TIME BUCKET SIZE AND LOT-SPLITTING APPROACH

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Som-theme A Primary processes within firms

ABSTRACT

We address the problem of lot splitting for various time bucket lengths in MRP systems. Two approaches for lot splitting can be applied: either use the same (equal) or a variable number of subbatches. Equal subbatching strategies have logistical and computational advantages. Literature states that variable batching strategies are only marginal better. However, these results do not take into account the sensitivity for changes in time bucket length. Managers have reduced time bucket lengths in planning systems. We examine the sensitivity of lot splitting for these changes. Our study reveals that it is not cost-effective to disregard time bucket length when deciding on the number of subbatches. Using the same number of subbatches per time bucket for all products results in substantial cost-differences, where the magnitude is affected by the discontinuity of the total cost curve. For a given time bucket length, a cost difference with a variable number of subbatches per operation of only 2.1% can be obtained if an appropriate, equal number of subbatches for each product can be found. Other equal subbatching strategies show much larger cost differences on average, ranging from 4-11%. In order to obtain these results, a new variable subbatch heuristic has been designed.

KEYWORDS: Lot sizing, Lot splitting, Time bucket size, Planning and Control

1 INTRODUCTION OF THE PROBLEM

MRP planning systems use time buckets for time-phasing the co-ordination of several stages of production, during each of which one or more operations are being performed. The length of the time bucket is an important design parameter, as it constrains the replanning frequency of the planning system and affects the length of internal work order lead times. Planned lead times in an MRP system are always a multiple of the length of the time bucket P. We would therefore expect determination of the time bucket length to have received much attention in literature. However, important and influential studies on understanding lead times in MRP systems - such as Kanet 1986, Karmarkar 1987, and Krajewski et al. 1987 - do not pay attention to the length of the time bucket at all. More recent studies by Matsuura et al. 1996, Portioli Staudacher 2000, Enns 2001, and Enns 2002 devote attention to the problem of setting MRP system design parameters. These studies consider the effect of time bucket size on lead time performance and on the costs of system operation. Several other papers have studied the effect of time bucket length variation on system performance, e.g. Luss 1989, Rees, Huang and Taylor 1989, Yang and Jacobs 1992, and Steele et al. 1995. These studies have shown that the length of a time bucket has a substantial effect on system performance. Many firms are aware of the benefits of time bucket length reduction and managers try to improve cycle times by reducing time buckets in the planning systems used (Burbidge 1996). Software developers for ERP systems have made it possible to exploit these benefits through the introduction of so-called ‘bucketless systems’. Behind the scenes, these systems still use discrete time units, but they allow various ordering intervals and make it easier to shorten the actual time bucket length that is used in operating the system. Setting the time bucket length is an important medium-term decision in planning system design.

A design parameter that has received much more attention in literature on planning system design is the lot size, see e.g. Wemmerlöv 1979, Karmarkar 1987, Wemmerlöv 1989, Shtub 1990, and Yeung, Wong and Ma 1998. The lot sizes used within an MRP system are either static or dynamic. Dynamic lot sizes cover demand during one or more time buckets. The minimum number of periods a lot size has to cover demand is one, in which case it is denoted as Lot for Lot ordering. The size of the lot may therefore not be smaller than the demand for this product during one time bucket, as the time bucket sets the minimum time between ordering moments. This condition holds true both for dynamic and static lot sizes. Therefore, lot sizes in an MRP system depend on the time bucket length P.

The size of the lot influences the amount of time necessary to produce the lot (see e.g. Kanet 1986). A decrease of the lot size results in a smaller production time and throughput time of the lot in the system. Lot size reduction can be obtained by decreasing the minimum time between ordering moments, the time bucket length P. Another way to shorten throughput times is by applying lot splitting. We define lot splitting as the division of the lot into nb subbatches that can be transferred to the next operation as soon as the former operation has been performed for all items in the subbatch. Lot splitting may have a substantial effect on the minimal required throughput time of the complete lot. Lot splitting affects the required number of stages N and hence the total amount of inventory in the system at a certain point in time.

The popularity of lot splitting has grown in practice since the success of cellular manufacturing. Olorunniwo 1996 reports that 45.6% of the firms moving towards cellular manufacturing apply lot splitting. About 90% of these firms indicate that such a change in planning and controlling the system significantly contributes to their success.

Several types of lot splitting have been described in the literature, for example, repetitive lots (Jacobs and Bragg 1988), overlapping operations (Graves and Kostreva 1986), lot streaming (Baker and Pyke 1990), transfer lot sizes (Trietsch 1989) and powered nested batching policies (Muckstadt and Roundy 1993).

A long-standing debate in this literature concerns the size of the subbatches. Size can be expressed either in the number of items in the subbatch or in the amount of time necessary to process the subbatch. Some authors (e.g., Veral 1995) focus on utilization of the equipment and argue that the number of items in the subbatches may change between operations if such a change helps maintain a constant amount of time necessary for the subbatch at the operations. Others (e.g., Muckstadt and Roundy 1993) use consistent subbatches but allow different subbatch combinations at the various operations.

A related point of discussion is the effectiveness of unequally sized subbatches. Some authors (e.g., Kropp and Smunt 1990) argue that the logistical complexity of various subbatches in the system should be avoided as much as possible. Usage of unequally sized subbatches for the products in the system decreases the transparency of the planning system. Other authors are more positive. Bogaschewsky, Buscher and Lindner 2001 state that for geometrically changing subbatch sizes at subsequent operations, total costs may be slightly lower. An important study by Baker and Pyke 1990 states that the advantage of unequally sized subbatches for flow time performance is only marginal (on average 4.6% reduction of make span in their test cases). They conclude that decision makers should trade off the complexity increase of unequally sized subbatches with these small savings.

However, in drawing these conclusions concerning the effectiveness of equally sized subbatches, the length of the time bucket has not been considered as an experimental design factor. In situations where managers reduce time buckets, they have also to decide on the appropriateness of the lot splitting approach used. Our study supports this decision and examines the effectiveness and appropriateness of an equal number of subbatches approach if time bucket length varies.

We expect that the effectiveness and near optimality of a strategy involving an equal number of subbatches depends on the length of the time bucket in the planning system and on the type of equal subbatch strategy applied. The objective of this study is therefore to explore the effect of time bucket size on the use of an equal subbatch strategy. We know that the length of the time bucket affects the total lot size and the amount of material transfers per year. An increase in the number of subbatches nb increases the number of material transfers within a time bucket P. This leads to larger transfer costs and possibly smaller holding costs. Therefore, a cost perspective will be useful when analysing the effectiveness of an equal number of subbatches strategy.

This paper is organised into six sections. Section 2 specifies the type of planning system explored and provides a mathematical model for the relations between the design parameters of this planning system. It develops an enumerative search heuristic for finding a variable subbatch solution. Section 3 presents the experimental design for evaluating several equal subbatch strategies, using a simulation approach. Section 4 presents the performance of the equal subbatch strategies for successive values of the time bucket length. Section 5 presents our conclusions.

2 MODELLING THE PLANNING SYSTEM

The planning system studied in this paper may be characterized according to two basic principles. First, for all products and parts this system uses an identical amount of time between two order releases (the reorder interval), which equals one time bucket *P*. Second, it is a lot for lot ordering system, i.e., the amount ordered equals the demand during the next period of length *P*. Lots consist of one type of end product; hence, the total throughput time of a lot equals the total throughput time of the products contained in this lot.

The number of stages *N* in the planning system is determined by the number of periods of length *P* a lot remains in the production system in various levels of completion. The throughput time of the system is the product of *N* and *P*. The basic idea is to determine the effect of time bucket length *P*, number of stages *N*, and number of subbatches nb_{hi} on three relevant cost factors: holding costs HC_h in the system, ordering or set-up costs SC_{hi}, and transfer costs TC_{hi}.

The number of stages *N* is a function of *P* and nb_{hi}. These factors directly influence the total throughput time TT_h needed for a batch q_h of product *h.* We assume for simplicity that the operations have to be performed in sequence (a linear product structure), so we can use as a lowerbound for TT_h :

$$TT_h \qquad \geq \max_{i=1}^{n_h} \left[s_{hi} + p_{hi} \cdot q_h + \sum_{t=i+1}^{n_h} \left\{ p_{ht} \cdot \left[\frac{q_h}{nb_{hi}} \right]^+ \right\} \right] \tag{1}$$

with s_{hi} *being* the set-up time and p_{hi} the unit processing time for operation i of product h (i=1..n_h). The number of stages N is an integer. We assume that the planning system uses N stages for all products h.

Formula (1) models a *variable subbatch strategy*, as it allows variation of the number and size of the subbatches *between* operations i and i+1.

An *equal subbatch strategy* uses the same number of subbatches for any operation. It can be modelled using an extra constraint with respect to nb_{hi}, as shown in Formula (2) and (3). Equal subbatch strategies of type A use the same number of subbatches for all products h. Type B strategies allow the number of subbatches per product to vary, but use the same number of subbatches for all operations of a product.

$$nb_{hi} = nb_{hi-1} \forall h, i > 1 \text{ (equal subbatch strategy type A)} \quad (2)$$

$$nb_{hi} = nb_{hi-1} \forall i > 1 \text{ (equal subbatch strategy type B)} \quad (3)$$

Holding costs are incurred from the moment that the required raw material is introduced into the system until the moment the finished product leaves the system. We assume that all required material is introduced during the first stage of the planning system. On average, the raw material will be introduced halfway through the first production stage and the final product will be sold or delivered halfway through stage N+1. The material worked into a lot q_h is therefore, on average, N stages of length P present in the system, irrespective of the actual progress in making the product. The average total inventory in the system is then $\sum_h N \cdot q_h = N \cdot P \cdot \sum_h D_h$. Holding one item of product h in stock during a standard time unit costs HC_h, so the total cost of holding this inventory during a standard time unit is $N \cdot P \cdot \sum_h HC_h \cdot D_h$.

The second cost factor is costs that will be involved every time a new cycle starts irrespective of the number of subbatches or the length of the time bucket. Examples of these costs are ordering and set-up costs. The number of cycles is 1/P per standard time unit. The set-up costs for an operation are the product of the set-up time for this operation s_{hi} and its set-up costs per standard time unit SC_{hi}. Each operation i of product h is performed once per cycle, unless the lot size in one of the cycles is zero. We therefore sum up the set-up times for all n_h operations of product h. The total set-up costs *per cycle of length P* are $\sum_{h=1..H} \sum_{i=1..nh} s_{hi} \cdot SC_{hi}$. Total set-up costs *per standard time unit* can be obtained through dividing this by the length of P.

Finally, we consider the costs of transferring subbatches to the next operation. These costs depend on the number of subbatches of a product h: nb_h. TC_{hi} reflects the cost of transportation and administration effort required at the successive operations. If the lot of product h is split at operation i into several subbatches (nb_{hi}>1), the total transfer costs increase. The transfer costs TC_{hi} may vary per product and operation. We assume that the transfer cost is linear with respect to the number of subbatches. Therefore, the transfer costs per standard time unit are $[\sum_{h=1..H} \sum_{i=1..nh} nb_{hi} \cdot TC_{hi}] / P$.

We obtain the following non-linear model for finding P and a variable subbatch strategy:

$$\underset{P,\, nb_{hi}\,(h=1..H,\, i=1..n_h)}{Minimize} \quad N \cdot P \cdot \sum_{h=1}^{H} \{D_h \cdot HC_h\} + \frac{\sum_{h=1}^{H} \sum_{i=1}^{n_h} \{s_{hi} \cdot SC_{hi} + nb_{hi} \cdot TC_{hi}\}}{P} \tag{4}$$

$$s.t. \qquad N \geq \max_{h=1}^{H} \left[\frac{TT_h}{P} \right]^+ \text{ with } TT_h \text{ as defined in Formula (1)} \tag{5}$$

The model searches for the optimal combination of a time bucket length P and a variable subbatch strategy nb_{hi} with respect to the sum of holding costs, set-up costs, and costs for the transfer of subbatches between operations. The model gives us insight into the combined effect of time bucket size and splitting a lot into nb_{hi} subbatches. However, no solution methods are available to optimally solve this model within polynomial time.

We can solve a simplified model that fixes P and tries to find a suitable subbatching strategy at this time bucket length. Repeatedly applying this model for successive values of P will yield an idea of the cost curves as a function of the time bucket length P. For fixed P, the problem of finding a suitable subbatching strategy can be reformulated as:

$$\underset{nb_{hi}(h=1..H, i=1..n_h)}{Minimize} \quad N \cdot \sum_{h=1}^{H} \{ q_h \cdot HC_h \} + \frac{\sum_{h=1}^{H} \sum_{i=1}^{n_h} \{ nb_{hi} \cdot TC_{hi} \}}{P} \qquad s.t. (1) \text{ and } (5) \tag{6}$$

For this problem, we determine the cost of various subbatch strategies at succeeding values of the time bucket length *P*. We develop an approximate solution approach to find a variable number of subbatches strategy and evaluate the effect of equal versus varying numbers of subbatches at different time bucket sizes.

2.1 Finding a variable subbatch solution

Graves and Kostreva (1986) discuss the problem of finding a suitable number of subbatches in a two-machine flow shop with equal processing times *p* and suggest a value equal to

$$Q_h^* \cdot \sqrt{p} \cdot \sqrt{\frac{HC_h}{TC_{hi}}} \qquad with\ Q_h^* = \sqrt{\frac{2 \cdot D_h \cdot \sum_{t=i}^{i+1} (s_{ht} \cdot SC_{ht})}{HC_h}} \tag{7}$$

A rounding procedure is proposed in order to find an integer valued number of subbatches for the batch of size Q^*. For the case of more than two operations, unequal processing times p_{hi} and fixed period length *P*, nb_{hi} has to reflect the number of subbatches per period of length *P*. We therefore transform the function to:

$$\begin{aligned} \tilde{n}b_{hi} &= P \cdot D_h \cdot \sqrt{\min[p_{hi}, p_{hi+1}]} \cdot \sqrt{\frac{HC_h}{TC_{hi}}} \\ nb_{hi} &= \lfloor \tilde{n}b_{hi} \rfloor \quad if \lfloor \tilde{n}b_{hi} \rfloor \cdot \lceil \tilde{n}b_{hi} \rceil \geq \tilde{n}b_{hi}^2 \quad (nb_{hi} \text{ is the largest integer} \leq \tilde{n}b_{hi}) \\ &= \lceil \tilde{n}b_{hi} \rceil \quad if \lfloor \tilde{n}b_{hi} \rfloor \cdot \lceil \tilde{n}b_{hi} \rceil < \tilde{n}b_{hi}^2 \quad (nb_{hi} \text{ is the smallest integer} \geq \tilde{n}b_{hi}) \end{aligned} \tag{8}$$

The reduction in throughput time due to the increase of the number of subbatches at operation *i* is a function of the minimum of the processing times at operations *i* and *i*+1:

$$\Delta TT_h \geq \left(\frac{q_h}{nb_{hi}+1} - \frac{q_h}{nb_{hi}} \right) \cdot \min\left[p_{hi}, p_{hi+1} \right] \tag{9}$$

We therefore propose to use the minimum of both processing times in Formula (8). Note that Formula (8) does not explicitly model the effect of throughput time reduction. Furthermore, it does not consider the interdependency between the decisions on the number of subbatches at the various operations. For the cost structure of our model, both factors have a strong impact. However, no solution approaches are known that solve this problem within polynomial time. We develop a heuristic that takes the effect of an increase in the number of subbatches at an operation on the expected throughput time into account.

2.2 Enumerative search heuristic

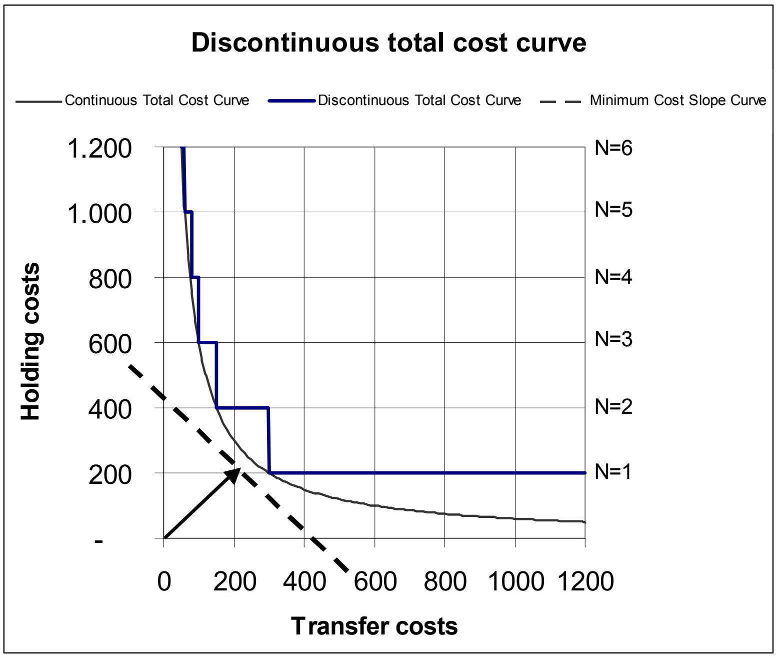

Figure 1 Discontinuous total cost curve for fixed *P*

The *enumerative search heuristic* tries to find the most cost effective variable batching strategy for a specific value of the time bucket length *P*. This batching strategy has to balance transfer costs (nb_{hi}) and holding costs (*N*). It takes into account that the actual total cost curve is discontinuous, as illustrated in Figure 1. Measures to increase the number of subbatches do therefore not directly pay off in a reduction of system throughput time, as would have been the case if the total cost curve had been continuous. Throughput time reduction is only achieved if the number of stages N decreases. If the associated holding cost reduction is still worthwhile depends on the increase in transfer costs necessary to achieve the throughput time reduction. The law of diminishing marginal returns holds here, illustrated by the shape of the total cost curve. The intersection of the minimum cost slope curve and the discontinuous total cost curve gives us the optimal balance between holding costs and transfer costs. Total costs can be read from the point where this minimum cost slope curve intersects with one of the axes.

The enumerative search heuristic subsequently evaluates suitable variable batching strategies for decreasing values of *N.* The heuristic begins with an initial (large) number of stages *N*, and a subbatch strategy in which all products have only one subbatch (nb_{hi}=1). The heuristic directs its attention to finding breakpoints in the cost curve - a function of *N* and nb_{hi} - compares the solutions for these breakpoints, and selects the batching strategy that results in the lowest total cost. The heuristic considers an increase in the number of subbatches per operation up to a user defined maximum number of subbatches at an operation. The decision to increase the number of subbatches at an operation is based on information related to the breakpoints' locations and the *estimated* reduction in the total throughput time. We use the lowerbound of Formula (9) for estimating this reduction. All products for which the throughput times have to be shortened are considered. The increase in the number of subbatches for an operation causes extra transfer costs, but a throughput time reduction will eventually reduce holding costs. The ratio between the two cost changes, $Cost_{hi}$, drives the selection of a suitable batching strategy. For $Cost_{hi}$, we use a ratio of holding costs and transfer costs similar to the one used in Formula (8).

1 $\quad Set\ nb_{hi} \Leftarrow 1 \quad \forall h, i$

2 $\quad Compute\ TT_h \quad \forall h \qquad (length\ of\ path\ TT_h\ is\ throughput\ time\ of\ product\ h)$

$$N \Leftarrow \frac{\max_h TT_h}{P}$$

$$Total\ cost[N] = N \cdot \sum_h q_h \cdot HC_h + \sum_h \sum_i nb_{hi} \cdot tc_{hi}$$

3 $\quad Consider\ if\ a\ solution\ with\ shorter\ throughput\ time\ (less\ stages)$
$and\ more\ subbatches\ has\ lower\ total\ costs:$

FOR ALL h $with\ TT_h > (N-1) \cdot P$

4 $$\Delta TT_{hi} = \left(\frac{q_h}{\max\left[nb^{\max}, nb_{hi}+1\right]} - \frac{q_h}{nb_{hi}} \right) \cdot \min\left[p_{hi}, p_{hi+1}\right]$$

$(\Delta TT_{hi}\ is\ a\ lowerbound\ on\ the\ throughput\ time\ reduction)$

5 $$Cost_{hi} = \frac{1 \cdot TC_{hi}}{\Delta TT_{hi} \cdot HC_h}$$

6 $\quad$ IF $Cost_{hi^*} = \min_i Cost_{hi} = \infty$: STOP, *return solution with N stages*

ELSE $\ nb_{hi^*} \Leftarrow nb_{hi^*} + 1$

$\qquad update\ TT_h$

7 $\quad$ REPEAT $steps$ 4 – 6 UNTIL $TT_h \le (N-1) \cdot P$

8 $$Total\ cost[N-1] = (N-1) \cdot \sum_h q_h \cdot HC_h + \frac{\sum_h \sum_i nb_{hi} \cdot TC_{hi}}{P}$$

IF $Total\ cost[N-1] \ge Total\ cost[N]$:STOP, *return solution with N stages*

ELSE $\ N \Leftarrow N-1$

$\qquad Goto\ step\ 3$

This enumerative search heuristic has been compared with the result of Formula (8) (Graves and Kostreva modified) for finding a variable subbatch strategy. The enumerative search heuristic outperformed Formula (8) in 89.4% of the cases, while in only 2% of the cases the modified Graves and Kostreva formula performed better. If the enumerative heuristic was better, the average cost difference was 5.3%. If it performed worse, cost difference was only marginal (less than 1%). We therefore use the result of the enumerative search heuristic when evaluating the effect of equal versus varying numbers of subbatches at different time bucket sizes.

2.3 Discontinuous total cost curve as a function of *P*

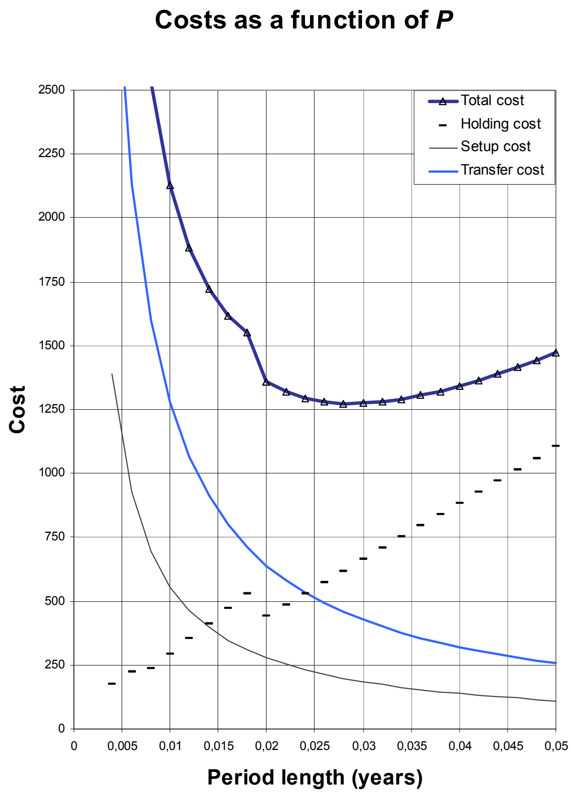

Figure 2 Partial and total cost curves as a function of *P*

This subsection discusses the cost behaviour for successive values of the time bucket length. Total costs are a function of P as shown in Formula (4). The contribution of set-up and transfer costs to the total costs diminishes if P increases. This is a rather familiar pattern, well known from inventory research (see e.g. Silver *et al.* 1998). The reason is that the number of set-ups and transfers are a linear function of the number of cycles per year, and this number is inversely proportional to the length of the time bucket.

Holding costs show a less familiar pattern. Usually, holding costs are a linear function of the batch size and hence of P as $q_h=P\cdot D_h$. However, holding costs also depend on the total time TT_h these units stay in the system. If P increases, possibly less stages are needed to cover the total throughput time of the batch. If a reduction in the number of stages occurs, this reduces the amount of items in the system and hence the holding costs. A tendency exists to require a lower number of stages for increasing values of P. Each reduction in the number of stages causes a breakpoint in the holding costs and total costs curves.

Figure 2 illustrates the various partial cost curves as well as the total cost curve as a function of P. We used the same number of subbatches for all values of P. Hence, the discontinuity of the holding cost curve is not caused by changes in the number of subbatches. It can only be addressed to changes in P.

From our analysis we expect that the length of the time bucket P has a strong effect on the total cost of a subbatch policy. IF P changes, a different number of stages may be required. Accordingly, a different number of subbatches may be appropriate. The decision on the number of subbatches is strongly interrelated with the choice of the time bucket length. Both choices have direct consequences for the required number of stages and resulting total costs. We should therefore examine the performance of subbatch strategies for various values of the time bucket length.

3 EXPERIMENTAL DESIGN

In order to investigate the cost differences between several types of equal subbatch strategies and a variable subbatch policy as a function of the time bucket length, a simulation study has been performed. The problem set contained 40 randomly generated problems for which several subbatch strategies have been evaluated at 80 different time bucket lengths. A problem consisted of three product types with each 12 operations that had to be performed successively. We used a fixed demand rate that remained constant over the experiments. We assumed that each operation had to be performed at a different machine, because scarce capacity was not an issue in our study. The time needed at an operation was therefore influenced only by the randomly generated set-up and processing time, the time bucket length (affecting q_h), and the subbatch policy applied. Operation processing and set-up times were normally distributed with mean as described in Table 1 and standard deviation equal to 1/3 of its mean.

Table 1 Set-up and processing time data used in experiments (years)

Operation number		Set-up time	Process time product 1	Process time product 2	Process time product 3
1	Mean	0.00125	0.00025	0.00020	0.00060
2		0.00225	0.00025	0.00025	0.00025
3		0.00050	0.00030	0.00030	0.00030
4		0.00050	0.00025	0.00025	0.00025
5		0.00200	0.00030	0.00025	0.00030
6		0.00000	0.00025	0.00025	0.00025
7		0.00000	0.00050	0.00050	0.00050
8		0.00100	0.00060	0.00075	0.00060
9		0.00050	0.00030	0.00030	0.00030
10		0.00100	0.00075	0.00075	0.00075
11		0.00100	0.00060	0.00060	0.00060
12		0.00000	0.00050	0.00050	0.00050
	Sum	0.01	0.00485	0.00490	0.00520

Table 2 gives details on the experimental design settings. Product 2 has a higher demand but a smaller mean processing time than product 3. It therefore depends on the randomly generated set-up and processing times and on the actual length of the time bucket which product will have the longest throughput time.

Table 2 Experimental design data

Demand (year)		Time bucket		Costs	
Product 1	1000	Minimum value (year)	0.001	Costs for holding one unit for one year	2.88
Product 2	1500	Step value (year)	0.001	Set-up costs per operation	720
Product 3	1250	Number of steps	80	Costs per transfer of a subbatch	0.20833

3.1 Types of equal subbatch strategies

The first equal subbatch policies that we consider do not consider the effect of changes in the time bucket size. No matter what time bucket length, the same number of subbatches is used. These policies are denoted by AI-nb=1, AI-nb=2, AI-nb=3 and AI-nb=4, indicating the number of subbatches that are used. These strategies are of type A, indicating that they use the same number of subbatches for all products, resulting in a batching policy that is very transparent and easy to use. However, type A policies may result in unnecessary high costs, as they do not distinguish between products for which throughput time reduction should or should not be realised through lot splitting. Type B policies consider equal subbatches that differ per product.

Second, an intuitively attractive strategy is introduced that considers the effect of time bucket size. It uses at the specified time bucket size the equal subbatch strategy that performs on average best. We denote the strategy as AII-MinAvg. The results of the AI strategies for all randomly generated problems were used to determine the on average best policy at a specific time bucket length. This policy presumes the existence of knowledge on what is in general optimal at a specific time bucket length, independent of the characteristics of the product to be made. Example of such knowledge is to use no more than two subbatches if the time bucket length is less than a week. Equal subbatch strategies like this are very often applied in practice.

Table 3 Subbatch strategies

Subbatch Strategy	Time bucket dependent	Problem dependent	Identical per product (A/B)	Identical per operation
AI-nb=x (x=1,2,3,4)	No	No (standard)	Yes (A)	Yes (Equal)
AII-MinAvg	Yes	No (average)	Yes (A)	Yes (Equal)
AII-AlmostMin	Yes	No (average)	Yes (A)	Yes (Equal)
AIII-Min	Yes	Yes	Yes (A)	Yes (Equal)
BIII-OptEqual	Yes	Yes	No (B)	Yes (Equal)
Variable Subbatch	Yes	Yes	No (B)	No (Variable)

The inherent danger of the above mentioned strategy is that unnoticed the wrong equal subbatch strategy is used. In practice, this may be caused when changes in product characteristics or time bucket length occur without reconsidering the subbatch strategy used. This results in cost differences, which we measure by considering the use of the on average second best equal subbatch strategy AII-AlmostMin.

The third type of equal subbatch strategy searches the appropriate number of subbatches dependent on the product characteristics. For each problem situation, instead of a general policy a specific equal batching strategy is being determined and applied to problems that belong to this category. We denote the results of this equal subbatch strategy as type III results. Two variants are distinguished: AIII-Min and BIII-OptEqual. AIII-Min uses the same number of subbatches for all products, while BIII-OptEqual allows the number of subbatches per product to vary.

Note that a variable subbatch policy allows variation between the operations of the same product, contrary to BIII-OptEqual. An optimal variable subbatch policy will lead to the lowest cost solution possible. Unfortunately, such a solution is both difficult and time-consuming to find, and we need to use a heuristic procedure to find an approximation of that solution. The enumerative search heuristic that we developed in order to find this approximation gives us a lowerbound on the cost increase if an equal subbatch strategy is used. We used it to obtain variable subbatch solutions for successive values of the time bucket length.

4 RESULTS

First, the performance of the four equal subbatch strategies AI-nb=1, AI-nb=2, .., AI-nb=4 was compared with the solution found with the heuristic for the problems generated.

Figure 3 and Table 4 show for all four equal subbatch strategies of type AI, at each time bucket size, the average distance to the cost of the variable subbatch strategy. For very small lengths of the time bucket (< 0.005 year) there is no difference in the effect of equal and variable subbatch strategies. This is not surprising, as the batch size in such a short time bucket is also very small and often cannot be split. The difference begins increasing at a time bucket length of 0.007 years. The variable subbatch strategy increases the number of subbatches at certain operations, which results in less costs than obtained when applying an equal subbatch strategy with nb_{hi}=1. The cost difference can only be attained by reducing the number of stages (total throughput time), as a consequence of the increased number of subbatches at these operations. Applying this increased number of subbatches for all operations (nb_{hi}=2) results in large cost differences (10-25%) for these small time bucket lengths. Hence, the flexibility of a variable subbatch strategy at small time bucket sizes pays.

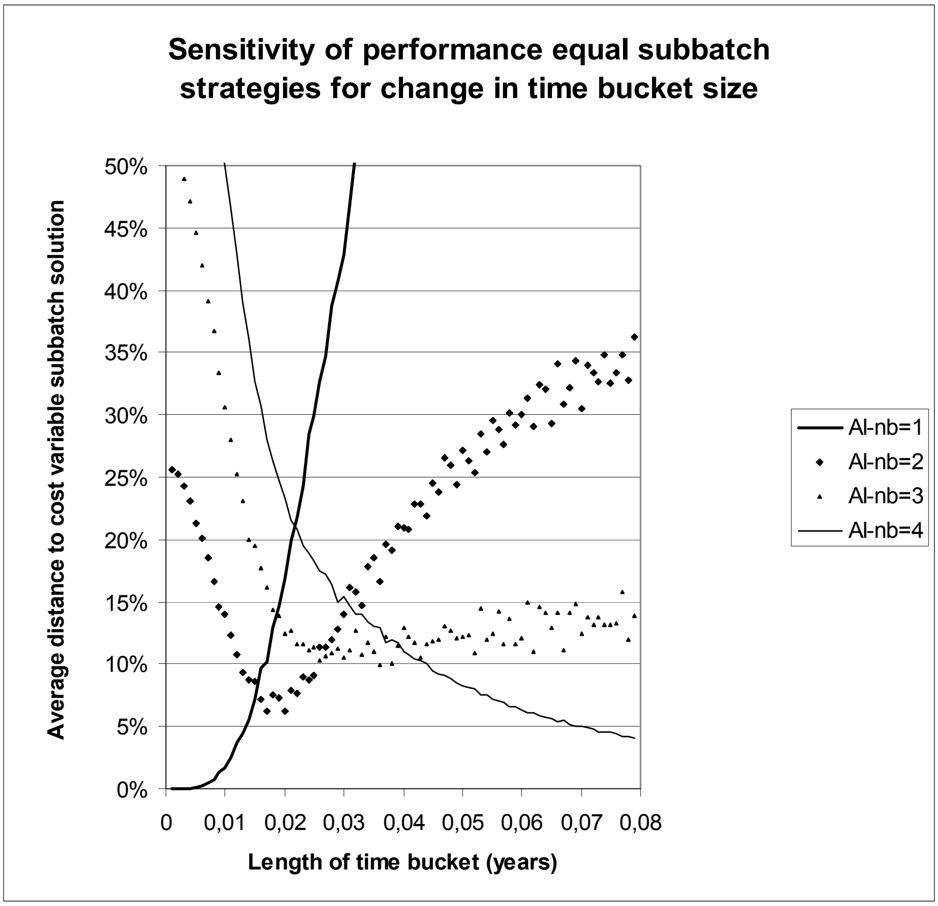

Figure 3 Cost effectiveness of equal subbatch strategies independent of *P*

For time bucket sizes of almost a week (*P*=0.015 year), lot splitting becomes for these products almost necessary in order to avoid huge cost differences. The use of an equal subbatch strategy of nb_{hi}=2, instead of a variable subbatch strategy, results in a cost difference of between 5 and 10%. For *P*=0.025 and larger, three equal subbatches result in still higher cost differences (>10%). However, if no change were made towards an increase in the number of subbatches (nb_{hi} still equal to 2), the cost difference would increase much faster. For large time bucket lengths, the cost differences with an equal subbatch strategy of nb_{hi}=4 tend to decrease. The variable subbatch policies at these high time bucket lengths do not change anymore, but the relative contribution of the transfer costs to the total costs steadily decreases.

Table 4 Sensitivity of equal subbatch strategies AI to time bucket size

From- P -To	AI-nb=1		AI-nb=2		AI-nb=3		AI-nb=4	
	Mean	St.Dev	Mean	St.Dev	Mean	St.Dev	Mean	St.Dev
0.001 0.005	0.02%	0.05%	23.87%	1.90%	48.44%	3.23%	73.26%	4.68%
0.006 0.010	0.88%	0.94%	16.75%	2.54%	36.35%	4.40%	57.48%	6.08%
0.011 0.015	4.67%	2.68%	9.95%	2.33%	23.14%	3.78%	39.43%	5.46%
0.016 0.020	12.85%	4.43%	6.88%	2.92%	14.92%	3.17%	26.64%	4.03%
0.021 0.025	24.85%	6.32%	8.47%	4.11%	11.68%	3.39%	19.80%	3.85%
0.026 0.030	37.94%	7.02%	12.28%	5.16%	10.72%	4.10%	16.29%	3.76%
0.031 0.035	51.49%	6.98%	16.62%	6.11%	11.46%	5.50%	13.82%	3.14%
0.036 0.040	62.51%	8.21%	19.49%	7.29%	11.32%	6.91%	11.87%	2.50%
0.041 0.045	71.80%	8.95%	22.59%	8.29%	11.61%	7.94%	10.19%	2.06%
0.046 0.050	80.82%	10.11%	25.58%	9.41%	12.41%	9.03%	8.76%	1.74%
0.051 0.055	87.07%	11.31%	27.34%	10.27%	12.41%	9.86%	7.68%	1.44%
0.056 0.060	93.10%	12.30%	29.15%	11.20%	12.64%	10.65%	6.73%	1.19%
0.061 0.065	98.31%	13.35%	30.83%	12.00%	13.52%	11.31%	5.91%	1.02%
0.066 0.070	102.54%	14.24%	32.38%	12.61%	13.34%	11.82%	5.23%	0.89%
0.071 0.075	105.96%	15.05%	33.46%	13.26%	13.39%	12.30%	4.66%	0.76%
0.076 0.080	108.93%	15.80%	34.30%	13.79%	13.73%	12.69%	4.21%	0.67%

We conclude that the length of the time bucket has a strong impact on the performance of the distinct equal subbatching strategies. The use of equal subbatches independent of the time bucket length results in substantial cost differences and is not effective.

We conclude further that the size of the cost difference with a variable subbatch strategy also depends on the length of the time bucket.

Table 5 Results of equal subbatch strategies dependent on time bucket size

	AII-MinAvg		AII-AlmostMin		AIII-Min		BIII-OptEqual	
From-P -To	Mean	St.Dev	Mean	St.Dev	Mean	St.Dev	Mean	St.Dev
0.001 0.005	0.02%	0.05%	23.87%	1.90%	0.02%	0.05%	0.02%	0.05%
0.006 0.010	0.88%	0.94%	16.75%	2.54%	0.88%	0.94%	0.87%	0.93%
0.011 0.015	4.67%	2.68%	9.95%	2.33%	4.63%	2.61%	3.27%	1.67%
0.016 0.020	6.88%	2.92%	11.81%	3.42%	6.74%	2.85%	3.28%	2.23%
0.021 0.025	8.47%	4.11%	11.68%	3.39%	8.14%	3.78%	3.34%	1.97%
0.026 0.030	10.72%	4.10%	12.28%	5.16%	9.51%	3.77%	3.23%	2.06%
0.031 0.035	11.46%	5.50%	13.82%	3.14%	8.76%	3.21%	2.95%	1.85%
0.036 0.040	10.94%	6.03%	12.25%	4.11%	7.49%	2.95%	2.29%	1.62%
0.041 0.045	10.19%	2.06%	11.61%	7.94%	6.50%	2.53%	2.05%	1.34%
0.046 0.050	8.76%	1.74%	12.41%	9.03%	5.75%	2.26%	1.72%	1.18%
0.051 0.055	7.68%	1.44%	12.41%	9.86%	4.99%	1.96%	1.53%	1.04%
0.056 0.060	6.73%	1.19%	12.64%	10.65%	4.39%	1.78%	1.35%	0.89%
0.061 0.065	5.91%	1.02%	13.52%	11.31%	3.96%	1.55%	1.23%	0.81%
0.066 0.070	5.23%	0.89%	13.34%	11.82%	3.46%	1.38%	1.06%	0.69%
0.071 0.075	4.66%	0.76%	13.39%	12.30%	3.07%	1.23%	0.96%	0.62%
0.076 0.080	4.21%	0.67%	13.73%	12.69%	2.80%	1.13%	0.85%	0.56%

The sensitivity to time bucket size changes does also occur for AII type policies. The mean and standard deviation of the distance of the AII-MinAvg solution to the best solution found is shown in the first column of Table 5. Changes in the number of subbatches took place at 0.015, 0.025, and 0.045 years. The results show clearly that the mean cost difference is not constant over a broad range of time bucket sizes. The average cost difference varies between 4 and 11%. Note also that variation increases if the average cost distance is higher. The largest cost difference we found in our experiments was 22%.

If product characteristics or time bucket have changed without notice, the chance exists that the ‘wrong’ equal subbatch strategy is being used. The strategy AII-AlmostMin indicates the cost of not using the best equal subbatch strategy. Table 5 shows that the cost difference with the best variable subbatch solution is generally around 12%, but is much higher for very small time bucket lengths (< P=0.010). The average cost difference does not fluctuate strongly with *P*, but its standard deviation does increase as a function of *P*. For specific product configurations, AII-AlmostMin proved to be more cost effective than the AII-MinAvg policy. Our experiments showed that the magnitude of the cost difference between the AII-MinAvg solution and the AII-AlmostMin solution was 34.85% in favour of the AII-AlmostMin solution (see Figure 4). Due to the fact that at very small time bucket sizes (P<0.03) AII-MinAvg almost always outperforms AII-AlmostMin, the 34.85% share will be realised in the mid and end range of time bucket sizes. Within this range, the inherent danger of applying AII-MinAvg is therefore substantial.

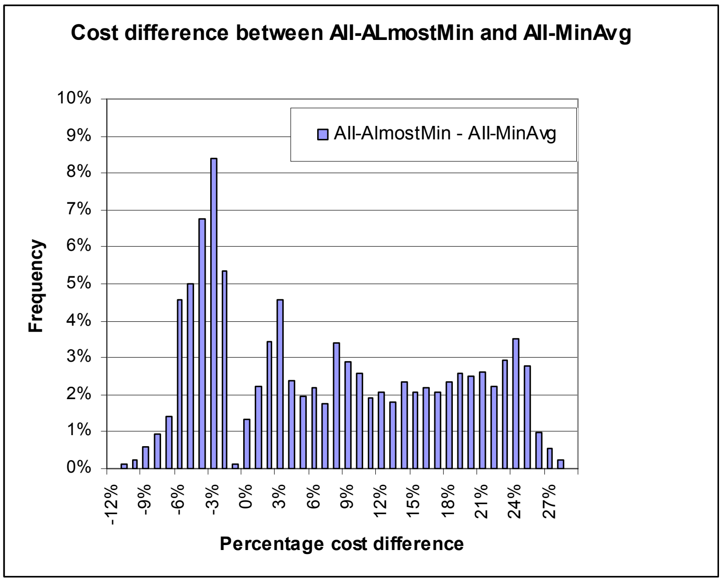

Figure 4 Cost difference between AII-MinAvg and AII-AlmostMin strategies

Finally, we performed an analysis using equal subbatch strategies that take product characteristics into account when they are used to determine the appropriate number of subbatches. We compared strategy AIII-Min with BIII-OptEqual. Note that B allows the number of subbatches per product to vary, which results in lower costs. Table 5 presents the results. Both strategies realise a significant cost difference in relation to the other equal subbatch strategies. If we compare AIII with AII-MinAvg, the cost difference is an average of approximatley 2%. The difference with BIII is higher, more than 5.5%. Taking product characteristics into account results, therefore, in lower costs. However, note that the efforts to achieve such a solution are also higher. The cost advantage of varying the number of equal subbatches per product (strategy B instead of A) is substantial. The largest average cost difference with the best known solution is around 4%, with a mean of 2%. It also has a positive effect on the standard deviation of the cost difference with the best known solution, indicating that the results it produces are more stable. Compared to the efforts needed for obtaining a variable subbatch strategy, equal subbatch strategies that allow different numbers of subbatches per product seem to be a good alternative.

5 CONCLUSION

The length of the time bucket has an important effect on the performance of equal subbatch strategies. The effectiveness of using the same equal subbatch is highly sensitive with respect to an increase in the time bucket length. Hence, from a cost perspective, it would be inappropriate to use equal subbatch strategies that specify the number of subbatches independent of the time bucket length.

If one changes to another equal subbatch strategy at appropriate lengths of the time bucket (i.e., equal subbatch strategies that depend on the time bucket length), large differences with a variable subbatch strategy can be avoided. In our experiments, changes at 0.015, 0.025 and 0.045 years were required for strategy AII-MinAvg. In that case, the average cost differences with the best solution found were still between 4 and 11 %. The problem is to determine at what length of the time bucket a change in the number of subbatches should be considered. If the change takes place too late, cost differences grow rapidly, as shown through strategy AII-AlmostMin. Thus, the inherent danger of this intuitively attractive strategy is that product characteristics or time bucket may have changed, while consequences for the number of equal subbatches have not been considered.

Equal subbatch strategies that take notice of the product characteristics at a specific time bucket size perform even better (on average 2.0%) than AII-MinAvg. We compared an equal subbatch strategy that used the same number of subbatches for all products with a strategy that allowed this number to vary. The cost difference between both strategies was substantial, on average 3.8%. The average cost advantage of a variable subbatch strategy, which also allows the number of subbatches per operation to vary, is 2.1%.

We obtained the variable subbatch solution using the heuristic search procedure, as described in Section 2.2. The presented cost difference is therefore a lowerbound for the cost difference with the unknown optimal solution. The heuristic outperformes the modified formula of Graves and Kostreva 1986 for finding a variable subbatch solution in almost 90% of the cases with an average cost difference of 4.7%.

We conclude that determination of the number of subbatches should receive careful attention of senior operations management in relation to the decision on the time bucket length. The cost difference with a variable subbatch solution depends strongly on the characteristics of the equal subbatch strategy used. Scientific studies comparing the performance of equal versus variable subbatch strategies should clearly delineate the assumptions behind the evaluated strategies. The performance differences that we found between the various equal subbatch strategies make general conclusions on the effectiveness of equal subbatch strategies worthless.

Summarizing our conclusion, our study reveals that:

- Using the same equal number of subbatches for various time bucket lengths results in substantial cost inefficiencies.
- Selecting the in general most cost effective equal subbatch strategy at a specific time bucket length interval still results in an average cost difference of between 4 and 11%, as compared to the result of our variable subbatch search heuristic.
- Equal subbatch strategies that determine the number of subbatches based on product characteristics and time bucket length perform 2.0% better, on average, than AII-MinAvg.
- Equal subbatch strategies that allow the number of subbatches per product to vary are the most cost effective. Mean cost difference with the best solution known is only 2.1%. Cost improvements compared to equal subbatch strategies that use the same number of subbatches for all products were, on average, 3.8%.

REFERENCES

Baker, K.R., D.F. Pyke. 1990. Solution procedures for the lot-streaming problem. *Decision Sci.* **21** 475-491.

Bogaschewsky, R.W., U.D. Buscher, G. Lindner. 2001. Optimizing multi-stage production with constant lot size and varying number of unequal sized batches. *Omega* **29** 183-191.

Burbidge, J.L. 1996. *Period Batch Control,* Oxford University Press, Clarendon Press, Oxford.

Enns, S.T. 2001. MRP performance effects due to lot size and planned lead time settings. *Internat. Production Res.* **39**(3) 461-480.

Enns, S.T. 2002. MRP performance effects due to forecast bias and demand uncertainty. *Eur. J. Oper. Res.* **138** 87-102.

Graves, S.C., M.M. Kostreva. 1986. Overlapping operations in material requirements planning. *J. Oper. Management* **6**(3) 283-294.

Jacobs, F.R., D.J. Bragg. 1988. Repetitive lots: flow-time reduction through sequencing and dynamic batch sizing. *Decision Sci.* **19** 281-294.

Kanet, J.J. 1986. Toward a better understanding of lead times in MRP systems. *J. Oper. Management* **6**(3) 305-315.

Karmarkar, U. 1987. Lot sizes, lead times and in-process inventories. *Management Sci.* **33(**3) 409-418.

Krajewski, L.J., B.E. King, L.P. Ritzman, D.S. Wong. 1987. Kanban, MRP and shaping the manufacturing environment. *Management Sci.* **33**(1), 39-57.

Kropp, D.H., T.L. Smunt. 1990. Optimal and heuristic models for lot splitting in a flow shop. *Decision Sci.* **21** 691-709.

Luss, H. 1989. Synchronized manufacturing at final assembly and feeder shops. *Internat. Production Res.* **27**(8) 1413-1426.

Matsuura, H., H. Tsubone, M. Kanezashi. 1996. Setting planned lead times for multi-operation jobs. *Eur. J. Oper. Res.* **88** 287-303.

Muckstadt, J.A., R.O. Roundy. 1993. Analysis of multistage production systems. 59-132 in: Graves, S.C., A.H.G. Rinnooy Kan, P.H. Zipkin (eds). 1993. *Logistics* *of production and inventory.* Handbooks in operations research and management science, vol. 4, North Holland, Amsterdam.

Olorunniwo, F.O. 1996. Changes in production planning and control systems with implementation of cellular manufacturing. *Prod. Inventory Management J.* **37**(1) 65-69.

Portiolo Staudacher, A. 2000. ISTS: a new procedure for the integrated scheduling of multi-stage systems. *Internat. Production Res.* **38**(11) 2489-2514.

Rees, L.P., P.Y. Huang, B.W. Taylor. 1989. A comparative analysis of an MRP lot-for-lot system and a Kanban system for a multistage production operation. *Internat. Production Res.* **27**(8) 1427-1443.

Shtub, A. 1990. Lot sizing in MRP/GT systems. *Production Planning & Control* **1**(1) 40-44.

Silver, E.A., D.F. Pyke, R. Peterson. 1998. *Inventory management and production planning and scheduling,* 3th edition. John Wiley & Sons, New York.

Steele, D.C., W.L. Berry, S.N. Chapman. 1995. Planning and control in multi-cell manufacturing. *Decision Sci.* **26**(1) 1-34.

Trietsch, D. 1989. Polynomial transfer lot sizing techniques for batch processing on consecutive machines. *Technical report NPS5489011* Naval Postgraduate School, Monterey, California, USA.

Veral, E. 1995. Using production and transfer batches in flow shops under MRP control. *J. Oper. Management* **12** 89-100.

Wemmerlöv, U. 1979. Design factors in MRP systems: a limited survey. *Prod. Inventory Management J.* **20** (4) 15-34.

Wemmerlöv, U. 1989. The behavior of lot-sizing procedures in the presence of forecast errors. *J. Oper. Management* **8**(1) 37-47.

Yang, K.K., F.R. Jacobs. 1992. Comparison of make-to-order job shops with different machine layouts and production control systems. *Internat. Production Res.* **30**(6) 1269-1283.

Yeung, J.H.Y., W.C.K. Wong, L. Ma. 1998. Parameters affecting the effectiveness of MRP systems: a review. *Internat. Production Res.* **36**(2) 313-331.