CentER

Discussion Paper

No. 2007–38

STRATEGIC R&D WITH KNOWLEDGE SPILLOVERS AND ENDOGENOUS TIME TO COMPLETE

By R. Lukach, P.M. Kort, J. Plasmans

June 2007

ISSN 0924-7815

Tilburg University

Strategic R&D with Knowledge Spillovers and Endogenous Time to Complete

Lukach, R.[1], Kort, P.M.[2,1], Plasmans, J.[1,2]
[1]Department of Economics, University of Antwerp,
[2]Department of Econometrics & Operations Research and CentER, Tilburg University.

November 22, 2006

Abstract[1]

We present a model where firms make competitive decisions about the optimal duration (or time to build) of their R&D projects. Choosing its project's duration, the firm can choose to become a leader or a follower, based on its R&D efficiency, the size of the R&D to be carried out and the degree of innovation, which this research will produce.

It is shown that asymmetry in R&D efficiency between firms is an important factor determining feasibility of the preemption and attrition scenarios in competitive R&D with time to build. Scenarios of attrition and preemption games are most likely to occur when competitors have similar R&D efficiencies. In case of largely asymmetric firms the games of attrition and preemption are very unlikely, thus the R&D duration choices of firms are determined by the actual trade-off between the benefits of earlier innovation and the costs of faster R&D project completion.

JEL Classification: C72, D21, O31
Keywords:R&D Investment, Competition, Preemption, Attrition.

[1]Corresponding Author: Ruslan Lukach, Department of Economics, University of Antwerp, Prinsstraat 13, 2000 Antwerp, BELGIUM. Tel: +32-3-220-4008. Fax: +3-3-220-4585. E-mail: ruslan.lukach@ua.ac.be.

1 Introduction

The host of the IO literature in the area of competitive/cooperative R&D investment regarding process innovation (Reinganum (1981), d'Aspremont and Jacquemin (1988) (referred below as A&J(88)), Suzumura (1992), and Petit, and Tolwinski (1999)) assumes that R&D investment has an immediate effect on the firm's production capacity and/or efficiency. This implies that the R&D project is completed instantaneously. Obviously, such a setup ignores an important property of R&D, which is that an R&D project requires time to complete.

Dutta (1997) investigates the problem of optimal budget management of an R&D project, which is carried out in several stages. The paper analyzes an optimization problem of a single decision maker. Our goal is to analyze the firms' R&D investment strategies in a duopolistic setting. We depart from the static competitive/cooperative R&D investment A&J(88) model.

In our model we combine the knowledge spillover idea of A&J(88) with an optimal R&D resource allocation problem setting. (Scherer (1967), Pacheco-de-Almedia and Zemsky (2003)). The Pacheco-de-Almedia and Zemsky's (2003) discrete-time model is timed in three stages: resolution of uncertainty of duration T, capacity investment also of duration T, and a production period of infinite length. We assume that the firms operate in a deterministic environment and, therefore, our model has two stages: R&D investment in order to develop a new production process where the firm has to determine the duration of the R&D process (while producing with old technology) and a production stage from the moment of R&D completion onwards. It can be assumed as well that uncertainty about the volume and the cost of R&D has been resolved in the previous stages of research, and the firm now considers its project duration decisions in the final stretch of the innovation process. Our analysis is carried out in a continous-time setting in the spirit of Scherer (1967).

In the presented model there is a positive relation between the degree of innovation and the amount of knowledge to be accumulated in order to achieve such an innovation. Although there is evidence that major product innovations do not necessarily require large amounts of resources and/or research effort (see Bercovitz *et al.* (1997)), it is almost definitely true for process innovations. The Toyota' successful introduction of its "lean" auto assembly process (Teece (1996) and Van Biesebroek (2003)) required a substantial amount of resources to redesign the whole production system and coordinate it with its numerous suppliers. Our study shows that, for bigger innovations a marginal increase in total R&D effort necessary to complete the innovation requires more additional time than in the case of a smaller innovation.

The firm's problem of determining the optimal duration of an R&D project resembles the ground breaking model of technology adoption timing of Fudenberg and Tirole (1985), which was extended by works of Dutta *et al.* (1995), Hoppe and Lehmann-Grube (2001) and (2005), and Hoppe (2000). But unlike in the setting of Fudenberg and Tirole (1985), where a new technology is available and becomes adopted against some cost, our model contains an R&D

process with endogenous time to complete, while the new technology can only become available after this process is completed. Intermediate innovations can not, thus, be used productively for the firm.

Also, unlike in studies of technology adoption timing of Katz and Shapiro (1987) and Hoppe and Lehmann-Grube (2001), where earlier adoption means usage of a less advanced technology, in this model the degree of innovation and R&D duration decisions are uncoupled. We find that our assumption fits better for a cost reducing process innovation, because the company deciding to modernize its production process almost always has a particular level of cost reduction in mind. As an example, consider Intel's computer chip production process, where each cycle of technological improvement has a goal of cutting the production cost of "putting" one transistor on the chip by one half (Marcyk (2002)).

With the degree of innovation not being dependent on time, this study shows that in the precommitment equilibrium the optimal R&D project duration of the leader (the firm, which finishes its R&D first) is the same as the optimal duration of the innovator in a duopoly, where only one firm conducts R&D; and the optimal R&D project duration of the follower (the firm, which finishes its R&D second) is the same as the optimal duration of the catching up innovator, facing the opponent who already possesses the new technology.

Furthermore, we investigate the competitive equilibria under conditions of knowledge spillovers, which create incentives for the firm to finish its R&D later than its competitor. Katz and Shapiro (1987) considered firms with different profit incentives for innovation and derived the corresponding preemption and waiting game equilibria under different spillover mechanisms such as licensing and imitation. We find that under conditions of weak knowledge spillovers, a preemption equilibrium is more likely to occur, while the attrition equilibrium will prevail when the spillovers are strong.

Firms' payoffs in the preemption equilibrium and in the "war of attrition" are different from the corresponding precommitment cases. In the precommitment scenarios with exogenous roles the optimal duration of R&D is determined by the trade-off between the extra profit gain from innovation and the additional R&D cost from finishing the project sooner. Firms do not consider strategic effects and the incentives for preemption or attrition.

In the preemption scenario the optimal project durations are set in a way that the follower is not able to preempt the leader. In such a situation the leader makes its decisions based on the follower's incentives to preempt (if the follower has such incentives). As a result, the winner (leader) has lower payoffs than in the corresponding precommitment case. In the attrition scenario the winner (follower) must neutralize the other firm's second mover advantage and give up part of its profits. But, unlike in the preemption case, where the deviating firm has the same payoff as in the original position, in the war of attrition the firm, which deviates, ends up with higher payoffs than in the original position. The looser in the war of attrition receives some extra benefits by getting a more extended period of technological leadership. This happens due to the fact that in our model adopting earlier does not mean that the leader will have a less advanced technology than the follower.

In the presented model we consider two asymmetric firms, which have different R&D efficiencies in the form of different per unit marginal cost of R&D. This asymmetry plays a role in determining who will become the leader and the follower, when feedback equilibria are considered. With a reasonable small asymmetry in R&D efficiency between firms, the game with endogenous roles can have preemption or attrition equilibria depending on different values of the parameters. But if one firm has a strong advantage in R&D efficiency over the other, a preemption or attrition game cannot occur. In such a situation the more R&D efficient firm is almost always capable of capturing the leading position.

This paper is structured as follows; the second section of this study presents the model of competitive R&D with time to build and knowledge spillovers. Sections 3, 4, 5 and 6 present solutions of the model for monopoly, one-innovator duopoly, catching-up innovator duopoly, and the two-innovators duopoly, respectively. Conclusions and discussions are presented in Section 7.

2 The Model of R&D with Time to Build

In this section we present the setting and main assumptions for the model of a competitive R&D in duopoly with time to build and knowledge spillovers.

Let us consider the following scenario. A firm is producing a good with a particular technology. This production technology can be improved through process innovation, which results in decreasing the marginal cost of production. In the initial state of development, this technology allows the firm to produce with marginal cost A. The firm has all necessary R&D capabilities to implement the required innovation, which will allow it to bring the marginal cost of production down to zero.

The firm knows exactly how much R&D is needed to reach a given level of production efficiency. By engaging in R&D the firm must realize a knowledge gain in order to obtain the new production technology. It takes time to complete each R&D project. The marginal cost decrease takes place only after the project is completed (i.e. the knowledge gain is fully realized). There is another competing firm in the market operating under the same set of rules, which makes it a duopoly.

Consider an A&J(88)-type problem, where the A&J(88) framework is extended by taking the above conditions into account. Assume that for firm i completion of an R&D project results in decreasing the marginal production cost from A to 0.

We assume that the two firms are symmetric in their marginal costs of production ($A_1 = A_2 = A$). Therefore, following the specification of the A&J(88) model, firm i's total individual knowledge gain must equal firm i's marginal cost of production before innovation, A. In this way we explicitly establish a positive correlation between the degree of process innovation and the knowledge gain necessary to achieve such an innovation.

To model the "time to complete", we assume that the firm subdivides the total R&D in per-period R&D efforts, so that a completed R&D project (in continuous space) satisfies

$$\int_0^n x_i(t)dt = A, \; i = 1, 2,$$

where $x_i(t)$ is the knowledge gain of firm i, and n is the total duration of the R&D project.

The firms are assumed to be asymmetric in R&D effectiveness, represented by different R&D efficiency ($\gamma_1 \neq \gamma_2$). The corresponding present value of firm i's total R&D spending of firm exhibits diminishing returns to R&D and is specified as

$$\Xi_i = \frac{\gamma_i}{2} \int_0^n x_i^2(t) e^{-rt} dt,$$

where r is the discount rate.

Currently we assume that the outcome of each step of the R&D project is deterministic. By spending Ξ_i dollars firm i achieves the projected decrease of the marginal production cost of A. After the R&D process is completed, the firm will continue producing with an upgraded technology from time n onwards.

There are knowledge spillovers present in the market, but unlike in A&J(88), where a part of one firm's R&D effort is utilized by the other firm immediately in the form of extra production cost benefit, in our case only one firm can benefit from a knowledge spillover, which is the firm that completes its R&D later than the other. Here we make a reasonable assumption that during the project implementation stage the firms are capable of protecting their knowledge so that no information flows occur. Yet, when one firm completes the project and the R&D output becomes realized (Grossman and Helpman (1990)), the other firm will be able to obtain some extra technological knowledge by observing the new technology or products produced with it.

We assume that this amount of knowledge equals a part of the other firm's total knowledge gain. For example, if firm i finishes its R&D as second, it will receive a knowledge spillover of βA at the moment when firm j completes its project, where parameter β $(0 < \beta < 1)$ here determines the strength or degree of such a knowledge spillover.

Here we address the problem of determining the optimal project duration time, given a certain amount of new knowledge needed to complete the project to be carried out during this time. Currently, we approach this task by considering a two-stage R&D/production game where the firm must carry out one innovation cycle to achieve one technological breakthrough.

Two situations can be distinguished: one situation with exogenous firm roles that are prescribed beforehand implicitely which firm will become the leader and finish the R&D process first (precommitment strategies as defined in Reinganum (1981)) In the second situation it is not established beforehand which of the two firms will finish its R&D first. In the latter case the firm roles are endogenous.

First, we derive the solution of the general problem of R&D with time to build. Then we consider an open-loop game setting with exogenous roles in cases of one-innovator duopoly, catching-up innovator duopoly, and the two innovators duopoly, respectively. The monopoly case is considered as a benchmark. The feedback model with endogenous roles is presented later.

3 Solution of General Problem of R&D with Time to Build

Let us consider a firm which produces with some technology generating the profit stream $\pi_0(q(t))$. If this firm invests in R&D it can obtain a new technology, which will generate the profit stream $\pi_1(q(t))$ after the innovation is completed. Innovation requires time to build as set in the above model description.

The problem can then be specified as:

$$\begin{aligned} \max_{\{n,q(t),x(t)\}} \pi &= \int_0^n [\pi_0(q(t)) - \gamma \frac{x^2(t)}{2}] e^{-rt} dt + \int_n^\infty \pi_1(q(t)) e^{-rt} dt, \qquad (1) \\ s.t. \int_0^n x(t)dt &= A. \end{aligned}$$

First we maximize this payoff function with respect to $q(t)$ and $x(t)$ for a given n. This requires maximization of the following corresponding Lagrangian functions:

$$\max_{\{q(t),x(t),\lambda\}} L_{t\in(0,n]} = [\pi_0(q(t)) - \gamma \frac{x^2(t)}{2}] e^{-rt} + \lambda \left(\int_0^n x(t)dt - A \right), \text{ and} \qquad (2)$$

$$\max_{\{q(t)\}} L_{t\in(n,\infty)} = \pi_1(q(t)) e^{-rt}. \qquad (3)$$

The FOCs for maximization of (2) and (3) with respect to $q(t)$ require:

$$\begin{aligned} \frac{\partial \pi_0}{\partial q} &= 0 \text{ and} \\ \frac{\partial \pi_1}{\partial q} &= 0, \end{aligned}$$

from which we obtain the optimal outputs $q_0^*(t)$ and $q_1^*(t)$. These optimal outputs yield π_0^* and π_1^*.

Maximizing (2) with respect to $x(t)$ and given the optimal π_0^* and π_1^* is equivalent to minimizing the total cost of R&D carrying out the project subject to the project completion constraint:

$$\min_{\{x(t),\lambda\}} H_{\Xi,m} = \gamma \frac{x^2(t)}{2} e^{-rt} - \lambda \left(\int_0^n x(t)dt - A \right). \qquad (4)$$

The set of first order conditions is:

$$\frac{\partial H}{\partial x} = \gamma x(t)e^{-rt} - \lambda n = 0, \tag{5}$$

$$\frac{\partial H}{\partial \lambda} = \int_0^n x(t)dt - A = 0. \tag{6}$$

From (5) we get $x(t) = \frac{\lambda n e^{rt}}{\gamma}$, which after substitution into (6) yields

$$\lambda = \frac{A\gamma r e^{-rn}}{n(1-e^{-rn})} \text{ and}$$

$$x^*(t)|_{t\in(0,n]} = \frac{Are^{-rn}}{1-e^{-rn}}e^{rt}. \tag{7}$$

The time schedule of the R&D effort presented in (7) implies the R&D effort increases during the project implementation phase. This result is in line with the finding of Grossman and Shapiro (1986) stating that, due to discounting, R&D expenditures increase.

Introducing our preliminary results into (1) and expanding the integrals we obtain:

$$\max_{\{n\}} \pi = \frac{1}{r}\pi_0 + \frac{e^{-rn}}{r}\Delta\pi - \frac{\gamma}{2}A^2\frac{re^{-rn}}{1-e^{-rn}}, \tag{8}$$

where $\Delta\pi = \pi_1 - \pi_0$ is the firm's profit gain from innovation.

The FOCs for solving (8) is:

$$e^{-rn}[\gamma A^2 r^2 - 2\Delta\pi(1-e^{-rn})^2] = 0. \tag{9}$$

Equation (9) has two nonzero roots in terms of e^{-rn}. But, we are interested only in non-negative values of n, which means that we consider only values $e^{-rn} < 1$. This leaves only one root of interest, which is:

$$(e^{-rn})^* = 1 - \frac{A}{\sqrt{\Delta\pi}}\sqrt{\frac{\gamma r^2}{2}} \tag{10}$$

and yields:

$$n^* = -\frac{1}{r}\ln\left(1 - \frac{A}{\sqrt{\Delta\pi}}\sqrt{\frac{\gamma r^2}{2}}\right). \tag{11}$$

4 Monopoly

Here we present a one-firm benchmark case. Consider the firm weighing a decision about the length of the R&D project n, which will result in achieving a marginal production cost decrease of A. After the project is completed, the

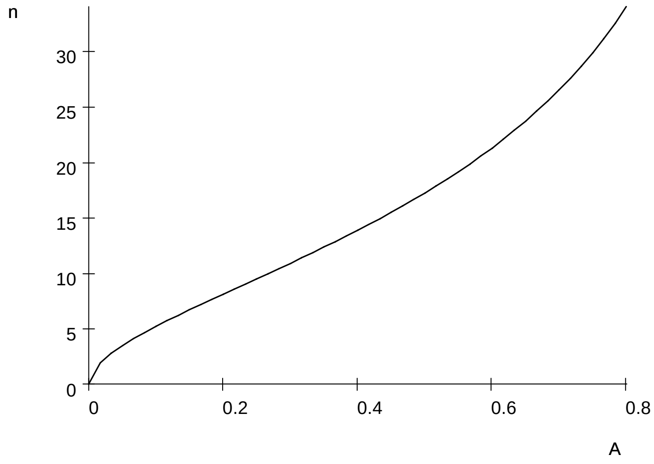

Figure 1: Monopolist's project duration as a function of the degree of innovation/knowledge gain ($r = 0.05$, $\gamma = 10$).

firm will be able to produce with a lower marginal production cost from time n onwards. The inverse market demand function is specified as

$$p = 1 - q.$$

Solving the monopolist's problem results in the following optimal values of output:

$$\begin{aligned} q^*_{0,m}(t) &= \frac{1-A}{2}, \\ q^*_{1,m}(t) &= \frac{1}{2}, \end{aligned} \tag{12}$$

with optimal R&D schedule:

$$x^*_m(t) = \frac{Are^{-rn^*_m}}{1-e^{-rn^*_m}} e^{rt},$$

and an optimal R&D project duration of:

$$n^*_m = -\frac{1}{r} \ln \left(1 - \sqrt{\frac{2A\gamma r^2}{2-A}} \right). \tag{13}$$

Additionally, the profits gain from the monopolist's innovation is $\Delta\pi_m = \frac{2A-A^2}{4}$.

As we see in Figure 1, the optimal duration of R&D exhibits an S-shaped relationship with respect to the degree of innovation/knowledge gain A. It has a concave shape for smaller values of A and becomes convex as A gets larger. To find the explanation of this fact we consider the effects of knowledge gain and degree of innovation apart from each other. We do this by allowing the knowledge gain to be equal to some arbitrary value ξ, which then gives us $n_m^* = -\frac{1}{r}\ln\left(1 - \frac{\xi}{\sqrt{\Delta\pi_m}}\sqrt{\frac{\gamma r^2}{2}}\right)$.

As we can see, the optimal duration of the R&D project is positively dependent on the amount of knowledge gain needed for innovation and negatively dependent on the square root of profit gain from this innovation. Thus, the degree of convexity(concavity) of the optimal duration with respect to the degree of innovation is determined by the relative size of the knowledge gain and the profit gain of innovation.

The profit gain from innovation creates an incentive to speed up the R&D process in order to start benefiting from the new technology sooner, thus pressing n^* down. As the R&D cost is convex in the knowledge gain, the value of additional knowledge gain necessary to complete R&D translates directly into additional research costs, which creates an incentive to extend the R&D process in order to decrease the present value of total R&D costs. The trade off between these two factors determines the optimal duration of the R&D project, which is also the case in the models of technology adoption timing (Fudenberg and Tirole (1985) and others).

Due to positive correlation between the degree of innovation and the necessary knowledge gain, the increase in parameter A leads to increase in both knowledge and profit gains of the firm. As A gets larger the downward pressure of the additional profit gain on the optimal R&D duration (incentive to speed up R&D) becomes weaker while the upward pressure of the necessary knowledge gain on n (incentive to extend the R&D project) becomes stronger. Thus, for more substantial innovations each additional knowledge gain for the monopolist requires more additional time than in the case of smaller innovation.

5 One Innovator Duopoly

In this section we present the case of one-innovator duopoly, where only one firm carries out R&D, while the other stays with the same technology. The role of innovator is assigned to firm *1* (denoted by *1* in the superscript along with a star). The non-innovating firm *2* maximizes with respect to output the profit function, which depends on firm i's R&D duration decision.

The inverse market demand function in a duopoly is given by:

$$p = 1 - (q_1 + q_2)$$

and the firms engage in a Cournot output competition. The non-innovating firm has:

$$\pi_2^{*1} = \int_0^{n_1} \pi_{0,2}^{*1} e^{-rt} dt + \int_{n_1}^{\infty} \pi_{1,2}^{*1} e^{-rt} dt,$$

where

$$\begin{aligned} \pi_{0,2}^{*1} &= \left(q_{0,2}^{*1}(t)\right)^2 = \left(\frac{1-A}{3}\right)^2, \\ \pi_{1,2}^{*1} &= \left(q_{1,2}^{*1}(t)\right)^2 = \left(\frac{1-2A}{3}\right)^2. \end{aligned}$$

The innovator firm in its turn has:

$$\pi_1^{*1} = \int_0^{n_1^{*1}} \left[\pi_{0,1}^{*1} - \gamma \frac{x_1^{*1}(t)}{2}\right] e^{-rt} dt + \int_{n_1^{*1}}^{\infty} \pi_{1,1}^{*1} e^{-rt} dt$$

with:

$$\begin{aligned} \pi_{0,1}^{*1} &= \left(q_{0,1}^{*1}(t)\right)^2 = \left(\frac{1-A}{3}\right)^2, \\ \pi_{1,1}^{*1} &= \left(q_{1,1}^{*1}(t)\right)^2 = \left(\frac{1+A}{3}\right)^2, \\ x_1^{*1}(t) &= \frac{Are^{-rn_1^{*1}}}{1-e^{-rn_1^{*1}}} e^{rt}, \text{ and} \\ n_1^{*1} &= -\frac{1}{r} \ln\left(1 - \sqrt{\frac{9A\gamma_1 r^2}{8}}\right) \end{aligned}$$

The optimal output levels in this scenario give rise to a profit gain from innovation $\Delta\pi_1^{*1} = \frac{4A}{9}$.

The relationship between the optimal duration of the R&D of one innovator and the degree of innovation/knowledge gain is presented in Figure 2. Here the shape of the relationship between knowledge gain/degree of innovation and the optimal duration of the project for the same parameter values is similar to that of the monopolist (Figure 1), but has a less prominent S-shape. Indeed, for relatively large innovations (with $A > \frac{2}{9}$) the innovator's profit gain is higher than the profit gain of the monopolist, resulting in stronger incentives to keep the project duration short. If the degree of innovation is small ($A \leq \frac{2}{9}$), the effect of the profit gain is stronger for the monopolist.

6 "Catching Up" Innovator Duopoly

Here we consider the case where one firm is already in possession of the new technology and the other firm decides about developing the new technology

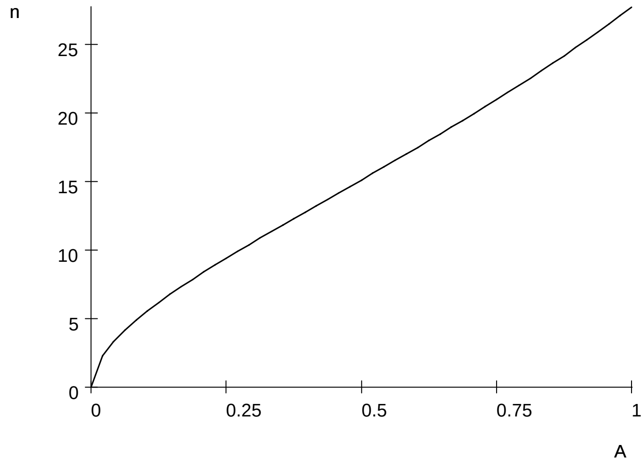

Figure 2: An innovator's project duration as a function of the degree of innovation/knowledge gain ($\delta = 0.05$, $\gamma_1 = 10$)

in the "catch up" mode. Let us assume that firm *1* produces with the new technology and firm *2* must develop one to be able to compete as equal. Firm *2* also benefits from the knowledge spillovers βA generated by firm *1*, so it must satisfy:

$$\int_0^{n_2} x_2(t)dt = (1-\beta)A.$$

The corresponding payoffs of the catching up innovator are (the order of indexes in the superscript indicates the order in which firms innovate):

$$\pi_2^{*1} = \int_0^{n_2^{*1}} \left[\pi_{0,2}^{*1} - \gamma \frac{x_2^{*1}(t)}{2}\right] e^{-rt}dt + \int_{n_2^{*1}}^{\infty} \pi_{1,2}^{*1} e^{-rt}dt,$$

where:

$$\begin{aligned}
\pi_{0,2}^{*1} &= \left(q_{0,2}^{*1}(t)\right)^2 = \left(\frac{1-2A}{3}\right)^2, \\
\pi_{1,2}^{*1} &= \left(q_{1,2}^{*1}(t)\right)^2 = \left(\frac{1}{3}\right)^2, \\
x_2^{*1}(t) &= \frac{(1-\beta)Are^{-rn_2^{*1}}}{1-e^{-rn_2^{*1}}}e^{rt}, \text{ and} \\
n_2^{*1} &= -\frac{1}{r}\ln\left(1-(1-\beta)\sqrt{\frac{9A\gamma_2 r^2}{8(1-A)}}\right)
\end{aligned}$$

and the corresponding profit gain from innovation of the catching-up innovator is $\Delta\pi_2^{*1} = \frac{4A(1-A)}{9}$.

The firm that already has the new technology has the following payoff:

$$\pi_1^{*1} = \int_0^{n_2^{*1}} \pi_{0,1}^{*1} e^{-rt} dt + \int_{n_2^{*1}}^{\infty} \pi_{1,1}^{*1} e^{-rt} dt,$$

where

$$\begin{aligned}
\pi_{0,1}^{*1} &= \left(q_{0,1}^{*1}(t)\right)^2 = \left(\frac{1+A}{3}\right)^2, \\
\pi_{1,1}^{*1} &= \left(q_{0,1}^{*1}(t)\right)^2 = \left(\frac{1}{3}\right)^2.
\end{aligned}$$

The relationship between the degree of innovation/knowledge gain and the optimal duration of R&D is presented in Figure 3. Here we must take into account that there is another parameter, which plays a role in determining the shape of the optimal project duration for the catching-up innovator. This parameter is the strength of knowledge spillovers. The stronger the knowledge spillover the smaller the amount of additional knowledge gain is needed, and, thus, the less additional time should be spent on R&D. In Figure 4 it is clearly visible that as knowledge spillovers get stronger, the relationship between the degree of innovation/knowledge gain becomes more concave.

In Figure 3 we observe that the optimal duration curve with respect to the knowledge gain and the degree of innovation in case of the catching-up innovator is S-shaped with a prominent convex interval. This is explained by the fact that the profit gain from innovation for the catching-up innovator ($\Delta\pi_2^{*12} = \frac{4A(1-A)}{9}$) is always smaller than the innovator's profit gain in the one innovator duopoly ($\Delta\pi_1^{*1} = \frac{4A}{9}$). Thus, the catching-up innovator has less incentives to conduct its R&D fast.

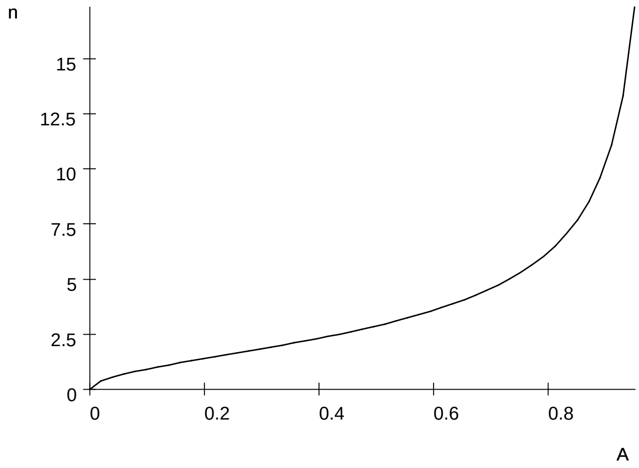

Figure 3: "Catching-up" innovator's project duration as a function of the degree of innovation/knowledge gain ($r = 0.05$, $\gamma_2 = 10$, $\beta = 0.2$)

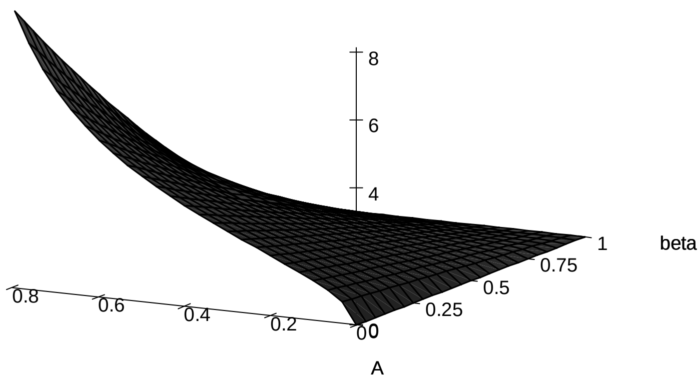

Figure 4: Optimal project duration of the catching-up innovator as a function of A and β ($r = 0.05$, $\gamma_2 = 10$).

7 Two Innovators with Different Project Durations

After studying the scenarios where only one firm must make the optimal R&D duration decision, we turn our attention to the case where both firms in duopoly innovate and set the optimal time to build for the new technology.

First of all, we want to determine the conditions under which two firms will choose different durations of their R&D, creating in this way the situation where one firm becomes the first investor, or the leader, and the other becomes the second inventor, or the follower. The follower then benefits from the knowledge spillovers generated by the leader.

Let us for now assume that firm *2* is going to spend more time developing its new technology: $n_2 > n_1$. Currently we consider $\gamma_1 \neq \gamma_2$, but we do not make any assumptions about which of these parameters is greater.

As firm *2*'s project duration is longer than that of firm *1* we take into account two main changes in the firms' profit functions. First, firm *2* will benefit from the knowledge spillovers generated at the moment after firm *1* completes its R&D project. Second, during the time period between n_1 and n_2 firm *1* will enjoy a technological advantage over firm *2*.

The leader's optimal payoff is expressed as:

$$\pi_1^L = \int_0^{n_1^L} \left[\pi_{0,1}^L - \gamma_1 \frac{x_1^L(t)}{2} \right] e^{-rt} dt + \int_{n_1^L}^{n_2^F} \pi_{1,1}^{*L} e^{-rt} dt + \int_{n_2^F}^{\infty} \pi_{1,1}^L e^{-rt} dt, \tag{14}$$

where

$$\pi_{0,1}^L = \left(q_{0,1}^L(t)\right)^2 = \left(\frac{1-A}{3}\right)^2,$$

$$\pi_{1,1}^{*L} = \left(q_{1,1}^{*L}(t)\right)^2 = \left(\frac{1+A}{3}\right)^2,$$

$$\pi_{1,1}^L = \left(q_{1,1}^L(t)\right)^2 = \left(\frac{1}{3}\right)^2,$$

$$x_1^L(t) = \frac{Are^{-rn_1^L}}{1-e^{-rn_1^L}} e^{rt}, \text{ and}$$

$$n_1^L = -\frac{1}{r} \ln\left(1 - \sqrt{\frac{9A\gamma_1 r^2}{8}}\right).$$

The follower's optimal payoff is:

$$\pi_2^F = \int_0^{n_1^L} \left[\pi_{0,2}^F - \gamma_2 \frac{x_2^F(t)}{2} \right] e^{-rt} dt + \int_{n_1^L}^{n_2^F} \left[\pi_{0,2}^{*F} - \gamma_2 \frac{x_2^F(t)}{2} \right] e^{-rt} dt + \int_{n_2^F}^{\infty} \pi_{1,2}^F e^{-rt} dt, \tag{15}$$

with

$$
\begin{aligned}
\pi^F_{0,2} &= \left(q^F_{0,2}(t)\right)^2 = \left(\frac{1-A}{3}\right)^2, \\
\pi^{*F}_{0,2} &= \left(q^{*F}_{0,2}(t)\right)^2 = \left(\frac{1-2A}{3}\right)^2, \\
\pi^F_{1,2} &= \left(q^F_{1,2}(t)\right)^2 = \left(\frac{1}{3}\right)^2, \\
x^F_2(t) &= \frac{(1-\beta)Are^{-rn^F_2}}{1-e^{-rn^F_2}}e^{rt}, \text{ and} \\
n^F_2 &= -\frac{1}{r}\ln\left(1-(1-\beta)\sqrt{\frac{9A\gamma_2 r^2}{8(1-A)}}\right).
\end{aligned}
$$

The corresponding profit gains from innovation of the two firms are

$$
\begin{aligned}
\Delta\pi^L_1 &= \frac{4A}{9}, \\
\Delta\pi^F_2 &= \frac{4A(1-A)}{9}.
\end{aligned}
$$

As the problem is constructed based on the assumption that $n_2 > n_1$, this result only holds under the parameters requirements, which ensure that $n^F_2 > n^L_1$. Given $\delta < 1$, it is clear that $n^F_2 > n^L_1$ if and only if

$$
\begin{aligned}
\frac{(1-\beta)^2\gamma_2}{1-A} &> \gamma_1, \\
&\text{or} \\
\frac{\gamma_2}{\gamma_1} &> \frac{1-A}{(1-\beta)^2}. \qquad (16)
\end{aligned}
$$

Interpreted directly, expression (16) presents the condition ensuring that the follower firm will still have to devote more time to complete its R&D than the leader even with the benefit of knowledge spillovers. Such a situation is most likely to occur if a notable asymmetry between firms is accompanied by weak knowledge spillovers and a modest degree of innovation.

If (16) does not hold, knowledge spillovers allow the follower to finish its R&D as soon as the leader completes the project and the knowledge spillovers get realized. In such a case the follower benefits from knowledge created by the leader and does not have a stream of lower profits associated with the period when it lags behind in developing the new technology. Such an outcome occurs in case of a smaller asymmetry, strong knowledge spillovers and a small degree of innovation, which ensure that the second-mover advantage (Hoppe (2000)) will be the strongest. Nonetheless, if the roles are exogenously predefined, the leader's optimal R&D duration choice is not affected by the fact that the follower will immediately bring up the new technology after the leader.

In several limit cases we make the following observations:

i)As $\lim\limits_{\substack{A\to 0\\ \beta\to 1}} \frac{1-A}{(1-\beta)^2} = \infty$ and $\lim\limits_{\substack{A\to 1\\ \beta\to 1}} \frac{1-A}{(1-\beta)^2} = \infty$, we conclude that under conditions of strong knowledge spillovers the follower will finish the R&D project at the same time as the leader.

ii) As $\lim\limits_{\substack{A\to 0\\ \beta\to 0}} \frac{1-A}{(1-\beta)^2} = 1$ and $\lim\limits_{\substack{A\to 1\\ \beta\to 0}} \frac{1-A}{(1-\beta)^2} = 0$, we see that with weak knowledge spillovers the feasible leader-follower arrangement is likely to hold for a broad range of asymmetries between firms. Moreover, in case of a large degree of innovation the leader must not be necessarily more efficient in R&D than the follower, due to a very strong incentive to preempt.

Above we have considered the condition for existence of a unique solution of the problem where the leader's role is assigned to firm *1*. If the leader's role is given to firm *2*, then the unique solution condition requires that

$$\frac{\gamma_1}{\gamma_2} > \frac{1-A}{(1-\beta)^2}. \tag{17}$$

In a situation where both (16) and (17) hold both firms will be able to perform the corresponding functions of the leader/follower, and the equilibrium will be the result of the game with endogenous roles.

Remark 1 *We observe that the leader's optimal R&D project duration is the same as the optimal duration of the innovator in the one innovator duopoly:* $n_1^L = n_1^{*1}$*; and the follower's optimal R&D project duration is the same as the optimal duration of the catching up innovator:* $n_2^F = n_2^{*1}$.

Observing results presented in Remark 1 and analyzing firms' payoff functions (14) and (15) we make the following observations. The leader's optimal R&D project duration based on the profit gain from innovation determined by the difference in profits in times $t \in (0, n_1]$ (duopoly with old technology) and times $t \in (n_1, n_2)$, where the leader has a technological advantage. Therefore, the decisive factor for the leader is the fact of technological leadership during a certain period of time. The profit stream generated in time periods after both firms acquire the new technology is not relevant for the leader's R&D duration decisions.

On the other hand, the follower's optimal R&D duration is based on the profit gain determined by the difference in profits in times $t \in (n_1, n_2]$ and $t \in (n_2, \infty)$ (duopoly with new technology). Thus, the follower disregards the profits during the time period when both firms have the old technology and takes into account the fact that it will be in the inferior position for some time, but will benefit from knowledge spillovers.

In other words, the technological leader will behave (regarding the duration of R&D) as if it is the only innovator in the market with the old technology. On the other hand, the follower will behave as the (catching-up) innovator in a duopoly market where the leader produces with a superior technology. This result is consistent with the findings of Katz and Shapiro (1987), who show that

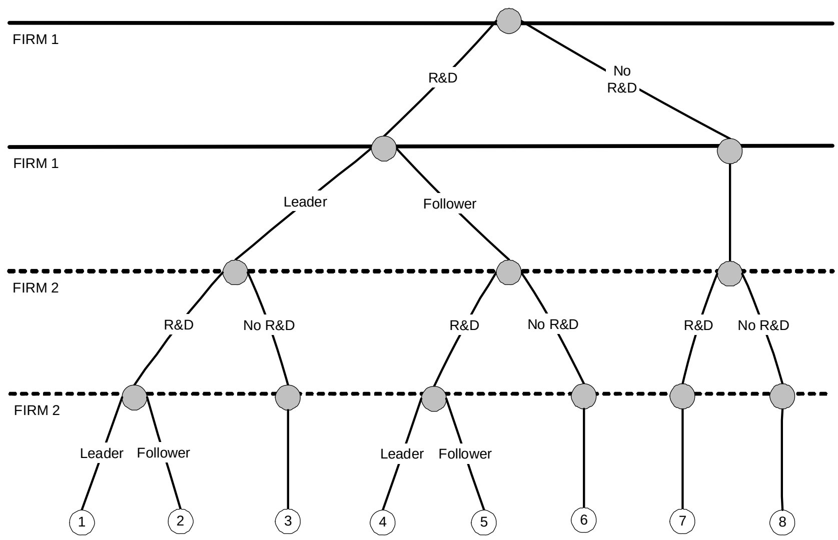

Figure 5: R&D Investment and Timing Decisions Tree.

under conditions of strong imitation risk, the timing of technology adoption by the leader is not influenced by imitation as long as its leader position is preserved.

In Appendix we also present the benchmark case of the two innovators duopoly with the same project duration, which results will be used in later discussions.

8 Strategy Choice in the Endogenous Roles Game

Here we analyze the firms' optimal decisions regarding carrying out the R&D and the mode in which this research must proceed (leader or follower). In the leader mode the firm will complete its R&D first, without benefitting from any positive knowledge spillovers. In the follower mode the firm will complete its innovation as second and will receive an additional benefit in the form of knowledge spillovers generated by the leader.

We derive the firms' payoff for each feasible outcome of different leader/follower decisions in Figure 5, and formulate the firms' optimal strategies. For simplicity purposes we assume that $A < \frac{1}{2}$, so that duopoly is preserved in case one firm does not invest in R&D.

It can be shown (see Appendix A**??**) that firms will finish their R&D projects simultaneously if and only if they have the same R&D efficiencies. Therefore, for asymmetric firms with $\gamma_1 \neq \gamma_2$ the game will not have the outcome with the simultaneous completion of R&D.

In this game we first consider only the decision paths where each firm chooses the leader/follower role compatible with its competitor's role, such as on the paths leading to the outcomes (2), (3), (4), (6), (7), and (8) in Figure 5. On the paths corresponding to the outcomes (1) and (5) both firms choose to be the leader or the follower, respectively. In these cases, one of them must switch its role for the equilibrium to exist. Outcome (1) is a preemption game and in the situation corresponding to outcome (5) a waiting game (war of attrition) takes place.

In this section we derive the firms' payoffs corresponding to the compatible role decisions first, followed by a description of the preemption game and the waiting game in the outcomes with incompatible role choices.

8.1 Compatible Role Decisions

The compatible role decisions of firms produce the equilibria derived in the same way as in the scenarios with exogenous roles shown in Sections 5 to 7.

In the outcome (2) the roles of two firms are clearly defined. Firm *1* is the leader and firm *2* is the follower with:

$$v(2) = \begin{pmatrix} \pi_1^L \\ \pi_2^F \end{pmatrix},$$

where

$$\begin{aligned}
\pi_1^L &= \left(\frac{1-A}{3}\right)^2 \frac{1-e^{-rn_1^L}}{r} - \frac{\gamma_1 A^2 r e^{-rn_1^L}}{2(1-e^{-rn_1^L})} + \left(\frac{1+A}{3}\right)^2 \left(\frac{e^{-rn_1^L} - e^{-rn_2^F}}{r}\right) \\
&\quad + \frac{e^{-rn_2^F}}{r}\left(\frac{1}{3}\right)^2, \\
\pi_2^F &= \left(\frac{1-A}{3}\right)^2 \frac{1-e^{-rn_1^L}}{r} - (1-\beta)^2 \frac{\gamma_2 A^2 r e^{-rn_2^F}}{2(1-e^{-rn_2^F})} + \left(\frac{1-2A}{3}\right)^2 \left(\frac{e^{-rn_1^L} - e^{-rn_2^F}}{r}\right) \\
&\quad + \frac{e^{-rn_2^F}}{r}\left(\frac{1}{3}\right)^2, \\
\text{with } n_1^L &= -\frac{1}{r}\ln\left(1 - \sqrt{\frac{9\gamma_1 A r^2}{8}}\right), \\
n_2^F &= -\frac{1}{r}\ln\left(1 - (1-\beta)\sqrt{\frac{9\gamma_2 A r^2}{8(1-A)}}\right).
\end{aligned}$$

The outcome (3) presents the situation when only firm *1* invests in R&D:

$$v(3) = \begin{pmatrix} \pi_1^{*1} \\ \pi_2^{*1} \end{pmatrix},$$

where

$$
\begin{aligned}
\pi_1^{*1} &= \left(\frac{1-A}{3}\right)^2 \left(\frac{1-e^{-rn_1^{*1}}}{r}\right) + \left(\frac{1+A}{3}\right)^2 \frac{e^{-rn_1^{*1}}}{r}, \\
\pi_2^{*1} &= \left(\frac{1-A}{3}\right)^2 \left(\frac{1-e^{-rn_1^{*1}}}{r}\right) + \left(\frac{1-2A}{3}\right)^2 \frac{e^{-rn_1^{*1}}}{r}, \\
\text{with } n_1^{*1} &= -\frac{1}{r}\ln(1-\sqrt{\frac{9\gamma_1 A r^2}{8}}).
\end{aligned}
$$

In the outcome (4) the roles are a reverse of the outcome (2) with firm *1* being the follower and firm *2* being the leader:

$$
v(4) = \begin{pmatrix} \pi_1^F \\ \pi_2^L \end{pmatrix},
$$

where

$$
\begin{aligned}
\pi_2^L &= \left(\frac{1-A}{3}\right)^2 \frac{1-e^{-rn_2^L}}{r} - \frac{\gamma_2 A^2 r e^{-rn_2^L}}{2(1-e^{-rn_2^L})} + \left(\frac{1+A}{3}\right)^2 \left(\frac{e^{-rn_2^L}-e^{-rn_1^F}}{r}\right) \\
&\quad + \frac{e^{-rn_1^F}}{r}\left(\frac{1}{3}\right)^2, \\
\pi_1^F &= \left(\frac{1-A}{3}\right)^2 \frac{1-e^{-rn_2^L}}{r} - (1-\beta)^2 \frac{\gamma_1 A^2 r e^{-rn_1^F}}{2(1-e^{-rn_1^F})} + \left(\frac{1-2A}{3}\right)^2 \left(\frac{e^{-rn_2^L}-e^{-rn_1^F}}{r}\right) \\
&\quad + \frac{e^{-rn_1^F}}{r}\left(\frac{1}{3}\right)^2, \\
\text{with } n_2^L &= -\frac{1}{r}\ln(1-\sqrt{\frac{9\gamma_2 A r^2}{8}}), \\
n_1^F &= -\frac{1}{r}\ln(1-(1-\beta)\sqrt{\frac{9\gamma_1 A r^2}{8(1-A)}}).
\end{aligned}
$$

The outcomes (7) and (8) are straightforward and are specified as:

$$
v(7) = \begin{pmatrix} \pi_1^{*2} \\ \pi_2^{*2} \end{pmatrix}
$$

where

$$
\begin{aligned}
\pi_2^{*2} &= \left(\frac{1-A}{3}\right)^2 \left(\frac{1-e^{-rn_2^{*2}}}{r}\right) + \left(\frac{1+A}{3}\right)^2 \frac{e^{-rn_2^{*2}}}{r}, \\
\pi_1^{*2} &= \left(\frac{1-A}{3}\right)^2 \left(\frac{1-e^{-rn_2^{*2}}}{r}\right) + \left(\frac{1-2A}{3}\right)^2 \frac{e^{-rn_2^{*2}}}{r}, \\
\text{with } n_2^{*2} &= -\frac{1}{r}\ln(1-\sqrt{\frac{9\gamma_2 A r^2}{8}}).
\end{aligned}
$$

In the outcome (8) both firms decide not to invest in R&D resulting in the standard Cournot equilibrium with:

$$\begin{aligned} q_1 &= q_2 = \frac{1-A}{3}, \\ \pi_1 &= \pi_2 = \left(\frac{1-A}{3}\right)^2 \left(\frac{1}{r}\right), \end{aligned}$$

and

$$v(8) = \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix}.$$

8.2 Preemption Game for Leadership

The decision path leading to the outcome (1) in the game corresponds to the scenario where both firms decide to invest in R&D and want to be the first to finish research. The profit-maximizing project durations of the two firms are:

$$\begin{aligned} n_1^L &= -\frac{1}{r} \ln \left(1 - \sqrt{\frac{9A\gamma_1 r^2}{8}}\right), \text{ and} \\ n_2^L &= -\frac{1}{r} \ln \left(1 - \sqrt{\frac{9A\gamma_2 r^2}{8}}\right). \end{aligned}$$

If $\gamma_1 < \gamma_2$, then $n_1^L < n_2^L$. Nonetheless firm *2* may attempt to preempt firm *1* by finishing its project a bit earlier as long as the profits obtained in the leader position are higher than the profits of the follower or abandoning the R&D. Firm *1* will follow the same logic and will definitely react to firm *2*'s preemption attempts. Such a preemption game is illustrated in Figure 6.

As it was mentioned in Remark 1 above, the leader's project duration in the two-innovator duopoly is the same as the duration in the case of one innovator ($n_i^L = n_i^{*i}$). The after-innovation profits stream of the only innovator is higher ($\pi_i^{*i} > \pi_i^L$), because the innovating firm has a technology advantage. Thus, a deviation of one firm from the R&D leadership decision (decision to become a follower or abandon R&D) will not change the other firm's decision about investing in R&D (it will always innovate).

Let us consider some particular value of the R&D project duration $\bar{n}$ such that $\bar{n} < n_1^L$ and $\bar{n} \leq n_2^L$. Thus, $\bar{n}$ represents the project duration decision by which one of the firms tries to preempt the other. Each firm can decrease the value of $\bar{n}$ until the profits from preempting become lower than the profits from becoming a follower or giving up the R&D.

Here the points of interest are the values of $\bar{n}$ where either firm *1* has:

$$\pi_1^L\big|_{\bar{n}} = \pi_1^F\big|_{n_2^L}, \text{ if } \pi_1^F \geq \pi_1^{*2}\big|_{n_2^L} \text{ and } \pi_1^L\big|_{\bar{n}} = \pi_1^{*2}\big|_{n_2^L}, \text{ if } \pi_1^{*2} > \pi_1^F\big|_{n_2^L} \quad (18)$$

or firm *2* has:

$$\pi_2^L\big|_{\bar{n}} = \pi_2^F\big|_{n_1^L}, \text{ if } \pi_2^F \geq \pi_2^{*1}\big|_{n_1^L} \text{ and } \pi_2^L\big|_{\bar{n}} = \pi_2^{*1}\big|_{n_1^L}, \text{ if } \pi_2^{*1} > \pi_2^F\big|_{n_1^L}. \quad (19)$$

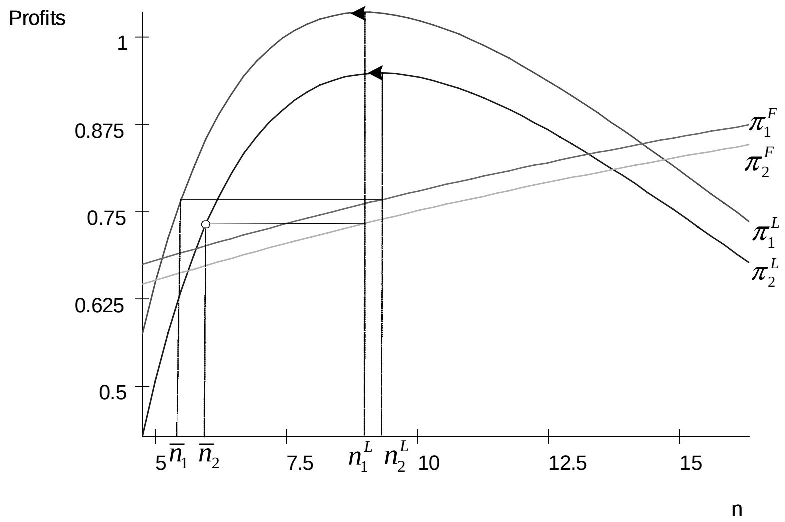

Figure 6: Preeemption Game Diagram ($A = 0.49$, $\gamma_1 = 95$, $\gamma_2 = 100$, $\beta = 0.01$, $r = 5\%$).

The equations in (18) represent the conditions where firm *1* will deviate from its leader's decision and will decide to become the follower or not to invest in the new technology at all. The equations in (19) represent the same conditions for firm *2*.

To derive which of the two firms will be able to preempt its competitor we need to explicitly calculate the critical values of duration where the firm will give up its leader role. These critical values are:

$$\bar{n}_1(n_2^L) = \max(\arg(\pi_1^L\big|_{\bar{n}} = \pi_1^F\big|_{n_2^L}), \arg(\pi_1^L\big|_{\bar{n}} = \pi_1^{*2}\big|_{n_2^L})), \text{ and} \tag{20}$$

$$\bar{n}_2(n_1^L) = \max(\arg(\pi_2^L\big|_{\bar{n}} = \pi_2^F\big|_{n_1^L}), \arg(\pi_2^L\big|_{\bar{n}} = \pi_2^{*1}\big|_{n_1^L})). \tag{21}$$

Deriving these critical values algebraically results in very complex and non-tractable expressions. Thus, in this study we will use numerical simulations to illustrate some scenarios.

The payoffs corresponding to the preemption outcomes are:

$$v(1)|_{\pi_2^{*1} > \pi_2^F} = \begin{pmatrix} \pi_1^{*1}(\bar{n}_2) \\ \pi_2^{*1}(\bar{n}_2) \end{pmatrix}, \text{ and } v(1)|_{\pi_2^F \geq \pi_2^{*1}} = \begin{pmatrix} \pi_1^L(\bar{n}_2) \\ \pi_2^F(\bar{n}_2) \end{pmatrix}, \text{ if } \bar{n}_1 < \bar{n}_2, \tag{22}$$

i.e. when firm *2* deviates first, and

$$v(1)|_{\pi_1^{*2} > \pi_1^F} = \begin{pmatrix} \pi_1^{*2}(\bar{n}_1) \\ \pi_2^{*2}(\bar{n}_1) \end{pmatrix}, \text{ and } v(1)|_{\pi_1^F \geq \pi_1^{*2}} = \begin{pmatrix} \pi_1^F(\bar{n}_1) \\ \pi_2^L(\bar{n}_1) \end{pmatrix}, \text{ if } \bar{n}_2 < \bar{n}_1, \tag{23}$$

i.e. when firm *1* deviates first.

The first set of the preemption payoffs corresponds to the case where firm *2* will give up its leader role first (the case shown in Figure 6) and the second set corresponds to the case of firm *1* abandoning its leadership. These outcomes are different from those obtained in the situations with exogenously defined roles.

In the precommitment scenarios with exogenous roles the optimal duration of R&D is determined by the trade-off between the extra profit gain from innovation and the additional R&D cost from finishing the project sooner. Firms do not consider strategic effects and the incentives for preemption.

In the endogenous roles scenario the optimal project durations are set in a way that the follower is not able to preempt the leader. In such a situation the leader makes its decisions based on the followers incentives to preempt (if the follower has such incentives). By speeding up its R&D the leader gives up a part of its profits to cancel out the preemption incentives of the competitor. As a result of the preemption game, the winner ends up with lower payoffs than in the corresponding precommitment case.

Figure 6 illustrates the situation where firm *1* will become effectively the leader and sets its R&D duration to $\bar{n}_2$ (going for an even shorter R&D project will further decrease firm *1*'s profits). Firm *2* will accept the follower's role and sets its R&D duration so that it maximizes its profits given the leader's duration $\bar{n}_2$. Correspondingly, in the situation leading to the second set of payoffs firm *2* will become the leader with duration $\bar{n}_1$.

Unlike in the exogenous roles scenarios, the leader's R&D duration decision directly depends on the follower's decision determined by conditions (20) and (21), respectively.

In general the set of firms' payoffs resulting from the preemption game will be denoted:

$$v(1) = \begin{pmatrix} \pi_1(\bar{n}) \\ \pi_2(\bar{n}) \end{pmatrix},$$

where $\bar{n}$ is determined by conditions in (18) and (19).

8.3 War of Attrition to Become the Follower

The second game's outcome with endogenous roles is the outcome (5) where both firms prefer to become the follower. In such a situation the firm wants to benefit from knowledge spillovers, thus trying to be the second to complete the project. If both firms are to choose to be a follower we must consider

$$\begin{aligned} n_2^F &= -\frac{1}{r}\ln\left(1-(1-\beta)\sqrt{\frac{9A\gamma_1 r^2}{8(1-A)}}\right), \\ n_2^F &= -\frac{1}{r}\ln\left(1-(1-\beta)\sqrt{\frac{9A\gamma_2 r^2}{8(1-A)}}\right). \end{aligned}$$

Firm *2* with a larger total R&D effort ($\gamma_2 > \gamma_1$) has $n_2^F > n_1^F$. Yet firm *1* may have some room for trying to "outwait" firm *2* in order to capture the follower

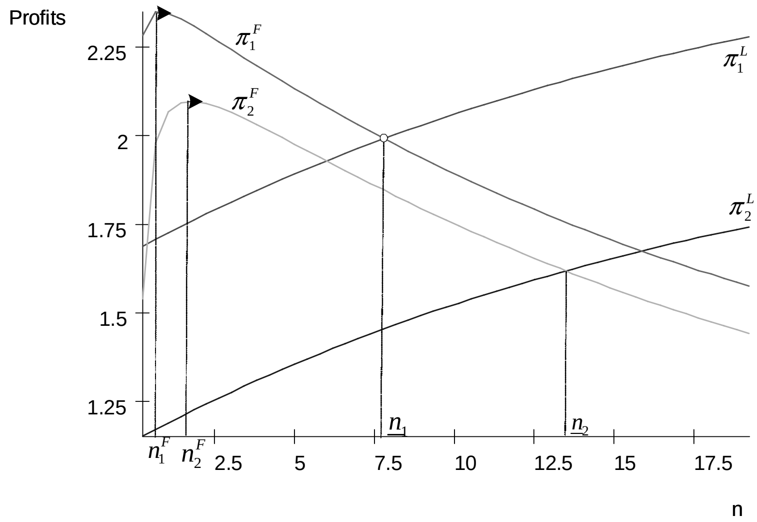

Figure 7: Attrition Game Diagram ($r = 5\%$, $A = 0.2$, $\gamma_1 = 150$, $\gamma_2 = 250$, $\beta = 0.8$).

position or make it abandon its R&D completely. In case the attrition game takes place, we conduct comparisons of firms' profits in different situations to figure out the outcomes.

Let us define $\underline{n}$ such that $\underline{n} > n_1^F$ and $\underline{n} > n_2^F$. Duration choice $\underline{n}$ represents the decision where one firm tries to extend the time to complete its R&D so that it will become the follower. As in the preemption game, the other firm may extend its project's duration as well in order to outwait its opponent (as shown in Figure 7).

It can be easily seen from (14) that the leader's payoff is an increasing and concave function in terms of the follower's optimal R&D project duration. Thus, the conditions where the firm *1* will give up the follower role in the attrition game are given by (note that $\pi_1 > \pi_1^{*2}$):

$$\pi_1^F\big|_{\underline{n}} = \pi_1^L\big|_{\underline{n}}, \text{ if } \pi_1^L \geq \pi_1^{*2} \text{ and } \pi_1^F\big|_{\underline{n}} = \pi_1^{*2}\big|_{\underline{n}}, \text{ if } \pi_1^{*2} > \pi_1^L. \tag{24}$$

Correspondingly for firm *2* these conditions are:

$$\pi_2^F\big|_{\underline{n}} = \pi_2^L\big|_{\underline{n}}, \text{ if } \pi_2^L \geq \pi_2^{*1} \text{ and } \pi_2^F\big|_{\underline{n}} = \pi_2^{*1}\big|_{\underline{n}}, \text{ if } \pi_2^{*1} > \pi_2^L. \tag{25}$$

Similarly to the previous case, here different conditions determine different scenarios and, thus, different solutions of the game. But, unlike in the preemption scenario, deviation of one firm from its decision to invest in R&D as the follower can lead to a change in the other firm's decision as well. This happens when one firm decides to abandon it R&D completely. In this case it becomes impossible for the other firm to be the follower, so it must invest in R&D is the only innovator or abandon its research too.

Let us define the critical values:

$$\begin{aligned} \underline{n}_1(n_2^L) &= \min(\arg(\pi_1^F = \pi_1^L\big|_{\underline{n}}), \arg(\pi_1^F = \pi_1^{*2}\big|_{\underline{n}})), \text{ and} \\ \underline{n}_2(n_1^L) &= \min(\arg(\pi_2^F = \pi_2^L\big|_{\underline{n}}), \arg(\pi_2^F = \pi_2^{*1}\big|_{\underline{n}})). \end{aligned}$$

Consider, for example, the situation when $\underline{n}_1 < \underline{n}_2$ (as illustrated in Figure 7). Here firm *1* will be the first to deviate from the follower role. If $\pi_1^L \geq \pi_1^{*2}$, then firm *1* will accept the leader role, with firm *2* as the follower and with firm *1* setting its R&D duration equal to $\underline{n}_1$. On the other hand, observing $\pi_1^{*2} > \pi_1^L$ firm *1* will abandon its R&D. This will make firm *2* to reconsider its own R&D decision. As becoming the follower is no longer feasible, there are two other options left: firm *2* may pursue the innovation alone (if $\pi_2^{*2} \geq \pi_2$), or abandon its R&D as well ($\pi_2^{*2} < \pi_2$). The similar logical sequence is used to determine firms' actions if $\underline{n}_2 < \underline{n}_1$.

The corresponding payoffs of the attrition game:

$$v(5)\big|_{\pi_1^L \geq \pi_1^{*2}} = \begin{pmatrix} \pi_1^L(\underline{n}_1) \\ \pi_2^F(\underline{n}_1) \end{pmatrix}, \quad v(5)\big|_{\substack{\pi_1^{*2} > \pi_1^L \\ \pi_2^{*2} \geq \pi_2}} = \begin{pmatrix} \pi_1^{*2}(\underline{n}_1) \\ \pi_2^{*2}(\underline{n}_1) \end{pmatrix},$$

$$\text{and} \quad v(5)\big|_{\substack{\pi_1^{*2} > \pi_1^L \\ \pi_2 > \pi_2^{*2}}} = \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix}, \text{ if } \underline{n}_1 < \underline{n}_2,$$

or

$$v(5)\big|_{\pi_1^L \geq \pi_1^{*2}} = \begin{pmatrix} \pi_1^F(\underline{n}_2) \\ \pi_2^L(\underline{n}_2) \end{pmatrix}, \quad v(5)\big|_{\substack{\pi_1^{*2} > \pi_1^L \\ \pi_2^{*2} \geq \pi_2}} = \begin{pmatrix} \pi_1^{*2}(\underline{n}_2) \\ \pi_2^{*2}(\underline{n}_2) \end{pmatrix},$$

$$\text{and} \quad v(5)\big|_{\substack{\pi_1^{*2} > \pi_1^L \\ \pi_2 > \pi_2^{*2}}} = \begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix}, \text{ if } \underline{n}_2 < \underline{n}_1.$$

Like in the preemption scenario we will use simplified notations for the firms' payoffs obtained in the game of attrition:

$$v(5)\big|_{\pi_1^L \geq \pi_1^{*2}} = \begin{pmatrix} \pi_1(\underline{n}) \\ \pi_2(\underline{n}) \end{pmatrix}$$

Firms' payoffs in the war of attrition are also different from the corresponding precommitment cases. The winner (follower) has to cancel out the other firm's incentives to extent its R&D (second mover advantage), thus it has to give up part of its profits. But, unlike in the preemption case, where the deviating firm has the same payoff as in the original position, in the war of attrition the firm, which deviates, ends up with the higher payoffs than in the original position. The looser (in this case the leader) receives some extra benefits in the form of a

Table 1: Process Innovation Game

Firm 2 / Firm 1	Leader	Follower	No R&D
Leader	$\pi_2(\bar{n})$ $\pi_1(\bar{n})$	π_2^F π_1^L	π_2^{*1} π_1^{*1}
Follower	π_2^L π_1^F	$\pi_2(\underline{n})$ $\pi_1(\underline{n})$	X
No R&D	π_2^{*2} π_1^{*2}	X	π_2 π_1

Table 2: Payoff matrix for the game with preemption equilibrium

Firm 2 / **Firm 1**	Leader	Follower	No R&D
Leader	$(0.73)\|_{\pi_2^F(\bar{n}_2)}$ $(0.85)\|_{\pi_1^L(\bar{n}_2)}$	(0.73) (1.04)	0.21 (3.36)
Follower	(0.95) 0.77	X	X
No R&D	(3.32) 0.22	X	0.58 0.58

more extended period of the technological advantage. This happens due to the fact that, unlike in the technology adoption literature mentioned in this study, in our model adopting earlier does not mean that the leader will have a less advanced technology than the follower.

8.4 Payoff Matrix

Thus, the game depicted in Figure 5 has seven feasible outcomes: (1), (2), (3), (4), (5), (7), (8). The corresponding game in the strategic form is presented in Table 1.

From the discussion above we see that, depending on values of the model parameters there can be different (unique and not unique) equilibria, and, thus, corresponding optimal strategies. Below we present some numerical examples identifying some interesting cases.

8.5 Example 1: Preemption Equilibrium

Let us consider the first possible scenario for the game with the following set of parameters: $A = 0.49$, $\gamma_1 = 95$, $\gamma_2 = 100$, $\beta = 0.01$, $r = 5\%$, which is also the same set used for the illustration in Figure 6. Here two firms face an innovation with a small difference in their R&D efficiencies and very weak knowledge spillovers. The calculated payoffs matrix is presented in Table 2.

The game produces two equilibria, both of which imply that firm *1* will become the leader and firm *2* will become the follower. One of this equilibria

Table 3: Payoff matrix for the game with attrition equilibrium

Firm 1 \ **Firm 2**	Leader	Follower	No R&D
Leader	X	(2.17) 1.42	0.98 (2.68)
Follower	1.20 (2.28)	$(1.73)\|_{\pi_2^F(\underline{n}_1)}$ $(1.79)\|_{\pi_1^L(\underline{n}_1)}$	X
No R&D	(2.53) 1.03	X	1.42 1.42

is equivalent to the precommitment situation and another is produced by a preemption game. In both equilibria the payoff of the follower is the same and the leader has a smaller payoff in the preemption equilibrium.

If the roles of the firms are determined endogenously, this particular parameter setting will produce one Nash equilibrium, which is the preemption equilibrium. In order to preempt its opponent firm *1* will have to speed up development resulting in a leader's payoff decrease of 18%.

8.6 Example 2: Attrition Equilibrium

The next parameters set is: $r = 5\%$, $A = 0.2$, $\gamma_1 = 150$, $\gamma_2 = 250$, $\beta = 0.8$, which corresponds to the illustration in Figure 7. In this scenario we see two firms with notable difference in the R&D production efficiency, which consider investing in a relatively small innovation under conditions of strong knowledge spillovers. Numerical calculations give us the payoffs matrix as presented in Table 3.

This game has a unique Nash equilibrium, corresponding to the outcome of a war of attrition. As a result of the waiting game the follower's payoff decreased by 25% and the leader's payoff increased by 26%. In case of precommitment strategies, the game would have two leader/follower equilibria and firms would face a coordination problem.

8.7 Example 3: No Preemption/Attrition Scenario

To find out the effect of larger asymmetry between firms we consider the following parameter set: $A = 0.49$, $\gamma_1 = 50$, $\gamma_2 = 100$, $\beta = 0.01$, $r = 5\%$, which is different from that in Example 1 only by a larger degree of asymmetry between firms. From Figures 8 and 9 we can clearly see that neither preemption nor war of attrition are possible.

A very large advantage of a more R&D efficient firm eliminates any preemption incentives for the less R&D efficient firm. On the other hand, the knowledge spillover is not strong enough to create the incentive to engage in a waiting game. Thus, it is obvious, that the only outcomes feasible in this game

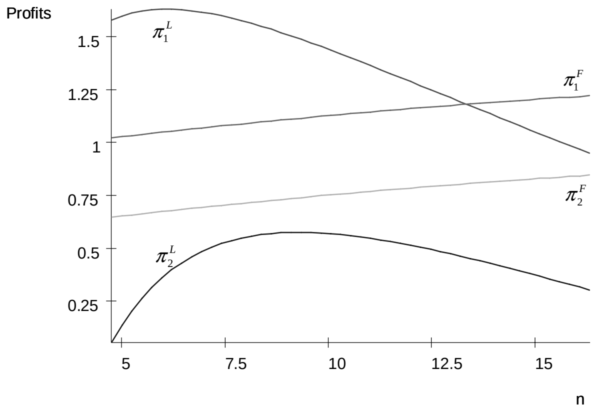

Figure 8: No preemption scenario ($A = 0.49$, $\gamma_1 = 50$, $\gamma_2 = 100$, $\beta = 0.01$, $r = 5\%$)

are those with compatible strategies, which provide the payoffs equivalent to the payoffs in equilibria with endogenous roles.

9 Conclusions

This study highlights the aspects distinguishing the model of competitive R&D with time to build from the model of technology adoption. First of all the model presented here emphasizes such property of R&D as a positive correlation between the degree of innovation and the amount of knowledge gain necessary to achieve the breakthrough. Our study show that for more substantial process innovations a marginal increase in total R&D effort necessary to complete the innovation requires more additional time than in the case of a smaller innovation.

When analyzing firms' competitive strategies we distinguish two types of equilibria: one type with exogenous firm roles prescribing beforehand which firm will become the leader and finish the R&D process first The second type implies that it is not established beforehand which of two firms will finish its R&D first. In the latter case the firm roles are endogenous.

It is observed that in the exogenously defined leader/follower duopoly the optimal R&D project duration of the leader is the same as the optimal duration of the innovator in the duopoly, where only one firm does R&D; and the optimal R&D project duration of the follower is the same as the optimal duration of the catching up innovator, facing the opponent who already possesses the new technology. While making the R&D duration decisions the leader disregards the profits after both firms acquire the new technology, concentrating solely on the fact of technological leadership. The follower, in its turn, disregards

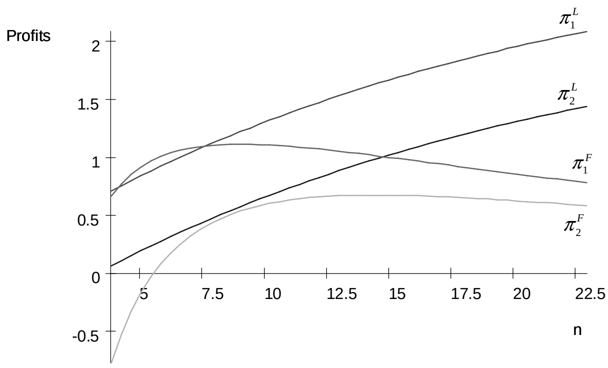

Figure 9: No attrition scenario ($A = 0.49$, $\gamma_1 = 50$, $\gamma_2 = 100$, $\beta = 0.01$, $r = 5\%$)

the profits where both firms have the old technology and takes into account its technological handicap resulting from slower innovation. This emphasizes the importance of strategic effects for the firms' R&D duration decisions.

We find that with weak knowledge spillovers the feasible leader-follower arrangement is likely to hold for a broad range of asymmetries between firms. Moreover, in case of a large degree of innovation the leader must not be necessarily more efficient in R&D than the follower, due to a very strong incentive to preempt.

With a reasonably small asymmetry in R&D efficiency between firms, the game with endogenous roles can have preemption or attrition equilibria depending on the parameter constellations. But if one firm has strong advantage in R&D efficiency over another, preemption game and/or attrition games become not feasible, thus the more R&D efficient firm is capable of securing its technological leadership.

10 APPENDIX

10.1 Two Innovators, Same Project Duration Duopoly Case

Consider firms' competitive R&D and production strategies provided that to complete the project they have different R&D efficiencies, but must carry out their R&D during the same period of time. In this case none of the firms is able to make use of knowledge spillovers occurring after completion of R&D. We, thus, assume that firms have $\gamma_1 \neq \gamma_2$, and $n_1 = n_2 = n$.

Solving the firm i's optimal time to build problem we obtain the optimal payoffs of:

$$\pi_i^* = \int_0^{n_i^*} \left[\pi_{0,i}^* - \gamma \frac{x_i^*(t)}{2} \right] e^{-rt} dt + \int_{n_i^*}^{\infty} \pi_{1,i}^* e^{-rt} dt, \quad i,j = 1,2, \quad i \neq j,$$

and where:

$$\begin{aligned} \pi_{0,i}^* &= \left(q_{0,i}^*(t)\right)^2 = \left(\frac{1-A}{3}\right)^2, \\ \pi_{1,i}^* &= \left(q_{1,i}^*(t)\right)^2 = \left(\frac{1}{3}\right)^2, \\ x_i^*(t) &= \frac{Are^{-rn_i^*}}{1-e^{-rn_i^*}} e^{rt}, \text{ and} \\ n_i^* &= -\frac{1}{r} \ln \left(1 - \sqrt{\frac{9A\gamma_i r^2}{8}}\right) \\ \text{for } i,j &= 1,2,\ i \neq j. \end{aligned}$$

It is clearly visible that if $\gamma_i \neq \gamma_j$, the model does not provide an equilibrium solution with $n_i^* = n_j^*$. Therefore we conclude that it is not optimal for the two firms with different R&D efficiency to choose the same R&D project durations.

11 References

d'Aspremont, C. and A. Jacquemin (1988), Cooperative and Noncooperative R&D in Duopoly with Spillovers, *The American Economic Review*, Vol. 78, pp. 1133-1137.

Bercovitz, J., J. de Figueiredo, and D. Teece (1997), Firm Capabilities and Managerial Decision Making: A Theory of Innovation Biases, in R. Garud, P. Nayyar, and Z. Shapira, (eds.), *Technological Innovation. Oversights and Foresights*, Cambridge University Press, Cambridge.

Dutta P. (1997), Optimal Management of an R&D Budget, *Journal of Economic Dynamics and Control*, Vol. 21, pp. 575-602.

Dutta, P., S. Lach, and A. Rustichini (1995), Better Late than Early: Vertical Differentiation in the Adoption of a New Technology, *Journal of Economics and Management Strategy*, Vol. 4, pp. 563–589.

Fudenberg, D. and J. Tirole (1985), Preemption and Rent Equalization in the Adoption of New Technology, *The Review of Economic Studies*, Vol. 52, pp. 383–401.

Grossman, G. and E. Helpman (1990), Trade, Innovation, and Growth, *American Economic Review*, Vol. 80, pp. 86-91.

Grossman, G. and C. Shapiro (1986), Optimal Dynamic R&D Programs, *RAND Journal of Economics*, Vol. 17, pp. 581-593.

Hoppe, H. (2000), Second-Mover Advantages in the Strategic Adoption of New Technology under Uncertainty, *International Journal of Industrial Organization*, Vol. 18, pp. 115-138.

Hoppe, H. and U. Lehmann-Grube (2001), Second-mover Advantages in Dynamic Quality Competition, *Journal of Economics and Management Strategy*, Vol. 10, pp. 419-433.

Hoppe, H. and U. Lehmann-Grube (2005), Innovation Timing Games: A General Framework with Applications, *Journal of Economic Theory*, Vol. 121, pp. 30-50.

Katz, M. and C. Shapiro (1987), R&D Rivalry with Licensing and Imitation, *The American Economic Review*, Vol. 77, pp. 402-420.

Marcyk, G. (2002), Silicon Technology: Scaling for the Second Half of the Decade, Logic Technology Development, Intel Corporation (http://intel.com/labs/).

Pacheco-de-Almedia, G. and P. Zemsky (2003), The Effect of Time-to-Build on Strategic Investment under Uncertainty, *RAND Journal of Economics*, Vol. 34, pp. 166-182.

Petit, M. L. and B. Tolwinski (1999), R&D Cooperation or Competition?, *European Economic Review*, Vol. 43, pp. 185-208.

Reinganum, J. (1981), Dynamic Games of Innovation, *Journal of Economic Theory*, Vol. 25, pp. 21-41.

Scherer, F (1967), Research and development resource allocation under rivalry, *The Quarterly Journal of Economics*, Vol. 81, pp. 359-394.

Suzumura, K. (1992), Cooperative and Noncooperative R&D in an Oligopoly with Spillovers, *The American Economic Review*, Vol. 82, pp. 1307-1320.

Teece, D. (1996), Firm Organization, Industrial Structure, and Technological Innovation, *Journal of Economic Behavior and Organization*, Vol. 31, pp. 193-224.

Van Biesebroek, J. (2003), Productivity Dynamics with Technology Choice: An Application to Automobile Assembly, *The Review of Economic Studies*, Vol. 70, pp. 167-198.