Pricing of Convertible Bonds with Hard Call Features[1]

Jolle O. Wever[2] and Peter P.M. Smid[3] and Ruud H. Koning[4]

SOM-theme E: Financial Markets and Institutions

[2]Kempen & Co, Beethovenstraat 300, 1077 WZ Amsterdam, The Netherlands.
[3]Department of Economics, PO Box 800, 9700 AV Groningen, The Netherlands.
[4]Corresponding author, Department of Economics, PO Box 800, 9700 AV Groningen, The Netherlands. Email: `r.h.koning@eco.rug.nl`.

Abstract

This paper discusses the development of a valuation model for convertible bonds with hard call features. We define a hard call feature as the possibility for the issuer to redeem a convertible bond before maturity by paying the call price to the bondholder.

We use the binomial approach to model convertible bonds with hard call features. By distinguishing between an equity and a debt component we incorporate credit risk of the issuer. The modelling framework takes (discrete) dividends that are paid during the lifetime of the convertible bond, into account. We show that incorporation of the entire zero-coupon yield curve is straightforward.

The performance of the binomial model is examined by calculating theoretical values of four convertible bonds. The measure used to compare theoretical values with is the average quote, equal to the average of bid and ask quotes provided by several financial institutions. We conclude that in general long historical volatilities and implied volatilities tend to give the best results. Moreover, we find that our model follows market movements very well. The impact of different dividend and interest rate scenarios is rather small.

Keywords: Convertible bonds, hard call, binomial trees

1 INTRODUCTION

The global convertible bond markets is very active at the moment, both in terms of issuance and interest from investors. Convertible bonds are popular financing vehicles for a diverse range of companies. Possible motives for a company to issue a convertible bond are, for example, discussed in Brennan (1986)[1]. Figure 1 gives an impression of the development of the global convertible bond market. The market has grown by 66% in eight years, with most growth occurring in the USA and Europe.
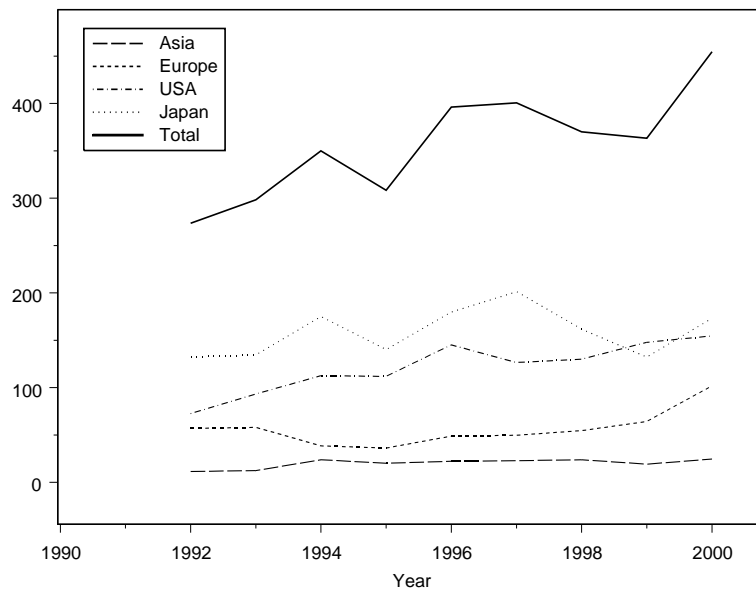

Figure 1: Value of the global convertible bond market in $ billion (source: Deutsche Bank (2001)).

There are different types of convertible bonds. A plain-vanilla convertible bond is a bond that gives the holder the right (but not the obligation) to exchange a bond for a fixed number of ordinary shares, usually of the issuer. Usually convertible bonds are redeemed at maturity and they have fixed coupon payments. The pricing of these bonds is well documented, see for example Tsiveriotis and Fernandes (1998). In the pricing of such a bond, the distinction between the debt part and the equity part plays an important role. When converting into shares, the right to receive (future) coupon

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[1] Other references are Barnea, Haugen, and Senbet (1985), Stern (1992), and Lewis, Ragolski, and Seward (1998).

payments forgoes. The bond part of a convertible bond is usually described in terms of Nominal value, Coupon, Number of coupon payments per year, Issue date and Maturity. The equity part comes up with the Conversion ratio, which gives the number of underlying shares into which the convertible bond can be converted. The Conversion ratio multiplied by the current share price is called the Conversion value. By dividing the Nominal value through the Conversion ratio we find the Conversion price: the price at which shares are effectively 'bought' upon conversion. A convertible bond is called 'in the money' if the share price is higher than the conversion price.

Market-observed convertible bonds are usually not the plain-vanilla type. Without being exhaustive, we mention premium redemption, putable, soft call, callable, step-up, mandatory, parity reset, IPO, multiple currency, perpetual, exotic, ranking, and reverse convertible bonds. In this paper we will discuss the pricing of convertible bonds with hard call features. A hard call feature allows the issuer of a convertible bond to redeem the convertible bond before maturity by paying a predetermined amount (the call price) to the investor.

Usually the issuer can exercise the hard call feature after a predetermined date. The period during which the issuer may not redeem a convertible bond under any circumstances is called the hard non-call period. In addition to the call price the issuer has to pay the accrued interest to the investor. As soon as the issuer announces to redeem a convertible bond a notice period starts. During this period (usually approximately 30 days) bondholders may decide to convert the convertible bond into shares. Often, the issuer tries to force bondholders to convert into shares in order to lessen the degree of leverage of the company. Another explanation for early redemption might be that current financing opportunities are more attractive than the convertible bond issue.

Convertible bonds are quoted as a percentage of the nominal value. Assuming a nominal value of € 1000, we find that the price of a convertible bond quoted as 91.35% is € 913.50, excluding accrued interest. Often the quote is simply 91.35. Call prices are quoted as a percentage of the nominal value, too.

Sometimes call prices follow a multi-stage scheme. For instance, a convertible bond may be redeemed at 105% during the third year, at 102.5% during the fourth year and at 100% during the fifth year.

In this paper we discuss a tractable model for the valuation of such convertible bonds with a hard call feature. Prices derived from this model can be used to value the bonds when they are issued and, perhaps more importantly, when they are non-traded. The model is a binomial valuation model, and the value of the bond depends on the characteristics of the underlying stock (especially volatility and dividend payments) and the term structure. The remainder of this paper is organized as follows. The theoretical model to value convertible bonds with hard call features is introduced in section 2. The model is implemented empirically in section 3, and section 4

concludes.

2   Modelling Convertible Bonds with Hard Calls Using a Binomial Tree

Since a convertible bond is a hybrid security that consists of a debt part and an equity part, it is intuitively logical to value a convertible bond as the sum of those two components. In this section we develop an option-like model that can be used to determine the current theoretical value of a convertible bond with a hard call feature. This value depends of course on the current values of its underlying debt component and equity component. The approach adopted here is the binomial tree method developed by Cox, Ross, and Rubinstein (1979) (the 'CRR approach').

The underlying equity of a convertible bond is usually that of the issuer. Since the issuer can always deliver his own stock, the equity part is not exposed to any credit risk. Following Tsiveriotis and Fernandes (1998), Hull (2000) therefore suggests that the total value of a convertible bond consists of two components: a risk-free and a risky part. The risk-free part represents the value of the convertible bond in case it ends as equity, while the risky part represents the value of the convertible in case it ends as a bond. Remember, we refer here to credit risk only. Of course, the equity part is risky because the pay-offs are uncertain, i.e. dependent on future circumstances. Summing the two components gives the total value of the convertible bond. If we apply the risk-neutral valuation approach, we should use the risk-free discount rate for the equity part. However, the debt part, which comprises all payments in cash due to principal and coupon payments, is subject to risk: cash payments depend on the issuer's timely access to the required cash amounts, and thus introduces credit risk. One possible way to incorporate credit risk is to reduce the expected payoff of the debt part. We follow a different approach: we increase the applicable interest rate. This implies that the debt part should be discounted using an interest rate that reflects the credit risk of the issuer. The risky interest rate can be determined by adding a credit spread (r_c) to the risk-free interest rate (r_f). This spread is the observable credit spread implied by non-convertible bonds of the same issuer for maturities similar to the convertible bond. Often the credit spread immediately follows from the credit rating given to a companies' debt by rating agencies like Standard & Poor's and Moody's.

The CRR approach is perfectly suited to model convertible bonds with hard calls. The stock is the underlying value. The life of the binomial tree should be set equal to the life of the convertible bond. The value of the convertible bond at the final nodes can be calculated by applying possible conversion options that the holder has at expiration. Provided that conversion is permitted, the bondholder converts into shares if the conversion value is greater than the final bond payment (usually the nominal value plus interest). If the holder does not convert, the final payment is the sum

of the nominal value and the final interest payment. Then, by applying the roll-back procedure, the current value of the convertible bond can be determined. The roll-back procedure has to be applied for both the risk-free and the risky part. When rolling back through the tree, at each node we have to determine whether conversion improves the bondholder's situation. Suppose that the node under review is node N. When rolling back, the calculated value of the equity part is equal to

$$E_N = e^{-r_f \Delta t} \left(p E_u + (1 - p) E_d \right), \tag{1}$$

while the value of the debt part is equal to

$$D_N = e^{-(r_f + r_c) \Delta t} \left(p D_u + (1 - p) D_d \right). \tag{2}$$

In these expressions, E_u and D_u are, respectively, the values of the equity and the debt part after an up move (relative to node N), while E_d and D_d represent the equity and debt values after a down move, and p is the risk-neutral probability of an upward movement of the stock price. Note that the credit spread r_c has entered expression (2). The total roll-back value R_N is equal to $R_N = E_N + D_N$.

Now suppose that the convertible bond can be converted into shares of stock. What happens exactly if the bondholder converts? Then the bondholder receives CR shares, with conversion value $CR \cdot S_N$, where CR is the conversion ratio and S_N represents the share price at node N. If the conversion value is greater than the roll-back value, it is favorable to convert; otherwise, the bondholder should not convert. From the discussion above it follows that the value at node N is equal to

$$\max(CR \cdot S_N, E_N + D_N). \tag{3}$$

If conversion takes place, the value of the conversion ($CR \cdot S_N$) is risk-free, so it has to be regarded as the equity part. This means that when the convertible bond is converted, the values of the different components at node N are:

$$E_N = CR \cdot S_N,$$
$$D_N = 0,$$
$$R_N = E_N + D_N = CR \cdot S_N. \tag{4}$$

Suppose that a bond is callable at 101% of the nominal value W, where $W = 1000$. This means that the issuer can buy back the convertible bond by paying 1010 to the bondholder. Depending on the share price, the investor may decide to convert into shares. The issuer's decision to call a convertible bond will be a consideration between the call price, the roll-back value ('do-nothing value'), and the conversion value.

How can this be formalized and be incorporated in a binomial tree? Let C_N be the call price. The issuer tries to minimize the payoff to the investor and tries to set the value at node N (I_N) equal to

$$I_N = \min(R_N, C_N). \tag{5}$$

Table 1: Total value at a node under different conditions of a convertible bond.

Conversion allowed	Calling allowed	Total value at node
Yes	Yes	$\max(\min(R_N, C_N), CR \cdot S_N)$
Yes	No	$\max(R_N, CR \cdot S_N)$
No	Yes	$\min(R_N, C_N)$
No	No	R_N

Table 2: Term sheet of imaginary convertible bond XYZ and characteristics underlying stock XYZ.

Characteristics convertible bond			
Nominal value (e)	1000	Conversion ratio	35.7
Coupon	3%	Redeems at	100.0%
Frequency	1	Call price	100.5%
Time to maturity (years)	3	Number of steps	3
Risk-free interest rate (per year, compounded once a year)	5.24%	Credit spread (basis points)	100

Characteristics underlying stock			
Stock price (e)	31.25	Volatility (per year)	35%

The bondholder is always allowed to convert if the issuer calls the convertible bond. Therefore, the bondholder will maximize his payoff H_N:

$$H_N = \max(I_N, CR \cdot S_N). \tag{6}$$

Substituting expression (5) into (6), we find that the total value at node N is equal to

$$\max(I_N, CR \cdot S_N) = \max(\min(R_N, C_N), CR \cdot S_N). \tag{7}$$

Table 1 summarizes the total value at a node for different conversion and calling possibilities.

The valuation procedure is best understood by using an example. The characteristics of the convertible bond XYZ and its underlying equity are given in table 2.

Consequently, the parameters of the binomial tree are as follows:

$$\Delta t = \frac{T}{n} = 1,$$

$$u = e^{\sigma\sqrt{\Delta t}} = 1.4191,$$
$$d = e^{-\sigma\sqrt{\Delta t}} = 0.7047,$$
$$p = \frac{e^{r\Delta t} - d}{u - d} = 0.4867,$$
$$d_{r_f} = \frac{1}{1.0524} = 0.9502,$$
$$d_r = \frac{1}{1.0524 + 0.0100} = 0.9413.$$

T is the time to maturity, Δt is the length of a time step, r is the risk-free interest rate , σ is the standard deviation of the return on the share, and u and d are the ratios by which the share can increase or decrease in value, respectively. Note that in these formulas the interest rate is transformed into a continuously compounded interest rate so r is 5.11% . Moreover, two discount factors are calculated: d_{r_f} is used for the risk-free equity part, while d_r is used for the risky debt part. Figure 2 shows the binomial tree. At each node four numbers are given: the share price, the equity part, the debt part and the total value of the convertible bond. At times $t = 1$ and $t = 2$ we add the coupon payment (equal to 3) to the debt component. At the two upper nodes at time $t = 3$ the convertible bond is converted into shares, while at the two other nodes the convertible bond is not converted. At the middle node at time 2 the roll-back value is equal to $R_{ud} = 73.22 + 52.76 = 125.98$. The issuer can reduce this value by calling the convertible bond at 100.50. In addition to the call price the issuer has to pay the accrued interest, which is equal to 3. Next, the bondholder's position can be improved by converting into shares. Therefore, E_{ud} becomes 111.56, while D_{ud} becomes zero. At the lower node at time $t = 1$ the roll-back value is $R_d = 51.60 + 51.29 = 102.89$. Calling the bond at 103.50 (the call price plus the accrued interest) does not improve the issuer's position. Finally, the current value of the convertible bond is $R_0 = 123.16$ (at this node, neither conversion nor calling is allowed). The pure bond value is $B = \sum_{t=1}^{3} \frac{3}{(1.0624)^t} + \frac{100}{(1.0624)^3} = 91.38$. This means that the value of the conversion option (net of the issuer's call option) is 31.78. Without the hard call feature the value of the convertible bond would have been 131.23, which can be calculated in a similar way.

The valuation procedure discussed so far does not take dividend payments into account. Dividend payments have a non-negligible impact on share prices, though, so now we incorporate dividend payments in the valuation tree. The dividend adjustment incorporates known dividends in the tree. Denote a dividend with ex-dividend date τ (assume $\tau = i\Delta t$) by the symbol D_τ. Straightforward incorporation would result in share prices $S_u = S_0 u - D_\tau$ and $S_d = S_0 d - D_\tau$ at time τ for a specific i. A tree without dividends has the recombining feature: $S_{du} = S_{ud}$. However, in case of a dividend payment at time τ these two nodes do not recombine. Assuming that $d = 1/u$, we find that the share price after a down move from node S_u
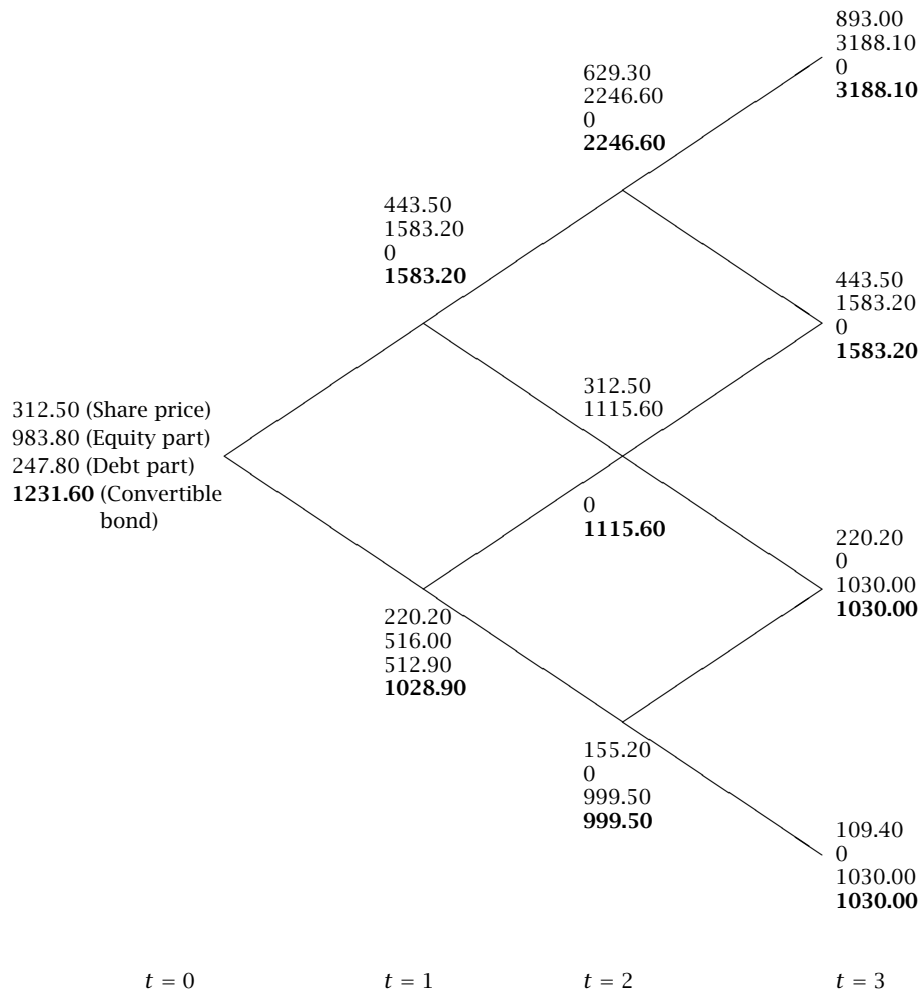
893.00
3188.10
0
3188.10

629.30
2246.60
0
2246.60

443.50
1583.20
0
1583.20

443.50
1583.20
0
1583.20

312.50 (Share price)
983.80 (Equity part)
247.80 (Debt part)
1231.60 (Convertible bond)

312.50
1115.60

0
1115.60

220.20
0
1030.00
1030.00

220.20
516.00
512.90
1028.90

155.20
0
999.50
999.50

109.40
0
1030.00
1030.00

$t = 0$ $t = 1$ $t = 2$ $t = 3$

Figure 2: Binomial tree for convertible bond.

becomes $S_{ud} = (S_0 u - D_\tau)d$, while the share price after an up move from node S_d becomes $S_{du} = (S_0 d - D_\tau)u$. Since $S_{ud} \neq S_{du}$, the tree does not re-combine. This direct approach to modelling dividend payments creates too many nodes: the number of nodes grows exponentially with the number of time periods until maturity. In order to reduce calculation time, the model should deal with dividend payments in a more efficient way.

Hull (2000) discusses a modification that overcomes this problem. The basic idea is to split the share price into two components: a certain part and an uncertain part that is the present value of all future dividends during the lifetime of the convertible bond. Define S^u and σ^u as the uncertain part of the stock and its volatility, respectively. Assume that $0 < \tau < T$, where T represents the expiration of the convertible bond. Then, at time $i\Delta t$ we have:

$$S^u = \begin{cases} S - D_\tau e^{-r_f(\tau - i\Delta t)} & \forall i\Delta t \leq \tau \\ S & \forall i\Delta t > \tau \end{cases} \tag{8}$$

After substituting σ^u in the expressions for u, d, and p, the tree can be calculated, following the normal roll-back procedure. Next, we have to add the present value of the dividend payment. The share price at time $t = i\Delta t$ is given by the following expression:

$$S = \begin{cases} S_0^u u^j d^{i-j} + D_\tau e^{-r_f(\tau - i\Delta t)} & j = 0, 1, \ldots i, \forall i\Delta t \leq \tau \\ S_0^u u^j d^{i-j} & j = 0, 1, \ldots i, \forall i\Delta t > \tau \end{cases} \tag{9}$$

This procedure can easily be extended to a stock paying more than one dividend.

Until now, we have assumed that the term structure of interest rates is flat. In reality, of course, this is not the case so we need to extend the valuation procedure to take the entire zero-coupon yield curve into account.

Assume that the zero-coupon curve consists of n points. Let the symbols t_i and R_i denote the maturity and the continuously compounded yield, on a yearly basis, of point i on the zero-coupon curve ($1 \leq i \leq n$). The concordant short-term interest rates in each time interval are derived from the expectations theory (see for example Cochrane (2001) for a recent exposition). The expectations theory states that long-term interest rates should reflect expected future short-term interest rates: the forward interest rate for a certain future period should be equal to the expected future zero-coupon rate for that period.

Let F_i denote the forward rate between time t_{i-1} and t_i. To calculate the forward rate F_i, the following equation must hold:

$$e^{t_{i-1}R_{i-1}}e^{(t_i - t_{i-1})F_i} = e^{t_i R_i}. \tag{10}$$

Solving for F_i gives

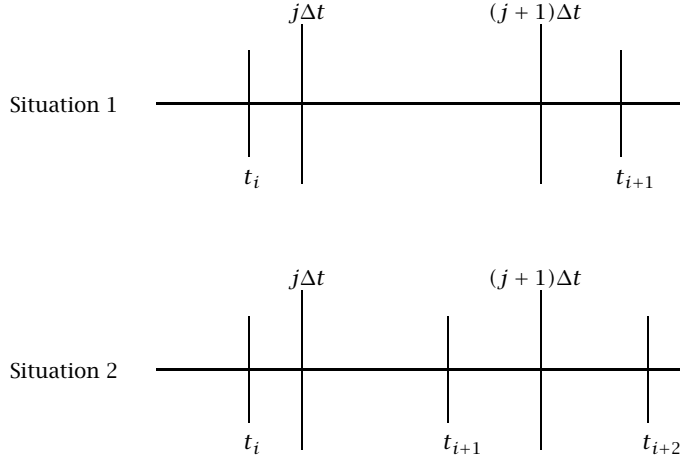$$F_i = \frac{t_i R_i - t_{i-1}R_{i-1}}{t_i - t_{i-1}}. \tag{11}$$

Figure 3: Two different situations that may arise when using forward rates in a binomial tree.

Expression (11) is used to calculate the forward rates between the nodes of a tree. Several situations may occur. In figure 3, two different situations are drawn.

In general the time intervals Δt are rather small. For this reason, situation 1 is most likely to occur. In this case, the forward interest rate between node j and node $j+1$ is equal to $F_{i+1} = \frac{t_{i+1}R_{i+1}-t_iR_i}{t_{i+1}-t_i}$. This means that the risk-neutral probability of an up move between node j and node $j+1$ is as follows:

$$p = \frac{e^{F_{i+1}\Delta t} - d}{u - d} \tag{12}$$

The second possibility is slightly more complicated. In that case, the maturity of a zero-coupon yield falls between two nodes, so that during this time interval we have two different zero-coupon yields. The forward interest rate in this interval is

$$F' = \frac{\left((j+1)\Delta t - t_{i+1}\right)F_{i+2} + \left(t_{i+1} - j\Delta t\right)F_{i+1}}{\Delta t}. \tag{13}$$

Forward rate F' should be used to calculate the probability p in equation (12). Obviously, the two situations given above do not cover all possibilities. For instance, another point of the zero-coupon curve can fall between two nodes. In this case, the forward rate should be calculated analogously to the calculation of F'.

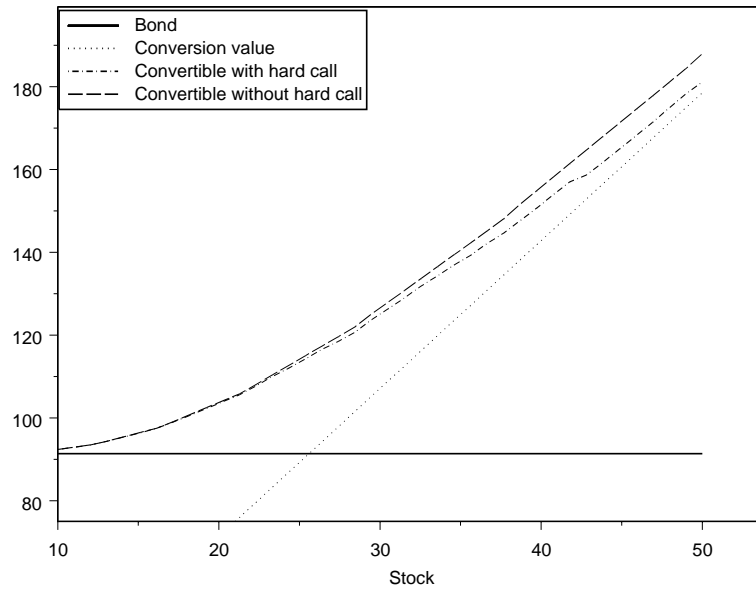Summarizing, we model the value of a convertible bond with a hard call feature by the following steps:

Figure 4: Value of convertible bond with and without hard call feature, conversion value and pure bond value versus share price.

- a CRR binomial tree was used to calculate the value of a convertible bond, thereby distinguishing between a risk-free and a risky component;

- the binomial tree was modified to include issuer's hard call features;

- dividends to be paid out during the lifetime of the convertible bond were incorporated;

- the model was adjusted to take the entire zero-coupon yield curve into account.

In the remainder of this section we show how the value of the bond depends on its characteristics. The characteristics of the convertible bond are specified in table 2.

In figure 4, the value of a convertible bond (with and without hard call feature) and the conversion value are drawn versus the share price. Also, the pure bond value is displayed. A convertible bond without a hard call feature has a higher value than a convertible bond with a hard call features: the hard call feature is unfavorable for the bondholder and therefore reduces the value of the convertible bond.
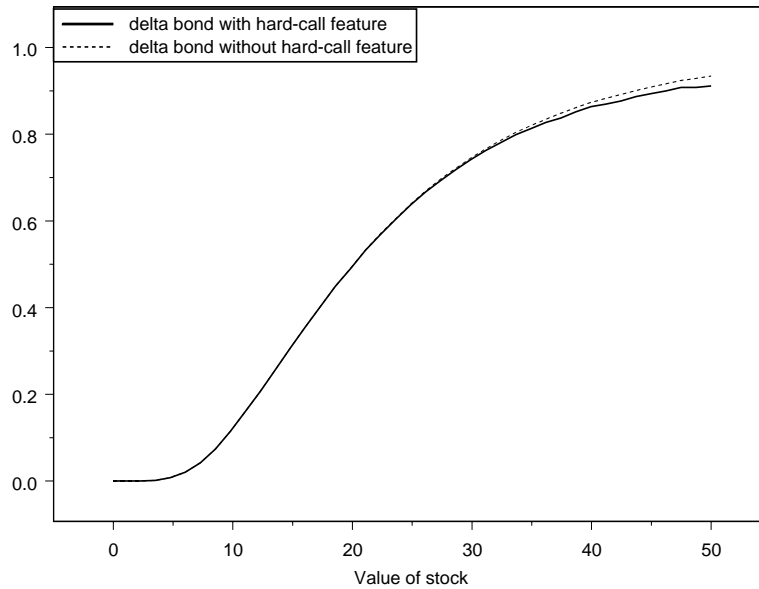
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Figure 5: Delta of convertible bond versus share price.

Figure 4 also shows that for relatively low share prices (compared to the conversion price) the values of the convertible bond with and without hard call are equal to the pure bond value. For low share prices the convertible bond behaves like a bond, while for high share prices the convertible bond behaves like the underlying equity (in this case, 35.7 shares). This can be concluded from the following figure, too: in figure 5 the delta of the convertible bond is drawn. (Delta is the rate of change of the price of a derivative with the price of the underlying equity, 35.7 shares in our example, so delta equals $\frac{\partial CB}{\partial (35.7S)}$.) For low share prices the delta is almost 0, while for high share prices the delta converges to 1.

Figure 6 shows the effect of an increase of the volatility of the underlying share price. A higher volatility increases the probability that a high share price will be reached, so that the value of the convertible bond increases. This can be understood as well by seeing the convertible bond in terms of an option and a pure bond, thereby neglecting the issuer's call. A higher volatility increases the value of the option and leaves the value of the bond unchanged, so that the value of the convertible bond increases. In figure 7 we show the impact of a longer time to maturity. Since the risky interest rate (6.24%) is higher than the coupon (3%), a longer time to maturity reduces the pure bond value. However, a longer time to maturity increases
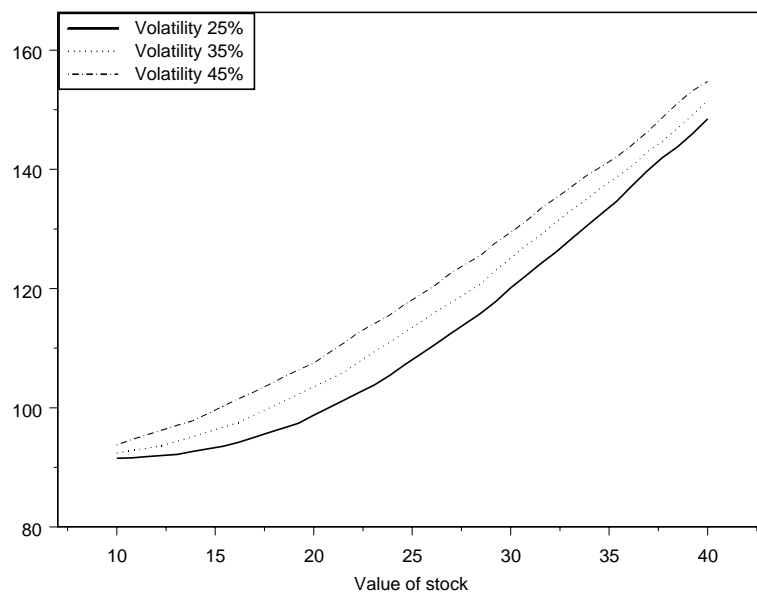
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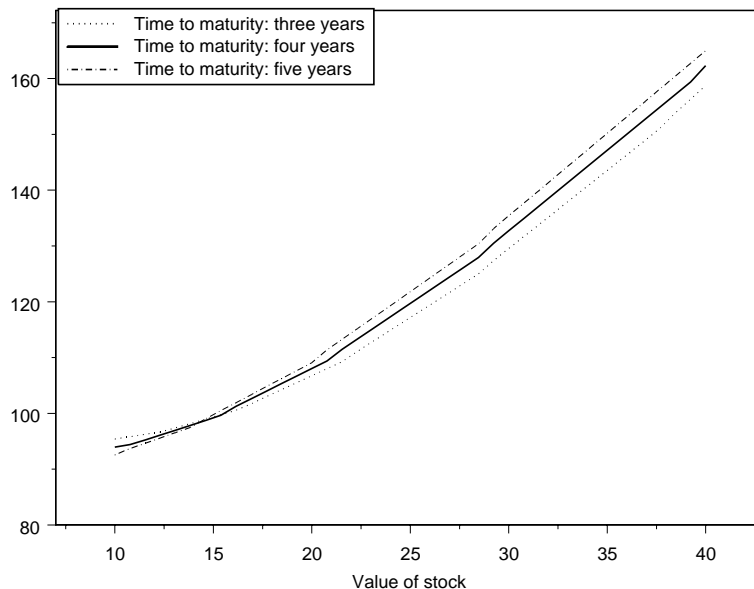Figure 6: Value convertible bond with hard call for different volatilities versus share price.

Figure 7: Value convertible bond with hard call for different times to maturity versus share price.

the value of the implied option. Figure 7 illustrates the net impact of these opposite effects. For low share prices, the debt part dominates the equity part, so the reduction of the pure bond value exceeds the increase of the value of the implied option. For high share prices the net impact of a longer time to maturity is the reverse: the value of the convertible bond increases.

In the next section we put our valuation model to test: we will calculate theoretical values of four convertible bonds. These theoretical values will be compared to market-observed prices.

3 EMPIRICAL RESULTS OF THE BINOMIAL VALUATION MODEL

In this section we use the valuation model of the previous section to calculate theoretical prices of three convertible bonds. Two remarks have to be made. First, in reality the time at which zero coupon rates and concordant forward rates are applicable, does not always coincide with the chosen time step at the binomial tree. Second, the timing of the observed payments does not always coincide with the points in time of the tree.

The bonds we consider are Ahold 3% 1998-2003, Ahold 4% 2000-2005,

and ASML 2.5% 1998-2005; details on these specific bonds can be found in Appendix A. We will compare these theoretical prices with those that were observed in practice, and we examine how sensitive the results of the model are to some of the parameters that need to be estimated.

The convertible bond market is not very liquid, and consequently market prices can be outdated. For this reason we compare the theoretical values with quotes given by financial institutions. These quotes are indicative prices against which banks are prepared to trade convertible bonds. Investment banks like ABN AMRO, UBS Warburg, and Morgan Stanley Dean Witter provide quotes on a real time basis. The measure used to compare theoretical values with is the average quote (Q). The average quote is the average of all bid and ask quotes provided by these banks:

$$Q = \frac{1}{2N} \left(\sum_i B_i + \sum_i A_i \right). \tag{14}$$

In this expression, B_i and A_i represent the bid and the ask quote of bank i, respectively, while N represents the total number of quotes used. Quotes not updated regularly were left out. Moreover, we omitted bad quotes. An example of a bad quote is one that is below the pure bond value. Most times, at least fourteen banks provide useful quotes. The data used for the empirical analysis consist of more than 1600 observations. During three weeks (23 October to 17 November 2000), every five minutes we stored all relevant parameters: the price of the underlying stock, risk-free interest rates, all quotes, etc.

Most of the parameters of the valuation model (for instance the time to maturity) are known with certainty. However, some parameters cannot be determined unambiguously and thus have to be estimated. The parameters to be estimated are the volatility, the dividends to be paid out during the lifetime of the convertible bond, and the risk-free interest rate. In the next three subsections, we will carry out a valuation analysis including a sensitivity analysis with respect to these three parameters.

3.1 Volatility Analysis

First, we consider estimation of the volatility. The volatility can be determined by either using historical data or by calculating the implied volatility from current option prices.

To estimate the volatility of a stock empirically from historical stock prices, the stock price has to be observed at fixed intervals of time. In this volatility analysis, we calculated the historical volatility based on closing prices of the last 5, 10, 20, 30, 50, and 100 trading days[2]. Let the continuously compounded return u_i on day i be calculated as $\ln(S_i + \delta_i D_i) - \ln S_{i-1}$,

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[2]Alternatively, we could estimate the volatility using a (G)ARCH-model, but we do not pursue that direction here.

with δ_i an indicator which is 1 if day i is a dividend day. The n-day volatility is now calculated as

$$\sigma_n = \frac{1}{TD} \sqrt{\frac{1}{n-1} \sum_i (u_i - \bar{u})^2}$$

with TD the length of a time interval in a year. TD can be measured in calendar days or in trading days. Fama (1965) and French (1980) show that volatility is far larger when the exchange is open than when it is closed. French and Roll (1986) conclude that volatility is largely caused by trading itself. These results suggest that we should measure TD in trading days. Since the average number of trading days over the last five years was 253, we use $TD = 1/253$.

Implied volatilities are derived from call options, with preferably the same time to maturity as the convertible bond. However, these series may not be very liquid, so we also consider option series with a shorter time to maturity. We distinguish between volatility implied by a short running option series (this series matures in October 2001), and the volatility implied by series maturing either in October 2003 (in the case of the Ahold 3% convertible) and October 2005 (in the case of the Ahold 4% convertible). For the analysis of the ASML 2.5% convertible, we used option series running until October 2001 and October 2005. We will refer to these volatility estimates as short and long implied volatility estimates.

Finally, dividend predictions were obtained from the merchant bank Kempen & Co[3], and we used two yield curves: a single yield curve and the actual zero-coupon yield curve (see also subsection 3.3).

Because of the high frequency nature of the data, we select two observations for each day, one at 12.00 am and one at 4.00 pm. This leaves us with 36 observations per case. The interpretation of the results is made easier by selecting these observations, since the original observations (taken at 5-minutes intervals) are strongly dependent, and the selected observations display much weaker dependence over time. Unless indicated otherwise, we use this set of 36 observations to assess the fit of the theoretical values to the observed prices.

In Figures 8-9, we graph the average prices of the convertible bonds and the theoretical values. `Average.mid` is the average of the bid and ask quotes (equation 14). Two choices for the volatility estimate are shown in the graphs: `zc.100` is based on historical volatility of the last 100 trading days, and `zc.ivl` is based on the volatility implied by the option series with duration until maturity of the bond. In both cases, the risk free interest is derived from the zero-coupon interest structure.

In Table 3, we list the correlation between the theoretical value and the average quote for all estimates of the volatility (all correlations are calculated using the 36 selected observations). The distribution of the relative

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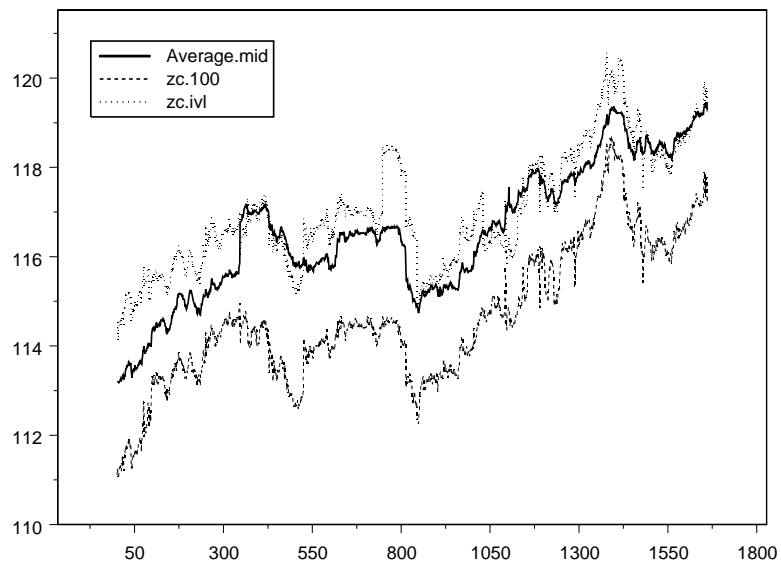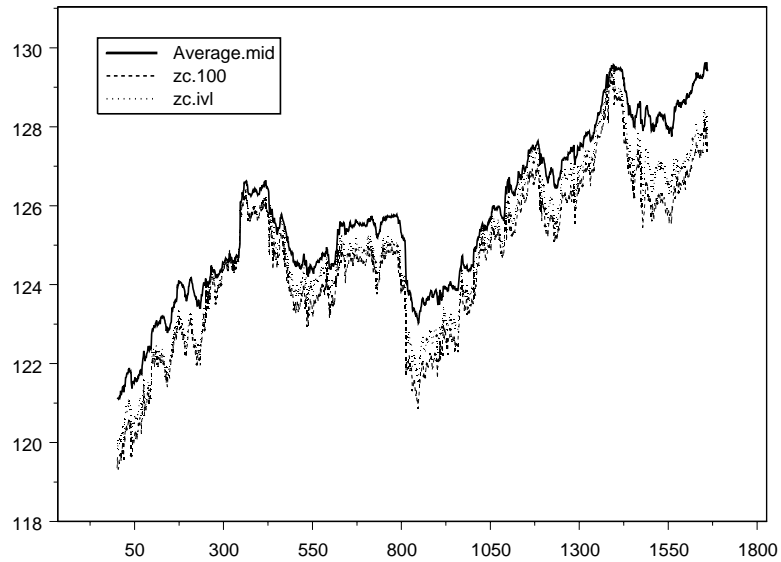
[3]This bank is now part of Dexia Securities.

Figure 8: Theoretical values and average quote Ahold 3% (top) and Ahold 4% (bottom) (zero-coupon interest structure).
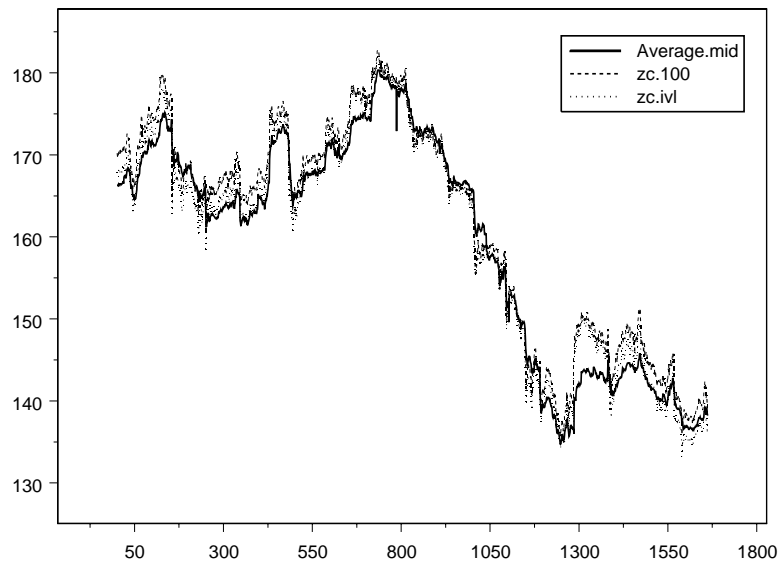
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Figure 9: Theoretical values and average quote ASML (zero-coupon interest structure).

	AHOLD 3%		AHOLD 4%		ASML	
	ZC	SY	ZC	SY	ZC	SY
Hist. 5	0.981	0.981	0.836	0.825	0.916	0.917
Hist. 10	0.987	0.987	0.961	0.957	0.949	0.950
Hist. 20	0.983	0.983	0.948	0.940	0.991	0.991
Hist. 30	0.980	0.944	0.941	0.946	0.991	0.991
Hist. 50	0.944	0.980	0.446	0.453	0.994	0.994
Hist. 100	0.972	0.972	0.954	0.953	0.993	0.993
IV short	0.981	0.981	0.871	0.873	0.947	0.985
IV long	0.983	0.983	0.929	0.932	0.993	0.993

Table 3: Correlation between average quotes and theoretical values for several choices of volatility.

errors are shown in the box-and-whisker plots in Figure 10[4].

We see from Table 3 and Figures 8-9 that the model tracks the observed average prices reasonably well. The correlation between the theoretical values and observed prices exceeds, for almost all choices of volatility, 0.90. There is no single volatility measure that gives the best estimate for the average quote. However, in general the long historical and implied volatilities give reasonably good results. This is shown in Figure 10, where error distributions corresponding to these choices of volatility estimates are more concentrated around zero. For the Ahold 3% and Ahold 4% convertible bonds, the average quotes tend to be underestimated by the model, for the ASML convertible bond the average quote is overestimated. In the case of the Ahold 3% convertible, this bias appears for all cases, that is, for all measures of volatility and for both the single yield and zero-coupon interest structure. In the case of the Ahold 4% convertible, the observed price of the bond is higher than the value resulting from the model for all choices of volatility estimate except the one based on the long implied volatility. In that case, the actual price is lower than the one from the binomial valuation model. From Table 3 we see that the correlation between the model value and the observed value is very low in case of the historic 50-day volatility (compared to the price correlations resulting from other choices of volatility estimates). This can be explained by the sharp rise of the price of the underlying share in the end of October. The price of the stock stabilized again early November. This caused a significant drop of the 50-day historical volatility. Historical volatilities based on less than 50 days adjusted more quickly than the other volatilities.

From Table 3 and Figure 10 we see that the correlation between the theoretical value and the average quote is also very high for the ASML 2.5% convertible bond. The distribution of the relative errors is more dispersed than those of the AHOLD convertibles. The average quote is overestimated on average, but the relative error is small if the volatility estimate is based

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[4]In these box-and-whisker plots, the lower side box indicates the first quartile, and the upper side the third quartile of the distribution of relative errors. The horizontal line in each box is the median. The whiskers indicate the minimum and maximum relative error.
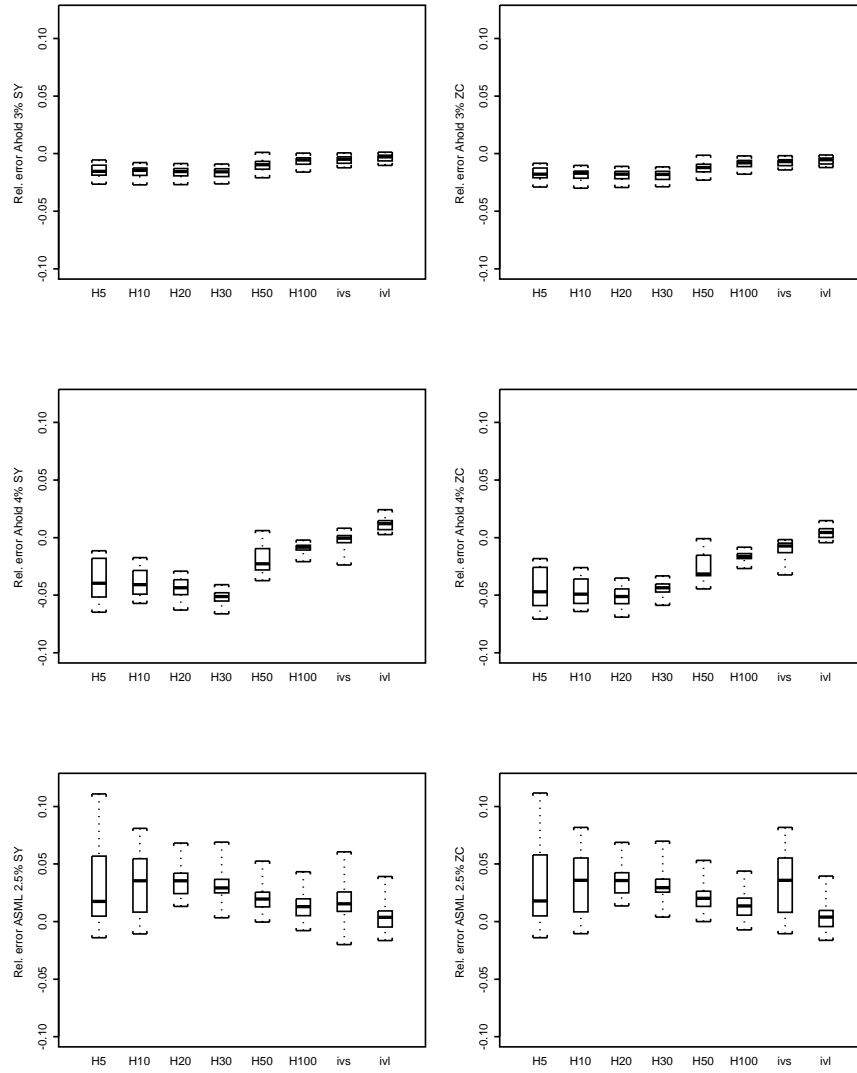
Figure 10: Boxplots of relative error of theoretical valuation.

on long historical data or the long implied volatility.

We conclude this subsection by summarizing our main findings. The binomial valuation model is reasonably well able to model the average quotes. There is no single volatility measure that estimates the average quote best, but in general long historical and implied volatilities give better results.

3.2 DIVIDEND ANALYSIS

In this subsection we examine how sensitive the valuations by the binomial valuation model are to different estimates of the dividends to be paid out during the lifetime of the convertible bond. Since ASML does not pay any dividends, we consider the two Ahold convertible bonds only.

The basic estimates of future dividends are provided by Kempen & Co Research. We have used these estimates in the previous subsection. To estimate impact of different dividend estimates on theoretical prices, we calculate four other scenarios in which the dividends are 80%, 90%, 110% and 120% of the Kempen & Co Research estimates. The volatility used in the analysis is the long implied volatility, which provided a reasonable fit as seen in the previous subsection. Somewhat arbitrarily, we consider four different dividend scenarios, in which the dividends are 80%, 90%, 110% and 120% of the baseline estimates.

Higher dividend payments reduce the value of the convertible bond. The reductions of the values of the convertible bonds due to an increase of the dividend payments are seen both for the Ahold 3% convertible and the Ahold 4% convertible, and do not depend on the interest structure chosen. The Ahold 4% convertible bond is most sensitive to changes of the dividend estimates: on average, the bond loses 0.5% in value when dividends increase by 10%. The same quantity is only 0.2% for the Ahold 3% convertible. This is due to the longer time to maturity (2005 versus 2003) of the Ahold 4% convertible.

3.3 INTEREST RATE ANALYSIS

The last parameter in the binomial valuation model that needs to be chosen is the risk-free interest rate. Previously, little attention was paid to the risk-free interest rates used to calculate theoretical values and in this section we are more precise. We distinguished between two different term structures of interest rates: a full zero-coupon term structure (ZC) and a single-yield term structure (SY). The single yield is the yield to maturity on Dutch government bonds. The expiration of this government bond is the expiration that matches best with the expiration of the convertible bond. The single yields are, for example, given by Reuters on a real time basis. The zero-coupon term structure can be found on Reuters as well and is derived from swap rates.

We use the zero-coupon curve based on the euro. To examine the impact of a different term structure, for both structures the interest rates are lowered and increased by 10 and 25 basis points ($\pm 0.10\%$, $\pm 0.25\%$). The

volatility used is the long implied volatility. In Figure 11 we display the relative error of the valuation under different terms structures of interest rates.

If we look at the valuations at different times, it turns out that an upward shift of the yield curve reduces the theoretical value of a convertible bond. Especially the Ahold 4% convertible bond is quite sensitive with respect to interest rate changes, which is explained by the long time to maturity. However, we do not observe this for the ASML convertible bond, which has more or less the same time to maturity. Since this convertible bond is far in-the-money, the equity part dominates the debt part. Consequently, the convertible bond behaves like equity and is, keeping all other factors constant, not very sensitive to interest rate changes. The relative error distributions in Figure 11 do not vary much by the yield curve that is chosen. Therefore, a shift of the yield curve does not change theoretical values enormously, especially if the convertible bond is in-the-money. For example, on October 23 2000, 16.00hrs, the model value of the Ahold 4% convertible with zero-coupon rates was 114.98, and with single yield rates 115.80 (both numbers are calculated using the long implied volatility estimate). Shifting the yield curve upwards by 25 basis points decreases the zero coupon valuation to 114.52, shifting it downwards by 25 basis points changes the zero coupon valuation to 115.33.

4   Conclusions and Directions for Further Research

In this paper we have proposed a binomial valuation model for convertible bonds with hard call features. The model distinguishes between a debt component and an equity component of the bond. It takes dividend payments during the life time of the bond into account, as well as the actual yield curve.

In section 3 we compared the theoretical valuations with the market prices of three different convertible bonds. In general, the binomial valuation model tracks the actual quotes quite well. The correlation between the theoretical value and the actual average quotes exceeds 0.95 for almost all choices of volatility estimates, dividend policy, and term structure of interest rates.

We examined in detail how sensitive the valuations are to the three parameters. It was found that the results are sensitive to the estimate of the volatility of the underlying stock, and much less so to the estimate of dividend payout and the choice of the specific term structure of interest rates. Of course, this depends on the level of the underlying stock price. Estimates of volatility based on long historic data of the stock, or on the volatility implied by call-options with a long life time tend to give the best results.

In future research we will use a similar approach to the valuation of convertible bonds with soft call features. These bonds are convertible only if the level of the stock price has exceeded a prescribed level for a certain amount of time. This creates state dependence in the binomial tree, and
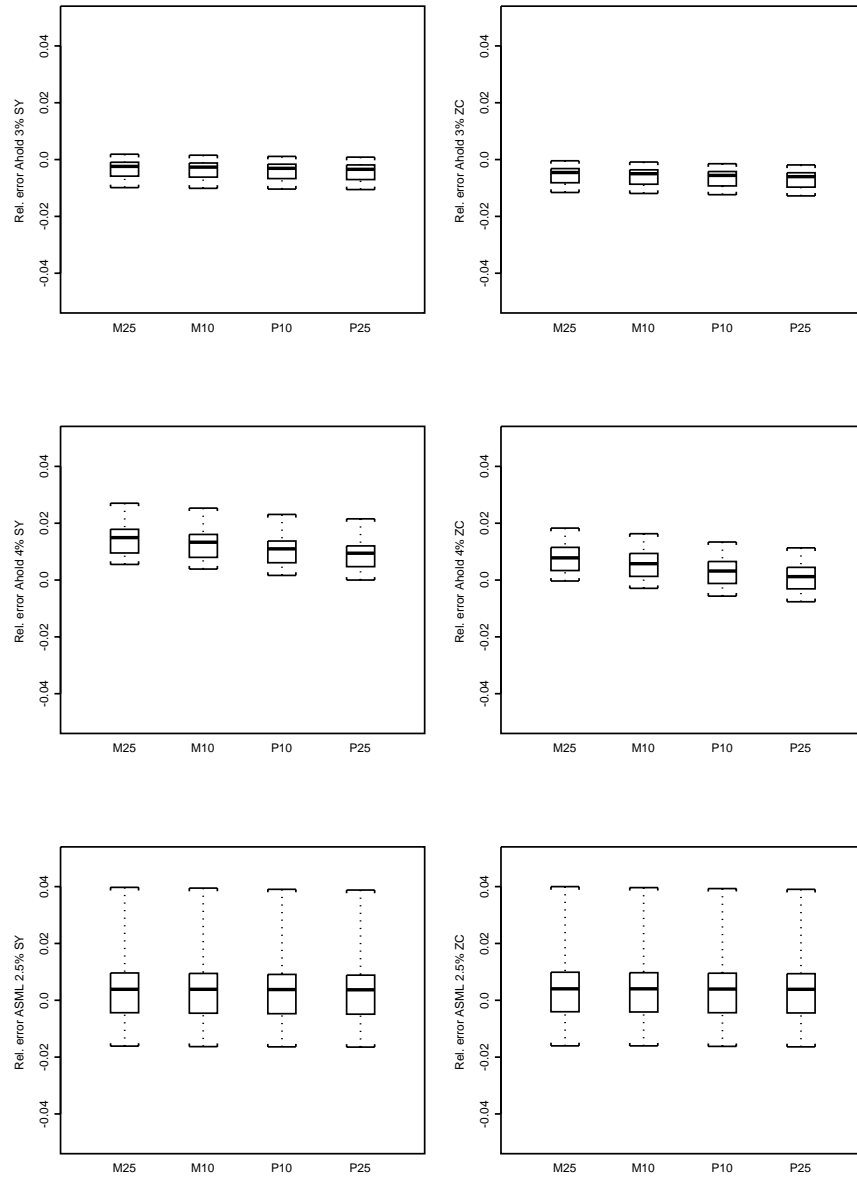
Figure 11: Boxplots of relative error of theoretical valuation.

therefore valuation using methodology based on partial differential equation seems to hold more promise.

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A   Details of the Convertible Bonds

	Ahold 3%	Ahold 4%	ASML
Issue price	100%	100%	100%
Issue date	10 September 1998	15 May 2000	19 March 1998
Issue amount	ƒ 1,495 million	e 920 million	ƒ 600 million
First settlement	30 September 1998	19 May 2000	9 April 1998
Denomination	ƒ 1000	e 1000	ƒ 1000
Redemption amount	100%	100%	100%
Redemption date	30 September 2003	19 May 2005	9 April 2005
Conversion ratio	16.3113	30.5437	23.3120
Conversion price	e 27.82	e 32.74	e 19.47
Coupon	3%	4%	2.5%
Coupon payable	Annual	Annual	Annual
Moody's rating	Baa2	Baa2	B2
S & P's rating	BBB	BBB	N/A
Credit spread	180 bps	180 bps	450 bps
Hard non-call	30 September 2001	19 May 2003	9 April 2001
Call price	100%	100%	100%
ISIN code	XS0090798405	XS0111182597	USN07059AA19

Sources: Deutsche Bank, Bloomberg