

MARKET PARTITIONING AND THE GEOMETRY OF THE RESOURCE SPACE

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SOM theme B: Inter-firm coordination and change

Abstract

The paper gives a geometry based explanation for organization ecology's resource partitioning theory. The original theory explains market histories of generalist and specialist organizations with scale economies. We show that the main predictions can be restated in terms of certain structural properties of the n -dimensional Euclidean resource space. We model customer demand elaboration with the increasing number of dimensions (taste descriptors), and demonstrate that the resulting change in spatial configurations increases market concentration and enhances resource partitioning. The original and the proposed models of resource partitioning are complementary: their predicted effects add up and drive the events towards the perceived market phases. Moreover, each approach answers questions that the other cannot address.

Keywords: market position, product differentiation, organizational ecology, resource partitioning, geometry

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1. Introduction

Multidimensional spaces are well understood tools of social scientists to represent objects with several attributes. We model our entities, organizations, as spatial bodies with certain geometric properties (volume, shape, symmetries) that stand for certain organizational traits. Spatial configurations of bodies represent relations between our objects, and geometry specifies constraints on their available configurations. For non-expert applicers of geometry like the authors, it was quite surprising to discover these constraints; to restate deeply influential effects for organizational populations in terms of clear-cut geometric considerations. Hence the objective our paper: to demonstrate that the claims of an empirically tested sociological theory, the resource partitioning model of organization ecology (Carroll 1985, 1987, 1997; Carroll and Hannan 1995) can be explained with structural properties of the multidimensional bodies.

The Euclidean space represents the market in resource partitioning theory; it is the scaffold on which organizational interactions take place. The n -dimensional space has different names in the different market models of organization sociology and economics. It is called *product characteristics space* when spatial axes stand for descriptors of commodities (Lancaster 1966). It is called *competence space* if production skills are in focus (Nooteboom 1994a; Péli and Nooteboom 1997). Its name is *resource space* in organizational ecology, when customers with different tastes constitute the key resource for organizational populations (Carroll 1985; Carroll and Hannan 1995).

We adopt the last, resource oriented interpretation: customer tastes are n -dimensional points in space, each featuring a certain amount of demand. Organizations are represented by the tastes they address, i.e. the subsets of resource space they exploit (niches, catchment areas). Competition is modeled as niche overlap (Hannan and Freeman 1977, 1989). Under certain conditions, markets (resource spaces) are partitioned between organizations that realize a more or less peaceful coexistence, just as the Earth's surface is partitioned between countries (Carroll 1985). Competitive and convivialist phases may follow each other as markets develop. Our work intends to contribute to the understanding of these resource partitioning histories.

The simplest and most widely studied way of market partitioning is to assign the same catchment area to each organization. A classic one-dimensional example is Hotelling's linear city model of product differentiation (1929) also readdressed as

"circular city" by Salop (1979). Nooteboom (1993) investigated the possibilities of the multidimensional generalization of the market partitioning problem. How can the n -dimensional Euclidean space be partitioned between congruent and regular *polytopes*, the n -dimensional generalizations of polyhedra? The goal was to specify multidimensional "honeycombs" with sphere-like cells, or in socio-economic terms, organizations with equal catchment areas in a product characteristics space. A survey of the mathematical literature revealed that the underlying *tessellation* problem has no regular and sphere-like solution beyond two dimensions (Coxeter 1948).¹

We address the space partitioning problem differently. We assume that catchment areas are not polytopes but n -dimensional spheres (*hyperspheres*). Even higher dimensional hyperspheres are handy objects, their volume and surface depend only on their radii and thus can be easily calculated.² However, spaces cannot be completely covered by spheres without overlap: if $n > 1$, then there is always some residual left between the hyperspheres. That is, some demand must be left unsatisfied by organizations with spherical niches. The ubiquitous presence of leftout space around spheres is a mathematical inconvenience in equipartition models. However, the same fact can have an explanatory function if one allows for the existence of organizations with small niches, called specialists in the organizational literature (Levins 1968; Brittain and Freeman 1980; Freeman and Hannan 1983; Hannan and Freeman 1989; Péli 1997). Specialist organizations can populate residual regions or "holes" between organizations of broader niches (generalists).

The paper concentrates on these holes. It shows that a field in geometry, known as the *sphere packing* problem (Section 3), provides useful and somewhat surprising inputs for the sociology of organizations. Applying the results of this domain to resource partitioning theory (Section 2), one can explain a lot from the dynamics of generalist-specialist markets. While the original model explains resource partitioning processes on the basis of scale economies, now the results obtain from structural properties of n -dimensional arrangements. The original and the proposed explanations are

¹ Sphere-likeness means that catchment areas have similar extensions in each direction. Soccer balls are sphere-like polyhedra, but they are not regular, their surface being composed of pentagons and hexagons. The only regular (but not sphere-like) polytopes that equipartition the space if $n \neq 2$ and $n \neq 4$ are the *hypercube*, the n -dimensional generalization of the cube, and the *hyper-octahedron*.

² For the sake of convenience, we use the words *hypersphere* and *sphere* synonymously, though literally the second denotes 3D objects.

complementary: first, both approaches specify effects that point in the same direction, thus reinforcing each other; second, each explains aspects for which the other, alone, could not give an account.

Resource inhomogeneity is a crucial assumption in the original, scale economy based resource partitioning model: demands have an uni- or polymodal distribution in space. On the contrary, the geometric approach goes along with the assumption that customer demand is homogeneously distributed. The two models can be seen as two layers of explanation for the same phenomena. The geometric explanation (flat demand distribution) serves as a background. The second layer adds complexity to the first: peaks in the demand distribution that yield scale economy advantages.³

The potential applications of the sphere packing problem in the social sciences go far beyond organizational ecology. Beside product differentiation and market positioning, the same approach may have a bearing for political sociology (e.g., for the line of research reported by Kollman, Miller and Page 1992). How should political parties position their catchment areas in the space of potential voters, minimizing both residual space and overlap? Since network structures are related to environmental resources, some structural holes in social networks (Burt 1992) may correspond to holes in resource exploitation.⁴ The presented approach may help at data evaluation. For example, the task of finding appropriate cluster centers in cluster analysis seems to be related to the *quantizer problem* in mathematical communication theory (Conway and Sloane 1988) which also goes back to the search of optimal sphere packings.

The paper is organized as follows. Section 2 summarizes the original resource partitioning theory: What are the main predictions and explanatory elements? What is its position in current organizational and economic theories? Section 3 comes up with the alternative geometric considerations: Similar phenomena, but a different explanatory structure. Section 4 assesses the strength and the shortcomings of the proposed explanation; methodological benefits, empirical ramifications are addressed.

³ These two layers were originally three, because first Glenn Carroll used homogeneous resource distributions and circular niches in two dimensions in his resource partitioning paper (1985).

⁴ Burt suggests a conceptualization of population niche in network terms; his rephrasing shifts the emphasis from resources to relationship patterns that provide access to these resources (1992:210).

2. Resource Partitioning with Scale Economies

2.1 Model and Ramifications

The resource partitioning model was delineated by Glenn Carroll who analyzed the history of American newspaper publishing (1985). The theory explains the long term history of markets composed of generalist and specialist organizations. The market is an n -dimensional Euclidean space with axes that denote taste descriptors; thus, each point in space stands for a certain customer taste. Generalist and specialist organizations are characterized with their niche width. Generalists make appeals to a broad range of customer tastes while specialists focus to specific ones. Accordingly, a generalist's niche is a broad region in the resource space, while specialists occupy small spots. A given customer type purchases if some organizations offer the kind of product that it needs. The taste distribution is uneven over the population, the market has a center (or a few centers) composed of mainstream tastes. Organizational resources (demand) are abundant in the center, so the incumbent organizations can grow big. The taste distribution is often conceptualized as normal along each taste axis (Carroll and Hannan 1995:217-219).

This setting gives rise to the following population history. Early in the market, the surviving firms are mainly generalists. To increase sales, generalists tend to differentiate themselves by differentiating their product offers, placing their niches apart from each other as much as possible. Product differentiation is also a way to reduce competition (Eaton and Lipsey 1989), since niche overlap measures the intensity of competition. The occupant organizations of the resourceful central regions grow bigger than the others, and the induced increase in size variation yields scale economy advantages: the big firms get even bigger, forcing the medium size generalists out from the market. The number of generalist organizations falls while their average size grows. Market concentration increases.

A crucial element in the model is that the life chances of emerging small specialist organizations is attached to the concentration level of generalists: high concentration opens little resource pockets for specialists. This happens as follows. As medium size organizations disappear, resources become unbound. The surviving generalists take the best chunks of the residual space, positioning themselves in market centers. As fights

between generalists die out, product differentiation loses its importance. Having no strong competitors around, the survivor generalists feel less pressure to make dedicated bids for specific customer tastes; they rather adjust their offers to the mainstream needs at the centers. In the newspaper industry, this means following a middle-of-the-road editorial policy (Carroll 1985). In industries like breweries and medical diagnostics (Swaminathan 1995a; Mitchell 1995) a market center can be also seen as a common denominator of tastes: some general products satisfy a great portion of needs for a broad range of customers.

The surviving generalists increase their niche width, taking over the best chunks of the extinct competitors' market segments. But as expanding generalists march towards market centers, they also leave some customers unsatisfied at the edges. Small specialist organizations may establish footholds in these market pockets. Furthermore, taste distributions often get flatter as markets develop, increasing resource abundance at the edges (Carroll and Hannan 1995). Specialists' life chances are tightly related to market concentration under this explanation. These organizations do not threaten the generalists who may let them survive in the deserted edges of the resource space. In the end, there is no competition but a kind of conviviality between the survivor generalists and the newcomer specialists. The latter are the scavengers of the resource space exploiting what is left behind by generalist predators.

Summarizing the stages of market history: mainly generalists populate early markets; their number decreases with time; the surviving generalists broaden their niche and position themselves into market centers; resource pockets open up for specialists.

2.2 Empirical Evidence and Questions

A rapidly growing set of research in a broad variety of industries gives empirical support to the outlined theoretical picture. Resource partitioning effects were detected in the brewing industry (Carroll and Swaminathan 1992, 1993; Swaminathan 1995a), in banking co-operatives (Freeman and Lomi 1994, Lomi 1995), in wine production (Swaminathan 1995b), in medical diagnostic imaging (Mitchell 1995), and in microprocessor production (Wade 1996). Earlier, Barnett and Carroll (1987) observed a symbiotic relation between telephone companies occupying different niches in the same location: these organizations exerted a positive influence on each other's fate. Recently, Dobrev (1997) analyzed the restructuring process of the Bulgarian newspaper industry

during the political transition, applying the resource partitioning framework to an environment substantially different from American markets. The research of Torres (1995) and Seidel (1997) investigate, respectively, the British automobile and the American Airline industry in a resource partitioning framework.

Studies on size-localized competition between organizations also revealed similar effects to those claimed by resource partitioning theory (Hannan and Freeman 1977; Hannan and Ranger-Moore 1990; Hannan, Ranger-Moore and Banaszak-Holl 1990). Organizations of very different sizes typically differ in strategy and structure, and competition tends to be stronger among structurally similar organizations. Since generalists and specialists can be quite different both in size and structure, the losers of size-localized competition are mostly the medium size generalists.⁵ Some current economic theories address similar concentration issues as the resource partitioning model. The study of Boone and Witteloostuijn (1995) analyses the connection between organizational ecology and the field of industrial organization, looking for overlaps and possibilities for cross-fertilization. The authors pointed out a series of similarities between resource partitioning model and Sutton's dual structure theory (1991) on industry concentration.

We mention some effects that are hard to explain exclusively with scale economy advantages. The first concerns market center. This notion involves a peak in the demand distribution. If only scale economies govern competition, then the Macbeth-effect applies: there is only one place at the top. Otherwise, the winner organizations might be of exactly the same size (which is quite improbable), because scale economy effects would magnify even minor differences, finally leaving a single organization in place. How can the model allow for the survival of more than one generalist in the long run? Remaining in the context of scale economy explanations, a possible solution is to assume polymodal taste distributions; then a handful of generalists may survive in the resulting landscape, each occupying a different market center. The low demand ditches around the resource heights may keep the incumbent organizations away from appropriating the neighbor's domain. Though such outcomes do occur, the proposed setting replicates the original problem: each local center can be occupied by only one player. More general solutions should require additional explanatory elements, for example, institutional

⁵ Taking into account other aspects, medium size generalists may have their chances. Investigating the California loan and savings industry, Haveman (1993) found that medium size organizations are the most willing to diversify their activities into new domains.

aspects (Meyer and Scott 1983; DiMaggio and Powell 1983; Meyer and Zucker 1989). We propose another solution, explaining resource partitioning without reference to abundant resource spots in space .

Two other effects to explain are specialists' presence in early markets and their occasional persistence in non-marginal market segments. How can specialists appear before the survivor generalists move to the market centers leaving behind resources at the margins? Why should the big generalist organizations tolerate specialists to gaze on the rich central parts of the resource base? The original model handles these, in fact rarely occurring, effects as disturbances. The geometric approach will show that some room does open for specialists in each segment of the market and even in early phases of the market history.

3. The Geometric Resource Partitioning Model

3.1 Spherical Niches

Organizations are represented by their niches in the resource space. Consider generalists as n -dimensional hyperspheres that populate the space. The market is a region in this space, sufficiently extended in each direction and filled up tightly with spheres without overlap (Figure 1). Spherical niche shape is a crucial aspect in our model, so we give a number of arguments to justify this assumption.

The first argument concerns *isotropy*, the invariance of directions in the resource space. If spatial directions do not count, then other things being equal, organizations develop the same niche breadth in any direction. However in reality, taste descriptors do differ in importance. This fact could be incorporated into the model by assigning a set of weights to the dimensions, and performing *affine transformations* along each axis with these weights. Instead of having hyperspheres, then we arrive to the n -dimensional analogues of ellipses (in 3D: rugby balls). Affine transformations do not affect volume ratios, therefore the geometric arguments in this paper will also apply when taste descriptors differ in importance. For the sake of convenience, we assume that all taste variables are *standardized*, and so proceed with spheres instead of ellipses. However, the shape of organizational niches can be also affected by the neighboring competitors. Organizations may also consider to extend their niches in certain directions to occupy

some close chunks of the residual space. We will discuss stabilizing effects in 3.5. Here we also mention some forces that penalize radius deviation.

One argument is clearly economic. It is reasonable to postulate a structure of producer costs with a variable cost as some increasing function of the niche volume, and a fixed capacity cost as an increasing function of the largest distance of customers from the center. The latter determines the "reach" that the producer has to serve distant customers (by product adaptation or sales support). Capacity cost then is determined by the largest distance that any customer has from the center. Any asymmetric niche then entails excess, unutilized capacity. Thus, in a consideration of asymmetrical niche extension there is a trade-off between the additional sales that it yields and the cost of unutilized capacity. The greater the share of fixed capacity cost, the sooner the trade-off will be in favor of maintaining a niche that is symmetrical in all directions, i.e., a hypersphere. This yields an empirical prediction: niches are spherical (vs. asymmetric) to the extent that fixed capacity costs (vs. variable costs) prevail. An example of low capacity cost would be the delivery of newspapers, where one pays independent delivery people a fee for every extra paper delivered. Here fixed capacity costs are zero, and every delivery man will explore extensions of the boundary in his area, regardless of what others do. Some may directly bump into competitors and stop, while others can proceed further until they meet opposition, so that irregular niches will obtain. A special interpretation of capacity cost is that extending reach has negative consequences for quality throughout the niche (principle of allocation, Levins 1968, Hannan & Freeman 1989), or makes brand image too diffuse. The argument then is that with asymmetric extension there is an increase of sales which is small relative to the loss of quality or brand image.

Another argument for spherical niche shape takes the surface of spatial objects into account. Spheres have the smallest surface among all bodies of the same volume, which is why soap bubbles and planets like Earth have the globe shape. This rule also holds for hyperspheres in any numbers dimensions higher than one. What is the problem with large niche surface? The points at the margins stand for customers whose tastes are least matched by the organization at hand. In lack of a fitting offer, the "buy or not to buy" question is less settled for these people than for others (inside zones). Moreover, they can also choose the products of neighboring organizations without a serious compromise in their taste preferences. Customers close to the surface are not stable customers, extra sales efforts may be necessary to attract and bind them. Thus, it may be economical to

minimize surface relative to volume, which yields the sphere.

Accepting that at least in a significant number of cases niches are spherical, the number of generalists in a market of given size and resources is determined by the sphere radius and by the manner of sphere packing. The first, sphere radius, is assumed to be similar for each generalist for the time being. The second aspect, spatial configuration, leads to the application of the sphere packing problem. The next subsection summarizes some important features of this field.

Table 1. The Known Densest Packings with Kissing Numbers and the Known Thinnest Coverings

<i>Dim.</i>	<i>Packing density</i>	<i>Kissing number</i>	<i>Thinnest covering</i>
n	Δ	τ	Θ
1	1	2	1
2	0.90690	6	1.2092
3	0.74048	12	1.4635
4	0.61685	24	1.7655
5	0.46526	40	2.1243
6	0.37295	72	2.5511
7	0.29530	126	3.0596
8	0.25367	240	3.6658
9	0.14577	272	4.3889
10	0.09962	372	5.2517
11	0.06624	519.78	6.2813
12	0.04945	756	7.5101
13	0.03201	1060.67	8.9768
14	0.02162	1422	10.727
15	0.01686	2340	12.817
16	0.01471	4320	15.311

17	0.008811	5346	18.288
18	0.005958	7398	21.841
19	0.004121	10668	26.082
20	0.003226	17400	31.143

Source: Conway and Sloane (1988: 15, 38). Note that τ is not always integer; if hyperspheres have different number of neighbors in a packing, then τ is calculated as an average.

Figure 1. Dense Sphere Packings in 1 - 3 Dimensions

Figure 1a

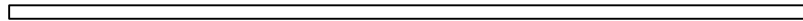
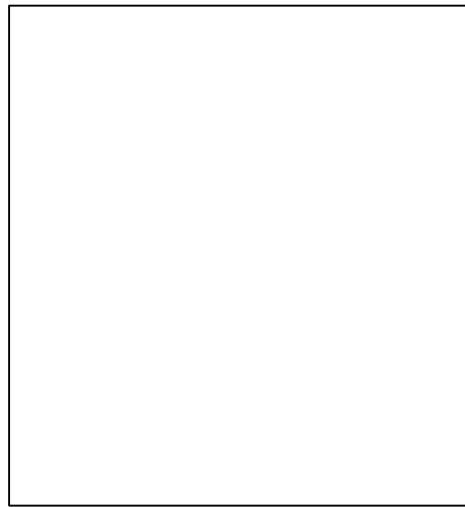


Figure 1b



Figure 1c



3.2 The Sphere Packing Problem

The sphere packing problem is concerned with ways of filling up the n -dimensional Euclidean space with hyperspheres of equal size (Conway and Sloane 1988). The main issue is to find dense packings, configurations where the space ratio occupied by spheres is high. For example: How can one heap the biggest number of melons on a trolley in the market place?

The efficiency of a sphere packing is measured by *packing density* (Δ), the ratio of the volume occupied by the spheres to total space volume ($0 \leq \Delta \leq 1$). Unfortunately, no general solution to the sphere packing problem is known yet, the densest packings are known for sure only up to two dimensions. In one dimension, the hyperspheres are sections of equal length along a line. If the neighboring sections meet, then packing density is unity (Figure 1a). In two dimensions, hyperspheres are circles. The densest is the hexagonal packing ($\Delta = 0.9069\dots$) that obtains by pulling circles into a honeycomb pattern (Figure 1b).

From $n = 3$ several dense packings are known, but not the densest ones. In three dimensions, intuition suggests that the cannon-ball packing is the densest (Figure 1c). Gauss proved in 1831 that this is the best among *lattice packings*, arrangements in

which a configuration of spheres repeats itself in space.⁶ But, even if the densest packings are not precisely known, some upper bounds on packing density can be calculated for any n . Fortunately, the known densest packings approximate these bounds quite well for not very high dimensions (Table 1). It is also proven for big n -s that adding an $n+1^{\text{th}}$ dimension to the space amounts to dividing the highest packing density in n with a number between 1.51 and 2 (Conway and Sloane 1988:20). That is, packing density converges pretty fast to zero as the number of spatial dimensions increases.

This result is just the opposite of what one might expect. Consider the following thought experiment! Replace the spheres with soft balloons in the assumed-to-be-best cannon ball packing (Figure 1c), and inflate them simultaneously. The balloons press against each other as pressure increases, gradually taking the shape of polyhedra equal in size and shape. Finally, the resulting polyhedra fill up the residual space. Applying a similar procedure to the densest packings in higher dimensions, one would end up in a set of convex (but not regular) polytopes that equipartition the space. These cells become more and more complex with n : more vertices, edges, surfaces, etc. Intuition might suggest that they gradually take the shape of hyperspheres as n goes to infinity, just like snowballs are formed by pressing a piece of snow from several directions. However, the partitioning cells cannot converge to hyperspheres with n , because the density of any complete space partitioning is 1 by definition, while the limit value of sphere packing density is zero if n goes to infinity. The following subsections discuss those properties of n -dimensional sphere packings that account for some core elements of resource partitioning theory.

3.3 Dimensional Expansion: New Opportunities for Specialists

We describe resource partitioning processes with a single explanatory variable: the increasing number of spatial dimensions. Since axes stand for taste descriptors, an increase in n reflects customer taste elaboration. We concentrate on markets where the appearance of new taste dimensions deeply influence purchases, like user friendliness in the computer industry in the eighties or car airbags in the nineties. After each increase in dimension, organizational bids fold out into the extended space. For example, circular niches take globe shape in moving to three dimensions.

⁶ The sphere centers form an additive group in lattice packings (Conway and Sloane 1988:3-4).

We do not address other forms of customer demand elaboration in this paper, though they may influence population histories. For example, scale extension (offering extra size products) can provide opportunities for specialists. Scale refinement (intermediate product sizes, qualities) may establish stepping-stones over the demarcation lines between remote market segments, enabling some generalists to invade the neighbors' domain. Something like this may have happened in 1983, when the mainframe computer producer IBM made its D-day introducing the XT to the personal computer market (Anderson 1994).

The most debated claim in resource partitioning theory is specialists' emergence in mature markets (Carroll 1997). How can these organizations persist in places densely packed with big generalists?⁷ Actually, this is also the most accessible aspect of the geometric explanation. Therefore, we address this claim first, beginning the story at the end when the market is already partitioned between surviving generalists. We consider the known tightest market packings, when generalists' spatial configuration is close to optimal. This is not a trivial assumption, since realizing dense packings may require lengthy reconfiguration. However, if many market segments were left unexploited, then specialists' presence would not require extra explanation.

Customer demands gradually become more diversified and elaborated, so in time the resource space extends into new dimensions. Packing density falls with n . The percentage of total demand accessible for generalists decreases (Figure 2), pockets open between the spheres in which specialists with narrow niche can make footholds. The increase of residual space is steep at each dimensional change. In one dimension, spheres are linear sections that can fill up the resource space without residue. There are usually no specialists in rough markets where products are differentiated only in one dimension (or in none, like in extremely underdeveloped regions, or in some classical shortage economies, Kornai 1980). Moving to two dimensions, the loss in generalist resource utilization is about ten percent (Table 1); so some resource pockets already open even in such poorly differentiated markets. When moving from two to three dimensions, generalists' maximal resource share falls to 75%. The next few dimensional shifts yield roughly 20% loss per step in occupied territory percentage. The decline even gets steeper

⁷ Other organizational niche theories assess competitive advantages via organizational fitness (Levins 1968; Hannan and Freeman 1989). Then, specialists may be superior competitors to generalists, fitting better their output to the needs of particular customer groups.

from $n = 8$. In ten dimensions, the spheres occupy only 10% of the resource space at best!

The present argument does not exploit the notion of market center: generalists leave unabsorbed resources in every segment of the market. So theoretically, specialists may make footholds at any region of taste. However, the geometric approach, alone, does not explain why specialists' occurrence is more frequent at non-mainstream tastes. Here the original and the proposed resource partitioning models complement each other. Imagine the homogeneous resource distribution in two dimensions as an elastic membrane and add inhomogeneities to the flat surface: humps will stand for market centers. The strong competition at the resource abundant areas outforce most of the specialists. So in the one hand, opportunities do open for specialists even at the market centers (geometric explanation), but their survival ratio is much higher at the margins (scale economy explanation).

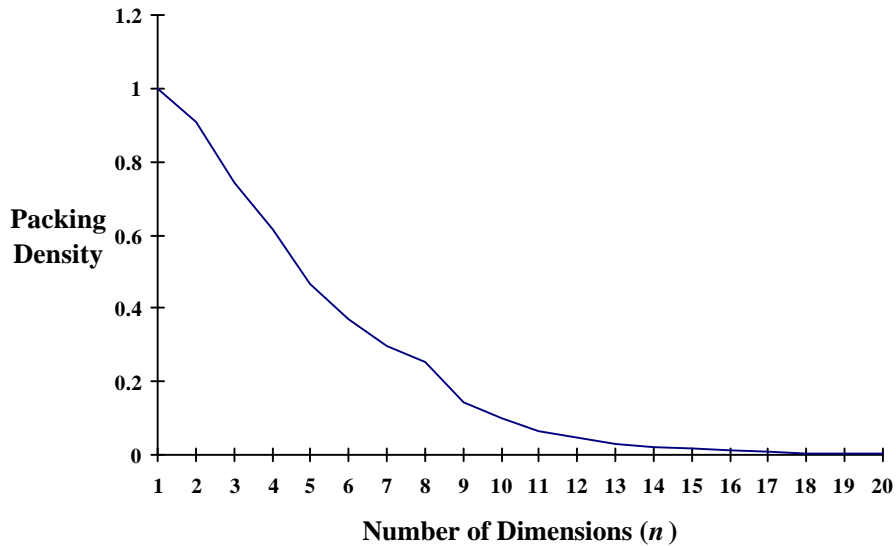
3.4 Increasing Niche width, Decreasing Number of Generalists

Resource partitioning theory predicts that the number of generalist decreases in time, the survivors increase their niche and move towards the market centers. Since homogeneous resource distributions have no centers, we only address the first two effects. Falling packing density as n increases explains the decreasing number of generalists: less and less percentage of the total resource base remains accessible for them. Some agents must quit. But what can be said about the survivors' radius?

Assume that total demand is by and large constant in time; then, each subsequent market phase represents the same purchasing power. Organizations extend their niche along the new demand dimension. One way of doing this is making bids in all taste categories of the new dimension. For example, this would mean that circular niches take cylindrical shape in moving from two to three dimensions. In Figure 3, the circles occupy the same ratio of the plane as the cylinders in the 3D box; consequently, packing density remains the same with this kind of dimensional expansion in place. However, having k taste categories along the added dimension would mean k times bigger product range than before. In car collision control, such a niche extension would mean offering each earlier model in several versions: without airbags, with driver bag, with two front bags, also with side bags, etc. The earlier argument against asymmetric niches applies: overextension in any direction is inefficient. It is better to assume that organizations

compromise on the width of the new taste dimension, and end up in a symmetric niche now in $n+1$ dimensions.

Figure 2. The Density of the Known Densest Packings



Folding out the circles into spheres with preserving their radius could leave plenty of unabsorbed space: Figure 3 may seem to show the emergence of new, unbound resources. However, purchasing power does not increase, rather, it is distributed along a higher number of taste "cells". Having constant total demand and more spatial dimensions, resource density gradually thins out. A sphere with a certain radius occupies a much lower share of the resource base than its ancestor circle with the same radius in 2D. Losing volume percentage means losing customers. If organizations want to preserve their market shares (or not surrender too much of it), then they have to increase their niche width following dimensional shifts. In the scale economy based model, the surviving generalists occupied new territories as a reward of the competition. Now, they must increase their niches not to lose sales.

How big is the radius change that counterbalances resource thinning out? Hypersphere volumes are proportional to r^n (where r is sphere radius), therefore a k

times sphere volume increase means only $\sqrt[n]{k}$ times bigger radius. The required niche extension is "dispersed" over the taste dimensions. Table 2 gives the degree of radius extensions required to preserve the spheres' volume percentage in the extended space (r_{n+1}/r_n). For example, if the added axis is two, three and five times longer, respectively, than sphere diameter (d), then the transition between five and five dimensions indicates 31%, 40% and 52% radius growth. That is, if the range of the new demand aspect is five times longer than d (what is a lot since generalists have broad niche), then the niche has to extend with a half to preserve the original market share. In higher dimensions, the extension is even less. However, the shift between lower dimensions requires quite big niche span increase (Table 2). That is, the appearance of a new taste dimension in early, undifferentiated markets may impel generalists to increase their niche considerably what also means strong competition.

Up till now, the geometric model gave an account for specialists' emergence and for increasing market concentration with time (fewer generalist organizations with broader niches). The specified effects are quite robust. Even a moderately increasing overall demand could not counterbalance the steep fall in packing density and resource thinning out. So, dimensional expansion may be a major cause of generalists' high concentration.

Table 2. The Required Niche Extension After Dimensional Change

$n \rightarrow n+1$	$h = 2d$	$h = 3d$	$h = 5d$
1 \rightarrow 2	1.68	2.05	2.65
2 \rightarrow 3	1.54	1.77	2.10
3 \rightarrow 4	1.42	1.57	1.80
4 \rightarrow 5	1.38	1.49	1.65
5 \rightarrow 6	1.31	1.40	1.52
6 \rightarrow 7	1.27	1.35	1.46
7 \rightarrow 8	1.23	1.31	1.39

8 → 9	1.27	1.33	1.40
9 → 10	1.86	1.28	1.39

Notes. h denotes demand range along the added $n+1^{\text{th}}$ dimension; d is generalist niche breadth given as sphere diameter. Cell contents show the niche extension (r_{n+1}/r_n) necessary to preserve the market share of a generalist in the extended space.

Figure 3. Cylindrical Niche Extension



3.5 Generalists Restrained?

How stable is a resource partitioning? If the importance of resource pockets between hyperspheres increases with time, so that they finally occupy the overwhelming majority of resource space, then why do not generalists absorb these spots? We introduce three additional explanatory notions: *covering density*, *deep hole* and *kissing number*. With these concepts, we formulate arguments that generalists most probably will leave some residual space for specialists, especially in higher dimensional resource spaces.

The residual space between spheres can only be absorbed if generalists modify their

niche. The change can take place in each direction simultaneously (radius increase), or in selected directions. Let us begin with the first option. If generalists increase their niches preserving their spherical form, then finally they will cover the whole resource space. The extended niches necessarily overlap: each generalist organization competes with all of its neighbors. When the covering of the resource space is complete, another mathematical notion, the *thickness of the covering* (Θ) applies. While the sphere packing problem is about densely filling up the space with spheres, the covering problem searches the thinnest covering of the space with hyperspheres of equal size (Figure 4). Covering thickness measures overlap, telling the average number of spheres that contain a given point in space. If each point belongs exactly to one sphere, then $\Theta = 1$. Such sphere covering is not possible beyond one dimensions, therefore $\Theta \geq 1$.

The thinnest coverings are only known up till two dimensions. The best center arrangement in 2D is the hexagonal, $\Theta = 1.2092$ (Figure 4).⁸ Fortunately, lower bounds on Θ are given for each dimension, and the known thinnest coverings well approximate these bounds if n is not high. Covering thickness steeply increases with the number of dimensions (Table 1, Figure 5). Beyond five dimensions, the minimal covering thickness exceeds two; then, more than two producers compete for a customer on average. So, the occupation of resource pockets via spherical niche extension is costly: it ignites strong competition as customer tastes become sophisticated.

The argument against niche overlap can be tightened as follows. A well-known formula from industrial organization, which indicates the relation between profitability and number of equal competitors (m), is as follows (Shapiro 1989):

$$L = (p-c)/p = 1/(m \cdot \varepsilon)$$

Where: L is the Lerner index of profitability, p is price, c is marginal cost, and ε is price elasticity in absolute value. If price discrimination is not feasible, one must offer one price to all customers. The price throughout the niche then equals the lowest price, which is the one offered in the niche overlap by the highest number of competitors. This implies that profitability, as measured by the Lerner index, is

⁸ Note that in general, radius extension in the densest sphere packing does not yield the thinnest covering.

lower than $1/(\Theta \cdot \epsilon)$. Since Θ increases steeply with the number of product dimensions, the loss of profitability due to niche overlap increases steeply.

Figure 4. The Thinnest Space Covering in Two Dimensions

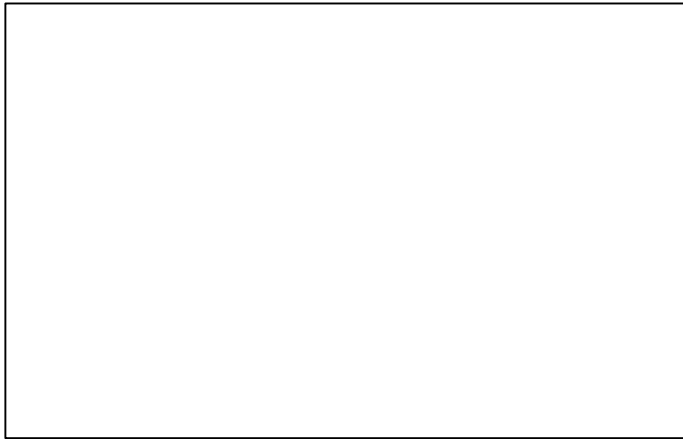
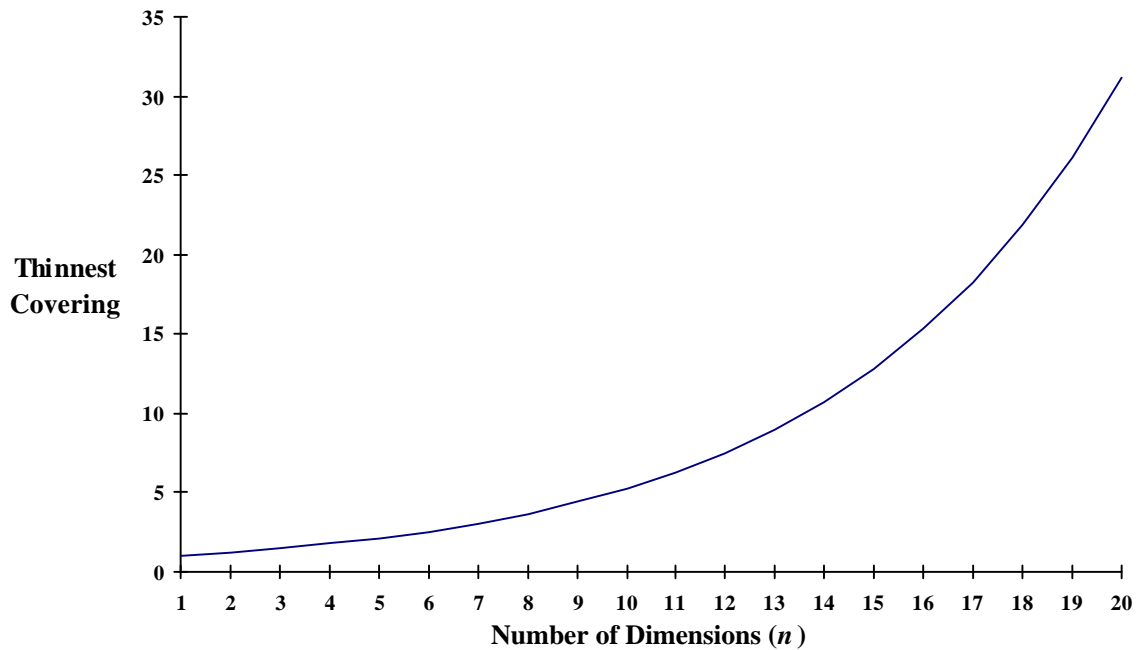


Figure 5. The Known Thinnest Coverings



The other way to occupy the residual space is that generalists extend their niches only in the directions of resource pockets, yielding non-spherical and often asymmetric niches. Obviously, all pockets can be covered without niche overlap if the constraint of spherical niche form is dropped. However, as we mentioned before, overstretching the niche asymmetrically has a price. How big an extension is necessary for a generalist to reach the heart of a neighboring pocket, the *deep hole* (Figure 6)?

The distance between a sphere center and a neighboring deep hole is *the covering radius* (R). The ratio of the covering and the packing radius, R/r , stands for the magnitude of the required overstretch. R/r varies over different packings; for example, in case of quite simple *square packings* (where niche centers form a lattice of hypercubes) it grows to infinity with the number of dimensions. The growth is much less dramatic in dense arrangements. Then, the covering radius R is less than

the double of r : otherwise another generalist could be inserted into the deep holes. However, a 40-70% overstretch required beyond two dimensions to reach the deep hole (Conway and Sloane 1988:158) may be too high a price for the extension to a single pocket. This is especially the case if the residual space is distributed along an increasingly fragmented pocket structure. These aspects lead to the next point.

Generalists have more potential competitors as time passes. In mathematical terms, the *kissing number* (τ), the number of immediate neighbors that touch a certain sphere increases rapidly with n (Figure 7). For example, the kissing number in the known best packings is 24, 72 and 240 at four, six and eight dimensions, respectively (Table 1). The question then is what grows faster: the value of the prey (resource pocket ratio) or the number of predators? The data in Table 1 suggest that the number of neighbors increases much faster than the leftout space ratio: e.g., meanwhile packing density becomes 10 times smaller between one and ten dimensions, the kissing number gets 186 time bigger. The steep growth of kissing numbers means that the residual space becomes fragmented: more and more resource pockets appear with n (an aspect pointed out by Glenn Carroll).

Massively growing covering thickness, overstretch costs and expansion of the kissing number; these structural effects make the occupation of resource pockets increasingly unprofitable for generalists relative to the potential benefits. This may allow for specialist “Switzerlands” to survive among generalist superpowers. Note moreover that organizational perception limits may also play a role in higher dimensions: as pockets around generalists rapidly proliferate, the chance of simply overlooking some of them increases.

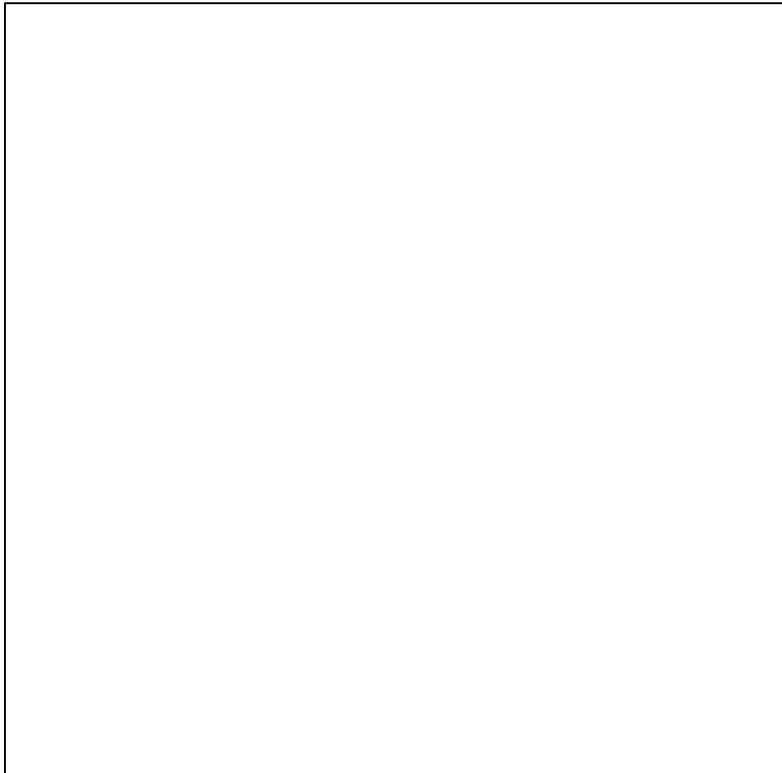
4. Discussion

4.1 Sphere Packings as Market Partition Models

The paper addressed some consequences of dimensional change on the organizations populating the resource space. The geometric properties of sphere packings and coverings offer new explanations for resource partitioning processes in generalist-

specialist markets. How do resource pockets open for specialists? Why does the number of generalists decrease, and why do generalist niches broaden with time?

Applying the sphere packing problem to the generalism-specialism problem, a handful of simplifications had to be made: organizations are able to realize dense market packings, niches are spherical and their widths are similar. Exact mathematical results on the one hand, and a possible lack of realism on the other hand. Are the outlined tendencies strong enough to prevail? Putting this differently, which are the socio-economic conditions under which the outlined tendencies prevail? What we demonstrated is that dimensional expansion has very robust structural effects on sphere packings and coverings, which are likely to have visible effects.



Stinchcombe (1991) calls *mechanisms* those pieces of scientific reasoning that connect lower and higher level entities in theories. He argues that objects at the lower can be conceptualized as very simple if this characterization provides sufficient explanation for the higher level outcomes (like gas molecules are modeled as little balls in classical gas theory). Organizational ecology, and hence the resource partitioning model exemplify multilevel theories: organizational events appear as cumulative outcomes at the population level. Therefore, we modeled our lower level objects with spheres in the belief that this choice provides a simple and powerful explanatory mechanism. Obviously, organizations are much more complex to be exhaustively represented with spheres. But the spherical approach may serve as a baseline model for generalizations. One can add variations to it and speculate about the qualitative outcomes, even if much of the mathematical rigor that characterized the original setting is lost. For example, loosening the constraint on resource homogeneity would introduce niche breadth variations among generalists. Resource abundance may make some extra niche extension profitable. Generalists positioned at the resourceful regions get bigger in niche and in size. The resulting scenery might resemble to soap foam: some of the bubbles are large, and the fluid between them is filled up with very little bubbles. This picture highlights that the ratio of abandoned resource space is very small when generalists' niche width is very much bigger than that of specialists: little balls can tightly fill up the residual place between the big balloons. Maybe generalists' high concentration is a form of efficient market exploitation?

Up till now we used the terms catchment area and niche synonymously. However, there is a conceptual difference between the two. One can speak about organizational catchment area if an organization addresses a certain range of customer taste with a single product. In geometric terms: products are located at sphere centers. Customers within the catchment area have to compromise on their specific demands, buying somewhat different products from what they would prefer. The same region in the resource space can be called niche if the organization addresses each taste in this area with a dedicated product. It may depend on the nature of the product which of the interpretations applies. A crucial question is if bids are sparsely or densely distributed in the market. The space industry is a good example for the catchment area approach: telecommunication firms have not too many options when choosing between satellite launching possibilities. The textile industry may exemplify the niche approach: each firm offers products in a broad variety of sizes, colors and qualities.

4.2. Topics for Empirical Justification

The basic objective of the paper was theoretical: to give new insights and to provide a tool for further theory building, placing the original resource partitioning model into a substantially different explanatory context. The geometric explanation is strong enough to predict the main phases and outcomes of resource partitioning histories, but so far it had little to say about the organizational procedures that finally end up in the specified outcomes. The geometric approach provided a few new predictions that are consistent with the field. It explains specialist organizations' presence in early markets. By pointing out the existence of resource pockets in non-marginal taste positions, we got closer to the explanation of the fact why can some specialists make footholds in demand abundant market segments in spite of generalists' presence.

We predicted a number of effects from an increase of the number of product characteristics. Leftout space rapidly increases its share and it fragments, thus increasing opportunities for small specialists. This yields a question for empirical testing: does the share of specialist organizations increase with the number of product dimensions? This being the case, it is in the interest of small outsiders to innovate products by adding to the number of product dimensions, and it is against the interest of large incumbent generalists. This may give a new twist to the debate on the innovativeness of large versus small firms (Nooteboom 1994b). Given spherical niches, the alternative to having small

specialists in the residual space between generalist hyperspheres is that generalists occupy leftout space and overlap their niches. We showed that the penalty in the form of lower profitability due to competition increases with the number of taste dimensions. To the extent that niches can profitably be non-spherical and non-symmetrical, generalists can occupy left-out space without niche overlap, so that opportunities for small specialists disappear. We proposed that the profitability of such non-spherical niches depends on the share of fixed “costs of reach”. In other words: in industries with low fixed costs of reach relative to variable costs, *ceteris paribus* (at a given number of product dimensions, and a given size of economy of scale), there will be a low share of small specialists. All these predictions can be tested empirically. Industries like book publishing and musical recording could be feasible candidates for investigation (Carroll 1997).

The distinction between catchment areas and niches may also yield topics for empirical research. The hypothesis is that the degree of product differentiation and the density of similar offers in an industry have a decisive role whether catchment areas or niches are formed. The scarcity of bids in the product characteristics space may correspond to catchment area formation: customers have to compromise on their tastes seriously. Conversely, the abundance of similar products in the market would facilitate niche formation. The interplay between these two processes could be studied in a temporal context in markets where the lack of product variety is suddenly replaced by an overwhelming abundance of offers. For example, Eastern Europe with its rapid socio-economic transition offers an excellent opportunity for this kind of research.

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