Collateral and Debt Maturity Choice:
A Signaling Model

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Abstract

This paper derives optimal loan policies under asymmetric information where banks offer loan contracts of long and short duration, backed or unbacked with collateral. The main novelty of the paper is that it analyzes a setting in which high quality firms use collateral as a complementary device along with debt maturity to signal their superiority. The least-cost signaling equilibrium depends on the relative costs of the signaling devices, the difference in firm quality and the proportion of good firms in the market. Model simulations suggest a non-monotonic relationship between firm quality and debt maturity, in which high quality firms have both long-term secured debt and short-term secured or non-secured debt.

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1. INTRODUCTION

The role of asymmetric information on debt maturity choices has been the subject of a debate for quite some time, both in the theoretical and empirical literature. Among other theoretical studies in this field (Robbins and Schatzberg, 1986; Kale and Noe, 1990 and Diamond, 1993), the models by Flannery (1986) and Diamond (1991) emphasize the signaling properties of the debt maturity choice. In both models, firms have private information about their two-period projects and firms may signal their quality by borrowing on a short-term basis. However, there are also considerable differences between the two models. Flannery analyzes a complete contracting model for high and low quality firms, while Diamond considers an incomplete contracting model for firms with different risk ratings. Unlike Flannery's model, the Diamond's model assumes short-term liquidity risk. The empirical implications of both models also differ. Flannery's model predicts debt maturity to be positively related to firms' quality: high quality firms will borrow on a long-term basis, whereas low quality firms will use short-term debt. The model by Diamond, on the other hand, predicts a non-monotonic relationship between firm risk and debt maturity. In his model, the extremely risky firms do not have access to long term debt and need to borrow short, the intermediate-risk firms will borrow long and the low risk firms will borrow short.

The empirical literature provides some support for the theoretical models proposed by Flannery and Diamond. Consistent with the predictions of Flannery and Diamond, Berger et al. (2004), Stohs and Mauer (1996), and Barclay and Smith (1995) find that firms with high bond ratings tend to use more short-term debt while firms with low bond ratings tend to have more long-term debt and firms without ratings have short-term debt. In contrast to the empirical predictions of both models, moreover, several empirical papers demonstrate that high quality firms do borrow on a long-term basis. For instance, Scherr and Hulbert (2001), using an accounting measure-Altman Z-score-to proxy for credit quality, find that high quality firms borrow on both a long- and short-term basis whereas low quality firms are restricted to long-term debt only. Furthermore, Molina and Penas (2004) provide evidence in favor of high quality firms that use long-term debt. Thus, unlike the predictions of the main signaling debt maturity models, the empirical literature suggests that high quality firms may borrow on both short- and long-term bases [1].

This paper contributes to the literature on signaling and debt maturity choice. In line with Flannery's and Diamond's work, our model considers a two-period asymmetric information setting between firms of different quality and a perfectly competitive bank. However, in contrast to all models we are aware of, we allow firms to signal

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[1] For other empirical papers testing the choice of debt maturity as a signaling tool, see e.g. Guedes and Opler (1996), Mitchell (1993), and Scherr and Hulburt (2001).

with two debt instruments. Specifically, we analyze the case where firms have the possibility to signal with collateral, in addition to debt maturity. In practice, debt contracts often contain clauses regarding both debt maturity and collateral. Our analysis therefore deals with an important policy issue, and is more in accordance with the observed regularities than are the models that analyze a single signaling instrument[2]. Our aim is to derive optimal loan policies under asymmetric information where banks offer loan contracts of long and short duration, backed or unbacked with collateral[3]. Our model provides a theoretical backing to a wide range of empirical outcomes. Further, in line with Diamond and Flannery, our model predicts the most risky firms to borrow on a long-term basis without collateral. However, for the less risky firms our model provides a justification for borrowing short-term debt, with or without collateral, and borrowing long-term debt without collateral. Thus, our analysis provides a broader justification for empirical regularities than most existing models.

[2] Since we focus on signaling properties of duration and collateral, we abstract from other factors associated with the debt maturity and collateral decisions. Therefore, we do not deal with e.g. tax-timing arguments of the debt maturity structure (see e.g. Brick and Palmon, 1992), maturity-matching arguments of duration, liquidity risk arguments of short-term debt (see Diamond, 1991) and the consequences of a firm's debt maturity decision on agency costs (see e.g. Myers, 1977). We also abstract from traditional trade-off theory arguments related to collateral, according to which a firms ability to obtain funds from banks is limited to the value of its collateralizable assets.

[3] The screening and signaling role of collateral has been theoretically well explored by Bester (1985, 1987), Besanko and Thakor (1987). See Coco (2000) for a more extensive survey.

We show that the choice for and the relevance of using either a particular signaling instrument or two signaling instruments at the same time, depends e.g. on the proportion of good firms in the market, the difference in quality between the firm willing to signal and the most risky firm in the market, and the relative costs of the available signaling possibilities. To better explain the empirical implications of our model, we describe a possible set of outcomes based on a model simulation. This simulation provides evidence suggesting a non-monotonic relationship between firm quality and debt maturity. For a particular parameter setting, we show that the most risky firm will borrow long without collateral, firms that are slightly less risky and the group with the lowest risk firms will borrow long with collateral, and the intermediate-risk firms will borrow short, with or without collateral. Most importantly, our analysis shows that the resulting equilibrium depends on a combination of a wide range of parameters, and therefore cannot be described by a simple rule, such as high quality firms will borrow short and low quality firms will borrow long. The crux of the matter is that the choice for short or long term debt also depends on the availability and costs of other signaling instruments. This seems obvious, but has never been taken into account in the existing empirical and theoretical debt maturity literature.

The paper is organized into 6 sections. Section 2 provides a general outline of the model. Section 3 derives the bank's optimal loan strategy in a full information setting. Section 4 introduces asymmetric information and examines the optimal loan policy. Section 5 sets out some empirical implications of the analyses. Section 6 summarizes our results and provides some areas for further research.

2. GENERAL OUTLINE OF THE MODEL

We consider a two-period model with firms and a competitive banking system embodied by a representative bank. At time t=0, firms are endowed with a risky investment project which lasts for two periods. If the investment project is carried out, all cash flows will occur at the end of period 2. Firms do not have initial wealth, which necessitates outside finance. All investment projects require a unit of investment, and thus a unit of external finance. The projects can be financed with short-term (*s*) or long-term (*l*) debt. The maturity time ($m \in (l,s)$) in the model should be interpreted as being defined relative to the timing of the cash flows, rather than in terms of calendar time (see Diamond, 1991). Firms need to pay an additional amount of fixed transaction cost *b* if they issue short-term debt instead of long-term debt.

There are two types of firms, good (*g*) and bad (*b*), who differ in their "up" probabilities – probabilities of success p_i, so the type of firm $i \in (g,b)$. The proportion of good firms is equal to θ. Under the asymmetric information setting, the bank only knows ex-ante the distribution of firms (i.e. the bank knows that a proportion θ of the firms are good borrowers), but the particular borrower's probability of success is private.

During each period there is a probability p_i that the project increases in value. M_1 M_2 are the interim values of the project at t=1. At t= 0 the bank and the firms know that the project's liquidating value at t =2 will be M_3 with probability p_i^2, M_4 with probability $2p_i(1-p_i)$ and 0 with probability $(1-p_i)^2$.

The time profile of the project's value is similar to the Flannery's (1986) profile, as described in Figure 1.

In addition to the interest factor (one plus the loan rate: R_i), the perfectly competitive bank may require collateral (C_i) and seize it in case the project's liquidation value is insufficient to repay the debt. Firms have other asset(s) which cannot be liquidated at t=0 or t=1 for financing purposes, but can be posted as collateral. Firms face the cost of collateralization, which is proportional to the amount of collateral they post. This cost can be recognized as the costs of collection and marketing of the collateralized assets (Barro, 1976), legal or monitoring cost (Chan and Kanatas, 1985) or dissipative cost in liquidating collateral (Boot, Thakor, Udell, 1991) and are entirely incurred by firms as borrowers.

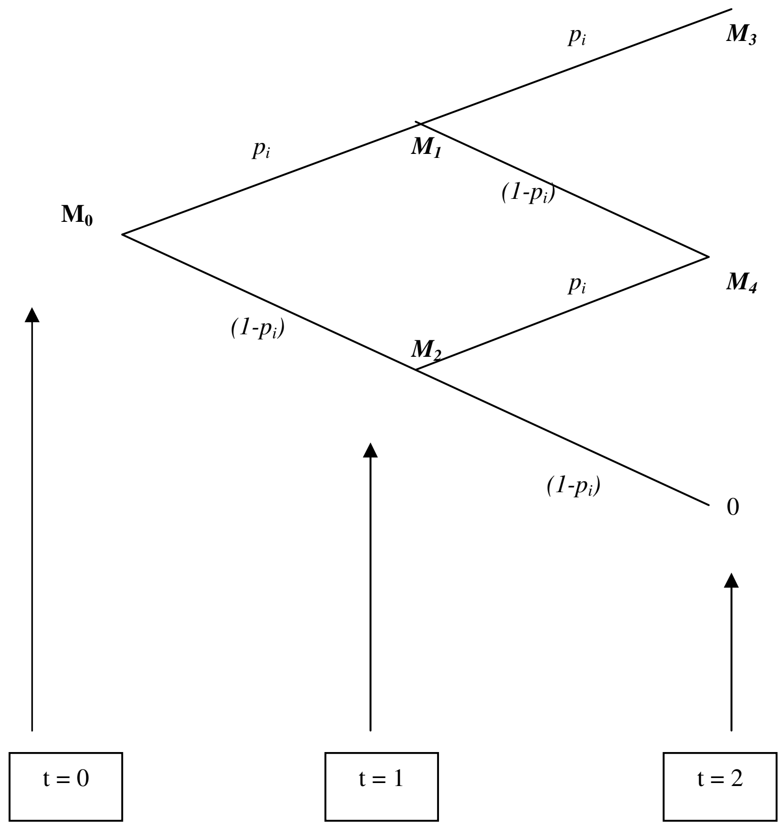

Figure 1: time profile of the model

We assume the following:

A.1: $0<p_b<p_g<1$

A.2: Firms and the bank are risk neutral

A.3: The bank makes zero profits

A.4: The risk-free interest rate (the opportunity cost of capital per loan) is zero.

A.5: M_1, M_2, M_3, and M_4 are larger than the promised debt repayments (principal plus interest).

A.6: The transaction costs (refinancing costs) of short-term debt equal *b*; the transaction costs of long-term debt equal 0.

A.7: Firms finance their investment projects with either short- or long-term debt. We ignore the possibility of a combination of the two forms of debt.

A.8: The costs of collateral are proportional to the amount of collateral by a factor *k*, so additional costs of collateralization equal *kC*.

A.9: $0 \leq C_i < 1$

A.10: The firms' collaterizable wealth *W* exceeds the needed collateral.

Most assumptions of our model are straightforward. Some, however, need additional explanation. A.5 implies that we assume that all debt maturing at t=1 is riskless (no liquidity risk), which is in line with Flannery's (1986) assumption. This assumption is made to not further complicate the model. An obvious drawback of this choice is that we ignore the possibility of firms' short-term liquidation. Several papers (e.g.

Diamond, 1991) emphasize that using short-term debt is advantageous since short-term debt may help to avoid strategic defaults by the threat of short-term liquidation of the firm. A.6 reflects the assumption that total transaction costs for short-term debt are higher than for long-term debt, the reason being that firms, which decide to finance with short-term debt need to consult more often (twice as much in our model) to a bank than firms who finance with long-term debt. The transaction costs *b* for short-term debt can therefore be interpreted as the additional costs of financing with short-term debt rather than with long-term debt. A.8 explains how we model the costly collateralization. Like Bester (1985), firms bear these costs, and banks do not take them into account when setting an interest rate. In many papers on this topic (Barro, 1976, Chan and Kanatas, 1985, Boot, Thakor and Udell, 1991), cost of collateralization creates a disparity in value of collateral between banks and firms[4]. A.9 implies that we rule out the uninteresting case where loans can become entirely riskless if they are backed by collateral. Finally, A.10 implies that we assume that firms are not wealth constrained since the value of the collaterizable assets W always exceeds the collateral requirement.

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[4] Some papers, e.g. Boot, Thakor and Udell (1991), assume that the dissipative costs of collateral are smaller for long-term debt than for short-term debt. The reason for this difference is that the bank has more timing flexibility in terms of when to force default with

3. FULL INFORMATION

To provide a benchmark, we start by assuming that the bank can identify the quality of the borrowers without costs. From Figure 1 we derive the valuation of a firm's equity if it borrows long (l) or short (s)[5], puts up a positive amount of collateral ($C_i > 0$) or zero collateral ($C_i = 0$). Assuming risk neutrality and a zero risk-free discount rate, we obtain the following results.

If a firm i, $i \in$ {g, b) borrows long the valuation of its equity (V_{li}) is equal to

$$(1)\quad V_{li} = p_i^2(M_3 - R_{li} - kC_{li}) + 2p_i(1-p_i)(M_4 - R_{li} - kC_{li}) - (1-p_i)^2(1+k)C_{li}$$

$$= p_i^2 M_3 + 2p_i(1-p_i)M_4 - p_i(2-p_i)R_{li} - (1-p_i)^2 C_{li} - kC_{li}$$

where R_{li} is the loan interest rate on long-run debt for borrower i.

If a firm i uses short-run debt, the equity value (V_{si}) is equal to

long-term debt than with short-term debt. However, this result is based on the idea of renegotiating possibilities, which we ignore.

[5] Recall that long-term debt is two-period debt issued at t=0 and that short-term debt is one-period debt issued at t=0 and t=1. Moreover, recall that we assume that short-term debt issued at t=0 is riskless (so that the lending rate equals 1) and that short-term debt issued at t=1 is risky.

$$V_{si} = p_i^2(M_3 - 1 - kC_{si}) + p_i(1-p_i)(M_4 - 1 - kC_{si})$$

$$(2) + p_i(1-p_i)(M_4 - R_{si} - kC_{si}) - b - (1-p_i)^2(1+k)C_{si}$$

$$= p_i^2 M_3 + 2p_i M_4(1-p_i) - p_i - (1-p_i)^2 C_{si} - p_i(1-p_i)R_{si} - kC_{si} - b$$

With perfect information, the bank knows the borrowers' probability of success (the "up" probabilities). Under the zero profit constraints, the short and long term loan rates are given by:

$$(3)\ 1 = R_{li}\left[p_i^2 + 2p_i(1-p_i)\right] + (1-p_i)^2 C_{li} \Rightarrow R_{li} = \frac{1-(1-p_i)^2 C_{li}}{p_i(2-p_i)}$$

$$(4)\ 1 = p_i R_{si} + (1-p_i)C_{si} \Rightarrow R_{si} = \frac{1 - C_{si}(1-p_i)}{p_i}$$

By substituting (3), and (4) in (1) and (2), respectively, the equity values for a firm *i* using short or long-term debt can be derived:

$$(5)\ V_{li} = p_i^2 M_3 + 2p_i(1-p_i)M_4 - 1 - kC_{li}$$

$$(6)\ V_{si} = p_i^2 M_3 + 2p_i(1-p_i)M_4 - 1 - kC_{si} - b$$

We also assume that the credit contracts are individually rational, i.e.,

A.11: $V_{im} > 0$ for $i \in \{g, b\}$ and $m \in \{l, s\}$

Proposition 1: Under A1-A11, the full information competitive equilibrium implies that both groups of firms borrow long without collateral. So, the full information equilibrium policy is given by $R_{li} = \frac{1}{p_i(2 - p_i)}$; $C_i = 0$ and $m_i = l$.

Proof**:** This solution of the optimal loan policy is straightforward. The bank optimizes each type of borrower's expected utility subject to the zero profit constraints and the participation constraints. It is obvious that the bank's optimal policy will never imply that a firm borrows short or that the loan is backed by collateral, since borrowing short and/ or securing a loan is costly. ■

4. ASYMMETRIC INFORMATION

We now assume that the bank does not know the type of the firm it faces, i.e. the probability of success of the firms is unknown to the bank. The bank will therefore set an average loan interest rate, which may cause the good firms to be undervalued and the risky firms to be overvalued. In this section, we attempt to examine whether good firms can signal their superior quality by using two debt instruments, maturity and collateral and how the signaling mechanism works. We aim to derive the optimal loan policy in equilibrium. Before proceeding we explain the equilibrium concept we employ.

4.1 THE EQUILIBRIUM CONCEPT

In our model, we focus on *Perfect Bayesian Equilibria (PBE)*[6], which can be classified into *separating PBE* and *pooling PBE*. *Separating PBE* occur when each type of firm chooses a different borrowing strategy; the observed signal therefore reflects the firm's type correctly. *Pooling PBE* occur when both types of firm opt for the same borrowing strategy; the observed signal therefore reveals no additional information about the firm's type. Since the *PBE* concept does not impose restrictions on out-of-equilibrium beliefs we follow the concept of the *Intuitive Criterion* (*IC*) formulated by Cho and Kreps (1987) to rule out perfect Bayesian equilibria that are upheld by unreasonable off-equilibrium beliefs.[7] In line with Spence (1973) and Riley (1979), the separating signaling equilibria we consider should satisfy the *Incentive Compatibility Constraint (ICC)* and the *Competitive Rationality Condition (CRC).* The *ICC* ensures that each agent is personally interested in accepting the contract designed for his type rather than the contract designed for the other type of agent. The *CRC* in our setting requires the credit market to be perfectly competitive and banks to have rational expectations so that in equilibrium they do not make profits and the loan

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[6] A *PBE* is defined as a set of strategies and beliefs such that (Rasmusen, 1989, p. 146): 1) the strategies for the remainder of the game are Nash given the beliefs and strategies of the other players; 2) the beliefs at each information set are rational given the evidence appearing thus far in the game. This means that along the equilibrium path beliefs are based on priors updated by Bayes' Rule, if possible. Off the equilibrium path, Bayes updating is not possible since the deviating action is taken with probability zero in equilibrium.

[7] The *IC* restricts the out-of-equilibrium beliefs by requiring that the uninformed player's belief must put zero probability on an informed player who would not benefit from the off-equilibrium action no matter what beliefs were held by the bank. We use the *IC* to rule out unreasonable perfect Bayesian pooling equilibria. Note that in the setting of Rotschild and Stiglitz (1976) there cannot be a pooling Nash equilibrium under asymmetric information.

interest rate correctly reflects firms' riskiness. Our model proposes a continuum of separating equilibria, but the *IC* restricts the separating equilibria to *least-cost separating equilibria.* These equilibria are such that the bad firms do not signal, and the good firms choose the minimum level of signaling that allows them to be separated without attracting the bad firms. These equilibria are the most efficient, perfect Bayesian equilibria, in that they entail the least wasteful signaling costs.

4.2 THE BORROWING AND SIGNALING POSSIBILITIES

Table 1. Different borrowing strategies

Good\Bad	L with C	L without C	S with C	S without C
L with C	1: P	2: S	3: S	4: S
L without C	5: S	6: P	7: S	8: S
S with C	9: S	10: S	11: P	12: S
S without C	13: S	14: S	15: S	16: P

Notes: P (S) means a candidate pooling (separating) equilibrium. L denotes long-term debt; S denotes short-term debt; C denotes collateral.

Table 1 presents all possible borrowing choices for both types of firms. The optimal borrowing strategy of each type depends on the behavior of the other type. If good firms succeed in signaling their quality by borrowing short, with or without collateral, or by borrowing long with collateral, a separating equilibrium may occur. However, bad firms may decide to mimic good firms and good firms can voluntarily decide not to signal their quality. By doing so, both groups of firms may opt for a pooling equilibrium if they achieve higher values.

4.3 THE SET OF PERFECT BAYESIAN EQUILIBRIA

We identify the set of *PBE* by ruling out borrowing possibilities that do not constitute a *PBE*. Under these possibilities, bad firms always tend to deviate from the original borrowing strategy to the perfect information strategy no matter what the bank believes. This rule allows us to discard immediately all separating possibilities where bad firms signal, depicted as cases 3, 4, 5, 7, 8, 9, 12, 13 and 15 in Table 1. If separation occurs, bad firms always prefer not to signal to avoid signaling costs. Thus, for any contract that is a candidate for a separating equilibrium, the contract offered by the bad firm should coincide with its full information contract (see, for instance, Macho-Stadler and Perez-Castrillo, 2001, p. 203, Result 5.6). Furthermore, we also eliminate conceivable pooling equilibria, where bad firms have incentives to switch to their full information contract. This may occur for all conceivable pooling equilibria with positive signaling costs. Thus, we derive conditions for which moving to the full information contract provides bad firms with a higher value than pooling. If such conditions do not hold, separating equilibria do not exist. In order to make the necessary calculations, we need to derive expressions for the common interest rates if firms decide to pool. There are several possibilities. They may pool by both borrowing long or by both borrowing short: in both cases they may back the loan with collateral. Under a pooling equilibrium, the bank knows that the proportion of good firms is θ and the proportion of bad firms is 1 - θ. To comply with the *Competitive Rationality Condition,* the *long pooling* loan rate is given by:

$$\begin{aligned} &1 = \theta R_{lp}\left[p_g^2 + 2p_g(1-p_g) \right] + \theta(1-p_g)^2 C_{lp} \\ (7)\quad &+(1-\theta)R_{lp}\left[p_b^2 + 2p_b(1-p_b) \right] + \theta(1-p_b)^2 C_{lp} \\ &\Rightarrow R_{lp} = \frac{1 - C_{lp}\left[\theta(1-p_g)^2 + (1-\theta)(1-p_b)^2 \right]}{\theta(2-p_g)p_g + (1-\theta)(2-p_b)p_b} \end{aligned}$$

The *short pooling* loan interest rate is determined as follows. We know that short-term debt issued at t = 0 is riskless since all firms reach either M_1 or M_2. However, short-term debt issued at t=1 is subject to default risk if the borrowers decrease in value at t=2. Since good and bad firms differ in their probabilities of reaching M_2, *(1-p_g)* of the good firms and *(1-p_b)* of the bad firms will arrive at state M_2. Therefore

$\left[\frac{\theta(1-p_g)}{1-(\theta p_g+(1-\theta)p_b)}\right]$ of good firms and $\left[\frac{(1-\theta)(1-p_b)}{1-(\theta p_g+(1-\theta)p_b)}\right]$ of bad firms borrow at t

= 1. Under *CRC* or the zero profit constraints, the loan rate then must satisfy

$$(8)\quad R_{sp}=\frac{1-(\theta p_g+(1-\theta)p_b)-C_{sp}\left[\theta(1-p_g)^2+(1-\theta)(1-p_b)^2\right]}{\theta(1-p_g)p_g+(1-\theta)(1-p_b)p_b}$$

The subscript p denotes pooling.

Lemma 1: (i) the candidate pooling equilibrium where both groups of firms borrow long with collateral is not a perfect Bayesian equilibrium if

$$(9)\, C_{lp}\geq\frac{\theta\left[p_g(2-p_g)-p_b(2-p_b)\right]}{(1+k)\theta\left[p_g(2-p_g)-p_b(2-p_b)\right]+kp_b(2-p_b)}$$

(ii) The candidate pooling equilibrium where both groups of firms borrow short without collateral is not a perfect Bayesian equilibrium if

$$(10)\, b_p\geq(1-p_b)\left[1-p_b\frac{1-(\theta p_g+(1-\theta)p_b)}{\theta p_g(1-p_g)+(1-\theta)p_b(1-p_b)}\right].$$

(iii) The candidate pooling equilibrium where both groups of firms borrow short with collateral is not a perfect Bayesian equilibrium if

$$(11)\ C_{sp} \geq \frac{\theta(1-p_g)(1-p_b)(p_g-p_b)-b\left[\theta p_g(1-p_g)+(1-\theta)p_b(1-p_b)\right]}{\theta(1-p_g)(1-p_b)(p_g-p_b)+k\left[\theta p_g(1-p_g)+(1-\theta)p_b(1-p_b)\right]}$$

<u>Proof</u>: See Appendix A.

The conditions (9), (10) and (11) determine when bad firms prefer separating long without collateral over pooling long with collateral, short without collateral and short with collateral, respectively. These conditions also provide the maximum levels of signaling costs for which bad firms can afford to pool with good firms under the three pooling possibilities. The three conditions are more likely to hold if the proportion of bad firms is very high (θ is low). In an extreme case, where $\theta \approx 0$, the conditions will always hold, irrespective of the other parameters, since the right-hand side of these expressions then becomes 0, 0 and $-b/k$, which are always smaller than the left-hand side of these expressions. For the remainder of the analysis, we introduce the above mentioned conditions as additional assumptions for the existence of separating signaling equilibria. We denote the conditions (9), (10) and (11) as A12, A13 and A14, respectively:

A12. $C \geq C_{lp}^{*}$ where C_{lp}^{*} is given by equality (9)

A13. $b \geq b_{p}^{*}$ where b_{p}^{*} is given by equality (10)

A14. $C \geq C^*_{sp}$ where C^*_{sp} is given by equality (11)

4.4 REFINEMENT OF *PBE* BY APPLYING THE INTUITIVE CRITERION.

Given the above-defined restrictions, the set of *PBE* is restricted to four borrowing strategies: pooling long without collateral, and three separating possibilities where bad firms do not signal and good firms can signal with different levels of collateral and short-term debt. In order to further restrict the set of equilibria by using the *IC*, we formulate the following lemma:

Lemma 2: The pooling equilibrium, where both types of firms borrow long without collateral, does not satisfy the *Intuitive Criterion* if one of the following conditions holds:

i) $$\begin{cases} (1-p_b)^2 C_{\lg} + p_b(2-p_b)R_{\lg} + kC_{\lg} \geq p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1 + kC_{\lg} \end{cases}$$

ii) $$\begin{cases} p_b + p_b(1-p_b)R_{sg} + b \geq p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1 + b \end{cases}$$

iii) $$\begin{cases} p_b + (1-p_b)^2 C_{sg} + kC_{sg} + p_b(1-p_b)R_{sg} + b \geq p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1 + kC_{sg} \end{cases}$$

where R_{lg}, R_{sg} and R_{lp} are given by equations (3), (4) and (7), respectively.

Under any of the above conditions, good firms are better off by deviating while bad firms are better off by staying at pooling. As a result, good firms tend to move away from the pooling equilibrium. The pooling long without collateral is therefore said to fail the *Intuitive Criterion*.

Proof: See Appendix B.

By precluding the pooling long without collateral, the above conditions guarantee the existence of separating equilibria.

4.5 INCENTIVE COMPATIBILITY CONSTRAINTS

Leaving out the borrowing possibilities that are not *PBE*, and ignoring the pooling equilibrium that does not satisfy the *Intuitive Criterion*, we are endowed with the set of candidate separating *PBE* presented by cases 2, 10 and 14 in Table 1. In equilibrium, good firms may exercise one of the following options to signal: borrowing long-term debt with collateral (Separation I), borrowing short-term debt without collateral (Separation II), or borrowing short-term debt with collateral (Separation III). A firm will choose the separating possibility that is the most efficient and gives the highest value, under the condition that A1-A14 hold and that the separating is incentive compatible.[8]

We denote $\hat{e}_g$ and $\hat{e}_b$ as the incentive compatible contract chosen by good and bad firms, respectively, and $\hat{V}_g$ and $\hat{V}_b$ as the values from incentive compatible contracts for good firms and bad firms, respectively. The *ICCs* are formulated as:

$$U(\hat{e}_g, g, \hat{V}_g) \geq \arg\max EU[e_b, g, V_b]$$
$$U(\hat{e}_b, b, \hat{V}_b) \geq \arg\max EU[e_g, b, V_g]$$

The *ICCs* require firms to be honest about their type in separation, i.e. the *ICCs* induce firms to prefer their own contract rather than mimicking through a choice of the contracts for the other type. The following lemma details the ICCs.

Lemma 3. The incentive compatibility requires the following conditions to hold

i) $\begin{cases} (1-p_b)^2 C_{\text{lg}} + p_b(2-p_b)R_{\text{lg}} + kC_{\text{lg}} \geq 1 \\ p_g(2-p_g)R_{lb} \geq 1 + kC_{\text{lg}} \end{cases}$ for Separation I,

[8] Flannery (1986) does not examine whether the separating equilibrium is incentive compatible. He derives parameter restrictions for different types of equilibria (pooling and separating) by simply comparing values for high-quality firms under different pooling possibilities with the value for high-quality firms under a separating equilibrium. Thus, the actual conditions that allow high-quality firms to separate themselves from low-quality firms by issuing short-term debt are expected to be much more restrictive than those given in Flannery (1986).

$$\text{ii)}\begin{cases} p_b + p_b(1-p_b)R_{sg} + b \geq 1 \\ p_g(2-p_g)R_{lb} \geq 1+b \end{cases} \qquad \text{for Separation II,}$$

$$\text{iii)}\begin{cases} p_b + (1-p_b)^2 C_{sg} + kC_{sg} + p_b(1-p_b)R_{sg} + b \geq 1 \\ p_g(2-p_g)R_{lb} \geq 1 + kC_{sg} + b \end{cases} \qquad \text{for Separation III}$$

where R_{lg} and R_{sg} are given by equation (3) and (4), respectively.

Conditions i), ii) and iii) ensure that under Separations I, II and III, respectively, both types of firm will present themselves rather than mimicking the other type.

Proof: See Appendix C.

To derive feasible incentive compatible separating equilibria, we should also consider the conditions for the existence of separation. More specifically, combining the conditions implied by the *IC* in Lemma 2, and the *ICC* conditions in Lemma 3, we establish the following lemma.

Lemma 4: The feasible incentive compatible separating equilibria require the following conditions to hold

$$\text{i)}\begin{cases} (1-p_b)^2 C_{\text{lg}} + p_b(2-p_b)R_{\text{lg}} + kC_{\text{lg}} \geq 1 \\ p_g(2-p_g)R_{lp} \geq 1 + kC_{\text{lg}} \end{cases} \qquad \text{for Separation I}$$

ii) $\begin{cases} p_b + p_b(1-p_b)R_{sg} + b \geq 1 \\ p_g(2-p_g)R_{lp} \geq 1+b \end{cases}$ for Separation II

iii) $\begin{cases} p_b + (1-p_b)^2 C_{sg} + kC_{sg} + p_b(1-p_b)R_{sg} + b \geq 1 \\ p_g(2-p_g)R_{lp} \geq 1 + kC_{sg} + b \end{cases}$ for Separation III

<u>Proof</u>: See Appendix D

Lemma 4 implies the following for each candidate of separation in equilibrium

For Separation I:

(12) $k_d \leq k \leq k_u$ with $$k_d = \frac{\left[(1-p_b)^2 - (1-p_g)^2\right](1-C_{lg})}{p_g(2-p_g)C_{lg}}$$ $$k_u = \frac{p_g(2-p_g)R_{lp} - 1}{C_{lg}}$$

From condition (12), we can derive the minimum level of collateral necessary for incentive compatibility to hold:

(13) $$C_{lgmin} = \frac{(1-p_b)^2 - (1-p_g)^2}{p_g(2-p_g)(1+k) - p_b(2-p_b)}$$

Note that this level of collateral satisfies assumption A12: $C_{lgmin} > C_{lp}*$.

<u>Proof</u>: see Appendix E1

If Separation I occurs, good firms minimize signaling costs if they offer the minimum level of collateral, C_{lgmin}. If they do so, the lower boundary of condition (12) will automatically be satisfied, since then $k_d = k$. A feasible solution also requires that the upper boundary of condition (12) exceeds the lower boundary. Given that they choose C_{lgmin}, it can be derived that Separation I may occur under the following condition:

$$(14)\ \theta \le \theta_1 = \frac{p_g(2-p_g)-p_b(2-p_b)(1-C_{\lg\min})}{p_g(2-p_g)+\left[p_g(2-p_g)-p_b(2-p_b)\right](1-C_{\lg\min})},$$

For Separation II the following should hold:

$$(15) \quad b_d < b < b_u \text{, with } \begin{array}{l} b_d = \dfrac{(p_g - p_b)(1-p_b)}{p_g} \\ b_u = p_g(2-p_g)R_{lp} - 1 \end{array}$$

Expression (15) implies that costs of short-term debt should exceed a certain threshold to make it unattractive for bad firms to mimic good firms, and should be lower than another threshold to induce good firms to be truthful. Note that b_d is always greater than b_p^* given in assumption A13.

Proof: see the appendix E2.

Condition (15) is feasible if $b_d < b_u$. This implies:

$$(16)\quad \theta \leq \theta_2 = \frac{p_g^2(2-p_g)-p_b(2-p_b)\left[(p_g-p_b)(1-p_b)+p_g\right]}{\left[p_g(2-p_g)-p_b(2-p_b)\right]\left[(p_g-p_b)(1-p_b)+p_g\right]}$$

Thus Separation II may result if (15) holds and a necessary (but not sufficient) condition that $\theta \leq \theta_2$.

For Separation III the following must hold:

$$(17)\quad b_d^* \leq b \leq b_u^* \quad \text{with} \quad \begin{aligned} b_d^* &= b_d - C_{sg}\left[k + b_d\right] \\ b_u^* &= b_u - kC_{sg} \end{aligned}$$

Note that condition (17) allows lower values of *b* as compared to condition (15). This can be explained by the fact that good firms simultaneously put up collateral and issue costly short-term debt under this separation. Therefore, even for $b < b_d$, separation may be incentive compatible as good firms now also signal with collateral. From condition (17), we can derive the minimum level of collateral that good firms need to offer for this separation:

$$(18)\, C_{sgmin} = \frac{b_d - b}{b_d + k}$$

Equation (18) clearly shows that the minimum level of collateral needed to make separation incentive compatible decreases if the costs of issuing short-term debt increase. Note that Csgmin > C_{sp}* , given by assumption A14, irrespective of all parameters.

<u>Proof</u>: see Appendix E3.

If Separation III occurs, good firms will offer collateral of C_{sgmin}. This guarantees that the lower boundary of condition (17) will be fulfilled since then $b_d^* = b$. To make the condition feasible, the upper boundary should exceed the lower boundary i.e. $b_d^* \leq b_u^*$. This requires:

$$(19)\quad \theta \leq \theta_3 = \frac{p_g(2-p_g) - p_b(2-p_b)\left[1+b_d(1-C_{sg\min})\right]}{\left[p_g(2-p_g) - p_b(2-p_b)\right]\left[1+b_d(1-C_{sg\min})\right]}$$

Note that for $b < b_{d,}$ $\theta_3 > \theta_2$ independent of the other parameter values. The reverse holds when $b > b_d$. This specification is relevant to distinguish among different separation possibilities, as will be analyzed in the following section.

4.6 THE LEAST-COST SEPARATING EQUILIBRIUM

The final point in our analysis of the optimal loan policy under asymmetric information is to solve for the least-cost separating equilibrium. It should be noticed that a separating equilibrium will only exist if the proportion of bad firms in the market is sufficiently high, $\theta < \max[\theta_1, \theta_2, \theta_3]$. Moreover, the conditions derived above imply that it is impossible that both Separations II and III are feasible. Separation III will only be incentive compatible (for positive collateral values) if $b<b_d$ whereas Separation II requires $b_d<b<b_u$. However, the conditions as specified in equations (12), (15) and (17) may satisfy either Separations I and II, or Separations I and III. The least-cost separating equilibrium then determines the overall optimum outcome.

We determine the least-cost separating equilibrium by comparing the value of good firms under the different separating equilibria. From equations (5) and (6), the values of good firms under the three alternative separating equilibria can be derived as follows, respectively:

$$V_{\lg c} = p_g^2 M_3 + 2p_g(1-p_g)M_4 - 1 - kC_{\lg \min}$$
$$V_{sg} = p_g^2 M_3 + 2p_g M_4(1-p_g) - 1 - b$$
$$V_{sgc} = p_g^2 M_3 + 2p_g M_4(1-p_g) - 1 - kC_{sg\min} - b$$

where C_{lgmin} and C_{sgmin} are given by equation (13) and (18). We can now formulate the following Lemma.

Lemma 5:

(i) Given A1 – A14, and if parameter values are such that Separation long with collateral and Separation short without collateral are both incentive compatible, good firms will opt to separate by borrowing short without collateral if $b < kC_{lgmin}$

(ii) Given A1 – A14, and if parameter values are such that Separation long with collateral and Separation short with collateral are both incentive compatible, good firms will opt for Separation short with collateral if $b + kC_{sgmin} < kC_{lgmin}$ or

$$b < k\left[\frac{C_{lg\min}(b_d + k)}{b_d} - 1\right]$$

From Lemma 5 it follows that $\theta_1 < \theta_2$ and $\theta_1 < \theta_3$ if conditions i) and ii) hold respectively.

Proof: see Appendix F

Lemma 5 provides a first step in determining the conditions for an overall optimal, i.e. least-cost equilibrium, given that the alternative Separations I, II and III are all incentive compatible. The following proposition serves to prepare a next step in determining the least-cost separating equilibria.

Proposition 2: Under A1-A14, different separating equilibria may occur conditionally on the following parameter restrictions.

- Separation I is optimal if

C1) $\theta < \theta_1$ and

C2) $b >$ max[b_d, kC_{lgmin}] or

C3) $k\left[\frac{C_{lg\min}(b_d+k)}{b_d}-1\right] < b < b_d$

The least-cost separating signaling equilibrium is then characterized by the following implication:

$$C_{lb}=0;\ C_{lg}=C_{lgmin}=\frac{(1-p_b)^2-(1-p_g)^2}{p_g(2-p_g)(1+k)-p_b(2-p_b)};\ m_g=1;\ m_b=1;$$

$$R_{lg}=\frac{1-(1-p_g)^2 C_{lgmin}}{p_g(2-p_g)} \text{ and } R_{lb}=\frac{1}{p_b(2-p_b)}$$

In equilibrium, both types of firms borrow long-term debt; good firms put up an amount C_{lgmin} of collateral while bad firms do not.

- Separation II is optimal if

C4) $b_d < b < b_u$ and $\theta_1 < \theta < \theta_2$ or

C5) $\theta < \theta_1 < \theta_2$ and $b_d < b <$ min[kC_{lgmin}, b_u]

The least-cost separating signaling equilibrium is then described by

C_b=0; C_g=0; m_g=s; m_b=l; $R_{sg} = \frac{1}{p_g}$ and $R_{lb} = \frac{1}{p_b(2-p_b)}$.

In equilibrium, bad firms choose long-term debt without collateral and good firms opt for short-term debt without collateral.

- Separation III is optimal if

C6) $\theta < \theta_3$ and $b < \min\left[k\left[\frac{C_{lg\min}(b_d+k)}{b_d} - 1\right], b_d\right]$

If C6 hold, the least cost separating equilibrium implies:

C_b=0; C_g=$C_{sgmin} = \frac{b_d - b}{b_d + k}$; m_g=s; m_b=l; $R_{sg} = \frac{1-(1-p_g)C_{sgmin}}{p_g}$ and

$R_{lb} = \frac{1}{p_b(2-p_b)}$.

The equilibrium entails bad firms having long-term debt without collateral and good firms having short-term debt with collateral.

Note that the conditions in proposition 2 are derived from the parameter restrictions given by equation (13) through (19). It is worthwhile to further discuss proposition 2. Condition C1 implies that Separation I is feasible. Separation I will be the least cost separating equilibrium if, in addition to C1, both Separations II and III are either infeasible or incur higher signaling costs than Separation I. Condition C2 indicates that Separation III is not feasible because $b > \max(b_d, kC_{lgmin})$ while Separation II is more costly. On the contrary to condition C2, C3 indicates that Separation II is not feasible and Separation III is more costly than Separation I.

Condition (C4) implies that Separation II is incentive compatible, automatically precluding Separation III. Condition (C4) also rules out Separation I because the values of θ are in excess of θ_1, making Separation II the least cost separating equilibrium. Moreover, if both Separations I and II are feasible, Separation II has the lowest cost if costs of borrowing short are lower than borrowing long with collateral. This is guaranteed by condition (C5). Similar restrictions for Separation III to be the least cost separating equilibrium are given by condition (C6).

5. Wealth constraints and Empirical implications

To better explain the empirical implications of our signaling model, in particular the impact of conditions C1 through C6, we describe a set of possible outcomes based on a model simulation. This allows us to specifically examine the impact of an increase in the difference between firm quality measured by $p_g - p_b$, and in the transaction costs of short-term debt b, for given values of k and θ. The graph below displays the results given a particular parameter setting with $k = 0.7$, $\theta = 0.2$ and $p_b = 0.49$. The curves b_u and b_d are given by equation (15); the curves kC_{lgmin} and $kC_{lgmin}(b_d+k)/(b_d)-k$ are given by conditions i) and ii) in Lemma 5. Notice that in this setting, separation always exists for all possible values of b and (p_g-p_b). Separation II occurs in the shaded area enclosed by the b_u, b_d and kC_{lgmin} schedules. Separation III occurs in the area below the b_d and $kC_{lgmin}(b_d+k)/(b_d)-k$ curves. Finally, Separation I occurs in the remaining area.

The graph shows that for a given b (below the intersection of the kC_{lgmin} and b_d schedules); see $\overline{b}$ for example, Separation I occurs for either small or large values of $(p_g - p_b)$. However, for intermediate values of $(p_g - p_b)$, either Separation II or Separation III results. This can be explained as follows. For small values of $(p_g - p_b)$, b_u and b_d are too small to make Separation II and 3 feasible. For large values of $(p_g - p_b)$, Separations II and III can be feasible but apparently more expensive than Separation I since the kC_{lgmin} and $kClgmin(b_d+k)/(b_d)-k$ schedules fall below b_d. This implies that two types of firms -slightly less risky firms and lowest risk firms- opt for secured debt at long maturities. Firms of intermediate risk will choose short maturities with or without collateral. Finally, the highest risk firms are settled with long-term, non-secured debt.

In addition, the graph also demonstrates how the signaling cost b influences the Separation at equilibrium. For a given value of $(p_g - p_b)$ (to the left of the intersection b_d and kC_{lgmin} schedules), good firms signal by borrowing short with collateral for low values of b. A rise in b such that b is in excess of b_d, leads good firms first to signal by borrowing short without collateral. However, if the transaction costs of short term debt rise dramatically, Separation I will outdo Separation II, since Separation II appears to be either unfeasible or more costly than Separation I.

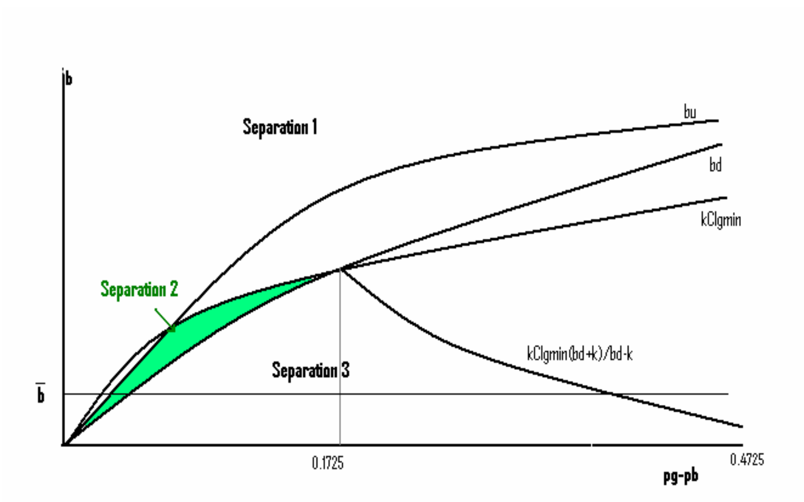

Figure 2. Different separating outcomes, given parameter values $k = 0.7$, $\theta = 0.2$ and $p_b = 0.49$.

In summary, our theoretical analysis and in particular the simulation results provide several empirical implications. Firstly, we show that the resulting separating or pooling equilibrium depends on a combination of parameters.

1) θ: higher values make separation less likely.

As previously noted, a separating equilibrium only exists if the proportion of good firms θ is sufficiently low, i.e. $\theta \le \max[\theta_1, \theta_2, \theta_3]$. By intuition, the higher the proportion of good firms in the credit market, the less rewarding for good firms to signal their true quality by separation.

2) *b:* higher values make signaling by long-term debt with collateral more likely

3) *k:* higher values make signaling by short-term debt without collateral more likely.

An increase in the costs of a given signaling mechanism, i.e. *b* of short-term debt and *k* of collateral, will induce the alternative mechanisms to be optimal in equilibrium.

4) $(p_g - p_b)$: higher values increase the threshold values for θ, below which one of the separating equilibria is feasible, thereby increasing the likelihood of separating.

Secondly, our signaling framework suggests a non-monotonic relationship between firm quality and debt maturity, with high quality firms having both long-term secured debt and short-term secured or non-secured debt.

Overall, the analysis shows that the signaling outcome cannot be described with a simple rule, such as high quality firms will offer collateral (as in the standard signaling models with collateral) or high quality firms will borrow short (as in the standard debt maturity signaling models). Rather, the use of a certain signaling device should be simultaneously determined under the interactions with other signaling devices. Our model provides a theoretical justification for a broad range of possible optimal debt contracts that allow good firms to signal their quality: they may borrow short - with or without collateral - or they may borrow long with collateral. The decision depends on the relative costs of the signaling devices, the difference in firm quality and the proportion of good firms in the market.

Finally, we consider one simple extension of the model. In the analysis so far we have assumed that there is no wealth constraint. However, in the context of small firm financing in developing countries this assumption does not seem to be realistic. A wealth constraint implies that the firms' collaterizable wealth W is smaller than the necessary level of collateral. From the previous analysis, we know that Separation I requires good firms to borrow long with collateral of C_{lgmin} and Separation III requires good firms to borrow short with collateral of C_{sgmin}, C_{lgmin} and C_{sgmin} as given by equations (13) and (18). It can be proved that $C_{sgmin} < C_{lgmin}$, irrespective of other parameters.

Proof: see Appendix G

So, there will be a wealth constraint if W < C_{lgmin}. If $W < C_{sgmin} < C_{lgmin}$, good firms do not have a sufficient amount of collateral to be able to separate by borrowing long with collateral or short with collateral. Separation II turns out to be the only signaling option, provided that the cost of short-term debt is sufficiently high. If $C_{sgmin} < W < C_{lgmin}$, good firms can still put up collateral to back their short-term debt, but not their long-term debt. Therefore, Separation I will never occur in equilibrium. Separation II or Separation III will result conditionally on the cost of short-term debt and other parameter values.

6. CONCLUSION

This paper develops a model in which firms may signal with two debt attributes, duration and collateral. The analysis shows that different separating equilibria may result. If Separation occurs, low-quality firms will always borrow long without collateral, while high-quality firms will borrow long with collateral or borrow short with or without collateral. The least-cost separating equilibrium depends on the relative signaling costs of the different signaling mechanism and the difference in firm quality. The analysis also indicates that separation will be more likely if the proportion of low-quality firms in the market is high. In addition, the simultaneous use of debt maturity and collateral as signaling devices plays a more significant role if the disparity in firm quality decreases. When a wealth constraint is imposed, the role of collateral as a signaling device is undermined, and high-quality firms have no choice but to signal with short-term debt. This is probably the most relevant outcome for developing countries where wealth constraints are severe.

Model simulations suggest a non-monotonic relationship between firm quality and debt maturity, with high quality firms having both long-term secured debt and short-term secured or non-secured debt. More importantly, the analysis clarifies that a proper empirical test of theoretical signaling models is not simple. Empirically testing the implications of signaling models requires at minimum that the relative costs of the different signaling devices be taken into account. To our knowledge this has not yet been done, but certainly is an important area for future empirical research.

The model we have developed concentrates on the signaling properties of debt maturity and collateral. Further research aims to enrich the analysis by including other factors associated with the debt maturity and collateral decisions. It may be interesting, for instance, to introduce liquidity risk of short-term debt, allowing for a costly transfer of ownership in case of liquidation.

APPENDIX A: PROOF OF LEMMA 1

We will prove here that for the conditions specified in Lemma 1, bad firms prefer switching to the full information contract (the outcome for these firms if separation occurs), irrespective of bank beliefs. We consider the case where banks perceive a deviating action to be carried out by firms of low quality. If bad firms prefer to deviate under this belief, they will certainly do so under other beliefs of banks (i.e. if the bank thinks that the deviating action is carried out by good firms).

If both types of firms *pool by borrowing long with some amount of collateral*, the model has a continuum of pooling equilibria, which are all Pareto inferior. Suppose the equilibrium level of collateral is C_{lp}, where each C_{lp} in the interval [0, C_{lp}*] supports a different equilibrium. We can now find the value of C_{lp}^*, the greatest possible level of collateral generated by a pooling equilibrium. The pooling equilibrium is defined as follows, where $C_{lp} \in [0, C_{lp}^*]$:

$$\begin{cases} m_g = m_b = l \\ C_g = C_b = C_{lp} \\ V_{lc}^p = p_i^2 M_3 + 2p_i(1-p_i)M_4 - p_i(2-p_i)R_{lpc} - (1-p_i)^2 C_{lp} - kC_{lp} \\ Prob(i = b \mid m = l, C = C_{lp}) = 1-\theta \\ Prob(i = b \mid m = l, C \neq C_{lp}) = 1 \end{cases}$$

Bad firms prefer to deviate from the candidate pooling equilibrium if the following holds: $V_{1b} = p_b^2 M_3 + 2p_b(1-p_b)M_4 - 1 > V_{lbc}^p$

where V_{lb} denotes the value of bad firms under Separation long without collateral, and V_{lbc}^p denotes the value of bad firms at pooling long with collateral.

This implies that: $C_{lp} \geq C^*_{lp} = \dfrac{\theta\left[p_g(2-p_g)-p_b(2-p_b)\right]}{(1+k)\theta\left[p_g(2-p_g)-p_b(2-p_b)\right]+kp_b(2-p_b)}$

If both types of firms *pool by issuing short-term debt without collateral,* the candidate pooling equilibrium is defined as:

$$\begin{cases} m_g = m_b = s \\ C_g = C_b = 0 \\ V^p_{sb} = p_i^2 M_3 + 2p_i(1-p_i)M_4 - p_i - p_i(1-p_i)R_{sp} - b \\ Prob(i=b \mid m=s, C=0) = 1-\theta \\ Prob(i=b \mid m=l, C=0) = 1 \end{cases}$$

Bad firms have incentives to deviate if pooling provides a lower value than the value of staying off-equilibrium: $V_{lb} = p_b^2 M_3 + 2p_b(1-p_b)M_4 - 1 \geq V^p_{sb}$

where V_{lb} denotes the value of bad firms under Separation long without collateral, and V^p_{sb} denotes the value of bad firms at pooling short without collateral.

The following is obtained: $b \geq b^*_p = (1-p_b)\left[1-p_b\dfrac{1-(\theta p_g+(1-\theta)p_b)}{\theta p_g(1-p_g)+(1-\theta)p_b(1-p_b)}\right].$

Finally, if both types of firms *pool by borrowing short with collateral,* again the model allows for a continuum of pooling equilibria. Suppose the equilibrium level of collateral is C_{sp} - where each C_{sp} in the interval [0, C_{sp}*] supports a different equilibrium- the pooling is characterized:

$$\begin{cases} m_g = m_b = s \\ C_g = C_b = C_{sp} \\ V_{sic}^{p} = p_i^2 M_3 + 2p_i(1-p_i)M_4 - p_i(1-p_i)R_{spc} - p_i - (1-p_i)^2 C_{sp} - kC_{sp} - b \\ Prob(i = b \mid m = s, C = C_{sp}) = 1 - \theta \\ Prob(i = b \mid m = l, C \neq C_{sp}) = 1 \end{cases}$$

We will determine C_{sp}* - the maximum level of collateral in equilibrium. Like the previous cases, bad firms prefer to deviate from the candidate pooling equilibrium if the following holds: $V_{lb} = p_b^2 M_3 + 2p_b(1-p_b)M_4 - 1 > V_{sbc}^{p}$

where V_{lb} denotes the value of bad firms under Separation long without collateral, and V_{sbc}^{p} denotes the value of bad firms at pooling short with collateral.

or $C \geq C_{sp}^{*} = \dfrac{\theta(1-p_g)(1-p_b)(p_g - p_b) - b\left[\theta p_g(1-p_g) + (1-\theta)p_b(1-p_b)\right]}{\theta(1-p_g)(1-p_b)(p_g - p_b) + k\left[\theta p_g(1-p_g) + (1-\theta)p_b(1-p_b)\right]}$

For any value of C_{sp} greater than C_{sp}*, bad firms prefer deviating over pooling, irrespective of the bank's belief.

APPENDIX B: PROOF OF LEMMA 2

Consider the following characterization of the perfectly Bayesian pooling equilibrium:

$$\begin{cases} m_g = m_b = l \\ C_g = C_b = C_{lp} = 0 \\ Prob(i = g \mid m = l, C = 0) = \theta \\ Prob(i = g \mid m = s) = 0 \\ Prob(i = g \mid C > 0) = 0 \\ V = V_{li}^{p} = p_i^2 M_3 + 2p_i(1-p_i)M_4 - p_i(2-p_i)R_{lp} \quad \text{if } m = l \text{ and } C = 0 \\ V = V_b \quad \text{if } m = s \text{ and/or } C > 0 \end{cases}$$

we show that this pooling equilibrium does not satisfy the Intuitive Criterion.

In equilibrium, both types of firms pool by issuing non-secured long-term debt, and thus no extra cost is incurred and signaling is uninformative. Rationally, the bank knows that the proportion of good firms is θ hence it will charge the pooling loan rate of R_{lp} to all borrowing firms. If we assume that the bank perceives the willingness to borrow short and/or a placement of collateral to be from low quality firms and accordingly offers a loan contract designed for bad firms, no firm will deviate from the pooling equilibrium, given the signaling cost they must pay and the lower value they will obtain ($V_b < V_{li}^{p}$). Therefore, the pooling long without collateral appears as a *PBE*, provided it is upheld by the bank's belief as specified

By introducing the *Intuitive Criterion*, we will consider whether or not such a belief is reasonable. If parameters exist such that the specified bank's off-equilibrium-path belief is not intuitive, the perfect Bayesian pooling equilibrium does not survive the *Intuitive Criterion* and thus will be precluded from the set of *PBE*.

Firms of any type have three options to deviate from pooling: (i) posting collateral to back their long-term debt, (ii) borrowing short-term debt without collateral; (iii) borrowing short-term debt with collateral. Firms bear some deviating costs, which may be costs of short-term debt or cost of collateralization. As to bad firms, by deviating they wish to fool the bank into believing them in their fake quality. As to good firms, by deviating they wish to convince the bank into believing in their true quality. If bad firms are indifferent about pooling and deviating, whereas good firms have incentives to deviate, it is reasonable for the bank to believe the deviating behavior to be carried out by good firms. If so, the belief as specified appears unreasonable and the pooling is said to fail the *IC*. We will now show that this holds in our model under certain parameters.

We first analyze the bad firms' behavior. Note that bad firms have the following options to deviate from pooling by mimicking good firms and providing the bank with one of the signals: (i) posting collateral; (ii) borrowing short without collateral;

(iii) borrowing short with collateral. Bad firms have to bear some deviating costs, which may be costs of short-term debt or cost of collateralization. In return, they will fool the bank into believing in their fake high quality. Bad firms' value with mimicking behavior are given by

$$\text{(B1)}\; V_{lbc}^{mimic} = p_b^2 M_3 + 2 p_b M_4 (1-p_b) - (1-p_b)^2 C_{\lg} - p_b (2-p_b) R_{\lg} - kC_{\lg}$$

$$\text{(B2)}\; V_{sb}^{mimic} = p_b^2 M_3 + 2 p_b M_4 (1-p_b) - p_b - p_b (1-p_b) R_{sg} - b$$

$$\text{(B3)}\; V_{sbc}^{mimic} = p_b^2 M_3 + 2 p_b M_4 (1-p_b) - p_b - (1-p_b)^2 C_{sg} - p_b (1-p_b) R_{sg} - kC_{sg} - b$$

The subscripts *lbc, sb, sbc* represent bad firms mimicking the behavior of good firms under the three above-mentioned alternatives. If bad firms pretend to be good firms, they will obtain the debt contract, i.e. the loan rate and the amount of collateral, designed for good firms. The interest rates R_{lg} and R_{sg} can be derived from equation (3) and (4).

If bad firms decide to pool, their value is:

$$\text{(B4)}\; V_{lb}^{p} = p_b^2 M_3 + 2 p_b (1-p_b) M_4 - p_b (2-p_b) R_{lp}$$

Bad firms are indifferent about pooling and deviating if the following holds.

(B5) $\begin{cases} V_{lbc}^{mimic} \leq V_{lb}^{p} \\ V_{sb}^{mimic} \leq V_{lb}^{p} \text{ or} \\ V_{sbc}^{mimic} \leq V_{lb}^{p} \end{cases}$

$$\begin{cases} (B5a)(1-p_b)^2 C_{lg} + p_b(2-p_b)R_{lg} + kC_{lg} \geq p_b(2-p_b)R_{lp} \\ (B5b)\, p_b + p_b(1-p_b)R_{sg} + b \geq p_b(2-p_b)R_{lp} \\ (B5c)\, p_b + (1-p_b)^2 C_{sg} + kC_{sg} + p_b(1-p_b)R_{sg} + b \geq p_b(2-p_b)R_{lp} \end{cases}$$

Next, we analyze the behavior of good firms. Good firms may also deviate by signaling if they benefit from doing so. Like bad firms, good firms also have three alternatives to deviate: i) posting collateral; (ii) borrowing short without collateral; (iii) borrowing short with collateral. Their values are equal to:

(B6) $V_{\text{lg}c} = p_g^2 M_3 + 2p_g(1-p_g)M_4 - 1 - kC_{\text{lg}}$

(B7) $V_{sg} = p_g^2 M_3 + 2p_g(1-p_g)M_4 - 1 - b$

(B8) $V_{sgc} = p_g^2 M_3 + 2p_g(1-p_g)M_4 - 1 - kC_{sg} - b$

If good firms pool, their value is:

(B9) $V_{lg}^{p} = p_g^2 M_3 + 2p_g(1-p_g)M_4 - p_g(2-p_g)R_{lp}$

Good firms prefer to deviate if the following holds:

$$(B10)\begin{cases} V_{lgc} \geq V_{lg}^{p} \\ V_{sg} \geq V_{lg}^{p} \quad \text{or} \\ V_{sgc} \geq V_{lg}^{p} \end{cases}$$

$$\begin{cases} (B10a)\, p_g(2-p_g)R_{lp} \geq 1+kC_{\text{lg}} \\ (B10b)\, p_g(2-p_g)R_{lp} \geq 1+b \\ (B10c)\, p_g(2-p_g)R_{lp} \geq 1+kC_{s\text{g}}+b \end{cases}$$

This system of inequalities implies that the deviating options are more profitable than pooling.

If parameters exist such that one equation in (B5) and its corresponding inequality in (B10) hold simultaneously, good firms are able to convince the bank that they are indeed better off by deviating than by staying on the equilibrium path. In order to support a deviating behavior by good firms, the reasonable off-equilibrium-path belief of the bank should be *Prob (i=g|m=s, C>0)* = 1. In other words, the off-equilibrium-path belief as specified in the pooling definition appears to be unreasonable. Hence, the pooling fails to meet the Cho-Kreps *Intuitive Criterion*.

Each equation in (B5) and its counterpart in (B10) hold simultaneously if the following conditions are satisfied:

i) $\begin{cases}(1-p_b)^2 C_{lg} + p_b(2-p_b)R_{lg} + kC_{lg} \geq p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1 + kC_{lg}\end{cases}$

ii) $\begin{cases}p_b + p_b(1-p_b)R_{sg} + b \geq p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1 + b\end{cases}$

iii) $\begin{cases}p_b + (1-p_b)^2 C_{sg} + kC_{sg} + p_b(1-p_b)R_{sg} + b \geq p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1 + kC_{sg}\end{cases}$

Under any of the above conditions, a pooling long without collateral does not survive the *Intuitive Criterion*. Accordingly, it will be discarded from the set of *PBE*.

APPENDIX C: PROOF OF LEMMA 3

For each separation, the incentive compatibility constraints (*ICCs*) guarantee firms to be honest about their type. In other words, the *ICCs* require the true value to be in excess of the mimicking value for firms of both types. We first derive the mimicking value equations under the three separation possibilities as follows:

$$V_{lbc}^{mimic} = p_b^2 M_3 + 2p_b M_4(1-p_b) - (1-p_b)^2 C_{lg} - p_b(2-p_b)R_{lg} - kC_{lg}$$

$$V_{sb}^{mimic} = p_b^2 M_3 + 2p_b M_4(1-p_b) - p_b - p_b(1-p_b)R_{sg} - b$$

$$V_{sbc}^{mimic} = p_b^2 M_3 + 2p_b M_4(1-p_b) - p_b - (1-p_b)^2 C_{sg} - p_b(1-p_b)R_{sg} - kC_{sg} - b$$

$$V_{lg}^{mimic} = p_g^2 M_3 + 2p_g M_4(1-p_g) - p_g(2-p_g)R_{lb}$$

Where V_{lbc}^{mimic}, V_{sb}^{mimic} and V_{sbc}^{mimic} respectively represent values of bad firms when mimicking the behavior of good firms under Separations I, II and III. Similarly, V_{lg}^{mimic} indicates the value of good firms when pretending to be bad firms under the three Separations. The *ICCs* imply

$$\begin{cases} V_{lb} \geq V_{lbc}^{mimic} \\ V_{lgc} \geq V_{lg}^{mimic} \end{cases} \text{ for Separation I,}$$

$$\begin{cases} V_{lb} \geq V_{sb}^{mimic} \\ V_{sg} \geq V_{lg}^{mimic} \end{cases} \text{ for Separation II,}$$

and $$\begin{cases} V_{lb} \geq V_{sbc}^{mimic} \\ V_{sgc} \geq V_{lg}^{mimic} \end{cases} \text{ for Separation III}$$

With V_{lb} and V_{lgc}, V_{sg}, V_{sgc} referring to the true values of bad firms and good firms under separation. We rewrite the above expressions as:

$$\text{i)} \begin{cases} (1-p_b)^2 C_{\text{lg}} + p_b(2-p_b)R_{\text{lg}} + kC_{\text{lg}} \geq 1 \\ p_g(2-p_g)R_{lb} \geq 1 + kC_{\text{lg}} \end{cases} \qquad \text{for Separation I,}$$

ii) $\begin{cases} p_b + p_b(1-p_b)R_{sg} + b \geq 1 \\ p_g(2-p_g)R_{lb} \geq 1+b \end{cases}$ for Separation II,

iii) $\begin{cases} p_b + (1-p_b)^2 C_{sg} + kC_{sg} + p_b(1-p_b)R_{sg} + b \geq 1 \\ p_g(2-p_g)R_{lb} \geq 1 + kC_{sg} + b \end{cases}$ for Separation III

APPENDIX D: PROOF OF LEMMA 4

The feasible incentive compatible separating equilibria may result if both conditions implied by the Intuitive Criterion and the Incentive Compatibility Constraint are satisfied. We rewrite the *IC* given by Lemma 2, and the *ICCs* given by Lemma 3 as follows:

IC1) $\begin{cases} (1-p_b)^2 C_{\lg} + p_b(2-p_b)R_{\lg} + kC_{\lg} > p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1 + kC_{\lg} \end{cases}$ for Separation I

IC2) $\begin{cases} p_b + p_b(1-p_b)R_{sg} + b > p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1+b \end{cases}$ for Separation II

IC3) $\begin{cases} p_b + (1-p_b)^2 C_{sg} + kC_{sg} + p_b(1-p_b)R_{sg} + b > p_b(2-p_b)R_{lp} \\ p_g(2-p_g)R_{lp} \geq 1 + kC_{sg} \end{cases}$ for Separation III

Under the *IC*, bad firms find pooling better than lying while good firms find separating more attractive than pooling.

$$ICC1)\begin{cases}(1-p_b)^2C_{\lg}+p_b(2-p_b)R_{\lg}+kC_{\lg}\geq 1\\ p_g(2-p_g)R_{lb}\geq 1+kC_{\lg}\end{cases}$$ for Separation I,

$$ICC2)\begin{cases}p_b+p_b(1-p_b)R_{s\mathrm{g}}+b>1\\ p_g(2-p_g)R_{lb}\geq 1+b\end{cases}$$ for Separation II

$$ICC3)\begin{cases}p_b+(1-p_b)^2C_{sg}+kC_{sg}+p_b(1-p_b)R_{s\mathrm{g}}+b>1\\ p_g(2-p_g)R_{lb}\geq 1+kC_{sg}+b\end{cases}$$ for Separation III

Under the *ICCs*, bad firms prefer truth-telling to lying whereas good firms prefer truth-telling to lying by pretending to be bad firms.

Now, the combination of both the *ICs* and the *ICCs* for each Separation result in the following conditions:

$$\text{i)}\begin{cases}(1-p_b)^2C_{\lg}+p_b(2-p_b)R_{\lg}+kC_{\lg}\geq 1\\ p_g(2-p_g)R_{lp}\geq 1+kC_{\lg}\end{cases}$$ for Separation I,

$$\text{ii)}\begin{cases}p_b+p_b(1-p_b)R_{s\mathrm{g}}+b>1\\ p_g(2-p_g)R_{lp}\geq 1+b\end{cases}$$ for Separation II,

$$\text{iii)}\begin{cases}p_b+(1-p_b)^2C_{sg}+kC_{sg}+p_b(1-p_b)R_{s\mathrm{g}}+b>1\\ p_g(2-p_g)R_{lp}\geq 1+kC_{sg}+b\end{cases}$$ for Separation III

Note that, the lower boundary of each condition is derived from the *ICCs*, and the upper boundary is derived from the *IC*. For bad firms, pooling is better than presenting as themselves at separation. For good firms, pooling is better than pretending to be bad firms at separation. The pooling possibility here refers to the zero-signaling cost pooling, which is ruled out by the *Intuitive Criterion* under certain conditions given in Lemma 2.

APPENDIX E1: PROOF $C_{lgmin} > C_{lp}^{*}$

Since $C_{lp}^{*} = \dfrac{\theta\left[p_g(2-p_g)-p_b(2-p_b)\right]}{(1+k)\theta\left[p_g(2-p_g)-p_b(2-p_b)\right]+kp_b(2-p_b)}$ increases in θ and

$C_{lp}^{*} = 0$ when $\theta = 0$ and $C_{lp}^{*} = C_{lgmin}$ when $\theta = 1$. So, $0 \le C_{lp}^{*} \le C_{\lg\min}$

APPENDIX E2: PROOF $b_d > b_p^{*}$

We have $b_p^{*} = (1-p_b)\left[1-p_b\dfrac{1-(\theta p_g+(1-\theta)p_b)}{\theta p_g(1-p_g)+(1-\theta)p_b(1-p_b)}\right]$ in an increasing

function of θ, $b_p^{*} = 0$ when $\theta = 0$ and $b_p^{*} = b_d$ when $\theta = 1$. This yields $b_p^{*} < b_d$.

APPENDIX E3: PROOF $C_{sgmin} > C_{sp}{}^{*}$

Since $$C_{sp}^{*} = \frac{\theta(1-p_g)(1-p_b)(p_g-p_b) - b\left[\theta p_g(1-p_g) + (1-\theta)p_b(1-p_b)\right]}{\theta(1-p_g)(1-p_b)(p_g-p_b) + k\left[\theta p_g(1-p_g) + (1-\theta)p_b(1-p_b)\right]}$$

increases in θ and $C_{sp}^{*} < 0$ when $\theta = 0$ and $C_{sp}^{*} = C_{sgmin}$ when $\theta = 1$. So, $C_{sp}^{*} \leq C_{sg\min}$

APPENDIX F1: PROOF IF $b_d < kC_{lgmin}$ then $\theta_1 \leq \theta_2$.

For the sake of simplicity, we rewrite the relevant expressions as follows:

$$C_{lgmin} = \frac{(1-p_b)^2 - (1-p_g)^2}{p_g(2-p_g)(1+k) - p_b(2-p_b)} \qquad b_d = \frac{(p_g - p_b)(1-p_b)}{p_g}$$

$$\theta_1 = \frac{p_g(2-p_g) - p_b(2-p_b)(1-C_{lgmin})}{p_g(2-p_g) + \left[p_g(2-p_g) - p_b(2-p_b)\right](1-C_{lgmin})}$$

$$\theta_2 = \frac{p_g(2-p_g) - p_b(2-p_b)[1+b_d]}{\left[p_g(2-p_g) - p_b(2-p_b)\right][1+b_d]}$$

Inserting C_{lgmin} into θ_1, we obtain

$$\theta_1 = \frac{(1+k)\left[p_g(2-p_g)-p_b(2-p_b)\right]}{(1+k)\left[p_g(2-p_g)-p_b(2-p_b)\right]+kp_g(2-p_g)}$$

Since θ_2 decreases in b_d, inserting $b_d \le kC_{lgmin}$, we have:

$$\theta_2 = \frac{p_g(2-p_g)-p_b(2-p_b)\left[1+b_d\right]}{\left[p_g(2-p_g)-p_b(2-p_b)\right]\left[1+b_d\right]}$$

$$\ge \frac{p_g(2-p_g)-p_b(2-p_b)\left[1+kC_{lg\min}\right]}{\left[p_g(2-p_g)-p_b(2-p_b)\right]\left[1+kC_{lg\min}\right]} = \frac{(1+k)\left[p_g(2-p_g)-p_b(2-p_b)\right]^2}{(1+k)\left[p_g(2-p_g)-p_b(2-p_b)\right]^2+p_g(2-p_g)\left[p_g(2-p_g)-p_b(2-p_b)\right]k} = \theta_1$$

So $\theta_2 > \theta_1$ always holds if $b_d \le kC_{lgmin}$

APPENDIX F2: PROOF IF $b \le k\left[\frac{C_{\lg\min}(b_d+k)}{b_d}-1\right]$ then $\theta_1 \le \theta_3$.

Recall expressions (18) and (19) with

$$C_{sgmin} = \frac{b_d - b}{b_d + k} \text{ and } \theta_3 = \frac{p_g(2-p_g)-p_b(2-p_b)\left[1+b_d(1-C_{sg\min})\right]}{\left[p_g(2-p_g)-p_b(2-p_b)\right]\left[1+b_d(1-C_{sg\min})\right]}$$

Inserting C_{sgmin}, we obtain

$$\theta_3 = \frac{p_g(2-p_g) - p_b(2-p_b)\left[1 + b_d(b+k)/(b_d+k)\right]}{\left[p_g(2-p_g) - p_b(2-p_b)\right]\left[1 + b_d(b+k)/(b_d+k)\right]}$$ is the decreasing function in b.

Therefore, with $b = k\left[\frac{C_{\lg\min}(b_d+k)}{b_d} - 1\right]$,

$$\theta_3 = \theta_{3\min} = \frac{\left[p_g(2-p_g) - p_b(2-p_b)\right]^2(1+k)}{\left[p_g(2-p_g) - p_b(2-p_b)\right]\left\{\left[p_g(2-p_g) - p_b(2-p_b)\right](1+k) + kp_g(2-p_g)\right\}}$$

$$= \frac{(1+k)\left[p_g(2-p_g) - p_b(2-p_b)\right]}{(1+k)\left[p_g(2-p_g) - p_b(2-p_b)\right] + kp_g(2-p_g)} = \theta_1 .$$

So, $\theta_1 \le \theta_3$ for all $b \le k\left[\frac{C_{\lg\min}(b_d+k)}{b_d} - 1\right]$.

APPENDIX G: PROOF $C_{sgmin} < C_{lgmin}$

From the analysis we have

$C_{lgmin} = \frac{(1-p_b)^2 - (1-p_g)^2}{p_g(2-p_g)(1+k) - p_b(2-p_b)}$ and $C_{sgmin} = \frac{b_d - b}{b_d + k}$ with

$$b_d = \frac{(p_g - p_b)(1-p_b)}{p_g}$$

We need to prove $C_{lgmin} > C_{sgmin}$ or $\frac{(1-p_b)^2 - (1-p_g)^2}{p_g(2-p_g)(1+k) - p_b(2-p_b)} > \frac{b_d - b}{b_d + k}$

If one rewrites this expression, the inequality becomes:

$$b\left[p_g(2-p_g)-p_b(2-p_b)+kp_g(2-p_g)\right]>k\left[\frac{(p_g-p_b)(1-p_b)}{p_g}p_g(2-p_g)-p_g(2-p_g)+p_b(2-p_b)\right]$$
$$=k(p_g-p_b)p_b(p_g-1)$$

Since the left-hand side is positive and right-hand side is negative, the inequality always holds.

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