein, 1998, pp. 109-110, and the references therein). So, the number of potential raiders might be related to, say, the number of competitors a firm has. Only these competitors may be capable to run the firm after takeover.

Intuitively, a higher number of potential raiders suggests a 'more active' market for corporate control. Thus, with more potential raiders one would expect the takeover threat to become stronger, and the manager to exert more effort. However, we show that this is not necessarily true. If two or more raiders are trying to take over the same firm, they engage in a bidding war. They bid up until the highest bid equals the value of the firm after takeover, and the winner earns zero profits (this is sometimes referred to as

the winner's curse). So, the return from trying to take over the firm depends on how many other raiders are trying to do the same thing. We refer to raiders actually monitoring the firm - with the aim to take over if the firm is not managed well - as *active raiders*.¹ If there is only one active raider, this raider can earn a profit from taking over the firm if it is badly managed. However, if the number of active raiders exceeds one, they all have zero net return. Now suppose there is a fixed cost for a raider to step in, i.e. to actively monitor the firm and acquire private information. Then an increase in the number of potential raiders implies that each individual raider expects to face increased competition when he becomes active, and expects to earn a negative return with an increased probability. This reduces his incentives to step in. Indeed, we show that the probability of zero active raiders *increases* with the number of potential raiders. This reduces the takeover threat that the manager is facing. As a result, in equilibrium, the manager chooses to exert less effort, which decreases the ex ante value of the firm.

The problem that a raider may face competition from other bidders, and a bidding war may result, is well known in the literature (see e.g. Scharfstein, 1988, p. 196; Hart, 1995, pp. 684-685; Allen and Gale, 2000, p. 99). In this paper, we provide a formal analysis of the problem and discuss the effects of the possibility of a bidding war on monitoring by potential raiders, and thereby on managerial effort and the ex ante value of the firm. In the literature, additional raiders are often assumed to be attracted by the initial raider's bid, which indicates to them that the firm is undervalued (Hart, 1995, pp. 684-685). This causes the bidding war. Instead, we assume that each raider must monitor the firm himself in order to obtain the relevant

¹In general, a raider who has monitored the firm does not necessarily want to take over whenever the manager performs badly. For example, in reality monitoring may not yield the required information with probability one. In our setup, we assume that it does, and we refer to an active raider as a raider who monitors the firm and makes a takeover bid whenever the manager shirks.

information privately. That is, even if an uninformed raider observes the initial raider's bid and concludes that the firm must be undervalued, there is no use in making a bid himself. If the uninformed raider were to take over the firm, he has no idea how to make it more profitable. See also Grossman and Hart (1980, p. 58), who argue that ex ante costs of monitoring tend to limit ex post competition between raiders since uninformed raiders cannot effectively compete with the initial raider.

The decision of potential raiders to become active is closely related to the entry decision in a Bertrand market, described by Elberfeld and Wolfstetter (1999). In a Bertrand market, profits drop to zero as soon as two or more entrants enter. Elberfeld and Wolfstetter show how an increase in the number of potential entrants increases the probability of a market breakdown. The coordination game described by Anderson and Engers (2002) is also related. They describe the game presented in the film 'A beautiful mind', where John Nash points out how he and his friends should direct attention to a number of women in a bar. One of the women is a particularly beautiful blonde. Whenever more than one of the friends go for the blonde, none of them will get her, nor will they get any of the other women since they do not like to be second choice. Note that both of these games, as well as the game described in this paper, resemble an all-pay auction: each (active) participant incurs a cost, but only one of them may win the prize. Several other studies use a related framework to describe free riding in the provision of a public good. Mukhopadhaya (2003) uses such a framework to illustrate why larger juries may make poorer decisions. Haan and Kooreman (2003) argue that majorities may lose elections in the presence of voting costs. Johnson (2002) describes open source software development as the provision of a public good and shows that an increase in the number of developers may lead to a smaller probability of development. Finally, Harrington (2001) explains why people are more reluctant to help a person in need when there are more people who

can help.

The remainder of the paper is organized as follows. In the next section, we present the basic setup of the model. In section 3, we discuss as a benchmark the model without raiders who may monitor the manager. Section 4 turns to the case with a single raider, and section 5 describes the general model with $n \geq 1$ potential raiders. In section 6 we study the effects of a change in the number of potential raiders on the takeover threat and thereby on managerial effort. We also examine the consequences for the value of the firm and the value of the shares. Section 7 concludes.

2 Setup of the model

Consider a firm that is run by a single manager and owned by shareholders. The manager must implement one of $m + 1$ projects, $m \geq 2$. Project i yields a return θ_i , $i = 0, \dots, m$, to the shareholders. The manager receives a fraction $0 < \alpha < 1$ of the return θ_i .² This assumption implies that the preferences of the manager and the shareholders over different projects are aligned. Returns are such that if a project $i \neq 0$ is implemented at random, the expected return is strictly smaller than the a priori known return of project 0, $\theta_0 > 0$. Thus, if no information is available, project $i = 0$ is preferred. If the manager exerts effort, he (privately) learns all information, i.e. learns all θ_i , $i = 0, \dots, m$. This allows him to implement the project that yields the highest return, $\bar{\theta} \equiv \max_i \theta_i$. The value $\bar{\theta}$ is common knowledge; however, the project i for which it obtains can only be learned by exerting effort. This project is optimal both for the shareholders and for the manager. Exerting effort comes with a cost $c > 0$ for the manager, though.

²Note that the total return of project i is given by $(1 + \alpha)\theta_i$; an amount θ_i flows to the shareholders and an amount $\alpha\theta_i$ to the manager in charge.

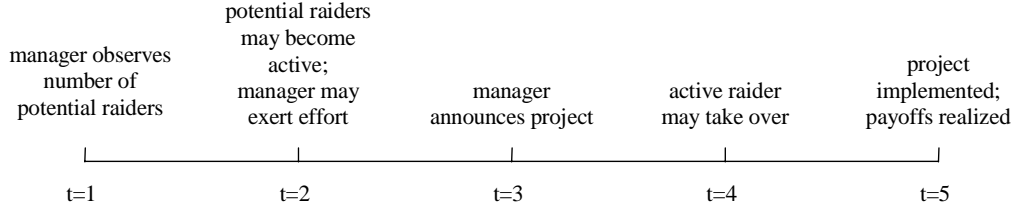


Figure 1: Timing of the model.

We assume that ownership is dispersed. This implies a free-riding problem for the shareholders. They do not have sufficient incentives to monitor the manager. In the general model, we assume that there are $n \geq 1$ potential raiders who may monitor the firm. The timing is as follows. At $t = 1$, the manager observes n , the number of potential raiders. At $t = 2$, each potential raider decides whether or not to pay a fixed cost I . By doing so, the raider becomes active and privately learns all the information about the projects. Simultaneously, the manager decides whether or not to exert effort. At $t = 3$, the manager announces the project i he will implement. At $t = 4$, an active raider can take over the firm and fire the manager. If there is only one active raider, he pays a premium $\rho > 0$ over the value of the shares without a takeover. (This premium will be discussed below.) If there is more than one active raider, there is a bidding war and the winner pays exactly the value of the firm after takeover, i.e. $\bar{\theta}$, to the shareholders. In both cases, the fired manager has utility U_F . At $t = 5$, the preferred project (of the manager if there was no takeover; of the winning raider if there was a takeover) is implemented, and payoffs are realized. Figure 1 summarizes the timing.

Note that in case of a takeover, again, there may be a free-riding problem. Small shareholders - that is, all shareholders in our setup - have an incentive not to tender, but keep their shares instead. By doing so, the individual shareholder could obtain his share of the capital gain from the takeover.

Thus, in this setup, the only successful bid would equal the value of the firm after takeover (Grossman and Hart, 1980; Hart, 1995). In order to avoid this problem, we simply assume that each shareholder believes that if he does not trade, the takeover will not take place (Huddart, 1993). An alternative solution could be that takeover law allows some expropriation of minority shareholders (Grossman and Hart, 1980).

The premium ρ does *not* refer to the takeover premium that is commonly used in the empirical literature. There, the takeover premium is the difference between the price paid by the raider, and the value of the shares before the announcement of the takeover. Here, instead, the premium ρ refers to the difference between the price paid by the raider, and the value of the shares when there is no takeover (as is the case in Haan and Riyanto, 2002). Clearly, the price paid by the raider in this setup depends on the effort exerted by the manager. Alternatively, we could assume that in case of a takeover a fixed price P is paid, independent of the manager's actions. This would not qualitatively change the results, as we will discuss in section 6.

In the next section, we discuss the benchmark model without a takeover threat ($n = 0$). We present two conditions on the parameters that ensure monitoring to increase effort. That is, under these conditions the manager does not exert effort if there is no monitoring, whereas with monitoring (say, if there is only one shareholder) he does exert effort. After deriving these conditions, we turn to the discussion of takeover threats and consider the case $n \geq 1$.

3 Benchmark model: No takeover threat

First consider the model with $n = 0$ as a benchmark. That is, there are no potential raiders - there is no takeover threat. Denote the manager's effort

by e_M . This variable takes the value one if the manager exerts effort, incurs cost c , and learns all information. Otherwise, it equals zero. The manager's utility U_M can then be written as³

$$U_M = \begin{cases} \alpha\bar{\theta} - c & \text{if } e_M = 1, \\ \alpha\theta_0 & \text{if } e_M = 0. \end{cases}$$

At $t = 2$, the manager maximizes U_M and will choose not to exert effort whenever $\alpha\bar{\theta} - c < \alpha\theta_0$.

Assumption 1 *We have $\alpha\bar{\theta} - c < \alpha\theta_0$.*

That is, we assume that the manager's utility of exerting effort falls below his utility of not exerting effort. Under this condition, without monitoring (either by shareholders or by raiders), the manager will choose $e_M = 0$ and implement project $i = 0$. So, without monitoring the manager will not exert any effort. The value of the firm, v_F , is then given by

$$v_F = \theta_0$$

and equals the value of the shares, denoted by v_S .⁴

For comparison, consider the case with a single, large shareholder who is able to monitor the firm. If this shareholder monitors for sure, the manager is sure to be fired when announcing project 0. The manager's utility now takes the form⁵

$$U_M = \begin{cases} \alpha\bar{\theta} - c & \text{if } e_M = 1, \\ U_F & \text{if } e_M = 0. \end{cases}$$

³Note that we could alternatively focus on p_M , the probability with which the manager exerts effort, and write $U_M = p_M(\alpha\bar{\theta} - c) + (1 - p_M)\alpha\theta_0$. This would yield the same condition for the manager not to exert effort (with probability one).

⁴We distinguish between the ex ante value of the firm and the expected value of the shares, because they may not be the same in our model in the presence of a takeover threat. The value of the shares is affected by the price paid by the raider in case of a takeover, whereas the value of the firm is only determined by the return of the project to be implemented.

⁵Again, we could alternatively focus on p_M , and write $U_M = p_M(\alpha\bar{\theta} - c) + (1 - p_M)U_F$. This would yield the same condition for the manager to exert effort (with probability one).

The manager now prefers to exert effort whenever $\alpha\bar{\theta} - c > U_F$.

Assumption 2 *We have $\alpha\bar{\theta} - c > U_F$.*

We assume that the manager's utility of exerting effort exceeds his utility of being fired (which occurs with probability one if he does not exert effort). Thus, under these assumptions, the manager prefers to exert effort ($e_M = 1$) and announce the project yielding $\bar{\theta}$, and monitoring clearly increases effort by the manager. Note that in this situation, $v_F = v_S = \bar{\theta}$.

We assume Assumptions 1 and 2 to be satisfied throughout the remainder of this paper. They are required for a takeover threat to possibly work as a governance mechanism: if Assumption 1 is violated, the manager will exert effort anyway; and if Assumption 2 is violated, even complete monitoring will not induce the manager to exert effort.

4 Takeover threat: Single raider

In this section we return to the case with dispersed ownership, in which there is no monitoring by shareholders. Now suppose that $n = 1$, that is, there is a single potential raider. At $t = 2$, the potential raider can either become active or not, and the manager can exert effort or not. Together, this yields four possible cases: (i) the raider is not active and the manager does not exert effort; (ii) the raider is not active but the manager does exert effort; (iii) the raider is active and the manager does not exert effort, so a takeover will occur and the manager will be fired; and (iv) the raider is active and the manager exerts effort and implements the preferred project, so there is no scope for a takeover. The payoffs of the manager and the raider in each of the four cases are presented in Table 1. In each cell, the first term gives the utility of the manager, U_M , and the second term represents the payoff or

		Raider	
		not active	active
Manager	no effort	$\alpha\theta_0$ 0	U_F $\Pi - I$
	effort	$\alpha\bar{\theta} - c$ 0	$\alpha\bar{\theta} - c$ $-I$

Table 1: Payoffs to manager and raider in the model with a single raider.

utility of the raider. Further, Π refers to the gross profits from a takeover when the manager did not exert effort. We assume that

$$\Pi \equiv \bar{\theta} - (1 + \rho) v_{S,NT},$$

where $v_{S,NT}$ denotes the value of the shares without a takeover. That is, we assume that when a takeover occurs, the raider buys the firms' shares at a premium ρ over $v_{S,NT}$ (for a discussion of ρ , see section 2). We assume $\Pi > I > 0$ to rule out the trivial case where a takeover never occurs and there is no takeover threat. This implies an assumption on parameter values that will be derived in detail below.

From Table 1, using Assumptions 1 and 2, it can be seen that no Nash equilibrium in pure strategies exists. Therefore, consider a mixed strategy equilibrium. In such an equilibrium, the manager exerts effort with probability $0 < p_M < 1$ and the raider becomes active with probability $0 < p_R < 1$. With this notation,

$$v_{S,NT} = p_M \bar{\theta} + (1 - p_M) \theta_0,$$

since with probability p_M the manager exerts effort and the shareholders get $\bar{\theta}$, whereas with probability $1 - p_M$ he does not exert effort and shareholders only get θ_0 . In equilibrium the manager must be indifferent between exerting

effort and not exerting effort, that is

$$\alpha\bar{\theta} - c = p_R U_F + (1 - p_R) \alpha\theta_0,$$

which can be rewritten to give the equilibrium value of p_R ,

$$p_R^* = \frac{\alpha(\theta_0 - \bar{\theta}) + c}{\alpha\theta_0 - U_F}. \quad (1)$$

We have $0 < p_R^* < 1$ by Assumptions 1 and 2. Further, the raider must be indifferent between entering and not entering:

$$(1 - p_M) \Pi - I = 0,$$

that is,

$$(1 - p_M) \left(\bar{\theta} - (1 + \rho) \left(p_M \bar{\theta} + (1 - p_M) \theta_0 \right) \right) - I = 0.$$

Solving for p_M yields

$$p_M = 1 - \frac{\rho\bar{\theta} \pm \sqrt{\rho^2\bar{\theta}^2 + 4I(1 + \rho)(\bar{\theta} - \theta_0)}}{2(1 + \rho)(\bar{\theta} - \theta_0)},$$

where we need the positive root since we require $p_M < 1$. Thus, in equilibrium,

$$p_M^* = 1 - \frac{\rho\bar{\theta} + \sqrt{\rho^2\bar{\theta}^2 + 4I(1 + \rho)(\bar{\theta} - \theta_0)}}{2(1 + \rho)(\bar{\theta} - \theta_0)}. \quad (2)$$

Equations (1) and (2) determine the equilibrium strategies of the potential raider and the manager in the model with $n = 1$.

A necessary condition for a takeover threat is that $\Pi > I$ in equilibrium, that is

$$\bar{\theta} - (1 + \rho) \left(p_M^* \bar{\theta} + (1 - p_M^*) \theta_0 \right) > I.$$

This can be rewritten to give

$$\rho < \frac{\bar{\theta} - \theta_0 - I}{\theta_0}.$$

This condition is necessary for the takeover to be profitable. It states that ρ , the premium paid over the value of the shares without a takeover, should not be too high. Note that if we would set $\theta_0 = 0$, the condition would be satisfied for all values of ρ , since the right hand side of this inequality approaches infinity as θ_0 approaches 0. The critical value for ρ is increasing in $\bar{\theta}$: the higher the firm's value after the takeover, the more the raider is willing to pay. It is decreasing in θ_0 as well as in I : the higher the value of the shares without a takeover, $v_{S,NT}$, (which is increasing in θ_0) and the higher the costs of monitoring the firm, the lower the premium over $v_{S,NT}$ the raider is willing to pay. Finally, note that the right hand side of this equality may become negative for specific values of $\bar{\theta}$, θ_0 , and I . In that case, there would be no ρ which ensures that the takeover is profitable. Evidently, we assume that this is not the case; instead we assume this necessary condition for a takeover to be satisfied.

The expected value of the firm with a single potential raider is given by

$$v_F = p_M \bar{\theta} + (1 - p_M) [p_R \bar{\theta} + (1 - p_R) \theta_0],$$

and the ex ante value of the shares is

$$v_S = p_M \bar{\theta} + (1 - p_M) [p_R (1 + \rho) (p_M \bar{\theta} + (1 - p_M) \theta_0) + (1 - p_R) \theta_0],$$

where (1) and (2) should be used to substitute for $p_R = p_R^*$ and $p_M = p_M^*$ to obtain the equilibrium values.

5 Takeover threat: Multiple raiders

Now we turn to the general model with $n \geq 1$ potential raiders. It is easy to see that there are n asymmetric equilibria in which precisely one raider becomes active, and all others stay out. We focus on the unique symmetric

equilibrium in which all raiders become active with the same probability p_R . Before deriving the equilibrium mixed strategies of the potential raiders and the manager, we first introduce some definitions (following closely Elberfeld and Wolfstetter, 1999).

Let p_0 denote the probability of no takeover. This refers to the probability that none of the raiders become active. This probability is given by

$$p_0 = (1 - p_R)^n. \quad (3)$$

Let p_1 denote the probability of a single active raider;

$$p_1 = np_R(1 - p_R)^{n-1}. \quad (4)$$

Finally, p_{2+} refers to the probability that multiple (at least two) raiders become active and is given by

$$\begin{aligned} p_{2+} &= 1 - p_0 - p_1 \\ &= 1 - (1 - p_R)^n - np_R(1 - p_R)^{n-1}. \end{aligned}$$

The condition for the manager to be indifferent between exerting effort and not exerting effort can now be written as

$$\alpha\bar{\theta} - c = (1 - p_0)U_F + p_0\alpha\theta_0. \quad (5)$$

A potential raider is indifferent between being active and not being active if

$$(1 - p_R)^{n-1}(1 - p_M)(\bar{\theta} - (1 + \rho)v_{S,NT}) - I = 0. \quad (6)$$

If the raider becomes active, he incurs a cost I for sure. If he is the only active raider, which happens with probability $(1 - p_R)^{n-1}$, he earns a gross return $\bar{\theta} - (1 + \rho)v_{S,NT}$ if and only if the manager does not exert effort, which happens with probability $1 - p_M$. Otherwise, his gross return is zero. This

yields the left hand side of condition (6). The right hand side is the net return of not becoming active, which is simply zero.

Solving simultaneously for p_M and p_R we find the equilibrium values

$$p_R^* = 1 - \sqrt[n]{\frac{\alpha\bar{\theta} - c - U_F}{\alpha\theta_0 - U_F}}, \quad (7)$$

and

$$p_M^* = 1 - \frac{\rho\bar{\theta} + \sqrt{\rho^2\bar{\theta}^2 + 4I\left(\frac{\alpha\theta_0 - U_F}{\alpha\bar{\theta} - c - U_F}\right)^{\frac{n-1}{n}}(1+\rho)(\bar{\theta} - \theta_0)}}{2(1+\rho)(\bar{\theta} - \theta_0)}. \quad (8)$$

Equations (7) and (8) describe the equilibrium mixed strategies of the manager and the potential raiders. Clearly, both p_M^* and p_R^* depend on n .

Again, a necessary condition for a takeover threat is that $\Pi > I$ in equilibrium, that is a single active raider should be able to make a profit when taking over. This condition can be written as

$$\bar{\theta} - (1+\rho)(p_M^*\bar{\theta} + (1-p_M^*)\theta_0) > I.$$

Rewriting as a condition on ρ , this is

$$\rho < \frac{\bar{\theta} - \theta_0 - I\sqrt[n]{\left(\frac{\alpha\bar{\theta} - c - U_F}{\alpha\theta_0 - U_F}\right)^{n-1}}}{\theta_0 - \bar{\theta}\left(1 - \sqrt[n]{\left(\frac{\alpha\bar{\theta} - c - U_F}{\alpha\theta_0 - U_F}\right)^{n-1}}\right)}. \quad (9)$$

This condition is necessary for the takeover to be profitable. It states that ρ , the premium paid over the value of the shares without a takeover, should not be too high. Note that the right hand side of this equality may become negative for specific parameter values, so there may be no ρ which ensures that the takeover is profitable. Evidently, we assume that this is not the case; instead we assume this necessary condition for a takeover to be satisfied.

Finally, we have

$$v_F = p_M \bar{\theta} + (1 - p_M) [p_0 \theta_0 + (1 - p_0) \bar{\theta}] \quad (10)$$

$$v_S = p_M \bar{\theta} + (1 - p_M) [p_0 \theta_0 + p_1 (1 + \rho) (p_M \bar{\theta} + (1 - p_M) \theta_0) + p_{2+} \bar{\theta}] \quad (11)$$

where (7) and (8) should be used to substitute for $p_R = p_R^*$ and $p_M = p_M^*$ to obtain the equilibrium values.

6 Effects of a change in the number of potential raiders

We now turn to a discussion of the comparative static effects of a change in the number of potential raiders, n . As we argued in the introduction, in general a ‘more active market for corporate control’ is thought to increase the takeover threat, which in turn increases managerial effort. In this section we explore the effects of a ‘more active market’ in the sense of more potential raiders on the takeover threat (represented by p_R or p_0) and managerial effort (p_M) in our model. We also consider the effects of a change in n on the expected value of the firm and the ex ante value of the shares.

Lemma 1 *In the mixed strategy equilibrium, the probability of zero active raiders, p_0^* , does not depend on n : $\frac{dp_0^*}{dn} = 0$.*

Proof. In equilibrium, condition (5) must hold for the manager to be indifferent between exerting effort and not exerting effort. From this condition, p_0^* can be solved to give

$$p_0^* = \frac{\alpha \bar{\theta} - c - U_F}{\alpha \theta_0 - U_F}. \quad (12)$$

From this expression, $\frac{dp_0^*}{dn} = 0$. ■

Note that from (12) and Assumptions 1 and 2, we have $0 < p_0^* < 1$ in equilibrium. Now consider the effects of a change in n on the potential raiders' equilibrium strategy.

Result 1 *In the mixed strategy equilibrium, the probability that a potential raider is active, p_R^* , is decreasing in n : $\frac{dp_R^*}{dn} < 0$.*

Proof. We have $p_0^* = (1 - p_R^*)^n$, so $p_R^* = 1 - \sqrt[n]{p_0^*}$, and using Lemma 1 we have

$$\frac{dp_R^*}{dn} = \frac{1}{n^2} \sqrt[n]{p_0^*} \ln(p_0^*),$$

which is negative since $0 < p_0^* < 1$. ■

Intuitively, for a given potential raider, an increase in the number of potential raiders implies increased competition. This decreases a potential raider's ex ante probability of winning, and therefore reduces the incentive to become active. That is, it reduces p_R . Clearly, this affects p_0 , the probability that no raider is active. The direct effect of n is to decrease p_0 ; however, the associated decrease in p_R tends to increase p_0 . Additionally, there will be an effect on p_0 via p_M . As Lemma 1 shows, in equilibrium p_0^* is constant. Now consider p_M^* .

Result 2 *In the mixed strategy equilibrium, the probability for the manager to exert effort, p_M^* , is decreasing in n : $\frac{dp_M^*}{dn} < 0$.*

Proof. From (8),

$$\begin{aligned} \frac{dp_M^*}{dn} &= -\frac{1}{2(1+\rho)(\bar{\theta} - \theta_0)} \frac{4I(1+\rho)(\bar{\theta} - \theta_0) \left(\frac{\alpha\theta_0 - U_F}{\alpha\theta - c - U_F}\right)^{\frac{n-1}{n}} \ln \frac{\alpha\theta_0 - U_F}{\alpha\theta - c - U_F}}{2n^2 \sqrt{\rho^2 \bar{\theta}^2 + 4I \left(\frac{\alpha\theta_0 - U_F}{\alpha\theta - c - U_F}\right)^{\frac{n-1}{n}} (1+\rho)(\bar{\theta} - \theta_0)}} \\ &= -\frac{I \left(\frac{\alpha\theta_0 - U_F}{\alpha\theta - c - U_F}\right)^{\frac{n-1}{n}} \ln \frac{\alpha\theta_0 - U_F}{\alpha\theta - c - U_F}}{n^2 \sqrt{\rho^2 \bar{\theta}^2 + 4I \left(\frac{\alpha\theta_0 - U_F}{\alpha\theta - c - U_F}\right)^{\frac{n-1}{n}} (1+\rho)(\bar{\theta} - \theta_0)}}, \end{aligned}$$

which is negative from Assumptions 1 and 2. ■

This can be seen as follows. For *given* p_M , p_R can be derived from (6) to give

$$p_R|_{p_M \text{ fixed}} = 1 - \sqrt[n-1]{X},$$

where

$$X \equiv \frac{I}{(1 - p_M) (\bar{\theta} - (1 + \rho) (p_M \bar{\theta} + (1 - p_M) \theta_0))},$$

and $0 < X < 1$. So,

$$p_0|_{p_M \text{ fixed}} = \sqrt[n-1]{X^n}$$

and an increase in n increases p_0 :

$$\left. \frac{dp_0}{dn} \right|_{p_M \text{ fixed}} = -\frac{1}{(n-1)^2} \sqrt[n-1]{X^n} \ln X > 0.$$

As we argued above, the direct effect of an increase in n is to decrease p_0 , but because of increased competition the potential raiders become active with a smaller probability, which increases p_0 . It turns out that the latter effect dominates (see also Elberfeld and Wolfstetter, 1999, who have a similar result for the probability of a market breakdown with Bertrand competition). Thus, an increase in n *decreases* the takeover threat that the manager is facing. This induces him to exert less effort, i.e. exert effort with a smaller probability p_M . In fact, the manager lowers p_M precisely to the level for which the takeover threat (indicated by p_0) is just as strong as it was before the increase in n (see Lemma 1).⁶

In equilibrium, the probability that a single raider becomes active (p_1) as well as the probability that at least two raiders become active (p_{2+}) also depend on n .

⁶Note that the increase in n also affects the condition for a takeover to be profitable, (9). The critical value for ρ may either increase or decrease, depending on the values of the parameters. In the latter case, the increase in n may cause the takeover threat to disappear completely, if (9) is violated for the new value of n .

Lemma 2 *In the mixed strategy equilibrium, the probability that exactly one raider becomes active, p_1^* , is decreasing in n : $\frac{dp_1^*}{dn} < 0$.*

Proof. From (4) and (3), in equilibrium we have

$$p_1^* = n \left(1 - \sqrt[n]{p_0^*} \right) \sqrt[n]{(p_0^*)^{n-1}} = n \left(\sqrt[n]{(p_0^*)^{n-1}} - p_0^* \right),$$

with p_0^* given by (12). Using Lemma 1, $\frac{dp_1^*}{dn}$ can be written as

$$\frac{dp_1^*}{dn} = \sqrt[n]{(p_0^*)^{n-1}} \left(1 + \frac{\ln p_0^*}{n} \right) - p_0^*.$$

For $n = 1$, $\frac{dp_1^*}{dn} < 0$. (This inequality follows from the property of the ln-function that $\ln x \leq x - 1$ for all x , and $\ln x < x - 1$ for all $x \neq 1$.) Using

$$\frac{d^2 p_1^*}{dn^2} = \frac{1}{n^3} (\ln p_0^*)^2 \sqrt[n]{(p_0^*)^{n-1}} > 0$$

and

$$\lim_{n \rightarrow \infty} \frac{dp_1^*}{dn} = \lim_{n \rightarrow \infty} \sqrt[n]{(p_0^*)^{n-1}} \left(1 + \frac{\ln p_0^*}{n} \right) - p_0^* = 0,$$

we see that $\frac{dp_1^*}{dn} < 0$ for all (finite) values of n . ■

Lemma 3 *In the mixed strategy equilibrium, the probability that two or more raiders become active, p_{2+}^* , is increasing in n : $\frac{dp_{2+}^*}{dn} > 0$.*

Proof. From $p_{2+}^* = 1 - p_0^* - p_1^*$ and Lemma 1, it easily follows that $\frac{dp_{2+}^*}{dn} = -\frac{dp_1^*}{dn}$. The lemma now follows using Lemma 2. ■

Finally, consider the effects of a change in n on the expected value of the firm (v_F) and the ex ante value of the shares (v_S).

Result 3 *The equilibrium expected value of the firm, v_F^* , is decreasing in n : $\frac{dv_F^*}{dn} < 0$.*

Proof. Using (7), (8), (10), and Lemma 1,

$$\begin{aligned}\frac{dv_F^*}{dn} &= \left(\bar{\theta} - [p_0^*\theta_0 + (1 - p_0^*)\bar{\theta}]\right) \frac{dp_M^*}{dn} \\ &= p_0^* (\bar{\theta} - \theta_0) \frac{dp_M^*}{dn},\end{aligned}$$

which is negative from Result 2. ■

This result is intuitive: an increase in n weakens the takeover threat and therefore decreases managerial effort, which in turn negatively affects firm value. A similar result holds for the value of the shares.

Result 4 *The equilibrium ex ante value of the shares, v_S^* , is decreasing in n : $\frac{dv_S^*}{dn} < 0$.*

Proof. From (7), (8), and (11),

$$\begin{aligned}\frac{dv_S^*}{dn} &= \left(\bar{\theta} - [\cdot] + (1 - p_M^*)p_1^*(1 + \rho)(\bar{\theta} - \theta_0)\right) \frac{dp_M^*}{dn} \\ &\quad + (1 - p_M^*)(1 + \rho)(p_M^*\bar{\theta} + (1 - p_M^*)\theta_0) \frac{dp_1^*}{dn} \\ &\quad + (1 - p_M^*)\bar{\theta} \frac{dp_{2+}^*}{dn},\end{aligned}$$

where $[\cdot]$ refers to the term in square brackets in (11), substituting the equilibrium values. From $\frac{dp_{2+}^*}{dn} = -\frac{dp_1^*}{dn}$ (see the proof of Lemma 3), we have

$$\begin{aligned}\frac{dv_S^*}{dn} &= \left(\bar{\theta} - [\cdot] + (1 - p_M^*)p_1^*(1 + \rho)(\bar{\theta} - \theta_0)\right) \frac{dp_M^*}{dn} \\ &\quad + (1 - p_M^*) \left((1 + \rho)(p_M^*\bar{\theta} + (1 - p_M^*)\theta_0) - \bar{\theta} \right) \frac{dp_1^*}{dn}.\end{aligned}$$

In this expression, the first term must be negative since $\bar{\theta} > [\cdot]$. Since by assumption $\Pi^* = \bar{\theta} - (1 + \rho)(p_M^*\bar{\theta} + (1 - p_M^*)\theta_0) > 0$, using Lemma 2 the second term is positive.

After some tedious calculations, the derivative $\frac{dv_S^*}{dn}$ can be rewritten in terms of p_0^* as

$$\frac{dv_S^*}{dn} = -F \left(1 + \sqrt[n]{p_0^*} \left(\frac{1}{n} (\ln p_0^*) \left(1 - \frac{1}{n} \frac{\bar{\theta} - \theta_0}{\sqrt{Y}} \right) - 1 \right) \right),$$

where

$$Y \equiv \rho^2 \bar{\theta}^2 + 4I (p_0^*)^{-\frac{n-1}{n}} (1 + \rho) (\bar{\theta} - \theta_0).$$

We must have $1 - \frac{1}{n} \frac{\bar{\theta} - \theta_0}{\sqrt{Y}} < 1$, so

$$1 + \sqrt[n]{p_0^*} \left(\frac{1}{n} (\ln p_0^*) \left(1 - \frac{1}{n} \frac{\bar{\theta} - \theta_0}{\sqrt{Y}} \right) - 1 \right) > 1 + \sqrt[n]{p_0^*} \left(\frac{1}{n} (\ln p_0^*) - 1 \right),$$

and the right hand side of this expression equals

$$1 - \sqrt[n]{p_0^*} + \frac{1}{n} (\ln p_0^*) \sqrt[n]{p_0^*}. \quad (13)$$

For $n = 1$, (13) equals

$$1 - p_0^* + (\ln p_0^*) p_0^* > 0$$

(the inequality can be verified by noting that the derivative with respect to p_0^* of this expression is given by $\ln p_0^* < 0$, and the limit for $p_0^* \rightarrow 1$ of the expression equals zero). Further, the derivative with respect to n of (13) is given by

$$-\frac{1}{n^3} (\ln p_0^*)^2 \sqrt[n]{p_0^*} < 0,$$

and we have

$$\lim_{n \rightarrow \infty} 1 - \sqrt[n]{p_0^*} + \frac{1}{n} (\ln p_0^*) \sqrt[n]{p_0^*} = 0.$$

Thus, the expression in (13) is strictly positive for any finite n . This implies $\frac{dv_S^*}{dn} < 0$ (for any finite n). ■

This result illustrates that the negative effects of an increase in n (decreasing managerial effort, and decreasing the price paid for the shares in the presence of a single active raider) dominate the positive effects (making the presence

of a single active raider less likely but that of more than one active raiders - in which case the winner pays a higher price for the shares - more likely). To see this, observe that the term between square brackets in (11) represents the expected value of the shares if the manager does not exert effort, and is strictly smaller than $\bar{\theta}$, the expected value if the manager does exert effort. An increase in n decreases managerial effort p_M^* , which puts more weight on the former, smaller term. This decreases v_S^* . Also, the price paid for the shares in case of a takeover with only one active raider falls, since $v_{S,NT}^*$ falls. This also decreases v_S^* . However, there are additional effects running via p_1^* and p_{2+}^* . An increase in n will make a single active raider (who pays $(1 + \rho)v_{S,NT}^*$ to the shareholders in case of a takeover) less likely, and two or more active raiders (who engage in a bidding war in case of a takeover, which yields $\bar{\theta} > (1 + \rho)v_{S,NT}^*$ to the shareholders) more likely. Combined with the result that p_0^* is independent of n , this increases v_S^* . It turns out that the sum of all these effects is negative; that is, the positive effects via p_1^* and p_{2+}^* are dominated by the negative effects running via p_M^* .

For completeness, we discuss here some asymptotic results. We consider what happens if the population of potential raiders grows large. Recall that p_0^* does not depend on n . It can easily be verified that both the potential raiders and the manager have little incentive to exert effort in the limit: both p_R^* and p_M^* converge to zero as $n \rightarrow \infty$. Note that these probabilities are never equal to zero though, since that cannot be a Nash equilibrium. A similar result holds for p_1^* , and thus p_{2+}^* has the limiting value $1 - p_0^*$. In the limit, the probability that there is exactly one active raider disappears. This implies that v_F^* and v_S^* have the same limiting value, which is equal to $p_0^*\theta_0 + (1 - p_0^*)\bar{\theta}$.

Finally, suppose that a fixed price P were paid in case of a takeover, instead of the amount $(1 + \rho)v_{S,NT}$. As we mentioned in section 2, this would not qualitatively change the results. This can be seen as follows. An increase in

the number of potential raiders, n , decreases the probability of a takeover, $1 - p_0$. This in turn implies a decrease in p_M , which in turn decreases $v_{S,NT}$. So in our model, if n increases, the takeover becomes cheaper. If it does not become cheaper, as is the case with a fixed takeover price P , a takeover will be even less likely. Thus, our results with respect to p_M , p_R , and v_F will continue to hold.

7 Concluding remarks

We have shown that when monitoring is costly an increase in the number of potential raiders may decrease managerial effort. For a given managerial effort level, a higher number of potential raiders implies that each individual raider faces increased competition, and therefore has less incentive to become active. It turns out that this decreases the takeover threat. This induces the manager to exert less effort. In turn, this tends to reduce the expected value of the firm as well as the ex ante value of the shares. Thus, one should be careful when asserting that an ‘active market for corporate control’ is required for takeover threats to function as a corporate governance mechanism. In our setup, a larger market in the sense of more potential raiders tends to reduce the effectiveness of this governance mechanism.

In our model, there is some probability that no raider becomes active, and a takeover will not occur. In equilibrium, this probability depends on exogenous parameters of the model only, and it is independent of the number of potential raiders. In the model, assuming that a takeover can be profitable, it is always strictly greater than zero. This may provide an explanation for ‘[t]he fact that companies can persist for long periods, operating publicly at profit levels substantially below maximum profit’ (Grossman and Hart, 1980, p. 58). Even if there is only one potential raider, this raider may follow a mixed strategy (as in the equilibrium of our model), which implies that

he does not necessarily invest in monitoring the firm. Thus, if the manager shirks and implements the ‘wrong’ project, a takeover will not necessarily follow. If the values of the parameters are such that the probability of having no active raiders is close to zero, in a repeated version of the model the manager may shirk for quite some time without the firm being subject to a takeover.

In some situations takeover threats may *decrease* managerial effort. For example, the possibility of getting fired lowers the incentive of a manager to invest in firm-specific human capital (Kahn and Huberman, 1988). Also, takeovers break the implicit contracts between managers and workers, and a takeover threat thus reduces worker’s incentives to engage in such contracts (Shleifer and Summers, 1988). Finally, takeover threats may decrease managerial effort if the manager derives private benefits and his preferences are not perfectly aligned with those of shareholders (Haan and Riyanto, 2002). Clearly, our results suggest that in these cases an increase in the number of potential raiders will increase effort by the manager.

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