Delegation with multiple instruments in a rent-seeking contest

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Abstract We consider delegation in a rent-seeking contest with two players, where delegates have more instruments at their disposal than the main players. We endogenize both the decision to hire a delegate and the contingent fee offered to the delegates. We characterize the situations when either no, one or two players hire a delegate in equilibrium. We show that the decision to hire a delegate depends in a non-monotone way on the size of the contested prize.

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1 Introduction

This paper investigates endogenous delegation in the well-known rent-seeking contest of Tullock (1980). We recall that in the standard Tullock contest two players compete for a single prize. Each player exerts effort in order to increase the probability that he wins the prize. Tullock's model has spawned a vast literature and has been applied in many areas, like lobbying, environmental regulation, litigation and sporting; see e.g. Nitzan (1994), Wärneryd (2000), Liston-Heyes (2001), Lockard and Tullock (2001) and Szymanski (2003). We consider here an extension of a two-player contest in which each (main) player has the option to either compete himself or to hire a delegate who competes on his behalf. Examples with delegation abound: firms can hire professional lobbyists to acquire monopoly rents from the government, or firms can hire lawyers to win lawsuits, etcetera.

Intuitively speaking, a reason why a player might decide to hire a delegate could be that the delegate in some sense has a larger proficiency than the player himself. We formalize this intuition by considering a model where a delegate has the option to compete with two instruments, whereas a player himself can use only one instrument. An example could be a case where a firm wants the government to change the law such that it acquires a monopoly rent. It might be that then the firm can only influence the government's decision by lobbying political parties in the parliament, while a professional lobbyist, in addition to this, might also put pressure on the government's decision by e.g. using her (direct) contacts with influential officials within the government or by advancing litigation for new precedents. The specification of our model borrows from Epstein and Hefeker (2003), who presented an extension of the standard two-player Tullock contest (without delegation) in which the players can compete with two instruments rather than with just one. Epstein and Hefeker show that the results derived with their model significantly differ from those derived with the standard Tullock contest. Hence, one cannot innocently replace two instruments by one instrument, effort.

We assume that a player cannot observe the efforts exerted by his delegate. In order to cope with the moral hazard, each player offers his delegate a contingent fee, i.e. a delegate obtains a deliberately chosen fee if she wins the contest and nothing otherwise. We investigate the following non-cooperative three-stage game. In stage 1, each player decides whether or not to hire a delegate. If a player decides to hire a delegate, then in stage 2 he selects the delegate's contingent fee. In stage 3 we have the actual contest for the prize. If a player has not hired a delegate, then he competes himself in this stage; otherwise, his delegate competes on his behalf.

We derive the conditions under which no, one or both players decide to hire a delegate in a (subgame-perfect pure-strategy) Nash equilibrium and give the corresponding contingent fees. We establish the interesting finding that the decision to hire a delegate does not depend in a monotone way on the size of the contested prize. This is related to the fact that in some equilibria delegates optimally use only one instrument, whereas in others they prefer to use two instruments.

Some other papers have studied delegation (where delegates have only one instrument at their disposal) in two-player contests. Baik and Kim (1997) assume that delegates have a greater so-called ability than the players. This means that, ceteris paribus, if a delegate exerts a certain effort, then this has a larger positive effect on the probability that the player associated to her will win the prize than if this player exerts the same effort level himself. Baik and Kim also endogenize the decision to hire a delegate – they show that if a player hires a delegate, then the ability of this delegate must exceed his own. However, in their analysis the payment schemes offered to the delegates are exogenously given. They assume that a delegate receives an exogenously given contingent fee, and in addition to that a fixed fee (which depends on her ability) regardless of the outcome of the contest.

Wärneryd (2000) investigates a two-player contest where it is exogenously given that both players must hire a delegate. He also assumes that the delegates and players have identical abilities. For this situation, he endogenously determines the equilibrium size of the contingent fee. In fact, it turns out from his analysis that if the players would be able to decide to hire a delegate or not, then they would face a prisoners' dilemma: i.e. not hiring a delegate would be a dominant strategy for each player, but both would benefit if both would hire a delegate. Delegation does not endogenously arise in equilibrium in such a case, however.[1] We stress that we endogenize both the decision to hire a delegate and the contingent fee. Moreover, we identify equilibria in which delegation does occur. We finally mention that Schoonbeek (2002) examines one-sided endogenous delegation in a contest where the two players have different risk-attitudes, while Schoonbeek (2004) analyses endogenous delegation in a contest between two groups of players.

The paper is further organized as follows. In Section 2 we present the model and derive the equilibria. We conclude in Section 3. All proofs are in the Appendix.

2 The model and the equilibria

Consider a contest with two risk-neutral players $i = 1, 2$, in which one of the players can win a prize of value $V > 0$. The contest is modelled as a non-cooperative three-stage game. In stage 1, each player decides whether or not he hires a risk-neutral delegate who will compete on his behalf in stage 3. If a player does not hire a delegate, he will compete himself in stage 3. In stage 3, the relevant contestants compete for the prize by exerting nonrefundable and nonnegative efforts. The effort of player i (if relevant) is denoted as e_i ($i = 1, 2$). A delegate, if hired, can compete with two instruments in stage 3. The effort levels of these two instruments chosen by delegate i (if relevant) are denoted as y_i and z_i ($i = 1, 2$). Depending on the players' decisions in stage 1, we distinguish four possible cases in the remainder of the game: in case (a) both players do not hire a delegate, in case (b) only player 1 hires a delegate, in case (c) only player 2 hires a delegate, and in case (d) both players hire a delegate. If player i decides to hire a delegate, then this player offers delegate i in stage 2 a contract that specifies her payment. There is moral hazard since player i cannot observe the effort of delegate i in stage 3. Delegate i is offered a contingent fee contract; she receives a fraction $0 \leq \gamma_i \leq 1$ of the prize V if the prize is won for player i, and nothing otherwise. Player i selects the value of γ_i. Delegate i accepts the contract if her expected payoff is nonnegative.

The probabilities that player 1 and player 2 receive the prize are denoted by, respectively, q and $1 - q$. The probabilities depend on the relative magnitudes of the efforts of the actual contestants. We use the specification proposed by Epstein and Hefeker (2003). In particular, if both players hire a delegate (case (d)), then

$$q = \frac{(1 + y_1)z_1}{(1 + y_1)z_1 + (1 + y_2)z_2} \tag{1}$$

if $z_1 + z_2 > 0$, whereas $q = 1/2$ if $z_1 + z_2 = 0$. Observe that (1) has the following attractive properties: (i) both instruments are complementary to each other, i.e. the second instrument y_i reinforces the effect of the first instrument z_i; (ii) delegate i does not have to use the second instrument (i.e. we can have $y_i = 0$) - she will only use it if doing so positively affects her expected payoff; (iii) if both delegates do not use their second instrument, then (1) reduces to the standard Tullock contest success function $q = z_1/(z_1 + z_2)$. Notice that if we would have in (1) the term $y_i z_i$ instead of the term $(1 + y_i)z_i$, then we would always have that $y_i = z_i$ in equilibrium, which is not very interesting. See further in Epstein and Hefeker (2003, p.

83). Proceeding, if only one player, say player i, does not hire a delegate (cases (b) and (c)), then we replace in (1) $(1+y_i)z_i$ with e_i. In the same spirit, if both players do not hire a delegate (case (a)), then $q = e_1/(e_1+e_2)$ if $e_1+e_2 > 0$, while $q = 1/2$ if $e_1+e_2 = 0$; i.e. then the situation boils down to the standard contest of Tullock (1980).[2]

We will analyse the (subgame-perfect pure-strategy Nash) equilibria. Using backward induction, we first investigate for each of the cases (a) to (d) (i.e. for given delegation decisions), the corresponding equilibrium efforts in stage 3 and the equilibrium contracts in stage 2 (if relevant). Next, combining the results of these four cases, we derive the equilibrium delegation decisions in stage 1.

Case (a). In this case both players compete for the prize themselves in stage 3. Given $e_j > 0$, player i maximizes his expected payoff, i.e. he solves

$$\max_{e_i \geq 0} \left(\frac{e_i}{e_i+e_j}\right) V - e_i, \tag{2}$$

with $i \neq j$ and $i, j = 1, 2$. This case corresponds to the standard Tullock (1980) contest. So, it is well-known that the equilibrium efforts of both players are $e_1^a = e_2^a = V/4$, while the expected payoffs are $\pi_1^a = \pi_2^a = V/4$.

Case (b). Now the contestants in stage 3 are delegate 1 and player 2. In stage 3, given $0 \leq \gamma_1 \leq 1$ and $e_2 > 0$, delegate 1 solves

$$\max_{y_1 \geq 0, z_1 \geq 0} \left(\frac{(1+y_1)z_1}{(1+y_1)z_1+e_2}\right) \gamma_1 V - y_1 - z_1, \tag{3}$$

while, given $z_1 > 0$, player 2 solves

$$\max_{e_2 \geq 0} \left(\frac{e_2}{(1+y_1)z_1+e_2}\right) V - e_2. \tag{4}$$

The next lemma characterizes the equilibrium in stage 3.

Lemma 1 *Consider case (b). For each $0 \leq \gamma_1 \leq 1$ there is a unique equilibrium in the corresponding contest between delegate 1 and player 2 in stage 3. The following holds in this equilibrium:*

(i) Delegate 1 uses one instrument if and only if $1/\gamma_1 \geq \sqrt{V} - 1$. The corresponding efforts are $y_1^b = 0$ and

$$z_1^b = \frac{\gamma_1^2 V}{(1+\gamma_1)^2}, \qquad e_2^b = \frac{\gamma_1 V}{(1+\gamma_1)^2}. \tag{5}$$

(ii) Delegate 1 uses both instruments if and only if $1/\gamma_1 < \sqrt{V} - 1$. *The corresponding efforts are* $y_1^b = z_1^b - 1$ *and*

$$z_1^b = \sqrt{V} - \frac{1}{\gamma_1}, \qquad e_2^b = \frac{1}{\gamma_1}\sqrt{V} - \frac{1}{\gamma_1^2}. \tag{6}$$

Hence, in case (b) delegate 1 uses both instruments if and only if, given $V > 1$, the size of γ_1 is large enough. Delegate 1 never uses both instruments if $V \leq 1$.

Turning to stage 2, we present the next result.

Lemma 2 *Consider case (b). Define* $V_0 \equiv \frac{1}{4}(\sqrt{2} - 1)^{-4} \approx 8.49$. *In stage 2 the following holds:*

(i) If $0 < V \leq V_0$, *then there is a unique equilibrium in stage 2, in which* $\gamma_1^b = \sqrt{2} - 1$. *Delegate 1 will use one instrument in the subsequent equilibrium in stage 3. The corresponding expected payoffs of player 1 and player 2 are:*

$$\pi_1^b = (\sqrt{2} - 1)^2 V, \qquad \pi_2^b = \frac{V}{2}. \tag{7}$$

(ii) If $V > V_0$, *then there is a unique equilibrium in stage 2, in which* $\gamma_1^b = V^{-\frac{1}{4}}$. *Delegate 1 will use two instruments in the subsequent equilibrium in stage 3. The corresponding expected payoffs of player 1 and player 2 are:*

$$\pi_1^b = (V^{\frac{1}{4}} - 1)^2 \sqrt{V}, \qquad \pi_2^b = \sqrt{V}. \tag{8}$$

Lemma 2 shows that if V is larger than the threshold V_0, then for player 1 it is profitable to offer delegate 1 a contract that induces her to compete with both instruments in stage 3. For smaller values of V, the contract offered to delegate 1 induces her to compete with one instrument only.

Case (c). Now the contestants in stage 3 are player 1 and delegate 2. Obviously, this is the counterpart of case (b), and we can state results similar to those of Lemma 1 and Lemma 2 by interchanging the indices 1 and 2. In particular, if $0 < V \leq V_0$, then there is an equilibrium in stage 2, in which delegate 2 uses one instrument, with the following expected payoffs for the players 1 and 2:

$$\pi_1^c = \frac{V}{2}, \qquad \pi_2^c = (\sqrt{2} - 1)^2 V. \tag{9}$$

If $V > V_0$, then there is an equilibrium in stage 2, in which delegate 2 uses both instruments, with associated expected payoffs

$$\pi_1^c = \sqrt{V}, \qquad \pi_2^c = (V^{\frac{1}{4}} - 1)^2 \sqrt{V}. \tag{10}$$

Case (d). Now the contestants in stage 3 are delegate 1 and delegate 2. The probability that player 1 wins the prize is given by (1). Given $0 \leq \gamma_1 \leq 1$ and $z_2 > 0$, the problem considered in stage 3 by delegate 1 is

$$\max_{y_1 \geq 0, z_1 \geq 0} \left(\frac{(1+y_1)z_1}{(1+y_1)z_1 + (1+y_2)z_2} \right) \gamma_1 V - y_1 - z_1, \tag{11}$$

while, given $0 \leq \gamma_2 \leq 1$ and $z_1 > 0$, the problem of delegate 2 reads

$$\max_{y_2 \geq 0, z_2 \geq 0} \left(\frac{(1+y_2)z_2}{(1+y_1)z_1 + (1+y_2)z_2} \right) \gamma_2 V - y_2 - z_2. \tag{12}$$

Investigating the situation in stage 3, we may assume without loss of generality that $\gamma_2 \geq \gamma_1 > 0$ and present the following result.

Lemma 3 *Consider case (d). For each $0 \leq \gamma_1 \leq 1$ and $0 \leq \gamma_2 \leq 1$, with $\gamma_2 = k\gamma_1 > 0$ and $k \geq 1$, there is a unique equilibrium in the contest between the two delegates in stage 3. The following holds in this equilibrium:*

(i) Both delegates use one instrument if and only if $\gamma_1 V \leq (1+k)^2/k^2$. The corresponding efforts are $y_1^d = 0$, $y_2^d = 0$ and

$$z_1^d = \frac{\gamma_1^2 \gamma_2 V}{(\gamma_1 + \gamma_2)^2}, \qquad z_2^d = \frac{\gamma_1 \gamma_2^2 V}{(\gamma_1 + \gamma_2)^2}. \tag{13}$$

(ii) Delegate 1 uses one instrument whereas delegate 2 uses two instruments if and only if $(1+k)^2/k^2 < \gamma_1 V \leq (1+k^2)^2/k^2$. The corresponding efforts are $y_1^d = 0$, $y_2^d = z_2^d - 1$ and

$$z_1^d = \left(\frac{\gamma_1}{\gamma_2}\right)\sqrt{\gamma_1 V} - \left(\frac{\gamma_1}{\gamma_2}\right)^2, \qquad z_2^d = \sqrt{\gamma_1 V} - \frac{\gamma_1}{\gamma_2}. \tag{14}$$

(iii) Both delegates use both instruments if and only if $\gamma_1 V > (1+k^2)^2/k^2$. The corresponding efforts are $y_1^d = z_1^d - 1$, $y_2^d = z_2^d - 1$ and

$$z_1^d = \frac{\gamma_1^3 \gamma_2^2 V}{(\gamma_1^2 + \gamma_2^2)^2}, \qquad z_2^d = \frac{\gamma_1^2 \gamma_2^3 V}{(\gamma_1^2 + \gamma_2^2)^2}. \tag{15}$$

Lemma 3 shows that, for given $k \geq 1$, both delegates use one instrument if the contingent fee $\gamma_1 V$ is small, whereas both delegates use both instruments if $\gamma_1 V$ is large. For intermediate values of $\gamma_1 V$, only delegate 2 – who has a larger contingent fee than delegate 1 – uses both instruments.

Notice that the effort levels (13) are identical to the equilibrium effort levels that would result in a standard Tullock contest between delegate 1 and delegate 2, i.e. if they only would be able to compete with the instruments z_1 and z_2. We further remark that Epstein and Hefeker (2003) also present the results of part (iii) of Lemma 3. However, they do not discuss those of part (ii).[3]

Proceeding with stage 2, we present Lemma 4. In turns out that in a number of situations the equilibrium in stage 2 is not unique. However, following common practice, we can obtain uniqueness in those cases by focusing on Pareto dominant equilibria. We call an equilibrium Pareto dominant if there does not exist another equilibrium in which both players are strictly better off.

Lemma 4 *Consider case (d). Define* $V_1 \equiv 3/(\sqrt{3}-1)^4 \approx 10.45$ *and let* $V_2 \approx 12.45$ *be the unique positive root of* $((3V)^{\frac{1}{2}}+9)^4 - 27V^3 = 0$. *In stage 2 the following holds:*

(i) If $0 < V \leq V_1$, *then there is a unique Pareto dominant equilibrium, in which* $\gamma_1^d = \gamma_2^d = \frac{1}{3}$. *(If* $0 < V \leq 8$, *this is the unique equilibrium.) Both delegates use one instrument in the subsequent equilibrium in stage 3. The corresponding expected payoffs of player 1 and player 2 are:*

$$\pi_1^d = \frac{V}{3}, \qquad \pi_2^d = \frac{V}{3}. \tag{16}$$

(ii) If $V_1 < V \leq V_2$, *then there are two Pareto dominant equilibria. In the first one, we have* $\gamma_1^d = \frac{1}{3}$ *and* $\gamma_2^d = (3V)^{-\frac{1}{4}}$. *Delegate 1 uses one instrument and delegate 2 uses both instruments in the subsequent equilibrium in stage 3. The corresponding expected payoffs of player 1 and player 2 are:*

$$\pi_1^d = 2 \times 3^{-\frac{5}{4}} \times V^{\frac{3}{4}}, \qquad \pi_2^d = \left(1 - (3V)^{-\frac{1}{4}}\right)^2 V. \tag{17}$$

In the second one, we have $\gamma_1^d = (3V)^{-\frac{1}{4}}$ *and* $\gamma_2^d = \frac{1}{3}$. *Delegate 1 uses both instruments and delegate 2 uses one instrument in the subsequent equilibrium in stage 3. The corresponding expected payoffs of player 1* *and player 2 are:*

$$\pi_1^d = \left(1 - (3V)^{-\frac{1}{4}}\right)^2 V, \qquad \pi_2^d = 2 \times 3^{-\frac{5}{4}} \times V^{\frac{3}{4}}. \tag{18}$$

(iii) *If $V > V_2$, then there is a unique equilibrium, in which $\gamma_1^d = \gamma_2^d = \frac{1}{2}$. Both delegates use both instruments in the subsequent equilibrium in stage 3. The corresponding expected payoffs of player 1 and player 2 are:*

$$\pi_1^d = \frac{V}{4}, \qquad \pi_2^d = \frac{V}{4}. \tag{19}$$

Using the above lemmas we turn to stage 1 and present our main result.

Proposition 1 *Let $V_1 \approx 10.45$ and $V_2 \approx 12.45$ be as defined in Lemma 4. We then have the following with respect to the equilibria of the model:*

(i) *If $0 < V < 9$, then there is a unique Pareto dominant equilibrium, in which both players do not hire a delegate in stage 1. (If $0 < V \leq 8$, this is the unique equilibrium.)*

(ii) *If $9 \leq V \leq V_1$, then there exist a unique Pareto dominant equilibrium, in which both players hire a delegate in stage 1. In this equilibrium both delegates use one instrument in stage 3, and their contingent fees are $\frac{V}{3}$.*

(iii) *If $V_1 < V < 16$, then there exists a unique Pareto dominant equilibrium, in which both players do not hire a delegate in stage 1. (If $V_2 < V < 16$, this is the unique equilibrium.)*

(iv) *If $V = 16$, then there exist equilibria in which both players, one player, or zero players hire a delegate in stage 1. If a delegate is hired, she uses two instruments in stage 3. If both delegates are hired, then their contingent fees are $\frac{V}{3}$; if only one delegate is hired, then her contingent fee is $(\sqrt{2} - 1)V$.*

(v) *If $V > 16$, then there is a unique equilibrium, in which both player 1 and player 2 hire a delegate in stage 1. In this equilibrium both delegates use both instruments in stage 3, and their contingent fees are $\frac{V}{2}$.*

The proposition shows that, depending on the size of the prize V, we have equilibria in which either no, one or both players hire a delegate. It is interesting to compare this with the corresponding standard benchmark case, where we have a contest in which delegates have only one instrument at their disposal. We recall from our discussion of Wärneryd (2000) that in that case there is always a unique equilibrium in which the players do not hire a delegate; in fact, in that case not hiring a delegate is always a dominant strategy for each player in stage 1. Hence, introducing the option that delegates use more than one instrument completely changes the set of possible equilibria.

We further see from Proposition 1 that the decision to hire a delegate does not depend in a monotone way on the size of V. In particular, we have (Pareto dominant) equilibria with delegation for both $9 \leq V \leq V_1$ and $V > 16$. This can be understood in an intuitive way as follows. First, for the case with relatively 'small' values of V, i.e. $9 \leq V \leq V_1$, we obtain a unique Pareto dominant equilibrium in which both players hire a delegate. We observe that in this equilibrium both delegates compete 'modestly', in the sense that they use only *one* instrument. Second, consider the case with 'intermediate' values of V, i.e. $V_1 < V < 16$, and suppose that both players hire a delegate in this case. It follows from the proof of Proposition 1 (see the Appendix) that we then have two subcases: (a) if $V_1 < V \leq V_2$, then one delegate uses one instrument whereas the other delegate competes 'aggressively' and uses two instruments; (b) if $V_2 < V < 16$, then both delegates compete 'aggressively' and use two instruments. It turns out that in subcase (a) it is profitable for the player associated with the modest delegate, to deviate unilaterally and not hire a delegate. Similarly, in subcase (b) a unilateral deviation by any player is profitable. In both subcases the prize is not large enough to sustain an equilibrium with delegation by both players. Third, take the case with relatively 'large' values of the prize, i.e. $V > 16$. We then have a unique equilibrium in which both players hire a delegate and where each delegate competes aggressively by using two instruments. Now V is so large that the aggressive behaviour of both delegates can be sustained in equilibrium.

Finally, it also follows from the proof of Proposition 1 (see the Appendix) that in the borderline case with $V = 16$, each player obtains a payoff equal to 4 in all possible situations. This explains part (iv) of Proposition 1.

3 Conclusion

We have analysed delegation in a two-person rent-seeking contest where delegates can use two instruments whereas the players themselves can only compete with one instrument. We endogenized both the decision to hire a delegate and the contingent fee offered to the delegates. It turns out that, depending on the size of the contested prize, we have (Pareto dominant) equilibria in which no, one or both players hire a delegate. Interestingly, the number of players that hire a delegate in equilibrium does not depend in a monotone way on the size of the contested prize. It depends in a non-trivial way on the interplay between the size of the contested prize and the decision of the delegates to compete with either one or two instruments.

Notes

[1] Baik (2006) similarly analyses a two-player contest with compulsory delegation where the delegates and players have identical abilities. In his model the payment schemes for the delegates consist of an endogenously determined contingent fee and a nonnegative fixed fee. Baik shows that in equilibrium the fixed fee is set equal to zero.

[2] Recalling our discussion of abilities in Section 1, we mention here for completeness the following. Take a standard two-player contest, and let the probability that player 1 wins the prize be $p = \lambda e_1/(\lambda e_1 + e_2)$, where $\lambda > 1$ is a given parameter. Then player 1 has a larger ability than player 2. Note that $p > 1/2$ if $e_1 = e_2$.

[3] We remark that Epstein and Hefeker (2003, Corollary 1) present a sufficient condition such that one contestant uses one instrument while the other contestant uses both instruments. Part (ii) of our Lemma 3 presents a necessary and sufficient condition. Epstein and Hefeker also do not give (14).

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Appendix: Proofs

Proof of Lemma 1 – There is no equilibrium in which $z_1 = 0$ and/or $e_2 = 0$. Hence, the Kuhn-Tucker (KT) conditions characterizing an equilibrium are

$$\frac{z_1 e_2 \gamma_1 V}{((1+y_1)z_1+e_2)^2} \leq 1 \text{ with an equality sign if } y_1 > 0, \tag{A.1}$$

$$\frac{(1+y_1)e_2\gamma_1 V}{((1+y_1)z_1+e_2)^2} = 1, \tag{A.2}$$

$$\frac{(1+y_1)z_1 V}{((1+y_1)z_1+e_2)^2} = 1. \tag{A.3}$$

First, consider $y_1^b = 0$. We then see from the KT-conditions that $z_1^b \leq 1$ and $z_1^b = \gamma_1 e_2^b$. Substituting the latter equality, we find (5). The condition $z_1^b \leq 1$ can be rewritten as $1/\gamma_1 \geq \sqrt{V} - 1$. Second, consider $y_1^b > 0$. The KT-conditions then imply $y_1^b = z_1^b - 1$ and $z_1^b = \gamma_1 e_2^b$. Substituting, we obtain (6). Using z_1^b of (6), we derive that $y_1^b > 0$ if and only if $1/\gamma_1 < \sqrt{V} - 1$. ∎

Proof of Lemma 2 – We present the proof in 3 steps. Using backward induction, we analyse in steps 1 and 2 the cases where delegate 1 uses, respectively, both instruments and one instrument in stage 3. We combine results in step 3.

— *Step 1:* If delegate 1 uses both instruments in stage 3, it follows from part (ii) of Lemma 1 that $q = 1 - \gamma_1^{-1} V^{-\frac{1}{2}}$. The problem of player 1 in stage 2 now becomes

$$\max_{0 \leq \gamma_1 \leq 1} (1 - \gamma_1^{-1} V^{-\frac{1}{2}})(1-\gamma_1)V. \tag{A.4}$$

In the optimum $\gamma_1^b = V^{-\frac{1}{4}}$. The corresponding expected payoff of player 1 equals $\pi_1^{b1} = (V^{\frac{1}{4}} - 1)^2 \sqrt{V}$. Next, we must guarantee that the condition mentioned in part (ii) of Lemma 1 is satisfied. We observe that $1/\gamma_1^b < \sqrt{V} - 1$ holds with the value of γ_1^b just derived if and only if $V^{\frac{1}{2}} - V^{\frac{1}{4}} - 1 > 0$, i.e. if and only if $V > (\frac{1+\sqrt{5}}{2})^4 \approx 6.85$. Remark that now the expected payoff of player 2 equals $\pi_2^{b1} = \sqrt{V}$.

— *Step 2:* If delegate 1 uses one instrument in stage 3, part (i) of Lemma 1 implies that $q = \gamma_1/(\gamma_1 + 1)$. In turn, the problem faced by player 1 in stage 2 is

$$\max_{0 \leq \gamma_1 \leq 1} (\frac{\gamma_1}{\gamma_1 + 1})(1-\gamma_1)V. \tag{A.5}$$

It follows that in the optimum we have $\gamma_1^b = -1 + \sqrt{2}$, while the expected payoff of player 1 is $\pi_1^{b2} = (\sqrt{2} - 1)^2 V$. Next, we must guarantee that the condition mentioned in part (i) of Lemma 1 is satisfied with this value of γ_1^b. Observe that $1/\gamma_1^b \geq \sqrt{V} - 1$ holds if and only if $V \leq 2/(\sqrt{2} - 1)^2 \approx 11.66$. Remark that now the expected payoff of player 2 is $\pi_2^{b2} = V/2$.

— *Step 3:* We see that for $0 < V \leq (\frac{1+\sqrt{5}}{2})^4$, π_1^{b1} is not relevant, while for $V > 2/(\sqrt{2} - 1)^2$, π_1^{b2} is not relevant. Comparing π_1^{b1} and π_1^{b2} for $(\frac{1+\sqrt{5}}{2})^4 < V \leq 2/(\sqrt{2} - 1)^2$, we obtain that $\pi_1^{b1} > \pi_1^{b2}$ if and only if $V > \frac{1}{4}(\sqrt{2} - 1)^{-4}$. ■

Proof of Lemma 3 – There cannot exist an equilibrium in which $z_1 = 0$ and/or $z_2 = 0$. Consequently, the KT-conditions characterizing an equilibrium are

$$\frac{(1+y_2)z_1 z_2 \gamma_1 V}{((1+y_1)z_1 + (1+y_2)z_2)^2} \leq 1 \text{ with an equality sign if } y_1 > 0, \quad \text{(A.6)}$$

$$\frac{(1+y_1)(1+y_2)z_2 \gamma_1 V}{((1+y_1)z_1 + (1+y_2)z_2)^2} = 1, \quad \text{(A.7)}$$

$$\frac{(1+y_1)z_1 z_2 \gamma_2 V}{((1+y_1)z_1 + (1+y_2)z_2)^2} \leq 1 \text{ with an equality sign if } y_2 > 0, \quad \text{(A.8)}$$

$$\frac{(1+y_1)(1+y_2)z_1 \gamma_2 V}{((1+y_1)z_1 + (1+y_2)z_2)^2} = 1. \quad \text{(A.9)}$$

Recall that $\gamma_2 = k\gamma_1$ with $k \geq 1$. Hence, in equilibrium we can only have three situations: both delegates use one instrument; delegate 1 uses one instrument whereas delegate 2 uses both instruments; or both delegates use both instruments.

First, suppose that both delegates use only one instrument, i.e. $y_1^d = y_2^d = 0$. The KT-conditions then show that $z_1^d \leq 1$, $z_2^d \leq 1$ and $z_1^d \gamma_2 = z_2^d \gamma_1$. Using the latter equality, we find z_1^d and z_2^d given in part (i). Next, remark that $z_1^d \leq 1$ if and only if $\gamma_1 V \leq (1+k)^2/k$, and $z_2^d \leq 1$ if and only if $\gamma_1 V \leq (1+k)^2/k^2$. The condition on $\gamma_1 V$ given in part (i) follows from $k \geq 1$. This proves part (i).

Second, suppose that delegate 1 uses one instrument, i.e. $y_1^d = 0$, whereas delegate 2 uses both instruments. The KT-conditions then yield $z_1^d \leq 1$, $z_2^d = 1 + y_2^d$ and $z_1^d \gamma_2 = z_2^d \gamma_1$. Using the latter two equalities, we derive z_1^d and z_2^d presented in part (ii). The condition on $\gamma_1 V$ given in part (ii) follows from the fact that $z_1^d \leq 1$ can be rewritten as $\gamma_1 V \leq (1+k^2)^2/k^2$, while $y_2^d > 0$ is equivalent to $\gamma_1 V > (1+k)^2/k^2$. This establishes part (ii).

Third, suppose that both delegates use both instruments. The KT-conditions then give $z_1^d = y_1^d + 1$, $z_2^d = y_2^d + 1$ and $z_1^d\gamma_2 = z_2^d\gamma_1$. Using this we derive z_1^d, y_1^d, z_2^d and y_2^d given in part (iii). Observe that $y_1^d > 0$ if and only if $\gamma_1 V > (1+k^2)^2/k^2$, and $y_2^d > 0$ if and only if $\gamma_1 V > (1+k^2)^2/k^3$. The condition on $\gamma_1 V$ given in part (iii) follows since $k \geq 1$. This proves part (iii). ■

Proof of Lemma 4 – Using backward induction, we give the proof in 4 steps.

— *Step 1:* Assume that both delegates use one instrument in stage 3. Without loss of generality, we focus on the case with $\gamma_2 \geq \gamma_1 > 0$. We then can derive from part (i) of Lemma 3 that $q = \gamma_1/(\gamma_1+\gamma_2)$. Considering the corresponding problems faced by players 1 and 2 in stage 2, we obtain the optimal values $\gamma_1^d = \gamma_2^d = 1/3$ and the resulting expected payoffs $\pi_1^{d1} = \pi_2^{d1} = V/3$. Remark that in this case the condition $\gamma_2^d \geq \gamma_1^d$ is trivially satisfied, while the condition $\gamma_1^d V \leq (1+k)^2/k^2$ of part (i) of Lemma 3 holds if and only if $0 < V \leq 12$.

— *Step 2:* Assume that delegate 1 uses one instrument while delegate 2 uses two instruments in stage 3. Without loss of generality, we focus on the case with $\gamma_2 \geq \gamma_1 > 0$. It then follows from part (ii) of Lemma 3 that $q = \gamma_2^{-1}\gamma_1^{\frac{1}{2}}V^{-\frac{1}{2}}$. The problem solved by player 1 in stage 2 now equals

$$\max_{0\leq\gamma_1\leq 1} \gamma_2^{-1}\gamma_1^{\frac{1}{2}}(1-\gamma_1)\sqrt{V}. \tag{A.10}$$

The optimal solution is $\gamma_1^d = 1/3$. The problem solved by player 2 in stage 2 reads

$$\max_{0\leq\gamma_2\leq 1} (1-\gamma_2^{-1}\gamma_1^{\frac{1}{2}}V^{-\frac{1}{2}})(1-\gamma_2)V. \tag{A.11}$$

Using $\gamma_1 = 1/3$, the optimal solution for player 2 is $\gamma_2^d = (3V)^{-\frac{1}{4}}$. The resulting expected payoffs are $\pi_1^{d2} = 2\times 3^{-\frac{5}{4}}\times V^{\frac{3}{4}}$ and $\pi_2^{d2} = \left(1-(3V)^{-\frac{1}{4}}\right)^2 V$. Observe that $\gamma_1^d = 1/3$ in both the present case and in the case of step 1. Comparing the profits of player 2, we obtain that $\pi_2^{d1} < \pi_2^{d2}$ if and only if $V > V_1$, where $V_1 = 3/(\sqrt{3}-1)^4 \approx 10.45$. Next, examine for the present case which restrictions must be imposed on V such that both $\gamma_2^d \geq \gamma_1^d$ and the conditions on $\gamma_1^d V$ given in part (ii) of Lemma 3 are satisfied. First, $\gamma_2^d \geq \gamma_1^d$ if and only if $V \leq 27$. Second, $\gamma_1^d V \leq (1+k^2)^2/k^2$ if and only if $V \leq V_2$, where $V_2 \approx 12.45$ is the unique positive root of $(\sqrt{3V}+9)^4 - 27V^3 = 0$. Third, $\gamma_1^d V > (1+k)^2/k^2$ if and only if $V > V_3$, where $V_3 \approx 9.37$ is the unique positive root of $9V^2 - ((3V)^{\frac{1}{4}}+3)^4 = 0$.

— *Step 3:* Assume that both delegates use both instruments in stage 3. Without loss of generality, we focus on the case with $\gamma_2 \geq \gamma_1 > 0$. Part (iii) of Lemma 3 then implies that $q = \gamma_1^2/(\gamma_1^2 + \gamma_2^2)$. The problem faced by player 1 in stage 2 is

$$\max_{0 \leq \gamma_1 \leq 1} \frac{\gamma_1^2}{\gamma_1^2 + \gamma_2^2}(1 - \gamma_1)V, \tag{A.12}$$

while player 2 faces

$$\max_{0 \leq \gamma_2 \leq 1} \frac{\gamma_2^2}{\gamma_1^2 + \gamma_2^2}(1 - \gamma_2)V. \tag{A.13}$$

One can verify that in stage 2 we now have $\gamma_1^d = \gamma_2^d = 1/2$. The corresponding expected payoffs of players 1 and 2 are $\pi_1^{d3} = \pi_2^{d3} = V/4$. Notice that in this case we trivially have that $\gamma_2^d \geq \gamma_1^d$, while the condition $\gamma_1^d V > (1 + k^2)^2/k^2$ mentioned in part (iii) of Lemma 3 is satisfied if and only if $V > 8$.
— *Step 4:* Combining results, we obtain Lemma 4. Remark that if $8 < V \leq V_1$, then there is an equilibrium with $\gamma_1^d = \gamma_2^d = \frac{1}{3}$ as well as an equilibrium with $\gamma_1^d = \gamma_2^d = \frac{1}{2}$. However, only the former one is Pareto dominant since $\pi_1^{d1} > \pi_1^{d3}$ and $\pi_2^{d1} > \pi_2^{d3}$ in this case. Further, if $V_1 < V \leq V_2$, then (focusing on $\gamma_2^d \geq \gamma_1^d > 0$) there is an equilibrium with $\gamma_1^d = \frac{1}{3}$ and $\gamma_2^d = (3V)^{-\frac{1}{4}}$ and an equilibrium with $\gamma_1^d = \gamma_2^d = \frac{1}{2}$. Only the former one is Pareto dominant because $\pi_1^{d2} > \pi_1^{d3}$ and $\pi_2^{d2} > \pi_2^{d3}$ in this case. Finally, the second equilibrium mentioned in part (ii) of Lemma 4 is the obvious counterpart of the first equilibrium mentioned there. ■

Proof of Proposition 1 – Using backward induction, the expected payoffs of case (a), Lemmas 2 and 4, and (9) and (10), we can construct the relevant 2×2 (normal form) payoff table associated with stage 1 for all possible values of V (in the tables 'nd' means no delegation, while 'd' means delegation).
— Let $0 < V \leq V_0$. Using (7), (9) and (16), we obtain Table A.1. We conclude that now there is a unique Pareto dominant equilibrium in which both players do not hire a delegate. If $0 < V \leq 8$, this is the unique equilibrium.
— Let $V_0 < V \leq V_1$. We use (8), (10) and (16) to find Table A.2. We conclude the following: if $V_0 < V < 9$, then there is a unique Pareto dominant equilibrium in which both players do not hire a delegate; if $9 \leq V \leq V_1$, then there is a unique Pareto dominant equilibrium in which both players hire a delegate. In the latter equilibrium, both delegates use one instrument.
— Let $V_1 < V \leq V_2$. We use (8) and (10), while we have to examine two possible cases in Lemma 4. First, using (17), we obtain Table A.3. Second,

using (18), we obtain Table A.4. In both cases we have a Pareto dominant equilibrium in which both players do not hire a delegate.
— Let $V > V_2$. Using (8), (10) and (19), we arrive at Table A.5 and reach the following conclusions. First, if $V_2 < V < 16$, then there is a unique equilibrium in which both players do not hire a delegate. Second, if $V = 16$, then any combination of decisions of players 1 and 2 in stage 1 constitutes an equilibrium (since each player receives payoff 4 in all cases). If a delegate is hired, she uses two instruments. Third, if $V > 16$, then there is a unique equilibrium in which both players hire a delegate, and where both delegates use both instruments. ■

		Player 2	
		nd	d
Player 1	nd	$\frac{V}{4}, \frac{V}{4}$	$\frac{V}{2}, (\sqrt{2}-1)^2 V$
	d	$(\sqrt{2}-1)^2 V, \frac{V}{2}$	$\frac{V}{3}, \frac{V}{3}$

Table A1: Expected payoffs for players 1 and 2 in stage 1 for $0 < V \leq V_0$, using (7), (9) and (16).

		Player 2	
		nd	d
Player 1	nd	$\frac{V}{4}, \frac{V}{4}$	$\sqrt{V}, (V^{\frac{1}{4}}-1)^2\sqrt{V}$
	d	$(V^{\frac{1}{4}}-1)^2\sqrt{V}, \sqrt{V}$	$\frac{V}{3}, \frac{V}{3}$

Table A2: Expected payoffs for players 1 and 2 in stage 1 for $V_0 < V \leq V_1$ using (8), (10) and (16).

		Player 2	
		nd	d
Player 1	nd	$\frac{V}{4}, \frac{V}{4}$	$\sqrt{V}, (V^{\frac{1}{4}}-1)^2\sqrt{V}$
	d	$(V^{\frac{1}{4}}-1)^2\sqrt{V}, \sqrt{V}$	$2 \times 3^{-\frac{5}{4}} \times V^{\frac{3}{4}}, (1-(3V)^{-\frac{1}{4}})^2 V$

Table A3: Expected payoffs for players 1 and 2 in stage 1 for $V_1 < V \leq V_2$ using (8), (10) and (17).

		Player 2: nd	Player 2: d
Player 1	nd	$\frac{V}{4}$, $\frac{V}{4}$	$\sqrt{V}$, $(V^{\frac{1}{4}} - 1)^2\sqrt{V}$
	d	$(V^{\frac{1}{4}} - 1)^2\sqrt{V}$, $\sqrt{V}$	$(1 - (3V)^{-\frac{1}{4}})^2 V$, $2 \times 3^{-\frac{5}{4}} \times V^{\frac{3}{4}}$

Table A4: Expected payoffs for players 1 and 2 in stage 1 for $V_1 < V \leq V_2$ using (8), (10) and (18).

		Player 2: nd	Player 2: d
Player 1	nd	$\frac{V}{4}$, $\frac{V}{4}$	$\sqrt{V}$, $(V^{\frac{1}{4}} - 1)^2\sqrt{V}$
	d	$(V^{\frac{1}{4}} - 1)^2\sqrt{V}$, $\sqrt{V}$	$\frac{V}{4}$, $\frac{V}{4}$

Table A5: Expected payoffs for players 1 and 2 in stage 1 for $V > V_2$ using (8), (10) and (19).