Trends in productivity: The case of capital shortage

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SOM-theme C Coordination and growth in economies

Abstract

This paper analyses the effect of rising wage rates and real interest rates on labour productivity and capital productivity in a situation of capital shortage. Furthermore, it shows the effect of rising wage rates and real interest rates on the capital intensity of the production process. This latter effect can not be determined unambiguously.

1. Introduction

In the period 1913-1949, the USA combined relatively low rates of GDP-growth – about 2.84% on average – with relatively high rates of growth of labour and capital productivity of 2.48% and 0.81%, respectively.[1] In the two decades after World War II, GDP-growth increased by a full percentage-point to 3.92% on average, while at the same time the rates of productivity growth remained high. However, after 1973 GDP-growth dropped to levels comparable to that of the period 1913-1949. Unlike the period 1913-1949 rates of productivity growth were low. These numbers, which

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[1] With respect to these numbers, Kuipers and Kuper (2001b) argue that the average total capital productivity growth as reported by Maddison (1995, Tables 2-1 and 2-6) does not fully characterise the entire 1919-1949 period (*cf.* Gordon, 2000). On the contrary, if one takes the effects of the Great Depression into account, the rise in the output-capital ratio only starts in the late 1930s. In the 1920s the output-capital ratio even falls. Obviously, Maddison's representation of the 1913-1949 period by reporting only the average growth rate and the values in the first and final year of the period is imprecise.

are presented in Table 1, reveal two puzzles. The first one relates to the Depression and is addressed also by Cole and Ohanian (2001). They try to answer the following question: Why is the economy on a lower steady-state growth path in the absence of large negative shifts in productivity? The second puzzle can be expressed as follows: Why is GDP-growth after 1973 comparable to that of the period 1913-1949, while at the same time rates of productivity growth after 1973 were considerably lower? In this paper we are interested in the second puzzle. Before we proceed, we briefly summarise the main findings of Cole and Ohanian which will be useful for our purposes.

Table 1 *Rates of growth (%) of GDP and productivity of labour and capital in the USA, 1870-1992*

Period	*GDP*	*Labour productivity*	*Capital productivity*
1870-1913	3.94	1.88	-1.51
1913-1950	2.84	2.48	0.81
1950-1973	3.92	2.74	0.63
1973-1992	2.39	1.11	-0.72

Source: Maddison (1995, p. 41)

Cole and Ohanian (2001) examine the Depression and the ensuing recovery from a neoclassical perspective without finding compelling answers. In another paper, Ohanian (2001) offers two possible interpretations. The first one is the measurement-error hypothesis, the second one the efficiency view. The first interpretation, obviously, focuses on the quality of the data, the latter interpretation has to do with organisational capital affecting production, distribution, marketing and inventory plans. In this paper, we take an efficiency view, and at the same time depart form the neoclassical perspective. This is our main contribution to this discussion.

Cole and Ohanian and others take the neoclassical perspective for granted, which – in our opinion – need not lead to the right answers per se. Changing the theoretical perspective opens alternative paths towards possible answers.

As mentioned before, we focus on the second puzzle. We explain the productivity puzzle by identifying the pre-World War II period as a period of capital abundance and the period after 1973 as a period of capital shortage. This relates to Burda (1988) who refers to the "capital shortage" hypothesis to explain the rise in the natural rate of unemployment in Europe after 1973. Moreover, we assign an important role to intangible assets. In that respect our analysis takes the efficiency view proposed by Ohanian (2001).

Our analysis is based on alternating long and prolonged periods of capital shortage with increasing unemployment and capital abundance. The underlying assumption is the lack of substitutability between capital and labour. This type of analysis implies that changes in productivity are the result of technical change. Changes in relative prices affects productivity only indirectly through their impact on investment in intangible assets (*cf.* organisational capital affecting production, distribution, marketing and inventory plans as in Ohanian's efficiency view). These intangible assets enable the entrepreneurs to reach the desired productivity levels.

Kuipers and Kuper (2001a) argue that in a situation of capital abundance (i) the capital intensity of production unambiguously falls if the real interest rate increases and (ii) the capital intensity of production is independent of the real wage rate. However, the case of capital shortage is not a trivial one. It is argued that the effect of rising real interest rates on capital intensity is not unambiguous if there is a shortage of capital. The situation of capital shortage will be analysed more rigorously in this paper. In the next section the model will be presented. Based on this model, the effects of changes in real wages and real interest rates are discussed in Section 3. Because of the earlier mentioned ambiguity, empirical analysis is called for. Results are in Section 4. Section 5 concludes.

2. The model in case of capital shortage

The simplest model to analyse situations of capital abundance and capital shortage starts from assuming that capital and labour are complementary inputs in a constant-returns-to-scale production process. Defining X as output at full capacity, K as the stock of capital (tangible assets), N as employment and $\overline{N}$ as the labour force we can write the production process as follows:

$$X = qK, \qquad q > 0, \tag{1}$$

$$N = \frac{1}{m}X, \qquad m > 0;\ N < \overline{N}. \tag{2}$$

Parameters q and m denote the average productivity of capital and labour respectively. These are defined as $q = X/K$, and $m = X/N$, respectively.

In our definition X includes not only consumption (C) and investment in tangible assets (I), but also the production of intangible assets J:

$$X = C + I + J. \tag{3}$$

Intangibles – like assets related to management, organisation and knowledge – are important in the sense that these assets allow entrepreneurs to adjust the average productivity of capital and labour to their desirable levels. H_1 is the stock of intangible assets used to adjust the average productivity of labour m and H_2 is the stock of intangible assets used to adjust the average productivity of capital q. Assuming decreasing returns to intangible assets, we write:

$$m = E_1\left(\frac{H_1}{K}\right)^{e_1}, \qquad E_1 > 0;\ 0 < e_1 < 1, \tag{4}$$

$$q = E_2\left(\frac{H_2}{K}\right)^{e_2}, \qquad E_2 > 0;\ 0 < e_2 < 1, \tag{5}$$

where E_i, e_i (i=1,2) are technical parameters and H is the total stock of intangible assets $H = H_1 + H_2$.

Optimization

Entrepreneurs choose H_1 and H_2 to maximise profits Π given K. Profits are defined as:

$$\Pi = X - rK - wN - vH \ ,$$

where r is the real interest rate, w is the real wage rate and v is the real price of intangible capital. Rewriting the equations for m and q defines H_1 and H_2 as

$H_1 = KE_1^{-1/e_1} m^{1/e_1}$ and $H_2 = KE_2^{-1/e_2} q^{1/e_2}$, E_i>0, 0<e_i<1.

Note that H is defined as the sum of H_1 and H_2:

$$H = H_1 + H_2 = KE_1^{-1/e_1} m^{1/e_1} + KE_2^{-1/e_2} q^{1/e_2} \ . \qquad (6)$$

Substituting Equations (1), (2), and (6) gives:

$$\Pi = X - rK - wN - vH = qK - rK - w\frac{q}{m}K - vKE_1^{-1/e_1} m^{1/e_1} - vKE_2^{-1/e_2} q^{1/e_2} \ ,$$

Maximising profits Π yields the FOC's:

$$\frac{\partial \Pi}{\partial m} = 0 \Rightarrow \frac{wq}{m^2}K - v\frac{1}{e_1}KE_1^{-1/e_1} m^{(-e_1+1)/e_1} = 0 \ , \qquad (7)$$

$$\frac{\partial \Pi}{\partial q} = 0 \Rightarrow K - \frac{w}{m}K - v\frac{1}{e_2}KE_2^{-1/e_2} q^{(-e_2+1)/e_2} = 0 \ . \qquad (8)$$

From these conditions we have to show the existence of an optimum which, however, is a bit complicated.

Existence

First, we rewrite the FOC's in a more compact way.

Defining $n_1 = \frac{1}{e_1}E_1^{-1/e_1} > 0$ and $n_2 = \frac{1}{e_2}E_2^{-1/e_2} > 0$, we can rewrite the FOC's as:

$$q = \left(\frac{mvn_2}{-w+m}\right)^{\frac{e_2}{-1+e_2}} \text{ and } m = \left(\frac{wq}{vn_1}\right)^{\frac{e_1}{1+e_1}} .$$

Defining $a = \frac{-e_2}{-1+e_2}$ and $c = \frac{e_1}{1+e_1}$, these equations can be written as:

$$q = \left(\frac{mvn_2}{-w+m}\right)^{-a} \text{ or } m = \frac{w}{1-vn_2 q^{1/a}} \text{ and}$$

$$m = \left(\frac{wq}{vn_1}\right)^{c}.$$

From these expressions we can see that m=m(q,v,w) and q=q(m,v,w). That is, the capital intensity $m/q=K/N$ depends on the real input prices v and w. What the signs of the parameters are is discussed later.

Without formally proving the existence of a solution it is possible to say a bit more. Above, the FOC's are written as:

$$m = \frac{w}{1-vn_2 q^{1/a}}, \tag{9}$$

and

$$m = \left(\frac{wq}{vn_1}\right)^{c}. \tag{10}$$

The first equation is convex in q, while the second equation is concave in q. Equation (9) implies: $q^* = \lim_{m\to\infty} q = (vn_2)^{-a}$ and m=w if q=0. Note that we restrict ourselves to the interval $(0, q^*)$ where m>w. Obviously, if the curves do not intersect there will be no solution. This is illustrated in the right graph in Figure 1. To find the conditions for which both curves intersect as is shown in the left graph in Figure 1, subtract Equation (9) from Equation (10), show that result is concave in q, and find the maximum. Next, evaluate the difference between Equations (10) and (9) at the maximum. If this value is positive, then Equations (9) and (10) intersect (twice) as Figure 2 illustrates.

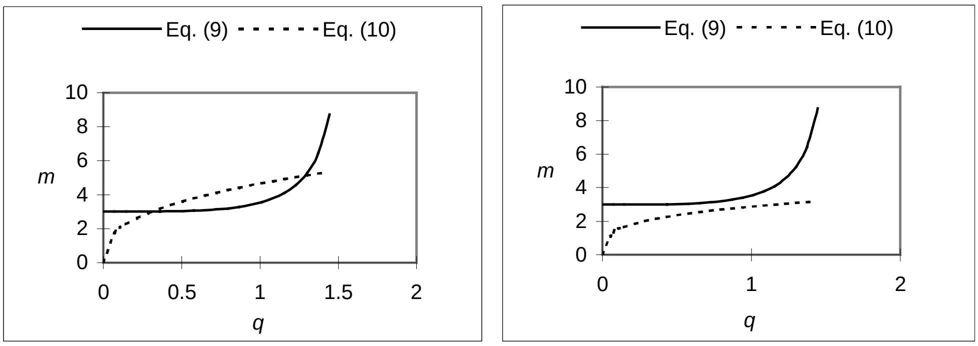

Figure 1 *Simulations using the following values: both graphs:* w=3; r=0.03; e_2=0.2; E_1=0.6; E_2=0.2. *Parameter* e_1=0.6 (*left*) *and* e_1=0.4 (*right*).

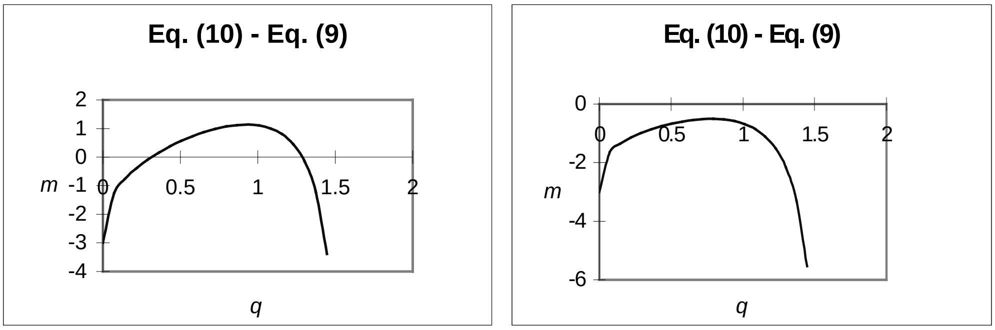

Figure 2 *Simulations using the following values: both graphs:* w=3; r=0.03; e_2=0.2; E_1=0.6; E_2=0.2. *Parameter* e_1=0.6 (*left*) *and* e_1=0.4 (*right*).

Assume that Equation (9) is fixed which means that w, v, n_2 and a are given. It is obvious that the curves intersect only if n_1 is not too high, *i.e.* if e_1 and/or E_1 are not too low. Likewise, if Equation (10) is fixed, the curves intersect only if e_2 and/or E_2 are not too low.

Second-order conditions

The second-order condition (necessary condition not sufficient) is that the Hessian H is negative semidefinite:

$$H = \begin{pmatrix} \frac{\partial^2 \Pi}{\partial q^2} & \frac{\partial^2 \Pi}{\partial q \partial m} \\ \frac{\partial^2 \Pi}{\partial m \partial q} & \frac{\partial^2 \Pi}{\partial m^2} \end{pmatrix} = \begin{pmatrix} \left(\frac{e_2 - 1}{e_2^{\ 2}}\right) vKE_2^{\frac{-1}{e_2}} q^{\frac{-2e_2+1}{e_2}} & \frac{w}{m^2}K \\ \frac{w}{m^2}K & \frac{-2wq}{m^3}K + \left(\frac{e_1 - 1}{e_1^{\ 2}}\right) vKE_1^{\frac{-1}{e_1}} m^{\frac{-2e_1+1}{e_1}} \end{pmatrix}.$$

A negative semidefinite matrix must have diagonal elements that are less than or equal to zero. H is negative semidefinite since $0<e_i<1$. So, the necessary condition for maximising profits is met. Sufficient conditions are a bit more complicated to derive.

The sufficient second-order condition is that the principal minors of the Hessian H must alternate in sign. It is important to note that $a>0$ and $0<c<1/2$. The Hessian can now be written as:

$$H = \begin{pmatrix} \frac{-1}{aq}\left(\frac{m-w}{m}\right) & \frac{w}{m^2} \\ \frac{w}{m^2} & -\frac{wq}{m^3}\left(2 + \frac{1}{c}\right) \end{pmatrix} \otimes K .$$

The sufficient conditions for profit maximisation are:

$$\left| \frac{-1}{aq}\left(\frac{m-w}{m}\right) \right| < 0 ,$$

and

$$\begin{vmatrix} \frac{-1}{aq}\left(\frac{m-w}{m}\right) & \frac{w}{m^2} \\ \frac{w}{m^2} & -\frac{wq}{m^3}\left(2 + \frac{1}{c}\right) \end{vmatrix} > 0 .$$

Since $a>0$ and $m>w$, the first condition definitely holds.

Defining $b = \frac{w}{m-w}>0$ or $\frac{m-w}{m} = \frac{1}{b}\frac{w}{m}$, the second condition is written as:

$$\begin{vmatrix} \frac{-1}{abq}\frac{w}{m} & \frac{w}{m^2} \\ \frac{w}{m^2} & -\frac{wq}{m^3}\left(2+\frac{1}{c}\right) \end{vmatrix} > 0 ,$$

which means that the sufficient second-order condition is

$$\frac{1}{ab}\left(2+\frac{1}{c}\right)\frac{w^2}{m^4}-\frac{w^2}{m^4}=\frac{w^2}{m^4}\left[\frac{1}{ab}\left(2+\frac{1}{c}\right)-1\right]>0 ,$$

or

$$\frac{1}{ab}\left(2+\frac{1}{c}\right)-1>0 \Rightarrow \frac{m}{w}-1=\frac{1}{b}>\frac{ac}{2c+1} ,$$

i.e. the sufficient second-order condition is $0<abc<2c+1$. This condition implies that $\frac{m}{w}>1+\frac{ac}{2c+1}$. This means that if $1<\frac{m}{w}<1+\frac{ac}{2c+1}$, the sufficient second-order condition does not hold.

3. Elasticities

In this section we try to find conditions for the size and the sign of the elasticities (assuming that there is a profit maximising solution to the optimisation problem).

Size and sign

The relevant equations (11), (12), (13), and (14), which are derived in Appendix A:

$$\frac{\partial m/m}{\partial v/v}=c\left[\frac{\partial q/q}{\partial v/v}-1\right], \tag{11}$$

$$\frac{\partial m/m}{\partial w/w}=c\left[1+\frac{\partial q/q}{\partial w/w}\right], \tag{12}$$

$$[1-abc]\frac{\partial q/q}{\partial v/v}=-a[1+bc], \tag{13}$$

$$[1-abc]\frac{\partial q/q}{\partial w/w}=ab(c-1), \tag{14}$$

where

$$a = \frac{-e_2}{-1 + e_2}, \quad b = \frac{w}{m - w}, \quad c = \frac{e_1}{1 + e_1}.$$

Since 0 < e_i < 1, it is easily seen that $0 < c < 1/2$ and $a > 0$. Furthermore $w<m$ implies that $b>0$. From Equations (11) and (12) we can draw some conclusions.

1. $\frac{\partial q/q}{\partial v/v} < 1 \Rightarrow \text{sgn}\left(\frac{\partial m/m}{\partial v/v}\right) = -\text{sgn}\left(\frac{\partial q/q}{\partial v/v}\right).$

2. $\frac{\partial q/q}{\partial w/w} < -1 \Rightarrow \text{sgn}\left(\frac{\partial m/m}{\partial w/w}\right) = -\text{sgn}\left(\frac{\partial q/q}{\partial w/w}\right).$

It can be shown that the right-hand-sides of Equations (13) and (14) are negative. Furthermore, given the sufficient second-order condition $0 < abc < 2c + 1$, the term between square brackets on the left-hand-sides of Equations (13) and (14) can be either negative ($1 < abc < 2c + 1$) or positive ($0 < abc < 1$). This leads to the following conclusion:

3. If $0 < abc < 1$, $\frac{\partial q/q}{\partial v/v} < 0$ and $\frac{\partial q/q}{\partial w/w} < 0$.

4. If $1 < abc < 2c + 1$, $\frac{\partial q/q}{\partial v/v} > 0$ and $\frac{\partial q/q}{\partial w/w} > 0$.

Table 2 below summarises these results.

Table 2 *The effects of changes in real wages (w) and real interest rates (v)*

	$\frac{\partial q/q}{\partial v/v}$	$\frac{\partial m/m}{\partial v/v}$	$\frac{\partial q/q}{\partial w/w}$	$\frac{\partial m/m}{\partial w/w}$
$0 < abc < 1$	<0	>0	>-1 and <0	<0
			<-1	>0
$1 < abc < 2c + 1$	>0 and <1	<0	>0	>0
	>1	>0		

The effect on capital intensity

How does the intensity of capital m/q react to changes in v and w? If the real interest rate rises, the average productivity of capital and labour drops. The overall effect on the capital intensity (m/q) can not be determined unambiguously. However, falling real wage rates causes the capital intensity (m/q) to drop unambiguously.

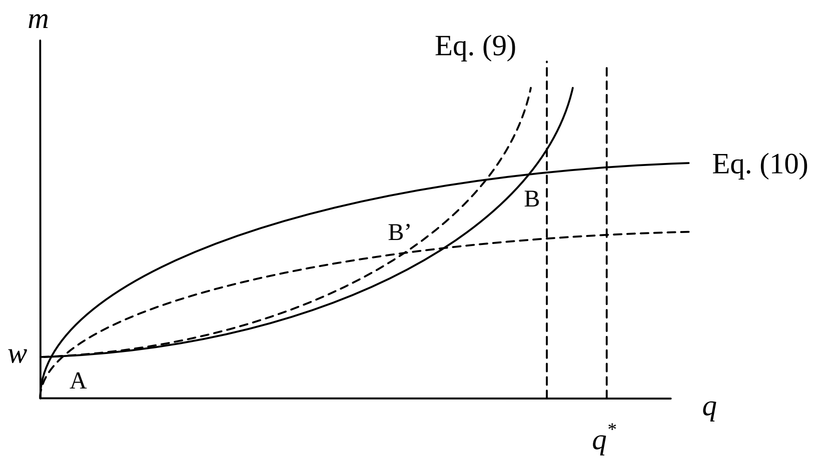

Figure 3 *The effect of rising real interest rates*

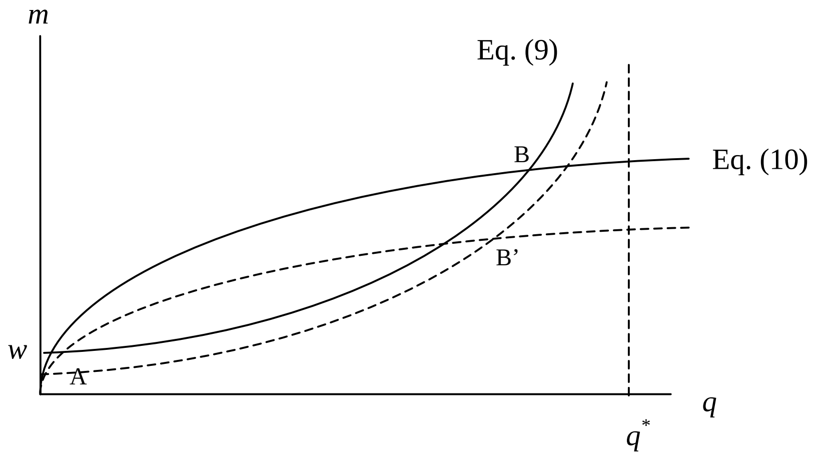

Figure 4 *The effect of falling real wage rates*

This effect on the capital intensity is more formally shown below. First, note that:

$$\frac{\partial(m/q)}{\partial v} = \frac{m}{qv}\left[\frac{\partial m/m}{\partial v/v} - \frac{\partial q/q}{\partial v/v}\right],$$

and

$$\frac{\partial(m/q)}{\partial w} = \frac{m}{qw}\left[\frac{\partial m/m}{\partial w/w} - \frac{\partial q/q}{\partial w/w}\right].$$

Using Equations (11), (12), (13), and (14) these equations are written as:

$$\frac{\partial(m/q)}{\partial v} = \frac{m}{qv}\left[(1-c)\frac{a(1+bc)}{1-abc} - c\right], \tag{15}$$

and

$$\frac{\partial(m/q)}{\partial w} = \frac{m}{qw}\left[(c-1)\frac{ab(c-1)}{1-abc} + c\right]. \tag{16}$$

Conclusions:

5. If $(1-c)a(1+bc) < c(1-abc)$, *i.e.* $abc < c - a(1-c)$, then $\frac{\partial(m/q)}{\partial v} < 0$.

6. If $ab(c-1)^2 > -c(1-abc)$, *i.e.* $abc < \frac{1}{2}(ab+c)$, then $\frac{\partial(m/q)}{\partial w} > 0$.

Note that, since $0 < c < \frac{1}{2}$, it is easily shown that $abc < \frac{1}{2}ab < \frac{1}{2}(ab+c)$ which means that the capital intensity drops unambiguously if the real wage rate falls.

4. Empirical testing

Here we try to test the hypothesis put forward in the previous sections for the USA in the period 1973-1996. The sources of the data we have used are listed in Appendix B.

Before presenting the estimation results, we show time series of the nominal interest rate (*i*), the real interest rate (*r*) and the rate of growth of the real wage rate (dlog *w*). From Figure 5 we conclude that

(1) In the period – we identify as a period of capital shortage (1973-1996) – the real interest rate increases until the mid 1980s. The nominal interest rate increases until the early 1980s and drops in later years.

(2) Compared to the 1950s and 1960s the rate of growth of the real wage rate is low at about 1% on average.

These observations are consistent with what our theory predicts in periods of capital shortage.

Because data on intangible assets are not available, we can only test the hypotheses in an indirect way. In the precious previous sections it was argued that in a situation of capital shortage *m* and *q* are dependent on *w* and *r*. This dependence, however, we can test.

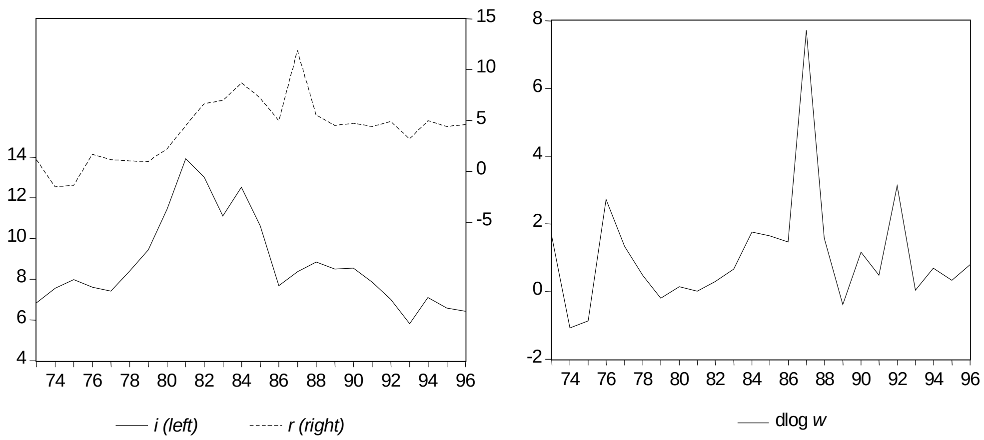

Figure 5 *The development of the nominal interest rate (i),the real interest rate (r) and the rate of growth of the real wage rate (dlog w) in the period 1973-1996*

In Equation (17) the stock of capital K accounts for changes in the utilisation rate. This equation, which includes a time trend, is estimated for the period 1976-1996 because the real interest rate turned negative in the period 1973-1975 (t-values between brackets).

$$\log K/N = 2.721 + 0.002\,t - 0.013 \log r + 0.403 \log w \quad (17)$$
$$(9.269) \quad (1.024) \quad (-2.379) \quad (3.441)$$

sample: 1976-1996

adjusted R^2 = 0.882

Durbin-Watson = 1.811

From the estimation results we conclude:

(1) log r and log w contribute significantly to the explanation of the capital intensity. The hypothesis that in a situation of capital shortage m and q both depend on r and w can not be rejected.

(2) The sign of log w is positive. This again is what theory predicted.

(3) The negative coefficient of log r indicates that investment in intangible assets increase the average productivity of labour more strongly than the average productivity of capital. We can roughly calculate the parameters e_1 and e_2, if we assume that the labour income ratio and the average productivity of capital is constant. In that case, e_1=0.66 and e_2=0.29[2,3]. This confirms our hypothesis that investment in intangible assets increases m more than q.

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[2] From Equations (9) and (10) we can derive:

$$\frac{m}{q} = \frac{e_1^{\frac{e_1}{1+e_1}} E_1^{\frac{1}{1+e_1}} w^{\frac{e_1}{1+e_1}} r^{\frac{-e_1}{1+e_1}} q^{\frac{e_1}{1+e_1}}}{e_2^{\frac{e_2}{1-e_2}} E_2^{\frac{1}{1-e_2}} \left(\frac{m-w}{m}\right)^{\frac{e_2}{1-e_2}} r^{\frac{-e_2}{1-e_2}}}$$

(4) The slowdown of the real wage rate in the 1973-1996 reduced the negative impact of increasing real interest rate on the average productivity of labour and, at the same time, mitigated the negative impact of the rising real interest rate on the average productivity of capital.

(5) The time trend is not significant. This implies that $d \log E_1 /(1+e_1)$ and $d \log E_2 /(1-e_2)$ do not differ significantly.

5. Conclusion

The case of capital shortage is not a trivial one. It can be expected that in this situation the real interest rate rises, after all capital is scarce. However, whether the capital intensity of the production process rises or drops depends on the elasticities in the model. Hence, the effect on the capital intensity of rising real interest rates is an empirical matter.

Our analysis leads us to the following conclusions. First of all, in situations of low growth, the growth rate of productivity of labour and capital can be either high or low, depending on the development of factor prices in situations of shortage or abundance of capital. Secondly, data for the USA show an increase in the real interest rate and slow growth in real wages. This supports our theoretical explanation of the productivity puzzle. The rise in the real interest rate in the period after 1973 reduces investment in intangible assets which reduces the productivity of both labour and capital unambiguously. Thirdly, the real wage rate plays a less prominent role. In a situation of capital shortage, the effect of the real wage rate on labour productivity is ambiguous. What is clear, however, is that in situations of capital shortage, lower real wage rates decrease capital intensity. For the USA in the period after 1973, lower real wages increased productivity of labour and capital.

[3] The average productivity of capital drops from a value of 0.399 in 1973 to 0.344 in 1981 and rises again to 0.397 in 1996.

Appendix A. Some arithmatic

From the FOC's it is clear that m=m(q,v,w) and q=q(m,v,w). Implicitly differentiating m with respect to v and w yields the following expressions:

$$\frac{\partial m}{\partial v}=\frac{e_1}{1+e_1}\left(\frac{wq}{vn_1}\right)^{\frac{-1}{1+e_1}}\left[\frac{vn_1 w\partial q/\partial v-wqn_1}{(vn_1)^2}\right]=\frac{e_1}{1+e_1}\left(\frac{wq}{vn_1}\right)^{\frac{-1}{1+e_1}}\frac{wq}{vn_1}\left[\frac{\partial q/q}{\partial v}-\frac{1}{v}\right]$$

$$=\frac{e_1}{1+e_1}\left(\frac{wq}{vn_1}\right)^{\frac{e_1}{1+e_1}}\frac{1}{v}\left[\frac{\partial q/q}{\partial v/v}-1\right]=\frac{e_1}{1+e_1}\frac{m}{v}\left[\frac{\partial q/q}{\partial v/v}-1\right],$$

or

$$\frac{\partial m/m}{\partial v/v}=\frac{e_1}{1+e_1}\left[\frac{\partial q/q}{\partial v/v}-1\right], \quad \text{(A1)}$$

and

$$\frac{\partial m}{\partial w}=\frac{e_1}{1+e_1}\left(\frac{wq}{vn_1}\right)^{\frac{-1}{1+e_1}}\left[\frac{vn_1(q+w\partial q/\partial w)}{(vn_1)^2}\right]=\frac{e_1}{1+e_1}\left(\frac{wq}{vn_1}\right)^{\frac{-1}{1+e_1}}\frac{wq}{vn_1}\left[\frac{1}{w}+\frac{\partial q/q}{\partial w}\right]$$

$$=\frac{e_1}{1+e_1}\left(\frac{wq}{vn_1}\right)^{\frac{e_1}{1+e_1}}\frac{1}{w}\left[1+\frac{\partial q/q}{\partial w/w}\right]=\frac{e_1}{1+e_1}\frac{m}{w}\left[1+\frac{\partial q/q}{\partial w/w}\right],$$

or

$$\frac{\partial m/m}{\partial w/w}=\frac{e_1}{1+e_1}\left[1+\frac{\partial q/q}{\partial w/w}\right]. \quad \text{(A2)}$$

Implicitly differentiating q with respect to v and w yields the following expressions:

$$\frac{\partial q}{\partial v}=\frac{e_2}{-1+e_2}\left(\frac{mvn_2}{-w+m}\right)^{\frac{1}{-1+e_2}}\left[\frac{(-w+m)(mn_2+vn_2\partial m/\partial v)-mvn_2\partial m/\partial v}{(-w+m)^2}\right]$$

$$=\frac{e_2}{-1+e_2}\left(\frac{mvn_2}{-w+m}\right)^{\frac{1}{-1+e_2}}\frac{mvn_2}{-w+m}\left[\frac{1}{v}+\frac{\partial m/m}{\partial v}-\frac{\partial m/\partial v}{-w+m}\right]$$

$$=\frac{e_2}{-1+e_2}\left(\frac{mvn_2}{-w+m}\right)^{\frac{e_2}{-1+e_2}}\frac{1}{v}\left[1+\frac{\partial m/m}{\partial v/v}-\frac{m}{-w+m}\frac{\partial m/m}{\partial v/v}\right]$$

$$=\frac{e_2}{-1+e_2}\frac{q}{v}\left[1+\left(1-\frac{m}{-w+m}\right)\frac{\partial m/m}{\partial v/v}\right]=\frac{e_2}{-1+e_2}\frac{q}{v}\left[1+\left(\frac{w}{w-m}\right)\frac{\partial m/m}{\partial v/v}\right],$$

or

$$\frac{\partial q/q}{\partial v/v}=\frac{e_2}{-1+e_2}\left[1+\left(\frac{w}{w-m}\right)\frac{\partial m/m}{\partial v/v}\right], \qquad \text{(A3)}$$

and

$$\frac{\partial q}{\partial w}=\frac{e_2}{-1+e_2}\left(\frac{mvn_2}{-w+m}\right)^{\frac{1}{-1+e_2}}\left[\frac{(-w+m)vn_2\partial m/\partial w-mvn_2(-1+\partial m/\partial w)}{(-w+m)^2}\right]$$

$$=\frac{e_2}{-1+e_2}\left(\frac{mvn_2}{-w+m}\right)^{\frac{1}{-1+e_2}}\frac{mvn_2}{-w+m}\left[\frac{\partial m/m}{\partial w}-\frac{1}{-w+m}\left(-1+\frac{\partial m}{\partial w}\right)\right]$$

$$=\frac{e_2}{-1+e_2}\left(\frac{mvn_2}{-w+m}\right)^{\frac{e_2}{-1+e_2}}\frac{1}{w}\left[\frac{\partial m/m}{\partial w/w}+\frac{w}{-w+m}\left(1-\frac{\partial m}{\partial w}\right)\right]$$

$$=\frac{e_2}{-1+e_2}\frac{q}{w}\left[\frac{\partial m/m}{\partial w/w}-\frac{w}{-w+m}\frac{\partial m}{\partial w}+\frac{w}{-w+m}\right]$$

$$=\frac{e_2}{-1+e_2}\frac{q}{w}\left[\left(1-\frac{m}{-w+m}\right)\frac{\partial m/m}{\partial w/w}+\frac{w}{-w+m}\right]=\frac{e_2}{-1+e_2}\frac{q}{w}\left[\frac{w}{w-m}\left(\frac{\partial m/m}{\partial w/w}-1\right)\right],$$

or

$$\frac{\partial q/q}{\partial w/w}=\frac{e_2}{-1+e_2}\left[\frac{w}{w-m}\left(\frac{\partial m/m}{\partial w/w}-1\right)\right]. \qquad \text{(A4)}$$

Substituting Equation (A1) in (A3) gives:

$$\frac{\partial q/q}{\partial v/v} = \frac{e_2}{-1+e_2}\left[1+\left(\frac{w}{w-m}\right)\left(\frac{e_1}{1+e_1}\left[\frac{\partial q/q}{\partial v/v}-1\right]\right)\right].$$

Rearrange:

$$\left[1-\frac{e_2}{-1+e_2}\left(\frac{w}{w-m}\right)\left(\frac{e_1}{1+e_1}\right)\right]\frac{\partial q/q}{\partial v/v} = \frac{e_2}{-1+e_2}\left[1-\left(\frac{w}{w-m}\right)\left(\frac{e_1}{1+e_1}\right)\right]. \quad \text{(A5)}$$

Substituting Equation (A2) in (A4) gives:

$$\frac{\partial q/q}{\partial w/w} = \frac{e_2}{-1+e_2}\left[\frac{w}{w-m}\left(\frac{e_1}{1+e_1}\left[1+\frac{\partial q/q}{\partial w/w}\right]-1\right)\right].$$

Rearrange:

$$\left[1-\frac{e_2}{-1+e_2}\left[\frac{w}{w-m}\left(\frac{e_1}{1+e_1}\right)\right]\right]\frac{\partial q/q}{\partial w/w} = \frac{e_2}{-1+e_2}\left[\frac{w}{w-m}\left(\frac{e_1}{1+e_1}-1\right)\right]. \quad \text{(A6)}$$

Appendix B. Data sources

The data on *real GDP* (in mln 1982-dollars), the *stock of capital* (in mln 1987-dollars), and *employment* are made available by the Groningen Growth and Development Centre. The *utilisation rate of capital* is based on two sources: the Board of Gouvernors of the Federal Reserve System (1967-1986) and the Federal Reserve Bulletin of January 1999 (1987-1996). Government bond yields (code USI61...) as a proxy for the *nominal interest rate*, GDP - Implicit price index uit (code S001000E) as proxy for *inflation* and GDP - Compensation of employees (code US002310B) as a proxy for the *wage bill* are taken from Datastream. *The real interest rate* is calculated as the difference between the nominal interest rate and inflation. The *real wage rate* is calculated as the wage bill divided by employment and deflated using inflation.

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