Reallocation of Beds to Reduce Waiting Time for Cardiac Surgery

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SOM theme A **Primary processes within firms**

Abstract

Waiting time for cardiac surgery is a significant problem in the current medical world. The fact that patients' length of stay varies considerably makes effective hospital operation a hard job. In this paper, the patients' length of stay is analyzed. Three scenarios for hospital management are presented and evaluated in two ways. First, the theoretical number of beds needed in each of these scenarios is analyzed using techniques from Markov chain theory. This analysis does not include the important variability in length of stay. Therefore, the second evaluation is based on simulation experiments to further investigate the variability. The aim of the analyses is to look at unused bed capacity in the hospital wards. By knowing the size of the unused bed capacity, it is possible to come to a more efficient reallocation of the beds. The results presented in this paper provide some insight in the relation between patients' length of stay, bed availability and hospital waiting lists. Finally, some ideas are raised as discussion points for further research.

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1 Introduction

In the Netherlands, there is a waiting list for cardiac surgery. One of the main reasons is the fact that Dutch hospitals have serious problems concerning the unavailability of intensive care beds, as is reported by Hautvast *et al.* (2001) in an extensive study of intensive care units.

In this paper, we analyze the situation in one of these hospitals. The department of cardiac surgery in this hospital has two hospital wards and one intensive care unit at its disposal. The unavailability of an intensive care bed after the operation is one of the main reasons for the increase of the waiting time. Whether there are beds available in the hospital wards does not seem to influence the waiting time. Before the increase of the waiting time, many beds in the hospital wards where not used.

In the literature little attention is paid to the variability of the length of stay (Gallivan *et al.*, 2002). In a recent article by Gallivan *et al.* (2002), the variability of patients' length of stay was stressed to be an important determinant of the effective hospital operation. Therefore, this determinant is taken into the model we are presenting. Harper and Shahani (2002) even state that the relationship between beds, occupancy, and refusals is complex and often overlooked by hospital managers. Ridge *et al.* (1998) also mention the uneven distribution of beds between hospitals and between levels of care in hospitals.

Based on our model, we present three scenarios and the consequences for the number of beds occupied in the hospital wards. The question is whether it is possible to come to a more efficient distribution of beds between the hospital wards and the intensive care unit. In this paper, we will try to give an answer to this question.

In Section 2, we first present a description of the current situation in the hospital. Then, in Section 3, we describe the scenarios used in our analysis. Section 4 subsequently provides some information on the data we used. The theoretical analysis of the number of beds needed is shown in Section 5. Following that, in Section 6, we present a simulation analysis to provide some more insight in the situation. Finally, in Sections 7 and 8, we present our conclusions and discuss the results of our study.

2 Current Situation

Before we analyze the length of stay in the hospital, we first have to describe the current situation. The patients are signed up for operation in the hospital by a family doctor, the emergency room, or the cardiologist. After the patient is signed up, the cardiologist and the cardiac surgeon judge if it is necessary that an operation is performed. After they decide that an operation is necessary, the patient is put on the waiting list. The rank of the patient on this list depends on the seriousness of his symptoms. The waiting list is managed by the admission agency of the department of heart surgery. When a patient is actually admitted depends on:

- the seriousness of the symptoms;
- the precedence for operation;
- the number of intensive care beds available;
- the number of beds in the hospital ward available;
- the capacity of the operation room.

The department of cardiac surgery has at its disposal:

- two hospital wards;
- one intensive care unit.

There are two hospital wards. The total number of beds is 50. Ward I has 32 beds and ward II 18 beds. The intensive care unit consists of 16 beds.

One day before the operation, the patient is admitted to one of the hospital wards. The only reason for having two wards is a constructional limitation to place all patients on one ward. So both wards are equal and therefore the admittance on a ward depends on where a bed is available. After the operation, the patient is transferred to the intensive care unit. When the patient has recovered, he will return to the hospital ward where he was admitted before the operation. After a number of days, there are two options:

- the patient can stay till he is entirely recovered from the operation and can go home;
- after 4 days, the patient is transferred to another hospital and stays in this hospital a number of days before going home.

The routing of the patients is illustrated in Figure 1.

3 Scenarios

There is a strong impression that there is unused bed capacity in the two wards. In this paper, three possible scenarios are analyzed. The aim of the three scenarios is to come to a reallocation of the unused bed capacity in the two wards. Maybe this unused capacity can be used to enlarge the intensive care unit.

In the first scenario, we maintain the current situation. By analyzing the first scenario, we try to gain a clear understanding of the unused bed capacity in the two wards. In the next two scenarios, two alternatives are analyzed in which the unused bed capacity can even be increased. In these two scenarios not only the center for cardiac surgery is involved, but also other hospitals. When patients are transferred to other hospitals in an early stage of their treatment, it might be possible that the center needs fewer beds in the hospital wards. The remaining beds are only used for those patients who need the intensive treatment and care of the center for cardiac surgery. Scenarios two and tree will analyze the size of the unused bed capacity.

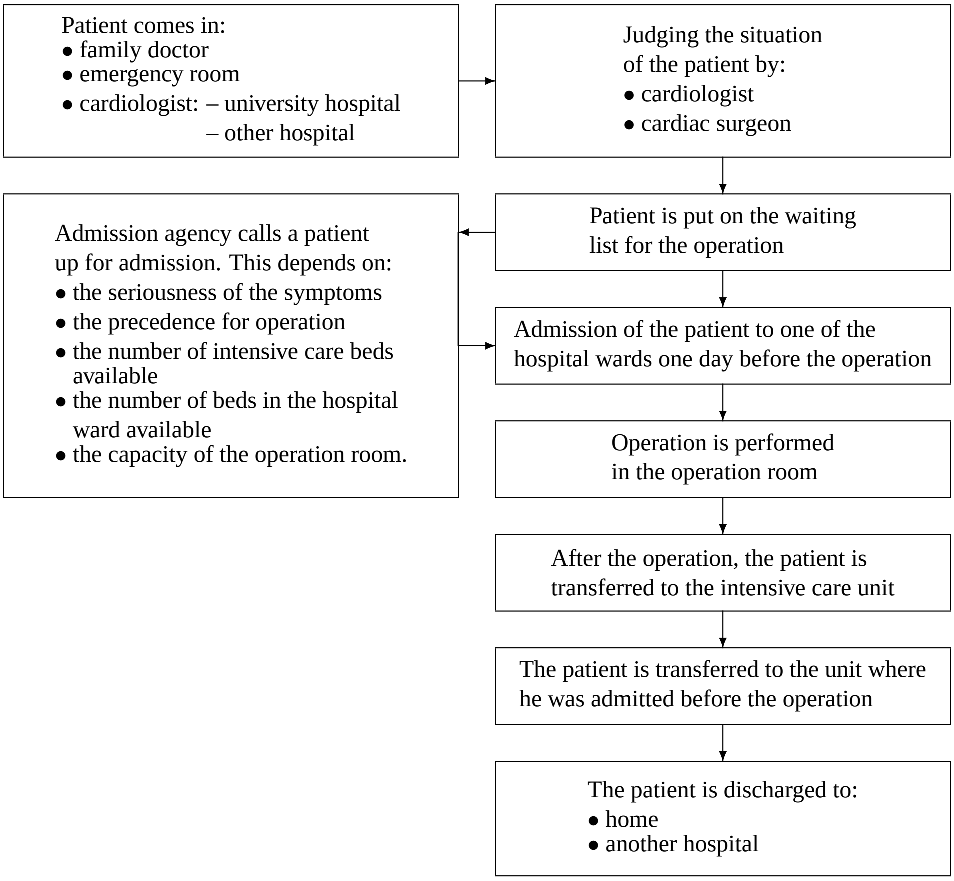


Figure 1. Patient routing

In the second scenario, all patients are admitted to one of the hospital wards. After the operation the external patients (patients who are admitted from another hospital before the operation) will placed back to this hospital after four days. The hospital-based patients (patients who have not been admitted to another hospital before the operation) will stay in the ward until they are entirely recovered to go home.

In the third and last scenario, all patients are admitted to the same hospital ward before the operation. After the operation, all patients are transferred to this hospital ward. After four days the external patient are transferred to the other hospital where they were admitted before they were transferred to the hospital. A percentage of the hospital-based patients are also transferred to another hospital. Several of the hospital-based patients are transferred to the other hospital ward.

In scenario 3, there is a clustering of complex patients and less complex patients. All patients are admitted in the same ward after the operation for four days. After these four days the patients need less attention from the nurses and they are transferred to another hospital or to the other ward. In scenario 2, there is no clustering of the patients; the complex and less complex patients are in the same hospital ward.

4 Description of Data

For the development of the model, we make use of hospital records from the period October 2000 until January 2001. In this period, 79.25% of all beds in ward I were used, and 54.29% of all beds in ward II were used (see Figure 2(a)).

The hospital records used in this paper consist of the following information for each patient admission:

1. Patients' identification number;
2. Patients' date of birth;
3. Hospital ward to which the patient was admitted;

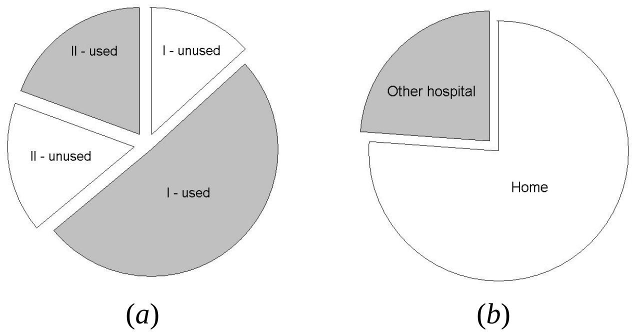


Figure 2. (*a*) Pie chart for the proportion of beds used on wards I and II in the period October 2000 until January 2001. (*b*) Pie chart for the destination of patients in the period October 2000 until January 2001.

4. Authority who signed up the patient for surgery;

5. Patients' release destination (home/other hospital);

6. Patients' length of stay.

Part of this information can be used for our analysis. The distribution of release destinations of patients is shown in Figure 2(*b*). As we can see, most patient stay in the hospital until their release. The last piece of information is the most interesting. Most patients stay 6 to 10 days in the wards, but the length of stay varies considerably. In Figure 3 the distribution of the number of days spent in hospital is presented for both the patients who go home after recovery and patients who go to another hospital for further recovery. Figure 4 adds both of these distributions together — taking into account the number of patients in each group (as in Figure 2(*b*)). The resulting distribution is influenced by the current situation. However, it is not likely that huge changes occur when the situation changes. Maybe a minor decrease in the number of days spent in hospital can be encountered when time spent on waiting lists becomes shorter. This could mean people are in better shape when admitted and recover more easily.

The data described in this section will be used throughout this paper as the basis for

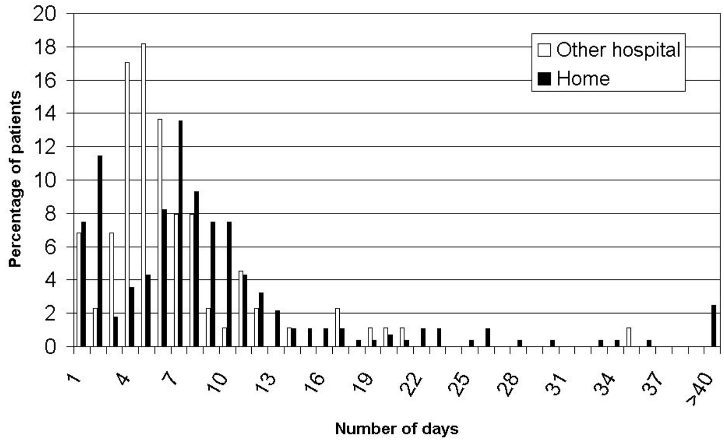


Figure 3. Distribution of the number of days spent in hospital for each release destination.

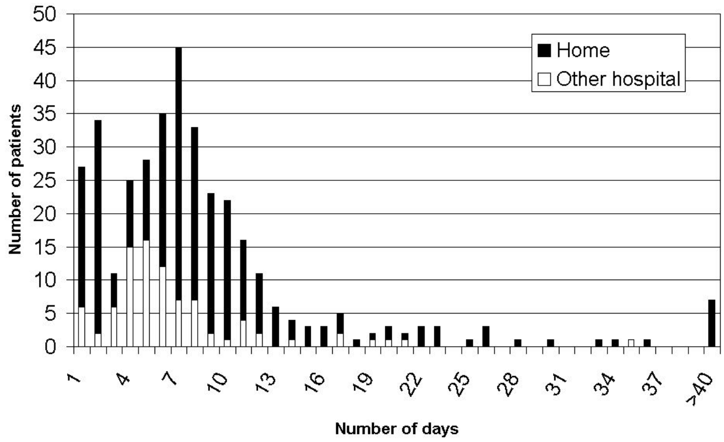


Figure 4. Number of days spent in the hospital wards for all patients.

the analysis presented. We realize that a couple of months is not a long period to base our analysis on. Cardiac surgery does not have any seasonal effects, especially under the presence of waiting lists. Because of this relative constant nature of cardiac surgery, we think that our data is representative and that the results of the analysis are valid.

5 Number of Beds Needed

In this section, we calculate the theoretical number of beds needed for each of the scenarios described in Section 3. To calculate these numbers, we analyze the length of stay in hospital wards I and II using Markov chains. Markov models are a very useful tool in analyzing the movement of entities in a system, in this case patients in a hospital. The state they occupy resembles the number of days they have been present in the hospital.

In the literature, we often find Markov models that consist of a number of compartments (see *e.g.*, Gorunescu *et al.*, 2002; MacKay, 2001; Taylor *et al.*, 2000). These compartments are mostly defined as short, medium, and long stay care. Most of these papers deal with geriatric departments. In our analysis — in cardiac surgery — we want to emphasize the number of days that a patient is in hospital. Therefore, the state space we use consists of the integers 1 to N, where N is the maximum number of days a patient will stay in hospital.

The transition probabilities — the probabilities of prolonging the stay based on the current length of stay — are obtained by the creation of an empirical distribution based on the available data described in Section 1 (see Figure 4). We can calculate the transition probabilities in the following way:

$$p_{i,j} = \begin{cases} K_j/K_i, & \text{if } j = i+1 \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

where K_i is the percentage of patients still in hospital after i days. These percentages can easily be obtained from Figure 4. Only transitions from state i to state $i+1$ are pos-

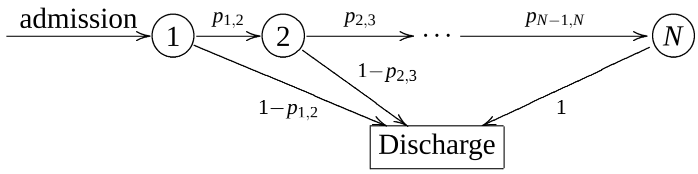


Figure 5. Graph with state spaces and transition probabilities.

sible; all other transition probabilities are equal to zero. In matrix form these transition probabilities can be presented as follows:

$$P = \begin{bmatrix} 0 & p_{1,2} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ \vdots & & & \ddots & p_{N-1,N} \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}. \tag{2}$$

In short, each day there is a possibility that a patient stays and a possibility that he is discharged. This is illustrated in Figure 5, where the state space and the transition probabilities are graphically represented. When a patient is discharged, he leaves the system; re-admissions are treated as new admissions.

5.1 Calculations

Using the probabilities from the empirical distribution, we can calculate the theoretical number of beds needed for each day. To denote the state on day t, we use the state vector $x(t)$, with $x_i(t)$ being the number of patients on the ith day of their stay.

The state of the system changes every day when the stay of patients is prolonged or they are discharged and M new patients are admitted. This can be calculated with the following equation:

$$x(t)' = x(t-1)'P + I_w(t)Me_1', \tag{3}$$

where e_i is a vector with a 1 on the ith place and zeros otherwise. The indicator function $I_w(t)$ is defined as follows:

$$I_w(t) = \begin{cases} 1, & \text{if day } t \text{ is a weekday} \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

The indicator function introduced in (4) is necessary to distinguish weekdays from week-ends, because new patients are only admitted on weekdays.

The state transition, described in (3), can be interpreted in the following way. To determine the state of the system on day t, we have to calculate the number of "old" patients and the number of "new" patients. The former is a certain percentage of the patients present on the previous day $(t-1)$, and is calculated by $x(t-1)'P$. The latter is a possible admission of new patients and is calculated by $I_w(t)Me_1'$, which adds M new patients to the top of the state vector $x(t)$, if day t is a weekday.

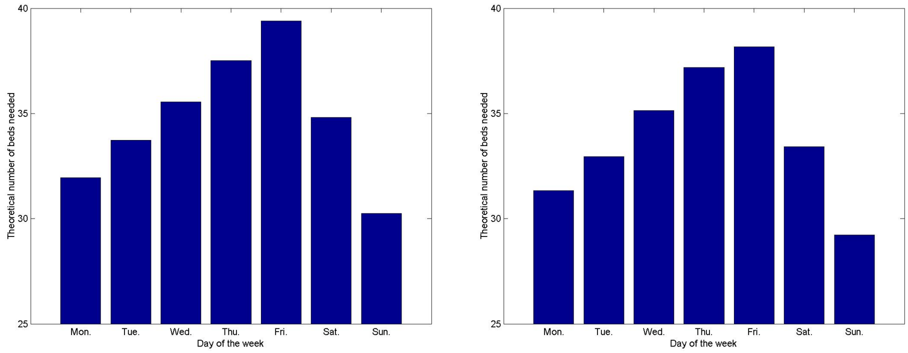
The theoretical number of beds needed varies during the week. This is — among other things — due to the fact that there are no admissions in the weekend. Therefore, We calculate the theoretical number of beds needed for each of the seven days of the week. These numbers result from the seven stationary states[1] (one for each day of the week), which we call $x^*_{(1)}, \ldots, x^*_{(7)}$. The sum of the elements in the vectors for the stationary states provide us with the theoretical number of beds needed for each day:

$$x^*_{(d)} = \sum_{i=1}^{N} x^*_{(d)}(i) = \sum_{i=1}^{N} \left(I_w\big(i + (d-1)\big) M \prod_{j=1}^{i-1} p_{j,j+1} \right), \qquad \text{for } 1 \leq d \leq 7. \tag{5}$$

5.2 Results

For each of the scenarios described in Section 3, we are now able to calculate the theoretical number of beds needed. To obtain the results, we use the distribution in Figure 4 as input data and use 50 days for N, the maximum length of stay.

[1] A stationary state is a state that does not change after a state transition. In this case we have weekly stationarity: every seven days we encounter — theoretically — the same state. This results in seven stationary states.



(a) (b)

Figure 6. Theoretical number of beds needed, based on the empirical distribution in Figure 4. (a) Scenario 1. (b) Scenario 2.

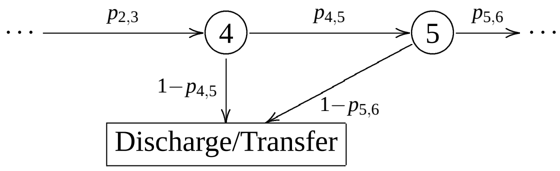


Figure 7. Graph with state spaces and transition probabilities for scenario 2.

In Figure 6(a) the result for scenario 1 is presented. The figure shows that the number of beds needed varies between 30 and 40. As was expected, there is an increase during weekdays and a decrease in the weekend, when no new patients are admitted.

For the calculation of the theoretical number of beds needed for scenario 2, we have to make a few adjustments to the transition probabilities. Because some of the patients are transferred to other hospitals after day 4, we make a distinction between the transition probabilities before, on, and after this day. Figure 7 shows the relevant states and transition probabilities. The new transition probabilities can be calculated in the

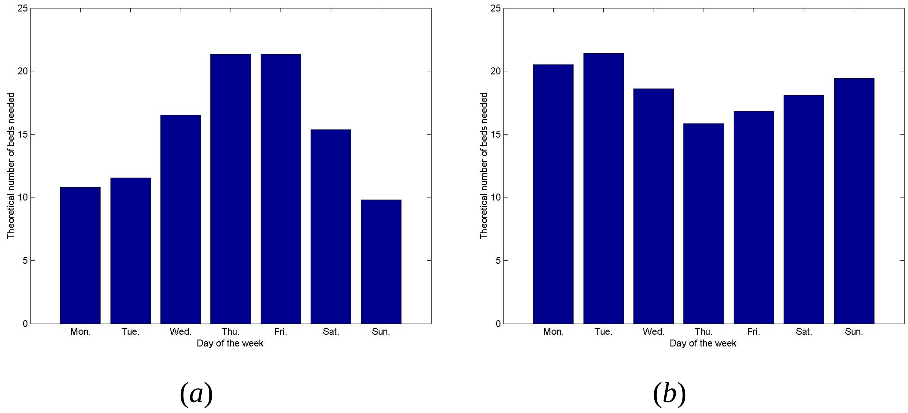


Figure 8. Theoretical number of beds needed in scenario 3. (*a*) Hospital ward I. (*b*) Hospital ward II.

following way:

$$p_{i,j} = \begin{cases} K_j/K_i, & \text{for } j = i+1 \text{ and } 1 \leq i \leq 3 \\ K_j^*/K_i, & \text{for } j = i+1 \text{ and } i = 4 \\ K_j^*/K_i^*, & \text{for } j = i+1 \text{ and } i \geq 5 \\ 0, & \text{otherwise,} \end{cases} \tag{6}$$

where K_i^* is the percentage of hospital-based patients still in hospital after i days, while K_i is the percentage of all patients still in hospital after i days. In Figure 6(*b*), the resulting theoretical number of beds needed is shown. As we can see, the number of beds needed is slightly less than in scenario 1.

Scenario 3 is completely different; a distinction is made between the wards I and II, while they were considered as one ward in the other strategies. Therefore, we calculated the theoretical number of beds needed for both wards. The results can be seen in Figure 8. If we add up the number of beds needed for both wards, we obtain the same result as in scenario 2 (see Figure 9). This result was expected because the only difference

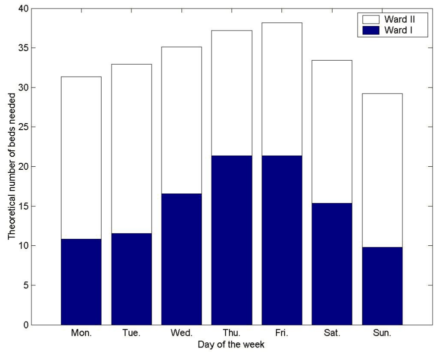


Figure 9. Theoretical number of beds needed in both wards in scenario 3.

between scenarios 2 and 3 is the separation of the wards.

6 Simulation Analysis

In this section, we perform some simulation experiments to obtain more insight in the number of beds needed in the hospital wards. In the previous section, we calculated the theoretical number of beds needed, but this does not provide enough insight in the bed-occupancy situation, which is highly unpredictable. Simulation is a very useful tool in this case, because it is mostly used to model uncertainty — a major characteristic of disease processes (Lowery, 1998). One of the often-mentioned reasons for using simulation as a tool is the experimentation with non-existing systems (Law and Kelton, 1991). In a lot of situations it is not possible to experiment with the real system, due to technical or financial problems. In our case, experimentation with the real hospital configuration would cause a lot of trouble for patients and staff.

6.1 Simulation Model

The simulation analysis we perform is based on the model used in the Markov chain analysis (see Figure 5). The transition probabilities are now used in comparison with random numbers drawn from the uniform distribution $U(0,1)$. In this way the system is updated on a daily basis. For each of the patients in the system, the outcome of the random number determines whether the patient's stay is prolonged. Prolongation of a patient's stay after day i can be determined with $u \in U(0,1)$ and the following procedure:

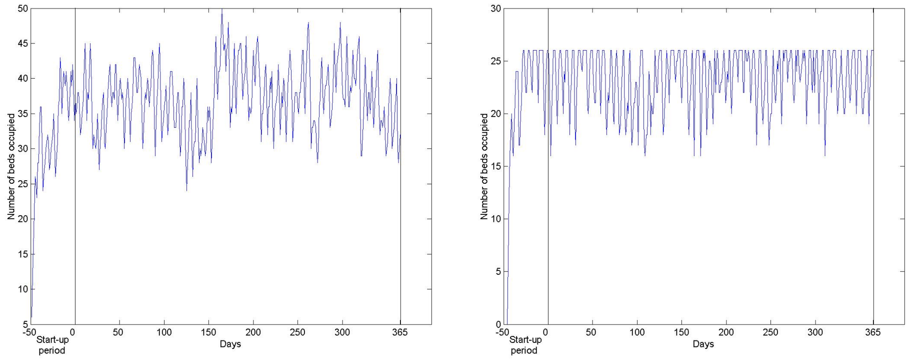
if $u < p_{i,j}$ **then** patient to day j;
else patient is discharged.

If we start simulating the system, we have to take into account that we start with an "empty" system — *i.e.*, a system without patients. In the simulation literature, this is called the problem of initial transient or the start-up problem (Law, 1983). Therefore, we will use initial data deletion to make sure the set of observations is really representative. We introduce a start-up period of N days. Normally, one uses a method from the literature to choose the start-up period; because we have a maximum length of stay in our model, the start-up period does not need to be longer than this maximum. To illustrate this start-up period the results of two simulations are shown in Figure 10. In this simulation a start-up period N of 50 days is used and a period of 365 days is added.

6.2 Output Data Analysis

We have performed simulations for all three scenarios; the results of these simulations are presented in this section.

For scenarios 1 and 2, we calculated the mean number of patients rejected due to a "full system". The results are based on 250 replications of a 1-year period. For scenario 3, we have to take a different approach. We can not judge the scenario results on the



(*a*) (*b*)

Figure 10. (*a*) Simulation results for 1 run based on scenario 1 (for both wards). (*b*) Simulation results for 1 run based on scenario 3 (for ward II).

number of rejected admissions, because patients are always admitted in this scenario. In this scenario, patients who would be rejected when they are internally transferred from ward I to ward II are transferred to another hospital. In Table 1 we show the resulting rejections and transfers for the three scenarios.

As we can see, there are almost no rejections in scenarios 1 and 2. The fact that scenario 1 shows almost no rejections corresponds to the current situation and therefore indicates the validity of the simulation model.

However, in scenario 3 we see a reasonable amount of transfers. In the 1-year period, used in the simulation, we have 6 patients per weekday for 52 week; this adds up to a total of 1560 patients for the full year. Of this 1560 patients, about 43% has already left because of discharge of transfer by the fourth day. This leaves about 890 patients of which another 168 (19%) have to be transferred. This comes down to the transfer of about three patients per week.

Table 1. Mean number of rejections or transfers encountered in 250 replications of a 1-year period for each of the scenarios.

	Rejections Scenario 1	Rejections Scenario 2	Transfers Scenario 3
Monday	0	0	56.3360
Tuesday	0.0120	0	66.0080
Wednesday	0.0400	0.0040	0
Thursday	0.1880	0.0680	0
Friday	0.6160	0.0960	0.9880
Saturday	0	0	9.8640
Sunday	0	0	35.0560
Overall	0.8560	0.1680	168.2520

7 Conclusion

There is a waiting list for cardiac surgery in the Netherlands. One of the reasons for the waiting time is the unavailability of beds in the intensive care unit. In our case, the waiting time is not influenced by the availability of beds in the hospital ward. In spite of the variability in length of stay, there are more then enough beds available in the ward. As we show in scenario 1, the number of beds is more than sufficient. One of the reasons is that a number of patients is transferred to other hospitals. Whether a patient is transferred sometimes depends on the number of beds that is available, and sometimes happens on request of the patient.

In scenario 2 we show what will happen when all the external patients are transferred to the other hospitals. In scenario 3, we present an idea of clustering complex and less complex patients in different hospital wards. In this scenario not only the external patients are transferred to another hospital, but also a part of the hospital-based patients are transferred. We show that the theoretical number of beds needed varies between 30 and 40 beds. This theoretical number of beds needed does not provide enough insight in the bed-occupancy situation. Therefore, we introduced a simulation model. With this model, we show that there are almost no rejections in scenario 1 and 2. In scenario 3, 19% of the hospital-based patients must be transferred to another hospital.

By analyzing the last two scenarios, we showed that the number of beds could be reduced. Only those patients who really need the special treatment of a center for cardiac surgery will be admitted. When the patients recover and do not need this type of care, they are transferred to another hospital. In this way the center needs less beds in the hospital wards. The beds can be reallocated in an efficient way. One of the possibilities is to increase the number of beds in the intensive care unit.

8 Discussion

In the current situation, the number of beds in the wards is 50. We show in this paper that this can be reduced by a number of beds. It is possible to come to a more efficient distribution of beds between the hospital wards and the intensive care unit. By decreasing the number of beds in the ward, it is possible to increase the number of beds on the intensive care. We know that this would partly be a discussion of the government.

However, we think there is a solution that can be initiated by the staff of the hospital. A number of patients are too complex to treat in the hospital ward. This group of patients stays in the intensive care unit. In fact, some of these patients do not need entirely intensive care. It would be possible to create a number of post-intensive care beds in a special unit in the hospital ward. In scenario 3, we present an idea of clustering complex and less complex patient. It is possible to create, for instance, a situation with:

- 16 intensive care beds (in the intensive care unit);
- 5 post intensive care beds (in one of the hospital ward);
- 40 hospital beds.

In this situation, patients who really need intensive care use the intensive care beds. The patients who need more care than can be offered in the wards are treated on the post-intensive care beds. With a more efficient distribution of beds between the intensive care unit and the beds in the ward, it is possible to create a number of post-intensive care beds. In this way, it may be possible to decrease the waiting time for cardiac patients.

In this paper, we only used the model for one hospital. We have no exact data from other hospitals, but the situation is probably comparable. The waiting time for cardiac surgery seems to be a problem for all the centers. However, we think that other hospitals — that provide no cardiac surgery — can help to decrease this time. In scenario 3 we suggest that more patients must be transferred to these other hospitals. This would be at least 19% of the hospital-based patients. When more patients are transferred to other hospitals, it is possible to decrease the amount of beds on the hospital wards more and create even more post-intensive care beds. In this way, all hospitals can contribute to solving the waiting list for cardiac surgery.

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