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By Joseph Plasmans, Jorge Fornero, Tomasz Michalak

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A microfounded sectoral model for open economies

Joseph Plasmans*
University of Antwerp and Tilburg University

Jorge Fornero
University of Antwerp

Tomasz Michalak
Universities of Antwerp and Liverpool

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Abstract

In this paper we derive a microfounded macro New Keynesian model for open economies, be them large or small. We consider habit formation in consumption, sectoral linkages for tradable and non-tradable goods, capital stock investments with variable capital utilization, domestic and foreign governments, imperfect (exchange rate) pass-through in import prices and incomplete international financial markets. Sticky nominal prices and wages are modeled in Calvo and Taylor staggered ways. The model economy is composed of a continuum of infinitely-lived consumers and producers of final and intermediate goods. We provide a very general log-linearization method, from which we can easily obtain various special cases, as trend inflation or steady-state log-linearizations.

Numerical simulations of the two-country sectoral model are provided for a relatively large number of structural shocks as domestic and foreign productivity shocks in final tradables and non-tradables, money demand shocks and a shock in the exchange rate. Such a model is well suited for monetary policy analysis at the international level and risk analysis.

Keywords: New Keynesian open economy model, tradable and non-tradable sectors, final and intermediate goods, log-linearization.

JEL codes: E31, D21, F41, P24.

1 Introduction

During the recent years the theoretical and empirical research in New Keynesian (NK) macroeconomics has been extended steadily and produced a whole new series of results and insights about the workings of the macroeconomy. Essential starting point of the NK approach is the explicit derivation of macroeconomic relations by their microeconomic foundations, in particular the inclusion of optimizing consumers and producers. It shares this principle with New Classical macroeconomics, although it differs from the latter by considering various imperfections in the goods and labor markets that contain an amount of flexibility, especially in the short run.

Recently, much interest is also directed at the modeling and testing of the effects and interactions produced by the foreign sector, e.g. rigidities of import and export pricing may be of significant importance to an open economy. Moreover, fluctuations in import prices of important intermediate goods such as oil and steel will have strong effects on domestic firms of an open economy. In that perspective, the exchange rate will play an important role in the transmission of such price fluctuations to the domestic economy. NK models with a worked out foreign sector are often referred to as *New Open Economy Macroeconomics* (NOEM) models.

This paper derives *a microfounded macroeconomic NK model for open economies*, which tries to extend the NK/NOEM literature in several respects. We provide a detailed modeling of the consumers' and producers' decisions. In particular, firms in this model produce two types of goods: *final* and *intermediate* goods. Final goods can be consumption and capital (investment) goods. Moreover, both types of goods can be either *tradable* or

*Corresponding author, Department of Economics, University of Antwerp, Prinsstraat 13, B2000 Antwerp; Phone: +32 (0)3 220 4149, Fax: +32 (0)3 220 4585. Authors' emails: joseph.plasmans@ua.ac.be, jorgealberto.fornero@ua.ac.be and tomasz.michalak@ua.ac.be. The authors thank Raf Wouters (National Bank of Belgium - NBB), Bas van Aarle (University of Maastricht), Luca Guerrieri (Federal Reserve Board), Eva Ortega (Bank of Canada), Vicente Tuesta (Reserve Bank of Peru) and Jean-Guillaume Sahuc (Banque de France) for interesting comments on a previous version of the paper. They also thank seminar participants at the International Conference on 'Price and Wage Rigidities in an Open Economy", NBB, 12 and 13 October 2006, for interesting discussions. Nevertheless, the authors are solely responsible for any remaining shortcomings.

non-tradable depending on whether they can be traded internationally.[1] Firms in every sector are assumed to be characterized by nested CES production functions whose arguments are technology, labor force, domestic tradable and non-tradable capital goods and intermediate goods and imported foreign capital goods and intermediate goods. It is assumed that intermediate goods producers do not use other intermediate goods as their inputs, but notice that capital goods producers may use all kinds of inputs. Each firm is assumed to possess some price-setting power on relevant markets, reflecting *monopolistic competition*. Consumers are assumed to purchase a bundle of domestically produced tradable and non-tradable final goods and imported final goods and are assumed to act as monopolistic suppliers of labor services (with sticky wages). Moreover, consumers own capital that is rented out to firms and the consumer budget constraint is subject to variable capital utilization costs. Consumption allocation is assumed to be shaped by *habit formation*. Financial markets are assumed to be complete domestically and incomplete internationally (see Benigno (2001)).[2] Domestic consumers' and firms' decisions, and financial markets build the model of the home economy. Altogether, we distinguish 21 (final and intermediate) goods markets with which domestic firms are confronted. The foreign economy is modeled in a parallel manner so that import/export prices and quantities and other relevant variables are endogenized in our approach. In particular, it makes the model more flexible since both small and large economy settings can be studied in this framework.

Such an extensive modeling of intermediate goods sectors is important especially in the context of exchange rate policies. Dellas (2005) points out that the presence of intermediate goods has vital consequences for the ability of monetary authorities to manipulate nominal exchange rates. When there is no production interdependence between countries (i.e. only consumption goods are traded) changes in a nominal exchange rate do not affect production costs. However, when there is trade in intermediate and capital goods as in the real world, an exchange rate depreciation/appreciation has adverse direct effects on the cost of domestic production. Consequently, it makes the exchange rate instrument less useful.[3]

An important feature of the model is the method of log-linearization. Almost the complete existing NOEM literature approximates the model by log-linearization around steady state values of variables. These studies usually assume that the long-run anchor for inflation expectation is zero, which is a very convenient but counterfactual assumption. Long-run inflation expectations should converge to the natural inflation of monetary policy, where there are no restrictions. This natural inflation is a (relatively small) positive rate in most modern industrialized economies, but typically time-varying. Consequently, some recent studies departure from the zero steady state inflation rate, e.g. Ascari (2004), who considers trend inflation, and Kozicki and Tinsley (2002) and Bakhshi *et al.* (2003), who also provide derivations of an NK Philips curve with a non-zero steady state inflation rate. We propose a more general approach of log-linearization around *time-dependent paths* of economic variables. Starting from this general formulation, we introduce several restrictions in increasing order of limitation. First, we assume that time-dependent variables around which we log-linearize satisfy the first order conditions, where there are no restrictions or rigidities on the variables. Such time-dependent variables are called *natural* (or *flexible*) *values*. Second, we restrict that all variables in natural values fluctuate at the same time-dependent rate. Such paths of variables are called *(time-)varying rate paths*. Third, we further restrict our attention to constant rate paths, i.e. to (constant) *trend paths* of (natural) variables (see, for example, Ascari's (2004) constant *trend inflation*). Finally, the conventional *steady state* is the simplest special case of all the above log-linearizations, where the constancy of natural values is assumed. Moreover, if also the equilibrium conditions are satisfied, time-dependent paths of variables are called *time-dependent equilibrium paths*. In this paper we log-linearize around time-dependent paths of variables and derive the above mentioned special cases from them. The log-linearization is performed in detail in the appendix.

In this paper we focus on *nominal* price and wage rigidities in all relevant markets.[4] Given adequate inter-

[1] The distinction between tradable and non-tradable goods in macroenomic models is still controversial. On the one hand, Chari *et al.* (2002) find that only 2% of the variance of real exchange rates is due to fluctuations in the relative prices of non-traded to traded goods using data for the United States and an aggregate of Europe. Consequently, they ignore the non-tradable sector in their model. On the other hand, our approach hinges upon the explicit introduction of non-tradable goods and incomplete markets with a stationary net foreign assets position. Betts and Kehoe (2005) and Burstein *et al.* (2005) stress that the variance of the real exchange rates explained by (that of) the relative prices of non-traded to traded goods is not 2% but about one third of the total variance (see also Selaive and Tuesta (2006), pp. 1 and 2). Hence, a modeling with a separate treatment of non-tradable goods seems to be important.

[2] The assumption that international financial markets are incomplete originates from the fact that a risk premium on external borrowing exists and is related to net foreign assets. Hence, the (exchange rate) pass-through is assumed to be incomplete due to underlying nominal rigidity in the buyer's currency position (see e.g. Adjémian *et al.* (2004)). The proposed specification in this paper is flexible enough to discriminate between the polar cases of Producer Currency Pricing (PCP), where the law of one price holds and there is perfect pass-through, and Local Currency Pricing (LCP), where the pass-through is zero in the short run.

[3] The relevance of intermediate goods in the production processes is nicely illustrated by the annual input-output tables of (for instance) the Dutch economy, where they account for up to more than 50% of all inputs in several industries.

[4] *Real* rigidities occur when relative prices react only slowly to changes in relative demand, implying that reallocation of (factors of) production is slow. There are numerous possible sources of real rigidities. A selection includes efficiency wages, wage indexation, hys-

pretation, *sticky wages* on labor markets can be modeled in a similar way as sticky prices on product markets. This way of modeling prices and wages is now widely used in empirical Dynamic Stochastic General Equilibrium (DSGE) models; see, for example, Altig *et al.* (2002), Amato and Laubach (2003), Smets and Wouters (2003, 2004), Adjémian *et al.* (2004) and Christiano *et al.* (2005).

Modeling nominal price and wage rigidities can generally be subdivided according to three main hypotheses (see also Klein (2002)):

(i) Grossman (1974), introduced the model of price adjustment, which was named the "P-bar" model by McCallum (1994). The idea is that the degree of disequilibrium in prices indicates inflationary pressure in a "sticky" price world and prices are set before trading; hence, price setters can only adjust at time t to the movements in equilibrium prices expected at $t-1$. This specification was used by Barro and Grossman (1976), McCallum (1979,1994), Mussa (1981a,1981b,1982), Obstfeld and Rogoff (1984), Flood and Hodrick (1986), and Chadha and Prasad (1993).

(ii) Taylor's (1980) staggered price (wage) setting model is a second well known tractable model of slow price (wage) adjustment. It is based on the theory of price and wage contracting, where periodically such contracts are renegotiated.

(iii)The Calvo-type staggered price setting (see Rotemberg (1982) and Calvo (1983)) allows a subset of firms to choose their own product price randomly with a constant probability. From this partial adjustment behavior price dispersion emerges. Calvo's price adjustment model is extremely tractable and also very popular in DSGE models such as Rotemberg (1987), Hairault and Portier (1993), Kimball (1995), King and Watson (1996), King and Wolman (1996), Rotemberg (1996), Yun (1996), Ireland (1997), Woodford (1998) and Kim (2000). A Calvo-type staggered wage setting by consumers can also be studied similarly; see e.g. Erceg *et al.* (2000) who present a Calvo model for both price and wage settings.

Interrelationships of these three price adjustment models can be found in Kiley (2002). In this paper we concentrate on the last two nominal price and wage adjustment models. In this respect, some recent studies have pointed out deficiencies of the Calvo (1983) approach and have challenged it. For example, when using the Calvo price setting model, it is typically assumed that steady state inflation is zero, which is a very restrictive assumption *per sé* as mentioned above. Moreover, Bakhshi *et al.* (2003) argue that firms in a low inflation environment behave in different ways than firms in a high inflation environment; consequently, inflation dynamics driven in the Calvo-type model by a constant probability of resetting prices should not be considered as constant. They show that for standard calibrations, the Calvo price setting model with constant probability of resetting prices can be used only when annual trend inflation is lower than 5.5%, a condition that is not met for inflation time-series data of several OECD countries over the last 25 years.

This paper is organized as follows. In Section 2 the domestic households' problem is defined, where habit formation in consumption is assumed and domestic households' consumption demands and investment supplies are derived in Subsections 2.1 and 2.2, respectively. The foreign households' consumption demands and investment supplies are analyzed in Appendices A.1 and A.2, respectively. Section 3 introduces a domestic government, while the foreign government's demands are summarized in Appendix A.3. The firms' supply linkages in production, distinguishing intermediate goods from final goods and tradable from non–tradable goods, are shortly discussed in Section 4. The domestic households' and firms' optimization problems are explicitly solved in Section 5. Equilibrium conditions for the home economy are derived in Section 6. In Section 7, the net foreign assets position of the aggregate domestic economy is derived. Regarding price and wage formation, we consider stickiness with Calvo and Taylor staggered price and wage settings in Section 8. Monetary policy rules are discussed in Section 9. Some numerical simulations demonstrating the functioning of the model will be discussed in Section 10. Finally, concluding remarks are in Section 11.

2 Domestic households

Household i is assumed to enjoy an expected level of intertemporal utility represented by a concave, differentiable and strongly separable function, which positively depends on present and past consumption (the latter due to habit

teresis, insider-outsider workers and other labor market institutions, distortionary taxation and other forms of government intervention and regulation. In a reduced form, real rigidities imply amongst others a low parameter in the price adjustment function of the real variables, principally output. In other words, the output supply function is flatter, i.e. shifts in demand lead to adjustment in output rather than in prices (see Blanchard and Fisher (1989)). Similarly, wages would not react much to unemployment. In addition, real rigidities may imply that long-run equilibrium is achieved at below-equilibrium full employment output. In the presence of (nominal and) real rigidities, the economy may therefore reach a steady state that is marked by both positive long-run inflation ("core" inflation) and divergences between long-run equilibrium real output and employment and potential output and employment.

formation), leisure and real money balances. Hence, household i's utility can be written as:[5]

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left[U_1 \left(C_t(i), C_{t-1}(i) \right) - U_2(L_t(i)) + U_3 \left(\frac{M_{t+1}(i)}{P_t^C(i)} \right) \right] \right], \tag{1}$$

where E_0 denotes the conditional expectations' operator depending on the information available at period 0; β is the household's discount factor satisfying $0 < \beta < 1$.

The first (utility) felicity $U_1(.)$ represents utility from consumption $C(i)$ of both domestic and foreign goods. We assume that household i is interested not only in current consumption but also in past values of it because of habit formation (see Abel (1990, 1999), Caputo (2003), Choi and Jung (2003), and Lindé *et al.* (2004)). The idea is that, if a household's consumption increases today owing to a shock, this household will experience a higher utility from an additional unit of consumption tomorrow. Intuitively, under habit formation, a consumer gets 'used' to a higher level of consumption and the marginal utility of consumption gets 'renormalized' at this higher reference level.

The second (utility) felicity $U_2(.)$ in (1) reflects a domestic consumer i's disutility derived from supplying labor services $L_t(i)$ to the (domestic) firms remunerated at the rate $W_t(i)$; hence, period t utility is a negative function of labor effort $L_t(i)$.[6] This labor effort can be subdivided across four different sectors that produce tradable final goods ($L_t^{FT}(i)$), non-tradable final goods ($L_t^{FN}(i)$), tradable intermediate goods ($L_t^{VT}(i)$) and non-tradable intermediate goods ($L_t^{VN}(i)$),[7] or

$$L_t(i) \equiv L_t^{FT}(i) + L_t^{FN}(i) + L_t^{VT}(i) + L_t^{VN}(i), \tag{2}$$

and corresponding nominal wages in each sector are $W_t^{FT}(i)$, $W_t^{FN}(i)$, $W_t^{VT}(i)$ and $W_t^{VN}(i)$, respectively. Hence, household i's gross nominal labor income equals,

$$W_t(i)L_t(i) = \sum_{m=FT,FN,VT,VN} W_t^m(i)L_t^m(i), \tag{3}$$

so that $W_t(i)$ is consumer i's weighted aggregate wage rate.

The third (utility) felicity $U_3(.)$ in (1) reflects utility from holding real money balances by consumer i at the beginning of period $t+1$, $\frac{M_{t+1}(i)}{P_t^C(i)}$, with $M_{t+1}(i)$ and $P_t^C(i)$ consumer i's nominal money balances at the beginning of period $t+1$ and consumer i's prices at (the end of) period t, respectively.

For sector $m = FT, FN, VT, VN$, let us assume that consumer i owns physical capital, $K_t^m(i)$, which is rented out to firms, and decides on its utilization rates, $\mathcal{Z}_t^m(i)$. Consequently, capital services (or rental services from capital) supplied by consumer i to sector m at period t are:[8]

$$\mathcal{K}_t^m(i) \equiv \mathcal{Z}_t^m(i)K_t^m(i). \tag{4}$$

The total supply of real investment goods by domestic consumer i is defined as:

$$I_t(i) \equiv \sum_{m=FT,FN,VT,VN} I_t^m(i), \tag{5}$$

where sector m's physical capital at the beginning of period $t+1$ is subject to the following laws of capital accumulation (see e.g. Smets and Wouters (2003, p. 1128) and Coenen and Straub (2005, p. 13)):

$$K_{t+1}^m(i) = (1 - d_t^m(i))K_t^m(i) + \left(1 - \Upsilon \left(\frac{I_t^m(i)}{I_{t-1}^m(i)} \right) \right) I_t^m(i) \;\text{ for } m = FT, FN, VT, VN, \tag{6}$$

[5] The infinite time horizon implied by utility function (1) is a simplication. It can be rationalized as if there are dynasties of individual consumers who are concerned about the future of their children.

[6] Thus, leisure $1 - L_t(i)$ is the residual of the individual's time endowment (with a normalized value of 1 for total time endowment).

[7] This sectoral split is motivated from Natalucci and Ravenna (2005) and Ortega and Rebei (2006). As pointed out in the Introduction, a modeling with a separate treatment of non-tradable goods seems to be very relevant.

[8] Even though the installed capital in sector m is utilized at a variable rate, it is costly to modify the utilization rate in the short run. Moreover, to expand the capital available in sector m, investors must face sector-specific installation costs. The implied asymmetric treatment of inputs, namely labor and capital in terms of their underemployment, hinges on the fact that we assume that labor can move freely among sectors, whereas capital stock is fixed once it is allocated in a sector (see Natalucci and Ravenna (2005)). For a model that introduces labor unemployment in the equilibrium integrating search and matching with NK models (by considering real frictions in the labor market), see Blanchard and Galí (2006).

where $d_t^m(i)$ is consumer i's depreciation rate (in perunages) for sector m, $I_t^m(i)$ is the investment expenditure in sector m by consumer i at period t and the function $\Upsilon(.)$ captures the (adjustment) costs due to changes in investment rates and acts as a proxy to installation costs of investment.[9]

Each household i is assumed to face a (nominal) consumer budget constraint (CBC) in home currency units:[10]

$$\begin{aligned} & P_t^C(i)C_t(i) + P_t^I(i)I_t(i) + Q_{h,t,t+1}B_{h,t+1}(i) + S_t\hat{Q}_{f,t,t+1}B_{f,t+1}(i) + M_{t+1}(i) \\ & +P_t^I(i)\left[\sum_{m=FT,FN,VT,VN}\Psi(\mathcal{Z}_t^m(i))K_t^m(i)\right] \leq (1-\tau_t^w)\,W_t(i)L_t(i) + T_t(i) \\ & +\left(1-\tau_t^k\right)\left[\sum_{m=FT,FN,VT,VN}R_t^m(i)\mathcal{K}_t^m(i)\right] + B_{h,t}(i) + S_tB_{f,t}(i) + M_t(i) + \mathcal{B}_t(i), \end{aligned} \tag{7}$$

where consumer i's outlays of resources are on the left hand side, while her disposable resources are on the right hand side; moreover, $P_t^C(i)$ and $P_t^I(i)$ indicate aggregate consumption and investment goods prices, respectively.[11]

Among the outlays we include nominal consumption, nominal investment, the portfolio of assets (bonds and stocks) and money holdings and the nominal cost of the underutilization of sectoral physical capital, which is given by the function $\Psi(\mathcal{Z}_t^m(i))$ and is zero when capacity is fully utilized.[12] Regarding assets, according to Woodford (2003) and Ambler *et al.* (2003, 2004), $Q_{h,t,t+1} \equiv (1+r_{t,t+1})^{-1}$ and $\hat{Q}_{f,t,t+1} \equiv (1+\hat{r}^*_{t,t+1})^{-1}$ are the domestic consumer's one-period ahead discount factors for (nominal) domestic and foreign asset payoffs, respectively.[13] Following Benigno (2001) financial markets are assumed to be complete domestically and incomplete internationally. Hence, a risk premium exists only on foreign assets so that the discount factor for foreign asset payoffs is stochastic and the expected (nominal) market price at date t of an asset portfolio, held at the beginning of period $t+1$, is given by $Q_{h,t,t+1}B_{h,t+1}(i) + E_t\left[S_t\hat{Q}_{f,t,t+1}B_{f,t+1}(i)\right]$, where $B_{h,t+1}(i)$ $(B_{f,t+1}(i))$ are consumer i's domestic (foreign) asset holdings denominated in local currency.[14]

Concerning the net resources, τ_t^w and τ_t^k are proportional tax rates applied on nominal labor and capital incomes, respectively, $R_t^m(i)$ is consumer i's rental rate on capital services supplied to sector m, $T_t(i)$ is consumer i's nominal lump-sum transfer from the government and $\mathcal{B}_t(i)$ are consumer i's net benefits from firms' ownerships.

Summarizing, households' behavior consists in an intertemporal smoothing of consumption, a supply of rental services from capital and a monopolistic supply of labor services under a staggered wage setting. This behavior will be analyzed in the following (sub)sections.

2.1 Domestic households' consumption demands

Reminding that subscript $h(f)$ denotes the home (foreign) economy and that, if necessary, the country of origin is indicated as a lower index and the country of destination (usage) as an upper index, the total domestic households' expenditure on consumption can be written as:

$$P_t^C C_t \equiv \int_0^1\int_0^1 P_{h,t}^{CT}(z,i)\,C_{h,t}^T(z,i)\,dzdi + \int_0^1\int_0^1 P_{h,t}^{CN}(z,i)\,C_{h,t}^N(z,i)\,dzdi + \int_0^1\int_0^1 P_{f,t}^{CT}(z,i)\,C_{f,t}^T(z,i)\,dzdi, \tag{8}$$

where $P_{h,t}^{CT}(z,i)$, $P_{h,t}^{CN}(z,i)$ and $P_{f,t}^{CT}(z,i)$ are the prices of the home produced tradable and non-tradable consumption goods and foreign produced tradable consumption goods purchased by domestic household i, which consumes domestically produced and imported goods z at quantities $C_{h,t}^T(z,i)$, $C_{h,t}^N(z,i)$ and $C_{f,t}^T(z,i)$, respectively. Consumer prices are assumed to be expressed in the home currency ('pricing to market' or LCP with zero (exchange rate) pass-through as opposed to PCP with perfect (exchange rate) pass-through). As suggested by Dixit and Stiglitz (1977), these consumption quantities can be aggregated through CES (constant elasticity of substitution)

[9] Smets and Wouters (2003) and Christiano *et al.* (2005) assume that $\Upsilon(.) = \Upsilon'(.) = 0$ in the steady state (no fixed adjustment costs). In addition, they assume that $\Upsilon''(.)$ is non-negative then.

[10] Notice that the nominal exchange rate (the price of one unit of foreign currency in home currency) S_t is used to translate asset returns both at the beginning and at the end of period t so that, *de facto*, it represents the average exchange rate during period t; see also equation (20) in Selaive and Tuesta (2006). Therefore, S_t is used to change both $B_{f,t+1}$ and $B_{f,t}$ into the home currency.

[11] See Smets and Wouters (2003, p. 1128) for the consideration of the (real) terms containing capital stocks and depreciation rates.

[12] Or, $\Psi(1) = 0$; see Christiano *et al.* (2005).

[13] Alternatively, these discount factors can also be considered as prices of one (home and foreign) asset paid by the domestic consumer (owner of assets) at time $t+1$.

[14] See Section 7 in this paper for an exact definition of the stochastic effective interest rate $\hat{r}^*_{t,t+1}$.

aggregators over a continuum $[0,1]$ of differentiated goods:

$$C_{k,t}^{l}(i) \equiv \left(\int_0^1 C_{k,t}^{l}(z,i)^{\frac{\theta_{Cl}^{(i)}-1}{\theta_{Cl}^{(i)}}} dz \right)^{\frac{\theta_{Cl}^{(i)}}{\theta_{Cl}^{(i)}-1}} \quad \text{for } (k,l) = (h,T),\ (h,N) \text{ and } (f,T), \tag{9}$$

with household i's intratemporal elasticity of substitution between two different goods given by $\theta_{Cl}^{(i)} > 1$.[15]

Consumer i takes the differentiated consumption goods prices as given and minimizes purchases of differentiated goods $C_{k,t}^{l}(z,i)$ over $[0,1]$ for each consumption category (k,l); the solutions to this minimization of total expenditure costs (8) subject to (9) are provided by the minimands:

$$\arg \min_{\{C_{k,t}^{l}(z,i)\}} \left\{ \int_0^1 P_{k,t}^{Cl}(z) C_{k,t}^{l}(z,i) dz + \lambda_{k,t}^{Cl}(i) \left(C_{k,t}^{l}(i) - \left(\int_0^1 C_{k,t}^{l}(z,i)^{\frac{\theta_{Cl}^{(i)}-1}{\theta_{Cl}^{(i)}}} dz \right)^{\frac{\theta_{Cl}^{(i)}}{\theta_{Cl}^{(i)}-1}} \right) \right\}, \tag{10}$$

where the Lagrange multiplier $\lambda_{k,t}^{Cl}(i)$ is household i's cost-minimizing (shadow) price of one unit of consumption goods in each particular category (k,l). Solving for the Lagrange parameters, we obtain consumer i's aggregate consumption price index for each consumption category (k,l) expressed in domestic currency:

$$\lambda_{k,t}^{Cl}(i) = P_{k,t}^{Cl}(i) = \left(\int_0^1 (P_{k,t}^{Cl}(z,i))^{1-\theta_{Cl}^{(i)}} dz \right)^{\frac{1}{1-\theta_{Cl}^{(i)}}} \quad \text{for } (k,l) = \{(h,T),(h,N),(f,T)\}. \tag{11}$$

Hence, household i's expenditure minimizing demands for each differentiated good z and each consumption category (k,l) are:

$$C_{k,t}^{l}(z,i) = \left(\frac{P_{k,t}^{Cl}(z)}{P_{k,t}^{Cl}(i)} \right)^{-\theta_{Cl}^{(i)}} \left(\left(\int_0^1 C_{k,t}^{l}(z,i)^{\frac{\theta_{Cl}^{(i)}-1}{\theta_{Cl}^{(i)}}} dz \right)^{\frac{\theta_{Cl}^{(i)}}{\theta_{Cl}^{(i)}-1}} \right) = \left(\frac{P_{k,t}^{Cl}(z)}{P_{k,t}^{Cl}(i)} \right)^{-\theta_{Cl}^{(i)}} C_{k,t}^{l}(i) \tag{12}$$

for $i, z \in [0,1]$ and $(k,l) = (h,T),(h,N)$ and (f,T).

Defining household i's aggregate consumption basket (index) of tradable and non-tradable goods as a nested CES specification (Dixit-Stiglitz aggregator of indices of traded and non-traded goods):

$$C_t(i) \equiv \left[\left(\alpha_{CT}^{(i)}\right)^{\frac{1}{\eta_{CT}^{(i)}}} \left(C_t^T(i)\right)^{\frac{\eta_{CT}^{(i)}-1}{\eta_{CT}^{(i)}}} + \left(1-\alpha_{CT}^{(i)}\right)^{\frac{1}{\eta_{CT}^{(i)}}} \left(C_{h,t}^N(i)\right)^{\frac{\eta_{CT}^{(i)}-1}{\eta_{CT}^{(i)}}} \right]^{\frac{\eta_{CT}^{(i)}}{\eta_{CT}^{(i)}-1}}, \tag{13}$$

where $C_t^T(i)$ is household i's aggregate consumption basket of tradable domestic and imported goods bundles, defined as $C_t^T(i) \equiv \left[\left(\alpha_{Ch}^{(i)}\right)^{\frac{1}{\eta_{Ch}^{(i)}}} \left(C_{h,t}^T(i)\right)^{\frac{\eta_{Ch}^{(i)}-1}{\eta_{Ch}^{(i)}}} + \left(1-\alpha_{Ch}^{(i)}\right)^{\frac{1}{\eta_{Ch}^{(i)}}} \left(C_{f,t}^T(i)\right)^{\frac{\eta_{Ch}^{(i)}-1}{\eta_{Ch}^{(i)}}} \right]^{\frac{\eta_{Ch}^{(i)}}{\eta_{Ch}^{(i)}-1}}$, with $\eta_{CT}^{(i)}$ and $\eta_{Ch}^{(i)}$ being domestic consumer i's intratemporal elasticities of substitution between (the bundle of) tradable consumption goods and (the bundle of) domestic non-tradable consumption goods, and between domestic and imported tradable consumption goods, respectively ($\eta_{Ch}^{(i)} > \eta_{CT}^{(i)} > 0$); $\alpha_{CT}^{(i)}$ and $\alpha_{Ch}^{(i)}$ are domestic consumer i's shares of tradable goods in her total domestic consumption and of domestically produced tradable goods in her total domestic tradable goods consumption, respectively.[16]

[15] The elasticity of substitution is larger than one to ensure that firms' markup factors are always positive.

[16] Hence, $\alpha_{CT}^{(i)}$ is household i's *tradable goods bias* in its total consumption, whereas $\alpha_{Ch}^{(i)}$ is its *home bias* in tradable consumption.

By using the definition of household i's total domestic consumption (13), we get the aggregate demand relationships of the optimal intratemporal consumption allocations (12):

$$C_{h,t}^{T}(i)=\alpha_{Ch}^{(i)}\left(\frac{P_{h,t}^{CT}(i)}{P_{t}^{CT}(i)}\right)^{-\eta_{Ch}^{(i)}}C_{t}^{T}(i)\quad,\quad C_{f,t}^{T}(i)=\left(1-\alpha_{Ch}^{(i)}\right)\left(\frac{P_{f,t}^{CT}(i)}{P_{t}^{CT}(i)}\right)^{-\eta_{Ch}^{(i)}}C_{t}^{T}(i),$$

$$C_{h,t}^{N}(i)=\left(1-\alpha_{CT}^{(i)}\right)\left(\frac{P_{h,t}^{CN}(i)}{P_{t}^{C}(i)}\right)^{-\eta_{CT}^{(i)}}C_{t}(i)\ \text{ and }\ C_{t}^{T}(i)=\alpha_{CT}^{(i)}\left(\frac{P_{t}^{CT}(i)}{P_{t}^{C}(i)}\right)^{-\eta_{CT}^{(i)}}C_{t}(i),\qquad(14)$$

where $P_{t}^{C}(i)$ is household i's aggregate consumption price index and $P_{t}^{CT}(i)$ ($P_{t}^{CN}(i)$) is household i's aggregate consumption price index of tradable (non-tradable) goods. These price indices can be directly derived by minimizing the cost of purchasing one unit of the aggregate nested consumption bundle by household i and are dual equations to CES specifications (13):

$$P_{t}^{C}(i)=\left[\alpha_{CT}^{(i)}\left(P_{t}^{CT}(i)\right)^{1-\eta_{CT}^{(i)}}+\left(1-\alpha_{CT}^{(i)}\right)\left(P_{h,t}^{CN}(i)\right)^{1-\eta_{CT}^{(i)}}\right]^{\frac{1}{1-\eta_{CT}^{(i)}}}\ \text{ and}\qquad(15)$$

$$P_{t}^{CT}(i)=\left[\alpha_{Ch}^{(i)}\left(P_{h,t}^{CT}(i)\right)^{1-\eta_{Ch}^{(i)}}+\left(1-\alpha_{Ch}^{(i)}\right)\left(P_{f,t}^{CT}(i)\right)^{1-\eta_{Ch}^{(i)}}\right]^{\frac{1}{1-\eta_{Ch}^{(i)}}}.\qquad(16)$$

2.2 Domestic households' investment supplies

Similarly to (8) for the total domestic households' consumption, the total domestic households' expenditure on investment can be written as:

$$P_{t}^{I}I_{t}\equiv\int_{0}^{1}\int_{0}^{1}P_{h,t}^{IT}(z,i)\,I_{h,t}^{T}(z,i)\,dzdi+\int_{0}^{1}\int_{0}^{1}P_{h,t}^{IN}(z,i)\,I_{h,t}^{N}(z,i)\,dzdi+\int_{0}^{1}\int_{0}^{1}P_{f,t}^{IT}(z,i)\,I_{f,t}^{T}(z,i)\,dzdi,\qquad(17)$$

where $I_{h,t}^{T}(z,i)$, $I_{h,t}^{N}(z,i)$ and $I_{f,t}^{T}(z,i)$, and corresponding prices $P_{h,t}^{IT}(z,i)$, $P_{h,t}^{IN}(z,i)$ and $P_{f,t}^{IT}(z,i)$ are the quantities and prices of the home produced tradable and non-tradable investment goods, and tradable foreign investment goods purchased by domestic household i, respectively.

Considering CES aggregators for individual investment quantities as for consumption quantities in (9) as constraints, the solutions of minimizing (17) over the interval $[0,1]$, with household i's intratemporal elasticity of substitution between two different investment goods given by $\theta_{Il}^{(i)}>1$, involve Lagrange multipliers $\lambda_{k,t}^{Il}(i)$ (similar to (11)).

Defining consumer i's aggregate investment basket of tradable and non-tradable investment goods similarly to (13) as:

$$I_{t}(i)\equiv\left[\left(\alpha_{IT}^{(i)}\right)^{\frac{1}{\eta_{IT}^{(i)}}}\left(I_{t}^{T}(i)\right)^{\frac{\eta_{IT}^{(i)}-1}{\eta_{IT}^{(i)}}}+\left(1-\alpha_{IT}^{(i)}\right)^{\frac{1}{\eta_{IT}^{(i)}}}\left(I_{h,t}^{N}(i)\right)^{\frac{\eta_{IT}^{(i)}-1}{\eta_{IT}^{(i)}}}\right]^{\frac{\eta_{IT}^{(i)}}{\eta_{IT}^{(i)}-1}},\qquad(18)$$

with household i's aggregate investment basket of tradable domestic and imported investment goods bundles, $I_{t}^{T}(i)\equiv\left[\left(\alpha_{Ih}^{(i)}\right)^{\frac{1}{\eta_{Ih}^{(i)}}}\left(I_{h,t}^{T}(i)\right)^{\frac{\eta_{Ih}^{(i)}-1}{\eta_{Ih}^{(i)}}}+\left(1-\alpha_{Ih}^{(i)}\right)^{\frac{1}{\eta_{Ih}^{(i)}}}\left(I_{f,t}^{T}(i)\right)^{\frac{\eta_{Ih}^{(i)}-1}{\eta_{Ih}^{(i)}}}\right]^{\frac{\eta_{Ih}^{(i)}}{\eta_{Ih}^{(i)}-1}}$, with $\eta_{IT}^{(i)}$ and $\eta_{Ih}^{(i)}$ being consumer i's intratemporal elasticities of substitution between tradable and non-tradable investment goods, and between domestic and imported tradable investment goods, respectively ($\eta_{Ih}^{(i)}>\eta_{IT}^{(i)}>0$); $\alpha_{IT}^{(i)}$ and $\alpha_{Ih}^{(i)}$ are domestic consumer i's shares of tradable goods in her total domestic investment and of domestically produced tradable goods in her total domestic tradable goods investment, respectively.[17]

Proceeding in a way similar to the previous subsection, the aggregate optimal intratemporal investment demands

[17] Parallel to the previous footnote, $\alpha_{IT}^{(i)}$ is household i's tradable goods bias in its total investment, whereas $\alpha_{Ih}^{(i)}$ is its home bias in tradable investment.

are:

$$I^T_{h,t}(i) = \alpha^{(i)}_{Ih}\left(\frac{P^{IT}_{h,t}(i)}{P^{IT}_t(i)}\right)^{-\eta^{(i)}_{Ih}} I^T_t(i) \quad , \quad I^T_{f,t}(i) = \left(1-\alpha^{(i)}_{Ih}\right)\left(\frac{P^{IT}_{f,t}(i)}{P^{IT}_t(i)}\right)^{-\eta^{(i)}_{Ih}} I^T_t(i),$$

$$I^N_{h,t}(i) = \left(1-\alpha^{(i)}_{IT}\right)\left(\frac{P^{IN}_{h,t}(i)}{P^I_t(i)}\right)^{-\eta^{(i)}_{IT}} I_t(i) \text{ and } I^T_t(i) = \alpha^{(i)}_{IT}\left(\frac{P^{IT}_t(i)}{P^I_t(i)}\right)^{-\eta^{(i)}_{IT}} I_t(i), \tag{19}$$

where $P^I_t(i)$ is household i's aggregate investment price index and $P^{IT}_t(i)$ ($P^{IN}_t(i)$) is household i's aggregate investment price index of tradable (non-tradable) goods. These price indices can be directly derived by minimizing the cost of purchasing one unit of the investment bundle $I_t(i)$ by household i as (these are the dual equations to the CES definitions (18)):

$$P^I_t(i) = \left[\alpha^{(i)}_{IT}\left(P^{IT}_t(i)\right)^{1-\eta^{(i)}_{IT}} + \left(1-\alpha^{(i)}_{IT}\right)\left(P^{IN}_{h,t}(i)\right)^{1-\eta^{(i)}_{IT}}\right]^{\frac{1}{1-\eta^{(i)}_{IT}}} \text{ and} \tag{20}$$

$$P^{IT}_t(i) = \left[\alpha^{(i)}_{Ih}\left(P^{IT}_{h,t}(i)\right)^{1-\eta^{(i)}_{Ih}} + \left(1-\alpha^{(i)}_{Ih}\right)\left(P^{IT}_{f,t}(i)\right)^{1-\eta^{(i)}_{Ih}}\right]^{\frac{1}{1-\eta^{(i)}_{Ih}}}. \tag{21}$$

The foreign consumers solve similar consumption and investment problems as the domestic consumers do. These problems are developed in detail in Appendices A.1 and A.2, respectively.

3 Domestic government

The expenditure of the domestic government usually comprises public consumption and public investment goods. According to the literature about DSGE models, the government only purchases consumption goods (see e.g. Smets and Wouters (2003)). Following Leith and Malley (2003), the domestic government considers the same differentiated consumption goods prices as the domestic private consumers do. Hence, it minimizes for each consumption category (k,l) purchases $G^l_{k,t}(z)$ of a differentiated good at a price $P^{Cl}_{k,t}(z)$ over the interval $[0,1]$. The solution to this total budget minimization subject to a CES aggregator similar to (9) results in minimands and Lagrange multipliers similar to (10) and (11), respectively. Thus, we get for $z \in [0,1]$ (in the sense of (12)):

$$G^l_{k,t}(z) = \left(\frac{P^{Cl}_{k,t}(z)}{P^{Cl}_{k,t}(G)}\right)^{-\theta^{(G)}_{Cl}} G^l_{k,t} \text{ for } (k,l) = (h,T),(h,N) \text{ and } (f,T) \tag{22}$$

with $\theta^{(g)}_{Cl} > 1$ the relevant elasticity of substitution and

$$P^{Cl}_{k,t}(G) = \left(\int_0^1 (P^{Cl}_{k,t}(z,G))^{1-\theta^{(G)}_{Cl}} dz\right)^{\frac{1}{1-\theta^{(G)}_{Cl}}} \tag{23}$$

the domestic government consumption price index.

In order to get the domestic government's aggregate demand, we propose CES aggregators, similar to equations (13) with $\eta^{(G)}_{CT}$ and $\eta^{(G)}_{Ch}$ and $\alpha^{(G)}_{CT}$ and $\alpha^{(G)}_{Ch}$ having similar interpretations as in Subsection 2.1; hence, the domestic government demand is:

$$G_t \equiv \left[\left(\alpha^{(G)}_{CT}\right)^{\frac{1}{\eta^{(G)}_{CT}}}\left(G^T_t\right)^{\frac{\eta^{(G)}_{CT}-1}{\eta^{(G)}_{CT}}} + \left(1-\alpha^{(G)}_{CT}\right)^{\frac{1}{\eta^{(G)}_{CT}}}\left(G^N_{h,t}\right)^{\frac{\eta^{(G)}_{CT}-1}{\eta^{(G)}_{CT}}}\right]^{\frac{\eta^{(G)}_{CT}}{\eta^{(G)}_{CT}-1}}, \tag{24}$$

where G^T_t is the aggregate domestic government demand of tradable home and imported goods bundles, defined as

$$G^T_t \equiv \left[\left(\alpha^{(G)}_{Ch}\right)^{\frac{1}{\eta^{(G)}_{Ch}}}\left(G^T_{h,t}\right)^{\frac{\eta^{(G)}_{Ch}-1}{\eta^{(G)}_{Ch}}} + \left(1-\alpha^{(G)}_{Ch}\right)^{\frac{1}{\eta^{(G)}_{Ch}}}\left(G^T_{f,t}\right)^{\frac{\eta^{(G)}_{Ch}-1}{\eta^{(G)}_{Ch}}}\right]^{\frac{\eta^{(G)}_{Ch}}{\eta^{(G)}_{Ch}-1}}.$$

Similarly to (14), the optimal demand equations for domestic government expenditure are:

$$G_{h,t}^{T}=\alpha_{Ch}^{(G)}\left(\frac{P_{h,t}^{CT}(G)}{P_{t}^{CT}(G)}\right)^{-\eta_{Ch}^{(G)}}G_{t}^{T},\quad G_{f,t}^{T}=\left(1-\alpha_{Ch}^{(G)}\right)\left(\frac{P_{f,t}^{CT}(G)}{P_{t}^{CT}(G)}\right)^{-\eta_{Ch}^{(G)}}G_{t}^{T},$$

$$G_{h,t}^{N}=\left(1-\alpha_{CT}^{(G)}\right)\left(\frac{P_{h,t}^{CN}(G)}{P_{t}^{C}(G)}\right)^{-\eta_{CT}^{(G)}}G_{t}\text{ and }G_{t}^{T}=\alpha_{CT}^{(G)}\left(\frac{P_{t}^{CT}(G)}{P_{t}^{C}(G)}\right)^{-\eta_{CT}^{(G)}}G_{t},\tag{25}$$

where $P_t^C(G)$ is the domestic government's aggregate consumption price index and $P_t^{CT}(G)$ ($P_t^{CN}(G)$) is the domestic government's aggregate consumption price index of tradable (non-tradable) goods, derived as:

$$P_{t}^{C}(G)=\left[\alpha_{CT}^{(g)}\left(P_{t}^{CT}(G)\right)^{1-\eta_{CT}^{(g)}}+\left(1-\alpha_{CT}^{(g)}\right)\left(P_{h,t}^{CN}(G)\right)^{1-\eta_{CT}^{(g)}}\right]^{\frac{1}{1-\eta_{CT}^{(g)}}}\text{ and}\tag{26}$$

$$P_{t}^{CT}(G)=\left[\alpha_{Ch}^{(g)}\left(P_{h,t}^{CT}(G)\right)^{1-\eta_{Ch}^{(g)}}+\left(1-\alpha_{Ch}^{(g)}\right)\left(P_{f,t}^{CT}(G)\right)^{1-\eta_{Ch}^{(g)}}\right]^{\frac{1}{1-\eta_{Ch}^{(g)}}}.\tag{27}$$

The domestic government is confronted with the government budget constraint (GBC), i.e. the total amount of taxes plus the net variation of the outstanding debt must equalize the total purchases in any period t, which is implied by (24) and (26). Formally, the GBC is given by:

$$\begin{aligned}&\tau_{t}^{w}\left[\sum_{m=FT,FN,VT,VN}\int_{0}^{1}W_{t}^{m}(i)L_{t}^{m}(i)di\right]+\tau_{t}^{k}\left[\sum_{m=FT,FN,VT,VN}\int_{0}^{1}R_{t}^{m}(i)\mathcal{K}_{t}^{m}(i)di\right]\\&+\int_{0}^{1}\left[M_{t+1}(i)-M_{t}(i)\right]di+Q_{h,t,t+1}\int_{0}^{1}B_{h,t+1}(i)di-\int_{0}^{1}B_{h,t}(i)di\\&+\hat{Q}_{h,t,t+1}^{*}\int_{0}^{1}B_{h,t+1}^{*}(i)di-\int_{0}^{1}B_{h,t}^{*}(i)di\geq\int_{0}^{1}T_{t}(i)di+P_{t}^{C}(G)G_{t}.\end{aligned}\tag{28}$$

Equation (28) includes on the left hand side labor and capital tax revenues, money creation and net domestic and foreign borrowing, while on the right hand side outlays of government revenues (transfers and goods purchases) are considered.

Similarly, the foreign government has a set of demands, which are considered in Appendix A.3, leading to a foreign GBC similar to (28).

4 Firms

Following Galí and Monacelli (2002), Smets and Wouters (2003), Choi and Jung (2003), Jung (2004) and Lindé *et al.* (2004), it is assumed that suppliers of inputs are price setters under profit maximization and demanders for inputs are price takers under cost minimization. All goods markets are characterized by monopolistic competition and profit maximization in the output markets. An alternative interpretation is that the firms' problem in open economies can be disentangled in two stages: one for final goods (being consumption and investment goods) and one for intermediate goods. The inspiration for an intermediate goods versus final goods situation can be found in Clarida *et al.* (2002), Smets and Wouters (2003) and Plasmans *et al.* (2004).[18] We assume that intermediate and final goods and tradable and non-tradable goods are produced in a sectoral framework.

Home goods are assumed to be produced in each sector by a continuum of monopolistically competitive firms, indexed by $j\in[0,1]$, while imported final and intermediate goods are bought (at marginal cost) in the foreign market by importing firms in import sectors MF and MV, respectively, and are repacked or rebranded and sold in the domestic market, also under monopolistic competition. Hence, firms in the monopolistically competitive import goods sectors turn the foreign goods, bought at their given world price marginal cost, into differentiated final and intermediate import goods.

Each domestic firm j is assumed to produce one differentiated final or intermediate good which can be either tradable or non-tradable. Six main sectors are distinguished: 1. the tradable final goods (FT) sector; 2. the non-tradable final goods (FN) sector; 3. the tradable intermediate goods (VT) sector; 4. the non-tradable intermediate

[18] Non-tradable intermediate goods include intermediate goods which are relatively too expensive to be transported, e.g. sand, water, various kinds of services.

goods (VN) sector; 5. the sector of imported (tradable) final goods (MF) and 6. the sector of imported (tradable) intermediate goods (MV). The latter two sectors are characterized by firms that import, repack or rebrand the imported good, adapting it for the home market.

In the **final goods** production sectors $m = FT, FN$, we assume variable returns to scale and a nested CES production technology with labor, $L_t^m(j)$, capital services, $\mathcal{Z}_t^m(i)K_t^m(j)$, and intermediate goods, $V_t^m(j)$, as inputs and symmetric production function parameters for all firms within the same sector:[19]

$$Y_t^m(j) \equiv (\Omega_{m,t} Z_t^m(j))^{\varpi_m} \equiv \left\{ \Omega_{m,t} \left[\begin{array}{c} \upsilon_{Lm}^{\frac{1}{\gamma_m}} (L_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}} + \upsilon_{Km}^{\frac{1}{\gamma_m}} (\mathcal{Z}_t^m K_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}} \\ + (1-\upsilon_{Km}-\upsilon_{Lm})^{\frac{1}{\gamma_m}} (V_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}} \end{array} \right]^{\frac{\gamma_m}{\gamma_m-1}} \right\}^{\varpi_m}, \tag{29}$$

where

$$K_t^m(j) \equiv \left[\kappa_{m1}^{\frac{1}{\epsilon_m}} \left(K_{h,t}^{T,m}(j) \right)^{\frac{\epsilon_m-1}{\epsilon_m}} + \kappa_{m2}^{\frac{1}{\epsilon_m}} \left(K_{h,t}^{N,m}(j) \right)^{\frac{\epsilon_m-1}{\epsilon_m}} + (1-\kappa_{m1}-\kappa_{m2})^{\frac{1}{\epsilon_m}} \left(K_{f,t}^{T,m}(j) \right)^{\frac{\epsilon_m-1}{\epsilon_m}} \right]^{\frac{\epsilon_m}{\epsilon_m-1}} \tag{30}$$

and

$$V_t^m(j) \equiv \left[\nu_{m1}^{\frac{1}{\chi_m}} \left(V_{h,t}^{T,m}(j) \right)^{\frac{\chi_m-1}{\chi_m}} + \nu_{m2}^{\frac{1}{\chi_m}} \left(V_{h,t}^{N,m}(j) \right)^{\frac{\chi_m-1}{\chi_m}} + (1-\nu_{m1}-\nu_{m2})^{\frac{1}{\chi_m}} \left(V_{f,t}^{T,m}(j) \right)^{\frac{\chi_m-1}{\chi_m}} \right]^{\frac{\chi_m}{\chi_m-1}}, \tag{31}$$

for $j \in [0,1]$, where γ_m, ϵ_m and χ_m are the (home country) intratemporal elasticities of substitution between the final goods inputs, different capital goods inputs and different intermediate goods inputs, respectively.

In (29-31), υ_{Lm} and υ_{Km} are the shares of labor and capital stock inputs in total input, κ_{m1} and κ_{m2} are the shares of domestically produced tradable and non-tradable capital stocks in total capital stock input of a domestic firm, while ν_{m1} and ν_{m2} are the shares of domestically produced tradable and non-tradable intermediate goods in total intermediate goods input of that firm in sector m; $\Omega_{m,t}$ is a domestic technology shock in period t, which, according to learning characteristics, is assumed to satisfy an $AR(1)$ process: $\ln \Omega_{m,t} \equiv \omega_{m,t} = \rho_{m,\omega}\omega_{m,t-1} + \xi_{m,\omega,t}$ with $-1 < \rho_{m,\omega} < 1$ and $\xi_{m,\omega,t}$ an independently and identically distributed (iid) error term; in addition, ϖ_m is the returns to scale parameter of the production function in sector m in the home country.

Moreover, the aggregate quantities $L_t^m(j)$, $K_{h,t}^{T,m}(j)$, $K_{h,t}^{N,m}(j)$, $K_{f,t}^{T,m}(j)$, $V_{h,t}^{T,m}(j)$, $V_{h,t}^{N,m}(j)$ and $V_{f,t}^{T,m}(j)$ are defined, using appropriate Dixit-Stiglitz CES aggregators similar to (9), as bundles over all labor, capital stock and intermediate goods types (varieties), respectively:

$$L_t^m(j) \equiv \left(\int_0^1 L_t^m(j,i)^{\frac{\varrho_{Lm}^{(j)}-1}{\varrho_{Lm}^{(j)}}} di \right)^{\frac{\varrho_{Lm}^{(j)}}{\varrho_{Lm}^{(j)}-1}}, \quad K_{k,t}^{l,m}(j) \equiv \left(\int_0^1 K_{k,t}^{l,m}(j,z)^{\frac{\varrho_{Kl,m}^{(j)}-1}{\varrho_{Km}^{(j)}}} dz \right)^{\frac{\varrho_{Kl,m}^{(j)}}{\varrho_{Kl,m}^{(j)}-1}} \tag{32}$$

and

$$V_{k,t}^{l,m}(j) \equiv \left(\int_0^1 V_{k,t}^{l,m}(j,z)^{\frac{\varrho_{Vl,m}^{(j)}-1}{\varrho_{Vl,m}^{(j)}}} dz \right)^{\frac{\varrho_{Vl,m}^{(j)}}{\varrho_{Vl,m}^{(j)}-1}} \quad \text{for } (k,l) = (h,T), (h,N) \text{ and } (f,T), \tag{33}$$

where for any category of final goods, $m = FT,\ FN$, $\varrho_{Lm}^{(j)}$, $\varrho_{Kl,m}^{(j)}$ and $\varrho_{Vl,m}^{(j)}$ are the intratemporal elasticities of substitution, which should be larger than one to assure positive markups, $L_t^m(j,i)$ is company j's demand for the labor supplied by household i at period t, $K_{k,t}^{l,m}(j,z)$ is company j's demand for capital stock produced by company z in regime (k,l) at time t or before and $V_{k,t}^{l,m}(j,z)$ is company j's demand for intermediate goods produced by company z in regime (k,l) at period t.

Intermediate good firms in production sectors $m = VT, VN$ combine capital stock and labor according to the production function with symmetric parameters for all firms within the same sector:

$$Y_t^m(j) \equiv (\Omega_{m,t} Z_t^m(j))^{\varpi_m} = \left\{ \Omega_{m,t} \left[\nu_{Lm}^{\frac{1}{\gamma_m}} (L_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}} + (1-\nu_{Lm})^{\frac{1}{\gamma_m}} (K_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}} \right]^{\frac{\gamma_m}{\gamma_m-1}} \right\}^{\varpi_m}, \tag{34}$$

[19] Notice that this assumed symmetry of production function parameters (for firms within the same sector) is similar to the usually assumed symmetry of preferential parameters in the household's optimization problem (*cf. ultra*).

Moreover, it is noted that the rate of utilization $Z_t^m$is not a firm j's choice variable so that it is independent of j.

where γ_m is the intratemporal elasticity of substitution between capital and labor and where $L_t^m(j)$ and $K_t^m(j)$ are defined as in (32) and (30), respectively, with ϖ_m being the returns to scale parameter in the production function for intermediate goods in sector m.

For simplicity, we assume no explicit production function for importers of final and intermediate goods because they are assumed to only rebrand and repack the same goods.

5 Optimization

5.1 The domestic households' optimization

Plugging household i's aggregate consumption demand functions (14) into its CBC (7), the constrained maximization of domestic household i's expected utility (1) leads to an Euler equation for consumption that is unsolvable.[20] Therefore, following, e.g. Ambler *et al.* (2004), we specify household i's period utility function in (1) as:[21]

$$U_1\left(C_t(i), C_{t-1}(i)\right) - U_2(L_t(i)) + U_3\left(\frac{M_{t+1}(i)}{P_t^C(i)}\right) = \frac{1}{1-\sigma}\left(\frac{C_t(i)}{\left(C_{t-1}(i)\right)^{\kappa}}\right)^{1-\sigma} - \frac{\left(L_t(i)\right)^{1+\phi}}{1+\phi} + \chi_M \frac{\left(\frac{M_{t+1}(i)}{P_t^C(i)}\right)^{1-\frac{1}{\chi}}}{\left(1-\frac{1}{\chi}\right)}, \quad (35)$$

where $\sigma > 0$ is a parameter of constant relative risk aversion of domestic households in the home country being equal to the inverse of the intertemporal elasticity of substitution in consumption, κ is the home consumers' habit persistence parameter,[22] and ϕ is the inverse of the elasticity of labor supply with respect to the real wage, $\frac{M_{t+1}(i)}{P_t^C(i)}$ is household i's real money balances at the beginning of period $t+1$, χ_M is a preferential constant for (real) money balances and χ is the elasticity of substitution of real money balances in the home country.

We maximize the expected discounted sum of household i's utility flows (35) subject to its CBC (7) and the law of motion (6) of its sectoral capital. In the domestic CBC we used definition (5) for consumer i's aggregate investment supply and expression (3) for consumer i's gross nominal labor income. The resulting first order conditions (FOCs) for $C_t(i)$, $L_t^m(i)$, $I_t^m(i)$, $B_{h,t+1}(i)$, $B_{f,t+1}(i)$, $M_{t+1}(i)$, $K_{t+1}^m(i)$ and $\mathcal{Z}_{t+1}^m(i)$ for $m = FT, FN, VT, VN$ are derived in Appendix B as follows:[23]

$$\Gamma_t(i) = \frac{U_{C_t(i)}}{P_t^C(i)} = \frac{\left(C_t(i)\right)^{-\sigma}}{P_t^C(i)\left(C_{t-1}(i)\right)^{\kappa(1-\sigma)}} - E_t\left[\beta\kappa\frac{\left(C_{t+1}(i)\right)^{1-\sigma}}{P_t^C(i)\left(C_t(i)\right)^{[\kappa(1-\sigma)+1]}}\right], \quad (36)$$

$$\frac{\varrho_{Lm}}{\varrho_{Lm}-1}\left(L_t(i)\right)^{\phi} = \Gamma_t(i)\left(1-\tau_t^w\right)W_t^m(i), \quad (37)$$

$$P_t^I(i)\Gamma_t(i)\text{-}\mathcal{Q}_t^m(i)\Gamma_t(i)\left[1\text{-}\Upsilon(.)\text{-}\Upsilon'(.)\frac{I_t^m(i)}{I_{t-1}^m(i)}\right] = \beta E_t\left[\mathcal{Q}_{t+1}^m(i)\Gamma_{t+1}(i)\left[\Upsilon'(.)\frac{\left(I_{t+1}^m(i)\right)^2}{\left(I_t^m(i)\right)^2}\right]\right], \quad (38)$$

$$E_t\left[\Gamma_t(i)Q_{h,t,t+1} - \beta\Gamma_{t+1}(i)\right] = 0, \quad (39)$$

$$E_t\left[\Gamma_t(i)\hat{Q}_{f,t,t+1}S_t - \beta\Gamma_{t+1}(i)S_{t+1}\right] = 0, \quad (40)$$

$$\chi_M\frac{1}{P_t^C(i)}\left[\frac{M_{t+1}(i)}{P_t^C(i)}\right]^{-\frac{1}{\chi}} = -\Gamma_t(i) + \beta E_t\left[\Gamma_{t+1}(i)\right], \quad (41)$$

$$\mathcal{Q}_t^m(i)\Gamma_t(i) = \beta E_t\left[\Gamma_{t+1}(i)\left((1\text{-}d_{t+1}^m(i))\mathcal{Q}_{t+1}^m(i) + (1\text{-}\tau_{t+1}^k)R_{t+1}^m(i)\mathcal{Z}_{t+1}^m(i)\text{-}P_{t+1}^I(i)\Psi\left(\mathcal{Z}_{t+1}^m(i)\right)\right)\right], \quad (42)$$

$$(1-\tau_{t+1}^k)R_{t+1}^m(i) = P_{t+1}^I(i)\Psi'\left(\mathcal{Z}_{t+1}^m(i)\right), \quad (43)$$

[20] Beyer and Farmer (2004) discuss the general lack of identification in linear rational expectations models and apply this to NK models.

[21] This is a Constant Rate of Risk Aversion (CRRA) utility function that allows for habit formation as in Kozicki and Tinsley (2002). Moreover, notice that following the international literature about DSGE models, we assume that the parameters in this utility function are equal over consumers.

[22] Instead of using an *ad hoc* rule of thumb as e.g. in Amato and Laubach (2003 and 2004), we propose a completely microfounded aggregate demand equation, based on the hypothesis of habit formation. Recent micro-level studies report mixed evidence of the impact of habit formation in consumption (see Ravina (2005)), while studies conducted with aggregate data find substantial evidence, e.g. Christiano *et al.* (2005) emphasize the role of habit persistence in the explanation of the hump-shaped behavior of aggregate consumption in response to a monetary policy shock. Notice that habit formation in consumption vanishes and consumption is as the usual CRRA ulitily if $\kappa = 0$.

[23] Any consumer can own homogeneous capital and can rent it out to firms in domestic industries. Capital services can be increased either by new capital or by augmenting the utilization rate of existing capital.

where the Lagrange multipliers $\Gamma_t(i)$ and $\mathcal{Q}_t^m(i)\Gamma_t(i)$ for $m = FT, FN, VT, VN$, are the marginal utility of household i's wealth and sector m's shadow prices of physical capital installed, respectively. Moreover, $\varrho_{Lm} > 1$ is the intratemporal elasticity of substitution for different types of labor demanded by firms in sector $m = FT, FN, VT, VN$. Equation (36) implies that the marginal utility of a particular household's consumption good equals the marginal utility of its wealth. Equation (37) relates the household i's marginal utility of leisure to its marginal utility of the nominal wage in every sector. Equation (38) refers to household i's investment decisions in sector m, which depend on the adjustment costs of investment growth in present and future periods; equations (39) and (40) refer to the household i's intertemporal decision involving the allocation of home and foreign financial assets and equation (41) is the optimal real money balances demand. Equation (42) states that the value of installed capital depends on its expected future value net of the depreciation rate and the expected rental rate multiplied by the utilization rate of capital. Finally, equation (43) equalizes the marginal adjustment cost of the utilization of capital to the (after tax) rental rate of capital.[24]

Combining FOCs (36) and (39), we may obtain a stochastic consumption Euler equation, which we may also derive by appropriate log-linearization. The complete log-linearization of FOCs (36) - (43) is worked out in Appendix D.

Each foreign consumer solves a similar problem as presented above, with the only difference providing an asterisk for the variables to denote foreign variables; foreign consumers must satisfy a set of FOCs analogous to (36 - 43).

5.2 The domestic firms' optimization: optimal demands for inputs

5.2.1 Final goods producers' input demands

Since demanders for inputs are price takers under minimization of costs, defined as $TC_t^m(j) \equiv W_t^m(j)L_t^m(j) + R_t^m(j)\mathcal{Z}_t^m K_t^m(j) + P_t^{V,m}(j)V_t^m(j)$, company j's derived demands for labor, capital stock and intermediate goods, given production function (29) subject to (30) and (31), satisfy (see Appendix E):

$$L_t^m(j) = \upsilon_{Lm}\left(\frac{W_t^m(j)}{P_t^{Z,m}(j)}\right)^{-\gamma_m}\frac{[Y_t^m(j)]^{\frac{1}{\varpi_m}}}{\Omega_{m,t}}, \tag{44}$$

$$K_t^m(j) = \upsilon_{Km}\left(\frac{\mathcal{Z}_t^m R_t^m(j)}{P_t^{Z,m}(j)}\right)^{-\gamma_m}\frac{[Y_t^m(j)]^{\frac{1}{\varpi_m}}}{\Omega_{m,t}} \quad \text{and} \tag{45}$$

$$V_t^m(j) = (1-\upsilon_{Lm}-\upsilon_{Km})\left(\frac{P_t^{V,m}(j)}{P_t^{Z,m}(j)}\right)^{-\gamma_m}\frac{[Y_t^m(j)]^{\frac{1}{\varpi_m}}}{\Omega_{m,t}}, \tag{46}$$

where price function $P_t^{Z,m}(j)$, dual to production function (29), satisfies for final goods sector $m = FT, FN$ at period t:

$$P_t^{Z,m}(j) = \left[\upsilon_{Lm}\left(W_t^m(j)\right)^{1-\gamma_m} + \upsilon_{Km}\left(\mathcal{Z}_t^m R_t^m(j)\right)^{1-\gamma_m} + (1-\upsilon_{Lm}-\upsilon_{Km})\left(P_t^{V,m}(j)\right)^{1-\gamma_m}\right]^{\frac{1}{1-\gamma_m}} \tag{47}$$

and $W_t^m(j)$ is the wage per unit of labor paid by company j and derived as equation (102) in Appendix B.2 as an intermediary step for deriving FOC (37). Similarly, the rental rate of one unit of capital stock for firm j, used as a measure for the user cost of one unit of capital stock demanded by firm j, is derived as $R_t^m(j) = \left(\int_0^1 R_t^m(j,i)^{1-\varrho_{Km}^{(j)}}di\right)^{\frac{1}{1-\varrho_{Km}^{(j)}}}$ and, finally, $P_t^{V,m}(j)$ is the intermediate good unit price for firm j. Both prices are derived as dual functions of the CES aggregators (30) and (31):

$$R_t^m(j) = \left[\kappa_{m1}\left(R_{h,t}^{T,m}(j)\right)^{1-\epsilon_m} + \kappa_{m2}\left(R_{h,t}^{N,m}(j)\right)^{1-\epsilon_m} + (1-\kappa_{m1}-\kappa_{m2})\left(R_{f,t}^{T,m}(j)\right)^{1-\epsilon_m}\right]^{\frac{1}{1-\epsilon_m}} \quad \text{and}$$
$$P_t^{V,m}(j) = \left[\nu_{m1}\left(P_{h,t}^{VT,m}(j)\right)^{1-\chi_m} + \nu_{m2}\left(P_{h,t}^{VN,m}(j)\right)^{1-\chi_m} + (1-\nu_{m1}-\nu_{m2})\left(P_{f,t}^{VT,m}(j)\right)^{1-\chi_m}\right]^{\frac{1}{1-\chi_m}}. \tag{48}$$

[24] In the interpretation of Smets and Wouters (2003) for instance, a higher rental rate of capital induces a higher utilization rate of it up to the point where the extra gains equalize the extra input costs.

From (30) and (31), the company j's optimal aggregate demands for tradable and non-tradable domestic capital and intermediate goods and imported foreign capital and intermediate goods are:

$$K_{k,t}^{l,m}(j) = \kappa_{m,kl}\left(\frac{R_{k,t}^{l,m}(j)}{R_t^m(j)}\right)^{-\epsilon_m} K_t^m(j) \text{ and } V_{k,t}^{l,m}(j) = \nu_{m,kl}\left(\frac{P_{k,t}^{Vl,m}(j)}{P_t^{V,m}(j)}\right)^{-\chi_m} V_t^m(j) \tag{49}$$

for $(k,l) = (h,T), (h,N)$ and (f,T), and where $\kappa_{m,kl}(\nu_{m,kl}) \equiv \kappa_{m1}(\nu_{m1}), \kappa_{m2}(\nu_{m2}), (1-\kappa_{m1}-\kappa_{m2})\,[(1-\nu_{m1}-\nu_{m2})]$, respectively. The corresponding cost-minimizing demands based on (32) and (33) for capital and intermediate goods produced by firm z are given by (similar to (12)):

$$K_{k,t}^{l,m}(j,z) = \left(\frac{R_{k,t}^{l,m}(z)}{R_{k,t}^{l,m}(j)}\right)^{-\varrho_{Kl,m}^{(j)}} K_{k,t}^{l,m}(j) \text{ and } V_{k,t}^{l,m}(j,z) = \left(\frac{P_{k,t}^{Vl,m}(z)}{P_{k,t}^{Vl,m}(j)}\right)^{-\varrho_{Vl,m}^{(j)}} V_{k,t}^{l,m}(j), \tag{50}$$

where, e.g., $P_{k,t}^{Vl,m}(z)$ is the price of the intermediate goods sold to sector m by a producer z in sector Vl.

5.3 Intermediate goods producers' input demands

From production function (34) the optimal demands for labor and capital stock inputs in the intermediate goods producing sectors $m = VT, VN$ are derived as (see also Appendix E):

$$L_t^m(j) = \nu_{Lm}\left(\frac{W_t^m(j)}{P_t^{Z,m}(j)}\right)^{-\gamma_m} \frac{[Y_t^m(j)]^{\frac{1}{\varpi_m}}}{\Omega_{m,t}} \text{ and} \tag{51}$$

$$K_t^m(j) = (1-\nu_{Lm})\left(\frac{\mathcal{Z}_t^m R_t^m(j)}{P_t^{Z,m}(j)}\right)^{-\gamma_m} \frac{[Y_t^m(j)]^{\frac{1}{\varpi_m}}}{\Omega_{m,t}}, \tag{52}$$

where $P_t^{Z,m}(j)$ is the aggregate input price derived as the dual price of the intermedate good production function (34):

$$P_t^{Z,m}(j) = \left[\nu_{Lm}\left(W_t^m(j)\right)^{1-\gamma_m} + (1-\nu_{Lm})\left(\mathcal{Z}_t^m R_t^m(j)\right)^{1-\gamma_m}\right]^{\frac{1}{1-\gamma_m}}. \tag{53}$$

6 Equilibrium conditions

Since sectoral firms are able to exert monopolistic power, they are also able to charge different prices according to the market they serve. First, domestic producers of tradable final goods sell their output to 5 domestic and to 2 foreign markets (foreign consumption and foreign investment goods). According to the destination of the tradable goods, we distinguish domestic markets of home tradable consumption and capital goods and foreign (wholesale) markets of home consumption and capital goods. The capital (stock) goods are bought by a domestic sector $m = FT, FN, VT, VN$ and by a foreign import sector. Since each buyer of final goods is characterized by an appropriate demand function, the domestic producer of tradable final goods may charge 7 monopolistic prices. We group these prices in the vector $\mathbf{P}_{h,t}^{FT}(j) \equiv [P_{h,t}^{CT}(j),\, P_{h,t}^{CT,X}(j),\, P_{h,t}^{IT,FT}(j),\, P_{h,t}^{IT,FN}(j),\, P_{h,t}^{IT,VT}(j),\, P_{h,t}^{IT,VN}(j),\, P_{h,t}^{IT,X}(j)]'$, where $P_{h,t}^{CT,X}(j)$ and $P_{h,t}^{IT,X}(j)$ are the export prices of domestic tradable consumption and investment goods, respectively. Second, a domestic producer of non-tradable final goods sells her products only to domestic sectors $m = FT, FN, VT, VN$ and to domestic consumers; hence, she is confronted with 5 prices assembled in the vector $\mathbf{P}_{h,t}^{FN}(j) \equiv [P_{h,t}^{CN}(j),\, P_{h,t}^{IN,FT}(j),\, P_{h,t}^{IN,FN}(j),\, P_{h,t}^{IN,VT}(j),\, P_{h,t}^{IN,VN}(j)]'$. Third, an intermediate tradable goods company sells its output to 4 markets (two domestic and two foreign final goods markets), charging the following set of prices $\mathbf{P}_{h,t}^{VT}(j) \equiv [P_{h,t}^{VT,FT}(j),\, P_{h,t}^{VT,FN}(j),\, P_{h,t}^{VT,X,FT}(j),\, P_{h,t}^{VT,X,FN}(j)]'$. Fourth, an intermediate non-tradable goods company sells its output only to the two domestic final goods markets; thus, $\mathbf{P}_{h,t}^{VN}(j) \equiv [P_{h,t}^{VN,FT}(j),\, P_{h,t}^{VN,FN}(j)]'$. Finally, a firm in the domestic import sector of final tradable goods charges $\mathbf{P}_t^{MF}(j) \equiv [P_{f,t}^{CT}(j), P_{f,t}^{IT,FT}(j), P_{f,t}^{IT,FN}(j),\, P_{f,t}^{IT,VT}(j),\, P_{f,t}^{IT,VN}(j)]$ and a firm in the domestic import sector of intermediate tradable goods charges $\mathbf{P}_t^{MV}(j) \equiv [P_{f,t}^{VT,FT}(j),\, P_{f,t}^{VT,FN}(j)]$. For notational convenience, we redefine all home price vectors in home currency: $\mathbf{S}_{h,t}^{FT}(j) \equiv [P_{h,t}^{CT}(j), P_{h,t}^{CT,X}(j), P_{h,t}^{IT,FT}(j), P_{h,t}^{IT,FN}(j), P_{h,t}^{IT,VT}(j), P_{h,t}^{IT,VN}(j), P_{h,t}^{IT,X}(j)]'$, $\mathbf{S}_{h,t}^{VT}(j) \equiv [P_{h,t}^{VT,FT}(j),\, P_{h,t}^{VT,FN}(j),\, P_{h,t}^{VT,X,FT}(j),\, P_{h,t}^{VT,X,FN}(j)]'$, $\mathbf{S}_{h,t}^{FN}(j) \equiv \mathbf{P}_{h,t}^{FN}(j)$, $\mathbf{S}_{h,t}^{VN}(j) \equiv \mathbf{P}_{h,t}^{VN}(j)$, $\mathbf{S}_t^{MF}(j) \equiv$

$[S_t P^{*CT,X}_{f,t}(j), S_t P^{*IT,X,FT}_{f,t}(j), S_t P^{*IT,X,FN}_{f,t}(j), S_t P^{*IT,X,VT}_{f,t}(j), S_t P^{*IT,X,VN}_{f,t}(j)]$ and $\mathbf{S}^{MV}_t(j) \equiv [S_t P^{*VT,X,FT}_{f,t}(j), S_t P^{*VT,X,FN}_{f,t}(j)]$.

Moreover, we require market clearing equilibrium conditions for all firms. The demand for the tradable final good produced by company j is the sum of domestic and foreign consumption, investment and government demands for this good. The demand for the non-tradable final good is equal to the sum of the domestic consumption, investment and government demands for this good. The demand for the tradable intermediate good is the sum of the domestic and foreign demands for this good in the domestic and foreign final goods sectors and the demand for the non-tradable intermediate good is the sum of the domestic demands for this good in the domestic final goods production sectors.

The domestic aggregate demand for tradable goods produced by domestic company j is the sum of consumption, investment, government purchases and the respective exports, where these aggregate components are $C^T_{h,t}(j) \equiv \int_0^1 C^T_{h,t}(j,i)di$, $I^{T,m}_{h,t}(j) \equiv \int_0^1 I^{T,m}_{h,t}(j,i)di$, $G^T_{h,t}(j) \equiv \int_0^1 G^T_{h,t}(j,i)di$, $C^{*T}_{h,t}(j) \equiv \int_0^1 C^{*T}_{h,t}(j,i)di$, $I^{*T,m}_{h,t}(j) \equiv \int_0^1 I^{*T,m}_{h,t}(j,i)di$ and $G^{*T}_{h,t}(j) \equiv \int_0^1 G^{*T}_{h,t}(j,i)di$, where $m = FT$, FN, VT, VN. The non-tradable domestic aggregate demand can be similarly derived. Hence, we can define the vector of relevant demands for the products of company j in sector FT as $\mathbf{D}^{FT}_{h,t}(j) \equiv [C^T_{h,t}(j) + G^T_{h,t}(j), C^{*T}_{h,t}(j) + G^{*T}_{h,t}(j), I^{T,FT}_{h,t}(j), I^{T,FN}_{h,t}(j), I^{T,VT}_{h,t}(j), I^{T,VN}_{h,t}(j), I^{*T}_{h,t}(j)]'$. Analogously, $\mathbf{D}^{FN}_{h,t}(j) \equiv [C^N_{h,t}(j)+G^N_{h,t}(j), I^{N,FT}_{h,t}(j), I^{N,FN}_{h,t}(j), I^{N,VT}_{h,t}(j), I^{N,VN}_{h,t}(j)]'$, $\mathbf{D}^{VT}_{h,t}(j) \equiv [V^{T,FT}_{h,t}(j), V^{T,FN}_{h,t}(j), V^{*T,FT}_{h,t}(j), V^{*T,FN}_{h,t}(j)]'$, $\mathbf{D}^{VN}_{h,t}(j) \equiv [V^{N,FT}_{h,t}(j), V^{N,FN}_{h,t}(j)]'$, $\mathbf{D}^{MF}_{h,t}(j) \equiv [C^T_{f,t}(j) + G^T_{f,t}(j), I^{T,FT}_{f,t}(j), I^{T,FN}_{f,t}(j), I^{T,VT}_{f,t}(j), I^{T,VN}_{f,t}(j)]'$ and $\mathbf{D}^{MV}_{h,t}(j) \equiv [V^{T,FT}_{f,t}(j), V^{T,FN}_{f,t}(j)]'$, where, for example, $V^{T,m}_{h,t}(j)$ represents the (domestic) demand for tradable intermediate goods produced by company j demanded by firms in production sector m.

Consequently, the equilibrium or market clearing conditions can be written as (see also Natalucci and Ravenna (2005)):[25]

$$Y^{FT}_{h,t}(j) = \left[\mathbf{D}^{FT}_{h,t}(j)\right]' \iota^{FT}, \tag{54}$$

$$Y^{FN}_{h,t}(j) = \left[\mathbf{D}^{FN}_{h,t}(j)\right]' \iota^{FN}, \tag{55}$$

$$Y^{VT}_{h,t}(j) = \left[\mathbf{D}^{VT}_{h,t}(j)\right]' \iota^{VT}, \tag{56}$$

$$Y^{VN}_{h,t}(j) = \left[\mathbf{D}^{VN}_{h,t}(j)\right]' \iota^{VN}, \tag{57}$$

$$Y^{MF}_{h,t}(j) = \left[\mathbf{D}^{MF}_{h,t}(j)\right]' \iota^{MF} \text{ and} \tag{58}$$

$$Y^{MV}_{h,t}(j) = \left[\mathbf{D}^{MV}_{h,t}(j)\right]' \iota^{MV}. \tag{59}$$

where ι^m is the unity vector containing an appropriate number of ones being equal to the number of markets in sector m. Furthermore, we disaggregate the total output of company j in every domestic sector as $\mathbf{Y}^{FT}_{h,t}(j) \equiv [Y^{CT}_{h,t}(j), Y^{CT,X}_{h,t}(j), Y^{IT,FT}_{h,t}(j), Y^{IT,FN}_{h,t}(j), Y^{IT,VT}_{h,t}(j), Y^{IT,VN}_{h,t}(j), Y^{IT,X}_{h,t}(j)]'$, and, analogously, $Y^{FN}_{h,t}(j) \equiv [Y^{CN}_{h,t}(j), Y^{IN,FT}_{h,t}(j), Y^{IN,FN}_{h,t}(j), Y^{IN,VT}_{h,t}(j), Y^{IN,VN}_{h,t}(j)]'$; $Y^{VT}_{h,t}(j) \equiv [Y^{VT,FT}_{h,t}(j), Y^{VT,FN}_{h,t}(j), Y^{VT,*FT}_{h,t}(j), Y^{VT,*FN}_{h,t}(j)]'$, $Y^{VN}_{h,t}(j) \equiv [Y^{VN,FT}_{h,t}(j), Y^{VN,FN}_{h,t}(j)]'$, $\mathbf{Y}^{MF}_{h,t}(j) \equiv \Big[Y^{CT}_{f,t}(j), Y^{*IT,FT}_{f,t}(j), Y^{*IT,FN}_{f,t}(j), Y^{*IT,VT}_{f,t}(j), Y^{*IT,VN}_{f,t}(j)$ and $\mathbf{Y}^{MV}_{h,t}(j) \equiv [Y^{*VT,FT}_{f,t}(j), Y^{*VT,FN}_{f,t}(j)]'$. In the equilibrium we assume that $\mathbf{Y}^{FT}_{h,t}(j) = \mathbf{D}^{FT}_{h,t}(j)$, $\mathbf{Y}^{FN}_{h,t}(j) = \mathbf{D}^{FN}_{h,t}(j)$, $\mathbf{Y}^{VT}_{h,t}(j) = \mathbf{D}^{VT}_{h,t}(j)$, $\mathbf{Y}^{VN}_{h,t}(j) = \mathbf{D}^{VN}_{h,t}(j)$, $\mathbf{Y}^{MF}_{h,t}(j) = \mathbf{D}^{MF}_{h,t}(j)$ and $\mathbf{Y}^{MV}_{h,t}(j) = \mathbf{D}^{MV}_{h,t}(j)$.

Analogous equilibrium conditions for the equality between foreign supplies and demands hold.

7 Net Foreign Assets

In Section 2 we introduced home and foreign stochastic discount factors and we linked them to $r_{t,t+1}$ and $\hat{r}_{t,t+1}$, respectively. In particular, the interest rate effectively paid by domestic consumers for foreign assets can be defined as:

$$\left[\hat{Q}_{f,t,t+1}\right]^{-1} \equiv (1 + \hat{r}^*_{t,t+1}) = F_{t+1}(.)(1 + r^*_{t,t+1}), \tag{60}$$

where the factor of proportionality $F_{t+1}(.)$ is a function which, according to Benigno (2001), depends on the real holdings of the corresponding domestic consumer i's foreign assets; this means that domestic households take this

[25] Some authors assume that producers do not know beforehand whether their goods will be used as a final good or as an input in production chain (see for example Leith and Malley (2002)). However, we follow the spirit of the General Equilibrium models à la Debreu where market clearing is defined for each (sub)market.

function as given when deciding on the optimal position in foreign assets. Benigno (2001) notes that this function is subject to the following two restrictions: $E_t\left[\digamma_{t+1}(0)\right]=1$ and takes the value 1 only if $B_{f,t+1}(i)=0$. Moreover, $\digamma_{t+1}(.)$ is a differentiable, decreasing function in the neighborhood of zero. Benigno (2001, p. 5) argues that function $\digamma_{t+1}(.)$ can be described in two ways. First, it captures the (intermediation) costs, for the domestic households of undertaking positions in the international bonds (assets) market. As borrowers, they will be charged a (risk) premium on the foreign interest rate; as lenders, they will receive a remuneration lower than the foreign interest rate. Second, an alternative way to describe this cost is to assume the existence of intermediaries in the foreign bonds (assets) market (which are owned by the foreign households), who can borrow from and lend to foreign households at rate $1+r^*_{t,t+1}$, but can borrow from and lend to domestic households at rate $\digamma_{t+1}(.)(1+r^*_{t,t+1})$.[26]

There are many functions $\digamma_t(.)$ that satisfy the above requirements. In the spirit of Ambler *et al.* (2003, 2004), household i's risk premium depends on its real foreign assets position as follows:[27]

$$\digamma_t(.)\equiv\exp\left(v_t\left[\exp\left(\frac{\bar{S}\bar{B}_f(i)}{\bar{P}(i)}-\frac{S_{t-1}B_{f,t}(i)}{P_t(i)}\right)-1\right]\right), \tag{61}$$

where $P_t(i)$ is consumer i's price index similarly derived as (15) and (20) and which is a CES aggregator of consumption and investment goods prices:[28]

$$P_t(i)=\left[\alpha_C^{(i)}\left(P_t^C(i)\right)^{1-\eta_C^{(i)}}+\left(1-\alpha_C^{(i)}\right)\left(P_t^I(i)\right)^{1-\eta_C^{(i)}}\right]^{\frac{1}{1-\eta_C^{(i)}}}, \tag{62}$$

$S_{t-1}B_{f,t}(i)$ is household i's nominal foreign assets position at the beginning of period t with $\bar{S}\bar{B}_f(i)$ being its steady-state value and v_t is an *iid* process centered in a certain parameter value δ. Therefore, once we allow either for perfect capital mobility or for a foreign assets position equal to zero, the familiar uncovered interest rate parity (UIP) hypothesis holds with purely temporary deviations. Notice that the implied arbitrage condition is operating for the returns of all foreign assets. In particular, substituting $\Gamma_t(i)$ in (40) by its value from (39) and dividing both sides by $\beta\Gamma_{t+1}(i)$, we obtain the UIP hypothesis between both countries, taking account of equation (60):

$$\frac{E_t\left[S_{t+1}\right]}{S_t}=E_t\left[\frac{\hat{Q}_{f,t,t+1}}{Q_{h,t,t+1}}\right]=E_t\left[\frac{\left[\digamma_{t+1}(.)(1+r^*_{t,t+1})\right]^{-1}}{(1+r_{t,t+1})^{-1}}\right]. \tag{63}$$

To relate the current account balance to the trade balance, we assume that the home government follows a balanced budget policy so that we can replace the equilibrium GBC from (28) into the active consumers' CBC from (7). The aggregate active CBC over all domestic consumers can be written as:[29]

$$\begin{aligned}&\int_0^1 P_t^C(i)C_t(i)di+\int_0^1 P_t^I(i)I_t(i)di+\int_0^1\left[Q_{h,t,t+1}B_{h,t+1}(i)di+S_t\hat{Q}_{f,t,t+1}B_{f,t+1}(i)\right]di+\int_0^1 M_{t+1}(i)di\\&+\int_0^1 P_t^I(i)\left[\sum_{m=FT,FN,VT,VN}\Psi(\mathcal{Z}_t^m(i))K_t^m(i)\right]di-(1-\tau_t^w)\int_0^1 W_t(i)L_t(i)di-\int_0^1 T_t(i)di\\&-\left(1-\tau_t^k\right)\int_0^1\left[\sum_{m=FT,FN,VT,VN}R_t^m(i)\mathcal{K}_t^m(i)\right]di-\int_0^1 B_{h,t}(i)di-S_t\int_0^1 B_{f,t}(i)di-\int_0^1 M_t(i)di-\int_0^1\mathcal{B}_t(i)di=0,\end{aligned} \tag{64}$$

[26] The property that in (60) only the factor of proportionality $\digamma_{t+1}(.)$ (and not $\hat{r}^*_{t,t+1}$) is consumer i-dependent is argued from this interpretation.

[27] Two alternative formulations of $\digamma_t(.)$ can be found in Schmitt-Grohé and Uribe (2003) and Malik (2005) on the one side and Erceg *et al.* (2005) on the other side.

[28] The aggretate price is the Lagrange multiplier from the total expenditure minimization (sum of consumption and investment) given that consumer i purchases a consumption-investment bundle. The problem is to $\min_{\{C_t(i),I_t(i)\}}\left\{P_t^C(i)C_t(i)+P_t^I(i)I_t(i)\text{ s.t. }\left[\left(\alpha_C^{(i)}\right)^{\frac{\eta_C^{(i)}-1}{\eta_C^{(i)}}}C_t(i)+\left(1-\alpha_C^{(i)}\right)^{\frac{\eta_C^{(i)}-1}{\eta_C^{(i)}}}I_t(i)\right]^{\frac{\eta_C^{(i)}}{\eta_C^{(i)}-1}}\right\}$, where $\eta_C^{(i)}$ is consumer i's elasticity of substitution between aggregate consumption and investment goods, while $\alpha_C^{(i)}$ is consumer i's weight of consumption in terms of the consumption and investment bundles (in other words, $\alpha_C^{(i)}$ is the consumption bias in consumer i's aggregate final goods expenditures). Hence, price function (62) is the dual function of this CES consumption-investment bundle.

[29] Recall that stock variables are measured at the beginning of a period, so that $M_{t+1}(i)$ and $B_{f,t+1}(i)$ are decision variables at the end of period t and are represented without conditional expectation operators.

so that subtracting the balanced budget (binding) version of GBC (28) from (64), we obtain the following NFAs equation:

$$
\begin{aligned}
0= & \int_0^1 P_t^C(i) C_t(i) di+\int_0^1 P_t^I(i) I_t(i) di+\int_0^1 P_t^I(i) \textstyle\sum_{m=FT,FN,VT,VN} \Psi(\mathcal{Z}_t^m(i)) K_t^m(i) di-\int_0^1 W_t(i) L_t(i) di \\
& -\left(\textstyle\sum_{m=FT,FN,VT,VN} \int_0^1 R_t^m(i) \mathcal{K}_t^m(i) di\right)+\int_0^1\left[S_t \hat{Q}_{f,t,t+1} B_{f,t+1}(i)\right] di-\int_0^1 S_t B_{f,t}(i) di \\
& -\left(\int_0^1\left[\hat{Q}_{h,t,t+1}^* B_{h,t+1}^*(i)\right] di-\int_0^1 B_{h,t}^*(i) di\right)-\int_0^1 \mathcal{B}_t(i) di+P_t^C(G) G_t,
\end{aligned} \tag{65}
$$

where $\int_0^1 \left[S_t \hat{Q}_{f,t,t+1} B_{f,t+1}(i)\right] di-\int_0^1 S_t B_{f,t}(i) di-\left(\int_0^1\left[\hat{Q}_{h,t,t+1}^* B_{h,t+1}^*(i)\right] di-\int_0^1 B_{h,t}^*(i) di\right)$ is the net variation of the home country's assets position. Let us introduce aggregates to rewrite (65), taking account of incomplete foreign financial markets, as, :

$$
\begin{aligned}
0= & P_t^C C_t+P_t^I I_t+P_t^I\left[\textstyle\sum_{m=FT,FN,VT,VN} \Psi(\mathcal{Z}_t^m) K_t^m\right]-W_t L_t-\left[\textstyle\sum_{m=FT,FN,VT,VN} R_t^m \mathcal{K}_t^m\right] \\
& +S_t E_t\left[\hat{Q}_{f,t,t+1} B_{f,t+1}\right]-S_t B_{f,t}-\left(E_t\left[\hat{Q}_{h,t,t+1}^* B_{h,t+1}^*\right]-B_{h,t}^*\right)-\mathcal{B}_t+P_t^C(G) G_t,
\end{aligned} \tag{66}
$$

where e.g. the aggregate nominal consumption is defined as $P_t^C C_t \equiv \int_0^1 P_t^C(i) C_t(i) di$, and so on (see e.g. Galí and Monacelli (2004)).

Assuming that the conditional expectations of the stochastic discount factors are mutually pairwise equal between countries, i.e. $E_t\left[\hat{Q}_{f,t,t+1}\right]=E_t\left[\hat{Q}_{h,t,t+1}^*\right]$,[30] we ultimately obtain the NFAs equation, using (3):

$$
\begin{aligned}
0= & P_t^C C_t+P_t^I I_t+P_t^I\left[\sum_{m=FT,FN,VT,VN} \Psi(\mathcal{Z}_t^m) K_t^m\right]-\left[\sum_{m=FT,FN,VT,VN} W_t^m L_t^m\right] \\
& -\left[\sum_{m=FT,FN,VT,VN} R_t^m \mathcal{K}_t^m\right]+S_t E_t\left[\hat{Q}_{f,t,t+1} B_{f*,t+1}\right]-S_t B_{f*,t}-\mathcal{B}_t+P_t^C(G) G_t,
\end{aligned} \tag{67}
$$

where $B_{f*,t+1} \equiv B_{f,t+1}-S_t^{-1} B_{h,t+1}^*$ are the foreign assets held by domestic consumers at the beginning of period $t+1$ and total benefits $\mathcal{B}_t$ are defined in equation (140) in Appendix F. Substituting these equations in (67), this equation results in the final NFAs equation:

$$
S_t E_t\left[\hat{Q}_{f,t,t+1} B_{f*,t+1}\right]=S_t B_{f*,t}+N X_{h,t}^{FT}+N X_{h,t}^{VT}, \tag{68}
$$

where $NX_{h,t}^{FT}$ and $NX_{h,t}^{VT}$ are the net domestic exports of the final and intermediate tradable goods. Note that the home producer can either sell the good in the home market or export it abroad. In both situations a markup is applied. Concentrating on the latter situation (the export case), it is important to notice that the export variables are measured at the border so that the relevant prices matter. At the border, goods are shipped abroad and the foreign importer could take the home importer's price as the relevant marginal cost and further applies a markup that fully exploits the elasticity of the foreign country demand. In that case, the foreign importer has an informational advantage over the home importer to set the appropriate price. It follows that two markups would been applied then. The mechanism is identical no matter the goods are final (investment or consumption) or intermediate.[31]

According to the LCP of the final goods as assumed for consumption goods in (8) and for investment goods in (17) and according to equilibrium condition (54), we can define the net exports for final tradable goods as:[32]

$$
NX_{h,t}^{FT} \equiv P_{h,t}^{*CT}\left(C_{h,t}^{*T}+G_{h,t}^{*T}\right)+P_{h,t}^{*IT} I_{h,t}^{*T}-S_t\left(P_{f,t}^{CT}\left(C_{f,t}^T+G_{f,t}^T\right)+P_{f,t}^{IT} I_{f,t}^T\right) \tag{69}
$$

[30] This equality means that foreign investor i obtains the same return by buying a home issued asset as the return that is obtained by the home investor from buying a foreign bond. Since the risk premium captures country-specific risks, the underlying interest rates can differ.

[31] See the first section of the internet appendix for the calculation of the importer price.

[32] See Plasmans *et al.* (2006b) for an example of an NFAs equation for a three-country setting and proper definitions of corresponding net exports.

and

$$NX_{h,t}^{VT} \equiv P_{h,t}^{VT,X,FT} V_{h,t}^{*T,FT} + P_{h,t}^{VT,X,FN} V_{h,t}^{*T,FN} - S_t \left(P_{f,t}^{*VT,X,FT} V_{f,t}^{T,FT} + P_{f,t}^{*VT,X,FN} V_{f,t}^{T,FN} \right), \tag{70}$$

respectively.

Expression (68) (with definitions (69) and (70)) is log-linearized in Appendix G.

8 Price and wage settings

This section is motivated from the stickiness observed both in prices and wages. Empirical evidence surveyed by Taylor (1998) stresses that even though casual observation might suggest that wages are more sticky than prices, detailed studies suggest that price and wage changes have about the same average frequency of 1 year with highly non-synchronized timings. It is also found that there is a great deal of heterogeneity in wage and price changes and that the overall duration depends on the average rate of inflation.

8.1 Sticky prices with staggered price setting

In order to generate price stickiness, let us assume that each domestic firm j sets its own product price according to Calvo's rule (see Calvo (1983)) or by adopting Taylor contracts (Taylor (1980)). We analyse this price stickiness in the following subsections.

8.1.1 Calvo staggered price setting

Following Calvo (1983), we assume that domestic firms (suppliers) adjust their price infrequently and in such an event, they reset prices according to "price signals", which follow an exogenous *iid* Poisson process with constant probability, independently of past history. For instance, for the first entry in the FT market this probability is $1 - \varphi_{CT}^{(j)}$, meaning that firm j in domestic sector $m = FT$ will not be able to adjust its price on its market CT with probability $\varphi_{CT}^{(j)}$. This probability is the so-called Calvo price parameter.[33]

Suppose that there is a continuum of independent firms indexed in the $(0,1)$ interval, so that the law of large numbers can be applied and, consequently, we can drop the Calvo price parameter's upper index j. Conditional on the fact that firm j does not receive a price signal and keeps prices constant in the future, each firm solves:

$$\max_{\left\{\mathbf{S}_{h,t}^m(j)\right\}} E_t \left\{ \sum_{a=0}^{\infty} \Delta_{t,t+a}(i) (\varphi_{\mathbf{m}})^a \left[\left[\mathbf{S}_{h,t}^m(j)\right]' \mathbf{Y}_{h,t+a}^m(j) - TC_{t+a}^m(j) \left(\left[\mathbf{Y}_{h,t+a}^m(j)\right]' \iota^m \right) \right] \right\}, \tag{71}$$

subject to appropriate demand functions, e.g., under equilibrium, the aggregate optimal demand for a domestic tradable consumption good $n = j$ becomes, similarly to (12),

$$Y_{h,t+a}^{CT}(j) = \left(\frac{P_{h,t}^{CT}(j)}{P_{h,t+a}^{CT}} \right)^{-\theta_{CT}} Y_{h,t+a}^{CT}, \tag{72}$$

taking into account equilibrium conditions in Section 6. The solution is a vector of domestic optimal prices for each producer j in sector m (see Ascari (2004)).

The shareholders-households' nominal discount factor, consistent with the profit-maximizing objective of the monopolistically competitive firm, is equal to $\Delta_{t,t+a}(i) = \beta^a \frac{E_t[\Gamma_{t+a}(i)]}{\Gamma_t(i)}$ in (71).[34] Hence, $\Delta_{t,t+a}(i)$ represents the (nominal) discount factor from t to $t+a$ applied by firm j to the stream of future profits, $\mathbf{S}_{h,t}^m(j)$ is defined in Section 6 for sector m and $TC_{t+a}^m(j)(.)$ is (nominal) total cost of production at period $t+a$ of firm j in domestic sector m, which is a function of firm j's total output during period t. Moreover, ι^m is the unity vector consisting of an appropriate number of ones which equals the number of markets and $(\varphi_{\mathbf{m}})^a$ is a vector of probabilities that

[33] Hence, the average duration between two subsequent price adjustments can be computed as $\frac{1}{1-\varphi_{CT}^{(j)}}$, since $0 < \varphi_{CT}^{(j)} < 1$. For example, a Calvo price parameter equal to 0.75 implies an average duration of 4 periods.

[34] This relationship comes from FOC (39) for home assets. Indeed, it can be claimed that from that FOC $(1 + r_{t,t+a}(i))^{-a} = \beta^a \frac{E_t[\Gamma_{t+a}(i)]}{\Gamma_t(i)}$. Note that this formulation does not include price indexation. This can be introduced by multiplying prices by the relevant sectoral inflation. Then, (71) remains the same, but inflation must correct prices $\mathbf{\Pi}_{h,t}(j)' \left[\mathbf{S}_{h,t}^m(j)\right]$ and also demands (72). For instance, the first entry of the inflation vector $\mathbf{\Pi}_{h,t}(j)$ is then $\Pi_{h,t}^{CT}(j) \equiv \frac{P_{h,t}^{CT}(j)}{P_{h,t-1}^{CT}(j)}$.

price vector $\mathbf{S}_{h,t}^{m}(j)$ remains unchanged for producer j in domestic sector m. Entries of this vector correspond to elements of relevant price vectors $\mathbf{P}_{h,t}^{FT}(j)$, $\mathbf{P}_{h,t}^{FN}(j)$, $\mathbf{P}_{h,t}^{VT}(j)$, $\mathbf{P}_{h,t}^{VN}(j)$, $\mathbf{P}_{h,t}^{MF}(j)$ and $\mathbf{P}_{h,t}^{MV}(j)$. For instance, $\varphi_{FT} \equiv [\varphi_{CT}, \varphi_{CT}^{X}, \varphi_{IT}^{FT}, \varphi_{IT}^{FN}, \varphi_{IT}^{VT}, \varphi_{IT}^{VN}, \varphi_{IT}^{X}]'$ and $\varphi_{VN} \equiv [\varphi_{VN}^{FT}, \varphi_{VN}^{FN}]'$.[35]

Solving (71) subject to demands (as e.g. (72)), we obtain the following optimality condition (see Appendix I):

$$\breve{\mathbf{P}}_{h,t}^{m}(j,i) = \begin{bmatrix} \frac{\theta_1}{(\theta_1-1)} \dfrac{E_t \sum_{a=0}^{\infty} (\varphi_1\beta)^a \frac{\Gamma_{t+a}(i)}{\Gamma_t(i)} \left[MC_{t+a}^{m}\left(Y_{h,t+a}^{m}(j)\right)\left(P_{h,t+a}^{1}\right)^{\theta_1} Y_{h,t+a}^{1} \right]}{E_t \sum_{a=0}^{\infty} (\varphi_1\beta)^a \frac{\Gamma_{t+a}(i)}{\Gamma_t(i)} \left[Y_{h,t+a}^{1}\left(P_{h,t+a}^{1}\right)^{\theta_1} \right]}, \\ \vdots \\ \frac{\theta_{n_m}}{(\theta_{n_m}-1)} \dfrac{E_t \sum_{a=0}^{\infty} (\varphi_{n_m}\beta)^a \frac{\Gamma_{t+a}(i)}{\Gamma_t(i)} \left[MC_{t+a}^{m}\left(Y_{h,t+a}^{m}(j)\right)\left(P_{h,t+a}^{n_m}\right)^{\theta_{n_m}} Y_{h,t+a}^{n_m} \right]}{E_t \sum_{a=0}^{\infty} (\varphi_{n_m}\beta)^a \frac{\Gamma_{t+a}(i)}{\Gamma_t(i)} \left[Y_{h,t+a}^{n_m}\left(P_{h,t+a}^{n_m}\right)^{\theta_{n_m}} \right]} \end{bmatrix}, \tag{73}$$

where firm j's prices $\breve{\mathbf{P}}_{h,t}^{m}(j,i)$ in domestic sector m are aggregated over consumers as it is done for wages in equation (102) in Appendix B.2, resulting in vectors $\breve{\mathbf{P}}_{h,t}^{m}(j)$.

Since any domestic price at period t, $P_{h,t}^{n}(j)$ $(n = 1, ..., n_m)$, is assumed to be a CES aggregator (see e.g. (15)) of the predetermined price $\left\{P_{h,t-1}^{n}(j)\right\}$ and the newly set price $\breve{P}_{h,t}^{n}(j)$ according to Calvo in (73), this domestic price index for a typical domestic company j in sector m operating on market n $(n = 1, ..., n_m)$ can be written as:

$$P_{h,t}^{n}(j) = \left[\varphi_n \left(P_{h,t-1}^{n}(j)\right)^{1-\theta_n} + (1-\varphi_n)\left(\breve{P}_{h,t}^{n}(j)\right)^{1-\theta_n}\right]^{\frac{1}{1-\theta_n}}. \tag{74}$$

Equation (74) is log-linearized in Appendix I.

8.1.2 Taylor staggered price setting

Following the Chari *et al.* (2000) notation for a Taylor staggered price setting model, we assume that in each sector $m = FT, FN, VT, VN, MF, MV$ firms set new prices for N periods. Taylor's (1980) staggered price setting assumes that the average fraction of resetting prices is $\frac{1}{N_m}$ with an equal fraction of firms distributed in each renegotiation period. Hence, the first fraction of producers or importers j in each sector m are indexed, $j \in [0, 1/N_m]$, and set new prices in periods 0, N_m, $2N_m$, ..., and producers or importers, indexed as $j \in [1/N_m, 2/N_m]$, set new prices in 1, $N_m + 1$, $2N_m + 1$, ... Altogether, there are N_m cohorts. Given the specification for $\Delta_{t,t+a}(i)$ from the previous subsection, the vector of domestic prices is derived from the solution of the expected profit maximization problem:

$$\max_{\mathbf{P}_{h,t}^{m}(j)} E_t \left\{ \sum_{a=t}^{t+N_m-1} \beta^a \frac{E_t\left[\Gamma_{t+a}(i)\right]}{\Gamma_t(i)} \left[\left[\mathbf{S}_{h,t}^{m}(j)\right]' \mathbf{Y}_{h,t+a}^{m}(j) - TC_{t+a}^{m}(j)\left(\left[\mathbf{Y}_{h,t+a}^{m}(j)\right]' \iota^m\right) \right] \right\}, \tag{75}$$

s.t. relevant demand functions as e.g. (72). The constrained maximization leads to an optimal price vector similar to (73); however, it differs in the fact that $\varphi_n = 1$ $(n = 1, 2, ..., n_m)$ and the upper limit of the summation being $t + N_m - 1$ instead of ∞:

$$\breve{\mathbf{P}}_{h,t}^{m}(j,i) = \begin{bmatrix} \frac{\theta_1}{(\theta_1-1)} \dfrac{E_t \sum_{a=t}^{t+N_m-1} \beta^a \frac{\Gamma_{t+a}(i)}{\Gamma_t(i)} \left[MC_{t+a}^{m}\left(Y_{h,t+a}^{m}(j)\right)\left(P_{h,t+a}^{1}\right)^{\theta_1} Y_{h,t+a}^{1} \right]}{E_t \sum_{a=t}^{t+N_m-1} \beta^a \frac{\Gamma_{t+a}(i)}{\Gamma_t(i)} \left[Y_{h,t+a}^{1}\left(P_{h,t+a}^{1}\right)^{\theta_1} \right]}, \\ \vdots \\ \frac{\theta_{n_m}}{(\theta_{n_m}-1)} \dfrac{E_t \sum_{a=t}^{t+N_m-1} \beta^a \frac{\Gamma_{t+a}(i)}{\Gamma_t(i)} \left[MC_{t+a}^{m}\left(Y_{h,t+a}^{m}(j)\right)\left(P_{h,t+a}^{n_m}\right)^{\theta_{n_m}} Y_{h,t+a}^{n_m} \right]}{E_t \sum_{a=t}^{t+N_m-1} \beta^a \frac{\Gamma_{t+a}(i)}{\Gamma_t(i)} \left[Y_{h,t+a}^{n_m}\left(P_{h,t+a}^{n_m}\right)^{\theta_{n_m}} \right]} \end{bmatrix}, \tag{76}$$

[35] The vector of probabilities has the same dimension as the number of markets that firm j supplies. For example, if firm j produces a final tradable good, then 7 different Calvo price probabilities are theoretically possible, e.g. the entry φ_{IT}^{FT} is the fraction of final-tradable-goods-producing firms that are not resetting their prices on the domestic market of tradable investment goods.

The other four vectors of Calvo price parameters are $\varphi_{FN} \equiv [\varphi_{CN}, \varphi_{IN}^{FT}, \varphi_{IN}^{FN}, \varphi_{IN}^{VT}, \varphi_{IN}^{VN}]'$, $\varphi_{VT} \equiv [\varphi_{VT}^{FT}, \varphi_{VT}^{FN}, \varphi_{*VT}^{FT}, \varphi_{*VT}^{FN}]'$, $\varphi_{MF} \equiv [\varphi_{MF}^{CT}, \varphi_{*IT}^{FT}, \varphi_{*IT}^{FN}, \varphi_{*IT}^{VT}, \varphi_{*IT}^{VN}]'$ and $\varphi_{MV} \equiv [\varphi_{*VT}^{FT}, \varphi_{*VT}^{FN}]'$. The last two vectors contain importers' Calvo price parameters for all markets they serve.

where, similarly to the previous subsection, we have to aggregate optimal Taylor prices over all consumers. The Taylor aggregate price indices are then given by:

$$P^n_{h,t}(j) = \left[\frac{1}{N_n}\sum_{a=t}^{t+N_n-1}\left(\breve{P}^n_{h,t-a}(j)\right)^{1-\theta_n}\right]^{\frac{1}{1-\theta_n}}, \tag{77}$$

which are log-linearized in Appendix J.

Comparing the Calvo and Taylor prices (74) and (77), we conclude that the pricing rule assumed has important consequences for the persistence of CPI inflation. The key difference in those forms of pricing is that in Calvo pricing, the event of not being allowed to reset prices for a large number of periods has a non-zero probability. Therefore, Calvo contracts pick up the randomness of price changes. The Taylor pricing rule leads to a price index that is an average of all the optimal prices allowed in the contemporaneous contracts.

8.2 Sticky wages with staggered wage setting

8.2.1 Calvo staggered wage setting

Given the monopolistically competitive structure of the labor market, forward-looking households set nominal wages in staggered contracts that are analogous to the price contracts described above. More specifically, the fraction of wages that are kept sticky is φ^W_m. Hence, in any period in which household i is able to reset its wage contract, it maximizes the expected utility (1) with period utility function (35) with respect to wage rates $W^m_t(i)$, subject to household i's total supply of labor (103) from Appendix B.2 and CBC (7). According to Appendix K, this maximization leads to:

$$\breve{W}^m_t(i) = \frac{\varrho_{Lm}}{(\varrho_{Lm}-1)}\frac{E_t\sum_{a=0}^{\infty}(\beta\varphi^W_m)^a\left[\left(W^m_{t+a}\right)^{\varrho_{Lm}}L^m_{t+a}\left(L_{t+a}(i)\right)^{\phi}\right]}{E_t\sum_{a=0}^{\infty}(\beta\varphi^W_m)^a\left[\left(W^m_{t+a}\right)^{\varrho_{Lm}}L^m_{t+a}\left(1-\tau^w_t\right)\Gamma_t(i)\right]}. \tag{78}$$

Since any (domestic) wage at period t, $W^m_t(i)$, is assumed to be determined by the CES aggregator (as in formula (15)) of the predetermined wage $W^m_{t-1}(i)$ and the newly set wage, $W^m_t(i)$, according to Calvo a wage index for sector m is:

$$W^m_t(i) = \left[\varphi^W_m\left(W^m_{t-1}(i)\right)^{1-\gamma_m} + \left(1-\varphi^W_m\right)\left(\breve{W}^m_t(i)\right)^{1-\gamma_m}\right]^{\frac{1}{1-\gamma_m}}, \tag{79}$$

which is log-linearized in Appendix K.

8.2.2 Taylor staggered wage setting

Similarly to Subsection 8.1.2, we assume that in each producing sector $m = FT, FN, VT, VN$ a fraction $\frac{1}{N^W_m}$ of workers set new wages for N^W_m periods. Workers in each sector m are indexed by $j \in [0, 1/N^W_m]$ and set new wages in periods 0, N^W_m, $2N^W_m$, ..., and so forth, similarly to Subsection 8.1.2. The optimal wage in sector m is derived from the solution of the expected utility maximization problem:

$$\max_{\mathbf{P}^m_{h,t}(j)} E_t\left\{\sum_{a=t}^{t+N^W_m-1}\beta^a\left[\frac{1}{1-\sigma}\left(\frac{C_{t+a}}{(C_{t+a-1})^{\kappa}}\right)^{1-\sigma} - \frac{\left(L^m_{t+a}(i)\right)^{1+\phi}}{1+\phi} + \chi_M\frac{\left(\frac{M_{t+a+1}(i)}{P^C_{t+a}}\right)^{1-\frac{1}{\chi}}}{\left(1-\frac{1}{\chi}\right)}\right]\right\}, \tag{80}$$

s.t. relevant demand functions for labor where the Taylor assumption is applied and subject to CBC (7) This constrained maximization leads to the optimal wages:

$$\breve{W}^m_t(i) = \frac{\varrho_{Lm}}{(\varrho_{Lm}-1)}\frac{E_t\sum_{a=t}^{t+N^W_m-1}\beta^a\left[\left(W^m_{t+a}\right)^{\varrho_{Lm}}L^m_{t+a}L^{\phi}_{t+a}\right]}{E_t\sum_{a=t}^{t+N^W_m-1}\beta^a\left[\left(W^m_{t+a}\right)^{\varrho_{Lm}}L^m_{t+a}\left(1-\tau^w_t\right)\Gamma_t(i)\right]}. \tag{81}$$

The Taylor aggregate wage indices are obtained analogously to Subsection 8.1.2:

$$W^m_t(i) = \left[\left(N^W_m\right)^{-1}\sum_{a=t}^{t+N^W_m-1}\left(\breve{W}^m_{t-a}(i)\right)^{1-\gamma_m}\right]^{\frac{1}{1-\gamma_m}}, \tag{82}$$

which is log-linearized in Appendix L. As in the previous subsection, a similar interpretation can be done; however, the idea of Taylor contracting seems to be more realistic.

9 Monetary policy rules

Designing monetary policy rules concerns the choice of (a) the monetary policy instruments, (b) the variables which are targeted and (c) their targeted values. In the literature, the most important variables that are targeted by a central bank (CB) are: (1) real output (gap), (2) (changes in) prices, (3) (changes in) wages, (4) (changes in) exchange rates, (5) (changes in) interest rates, (6) a combination of real output and prices in the form of nominal GDP.

Kydland and Prescott (1977) claim that monetary policy effectiveness depends not only on policy actions undertaken but also on the public perception about these actions and its expectations about future actions. Consequently, policy is more effective when future actions are predictable so that a monetary authority can commit itself to a certain course of policies. As Atoian *et al.* (2004) argue, commitment permits the CB to distribute 'policy medicine' over time. For example, when the CB wishes to offset inflation that will result from a supply shock, under commitment, it can raise interest rates moderately provided that it maintains higher rates for a period of time. In contrast, in the case of lack of commitment, a higher initial rate increase will be necessary because of the public doubts that the CB will sustain this interest rate increase. Atoian *et al.* (2004) also argue that optimal commitment does not need to take the form of a reaction function with fixed coefficients. In general, an optimal commitment rule has the form of a state-contingent plan that presents the instrument setting as a function of the history of exogenous shocks. However, optimal commitment is not practical because, first, as noted by Woodford (2003), it is not feasible to provide an advance listing of all relevant contingencies and, second, it is difficult for the public to distinguish between discretion and a complicated contingency rule. Both problems are avoided when the CB commits to a rule with fixed coefficients.

Which form should such a rule with fixed coefficients take? Most CBs use the short-term nominal interest rate as their control variable, depending on economic conditions. The most famous and widely used examples of interest rate rules are those proposed by John Taylor. The log-linearized **standard Taylor rule** (see Taylor (1993)) relates the interest rate to inflation and (logarithmic) output gap:

$$r_{t,t+1} = \vartheta_0 + \vartheta_1 \pi_{h,t} + \vartheta_2 y_{h,t} + \varepsilon_t, \tag{83}$$

where $\pi_{h,t}$ and $y_{h,t}$ are annualized inflation and (logarithmic) deviations of output w.r.t its steady state value, which are assumed to be the target variables of a monetary authority. Taylor (1993) assigns coefficient values consistent with an accurate description of Federal Reserve policy for quarterly data and domestic annualized inflation (so instead of, $\pi_{h,t}$, we use $\pi_{h,t}^{(4)} \equiv \sum_{j=0}^{3} \pi_{h,t-j}$) as $\vartheta_1 = 1.5$ and $\vartheta_2 = 0.5$. The intuition for the value of the former reaction parameter is that the CB must raise the interest rate by more than any increase in inflation in order to raise the real rate of interest, cool the economy, and move inflation back toward its target.

We can study two institutional arrangements concerning the degree of integration of monetary authorities: (i) each CB is autonomous and follows its own monetary policy rule or (ii) an institutional setting in which the home country and the foreign country already agreed in a common monetary policy rule. Therefore, the common CB is going to set the nominal interest rate taking weighted aggregates into account (as, for instance, in the Maastricht Treaty). Define n_h as the share of the home country in the world GDP and the counterpart $1 - n_h$ as the share of the foreign country in the world GDP. In any of the institutional arrangements, given the two-level production functions (29)-(31) and (34) of our model, it is assumed that the CB targets only the final goods deviations from the steady state production. Since a CB is in general primarily interested in targeting CPI inflation, which is close to targeting inflation of final goods, we do not consider outputs of intermediate goods in the proposed monetary policy rules (although these intermediate goods are important for the whole economy as the before-mentioned Dutch example illustrates).

Therefore, we consider the following monetary policy rules:[36]

(I) The **Henderson-McKibbin and Taylor (HMT) rule for the monetary union**, which is a direct extension of the standard Taylor rule (83) for a setup with weighted tradable and non-tradable outputs (see Collard and Dellas (2004)):

$$r_{t,t+1} = \vartheta_0^I + \vartheta_1^I \pi_{h,t}^{(4)} + \vartheta_2^I y_{h,t}^T + \vartheta_3^I y_{h,t}^N + \varepsilon_{1t}, \tag{84}$$

where y_h^T and y_h^N are the logarithmic deviations of domestic outputs of tradable and non-tradable final goods from their steady state values. Since only final goods are considered, it is expected that the sum of the values ϑ_2^I and ϑ_3^I is (much) smaller than 0.5 (as found by Taylor (1993))

[36] These monetary policy rules can be easily extended to an institutional setting with a monetary union. See Plasmans *et al.* (2006b) for an application with three countries, where two of them are members of a monetary union.

(II) Taylor (1999) suggests another alternative for the standard Taylor rule (83) that allows for **interest-rate smoothing**:

$$r_{t,t+1} = \vartheta_0^{II} + \vartheta_1^{II}\pi_{h,t}^{(4)} + \vartheta_2^{II}y_{h,t}^T + \vartheta_3^{II}y_{h,t}^N + \vartheta_4^{II}r_{t-1,t} + \varepsilon_{2t}, \tag{85}$$

where $\vartheta_4^{II} > 0$ is a smoothing parameter.

Finally, we are also interested in (III) a **Taylor-type rule with wage inflation:**

$$r_{t,t+1} = \vartheta_0^{III} + \vartheta_1^{III}\pi_{h,t}^{(4)} + \vartheta_2^{III}\pi_{h,t}^{W(4)} + \vartheta_3^{III}y_{h,t}^T + \vartheta_4^{III}y_{h,t}^N + \vartheta_5^{III}r_{t-1,t} + \varepsilon_{3t}, \tag{86}$$

where $\pi_{h,t}^{W(4)} \equiv \sum_{j=0}^{3}\pi_{h,t-j}^W$ is the domestic annualized wage inflation.[37]

The foreign country CB is committed to similar Taylor-type rules as specified in (84), (85) and (86) with the instrument being r_t^* depending on its country variables such as (price) inflation, wage inflation and tradable and non-tradable output gaps.

Mc Callum (1997) argues that the policymakers' reaction is more accurate if it is based on lagged and not on current values of output and inflation. In response, Taylor (1999) suggests an alternative form of his rules where lagged values of output and inflation replace the current values in (85). In contrast, Clarida *et al.* (1998) and others argue that rules in which the CB reacts to forward-looking variables are optimal in the case of a quadratic objective function for the monetary authorities, which will be also utilized in this paper. The difference between backward-looking, contemporaneous and forward-looking monetary rules relates primarily to the information set at the disposal of the monetary policymakers. For instance, in the case of a contemporaneous rule the actual inflation rate, on which the CB is assumed to have adequate information, is targeted.

10 Numerical simulations

Given the large number of parameters involved in our DSGE model, it might not be trivial to know *a priori* the set of coefficients that assure the rank condition for the solution of forward-looking (jump) variables (see Blanchard and Kahn (1980)).[38] To assure a unique solution we parameterize the model following the recent literature. The model is transformed into the state space form in order to make the following numerical (stochastic) simulation experiment, where some of the endogenous variables are considered as unobserved. The model is estimated using (observable) variables from national accounts, though these resulting econometric estimations are beyond the scope of this paper.[39] Our computations are performed using the DYNARE toolbox for Matlab (see Juillard (2005)).

1. In Table 1 in Appendix M, we inform about priors of 'deep' structural parameters, AR coefficients of auto-correlated shocks and standard deviations of these shocks used in the stochastic simulation experiment. The CB targets aggregate zero inflation and final output deviations from the steady state and it is assumed that both CBs in a two-country setting are fully committed to rule II (equation (85)). In addition, price and wage settings are assumed à la Calvo. In order to check the accuracy of the specification, we simulate the full model and two reduced versions, namely, (i) a specification without capital goods and (ii) a specification that disregards capital and intermediate goods. These alternative specifications give rise to different responses in the form of impulse response functions (IRFs) that are denoted by dotted black lines for the full model, green full lines for the model without capital and blue full lines for the model without capital and intermediate goods. In the following subsections we analyze these key model variables' responses (expressed in logarithmic deviations from the corresponding steady state) when the home economy is hit by: (i) a productivity shock in the production of tradable and non-tradable goods, (ii) a shock on home money demand that increases the domestic interest rate and (iii) a shock in the exchange rate.

10.1 Technological improvement in the tradable and non-tradable goods production

Figures 1 and 2 in Appendix N depict the IRFs of a productivity improvement in the domestic tradable goods production technology. As a result, marginal costs of tradable goods go down and prices of domestic tradable consumption and investment goods as well (deflation, but temporary for the full model specification). Thus, the

[37] The reader could wonder why the CB would target also wage inflation if it already targets inflation. Concern about wage inflation could result for various reasons, among which: (i) wage inflation could lead to a wage-price spiral and (ii) it could target real wages, e.g. to secure competitiveness and constrain demand-pull inflation it could try to keep real wages low.

[38] Ratto *et al.* (2005, p.14) check systematically for all parameters and determine which of them are more likely to lead to indeterminacy.

[39] Econometric model estimations for a three-country model, including maximum likelihood and Bayesian ones, are included in Plasmans *et al.* (2006b).

domestic demand for consumption goods increases. Sectoral unemployment increases in all specifications, but whereas total unemployment goes up in the full model, the opposite is the case in the model without capital and intermediate goods. In the foreign country, aggregate consumption goes up, while aggregate inflation goes down. The exchange rate appreciates and the positive domestic net exports accumulate into a positive domestic NFAs position that is quite persistent. The home interest rate is shifted down by the domestic CB as a reaction to the deflation that is present in the home economy and also the foreign CB decreases the foreign interest rate, leaving room for possible monetary policy coordination. This is because the domestic deflation gives rise to foreign deflation due to the tradability of the good.

A remarkable characteristic that shows up across all graphs in Figures 1 and 2 is that the full model variables react in a proportionally more active way. This is rather important in the case of the NFAs because the exports of tradable investment goods benefit from technological improvement (a larger domestic NFAs position).

Some other variables are not reported, but are worth mentioning. Consumption of tradable goods goes up, while that of non-tradable goods goes down since the new relative price favors the former goods. Concerning capital, after the shock, capacity utilization and demand for capital in the production of final and intermediate tradable goods go down. However, the capacity utilization and the capital demand in the production of non-tradable goods go up. In addition, the technology shock makes it more attractive to invest in the non-tradable goods sector since a higher demand for these goods is expected. Moreover, disinvestment is verified in the aggregate domestic economy and in the intermediate goods sectors. Aggregate investment increases in the foreign economy.

Real wages and real rental rates of non-tradable capital goods are unambiguously higher, while rental rates of tradable capital goods are lower. The nominal wage increases in the tradable goods production, while it decreases in the non-tradable goods production.

In Figures 3 and 4 in Appendix N, we illustrate the effects on key variables derived from a domestic shock in the productivity of the non-tradable final goods sector. In that instance, domestic consumption goes up and foreign consumption goes down. Deflation is present in the home country but inflation appears in the foreign country. The tradable final goods production decreases in both countries, while the non-tradable final goods production increases domestically, resulting in higher employment on the aggregate level. Final goods production in the foreign country goes down. The domestic NFA position is positive and substantially large in the case of the full model specification (with capital goods). In addition, the home country becomes less competitive since the exchange rate depreciates. Regarding the instrument of the CBs, it is optimal to reduce the domestic interest rate given the deflation of prices. In the foreign country the interest rate is in the very short run lower, but this interest rate is adjusted upward to decrease foreign inflation. Exports of the domestic country decrease less than imports.

10.2 Money demand shock

When a money demand shock hits the home economy as in Figures 5 and 6 of Appendix N, the first effect is a relative scarcity of money that increases the interest rates of the domestic and foreign economies due to motives not explained by the Taylor rule. This nominal shock has consequences for the real variables of the model because of rigidities assumed to be operating in the model. The money demand shock results in a consumption and a domestic production decrease in the short run, while the foreign production increases. Un(der)employment of labor is generated in the short run. The domestic CB raises the interest rates to mitigate the effects of the shock. Also the foreign CB raises the interest rate in the short run. Since agents are forward-looking, negative savings and a negative NFA position emerge. In addition the exchange rate appreciates and domestic exports decrease.

10.3 Exchange rate shock (UIP shock)

Consider the effect of an exchange rate shock (expected depreciation) in Figures 7 and 8 of Appendix N. We observe a positive impact in domestic consumption and the opposite effect in the foreign country, because the depreciation produces an alteration of relative prices and, consequently, an expenditure switching effect that benefits the home country. Notice that since more complex relationships are captured by the fully-specified model, the expenditure-switching effect is mitigated because the depreciation magnifies marginal costs of intermediate imported goods in the home economy. Extra inflation is appearing in the home country and deflation in the foreign country. The home country exports less and the NFA position becomes negative. Because of the inflationary pressure, the domestic CB adjusts the interest rate downwards (and the foreign CB adjusts upwards).

10.4 Sensitivity analysis

In this section we do sensitivity analysis and compare the responses of key models' variables. First, we briefly examine how different monetary policy rules, i.e. Rules I, II and III, discussed in Section 9, affect the response of the interest rate provided that the CB is fully committed to the assumed rule. Second, we briefly analyze the impact of the Taylor contracts in the wage setting in the IRFs of consumption, inflation, tradable and non-tradable output and home aggregate employment.*Jorge or Tomasz, can you give more information about the sensitivity results??*

10.4.1 Monetary policy rules

We examine how the interest rate is adjusted by both fully committed domestic and foreign CBs. To do so, we compute IRFs, which are illustrated in Figures 17 to 28 in Appendix **??**.[40]

From the simulation of all the model specifications and all shocks we conclude that its evolution is similar in all the specifications, though in some special cases the interest rate is evolving in an upper or lower level. In particular, in the model that explicitly accounts for intermediate and capital goods rule III (equation (86)) is responsible for interest rate responses that are more stable and close to the zero line. The previous result is not maintained in simpler model versions: sometimes rule II performs better than rule III in specifications without capital goods and in specifications without capital and intermediate goods. In addition, magnitudes are in accordance with the common wisdom, e.g. given a technological shock in the domestic tradable final goods sector, the foreign CB must decrease the interest rate more in the full model specification (because of higher trade flows in intermediate and capital goods).

10.4.2 Pricing strategies

In Figures 29 to 36 in Appendix **??**,[41] we introduce in the model without intermediates and capital goods alternative price and wage setting assumptions to find out possible differences in the evolution of key endogenous variables.

We simulate three alternative specifications: (i) price and wage settings according to the Calvo rule, (ii) price setting à la Calvo but wages set by yearly contracts ($N_m^W = 4$ for $m = FT, FN, VT, VN, MF, MV$) and (iii) price and wage settings regulated through Taylor contracts ($N_m = N_m^W = 4$ for $m = FT, FN, VT, VN, MF, MV$).

As expected, Taylor contracts in prices and wages tend to produce more fluctuations with a periodicity of around 4 quarters. Given a technological shock in the FT sector, higher fluctuations are remarkably found in domestic and foreign interest rates and imported inflation at home. Moreover, we find higher fluctuations (recession) in domestic tradable output in the case of Taylor price and wages (because of the incidence of yearly contracts). Higher fluctuations are found when the technological shock is in the FN sector; however we do not find notorious differences in responses in the case of monetary and exchange rate shocks. Responses are quite comparable.

11 Concluding remarks

In this paper, a New-Keynesian open economy model was constructed with a detailed treatment of demand and supply relationships. For households the emphasis was put on the presence of habit formation and their supply of capital goods with variable capital utilization to firms, whereas in the production part we focused on different sectors in the economy and linkages between them. In particular, we distinguished between intermediate goods and final goods and between tradable and non-tradable goods (like services). We introduced two models of staggered price and wage setting, Calvo (1983) and Taylor (1980), into our framework. Whereas the theoretical discussion and comparison of these models can be found in Kiley (2002), we also compared them numerically in our setting. The foreign economy is modeled in a parallel manner; consequently, import/export prices and quantities and other relevant variables are endogenized in our approach. In particular, this makes the model more flexible since both small and large economy settings can be studied in this framework.

Responses of alternative monetary policy rules on major model variables were mutually compared. Which simple rule stabilizes most the interest rate? From the stochastic simulation of all the model specifications and all shocks we conclude that interest rate evolution is quite similar, though in some special cases the interest rate is evolving in an upper or lower level.

(*to improve*)As expected, Taylor contracts in prices and wages tend to produce more fluctuations with a periodicity of around 4 quarters. Given a technological shock in the domestic tradable final goods sector, higher

[40] Available on http://www.ua.ac.be/download.aspx?c=joseph.plasmans&n=13825&ct=009362&e=117058.

[41] Available on http://www.ua.ac.be/download.aspx?c=joseph.plasmans&n=13825&ct=009362&e=117058.

fluctuations are remarkably found in domestic and foreign interest rates and imported inflation in the home country. Moreover, we find higher fluctuations (recession) in home tradable output in the case of Taylor price and wages (because of the incidence of yearly contracts). Higher fluctuations are found when the technological shock is in domestic non-tradable final goods sector; however, we do not find notorious differences in responses in the case of monetary and exchange rate shocks. Responses are quite comparable.

Appendix

A The foreign households and government

A.1 Foreign households' consumption demands

The foreign households are assumed to face a problem similar to the one sketched in Section 2. The only (possible) extension exists in the fact that the home country may be a small open economy, so that a somewhat negligible weight is assigned to consumption goods produced in that small economy; then, $\alpha_{Cf}^{*(i)} \simeq 1$ in the foreign consumer i's aggregate consumption function (see (13)):

$$C_t^*(i) \equiv \left[\left(\alpha_{CT}^{*(i)}\right)^{\frac{1}{\eta_{CT}^{*(i)}}} \left(C_t^{*T}(i)\right)^{\frac{\eta_{CT}^{*(i)}-1}{\eta_{CT}^{*(i)}}} + \left(1-\alpha_{CT}^*(i)\right)^{\frac{1}{\eta_{CT}^{*(i)}}} \left(C_{f,t}^{*N}(i)\right)^{\frac{\eta_{CT}^{*(i)}-1}{\eta_{CT}^{*(i)}}} \right]^{\frac{\eta_{CT}^{*(i)}}{\eta_{CT}^{*(i)}-1}}, \tag{87}$$

where $C_t^{*T}(i) \equiv \left[\left(1-\alpha_{Cf}^{*(i)}\right)^{\frac{1}{\eta_{Cf}^{*(i)}}} \left(C_{h,t}^{*T}(i)\right)^{\frac{\eta_{Cf}^{*(i)}-1}{\eta_{Cf}^{*(i)}}} + \left(\alpha_{Cf}^{*(i)}\right)^{\frac{1}{\eta_{Cf}^{*(i)}}} \left(C_{f,t}^{*T}(i)\right)^{\frac{\eta_{Cf}^{*(i)}-1}{\eta_{Cf}^{*(i)}}} \right]^{\frac{\eta_{Cf}^{*(i)}}{\eta_{Cf}^{*(i)}-1}} \simeq C_{f,t}^{*T}(i)$, the latter when the home country is a small open economy.

Total cost-minimizing foreign household i's demands for differentiated goods, their foreign aggregates and belonging foreign prices can be derived in a way similar to (12), (14), (15) and (16), respectively. Hence, minimizing the total foreign consumption expenditure as in (8):

$$P_t^{*C} C_t^* \equiv \int_0^1 \int_0^1 P_{f,t}^{*CT}(z)\, C_{f,t}^{*T}(z,i)\, dzdi + \int_0^1 \int_0^1 P_{f,t}^{*CN}(z)\, C_{f,t}^{*N}(z,i)\, dzdi + \int_0^1 \int_0^1 P_{h,t}^{*CT}(z)\, C_{h,t}^{*T}(z,i)\, dzdi$$

with respect to $C_{f,t}^{*T}(z,i)$, $C_{f,t}^{*N}(z,i)$ and $C_{h,t}^{*T}(z,i)$, subject to $C_{k,t}^{*l}(i) \equiv \left(\int_0^1 C_{k,t}^{*l}(z,i)^{\frac{\theta_{Cl}^{*(i)}-1}{\theta_{Cl}^{*(i)}}} dz \right)^{\frac{\theta_{Cl}^{*(i)}}{\theta_{Cl}^{*(i)}-1}}$ for $(k,l) =$ (f,T), (f,N) and (h,T), with $\theta_{Cl}^{*(i)} > 1$ being the foreign household i's intratemporal elasticity of substitution among consumption goods, straightforwardly yields foreign consumer i's consumption demands for differentiated goods (see (14)):

$$C_{f,t}^{*T}(i) = \alpha_{Cf}^{*(i)} \left(\frac{P_{f,t}^{*CT}(i)}{P_t^{*CT}(i)} \right)^{-\eta_{Cf}^{*(i)}} C_t^{*T}(i) \quad , \quad C_{h,t}^{*T}(i) = \left(1-\alpha_{Cf}^{*(i)}\right) \left(\frac{P_{h,t}^{*CT}(i)}{P_t^{*CT}(i)} \right)^{-\eta_{Cf}^{*(i)}} C_t^{*T}(i),$$

$$C_{f,t}^{*N}(i) = \left(1-\alpha_{CT}^{*(i)}\right) \left(\frac{P_{f,t}^{*CN}(i)}{P_t^{*C}(i)} \right)^{-\eta_{CT}^{*(i)}} C_t^*(i) \text{ and } C_t^{*T}(i) = \alpha_{CT}^{*(i)} \left(\frac{P_t^{*CT}(i)}{P_t^{*C}(i)} \right)^{-\eta_{CT}^{*(i)}} C_t^*(i) \tag{88}$$

with aggregate price indices derived as (see also (15) and (16)):

$$P_t^{*C}(i) = \left[\alpha_{CT}^{*(i)} \left(P_t^{*CT}(i)\right)^{1-\eta_{CT}^{*(i)}} + \left(1-\alpha_{CT}^{*(i)}\right) \left(P_{f,t}^{*CN}(i)\right)^{1-\eta_{CT}^{*(i)}} \right]^{\frac{1}{1-\eta_{CT}^{*(i)}}} \text{ and} \tag{89}$$

$$P_t^{*CT}(i) = \left[\alpha_{Cf}^{*(i)} \left(P_{f,t}^{*CT}(i)\right)^{1-\eta_{Cf}^{*(i)}} + \left(1-\alpha_{Cf}^{*(i)}\right) \left(P_{h,t}^{*CT}(i)\right)^{1-\eta_{Cf}^{*(i)}} \right]^{\frac{1}{1-\eta_{Cf}^{*(i)}}} \tag{90}$$

with $P_t^{*CT}(i) \simeq P_{f,t}^{*CT}(i)$ when the home country is a small open economy.

A.2 Foreign households' investment supplies

The total foreign households' expenditure on investment can be written as (see (17)):

$$P_t^{*I} I_t^* \equiv \int_0^1 \int_0^1 P_{f,t}^{*IT}(z)\, I_{f,t}^{*T}(z,i)\, dzdi + \int_0^1 \int_0^1 P_{f,t}^{*IN}(z)\, I_{f,t}^{*N}(z,i)\, dzdi + \int_0^1 \int_0^1 P_{h,t}^{*IT}(z)\, I_{h,t}^{*T}(z,i)\, dzdi, \tag{91}$$

which is minimized w.r.t. quantities $I^{*T}_{f,t}(z,i)$, $I^{*N}_{f,t}(z,i)$ and $I^{*T}_{h,t}(z,i)$ and where $P^{*IT}_{f,t}(z)$, $P^{*IN}_{f,t}(z)$ and $P^{*IT}_{h,t}(z)$ are the corresponding prices of foreign produced tradable and non-tradable investment goods, and imported tradable investment goods by foreign household i, respectively.

Foreign consumer i's aggregate investment baskets are defined as nested CES specifications (see (18)):

$$I^*_t(i) \equiv \left[\left(\alpha^{*(i)}_{IT}\right)^{\frac{1}{\eta^{*(i)}_{IT}}} \left(I^{*T}_t(i)\right)^{\frac{\eta^{*(i)}_{IT}-1}{\eta^{*(i)}_{IT}}} + \left(1-\alpha^{*(i)}_{IT}\right)^{\frac{1}{\eta^{*(i)}_{IT}}} \left(I^{*N}_{f,t}(i)\right)^{\frac{\eta^{*(i)}_{IT}-1}{\eta^{*(i)}_{IT}}} \right]^{\frac{\eta^{*(i)}_{IT}}{\eta^{*(i)}_{IT}-1}}, \tag{92}$$

with foreign tradable investment, $I^{*T}_t(i)$, by foreign consumer i's as:

$$I^{*T}_t(i) \equiv \left[\left(\alpha^{*(i)}_{If}\right)^{\frac{1}{\eta^{*(i)}_{If}}} \left(I^{*T}_{f,t}(i)\right)^{\frac{\eta^{(i)}_{If}-1}{\eta^{(i)}_{If}}} + \left(1-\alpha^{*(i)}_{If}\right)^{\frac{1}{\eta^{(i)}_{If}}} \left(I^{*T}_{h,t}(i)\right)^{\frac{\eta^{*(i)}_{If}-1}{\eta^{*(i)}_{If}}} \right]^{\frac{\eta^{*(i)}_{If}}{\eta^{*(i)}_{If}-1}}$$

with $\eta^{*(i)}_{IT}$ and $\eta^{*(i)}_{If}$ being foreign consumer i's intratemporal elasticities of substitution between tradable and non-tradable investment goods, and between foreign and imported investment goods, respectively ($\eta^{*(i)}_{If} > \eta^{*(i)}_{IT} > 0$); $\alpha^{*(i)}_{IT}$ and $\alpha^{*(i)}_{If}$ are foreign household i's shares of tradable goods in its total foreign investment and of foreign produced tradable goods in her total foreign investment goods, respectively.

Similar to household i's optimal intratemporal aggregate investment supplies, we get from the minimization of (91):

$$I^{*T}_{f,t}(i) = \alpha^{*(i)}_{If} \left(\frac{P^{*IT}_{f,t}(i)}{P^{*IT}_t(i)}\right)^{-\eta^{*(i)}_{If}} I^{*T}_t(i) \quad , \quad I^{*T}_{h,t}(i) = \left(1-\alpha^{*(i)}_{If}\right) \left(\frac{P^{*IT}_{h,t}(i)}{P^{*IT}_t(i)}\right)^{-\eta^{*(i)}_{If}} I^{*T}_t(i),$$

$$I^{*N}_{f,t}(i) = \left(1-\alpha^{*(i)}_{IT}\right) \left(\frac{P^{*IN}_{f,t}(i)}{P^{*I}_t(i)}\right)^{-\eta^{*(i)}_{IT}} I^*_t(i) \text{ and } I^{*T}_t(i) = \alpha^{*(i)}_{IT} \left(\frac{P^{*IT}_t(i)}{P^{*I}_t(i)}\right)^{-\eta^{*(i)}_{IT}} I^*_t(i), \tag{93}$$

where $P^{*I}_t(i)$ is foreign household i's aggregate investment price index and $P^{*IT}_t(i)$ ($P^{*IN}_t(i)$) is foreign household i's aggregate investment price index of tradable (non-tradable) goods. These prices are derived by minimizing the cost of purchasing one unit of the investment bundle $I^{*T}_t(i)$ as:

$$P^{*I}_t(i) = \left[\alpha^{*(i)}_{IT} \left(P^{*IT}_t(i)\right)^{1-\eta^{*(i)}_{IT}} + \left(1-\alpha^{*(i)}_{IT}\right) \left(P^{*IN}_{f,t}(i)\right)^{1-\eta^{*(i)}_{IT}}\right]^{\frac{1}{1-\eta^{*(i)}_{IT}}} \text{ and} \tag{94}$$

$$P^{*IT}_t(i) = \left[\alpha^{*(i)}_{If} \left(P^{*IT}_{f,t}(i)\right)^{1-\eta^{*(i)}_{If}} + \left(1-\alpha^{*(i)}_{If}\right) \left(P^{*IT}_{h,t}(i)\right)^{1-\eta^{*(i)}_{If}}\right]^{\frac{1}{1-\eta^{*(i)}_{If}}}. \tag{95}$$

A.3 Foreign government

Following Leith and Malley (2003), the foreign government solves a cost-minimization problem similar to the domestic one (see Section 3), leading to demand equations for $z \in [0,1]$:

$$G^{*l}_{k,t}(z) = \left(\frac{P^{*Cl}_{k,t}(z)}{P^{*Cl}_{k,t}(G)}\right)^{-\theta^{*(G)}_{Cl}} \left(\left(\int_0^1 G^{*l}_{k,t}(z)^{\frac{\theta^{*(G)}_{Cl}-1}{\theta^{*(G)}_{Cl}}} dz \right)^{\frac{\theta^{*(G)}_{Cl}}{\theta^{*(G)}_{Cl}-1}} \right) = \left(\frac{P^{*Cl}_{k,t}(z)}{P^{*Cl}_{k,t}(G)}\right)^{-\theta^{*(G)}_{Cl}} G^{*l}_{k,t} \quad (k,l) = (f,T), (f,N) \text{ and } (h,T), \tag{96}$$

with similar interpretations for $\eta^{*(G)}_{CT}$, $\eta^{*(G)}_{Cf}$, $\alpha^{*(G)}_{CT}$ and $\alpha^{*(G)}_{Cf}$ and similar CES aggregators as in Section 3, the foreign aggregate government demands have similar expressions as in (25):

$$G^{*T}_{f,t} = \alpha^{*(G)}_{Cf} \left(\frac{P^{*CT}_{f,t}(G)}{P^{*CT}_t(G)}\right)^{-\eta^{*(G)}_{Cf}} G^{*T}_t, \quad G^{*T}_{h,t} = \left(1-\alpha^{*(G)}_{Cf}\right) \left(\frac{P^{*CT}_{h,t}(G)}{P^{*CT}_t(G)}\right)^{-\eta^{*(G)}_{Cf}} G^{*T}_t,$$

$$G^{*N}_{f,t} = \left(1-\alpha^{*(G)}_{CT}\right) \left(\frac{P^{*CN}_{f,t}(G)}{P^{*C}_t(G)}\right)^{-\eta^{*(G)}_{CT}} G^*_t \text{ and } G^{*T}_t = \alpha^{*(G)}_{CT} \left(\frac{P^{*CT}_t(G)}{P^{*C}_t(G)}\right)^{-\eta^{*(G)}_{CT}} G^*_t \tag{97}$$

with foreign aggregate prices similar to expressions (26) and (27).

B Domestic consumers' constrained maximization

The following Lagrangian, belonging to the constrained maximization of (1) with (35) as period utility function subject to CBC (7) and sectoral law of motion (6) of capital,

$$\mathcal{L}_0(i) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma} \left(\frac{C_t(i)}{(C_{t-1}(i))^{\kappa}} \right)^{1-\sigma} - \frac{\left(\sum_{m=FT,FN,VT,VN} L_t^m(i) \right)^{1+\phi}}{1+\phi} + \chi_M \frac{\left(\frac{M_{t+1}(i)}{P_t^C(i)} \right)^{1-\frac{1}{\chi}}}{\left(1-\frac{1}{\chi}\right)} \right]$$
$$+ E_0 \sum_{t=0}^{\infty} \beta^t \left[\Gamma_t(i) \left(\begin{array}{c} P_t^C(i) C_t(i) + P_t^I(i) \left(\sum_{m=FT,FN,VT,VN} I_t^m(i) \right) + Q_{h,t,t+1} B_{h,t+1}(i) \\ + S_t \hat{Q}_{f,t,t+1} B_{f,t+1}(i) + M_{t+1}(i) + P_t^I(i) \left[\sum_{m=FT,FN,VT,VN} \Psi(\mathcal{Z}_t^m(i)) K_t^m(i) \right] \\ - (1-\tau_t^w) \left(\sum_{m=FT,FN,VT,VN} W_t^m(i) L_t^m(i) \right) - (1-\tau_t^k) \left[\sum_{m=FT,FN,VT,VN} R_t^m(i) \mathcal{K}_t^m(i) \right] \\ - T_t(i) - B_{h,t}(i) - S_t B_{f,t}(i) - M_t(i) - \mathcal{B}_t(i) \end{array} \right) \right]$$
$$+ E_0 \sum_{m=FT,FN,VT,VN} \sum_{t=0}^{\infty} \beta^t \left(\mathcal{Q}_t^m(i) \Gamma_t(i) \right) \left[K_{t+1}^m(i) - (1-d_t^m(i)) K_t^m(i) - \left(1 - \Upsilon \left(\frac{I_t^m(i)}{I_{t-1}^m(i)} \right) \right) I_t^m(i) \right], \qquad (98)$$

has to be differentiated with respect to $C_t(i)$, $L_t^m(i)$, $I_t^m(i)$, household i's nominal domestic and foreign assets (consisting of bonds and stocks) at the beginning of period $t+1$, i.e. $B_{h,t+1}(i)$ and $B_{f,t+1}(i)$, $M_{t+1}(i)$, $K_{t+1}^m(i)$, $\mathcal{Z}_{t+1}^m(i)$, $\Gamma_t(i)$ and $\mathcal{Q}_t^m(i)\Gamma_t(i)$ and the resulting 26 equations have to be equalized to zero (FOCs). To do this, consider that two subsequent elements of Lagrangian (98) sum up to:

$$\mathcal{L}_0^-(i) \equiv E_0 \beta^t \left[\frac{1}{1-\sigma} \left(\frac{C_t(i)}{(C_{t-1}(i))^{\kappa}} \right)^{1-\sigma} - \frac{\left(\sum_{m=FT,FN,VT,VN} L_t^m(i) \right)^{1+\phi}}{1+\phi} + \chi_M \frac{\left(\frac{M_{t+1}(i)}{P_t^C(i)} \right)^{1-\frac{1}{\chi}}}{\left(1-\frac{1}{\chi}\right)} \right]$$
$$+ E_0 \beta^{t+1} \left[\frac{1}{1-\sigma} \left(\frac{C_{t+1}(i)}{(C_t(i))^{\kappa}} \right)^{1-\sigma} - \frac{\left(\sum_{m=FT,FN,VT,VN} L_{t+1}^m(i) \right)^{1+\phi}}{1+\phi} + \chi_M \frac{\left(\frac{M_{t+2}(i)}{P_{t+1}^C(i)} \right)^{1-\frac{1}{\chi}}}{\left(1-\frac{1}{\chi}\right)} \right]$$
$$+ E_0 \beta^t \Gamma_t(i) \left(\begin{array}{c} P_t^C(i) C_t(i) + P_t^I(i) \left(\sum_{m=FT,FN,VT,VN} I_t^m(i) \right) + Q_{h,t,t+1} B_{h,t+1}(i) \\ + S_t \hat{Q}_{f,t,t+1} B_{f,t+1}(i) + M_{t+1}(i) + P_t^I(i) \left[\sum_{m=FT,FN,VT,VN} \Psi(\mathcal{Z}_t^m(i)) K_t^m(i) \right] \\ - (1-\tau_t^w) \left(\sum_{m=FT,FN,VT,VN} W_t^m(i) L_t^m(i) \right) - (1-\tau_t^k) \left[\sum_{m=FT,FN,VT,VN} R_t^m(i) \mathcal{K}_t^m(i) \right] \\ - T_t(i) - B_{h,t}(i) - S_t B_{f,t}(i) - M_t(i) - \mathcal{B}_t(i) \end{array} \right)$$
$$+ E_0 \beta^{t+1} \Gamma_{t+1}(i) \left(\begin{array}{c} P_{t+1}^C(i) C_{t+1}(i) + P_{t+1}^I(i) \left(\sum_{m=FT,FN,VT,VN} I_{t+1}^m(i) \right) + Q_{h,t+1,t+2} B_{h,t+2}(i) + M_{t+2}(i) \\ + S_{t+1} \hat{Q}_{f,t+1,t+2} B_{f,t+2}(i) + P_{t+1}^I(i) \left[\sum_{m=FT,FN,VT,VN} \Psi(\mathcal{Z}_{t+1}^m(i)) K_{t+1}^m(i) \right] \\ - (1-\tau_{t+1}^w) \left(\sum_{m=FT,FN,VT,VN} W_{t+1}^m(i) L_{t+1}^m(i) \right) - (1-\tau_{t+1}^k) \left[\sum_{m=FT,FN,VT,VN} R_{t+1}^m(i) \mathcal{K}_{t+1}^m(i) \right] \\ - T_{t+1}(i) - B_{h,t+1}(i) - S_{t+1} B_{f,t+1}(i) - M_{t+1}(i) - \mathcal{B}_{t+1}(i) \end{array} \right)$$
$$+ E_0 \sum_{m=FT,FN,VT,VN} \beta^t \left(\mathcal{Q}_t^m(i) \Gamma_t(i) \right) \left[K_{t+1}^m(i) - (1-d_t^m(i)) K_t^m(i) - \left(1 - \Upsilon \left(\frac{I_t^m(i)}{I_{t-1}^m(i)} \right) \right) I_t^m(i) \right]$$
$$+ E_0 \sum_{m=FT,FN,VT,VN} \beta^{t+1} \left(\mathcal{Q}_{t+1}^m(i) \Gamma_{t+1}(i) \right) \left[K_{t+2}^m(i) - (1-d_{t+1}^m(i)) K_{t+1}^m(i) - \left(1 - \Upsilon \left(\frac{I_{t+1}^m(i)}{I_t^m(i)} \right) \right) I_{t+1}^m(i) \right]. \qquad (99)$$

B.1 Consumption

Equation (99) can be rewritten as a function of consumption as (omitting irrelevant terms and letting $K_t(i) \equiv \frac{C_t(i)}{(C_{t-1}(i))^{\kappa}}$):

$$\mathcal{L}_0^-(i) = E_0\left[\beta^t \frac{1}{1-\sigma}(K_t(i))^{1-\sigma}\right] + E_0\left[\beta^{t+1}\frac{1}{1-\sigma}(K_{t+1}(i))^{1-\sigma}\right] + E_0\left[\beta^t \Gamma_t(i) P_t^C(i) C_t(i)\right] + ... \tag{100}$$

Differentiating (100) w.r.t. the current level of household i's consumption $C_t(i)$ and setting equal to 0 yields, taking account of the property that $\frac{\partial K_t(i)}{\partial C_t(i)} = \frac{1}{C_t(i)} K_t(i)$ and $\frac{\partial K_{t+1}(i)}{\partial C_t(i)} = -\frac{\kappa}{C_t(i)} K_{t+1}(i)$:

$$\frac{\partial \mathcal{L}_0^-(i)}{\partial C_t(i)} = E_0\left\{\beta^t\left[(K_t(i))^{-\sigma}\frac{1}{C_t(i)}K_t(i) + \Gamma_t(i)P_t^C(i)\right] + \beta^{t+1}\left[-\frac{\kappa}{C_t(i)}(K_{t+1}(i))^{-\sigma}K_{t+1}(i)\right]\right\} = 0,$$

which can be simplified to:

$$E_0\left\{\left[\frac{1}{C_t(i)}(K_t(i))^{1-\sigma} - \kappa\beta\frac{1}{C_t(i)}(K_{t+1}(i))^{1-\sigma}\right] + \Gamma_t(i)P_t^C(i)\right\} = 0$$

and which is, after substitution of $K_t(i)$, FOC (36) in the main text.

B.2 Labor

It is assumed that each household is a monopolistic supplier of its own labor. Thus, the demand for labor i by company j in sector $m = FT, FN, VT, VN$ is $L_t^m(j,i)$. This demand results from cost minimization of the nominal wage bill paid by firm j, i.e. by $W_t^m(j)L_t^m(j) \equiv \int_0^1 W_t^m(j,i)L_t^m(j,i)di$. The optimization of this wage bill subject to equation (32) yields an expression compatible to (12):

$$L_t^m(j,i) = \left(\frac{W_t^m(j,i)}{W_t^m(j)}\right)^{-\varrho_{Lm}^{(j)}} L_t^m(j), \tag{101}$$

where for any category of final goods, $\varrho_{Lm}^{(j)} > 1$ is the intratemporal elasticity of substitution for different types of labor demanded by firm j in sector m. Moreover, firm j's wage index $W_t^m(j)$ of all demanded labor inputs is derived in analogy to (11) as the minimum cost of one unit of the labor bundle $L_t^m(j,i)$:

$$W_t^m(j) = \left(\int_0^1 W_t^m(j,i)^{1-\varrho_{Lm}^{(j)}} di\right)^{\frac{1}{1-\varrho_{Lm}^{(j)}}}. \tag{102}$$

Taking account of the assumptions of continuity and (perfect) competition among firms for workers (workers are assumed to supply labor monopolistically) in sector $m = FT, FN, VT, VN$, i.e. $W_t^m(j) = W_t^m$ and $\varrho_{Lm}^{(j)} = \varrho_{Lm}$ (constant rate of labor substitution within an industry), total household i's supply of labor to sector m is:

$$L_t^m(i) \equiv \int_0^1 L_t^m(j,i)dj = \left(\frac{W_t^m(i)}{W_t^m}\right)^{-\varrho_{Lm}} \int_0^1 L_t^m(j)dj = \left(\frac{W_t^m(i)}{W_t^m}\right)^{-\varrho_{Lm}} L_t^m, \tag{103}$$

where $L_t^m \equiv \int_0^1 L_t^m(j)dj = \left[\int_0^1 L_t^m(i)^{\frac{\varrho_{Lm}-1}{\varrho_{Lm}}} di\right]^{\frac{\varrho_{Lm}}{\varrho_{Lm}-1}}$ (see also (9)).

Using (103) we may rewrite household i's gross nominal labor income (3) as:

$$\begin{aligned} W_t(i)L_t(i) &= \sum_{m=FT,FN,VT,VN} W_t^m(i)L_t^m(i) = \sum_{m=FT,FN,VT,VN} W_t^m(i)\left[\frac{W_t^m(i)}{W_t^m}\right]^{-\varrho_{Lm}} L_t^m \\ &= \sum_{m=FT,FN,VT,VN} \left[W_t^m(i)\right]^{1-\varrho_{Lm}} \left(W_t^m\right)^{\varrho_{Lm}} L_t^m. \end{aligned}$$

Rewriting (103) as $W_t^m(i) = \left(\frac{L_t^m(i)}{L_t^m}\right)^{-\frac{1}{\varrho_{Lm}}} W_t^m$ and substituting this expression in (99), differentiating w.r.t. household i's current labor supply $L_t^m(i)$, so that for $A \equiv -\frac{\left[\sum_{m=FT,FN,VT,VN} L_t^m(i)\right]^{1+\phi}}{1+\phi}$ and $B \equiv \Gamma_t(i)\left(1-\tau_t^w\right)\left[\sum_{m=FT,FN,VT,VN}\left[\frac{L_t^m(i)}{L_t^m}\right]^{1-\frac{1}{\varrho_{Lm}}} W_t^m L_t^m\right]$,[42] we compute $\frac{\partial A}{\partial L_t^m(i)} = -\left[L_t(i)\right]^{\phi}$ and

[42]Notice that the tax rate τ_t^won labor income is assumed to be time-dependent but exogenous.

$\frac{\partial B}{\partial L_t^m(i)} = \Gamma_t(i)\left(1-\tau_t^w\right)\left[\left(1-\frac{1}{\varrho_{Lm}}\right)\left[\frac{L_t^m(i)}{L_t^m}\right]^{-\frac{1}{\varrho_{Lm}}} W_t^m\right] = \Gamma_t(i)\left(1-\tau_t^w\right)\left[\left(1-\frac{1}{\varrho_{Lm}}\right) W_t^m(i)\right]$. Hence,
$\frac{\partial \mathcal{L}_0^-(i)}{\partial L_t^m(i)} = -\left[L_t(i)\right]^{\phi} - \Gamma_t(i)\left(1-\tau_t^w\right)\left[\left(1-\frac{1}{\varrho_{Lm}}\right) W_t^m(i)\right] = 0$ or
$\left(\frac{\varrho_{Lm}}{1-\varrho_{Lm}}\right)\left[L_t(i)\right]^{\phi} = \Gamma_t(i)\left(1-\tau_t^w\right) W_t^m(i)$, which is equation (37) in the main text.

B.3 Investment

The FOC of domestic consumer i's supply of investment goods is derived from (99) as:

$$\frac{\partial \mathcal{L}_0^-(i)}{\partial I_t^m(i)} = 0 \Leftrightarrow P_t^I(i)\Gamma_t(i)\text{-}\mathcal{Q}_t^m(i)\Gamma_t(i)+\mathcal{Q}_t^m(i)\Gamma_t(i)\left[\Upsilon(.)+\Upsilon'(.)\frac{I_t^m(i)}{I_{t-1}^m(i)}\right]\text{-}\beta E_t\left\{\mathcal{Q}_{t+1}^m(i)\Gamma_{t+1}(i)\left[\Upsilon'(.)\frac{\left(I_{t+1}^m(i)\right)^2}{\left(I_t^m(i)\right)^2}\right]\right\} = 0$$

$$\Leftrightarrow P_t^I(i)\Gamma_t(i)\text{-}\mathcal{Q}_t^m(i)\Gamma_t(i)\left[1\text{-}\Upsilon(.)\text{-}\Upsilon'(.)\frac{I_t^m(i)}{I_{t-1}^m(i)}\right] = \beta E_t\left\{\mathcal{Q}_{t+1}^m(i)\Gamma_{t+1}(i)\left[\Upsilon'(.)\frac{\left(I_{t+1}^m(i)\right)^2}{\left(I_t^m(i)\right)^2}\right]\right\}, \tag{104}$$

for $m = FT, FN, VT, VN$, which is equal to FOC expression (38) in the main text.

B.4 Capital

The FOC of domestic consumer i's supply of domestic capital stock at the beginning of period $t+1$ is derived from (99), taking account of definition (4):

$$\begin{aligned}\frac{\partial \mathcal{L}_0^-(i)}{\partial K_{t+1}^m(i)} &= 0 \Leftrightarrow \beta E_t\left\{\Gamma_{t+1}(i)P_{t+1}^I(i)\Psi(.) - \Gamma_{t+1}(i)\left(1-\tau_{t+1}^k\right)R_{t+1}^m(i)\mathcal{Z}_{t+1}^m(i)\right\}\\ &+ \mathcal{Q}_t^m(i)\Gamma_t(i) - \beta E_t\left\{\mathcal{Q}_{t+1}^m(i)\Gamma_{t+1}(i)\left(1-d_{t+1}^m(i)\right)\right\} = 0\\ &\Leftrightarrow \mathcal{Q}_t^m(i)\Gamma_t(i) = \beta E_t\left\{\mathcal{Q}_{t+1}^m(i)\Gamma_{t+1}(i)\left(1-d_{t+1}^m(i)\right) - \Gamma_{t+1}(i)P_{t+1}^I(i)\Psi(.)\right.\\ &\left.+\Gamma_{t+1}(i)\left(1-\tau_{t+1}^k\right)R_{t+1}^m(i)\mathcal{Z}_{t+1}^m(i)\right\} \quad \text{for } m = FT, FN, VT, VN,\end{aligned} \tag{105}$$

which is FOC (42) that appears in the main text.

B.5 Capital utilization rate

Finally, FOC (43) for the utilization rate of physical capital stock is derived from (99) as:

$$\begin{aligned}\frac{\partial \mathcal{L}_0^-(i)}{\partial \mathcal{Z}_{t+1}^m(i)} &= 0 \Leftrightarrow E_t\left[\Gamma_{t+1}(i)P_{t+1}^I(i)\Psi'(.)K_{t+1}^m(i) - \Gamma_{t+1}(i)\left(1-\tau_{t+1}^k\right)R_{t+1}^m(i)K_{t+1}^m(i)\right] = 0\\ &\Leftrightarrow P_{t+1}^I(i)\Psi'(\mathcal{Z}_{t+1}^m(i)) = \left(1-\tau_{t+1}^k\right)R_{t+1}^m(i) \quad \text{for } m = FT, FN, VT, VN.\end{aligned} \tag{106}$$

C Methods of log-linearization

C.1 Log-linearization around time-dependent paths

Let X_t be any strictly positive variable (e.g. output) and $\bar{X}_t$ its time-dependent value (e.g. natural value or natural output in the absence of nominal rigidities) around which we would like to log-linearize. Then, x_t is defined as the logarithmic deviation from this time-dependent value (e.g. the natural output gap) so that a first order Taylor approximation at this time-dependent value yields:

$$X_t \equiv \bar{X}_t\left(\frac{X_t}{\bar{X}_t}\right) = \bar{X}_t e^{\ln\left(X_t/\bar{X}_t\right)} = \bar{X}_t e^{x_t} \simeq \bar{X}_t e^0 + \bar{X}_t e^0\left(x_t - 0\right) = \bar{X}_t(1+x_t).$$

Hence, log-linearization around a time-dependent value satisfies $X_t \simeq \bar{X}_t(1+x_t)$, $X_tY_t \simeq \bar{X}_t\bar{Y}_t(1+x_t+y_t+x_ty_t)$ with $x_ty_t \simeq 0$ since x_t and y_t are numbers close to zero, and
$f\left(X_t\right) \simeq f\left(\bar{X}_t\right)+f'\left(\bar{X}_t\right)\left(X_t-\bar{X}_t\right)=f\left(\bar{X}_t\right)+f'\left(\bar{X}_t\right)\bar{X}_t(X_t/\bar{X}_t-1) \simeq f\left(\bar{X}_t\right)+f\left(\bar{X}_t\right)\eta_t\left(1+x_t-1\right)=f\left(\bar{X}_t\right)\left(1+\eta_tx_t\right)$,
where $\eta_t \equiv \frac{\partial \ln\left[f\left(\bar{X}_t\right)\right]}{\partial \ln\left(\bar{X}_t\right)}$.

For instance, applying these properties on a consumption Euler equation of the simple type $E_t\left[R_{t+1}\beta\left(C_{t+1}/C_t\right)^{-\sigma}\right]$ $= 1$ and assuming that this Euler equation also holds for time-dependent paths, we get:

$$1 \simeq E_t\left[\bar{R}_{t+1}\beta\left(\bar{C}_{t+1}/\bar{C}_t\right)^{-\sigma}\right]\left(1+E_t\left[r_{t+1}\right]-\sigma\left(E_t\left[c_{t+1}\right]-c_t\right)\right),$$
$$\text{or } 0 \simeq E_t\left[r_{t+1}\right]-\sigma\left(E_t\left[c_{t+1}\right]-c_t\right).$$

A more general log-linearization procedure for a function of (at least) two variables, utilized in most places of this paper, is the following. The equation

$$f(X_t, Y_t) = g\left(Z_t\right) \tag{107}$$

of strictly positive variables X_t, Y_t and Z_t can be rewritten in logarithmic form as:

$$\ln[f(X_t, Y_t)] = \ln\left[f(e^{\ln X_t}, e^{\ln Y_t})\right] = \ln\left[g(e^{\ln Z_t})\right] = \ln[g\left(Z_t\right)]. \tag{108}$$

Taking a first order Taylor approximation at the logarithms of the variables' time-dependent paths, i.e. at $\ln\bar{X}_t, \ln\bar{Y}_t$ and $\ln\bar{Z}_t$, the two parts of equation (108) become:

$$\ln\left[f(e^{\ln X_t}, e^{\ln Y_t})\right] \simeq \ln\left[f(\bar{X}_t,\bar{Y}_t)\right] + \frac{1}{f(\bar{X}_t,\bar{Y}_t)}\left[f_{\bar{X}_t}(\bar{X}_t,\bar{Y}_t)\bar{X}_t\left(\ln(X_t)-\ln(\bar{X}_t)\right) + f_{\bar{Y}_t}(\bar{X}_t,\bar{Y}_t)\bar{Y}_t\left(\ln(Y_t)-\ln(\bar{Y}_t)\right)\right] \tag{109}$$

and

$$\ln\left[g(e^{\ln Z_t})\right] \simeq \ln\left[g\left(\bar{Z}_t\right)\right] + \frac{1}{g\left(\bar{Z}_t\right)}\left[g_{\bar{Z}_t}\left(\bar{Z}_t\right)\bar{Z}_t\left(\ln(Z_t)-\ln(\bar{Z}_t)\right)\right], \tag{110}$$

where $f_{\bar{X}_t}(\bar{X}_t,\bar{Y}_t)$ stands for the partial derivative of $f(\bar{X}_t,\bar{Y}_t)$ w.r.t. $\bar{X}_t$ (and similarly for $f_{\bar{Y}_t}$ and $g_{\bar{Z}_t}$).

Equating (109) to (110) and using the fact that (107) also holds for the time-dependent paths considered, i.e. $f(\bar{X}_t,\bar{Y}_t) = g\left(\bar{Z}_t\right)$, we obtain in terms of the logarithmic deviations of the three variables:

$$f_{\bar{X}_t}(\bar{X}_t,\bar{Y}_t)\bar{X}_t x_t + f_{\bar{Y}_t}(\bar{X}_t,\bar{Y}_t)\bar{Y}_t y_t \simeq g_{\bar{Z}_t}\left(\bar{Z}_t\right)\bar{Z}_t z_t. \tag{111}$$

This result can be generalized for functions of more than two variables. In general, the log-linearization of $f(X_t^{(1)}, X_t^{(2)}, ..., X_t^{(n)}) = g\left(Y_t^{(1)}, Y_t^{(2)}, ..., Y_t^{(m)}\right)$ is:

$$\textstyle\sum_{i=1}^{n} f_{\bar{X}_t^{(i)}}(\bar{X}_t^{(1)}, \bar{X}_t^{(2)}, ..., \bar{X}_t^{(n)})\bar{X}_t^{(i)} x_t^{(i)} \simeq \sum_{i=1}^{m} f_{\bar{Y}_t^{(i)}}(\bar{Y}_t^{(1)}, \bar{Y}_t^{(2)}, ..., \bar{Y}_t^{(m)})\bar{Y}_t^{(i)} y_t^{(i)}. \tag{112}$$

C.2 Log-linearization around (time-)varying rate paths

Assuming that a time-dependent value $\bar{X}_t$ fluctuates at a period to period (growth) rate λ_t, we may write this value as $\bar{X}_t = \bar{X}_{t-1}\lambda_t = \bar{X}_0\prod_{k=1}^{t}\lambda_k$. By assuming a (time-)varying trend we will assume that at each period t all time-dependent values in the model fluctuate at the same rate λ_t. Since log-linearization around such values is a special case of log-linearization around time-dependent values we can substitute $\bar{X}_t^{(1)} = \bar{X}_0^{(1)}\prod_{k=1}^{t}\lambda_k$, $\bar{X}_t^{(2)} = \bar{X}_0^{(2)}\prod_{k=1}^{t}\lambda_k$, ..., $\bar{X}_t^{(n)} = \bar{X}_0^{(n)}\prod_{k=1}^{t}\lambda_k$, $\bar{Y}_t^{(1)} = \bar{Y}_0^{(1)}\prod_{k=1}^{t}\lambda_k$, $\bar{Y}_t^{(2)} = \bar{Y}_0^{(2)}\prod_{k=1}^{t}\lambda_k$, ..., $\bar{Y}_t^{(m)} = \bar{Y}_0^{(m)}\prod_{k=1}^{t}\lambda_k$ into formula (112), which yields the desired approximation around (time-)varying rate paths.

C.3 Log-linearization around a constant trend

Such a log-linearization is a special case of that in Subsection C.2. Assuming a constant trend $\lambda_t = \lambda$ we may rewrite each time-dependent value as $\bar{X}_t = \bar{X}_0\lambda^t$ and substitute it into (112).

C.4 Log-linearization around the steady state

Such a log-linearization is a special case of that in Subsection C.3. Assuming that $\lambda = 1$ we may rewrite each time-dependent value as $\bar{X}_t = \bar{X}$ and substitute it into (112).

D Log-linearization of the consumers' FOCs (36)-(43)

D.1 FOC (36) of consumption

D.1.1 Log-linearization around time-dependent paths

The partial derivatives needed to log-linearize FOC (36) according to equation (112) are:
$f_{\bar{C}_{t-1}(i)}(\bar{C}_{t-1}(i),\bar{C}_t(i),\bar{C}_{t+1}(i),\bar{\Gamma}_t(i),\bar{P}_t^C(i)) = -\kappa\,(1-\sigma)\,\frac{(\bar{C}_t(i))^{-\sigma}}{(\bar{C}_{t-1}(i))^{\kappa(1-\sigma)+1}}$,
$f_{\bar{C}_t(i)}(\bar{C}_{t-1}(i),\bar{C}_t(i),\bar{C}_{t+1}(i),\bar{\Gamma}_t(i),\bar{P}_t^C(i)) = -\sigma\frac{(\bar{C}_t(i))^{-\sigma-1}}{(\bar{C}_{t-1}(i))^{\kappa(1-\sigma)}} + [\kappa\,(1-\sigma)+1]\,\beta\kappa E_t\left[\frac{(\bar{C}_{t+1}(i))^{1-\sigma}}{(\bar{C}_t(i))^{[\kappa(1-\sigma)+2]}}\right]$,
$f_{\bar{C}_{t+1}(i)}(\bar{C}_{t-1}(i),\bar{C}_t(i),\bar{C}_{t+1}(i),\bar{\Gamma}_t(i),\bar{P}_t^C(i)) = -\beta\kappa\,(1-\sigma)\,E_t\left[\frac{(\bar{C}_{t+1}(i))^{-\sigma}}{(\bar{C}_t(i))^{[\kappa(1-\sigma)+1]}}\right]$,
$f_{\bar{\Gamma}_t(i)}(\bar{C}_{t-1}(i),\bar{C}_t(i),\bar{C}_{t+1}(i),\bar{\Gamma}_t(i),\bar{P}_t^C(i)) = \bar{P}_t^C(i)$ and
$f_{\bar{P}_t^C(i)}(\bar{C}_{t-1}(i),\bar{C}_t(i),\bar{C}_{t+1}(i),\bar{\Gamma}_t(i),\bar{P}_t^C(i)) = \bar{\Gamma}_t(i)$.

Log-linearizing (36) around the time-dependent (equilibrium) paths using equation (112), we obtain:

$$f_{\bar{C}_{t-1}(i)}(.)\bar{C}_{t-1}(i)c_{t-1}(i) + f_{\bar{C}_t(i)}(.)\bar{C}_t(i)c_t(i) + f_{\bar{C}_{t+1}(i)}(.)\bar{C}_{t+1}(i)c_{t+1(i)} + f_{\bar{\Gamma}_t(i)}(.)\bar{\Gamma}_t(i)\gamma_t(i) + f_{\bar{P}_t^C(i)}(.)\bar{P}_t^C(i)p_t^C(i) \simeq 0, \tag{113}$$

or substituting the partial derivatives:
$$\left[-\kappa\,(1-\sigma)\,\frac{(\bar{C}_t(i))^{-\sigma}}{(\bar{C}_{t-1}(i))^{\kappa(1-\sigma)+1}}\right]\bar{C}_{t-1}(i)c_{t-1}(i) + \left[-\sigma\frac{(\bar{C}_t(i))^{-\sigma-1}}{(\bar{C}_{t-1}(i))^{\kappa(1-\sigma)}} + [\kappa\,(1-\sigma)+1]\,\beta\kappa E_t\left[\frac{(\bar{C}_{t+1}(i))^{1-\sigma}}{(\bar{C}_t(i))^{[\kappa(1-\sigma)+2]}}\right]\right]\bar{C}_t(i)c_t(i)$$
$$+\left[-\beta\kappa\,(1-\sigma)\,E_t\left[\frac{(\bar{C}_{t+1}(i))^{-\sigma}}{(\bar{C}_t(i))^{[\kappa(1-\sigma)+1]}}\right]\right]\bar{C}_{t+1}(i)c_{t+1}(i) + \bar{P}_t^C(i)\bar{\Gamma}_t(i)\left[\gamma_t(i)+p_t^C(i)\right] \simeq 0.$$

Rearranging, we get the log-linearized form of the FOC w.r.t. consumption:

$$E_t\left[-\sigma\frac{(\bar{C}_t(i))^{-\sigma}}{(\bar{C}_{t-1}(i))^{\kappa(1-\sigma)}} + [\kappa\,(1-\sigma)+1]\,\beta\kappa\frac{(\bar{C}_{t+1}(i))^{1-\sigma}}{(\bar{C}_t(i))^{[\kappa(1-\sigma)+1]}}\right]c_t - \left[\kappa\,(1-\sigma)\,\frac{(\bar{C}_t(i))^{-\sigma}}{(\bar{C}_{t-1}(i))^{\kappa(1-\sigma)}}\right]c_{t-1}$$
$$-E_t\left[\beta\kappa\,(1-\sigma)\,\frac{(\bar{C}_{t+1}(i))^{1-\sigma}}{(\bar{C}_t(i))^{[\kappa(1-\sigma)+1]}}\right]c_{t+1} + E_t\left[\frac{(\bar{C}_t(i))^{-\sigma}}{(\bar{C}_{t-1}(i))^{\kappa(1-\sigma)}} - \beta\kappa\frac{(\bar{C}_{t+1}(i))^{1-\sigma}}{(\bar{C}_t(i))^{[\kappa(1-\sigma)+1]}}\right]\left[\gamma_t(i)+p_t^C(i)\right] \simeq 0. \tag{114}$$

D.1.2 Varying rate

Assuming that all domestic consumer i's time-dependent values fluctuate at the same time-varying (growth) rate $\lambda_t^{(i)}$, we may write

$$\bar{C}_{t-1}(i) = \bar{C}_0(i)\textstyle\prod_{k=1}^{t-1}\lambda_k^{(i)},\ \bar{C}_t(i) = \bar{C}_0(i)\prod_{k=1}^{t}\lambda_k^{(i)} \text{ and } \bar{C}_{t+1}(i) = \bar{C}_0(i)\prod_{k=1}^{t+1}\lambda_k^{(i)}.$$

Substitution into (114) and then dividing both sides by $\frac{\left(\bar{C}_0(i)\prod_{k=1}^{t}\lambda_k^{(i)}\right)^{-\sigma}}{\left(\bar{C}_0(i)\prod_{k=1}^{t}\lambda_k^{(i)}\right)^{\kappa(1-\sigma)}}$, yields:

$$\left[-\sigma\left(\lambda_t^{(i)}\right)^{\kappa(1-\sigma)} + [\kappa\,(1-\sigma)+1]\,\beta\kappa E_t\left[\left(\lambda_{t+1}^{(i)}\right)^{1-\sigma}\right]\right]c_t(i) - \left[\kappa\,(1-\sigma)\left(\lambda_t^{(i)}\right)^{\kappa(1-\sigma)}\right]c_{t-1}(i)$$
$$-E_t\left[\beta\kappa\,(1-\sigma)\left(\lambda_{t+1}^{(i)}\right)^{1-\sigma}c_{t+1}(i)\right] + E_t\left[\left(\lambda_t^{(i)}\right)^{\kappa(1-\sigma)} - \beta\kappa\left(\lambda_{t+1}^{(i)}\right)^{1-\sigma}\right]\left[\gamma_t(i)+p_t^C(i)\right] \simeq 0. \tag{115}$$

If targeted growth rates are equal across consumers, or $\lambda_t^{(i)} = \lambda_t$, expression (115) can be simplified such that only the log-linearized deviations of consumption, the Lagrange multiplier and the aggregate consumption price index vary over individual consumers.

D.1.3 Constant trend

Assuming $\lambda_t^{(i)} = \lambda^{(i)}$ and substituting into (115) we get

$$\left[-\sigma\left(\lambda^{(i)}\right)^{\kappa(1-\sigma)} + \left[\kappa\left(1-\sigma\right)+1\right]\beta\kappa\left(\lambda^{(i)}\right)^{1-\sigma}\right]c_t(i) - \left[\kappa\left(1-\sigma\right)\left(\lambda^{(i)}\right)^{\kappa(1-\sigma)}\right]c_{t-1}(i)$$
$$-E_t\left[\left[\beta\kappa\left(1-\sigma\right)\left(\lambda^{(i)}\right)^{1-\sigma}\right]c_{t+1}(i)\right] + \left[\left(\lambda^{(i)}\right)^{\kappa(1-\sigma)} - \beta\kappa\left(\lambda^{(i)}\right)^{1-\sigma}\right]\left[\gamma_t(i) + p_t^C(i)\right] \simeq 0, \tag{116}$$

which is the log-linearized form of (36) around the steady state.

D.1.4 Steady state

Assuming $\lambda^{(i)} = \lambda = 1$ and substituting into (116) we get:

$$\left[-\sigma + \left[\kappa\left(1-\sigma\right)+1\right]\beta\kappa\right]c_t(i) - \left[\kappa\left(1-\sigma\right)\right]c_{t-1}(i) - \left[\beta\kappa\left(1-\sigma\right)\right]E_t\left[c_{t+1}(i)\right] + \left[1-\beta\kappa\right]\left[\gamma_t(i) + p_t^C(i)\right] \simeq 0. \tag{117}$$

Note that simplifying and rewriting (117), we obtain for the logarithmic deviations from the steady state:

$$c_t(i) \simeq \frac{\kappa\left[(\sigma-1)\right]}{\left[\sigma + \left[\kappa\left(\sigma-1\right)-1\right]\beta\kappa\right]}c_{t-1}(i) + \beta\frac{\left[\kappa\left(\sigma-1\right)\right]}{\left[\sigma + \left[\kappa\left(\sigma-1\right)-1\right]\beta\kappa\right]}E\left[c_{t+1}(i)\right] - \frac{\left[1-\beta\kappa\right]}{\left[\sigma + \left[\kappa\left(\sigma-1\right)-1\right]\beta\kappa\right]}\gamma_t(i),$$

so that, defining $a_1 \equiv \frac{\kappa[(\sigma-1)]}{[\sigma+[\kappa(\sigma-1)-1]\beta\kappa]}$ and $a_2 \equiv \frac{[1-\beta\kappa]}{[\sigma+[\kappa(\sigma-1)-1]\beta\kappa]}$, this log-linearized consumption equation can be rewritten as:

$$c_t(i) \simeq a_1 c_{t-1}(i) + \beta a_1 c_{t+1}(i) - a_2\gamma_t(i), \tag{118}$$

which is the log-linearized FOC (36) around the steady state.

D.2 FOC (37) of labor

To log-linearize the implicit form of FOC (37) of labor supply around time-dependent paths using formula (112), the following partial derivatives are needed:

$$f_{\bar{L}_t(i)}\left(\bar{L}_t(i), \bar{\Gamma}_t(i), \bar{W}_t^m(i)\right) = \frac{\varrho_{Lm}}{\varrho_{Lm}-1}\phi\left[\bar{L}_t(i)\right]^{\phi-1},$$
$$f_{\bar{\Gamma}_t(i)}\left(\bar{L}_t(i), \bar{\Gamma}_t(i), \bar{W}_t^m(i)\right) = -\left(1-\tau_t^w\right)\bar{W}_t^m(i) \text{ and}$$
$$f_{\bar{W}_t^m(i)}\left(\bar{L}_t(i), \bar{\Gamma}_t(i), \bar{W}_t^m(i)\right) = -\left(1-\tau_t^w\right)\bar{\Gamma}_t(i),$$

so that $f_{\bar{L}_t(i)}\bar{L}_t(i)l_t(i) - f_{\bar{\Gamma}_t(i)}\bar{\Gamma}_t(i)\gamma_t(i) - f_{\bar{W}_t^m(i)}\bar{W}_t^m(i)w_t^m(i) \simeq 0$,[43] or

$$\frac{\varrho_{Lm}}{\varrho_{Lm}-1}\phi\left[\bar{L}_t(i)\right]^{\phi}l_t(i) - \left(1-\tau_t^w\right)\bar{W}_t^m(i)\bar{\Gamma}_t(i)\gamma_t(i) - \left(1-\tau_t^w\right)\bar{\Gamma}_t(i)\bar{W}_t^m(i)w_t^m(i) \simeq 0. \tag{119}$$

Using the fact that equations (37) are also true for time-dependent paths as natural values, or $\frac{\varrho_{Lm}}{\varrho_{Lm}-1}\left[\bar{L}_t(i)\right]^{\phi} = \bar{\Gamma}_t(i)\left(1-\tau_t^w\right)\bar{W}_t^m(i)$, we can further rewrite equation (119) as:

$$\phi\bar{\Gamma}_t(i)\left(1-\tau_t^w\right)\bar{W}_t^m(i)l_t(i) - \left(1-\tau_t^w\right)\bar{W}_t^m(i)\bar{\Gamma}_t(i)\gamma_t(i) - \left(1-\tau_t^w\right)\bar{\Gamma}_t(i)\bar{W}_t^m(i)w_t^m(i) \simeq 0.$$

Dividing all terms by $\bar{\Gamma}_t(i)\left(1-\tau_t^w\right)\bar{W}_t^m(i)$ we obtain:

$$\phi l_t(i) - w_t^m(i) - \gamma_t(i) \simeq 0, \tag{120}$$

where condition (120) implies that the logarithmic deviations of nominal wages are equal across sectors. Any of the proposed methods of log-linearization as varying rate, constant trend and steady state will produce the same results.

[43] Recall that tax rate τ_t^w on labor income is assumed to be time-dependent and exogenous.

D.3 FOC (38) of home investment

Since FOC (38) is also true for time-dependent paths as natural values, or:

$$\bar{P}_t^I(i)\bar{\Gamma}_t(i)-\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)\left[1\text{-}\Upsilon\left(\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)\text{-}\Upsilon'\left(\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right]=\beta E_t\left[\bar{\mathcal{Q}}_{t+1}^m(i)\bar{\Gamma}_{t+1}(i)\left[\Upsilon'\left(\frac{\bar{I}_{t+1}^m(i)}{\bar{I}_t^m(i)}\right)\frac{\left(\bar{I}_{t+1}^m(i)\right)^2}{\left(\bar{I}_t^m(i)\right)^2}\right]\right], \tag{121}$$

for $m = FT, FN, VT, VN$, so that we get according to log-linearization formula (112):

$$\begin{aligned}
&\bar{\Gamma}_t(i)\bar{P}_t^I(i)\left[\gamma_t(i)+p_t^I(i)\right]-\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)\left[1-\Upsilon\left(\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)-\Upsilon'\left(\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right]\left[q_t^m(i)+\gamma_t(i)\right]\\
&\quad-\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)\left[-2\Upsilon'\left(\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right]i_t^m(i)-\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)\left[-\Upsilon''\left(\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)\tfrac{\left(\bar{I}_t^m(i)\right)^2}{\left(\bar{I}_{t-1}^m(i)\right)^2}\right]i_t^m(i)\\
&\quad-\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)\left[2\Upsilon'\left(\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right]i_{t-1}^m(i)-\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)\left[\Upsilon''\left(\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)\tfrac{\left(\bar{I}_t^m(i)\right)^2}{\left(\bar{I}_{t-1}^m(i)\right)^2}\right]i_{t-1}^m(i)\\
&\simeq\beta E_t\left\{\bar{\mathcal{Q}}_{t+1}^m(i)\bar{\Gamma}_{t+1}(i)\left[\Upsilon'\left(\tfrac{\bar{I}_{t+1}^m(i)}{\bar{I}_t^m(i)}\right)\tfrac{\left(\bar{I}_{t+1}^m(i)\right)^2}{\left(\bar{I}_t^m(i)\right)^2}\right]\left[q_{t+1}^m(i)+\gamma_{t+1}(i)\right]\right\}\\
&\quad+\beta E_t\left\{\bar{\mathcal{Q}}_{t+1}^m(i)\bar{\Gamma}_{t+1}(i)\left[\Upsilon''\left(\tfrac{\bar{I}_{t+1}^m(i)}{\bar{I}_t^m(i)}\right)\tfrac{\left(\bar{I}_{t+1}^m(i)\right)^3}{\left(\bar{I}_t^m(i)\right)^3}\right]+\bar{\mathcal{Q}}_{t+1}^m(i)\bar{\Gamma}_{t+1}(i)\left[2\Upsilon'\left(\tfrac{\bar{I}_{t+1}^m(i)}{\bar{I}_t^m(i)}\right)\tfrac{\left(\bar{I}_{t+1}^m(i)\right)^2}{\left(\bar{I}_t^m(i)\right)^2}\right]\right\}E_t i_{t+1}^m(i)\\
&\quad+\beta E_t\left\{-\bar{\mathcal{Q}}_{t+1}^m(i)\bar{\Gamma}_{t+1}(i)\left[\Upsilon''\left(\tfrac{\bar{I}_{t+1}^m(i)}{\bar{I}_t^m(i)}\right)\tfrac{\left(\bar{I}_{t+1}^m(i)\right)^3}{\left(\bar{I}_t^m(i)\right)^3}\right]-\bar{\mathcal{Q}}_{t+1}^m(i)\bar{\Gamma}_{t+1}(i)\left[2\Upsilon'\left(\tfrac{\bar{I}_{t+1}^m(i)}{\bar{I}_t^m(i)}\right)\tfrac{\left(\bar{I}_{t+1}^m(i)\right)^2}{\left(\bar{I}_t^m(i)\right)^2}\right]\right\}i_t^m(i).
\end{aligned}$$

Simplifying, we get:

$$\begin{aligned}
&\bar{\Gamma}_t(i)\bar{P}_t^I(i)\left[\gamma_t(i)+p_t^I(i)\right]-\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)\left[1-\Upsilon\left(\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)-\Upsilon'\left(\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right]\left[q_t^m(i)+\gamma_t(i)\right]\\
&\quad+\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)\left[2\Upsilon'\left(\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right]\left[i_t^m(i)-i_{t-1}^m(i)\right]+\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)\left[\Upsilon''\left(\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)\tfrac{\left(\bar{I}_t^m(i)\right)^2}{\left(\bar{I}_{t-1}^m(i)\right)^2}\right]\left[i_t^m(i)-i_{t-1}^m(i)\right]\\
&\simeq\beta E_t\left\{\bar{\mathcal{Q}}_{t+1}^m(i)\bar{\Gamma}_{t+1}(i)\left[\Upsilon'\left(\tfrac{\bar{I}_{t+1}^m(i)}{\bar{I}_t^m(i)}\right)\tfrac{\left(\bar{I}_{t+1}^m(i)\right)^2}{\left(\bar{I}_t^m(i)\right)^2}\right]\left[q_{t+1}^m(i)+\gamma_{t+1}(i)\right]\right\}\\
&\quad+\beta E_t\left\{\bar{\mathcal{Q}}_{t+1}^m(i)\bar{\Gamma}_{t+1}(i)\left[\Upsilon''\left(\tfrac{\bar{I}_{t+1}^m(i)}{\bar{I}_t^m(i)}\right)\tfrac{\left(\bar{I}_{t+1}^m(i)\right)^3}{\left(\bar{I}_t^m(i)\right)^3}\right]\left[i_{t+1}^m(i)-i_t^m(i)\right]\right\}\\
&\quad+\beta E_t\left\{\bar{\mathcal{Q}}_{t+1}^m(i)\bar{\Gamma}_{t+1}(i)\left[2\Upsilon'\left(\tfrac{\bar{I}_{t+1}^m(i)}{\bar{I}_t^m(i)}\right)\tfrac{\left(\bar{I}_{t+1}^m(i)\right)^2}{\left(\bar{I}_t^m(i)\right)^2}\right]\left[i_{t+1}^m(i)-i_t^m(i)\right]\right\}.
\end{aligned}$$

Replacing (121) into the above expression and grouping common terms, we get:

$$\begin{aligned}
&\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)\left[1\text{-}\Upsilon\left(\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)\text{-}\Upsilon'\left(\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right]\left[p_t^I(i)\text{-}q_t^m(i)\right]\\
&\quad+\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)\left[2\Upsilon'\left(\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right]\left[i_t^m(i)-i_{t-1}^m(i)\right]+\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)\left[\Upsilon''\left(\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)\tfrac{\left(\bar{I}_t^m(i)\right)^2}{\left(\bar{I}_{t-1}^m(i)\right)^2}\right]\left[i_t^m(i)-i_{t-1}^m(i)\right]\\
&\simeq\beta E_t\left\{\bar{\mathcal{Q}}_{t+1}^m(i)\bar{\Gamma}_{t+1}(i)\left[\Upsilon'\left(\tfrac{\bar{I}_{t+1}^m(i)}{\bar{I}_t^m(i)}\right)\tfrac{\left(\bar{I}_{t+1}^m(i)\right)^2}{\left(\bar{I}_t^m(i)\right)^2}\right]\left[q_{t+1}^m(i)+\gamma_{t+1}(i)-\gamma_t(i)-p_t^I(i)\right]\right\}\\
&\quad+\beta E_t\left\{\bar{\mathcal{Q}}_{t+1}^m(i)\bar{\Gamma}_{t+1}(i)\left[\Upsilon''\left(\tfrac{\bar{I}_{t+1}^m(i)}{\bar{I}_t^m(i)}\right)\tfrac{\left(\bar{I}_{t+1}^m(i)\right)^3}{\left(\bar{I}_t^m(i)\right)^3}\right]\left[i_{t+1}^m(i)-i_t^m(i)\right]\right\}\\
&\quad+\beta E_t\left\{\bar{\mathcal{Q}}_{t+1}^m(i)\bar{\Gamma}_{t+1}(i)\left[2\Upsilon'\left(\tfrac{\bar{I}_{t+1}^m(i)}{\bar{I}_t^m(i)}\right)\tfrac{\left(\bar{I}_{t+1}^m(i)\right)^2}{\left(\bar{I}_t^m(i)\right)^2}\right]\left[i_{t+1}^m(i)-i_t^m(i)\right]\right\}.
\end{aligned}$$

Finally, defining the elasticity of cost adjustment of investment as $\Theta_t(i)\equiv\frac{\Upsilon''\left(\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)}{\Upsilon'\left(\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)}$, we isolate sectoral investment, so that:[44]

$$\begin{aligned}
&\left\{1+\tfrac{\Theta_t(i)}{2}\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}+\tfrac{\beta}{2\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}}E_t\left\{\bar{\mathcal{Q}}_{t+1}^m(i)\bar{\Gamma}_{t+1}(i)\left[\Theta_t(i)\frac{\Upsilon''\left(\frac{\bar{I}_{t+1}^m(i)}{\bar{I}_t^m(i)}\right)}{\Upsilon''\left(\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)}\frac{\left(\bar{I}_{t+1}^m(i)\right)^3}{\left(\bar{I}_t^m(i)\right)^3}+2\frac{\Upsilon'\left(\frac{\bar{I}_{t+1}^m(i)}{\bar{I}_t^m(i)}\right)}{\Upsilon'\left(\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)}\frac{\left(\bar{I}_{t+1}^m(i)\right)^2}{\left(\bar{I}_t^m(i)\right)^2}\right]\right\}\right\}i_t^m(i)\\
&\simeq-\left[\frac{\left[1-\Upsilon\left(\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)\right]}{2\Upsilon'\left(\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}}-\frac{1}{2}\right]\left[p_t^I(i)\text{-}q_t^m(i)\right]+\left(1+\tfrac{\Theta_t(i)}{2}\tfrac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)i_{t-1}^m(i)\\
&+\tfrac{\beta}{2\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}}E_t\left\{\bar{\mathcal{Q}}_{t+1}^m(i)\bar{\Gamma}_{t+1}(i)\left[\frac{\Upsilon'\left(\frac{\bar{I}_{t+1}^m(i)}{\bar{I}_t^m(i)}\right)}{\Upsilon'\left(\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)}\frac{\left(\bar{I}_{t+1}^m(i)\right)^2}{\left(\bar{I}_t^m(i)\right)^2}\right]\left[q_{t+1}^m(i)+\gamma_{t+1}(i)-\gamma_t(i)-p_t^I(i)\right]\right\}
\end{aligned}$$

[44] Dividing both sides by $\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)\left[2\Upsilon'\left(\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right]$.

$$+\frac{\beta}{2\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}}E_t\left\{\bar{\mathcal{Q}}_{t+1}^m(i)\bar{\Gamma}_{t+1}(i)\left[\Theta_t(i)\frac{\Upsilon''\left(\frac{\bar{I}_{t+1}^m(i)}{\bar{I}_t^m(i)}\right)}{\Upsilon''\left(\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)}\frac{\left(\bar{I}_{t+1}^m(i)\right)^3}{\left(\bar{I}_t^m(i)\right)^3}+2\frac{\Upsilon'\left(\frac{\bar{I}_{t+1}^m(i)}{\bar{I}_t^m(i)}\right)}{\Upsilon'\left(\frac{\bar{I}_t^m(i)}{\bar{I}_{t-1}^m(i)}\right)}\frac{\left(\bar{I}_{t+1}^m(i)\right)^2}{\left(\bar{I}_t^m(i)\right)^2}\right]i_{t+1}^m(i)\right\}.$$

D.3.1 Steady state

In the steady steady state, equation (121) becomes,

$$\bar{P}^I(i)\bar{\Gamma}(i)-\bar{\mathcal{Q}}^m(i)\bar{\Gamma}(i)\left[1\text{-}\Upsilon(1)\text{-}\Upsilon'(1)\right]=\beta\bar{\mathcal{Q}}^m(i)\bar{\Gamma}(i)\Upsilon'(1), \tag{122}$$

so that, applying the basic log-linearization rule (112) w.r.t. this steady state, we obtain:[45]

$\bar{P}^I(i)\bar{\Gamma}(i)\left[p_t^I(i)+\gamma_t(i)\right]-\bar{\mathcal{Q}}^m(i)\bar{\Gamma}(i)\left[q_t^m(i)+\gamma_t(i)\right]+\bar{\mathcal{Q}}^m(i)\bar{\Gamma}(i)\left[2\Upsilon'(1)\frac{1}{\bar{I}^m(i)}+\Upsilon''(1)\frac{1}{\bar{I}^m(i)}\right]\bar{I}^m(i)i_t^m(i)$
$+\bar{\mathcal{Q}}^m(i)\bar{\Gamma}(i)\left[-2\Upsilon'(1)\frac{1}{\bar{I}^m(i)}-\Upsilon''(1)\frac{1}{\bar{I}^m(i)}\right]\bar{I}^m(i)i_{t-1}^m(i)\simeq\beta\bar{\mathcal{Q}}^m(i)\bar{\Gamma}(i)\left[\Upsilon'(1)\right]\left[q_{t+1}^m(i)+\gamma_{t+1}(i)\right]$
$+\beta\bar{\mathcal{Q}}^m(i)\bar{\Gamma}(i)\left[\Upsilon''(1)\frac{1}{\bar{I}^m(i)}+2\Upsilon'(1)\frac{1}{\bar{I}^m(i)}\right]\bar{I}^m(i)E_ti_{t+1}^m(i)-\beta\bar{\mathcal{Q}}^m(i)\bar{\Gamma}(i)\left[\Upsilon''(1)\frac{1}{\bar{I}^m(i)}+2\Upsilon'(1)\frac{1}{\bar{I}^m(i)}\right]\bar{I}^m(i)i_t^m(i).$

Simplifying, we get:
$\bar{P}^I(i)\bar{\Gamma}(i)\left[p_t^I(i)+\gamma_t(i)\right]-\bar{\mathcal{Q}}^m(i)\bar{\Gamma}(i)\left[q_t^m(i)+\gamma_t(i)\right]+\bar{\mathcal{Q}}^m(i)\bar{\Gamma}(i)\Upsilon''(1)\left[i_t^m(i)-i_{t-1}^m(i)\right]$
$\simeq\beta\bar{\mathcal{Q}}^m(i)\bar{\Gamma}(i)\Upsilon''(1)\left[E_ti_{t+1}^m(i)-i_t^m(i)\right].$

Consider again (122), or $\bar{P}^I(i)\bar{\Gamma}(i)=\bar{\mathcal{Q}}^m(i)\bar{\Gamma}(i)$, so that:
$\bar{P}^I(i)\bar{\Gamma}(i)\left[p_t^I(i)+\gamma_t(i)\right]-\bar{P}^I(i)\bar{\Gamma}(i)\left[q_t^m(i)+\gamma_t(i)\right]+\bar{P}^I(i)\bar{\Gamma}(i)\Upsilon''(1)\left[i_t^m(i)-i_{t-1}^m(i)\right]$
$\simeq\beta\bar{P}^I(i)\bar{\Gamma}(i)\Upsilon''(1)\left[E_ti_{t+1}^m(i)-i_t^m(i)\right],$
or dividing both sides by $\bar{P}^I(i)\bar{\Gamma}(i)$:
$p_t^I(i)-q_t^m(i)+\Upsilon''(1)\left[i_t^m(i)-i_{t-1}^m(i)\right]\simeq\beta\Upsilon''(1)\left[E_ti_{t+1}^m(i)-i_t^m(i)\right].$

In other words, grouping terms in logarithmic investment deviations:
$p_t^I(i)-q_t^m(i)-\Upsilon''(1)\,i_{t-1}^m(i)\simeq\beta\Upsilon''(1)\,E_ti_{t+1}^m(i)-(1+\beta)\,\Upsilon''(1)\,i_t^m(i),$
or expressing explicitly for current logarithmic investment deviations:

$$i_t^m(i)\simeq\frac{1}{(1+\beta)\,\Upsilon''(1)}\left[q_t^m(i)-p_t^I(i)+\Upsilon''(1)\,i_{t-1}^m(i)+\beta\Upsilon''(1)\,E_t\left\{i_{t+1}^m(i)\right\}\right]. \tag{123}$$

D.4 FOC (39) of home portfolio

FOC (39) can be rearranged as:

$$E_t\left[\Gamma_t(i)-\beta(1+r_{t,t+1})\Gamma_{t+1}(i)\right]=0, \tag{124}$$

where $(1+r_{t,t+1})\equiv\left[Q_{h,t,t+1}\right]^{-1}$. Log-linearizing (124) around time-dependent paths as natural values yields according to formula (112):

$$E_t\left[\beta\bar{\Gamma}_{t+1}(i)(1+\bar{r}_{t,t+1})r_{t,t+1}+\beta\bar{\Gamma}_{t+1}(i)(1+\bar{r}_{t,t+1})\gamma_{t+1}(i)\right]\simeq\bar{\Gamma}_t(i)\gamma_t(i).$$

Using the fact that (124) holds for any time-dependent path, or $E_t\left[\bar{\Gamma}_t(i)-\beta\bar{R}_{t,t+1}\bar{\Gamma}_{t+1}(i)\right]=0$, we get:

$$E_t\left[\beta\bar{\Gamma}_{t+1}(i)(1+\bar{r}_{t,t+1})r_{t,t+1}+\beta\bar{\Gamma}_{t+1}(i)(1+\bar{r}_{t,t+1})\gamma_{t+1}(i)\right]\simeq E_t\left[\beta(1+\bar{r}_{t,t+1})\bar{\Gamma}_{t+1}(i)\gamma_t(i)\right].$$

Dividing both sides by $\beta(1+\bar{r}_{t,t+1})\bar{\Gamma}_{t+1}(i)$, we obtain:

$$\gamma_t(i)\simeq E_t\left[r_{t,t+1}+\gamma_{t+1}(i)\right]. \tag{125}$$

D.5 FOC (40) of foreign portfolio

FOC (40) can be rearranged as:

$$E_t\left[\Gamma_t(i)S_t-\beta\Gamma_{t+1}(i)(1+\hat{r}_{t,t+1}^*)S_{t+1}\right]=0, \tag{126}$$

[45] Since there are no costs of investment adjustment in the steady state, the assumed cost hypothesis in Christiano *et al.* (2005) implies that $\Upsilon(1)=0$, $\Upsilon'(1)=0$ and $\Upsilon''(1)>0$. (which are the properties of a quadratic adjustment cost function through the origin). See also footnote 9.

where $(1+\hat{r}^*_{t,t+1}) \equiv \left[\hat{Q}_{f,t,t+1}\right]^{-1}$. Log-linearizing (126) around time-dependent paths yields according to formula (112):

$$E_t \left[\begin{array}{c} \bar{S}_t\bar{\Gamma}_t(i)\gamma_t(i) - \beta(1+\overline{\hat{r}^*_{t,t+1}})\bar{S}_{t+1}\bar{\Gamma}_{t+1}(i)\gamma_{t+1}(i) \\ -\beta\bar{\Gamma}_{t+1}(i)\bar{S}_{t+1}(1+\overline{\hat{r}^*_{t,t+1}})r^*_{t,t+1} + \bar{\Gamma}_t(i)\bar{S}_t s_t - \beta\bar{\Gamma}_{t+1}(i)(1+\overline{\hat{r}^*_{t,t+1}})\bar{S}_{t+1}s_{t+1} \end{array} \right] \simeq 0. \tag{127}$$

Given that (126) holds for time-dependent variables as natural values and substituting in (127), we obtain:

$$E_t \left[\begin{array}{c} \beta\bar{\Gamma}_{t+1}(i)(1+\overline{\hat{r}^*_{t,t+1}})\bar{S}_{t+1}\gamma_t(i) - \beta(1+\overline{\hat{r}^*_{t,t+1}})\bar{S}_{t+1}\bar{\Gamma}_{t+1}(i)\gamma_{t+1}(i) \\ -\beta\bar{\Gamma}_{t+1}(i)\bar{S}_{t+1}(1+\overline{\hat{r}^*_{t,t+1}})r^*_{t,t+1} + \beta\bar{\Gamma}_{t+1}(i)(1+\overline{\hat{r}^*_{t,t+1}})\bar{S}_{t+1}s_t - \beta\bar{\Gamma}_{t+1}(i)(1+\overline{\hat{r}^*_{t,t+1}})\bar{S}_{t+1}s_{t+1} \end{array} \right] \simeq 0.$$

We get (after dividing both sides by $\beta(1+\overline{\hat{r}^*_{t,t+1}})\bar{S}_{t+1}\bar{\Gamma}_{t+1}(i)$):

$$\gamma_t(i) \simeq E_t\left[r^*_{t,t+1} + \Delta s_{t+1} + \gamma_{t+1}(i)\right]. \tag{128}$$

Both relations (125) and (128) can conveniently be rewritten (using the transversality condition $\Delta\gamma_{t+\infty}(i) = 0$), by solving forwardly, as:[46]

$$\gamma_t(i) \simeq \textstyle\sum_{j=0}^{\infty} r_{t,t+j} \simeq \sum_{j=0}^{\infty}\left(r^*_{t,t+j} + \Delta s_{t+j}\right). \tag{129}$$

D.6 FOC (41) for real money balances

D.6.1 Log-linearization around time-dependent paths

Rewriting FOC (41) for time-dependent paths as natural values:

$$f(.) = \chi_M\left(\bar{M}_{t+1}(i)\right)^{-\frac{1}{\chi}}\left(\bar{P}^C_t(i)\right)^{\frac{1-\chi}{\chi}} + \bar{\Gamma}_t(i) - \beta E_t\left[\bar{\Gamma}_{t+1}(i)\right] = 0. \tag{130}$$

Log-linearizing (41) around natural value paths, we find according to log-linearization formula (112):

$$\left[-\frac{1}{\chi}m_{t+1}(i) + \frac{(1-\chi)}{\chi}p^C_t(i)\right]\chi_M\left(\bar{M}_{t+1}(i)\right)^{-\frac{1}{\chi}}\left(\bar{P}^C_t(i)\right)^{\frac{1-\chi}{\chi}} + \bar{\Gamma}_t(i)\gamma_t(i) - \beta E_t\left[\bar{\Gamma}_{t+1}(i)\gamma_{t+1}(i)\right] \simeq 0.$$

Rearranging this log-linearized form and using (130), we get:

$$\frac{1}{\chi}\left[m_{t+1}(i) - (1-\chi)\,p^C_t(i)\right]\left(\bar{\Gamma}_t(i) - \beta E_t\left[\bar{\Gamma}_{t+1}(i)\right]\right) + \bar{\Gamma}_t(i)\gamma_t(i) - \beta E_t\left[\bar{\Gamma}_{t+1}(i)\gamma_{t+1}(i)\right] \simeq 0.$$

Finally, we obtain:

$$\frac{1}{\chi}\left[m_{t+1}(i) - (1-\chi)\,p^C_t(i)\right] + \frac{\bar{\Gamma}_t(i)\gamma_t(i) - \beta E_t\left[\bar{\Gamma}_{t+1}(i)\gamma_{t+1}(i)\right]}{\left(\bar{\Gamma}_t(i) - \beta E_t\left[\bar{\Gamma}_{t+1}(i)\right]\right)} \simeq 0. \tag{131}$$

FOCs for the foreign households' problem can be log-linearized in a similar way as above leading to a log-linearized foreign Euler equation analogous to (114) and to a log-linearized foreign labor supply curve analogous to (120).

D.6.2 Varying, constant trend and steady state

Since $\bar{\Gamma}_t(i) = \bar{\Gamma}_0(i)\prod_{k=1}^{t}\lambda_k^{(i)}$ and $\bar{\Gamma}_{t+1}(i) = \bar{\Gamma}_0(i)\prod_{k=1}^{t+1}\lambda_k^{(i)} = \bar{\Gamma}_t(i)\lambda_{t+1}^{(i)}$, we can reformulate (131) cancelling out $\bar{\Gamma}_t(i)$, to get:

$$\frac{1}{\chi}\left[m_{t+1}(i) - (1-\chi)\,p^C_t(i)\right] + E_t\left[\frac{\gamma_t(i) - \beta\lambda_{t+1}^{(i)}\gamma_{t+1}(i)}{\left(1 - \beta\lambda_{t+1}^{(i)}\right)}\right] \simeq 0,$$

[46] Substituting (129) into the log-linearized domestic consumption equation (118), it can be shown that this consumption equation is similar to Caputo (2003) and, moreover, in the absence of habit formation, i.e. $\kappa = 0$, equation (118) collapses to $c_t(i) \simeq -\frac{1}{\sigma}\gamma_t \simeq c_t$, which is similar to the forward-looking expression for consumption in Galí and Monacelli (2002).The (slight) difference originates from the different log-linearization procedure in Caputo (2003) and Galí and Monacelli (2002), where a constant discount factor β remains in the log-linearized expression (129).

which under the assumption of constant trend can be rewritten as:

$$\frac{1}{\chi}\left[m_{t+1}(i)-(1-\chi)\,p_t^C(i)\right]+E_t\left[\frac{\gamma_t(i)-\beta\lambda^{(i)}\gamma_{t+1}(i)}{\left(1-\beta\lambda^{(i)}\right)}\right]\simeq 0. \tag{132}$$

In the steady state $\lambda^{(i)}=1$ for all i so that (132) reduces to:

$$\frac{1}{\chi}\left[m_{t+1}(i)-(1-\chi)\,p_t^C(i)\right]+E_t\left[\frac{(\gamma_t(i)-\beta\gamma_{t+1}(i))}{(1-\beta)}\right]\simeq 0. \tag{133}$$

D.7 FOC (42) of home capital

Rewritting the FOC w.r.t. the home physical capital evaluated at the **natural values**, as:[47]

$$\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)=\beta E_t\left\{\bar{\Gamma}_{t+1}(i)\left[\begin{array}{c}(1-d_{t+1}^m(i))\bar{\mathcal{Q}}_{t+1}^m(i)+(1-\tau_{t+1}^k)\bar{R}_{t+1}^m(i)\bar{\mathcal{Z}}_{t+1}^m(i)\\ -\bar{P}_{t+1}^I(i)\Psi\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right)\end{array}\right]\right\}\text{ for }m{=}FT,FN,VT,VN. \tag{134}$$

Using formula (112) and gathering variables in terms of log-deviations, we get:

$$\begin{aligned}&\bar{\Gamma}_t(i)\bar{\mathcal{Q}}_t^m(i)\left[q_t^m(i){+}\gamma_t(i)\right]\simeq\beta E_t\left\{\bar{\Gamma}_{t+1}(i)\gamma_{t+1}(i)\left[(1\text{-}d_{t+1}^m(i))\bar{\mathcal{Q}}_{t+1}^m(i){+}(1\text{-}\tau_{t+1}^k)\bar{R}_{t+1}^m(i)\bar{\mathcal{Z}}_{t+1}^m(i)-\bar{P}_{t+1}^I(i)\Psi\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right)\right]\right\}\\ &+\beta E_t\left\{\bar{\Gamma}_{t+1}(i)\left[\begin{array}{c}(1-d_{t+1}^m(i))\bar{\mathcal{Q}}_{t+1}^m(i)q_{t+1}^m(i)+(1-\tau_{t+1}^k)\bar{\mathcal{Z}}_{t+1}^m(i)\bar{R}_{t+1}^m(i)r_{t+1}^m(i)\\ +\left[(1-\tau_{t+1}^k)\bar{R}_{t+1}^m(i)-\bar{P}_{t+1}^I(i)\Psi'\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right)\right]\bar{\mathcal{Z}}_{t+1}^m(i)\mathsf{z}_{t+1}^m(i)-\Psi\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right)\bar{P}_{t+1}^I(i)p_{t+1}^I(i)\end{array}\right]\right\},\end{aligned}$$

Considering (134), the last expression can be rewritten as:

$$\begin{aligned}&\bar{\Gamma}_t(i)\bar{\mathcal{Q}}_t^m(i)\left[q_t^m(i)+\gamma_t(i)\right]\simeq\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)E_t\gamma_{t+1}(i)\\ &+\beta E_t\bar{\Gamma}_{t+1}(i)\left[\begin{array}{c}\left[\frac{\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)}{\beta E_t\bar{\Gamma}_{t+1}(i)}-E_t\left\{(1-\tau_{t+1}^k)\bar{R}_{t+1}^m(i)\bar{\mathcal{Z}}_{t+1}^m(i)-\bar{P}_{t+1}^I(i)\Psi\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right)\right\}\right]E_tq_{t+1}^m(i)\\ +\left[\frac{\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)}{\beta E_t\bar{\Gamma}_{t+1}(i)}-\left((1-d_{t+1}^m(i))\bar{\mathcal{Q}}_{t+1}^m(i)-\bar{P}_{t+1}^I(i)\Psi\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right)\right)\right]r_{t+1}^m(i)\\ +\left[\frac{\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)}{\beta E_t\bar{\Gamma}_{t+1}(i)}-\left((1-d_{t+1}^m(i))\bar{\mathcal{Q}}_{t+1}^m+(1-\tau_{t+1}^k)\bar{R}_{t+1}^m(i)\bar{\mathcal{Z}}_{t+1}^m(i)\right)\right]p_{t+1}^I(i)\\ +\left[\frac{\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)}{\beta E_t\bar{\Gamma}_{t+1}(i)}-\left((1-d_{t+1}^m(i))\bar{\mathcal{Q}}_{t+1}^m(i)-\bar{P}_{t+1}^I(i)\Psi\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right)\right)\right]\mathsf{z}_{t+1}^m(i)\\ +\left[\frac{\bar{\mathcal{Q}}_t^m\bar{\Gamma}_t(i)}{\beta E_t\bar{\Gamma}_{t+1}(i)}-(1-d_{t+1}^m(i))\bar{\mathcal{Q}}_{t+1}^m-(1-\tau_{t+1}^k)\bar{R}_{t+1}^m(i)\bar{\mathcal{Z}}_{t+1}^m(i)\right]\frac{\Psi'\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right)}{\Psi\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right)}\bar{\mathcal{Z}}_{t+1}^m(i)\mathsf{z}_{t+1}^m(i)\end{array}\right].\end{aligned}$$

Grouping common terms and dividing both sides by $\bar{\Gamma}_t(i)\bar{\mathcal{Q}}_t^m(i)$, we obtain

$$\begin{aligned}&q_t^m(i)+\gamma_t(i)\simeq E_t\gamma_{t+1}(i)\\ &+\left[\begin{array}{c}\left[1-\frac{\beta E_t\bar{\Gamma}_{t+1}(i)}{\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)}E_t\left[(1-\tau_{t+1}^k)\bar{R}_{t+1}^m(i)\bar{\mathcal{Z}}_{t+1}^m(i)-\bar{P}_{t+1}^I(i)\Psi\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right)\right]\right]E_tq_{t+1}^m(i)\\ +\left[1-\frac{\beta E_t\bar{\Gamma}_{t+1}(i)}{\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)}E_t\left[\left((1-d_{t+1}^m(i))\bar{\mathcal{Q}}_{t+1}^m(i)-\bar{P}_{t+1}^I(i)\Psi\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right)\right)\right]\right]E_t\left[r_{t+1}^m(i)+\mathsf{z}_{t+1}^m(i)\right]\\ +\left[1-\frac{\beta E_t\bar{\Gamma}_{t+1}(i)}{\bar{\mathcal{Q}}_t^m(i)\bar{\Gamma}_t(i)}E_t\left[\left((1-d_{t+1}^m(i))\bar{\mathcal{Q}}_{t+1}^m+(1-\tau_{t+1}^k)\bar{R}_{t+1}^m(i)\bar{\mathcal{Z}}_{t+1}^m(i)\right)\right]\right]E_t\left[p_{t+1}^I(i)+\frac{\Psi'\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right)}{\Psi\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right)}\bar{\mathcal{Z}}_{t+1}^m(i)\mathsf{z}_{t+1}^m(i)\right]\end{array}\right].\end{aligned}$$

Since progressing further yields only marginal simplifications, we will derive the log-linearized FOC (42) around its **steady state**:[48]

$$\bar{\mathcal{Q}}^m(i)=\beta\left[(1-d^m(i))\bar{\mathcal{Q}}^m(i)+(1-\tau^k)\bar{R}^m(i)\right]. \tag{135}$$

Using log-linearization rule (112), we get:

$$\begin{aligned}&\bar{\mathcal{Q}}^m(i)\bar{\Gamma}(i)\left[q_t^m(i)+\gamma_t(i)\right]\simeq\beta\bar{\Gamma}(i)\left[(1-d^m(i))\bar{\mathcal{Q}}^m(i)+(1-\tau^k)\bar{R}^m(i)\right]E_t\gamma_{t+1}(i)\\ &+\beta\bar{\Gamma}(i)\left[\begin{array}{c}(1-d^m(i))\bar{\mathcal{Q}}^m(i)E_tq_{t+1}^m(i)+(1-\tau^k)\bar{R}^m(i)E_tr_{t+1}^m(i)\\ +(1-\tau^k)\bar{R}^m(i)E_tz_{t+1}^m(i)-\bar{P}^I(i)\Psi'(1))\,E_tz_{t+1}^m(i)\end{array}\right].\end{aligned}$$

Dividing both sides by $\bar{\Gamma}(i)$ we get:

$$\begin{aligned}&\bar{\mathcal{Q}}^m(i)\left[q_t^m(i)+\gamma_t(i)\right]\simeq\beta\left[(1-d^m(i))\bar{\mathcal{Q}}^m(i)+(1-\tau^k)\bar{R}^m(i)\right]E_t\gamma_{t+1}(i)\\ &+\beta\left[\begin{array}{c}(1-d^m(i))\bar{\mathcal{Q}}^m(i)E_tq_{t+1}^m(i)+(1-\tau^k)\bar{R}^m(i)E_tr_{t+1}^m(i)\\ +(1-\tau^k)\bar{R}^m(i)E_tz_{t+1}^m(i)-\bar{P}^I(i)\Psi'(1))\,E_tz_{t+1}^m(i)\end{array}\right].\end{aligned}$$

Using (135) and replacing it in the previous expression, we get:

$$\begin{aligned}&\left[q_t^m(i)+\gamma_t(i)\right]\bar{\mathcal{Q}}^m(i)\simeq\bar{\mathcal{Q}}^m(i)E_t\gamma_{t+1}(i)+\beta(1-d^m(i))\bar{\mathcal{Q}}^m(i)E_tq_{t+1}^m(i)+\\ &\beta(1-\tau^k)\bar{R}^m(i)E_tr_{t+1}^m(i)+\bar{\mathcal{Q}}^m(i)\left[1-\beta(1-d^m(i))\right]E_tz_{t+1}^m(i)-\beta\bar{P}^I(i)\Psi'(1))\,E_tz_{t+1}^m(i),\end{aligned}$$

or dividing both sides by $\bar{\mathcal{Q}}^m(i)$:

[47] Notice that τ_t^k and $d_t^m(i)$ are assumed to be time-dependent and exogenous.

[48] Recall that the steady state $\bar{Z}^m(i)=1$ and that $\Psi(1)=0$ (see footnote 12). Notice that in the steady state the depreciation rate and the capital tax rate are assumed to be constant and exogenous parameters.

$q_t^m(i) + \gamma_t(i) \simeq E_t\gamma_{t+1}(i) + \beta(1-d^m(i))E_t q_{t+1}^m(i)$
$\quad + [1-\beta(1-d^m(i))]\, E_t r_{t+1}^m(i) - \frac{\beta\left[(1-\tau^k)\bar{R}^m(i) - \bar{P}^I(i)\Psi'(1))\right]}{\bar{Q}^m(i)} E_t z_{t+1}^m(i).$

Operating algebraically and given that $(1-\tau^k)\bar{R}_{t+1}^m(i) = \bar{P}^I(i)\Psi'(1))$ from the FOC (43) in the steady state, we obtain:

$q_t^m(i) + \gamma_t(i) \simeq E_t\gamma_{t+1}(i) + \beta(1-d^m(i))E_t q_{t+1}^m(i) + [1-\beta(1-d^m(i))]\, E_t r_{t+1}^m(i)$

Finally, taking (125) into account, we get:

$$q_t^m(i) \simeq -E_t r_{t,t+1} + \beta(1-d^m(i))E_t q_{t+1}^m(i) + [1-\beta(1-d^m(i))]\, E_t r_{t+1}^m(i), \tag{136}$$

which is similar to equation (51) in Adjémian *et al.* (2004) with the only difference being that inflation does not show up since our CBC (7) is in nominal terms.

D.8 FOC (43) of home capital utilization

The FOC w.r.t. the utilization rate of physical capital, evaluated at time-dependent paths as natural values is:

$$(1-\tau_{t+1}^k)\bar{R}_{t+1}^m(i) = \bar{P}_{t+1}^I(i)\Psi'\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right) \quad \text{for } m = FT, FN, VT, VN \tag{137}$$

and using formula (112), we obtain:

$$(1-\tau_{t+1}^k)\bar{R}_{t+1}^m(i) r_{t+1}^m(i) \simeq \Psi'\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right)\bar{P}_{t+1}^I(i) p_{t+1}^I(i) + \Psi''\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right)\bar{P}_t^I(i)\bar{\mathcal{Z}}_{t+1}^m(i)\mathsf{z}_{t+1}^m(i),$$

using expression (137) that holds in the natural path equilibrium, after some algebraic operations, we get:

$$r_{t+1}^m(i) \simeq p_{t+1}^I(i) + \frac{\Psi''\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right)}{\Psi'\left(\bar{\mathcal{Z}}_{t+1}^m(i)\right)}\bar{\mathcal{Z}}_{t+1}^m(i)\mathsf{z}_{t+1}^m(i) \quad \text{for } m = FT, FN, VT, VN.$$

In the steady state the capital is fully utilized, $\bar{\mathcal{Z}}_{t+1}^m(i) = 1$ and the cost function is $\Psi(1) = 0$ with slope $\Psi'(1) < 0$, then:

$$r_{t+1}^m(i) \simeq p_{t+1}^I(i) + \frac{\Psi''(1)}{\Psi'(1)}\mathsf{z}_{t+1}^m(i) \quad \text{for } m = FT, FN, VT, VN,$$

where $\frac{\Psi''(1)}{\Psi'(1)}$ is the elasticity of the capital utilization cost function.

D.9 Log-linearization of the law of motion of capital

Consider the log-linearized law of motion of capital, originating from equation (6), around the steady state:

$\bar{K}^m(i) = (1-d^m(i))\bar{K}^m(i) + (1-\Upsilon(1))\,\bar{I}^m(i)$ or

$\bar{K}^m(i) = \frac{(1-\Upsilon(1))}{d^m(i)}\bar{I}^m(i).$

Using the log-linearization formula (112), we get:

$\bar{K}^m(i)k_{t+1}^m(i) = (1-d^m(i))\bar{K}^m(i)k_t^m(i) + (1-\Upsilon(1))\,\bar{I}^m(i)i_t^m(i) - \Upsilon'(1)\frac{\bar{I}^m(i)}{\bar{I}^m(i)}i_t^m(i) + \Upsilon'(1)\frac{\bar{I}^m(i)}{\left[\bar{I}^m(i)\right]^2}\bar{I}^m(i)i_{t-1}^m(i).$

Simplyfing, given that $\Upsilon'(1) = 0$:

$\bar{K}^m(i)k_{t+1}^m(i) = (1-d^m(i))\bar{K}^m(i)k_t^m(i) + (1-\Upsilon(1))\,\bar{I}^m(i)i_t^m(i),$

$\frac{(1-\Upsilon(1))}{d^m(i)}\bar{I}^m(i)k_{t+1}^m(i) = (1-d^m(i))\frac{(1-\Upsilon(1))}{d^m(i)}\bar{I}^m(i)k_t^m(i) + (1-\Upsilon(1))\,\bar{I}^m(i)i_t^m(i),$

$$k_{t+1}^m(i) = (1-d^m(i))k_t^m(i) + d^m(i)i_t^m(i). \tag{138}$$

D.10 Modigliani-Miller theorem

The rental rate can be approximated from Plasmans (1975) as:

$R_t^m(i) \cong P_t^{I,m}(i)\left[d^m(i) + r_t\right],$

which in the steady state turns out to be, after using (112):

$\bar{R}^m(i) = \bar{P}^{I,m}(i)\left[d^m(i) + \bar{r}\right]$ and $\beta = \frac{1}{1+\bar{r}} \Rightarrow \bar{r} = \frac{1-\beta}{\beta}$; then: $\bar{R}^m(i) = \bar{P}^{I,m}(i)\left[d^m(i) + \frac{1-\beta}{\beta}\right].$

Log-linearizing around the steady state, we get:

$\bar{R}^m(i)r_t^m(i) = \bar{P}^{I,m}(i)p_t^{I,m}(i)\left[d^m(i) + \bar{r}\right] + \bar{P}^{I,m}(i)\left(\frac{1-\beta}{\beta}\right)r_t$

and replacing $\bar{R}^m(i)$ by $\bar{P}^{I,m}(i)\left[d^m(i)+\frac{1-\beta}{\beta}\right]$, we obtain:
$\bar{P}^{I,m}(i)\left[d^m(i)+\frac{1-\beta}{\beta}\right]r_t^m(i)=\bar{P}^{I,m}(i)p_t^{I,m}(i)\left[d^m(i)+\frac{1-\beta}{\beta}\right]+\bar{P}^{I,m}(i)\left(\frac{1-\beta}{\beta}\right)r_t$.
Finally, we get:

$$\left[d^m(i)+\frac{1-\beta}{\beta}\right]r_t^m(i)=p_t^{I,m}(i)\left[d^m(i)+\frac{1-\beta}{\beta}\right]+\left(\frac{1-\beta}{\beta}\right)r_t. \tag{139}$$

E Derived demand functions

Given production function (29), we derive the optimal demands for each input such that firm j in the final goods producing sector $m=FT,FN$ minimizes total costs $TC_t^m(j)(.)$ subject to $Y_t^m(j)\equiv(\Omega_{m,t}Z_t^m(j))^{\varpi_m}$.

Constructing the Lagrangian at period t:
$\pounds_t^m(j)\equiv W_t^m(j)L_t^m(j)+R_t^m(j)\mathcal{K}_t^m(j)+P_t^{V,m}(j)V_t^m(j)-\Lambda_t^m(j)\left((\Omega_{m,t}Z_t^m(j))^{\varpi_m}-Y_t^m(j)\right)$,
yields the following FOCs from the unconstrained minimization of this Lagrangian:
$\frac{\partial\pounds_t^m(j)}{\partial L_t^m(j)}=0\Leftrightarrow W_t^m(j)-\Lambda_t^m(j)\varpi_m(\Omega_{m,t}Z_t^m(j))^{\varpi_m-1}\frac{\partial Z_t^m(j)}{\partial L_t^m(j)}=0$,
$\frac{\partial\pounds_t^m(j)}{\partial K_t^m(j)}=0\Leftrightarrow \mathcal{Z}_t^m R_t^m(j)-\Lambda_t^m(j)\varpi_m(\Omega_{m,t}Z_t^m(j))^{\varpi_m-1}\frac{\partial Z_t^m(j)}{\partial K_t^m(j)}=0$,
$\frac{\partial\pounds_t^m(j)}{\partial V_t^m(j)}=0\Leftrightarrow P_t^{V,m}(j)-\Lambda_t^m(j)\varpi_m(\Omega_{m,t}Z_t^m(j))^{\varpi_m-1}\frac{\partial Z_t^m(j)}{\partial V_t^m(j)}=0$ and
$\frac{\partial\pounds_t^m(j)}{\partial\Lambda_t^m(j)}=0\Leftrightarrow(\Omega_{m,t}Z_t^m(j))^{\varpi_m}-Y_t^m(j)=0$.

Considering the partial derivatives of $Z_t^m(j)$ w.r.t. all inputs and defining $\upsilon_{Vm}\equiv1-\upsilon_{Km}-\upsilon_{Lm}$, we get:
$\frac{\partial Z_t^m(j)}{\partial L_t^m(j)}=\left[\upsilon_{Lm}^{\frac{1}{\gamma_m}}(L_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}+\upsilon_{Km}^{\frac{1}{\gamma_m}}(\mathcal{K}_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}+\upsilon_{Vm}^{\frac{1}{\gamma_m}}(V_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}\right]^{\frac{1}{\gamma_m-1}}\upsilon_{Lm}^{\frac{1}{\gamma_m}}(L_t^m(j))^{-\frac{1}{\gamma_m}}$; $\frac{\partial Z_t^m(j)}{\partial K_t^m(j)}$ and $\frac{\partial Z_t^m(j)}{\partial V_t^m(j)}$ are analogously derived.

Substitution of these partial derivatives in the above FOCs leads to the following input price expressions:
$$W_t^m(j)=\Lambda_t^m(j)\varpi_m\left\{\Omega_{m,t}\left[\upsilon_{Lm}^{\frac{1}{\gamma_m}}(L_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}+\upsilon_{Km}^{\frac{1}{\gamma_m}}(\mathcal{K}_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}+\upsilon_{Vm}^{\frac{1}{\gamma_m}}(V_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}\right]^{\frac{\gamma_m}{\gamma_m-1}}\right\}^{\varpi_m-1}$$
$$\left[\upsilon_{Lm}^{\frac{1}{\gamma_m}}(L_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}+\upsilon_{Km}^{\frac{1}{\gamma_m}}(\mathcal{K}_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}+\upsilon_{Vm}^{\frac{1}{\gamma_m}}(V_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}\right]^{\frac{1}{\gamma_m-1}}\upsilon_{Lm}^{\frac{1}{\gamma_m}}(L_t^m(j))^{-\frac{1}{\gamma_m}},$$
$$\mathcal{Z}_t^m R_t^m(j)=\Lambda_t^m(j)\varpi_m\left\{\Omega_{m,t}\left[\upsilon_{Lm}^{\frac{1}{\gamma_m}}(L_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}+\upsilon_{Km}^{\frac{1}{\gamma_m}}(\mathcal{K}_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}+\upsilon_{Vm}^{\frac{1}{\gamma_m}}(V_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}\right]^{\frac{\gamma_m}{\gamma_m-1}}\right\}$$
$$\left[\upsilon_{Lm}^{\frac{1}{\gamma_m}}(L_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}+\upsilon_{Km}^{\frac{1}{\gamma_m}}(\mathcal{K}_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}+\upsilon_{Vm}^{\frac{1}{\gamma_m}}(V_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}\right]^{\frac{1}{\gamma_m-1}}\upsilon_{Km}^{\frac{1}{\gamma_m}}(\mathcal{K}_t^m(j))^{-\frac{1}{\gamma_m}}$$
and
$$P_t^{V,m}(j)=\Lambda_t^m(j)\varpi_m\left\{\Omega_{m,t}\left[\upsilon_{Lm}^{\frac{1}{\gamma_m}}(L_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}+\upsilon_{Km}^{\frac{1}{\gamma_m}}(\mathcal{K}_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}+\upsilon_{Vm}^{\frac{1}{\gamma_m}}(V_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}\right]^{\frac{\gamma_m}{\gamma_m-1}}\right\}$$
$$\left[\upsilon_{Lm}^{\frac{1}{\gamma_m}}(L_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}+\upsilon_{Km}^{\frac{1}{\gamma_m}}(\mathcal{K}_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}+\upsilon_{Vm}^{\frac{1}{\gamma_m}}(V_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}\right]^{\frac{1}{\gamma_m-1}}\upsilon_{Vm}^{\frac{1}{\gamma_m}}(V_t^m(j))^{-\frac{1}{\gamma_m}},$$
so that
$$\frac{W_t^m(j)}{\upsilon_{Lm}^{\frac{1}{\gamma_m}}(L_t^m(j))^{-\frac{1}{\gamma_m}}}=\frac{\mathcal{Z}_t^m R_t^m(j)}{\upsilon_{Km}^{\frac{1}{\gamma_m}}(\mathcal{K}_t^m(j))^{-\frac{1}{\gamma_m}}}=\frac{P_t^{V,m}(j)}{\upsilon_{Vm}^{\frac{1}{\gamma_m}}(V_t^m(j))^{-\frac{1}{\gamma_m}}}.$$

Hence,
$$L_t^m(j)=\frac{\upsilon_{Lm}}{\upsilon_{Km}}\left(\frac{\mathcal{Z}_t^m R_t^m(j)}{W_t^m(j)}\right)^{\gamma_m}\mathcal{K}_t^m(j)\text{ and }V_t^m(j)=\frac{\upsilon_{Vm}}{\upsilon_{Km}}\left(\frac{\mathcal{Z}_t^m R_t^m(j)}{P_t^{V,m}(j)}\right)^{\gamma_m}\mathcal{K}_t^m(j),$$

so that, replacing these expressions for labor and intermediate goods into the production function constraint (29), we get:
$$Y_t^m(j)=\left\{\Omega_{m,t}\left[\begin{array}{c}\upsilon_{Lm}^{\frac{1}{\gamma_m}}\left(\frac{\upsilon_{Lm}}{\upsilon_{Km}}\left(\frac{\mathcal{Z}_t^m R_t^m(j)}{W_t^m(j)}\right)^{\gamma_m}\mathcal{K}_t^m(j)\right)^{\frac{\gamma_m-1}{\gamma_m}}+\upsilon_{Km}^{\frac{1}{\gamma_m}}(\mathcal{K}_t^m(j))^{\frac{\gamma_m-1}{\gamma_m}}\\+\upsilon_{Vm}^{\frac{1}{\gamma_m}}\left(\frac{\upsilon_{Vm}}{\upsilon_{Km}}\left(\frac{\mathcal{Z}_t^m R_t^m(j)}{P_t^{V,m}(j)}\right)^{\gamma_m}\mathcal{K}_t^m(j)\right)^{\frac{\gamma_m-1}{\gamma_m}}\end{array}\right]^{\frac{\gamma_m}{\gamma_m-1}}\right\}^{\varpi_m}$$

$$= \left\{ \Omega_{m,t} \left[\upsilon_{Lm}^{\frac{1}{\gamma_m}} \left(\frac{\upsilon_{Lm}}{\upsilon_{Km}} \left(\frac{\mathcal{Z}_t^m R_t^m(j)}{W_t^m(j)} \right)^{\gamma_m} \right)^{\frac{\gamma_m - 1}{\gamma_m}} + \upsilon_{Km}^{\frac{1}{\gamma_m}} + \upsilon_{Vm}^{\frac{1}{\gamma_m}} \left(\frac{\upsilon_{Vm}}{\upsilon_{Km}} \left(\frac{\mathcal{Z}_t^m R_t^m(j)}{P_t^{V,m}(j)} \right)^{\gamma_m} \right)^{\frac{\gamma_m - 1}{\gamma_m}} \right]^{\frac{\gamma_m}{\gamma_m - 1}} \mathcal{K}_t^m(j) \right\}^{\varpi_m}.$$

We obtain the (optimal) demand for capital stock by firm j as:

$\mathcal{K}_t^m(j) =$

$$= \frac{1}{\left[\upsilon_{Lm}^{\frac{1}{\gamma_m}} \left(\frac{\upsilon_{Lm}}{\upsilon_{Km}} \left(\frac{W_t^m(j)}{\mathcal{Z}_t^m R_t^m(j)} \right)^{-\gamma_m} \right)^{\frac{\gamma_m - 1}{\gamma_m}} + \upsilon_{Km}^{\frac{1}{\gamma_m}} + \upsilon_{Vm}^{\frac{1}{\gamma_m}} \left(\frac{\upsilon_{Vm}}{\upsilon_{Km}} \left(\frac{P_t^{V,m}(j)}{\mathcal{Z}_t^m R_t^m(j)} \right)^{-\gamma_m} \right)^{\frac{\gamma_m - 1}{\gamma_m}} \right]^{\frac{\gamma_m}{\gamma_m - 1}}} \frac{\left[Y_t^m(j) \right]^{\frac{1}{\varpi_m}}}{\Omega_{m,t}}$$

$$= \frac{\left(\left[\left(\mathcal{Z}_t^m R_t^m(j) \right)^{-\gamma_m} \right]^{\frac{\gamma_m - 1}{\gamma_m}} \right)^{\frac{\gamma_m}{\gamma_m - 1}}}{\left(\left[\upsilon_{Lm}^{\frac{1}{\gamma_m}} \left(\frac{\upsilon_{Lm}}{\upsilon_{Km}} \left(\frac{W_t^m(j)}{\mathcal{Z}_t^m R_t^m(j)} \right)^{-\gamma_m} \right)^{\frac{\gamma_m - 1}{\gamma_m}} + \upsilon_{Km}^{\frac{1}{\gamma_m}} + \upsilon_{Vm}^{\frac{1}{\gamma_m}} \left(\frac{\upsilon_{Vm}}{\upsilon_{Km}} \left(\frac{P_t^{V,m}(j)}{\mathcal{Z}_t^m R_t^m(j)} \right)^{-\gamma_m} \right)^{\frac{\gamma_m - 1}{\gamma_m}} \right] \left(\mathcal{Z}_t^m R_t^m(j) \right)^{1-\gamma_m} \right)^{\frac{\gamma_m}{\gamma_m - 1}}} \frac{\left[Y_t^m(j) \right]^{\frac{1}{\varpi_m}}}{\Omega_{m,t}}$$

$$= \frac{\left(\mathcal{Z}_t^m R_t^m(j) \right)^{-\gamma_m}}{\left(\left[\upsilon_{Lm}^{\frac{1}{\gamma_m}} \left(\frac{\upsilon_{Lm}}{\upsilon_{Km}} \left(W_t^m(j) \right)^{-\gamma_m} \right)^{\frac{\gamma_m - 1}{\gamma_m}} + \upsilon_{Km}^{\frac{1}{\gamma_m}} \left(\mathcal{Z}_t^m R_t^m(j) \right)^{1-\gamma_m} + \upsilon_{Vm}^{\frac{1}{\gamma_m}} \left(\frac{\upsilon_{Vm}}{\upsilon_{Km}} \left(P_t^{V,m}(j) \right)^{-\gamma_m} \right)^{\frac{\gamma_m - 1}{\gamma_m}} \right] \right)^{\frac{\gamma_m}{\gamma_m - 1}}} \frac{\left[Y_t^m(j) \right]^{\frac{1}{\varpi_m}}}{\Omega_{m,t}}$$

$$= \frac{\left(\mathcal{Z}_t^m R_t^m(j) \right)^{-\gamma_m}}{\left(\left[\upsilon_{Lm} \left(\frac{1}{\upsilon_{Km}} \left(W_t^m(j) \right)^{-\gamma_m} \right)^{\frac{\gamma_m - 1}{\gamma_m}} + \upsilon_{Km}^{\frac{1}{\gamma_m}} \left(\mathcal{Z}_t^m R_t^m(j) \right)^{1-\gamma_m} + \upsilon_{Vm} \left(\frac{1}{\upsilon_{Km}} \left(P_t^{V,m}(j) \right)^{-\gamma_m} \right)^{\frac{\gamma_m - 1}{\gamma_m}} \right] \right)^{\frac{\gamma_m}{\gamma_m - 1}}} \frac{\left[Y_t^m(j) \right]^{\frac{1}{\varpi_m}}}{\Omega_{m,t}}$$

$$= \frac{\left(\mathcal{Z}_t^m R_t^m(j) \right)^{-\gamma_m} \upsilon_{Km}}{\left(\left[\upsilon_{Lm} \left(W_t^m(j) \right)^{1-\gamma_m} + \left(\upsilon_{Km} \right)^{\frac{1}{\gamma_m}} \left(\upsilon_{Km} \right)^{\frac{\gamma_m - 1}{\gamma_m}} \left(\mathcal{Z}_t^m R_t^m(j) \right)^{1-\gamma_m} + \upsilon_{Vm} \left(P_t^{V,m}(j) \right)^{1-\gamma_m} \right] \right)^{\frac{\gamma_m}{\gamma_m - 1}}} \frac{\left[Y_t^m(j) \right]^{\frac{1}{\varpi_m}}}{\Omega_{m,t}}$$

Finally, we obtain:

$$K_t^m(j) = \frac{\upsilon_{Km} \left(\mathcal{Z}_t^m R_t^m(j) \right)^{-\gamma_m}}{\mathcal{Z}_t^m \left(\left[\upsilon_{Lm} \left(W_t^m(j) \right)^{1-\gamma_m} + \upsilon_{Km} \left(\mathcal{Z}_t^m R_t^m(j) \right)^{1-\gamma_m} + \upsilon_{Vm} \left(P_t^{V,m}(j) \right)^{1-\gamma_m} \right] \right)^{\frac{\gamma_m}{\gamma_m - 1}}} \frac{\left[Y_t^m(j) \right]^{\frac{1}{\varpi_m}}}{\Omega_{m,t}}.$$

which is, taking account of the dual price function (47) of the inputs, the derived demand for capital (45) in the main text.

Proceeding in the same way (replacing the relevant marginal conditions) it is straightforward to show that the other (optimal) input demands are:

$$L_t^m(j) = \frac{\upsilon_{Lm} \left(W_t^m(j) \right)^{-\gamma_m}}{\left(\left[\upsilon_{Lm} \left(W_t^m(j) \right)^{1-\gamma_m} + \upsilon_{Km} \left(\mathcal{Z}_t^m R_t^m(j) \right)^{1-\gamma_m} + \upsilon_{Vm} \left(P_t^{V,m}(j) \right)^{1-\gamma_m} \right] \right)^{\frac{\gamma_m}{\gamma_m - 1}}} \frac{\left[Y_t^m(j) \right]^{\frac{1}{\varpi_m}}}{\Omega_{m,t}}$$

and

$$V_t^m(j) = \frac{\upsilon_{Vm} \left(P_t^{V,m}(j) \right)^{-\gamma_m}}{\left(\left[\upsilon_{Lm} \left(W_t^m(j) \right)^{1-\gamma_m} + \upsilon_{Km} \left(\mathcal{Z}_t^m R_t^m(j) \right)^{1-\gamma_m} + \upsilon_{Vm} \left(P_t^{V,m}(j) \right)^{1-\gamma_m} \right] \right)^{\frac{\gamma_m}{\gamma_m - 1}}} \frac{\left[Y_t^m(j) \right]^{\frac{1}{\varpi_m}}}{\Omega_{m,t}},$$

so that equations (44), (45) and (46) in the main text are verified.

The cost minimizing input demands for the intermediate goods producing sectors $m = VT, VN.$ is derived analogously from minimizing total costs in labor and capital costs subject to production function (34) yielding derived demand equations (51) and (52) in the main text.

F Total consumers' benefits

Total consumers' benefits from the firms' ownership are defined as follows:

$$\mathcal{B}_t \equiv \mathcal{B}_t^{FT} + \mathcal{B}_t^{FN} + \mathcal{B}_t^{VT} + \mathcal{B}_t^{VN} + \mathcal{B}_t^{MF} + \mathcal{B}_t^{MV}, \tag{140}$$

where specific sectoral benefits are:[49]

$$\mathcal{B}_{h.t}^{FT} \equiv P_{h,t}^{CT}\left(C_{h,t}^{T} + G_{h,t}^{T}\right) + S_t P_{h,t}^{CT,X}\left(C_{h,t}^{*T} + G_{h,t}^{*T}\right) + P_{h,t}^{IT} I_{h,t}^{T} + S_t P_{h,t}^{IT,X} I_{h,t}^{*T} - W_t^{FT} L_t^{FT} - P_{h,t}^{VT} V_t^{FT} - R_t^{FT} \mathcal{Z}_t^{FT} K_t^{FT} + P_t^{I} \Psi(\mathcal{Z}_t^{FT}) K_t^{FT}, \tag{141}$$

where the total revenue from serving domestic is defined as:
$P_{h,t}^{IT} I_{h,t}^{T} \equiv P_{h,t}^{IT,FT} I_{h,t}^{T,FT} + P_{h,t}^{IT,FN} I_{h,t}^{T,FN} + P_{h,t}^{IT,VT} I_{h,t}^{T,VT} + P_{h,t}^{IT,VN} I_{h,t}^{T,VN}$ and

$$\mathcal{B}_{h.t}^{FN} \equiv P_{h,t}^{CN}\left(C_{h,t}^{N} + G_{h,t}^{N}\right) + P_{h,t}^{IN} I_{h,t}^{N} - W_t^{FN} L_t^{FN} - P_{h,t}^{VN} V_t^{FN} - R_t^{FN} \mathcal{Z}_t^{FN} K_t^{FN} + P_t^{I} \Psi(\mathcal{Z}_t^{FN}) K_t^{FN}, \tag{142}$$

where the total revenue from serving non-tradable investment markets can be defined as:
$P_{h,t}^{IN} I_{h,t}^{N} \equiv P_{h,t}^{IN,FT} I_{h,t}^{N,FT} + P_{h,t}^{IN,FN} I_{h,t}^{N,FN} + P_{h,t}^{IN,VT} I_{h,t}^{N,VT} + P_{h,t}^{IN,VN} I_{h,t}^{N,VN}$;

$$\mathcal{B}_{h,t}^{VT} \equiv P_{h,t}^{VT,FT} V_{h,t}^{T,FT} + S_t P_{h,t}^{VT,X,FT} V_{h,t}^{*T,FT} + P_{h,t}^{VT,FN} V_{h,t}^{T,FN} + S_t P_{h,t}^{VT,X,FN} V_{h,t}^{*T,FN} - W_t^{VT} L_t^{VT} - R_t^{VT} \mathcal{Z}_t^{VT} K_t^{VT} + P_t^{I} \Psi(\mathcal{Z}_t^{VT}) K_t^{VT}, \tag{143}$$

$$\mathcal{B}_{h,t}^{VN} \equiv P_{h,t}^{VN,FT} V_{h,t}^{N,FT} + P_{h,t}^{VN,FN} V_{h,t}^{N,FN} - W_t^{VN} L_t^{VN} - R_t^{VN} \mathcal{Z}_t^{VN} K_t^{VN} + P_t^{I} \Psi(\mathcal{Z}_t^{VN}) K_t^{VN}, \tag{144}$$

$$\mathcal{B}_{h,t}^{MF} \equiv P_{f,t}^{CT}\left(C_{f,t}^{T} + G_{f,t}^{T}\right) + P_{f,t}^{IT,FT} I_{f,t}^{T,FT} + P_{f,t}^{IT,FN} I_{f,t}^{T,FN} + P_{f,t}^{IT,VT} I_{f,t}^{T,VT} + P_{f,t}^{IT,VN} I_{f,t}^{T,VN} - S_t P_{f,t}^{CT,X} C_{f,t}^{T} - S_t P_{f,t}^{IT,X} I_{f,t}^{T} \tag{145}$$

and (146)

$$\mathcal{B}_{h,t}^{MV} \equiv P_{f,t}^{VT,FT} V_{f,t}^{T,FT} + P_{f,t}^{VT,FN} V_{f,t}^{T,FN} - S_t P_{f,t}^{VT,X,FT} V_{f,t}^{T,FT} - S_t P_{f,t}^{VT,X,FN} V_{f,t}^{T,FN}. \tag{147}$$

G Net Foreign Assets log-linearization

Log-linearizing equation (63) in the main text with respect to an expected zero depreciation of the exchange rate, we get:

$$E_t\left[\Delta s_{t+1}\right] = r_{t,t+1} - r_{t,t+1}^{*} - \ln E_t\left[\digamma_{t+1}(.)\right], \tag{148}$$

where $\ln E_t\left[\digamma_{t+1}(.)\right]$ is the (logarithmic) expectation of risk premium (61).

In order to evaluate $\ln E_t\left[\digamma_{t+1}(.)\right]$ in (148), we derive from (61) that:

$$\ln E_t\left[\digamma_{t+1}(.)\right] = E_t\left\{v_{t+1}\right\}\left[\exp\left(\frac{\bar{S}\bar{B}_f(i)}{\bar{P}(i)} - \frac{S_t B_{f,t+1}(i)}{P_{t+1}(i)}\right) - 1\right] = \delta\left[\exp\left(\frac{\bar{S}\bar{B}_f(i)}{\bar{P}(i)} - \frac{S_t B_{f,t+1}(i)}{P_{t+1}(i)}\right) - 1\right], \tag{149}$$

where consumer i's aggregate price index $P_t(i)$ is given by (62).

Aggregating over households as in Section 7 and rewriting NFAs equation (68) in real terms, i.e. deflating it with (62), as:

$$E_t\left[\hat{Q}_{f,t,t+1}\right]\frac{\left(S_t B_{f,t+1} - B_{h,t+1}^{*}\right)}{P_{t+1}} + \frac{\left(B_{h,t}^{*} - S_t B_{f,t}\right)}{P_{t+1}} = \frac{1}{P_{t+1}}\left(NX_{h,t}^{FT} + NX_{h,t}^{VT}\right), \tag{150}$$

which can be log-linearized by considering the behavior of domestic and foreign assets in the **steady state**. To this end, we rewrite the aggregated versions of first order conditions (39-40) for home and foreign assets in the steady state, taking account of the risk premium definition (61), as:

$$\frac{\bar{\Gamma}\beta^{*}\bar{S}}{\digamma\left(\frac{\bar{S}\bar{B}_f}{\bar{P}}\right)} - \beta\bar{\Gamma}\bar{S} = 0.$$

[49] Recall that the foreign importers of final goods have as costs the amount $S_t^{-1} P_{h,t}^{X,IT} I_{h,t}^{*T} \equiv S_t^{-1}\left(P_{h,IT,t}^{X,FT} I_{h,t}^{*T,FT} + P_{h,IT,t}^{X,FN} I_{h,t}^{*T,FN} + P_{h,IT,t}^{X,VT} I_{h,t}^{*T,VT} + P_{h,IT,t}^{X,VN} I_{h,t}^{*T,VN}\right)$.

Defining the real asset value and $B_{f,t}^{(R)} \equiv \frac{S_{t-1}B_{f,t}}{P_t}$, and considering that $\beta^* = \frac{1}{1+\bar{r}^*}$ and the assumption that the intertemporal discount rates are equal over the world in the steady state, i.e. $\beta = \beta^* \equiv \beta$, it follows that (61) in the steady state are one, or:

$$F\left(\bar{B}_f^{(R)}\right) = 1. \tag{151}$$

The assumptions concerning the risk function and $F(.)$ imply that $\bar{B}_f^{(R)} = 0$, or foreign assets owned by domestic consumers are zero in the steady state. Similarly for foreign households. Hence, the steady state of the real NFAs equation (150) is:

$$0 = \overline{NX}_h^{FT} + \overline{NX}_h^{VT}. \tag{152}$$

Applying the log-linearization rules from Appendix C, where the term between square brackets in the aggregate version of equation (149) is approximated by $b_{f,t+1}^{(R)} \equiv \ln \frac{\bar{S}\bar{B}_f}{\bar{P}} - \ln\left(\frac{S_t B_{f,t+1}}{P_{t+1}(i)}\right)$, so that (148) becomes the familiar UIP condition:

$$E_t\left[\Delta s_{t+1}\right] \simeq r_{t,t+1} - r_{t,t+1}^* - \delta b_{f,t+1}^{(R)}. \tag{153}$$

Equation (150) can be log-linearized around its steady state (152), taking account of $\beta = \beta^*$, as:

$$\beta\left(b_{f,t+1}^{(R)} - b_{h,t+1}^{*(R)}\right) - \left(b_{f,t}^{(R)} - b_{h,t}^{*(R)}\right) \simeq \frac{\overline{NX}_h^{FT}}{\bar{P}}\left(nx_{h,t}^{FT} - p_{t+1}\right) + \frac{\overline{NX}_h^{VT}}{\bar{P}}\left(nx_{h,t}^{VT} - p_{t+1}\right), \tag{154}$$

where from definitions (69) and (70) of final and intermediate goods net exports, we get the following log-linearized net export terms:[50]

$$\begin{aligned}\frac{\overline{NX}_h^{FT}}{\bar{P}}\left(nx_{h,t}^{FT} - p_{t+1}\right) =& \frac{\bar{P}_h^{CT,X}}{\bar{P}}\bar{\mathcal{C}}_h^{*T}\left(p_{h,t}^{CT,X} + \mathsf{c}_{h,t}^{*T} - p_{t+1}\right) + \frac{\bar{P}_{h,t}^{IT,X}}{\bar{P}}\bar{I}_h^{*T}\left(p_{h,t}^{IT,X} + i_{h,t}^{*T} - p_{t+1}\right)\\ &- \frac{\bar{S}\bar{P}_f^{*CT,X}}{\bar{P}}\bar{\mathcal{C}}_f^{T}\left(s_t + p_{f,t}^{*CT,X} + \mathsf{c}_{f,t}^{T} - p_{t+1}\right) - \frac{\bar{S}\bar{P}_f^{*IT,X}}{\bar{P}}\bar{I}_f\left(s_t + p_{f,t}^{*IT,X} + i_{f,t}^{T} - p_{t+1}\right)\end{aligned} \tag{155}$$

and

$$\begin{aligned}\frac{\overline{NX}_h^{VT}}{\bar{P}}\left(nx_{h,t}^{VT} - p_{t+1}\right) =& \frac{\bar{P}_h^{VT,X,FT}}{\bar{P}}\bar{V}_h^{*T,FT}\left(p_{h,t}^{VT,X,FT} + v_{h,t}^{*T,FT} - p_{t+1}\right)\\ &+ \frac{\bar{P}_h^{VT,X,FN}}{\bar{P}}\bar{V}_h^{*T,FN}\left(p_{h,t}^{VT,X,FN} + v_{h,t}^{*T,FN} - p_{t+1}\right)\\ &- \frac{\bar{S}\bar{P}_f^{*VT,X,FT}}{\bar{P}}\bar{V}_f^{T,FT}\left(s_t + p_{f,t}^{*VT,X,FT} + v_{f,t}^{T,FT} - p_{t+1}\right)\\ &- \frac{\bar{S}\bar{P}_f^{*VT,X,FN}}{\bar{P}}\bar{V}_f^{T,FN}\left(s_t + p_{f,t}^{*VT,X,FN} + v_{f,t}^{T,FN} - p_{t+1}\right).\end{aligned} \tag{156}$$

Notice again that we defined total (private and public) consumption home exports as $\bar{\mathcal{C}}_h^{*T} \equiv \bar{C}_h^{*T} + \bar{G}_h^{*T}$, while consumption home imports were defined as $\bar{\mathcal{C}}_f^{T} \equiv \bar{C}_f^{T} + \bar{G}_f^{T}$. Total consumption exports and imports deviations were defined as $\mathsf{c}_{h,t}^{*T} \equiv \phi_X c_h^{*T} + (1-\phi_X)\, g_h^{*T}$ and $\mathsf{c}_f^{T} \equiv \phi_M c_f^{T} + (1-\phi_M)\, g_f^{T}$, respectively. The shares ϕ_X and ϕ_M are estimated using long-run data.

In (155-156), all the steady state terms, as e.g. $\frac{\bar{P}_h^{CT,X}\bar{\mathcal{C}}_h^{*T}}{\bar{P}}$ and $\frac{\bar{P}_h^{VT,X,FT}\bar{V}_h^{*T,FT}}{\bar{P}}$, must be expressed in terms of model parameters. These steady state terms are derived in a separate Internet Appendix available on *http://www.ua.ac.be/joseph.plasmans* (click "documents" and "files").

H Marginal costs

Final tradable and non-tradable goods marginal costs, or $MC_{h,t}^m(j)$ for $m = FT, FN$, can be derived from the inversion of the production function (29) taking into account the price $P_t^{Z,m}(j)$ from (48). Analogously, the marginal costs for domestic intermediate goods, comes from the inversion of the intermediate goods production

[50] This analysis is done irrespective of government expenditure. Hence, according to (69), consumption stands for the sum of private consumption and government expenditure here.

function (34) taking into account the price $P_t^{Z,m}(j)$ from (48) for $m = VT, VN$. Consequently, firm j's sectoral marginal cost is:

$$MC_{h,t}^m(j) = \frac{1}{\varpi_m} P_t^{Z,m}(j) \frac{[Y_t^m(j)]^{\frac{1}{\varpi_m}-1}}{(\Omega_{m,t})^{\varpi_m}}, \text{ for } m = FT, FN, VT, VN. \quad (157)$$

We derive the log-linearized version of (157). To do so, we rewrite (157) at time-dependent equilibrium paths as natural values as an implicit function $f_t(j) = 0$, or

$$f_t(j) \equiv \overline{MC}_{h,t}^m(j) - P_t^{Z,m}(j) \frac{\left[\bar{Y}_t^m(j)\right]^{\frac{1-\varpi_m}{\varpi_m}}}{\varpi_m \bar{\Omega}_{m,t}} = 0. \quad (158)$$

We consider derivatives: $f_{\overline{MC}_{h,t}^m(j)}(.)$, $f_{P_t^{Z,m}(j)}(.)$, $f_{\bar{Y}_t^m(j)}(.)$ and $f_{\bar{\Omega}_{m,t}}(.)$ and use formula (112) to obtain:

$$\overline{MC}_{h,t}^m(j) mc_{h,t}^m(j) - \frac{\left[\bar{Y}_t^m(j)\right]^{\frac{1-\varpi_m}{\varpi_m}}}{\varpi_m \bar{\Omega}_{m,t}} P_t^{Z,m}(j) P_t^{Z,m}(j) - \frac{1-\varpi_m}{\varpi_m} P_t^{Z,m}(j) \frac{\left[\bar{Y}_t^m(j)\right]^{\frac{1-\varpi_m}{\varpi_m}}}{\varpi_m \bar{\Omega}_{m,t}} y_t^m(j) + P_t^{Z,m}(j) \frac{\left[\bar{Y}_t^m(j)\right]^{\frac{1-\varpi_m}{\varpi_m}}}{\varpi_m \bar{\Omega}_{m,t}} \omega_{m,t} \simeq 0.$$

Taking (158) into account to eliminate variables in natural values, we get log-linear forms of marginal costs of final and intermediate goods:

$$mc_{h,t}^m(j) \simeq P_t^{Z,m}(j) + \frac{1-\varpi_m}{\varpi_m} y_t^m(j) - \omega_{m,t} \text{ for } m = FT,\ FN \text{ and} \quad (159)$$

$$mc_{h,t}^m(j) \simeq P_t^{Z,m}(j) + \frac{1-\varpi_m}{\varpi_m} v_t^m(j) - \omega_{m,t} \text{ for } m = VT,\ VN. \quad (160)$$

I Optimality condition in Calvo-price setting

FOC of maximization problem (71) s.t. the relevant demand function on an arbitrary market n ($n = 1, ..., n_m$),[51] e.g s.t. $Y_{h,t+a}^n(j) = \left(\frac{P_{h,t}^n(j)}{P_{h,t+a}^n}\right)^{-\theta_n} Y_{h,t+a}^n$, is derived as follows:

$$0 = E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \frac{\Gamma_{t+a}(i)}{\Gamma_t(i)} \left[(1-\theta_n) \frac{\left(\breve{P}_{h,t}^n(j)\right)^{-\theta_n}}{\left(P_{h,t+a}^n\right)^{-\theta_n}} Y_{h,t+a}^n + \theta_n MC_{t+a}^m (Y_{h,t+a}^m(j)) \frac{\left(\breve{P}_{h,t}^n(j)\right)^{-\theta_n-1}}{\left(P_{h,t+a}^n\right)^{-\theta_n}} Y_{h,t+a}^n \right]$$

$$= E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \frac{\Gamma_{t+a}(i)}{\Gamma_t(i)} \left[(1-\theta_n) \frac{Y_{h,t+a}^n}{\left(P_{h,t+a}^n\right)^{-\theta_n}} + \theta_n MC_{t+a}^n \left(Y_{h,t+a}^n(j)\right) \frac{1}{\left(P_{h,t+a}^n\right)^{-\theta_n}} \frac{Y_{h,t+a}^n}{\breve{P}_{h,t}^n(j)} \right]$$

$$= E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \frac{\Gamma_{t+a}(i)}{\Gamma_t(i)} \left[(1-\theta_n) Y_{h,t+a}^n \left(P_{h,t+a}^n\right)^{\theta_n} \right]$$
$$+ \frac{1}{\breve{P}_{h,t}^n(j)} E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \frac{\Gamma_{t+a}(i)}{\Gamma_t(i)} \left[\theta_n MC_{t+a}^m \left(Y_{h,t+a}^m(j)\right) \left(P_{h,t+a}^n\right)^{\theta_n} Y_{h,t+a}^n \right].$$

Rearranging, we obtain:

$$\breve{P}_{h,t}^n(j) = \frac{\theta_n}{(\theta_n - 1)} \frac{E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \frac{\Gamma_{t+a}(i)}{\Gamma_t(i)} \left[MC_{t+a}^m \left(Y_{h,t+a}^m(j)\right) Y_{h,t+a}^n \left(P_{h,t+a}^n\right)^{\theta_n} \right]}{E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \frac{\Gamma_{t+a}(i)}{\Gamma_t(i)} \left[Y_{h,t+a}^n \left(P_{h,t+a}^n\right)^{\theta_n} \right]},$$

so that optimality condition (73) in the main text is derived. The log-linearization of equation (74), in deviations from natural values is:

$$p_{h,t}^n(j) \simeq \varphi_n \left(p_{h,t-1}^n(j)\right)^{1-\theta_n} + (1-\varphi_n) \left(\breve{p}_{h,t}^n(j)\right)^{1-\theta_n}. \quad (161)$$

[51] Note that, since marginal cost is (only) sector-specific, it considers sector-superscript m, while prices and (sub)outputs are denoted with market-superscript n.

I.1 Time-dependent equilibrium paths as natural values

Rewriting (73) in natural paths yields for a typical component $\breve{P}^n_{h,t}(j)$:

$$\bar{P}^n_{h,t}(j) E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \frac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)} \left[\bar{Y}^n_{h,t+a} \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \right] = \frac{\theta_n}{(\theta_n\text{-}1)} E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \frac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(j)} \left[\overline{MC}^m_{t+a} \left(\bar{Y}^m_{h,t+a}(j) \right) \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \bar{Y}^n_{h,t+a} \right]. \quad (162)$$

Defining:

$$\text{LHS of (162):} \ f_t(j) \equiv \bar{P}^n_{h,t}(j) E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \frac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)} \left[\bar{Y}^n_{h,t+a} \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \right],$$

$$\text{RHS of (162):} \ g_t(j) \equiv \frac{\theta_n}{(\theta_n - 1)} E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \frac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)} \left[\overline{MC}^m_{t+a} \left(\bar{Y}^m_{h,t+a}(j) \right) \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \bar{Y}^n_{h,t+a} \right],$$

and applying log-linearization rules from Appendix C, we obtain the following formulas for (sums of) derivatives:

$$f_{\bar{P}^n_{h,t}(j)} \bar{P}^n_{h,t}(j) p^n_{h,t}(j) = p^n_{h,t}(j) \bar{P}^n_{h,t}(j) E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \tfrac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)} \left[\bar{Y}^n_{h,t+a} \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \right]$$

$$E_t \sum_{a=0}^{\infty} f_{\bar{Y}^n_{h,t+a}} \bar{Y}^n_{h,t+a} y^n_{h,t+a} = \bar{P}^n_{h,t}(j) E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \tfrac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)} \left[\left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \bar{Y}^n_{h,t+a} y^n_{h,t+a} \right]$$

$$E_t \sum_{a=0}^{\infty} f_{\bar{P}^n_{h,t+a}} \bar{P}^n_{h,t+a} p^n_{h,t+a} = \bar{P}^n_{h,t}(j) E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \tfrac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)} \left[\theta_n \bar{Y}^n_{h,t+a} \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} p^n_{h,t+a} \right]$$

$$E_t \sum_{a=0}^{\infty} f_{\bar{\Gamma}_{t+a}}(j) \bar{\Gamma}_{t+a}(i) \gamma_{t+a}(i) = \bar{P}^n_{h,t}(j) E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \left[\bar{Y}^n_{h,t+a} \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \right] \tfrac{\bar{\Gamma}_{t+a}(i) \gamma_{t+a}(i)}{\bar{\Gamma}_t(i)}$$

$$E_t \sum_{a=0}^{\infty} f_{\bar{\Gamma}_t}(i) \bar{\Gamma}_t(i) \gamma_t(i) = \bar{P}^n_{h,t}(j) E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \left(-\tfrac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)} \right) \gamma_t(i) \left[\bar{Y}^n_{h,t+a} \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \right]$$

$$E_t \sum_{a=0}^{\infty} g_{\overline{MC}^m_{t+a}(j)} \overline{MC}^m_{t+a} (\bar{Y}^m_{h,t+a}(j)) mc^m_{t+a}(.) = \tfrac{\theta_n}{(\theta_n\text{-}1)} E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \tfrac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)} \left[\bar{Y}^n_{h,t+a} \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \overline{MC}^m_{t+a} (\bar{Y}^m_{h,t+a}(j)) mc^m_{t+a}(.) \right]$$

$$E_t \sum_{a=0}^{\infty} g_{\bar{Y}^n_{h,t+a}} \bar{Y}^n_{h,t+a} y^n_{h,t+a} = \tfrac{\theta_n}{(\theta_n - 1)} E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \tfrac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)} \left[\overline{MC}^m_{t+a} (\bar{Y}^m_{h,t+a}(j)) \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \bar{Y}^n_{h,t+a} y^n_{t+a} \right]$$

$$E_t \sum_{a=0}^{\infty} g_{\bar{P}^n_{h,t+a}} \bar{P}^n_{h,t+a} p^n_{h,t+a} = \tfrac{\theta_n}{(\theta_n - 1)} E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \tfrac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)} \left[\theta_n \overline{MC}^m_{t+a} (\bar{Y}^m_{h,t+a}(j)) \bar{Y}^n_{h,t+a} \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} p^n_{t+a} \right]$$

$$E_t \sum_{a=0}^{\infty} g_{\bar{\Gamma}_{t+a}(i)} \bar{\Gamma}_{t+a}(i) \gamma_{t+a}(i) = \tfrac{\theta_n}{(\theta_n - 1)} E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \tfrac{\bar{\Gamma}_{t+a}(i) \gamma_{t+a}(i)}{\bar{\Gamma}_t(i) \bar{\Pi}_{t+a}} \left[\overline{MC}^m_{t+a} (\bar{Y}^m_{h,t+a}(j)) \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \bar{Y}^n_{h,t+a} \right]$$

$$E_t \sum_{a=0}^{\infty} g_{\bar{\Gamma}_t(i)} \bar{\Gamma}_t(i) \gamma_t(i) = \tfrac{\theta_n}{(\theta_n - 1)} E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \tfrac{-\bar{\Gamma}_{t+a}(i) \gamma_t(i)}{\bar{\Gamma}_t(i)} \left[\overline{MC}^m_{t+a} (\bar{Y}^m_{h,t+a}(j)) \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \bar{Y}^n_{h,t+a} \right]$$

Hence, using formula (112), we get:

$$p^n_{h,t}(j) \bar{P}^n_{h,t}(j) E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \tfrac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)} \left[\bar{Y}^n_{h,t+a} \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \right] + \bar{P}^n_{h,t}(j) E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \tfrac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)} \left[\left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \bar{Y}^n_{h,t+a} y^n_{h,t+a} \right]$$
$$+ \bar{P}^n_{h,t}(j) E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \tfrac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)} \left[\theta_n \bar{Y}^n_{h,t+a} \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} p^n_{h,t+a} \right] + \bar{P}^n_{h,t}(j) E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \left[\bar{Y}^n_{h,t+a} \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \right] \tfrac{\bar{\Gamma}_{t+a}(i) \gamma_{t+a}(i)}{\bar{\Gamma}_t(i)}$$
$$+ \bar{P}^n_{h,t}(j) E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \tfrac{-\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)} \gamma_t(i) \left[\bar{Y}^n_{h,t+a} \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \right] \simeq \tfrac{\theta_n}{(\theta_n - 1)} E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \tfrac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)}$$
$$\left[\bar{Y}^n_{h,t+a} \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \overline{MC}^m_{t+a} (\bar{Y}^m_{h,t+a}(j)) mc^m_{t+a}(.) \right]$$
$$+ \tfrac{\theta_n}{(\theta_n - 1)} E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \tfrac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)} \left[\overline{MC}^m_{t+a} (\bar{Y}^m_{h,t+a}(j)) \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} \bar{Y}^n_{h,t+a} y^n_{t+a} \right]$$
$$+ \tfrac{\theta_n}{(\theta_n - 1)} E_t \sum_{a=0}^{\infty} (\varphi_n \beta)^a \tfrac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)} \left[\theta_n \overline{MC}^m_{t+a} (\bar{Y}^m_{h,t+a}(j)) \bar{Y}^n_{h,t+a} \left(\bar{P}^n_{h,t+a} \right)^{\theta_n} p^n_{t+a} \right]$$

$+\frac{\theta_n}{(\theta_n-1)}E_t\sum_{a=0}^{\infty}(\varphi_n\beta)^a\frac{\bar{\Gamma}_{t+a}(i)\gamma_{t+a}(i)}{\bar{\Gamma}_t(i)}\left[\overline{MC}^m_{t+a}(\bar{Y}^m_{h,t+a}(j))\left(\bar{P}^n_{h,t+a}\right)^{\theta_n}\bar{Y}^n_{h,t+a}\right]$
$+\frac{\theta_n}{(\theta_n-1)}E_t\sum_{a=0}^{\infty}(\varphi_n\beta)^a\frac{-\bar{\Gamma}_{t+a}(i)\gamma_t(i)}{\bar{\Gamma}_t(i)}\left[\overline{MC}^m_{t+a}(\bar{Y}^m_{h,t+a}(j))\left(\bar{P}^n_{h,t+a}\right)^{\theta_n}\bar{Y}^n_{h,t+a}\right]$

Dividing both sides by $\bar{P}^n_{h,t}(j)$ and using (162), we obtain:

$$\breve{p}^n_{h,t}(j)\simeq\frac{E_t\sum_{a=0}^{\infty}(\varphi_n\beta)^a\left[\begin{array}{c}\frac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)}\bar{Y}^n_{h,t+a}\left(\bar{P}^n_{h,t+a}\right)^{\theta_n}\overline{MC}^m_{t+a}(\bar{Y}^m_{h,t+a}(j))\times\\ \left(\theta_n p^n_{t+a}+y^n_{t+a}+mc^m_{t+a}(\bar{Y}^m_{h,t+a}(j))+\gamma_{t+a}(i)-\gamma_t(i)\right)\end{array}\right]}{E_t\sum_{a=0}^{\infty}(\varphi_n\beta)^a\frac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)}\left[\overline{MC}^m_{t+a}(\bar{Y}^m_{h,t+a}(j))\bar{Y}^n_{h,t+a}\left(\bar{P}^n_{h,t+a}\right)^{\theta_n}\right]}$$
$$-\frac{E_t\sum_{a=0}^{\infty}(\varphi_n\beta)^a\frac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)}\left[\bar{Y}^n_{h,t+a}\left(\bar{P}^n_{h,t+a}\right)^{\theta_n}\left(\theta_n p^n_{h,t+a}+y^n_{h,t+a}+\gamma_{t+a}(i)-\gamma_t(i)\right)\right]}{E_t\sum_{a=0}^{\infty}(\varphi_n\beta)^a\frac{\bar{\Gamma}_{t+a}(i)}{\bar{\Gamma}_t(i)}\left[\bar{Y}^n_{h,t+a}\left(\bar{P}^n_{h,t+a}\right)^{\theta_n}\right]}. \tag{163}$$

I.2 Varying rate

Assuming a varying trend λ_t for all variables we can rewrite variables in natural equilibrium values as follows (for $a>0$):

$\bar{Y}^n_{h,t+a}=\bar{Y}^n_{h,t}\prod_{k=1}^a\lambda_k$, $\overline{MC}^m_{h,t+a}(.)=\overline{MC}^n_{h,t}(.)\prod_{k=1}^a\lambda_k$, $\bar{P}^n_{h,t+a}=\bar{P}^n_{h,t}\prod_{k=1}^a\lambda_k$ and $\bar{\Gamma}_{t+a}(i)=\bar{\Gamma}_t(i)\prod_{k=1}^a\lambda_k$.

Hence, we rewrite (163) as:

$$\breve{p}^n_{h,t}(j)\simeq\frac{E_t\sum_{a=0}^{\infty}(\varphi_n\beta)^a\left[\begin{array}{c}\left[\prod_{k=1}^a\lambda_k\right]\left[\bar{Y}^n_{h,t}\prod_{k=1}^a\lambda_k\right]\left[\overline{MC}^m_{h,t}(\bar{Y}^m_{h,t+a}(j))\prod_{k=1}^a\lambda_k\right]\left[\bar{P}^n_{h,t}\prod_{k=1}^a\lambda_k\right]^{\theta_n}\times\\ \left(\theta_n p^n_{t+a}+y^n_{t+a}+mc^m_{t+a}(\bar{Y}^m_{h,t+a}(j))+\gamma_{t+a}(i)-\gamma_t(i)\right)\end{array}\right]}{E_t\sum_{a=0}^{\infty}(\varphi_n\beta)^a\left[\prod_{k=1}^a\lambda_k\right]\left[\bar{Y}^n_{h,t}\prod_{k=1}^a\lambda_k\right]\left[\overline{MC}^m_{h,t}(\bar{Y}^m_{h,t+a}(j))\prod_{k=1}^a\lambda_k\right]\left[\bar{P}^n_{h,t}\prod_{k=1}^a\lambda_k\right]^{\theta_n}}$$
$$-\frac{E_t\sum_{a=0}^{\infty}(\varphi_n\beta)^a\left[\prod_{k=1}^a\lambda_k\right]\left[\left[\bar{Y}^n_{h,t}\prod_{k=1}^a\lambda_k\right]\left[\bar{P}^n_{h,t}\prod_{k=1}^a\lambda_k\right]^{\theta_n}\left(\theta_n p^n_{h,t+a}+y^n_{h,t+a}+\gamma_{t+a}(i)-\gamma_t(i)\right)\right]}{E_t\sum_{a=0}^{\infty}(\varphi_n\beta)^a\left[\prod_{k=1}^a\lambda_k\right]\left[\bar{Y}^n_{h,t}\prod_{k=1}^a\lambda_k\right]\left[\bar{P}^n_{h,t}\prod_{k=1}^a\lambda_k\right]^{\theta_n}}. \tag{164}$$

Simplifying:

$$\breve{p}^n_{h,t}(j)\simeq\frac{E_t\sum_{a=0}^{\infty}(\varphi_n\beta)^a\left[\prod_{k=1}^a\lambda_k\right]\left[\left[\prod_{k=1}^a\lambda_k^{\theta_n+2}\right]\left(\theta_n p^n_{t+a}+y^n_{t+a}+mc^m_{t+a}(Y^m_{h,t+a}(j))+\gamma_{t+a}(i)-\gamma_t(i)\right)\right]}{E_t\sum_{a=0}^{\infty}(\varphi_n\beta)^a\left[\prod_{k=1}^a\lambda_k\right]\left[\prod_{k=1}^a\lambda_k^{\theta_n+2}\right]}$$
$$-\frac{E_t\sum_{a=0}^{\infty}(\varphi_n\beta)^a\left[\prod_{k=1}^a\lambda_k\right]\left\{\left[\prod_{k=1}^a\lambda_k^{\theta_n+1}\right]\left(\theta_n p^n_{h,t+a}+y^n_{h,t+a}+\gamma_{t+a}(i)-\gamma_t(i)\right)\right\}}{E_t\sum_{a=0}^{\infty}(\varphi_n\beta)^a\left[\prod_{k=1}^a\lambda_k\right]\left[\prod_{k=1}^a\lambda_k^{\theta_n+1}\right].}. \tag{165}$$

I.3 Constant trend

Assuming constant trend $\lambda_t=\lambda$ we can rewrite variables in time-dependent paths as follows:

$$\bar{Y}^n_{h,t+a}=\bar{Y}^n_{h,t}\lambda^a,\ \overline{MC}^m_{h,t+a}(.)=\overline{MC}^m_{h,t}(.)\lambda^a,\ \bar{P}^n_{h,t+a}(.)=\bar{P}^n_{h,t}\lambda^a \text{ and } \bar{\Gamma}_{t+a}(i)=\bar{\Gamma}_t(i)\lambda^a.$$

Approximation (165) becomes:

$$\breve{p}^n_{h,t}(j) \simeq \frac{E_t\sum_{a=0}^{\infty}(\varphi_n\beta\lambda^{\theta_n+3})^a\left[\theta_n p^n_{t+a}+y^n_{t+a}+mc^m_{t+a}(.)+\gamma_{t+a}(i)-\gamma_t(i)\right]}{E_t\sum_{a=0}^{\infty}(\varphi_n\beta\lambda^{\theta_n+3})^a} - \frac{E_t\sum_{a=0}^{\infty}(\varphi_n\beta\lambda^{\theta_n+2})^a\left[\theta_n p^n_{h,t+a}+y^n_{h,t+a}+\gamma_{t+a}(i)-\gamma_t(i)\right]}{E_t\sum_{a=0}^{\infty}(\varphi_n\beta\lambda^{\theta_n+2})^a}. \tag{166}$$

Finally:

$$\breve{p}^n_{h,t}(j) \simeq \left(1-\varphi_n\beta\lambda^{\theta_n+3}\right)E_t\sum_{a=0}^{\infty}(\varphi_n\beta\lambda^{\theta_n+3})^a\left[\theta_n p^n_{t+a}+y^n_{t+a}+mc^m_{t+a}(\bar{Y}^m_{h,t+a}(j))+\gamma_{t+a}(i)-\gamma_t(i)\right] - \left(1-\varphi_n\beta\lambda^{\theta_n+2}\right)E_t\sum_{a=0}^{\infty}(\varphi_n\beta\lambda^{\theta_n+2})^a\left[\theta_n p^n_{h,t+a}+y^n_{h,t+a}+\gamma_{t+a}(i)-\gamma_t(i)\right]. \tag{167}$$

I.4 Steady state

To obtain log-linearization around steady state, we substitute $\lambda = 1$ into (167), which yields:

$$\breve{p}^n_{h,t}(j) \simeq (1\text{-}\varphi_n\beta)\,E_t\sum_{a=0}^{\infty}(\varphi_n\beta)^a\left[\theta_n p^n_{t+a}+y^n_{t+a}+mc^m_{t+a}(\bar{Y}^m_{h,t+a}(j))\right] \text{-} (1\text{-}\varphi_n\beta)\,E_t\sum_{a=0}^{\infty}(\varphi_n\beta)^a\left[\theta_n p^n_{h,t+a}+y^n_{h,t+a}\right] \tag{168}$$

and simplifying:

$$\breve{p}^n_{h,t}(j) \simeq (1-\varphi_n\beta)\,E_t\sum_{a=0}^{\infty}(\varphi_n\beta)^a\left[mc^m_{t+a}(\bar{Y}^m_{h,t+a}(j))\right]. \tag{169}$$

J Log-linearization of Taylor-pricing formula

The aggregated domestic price index in period t, i.e. equation (77), can be log-linearized around $\theta_n = 1$ for $(n = 1, 2, ...n_m)$ as:

$$\ln P^n_{h,t}(j) \simeq \sum_{a=t}^{t+N_m-1}\ln \breve{P}^n_{h,t-a}(j), \tag{170}$$

which also holds in natural values, so that subtracting this natural value expression from (170), we obtain the log-linearization of (77) in deviations from their natural values (see, for instance, Collard and Dellas (2003)):

$$p^n_{h,t}(j) \simeq \sum_{a=t}^{t+N_m-1}\breve{p}^n_{h,t-a}(j). \tag{171}$$

K Optimality condition in Calvo-wage setting

Taking Calvo-assumptions into account and maximizing (1) with (35) as the period utility function, w.r.t. wage rates $W^m_t(i)$, s.t. the relevant aggregate supply for labor (103), $L^m_{t+a}(i) = \left(\frac{W^m_t(i)}{W^m_{t+a}}\right)^{-\varrho_{Lm}} L^m_{t+a}$ (see Appendix B.2), CBC (7) and the law of motion of capital (6) for every sector $m = FT, FN, VT, VN$, we get a Lagrangian similar to (98) with the only difference that it is weighted by the probability φ^W_m. We differentiate it w.r.t. wage rate $W^m_t(i)$ and setting equal to 0, we obtain the following FOC:

$$E_t\left\{\sum_{a=0}^{\infty}(\beta\varphi^W_m)^a\left[\varrho_{Lm}\left(L_{t+a}(i)\right)^{\phi}\left(\frac{\breve{W}^m_t(i)}{W^m_{t+a}}\right)^{-\varrho_{Lm}-1}\frac{1}{W^m_{t+a}}L^m_{t+a}+\Gamma_{t+a}(i)\,(1\text{-}\tau^w_t)\,(\varrho_{Lm}\text{-}1)\left(\frac{\breve{W}^m_t(i)}{W^m_{t+a}}\right)^{-\varrho_{Lm}}L^m_{t+a}\right]\right\} = 0.$$

Rearranging and dividing by $(\varrho_{Lm} - 1)$:

$$E_t\left\{\sum_{a=0}^{\infty}(\beta\varphi^W_m)^a\left(\frac{\breve{W}^m_t(i)}{W^m_{t+a}}\right)^{-\varrho_{Lm}}L^m_{t+a}\left[\frac{\varrho_{Lm}}{(\varrho_{Lm}-1)}\frac{(L_{t+a}(i))^{\phi}}{\breve{W}^m_t(i)}+\Gamma_{t+a}(i)\,(1-\tau^w_t)\right]\right\} = 0,$$

and further dividing both sides by $\left[\breve{W}^m_t(i)\right]^{-\varrho_{Lm}}$ yields:

$$E_t\sum_{a=0}^{\infty}(\beta\varphi^W_m)^a\left(W^m_{t+a}\right)^{\varrho_{Lm}}L^m_{t+a}\frac{\varrho_{Lm}}{(\varrho_{Lm}-1)}\frac{(L_{t+a}(i))^{\phi}}{\breve{W}^m_t(i)} + E_t\sum_{a=0}^{\infty}(\beta\varphi^W_m)^a\left(W^m_{t+a}\right)^{\varrho_{Lm}}L^m_{t+a}\Gamma_{t+a}(i)\,(1-\tau^w_t) = 0.$$

Expressing for consumer i's optimal Calvo-wage we obtain equation (78) in the main text. We log-linearize the wage index (79) around $\gamma_m = 1$ as:

$$\ln W_t^m(i) \simeq \varphi_m^W \ln W_{t-1}^m(i) + \left(1 - \varphi_m^W\right) \ln \breve{W}_t^m(i), \tag{172}$$

which also holds in natural values, so that subtracting this natural value expression from (172) we obtain the log-linearization of (79) in deviations from natural values:

$$w_t^m(i) \simeq \varphi_m^W w_{t-1}^m(i) + \left(1 - \varphi_m^W\right) \breve{w}_t^m(i). \tag{173}$$

K.1 Natural equilibrium paths

Rewriting (78) in natural (equilibrium) paths yields:

$$\bar{W}_t^m(i) E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{\Gamma}_{t+a}(i)\right] = \frac{\varrho_{Lm}}{(\varrho_{Lm} - 1)(1 - \tau_t^w)} E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{L}_{t+a}^{\phi}\right]. \tag{174}$$

Defining:

$$LHS \text{ of } (174) : f_t(j) \equiv \bar{W}_t^m(i) E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{\Gamma}_{t+a}(i)\right]$$

$$RHS \text{ of } (174) : g_t(j) \equiv \frac{\varrho_{Lm}}{(\varrho_{Lm} - 1)(1 - \tau_t^w)} E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{L}_{t+a}^{\phi}\right]$$

and applying log-linearization rules from Appendix C we obtain the following formulas for (sums of) derivatives:

$$f_{\bar{W}_t^m(i)} \bar{W}_t^m(i) \breve{w}_t^m(i) = \bar{W}_t^m(i) \breve{w}_t^m(i) E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{\Gamma}_{t+a}(i)\right]$$

$$\sum_{a=0}^{\infty} f_{\bar{W}_{t+a}^m} \bar{W}_{t+a}^m w_{t+a}^m = \varrho_{Lm} \bar{W}_t^m(i) E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{\Gamma}_{t+a}(i) w_{t+a}^m\right]$$

$$\sum_{a=0}^{\infty} f_{\bar{L}_{t+a}^m} \bar{L}_{t+a}^m l_{t+a}^m = \bar{W}_t^m(i) E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{\Gamma}_{t+a}(i) l_{t+a}^m\right]$$

$$\sum_{a=0}^{\infty} f_{\bar{\Gamma}_{t+a}(i)} \bar{\Gamma}_{t+a}(i) \gamma_{t+a}(i) = \bar{W}_t^m(i) E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{\Gamma}_{t+a}(i) \gamma_{t+a}(i)\right]$$

$$\sum_{a=0}^{\infty} g_{\bar{W}_{t+a}^m} \bar{W}_{t+a}^m w_{t+a}^m = \varrho_{Lm} \tfrac{\varrho_{Lm}}{(\varrho_{Lm}-1)(1-\tau_t^w)} E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{L}_{t+a}^{\phi} w_{t+a}^m\right]$$

$$\sum_{a=0}^{\infty} g_{\bar{L}_{t+a}^m} \bar{L}_{t+a}^m l_{t+a}^m = \tfrac{\varrho_{Lm}}{(\varrho_{Lm}-1)(1-\tau_t^w)} E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{L}_{t+a}^{\phi} l_{t+a}^m\right]$$

$$\sum_{a=0}^{\infty} g_{\bar{L}_{t+a}(i)} \bar{L}_{t+a} l_{t+a} = \phi \tfrac{\varrho_{Lm}}{(\varrho_{Lm}-1)(1-\tau_t^w)} E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{L}_{t+a}^{\phi} l_{t+a}\right]$$

Hence, using log-linearization formula (112) we get:

$$\bar{W}_t^m(i) \breve{w}_t^m(i) E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{\Gamma}_{t+a}(i)\right] + \bar{W}_t^m(i) E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{\Gamma}_{t+a}(i) \varrho_{Lm} w_{t+a}^m\right]$$

$$+\bar{W}_t^m(i) E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{\Gamma}_{t+a}(i) l_{t+a}^m\right] + \bar{W}_t^m(i) E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{\Gamma}_{t+a}(i) \gamma_{t+a}(i)\right] =$$

$$\simeq \tfrac{\varrho_{Lm}}{(\varrho_{Lm}-1)(1-\tau_t^w)} E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{L}_{t+a}^{\phi} \varrho_{Lm} w_{t+a}^m\right] + \tfrac{\varrho_{Lm}}{(\varrho_{Lm}-1)(1-\tau_t^w)} E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{L}_{t+a}^{\phi} l_{t+a}^m\right]$$

$$+ \tfrac{\varrho_{Lm}}{(\varrho_{Lm}-1)(1-\tau_t^w)} E_t \sum_{a=0}^{\infty} (\beta\varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m\right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{L}_{t+a}^{\phi} \phi l_{t+a}\right]$$

Using (174) and rearranging, we get:

$$\breve{w}_t^m(i) \simeq \frac{E_t \sum_{a=0}^{\infty} (\beta \varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m \right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{L}_{t+a}^{\phi} \left(\varrho_{Lm} w_{t+a}^m + l_{t+a}^m + \phi l_{t+a} \right) \right]}{E_t \sum_{a=0}^{\infty} (\beta \varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m \right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{L}_{t+a}^{\phi} \right]} - \frac{E_t \sum_{a=0}^{\infty} (\beta \varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m \right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{\Gamma}_{t+a}(i) \left(\varrho_{Lm} w_{t+a}^m + l_{t+a}^m + \gamma_{t+a}(i) \right) \right]}{E_t \sum_{a=0}^{\infty} (\beta \varphi_m^W)^a \left[\left(\bar{W}_{t+a}^m \right)^{\varrho_{Lm}} \bar{L}_{t+a}^m \bar{\Gamma}_{t+a}(i) \right]}. \tag{175}$$

K.2 Varying rate

Assuming a varying trend λ_t for all variables and all consumers we can rewrite the following variables in natural (equilibrium) values as follows (for $a > 0$):

$$\bar{W}_{t+a}^m = \bar{W}_t^m \prod_{k=1}^a \lambda_k, \ \bar{L}_{t+a}^m = \bar{L}_t^m \prod_{k=1}^a \lambda_k, \ \bar{L}_{t+a}(.) = \bar{L}_t \prod_{k=1}^a \lambda_k \text{ and } \bar{\Gamma}_{t+a}(i) = \bar{\Gamma}_t(i) \prod_{k=1}^a \lambda_k.$$

Hence, we rewrite (175) as:

$$\breve{w}_t^m(i) = \frac{E_t \sum_{a=0}^{\infty} (\beta \varphi_m^W)^a \left[\left(\bar{W}_t^m \prod_{k=1}^a \lambda_k \right)^{\varrho_{Lm}} \bar{L}_t^m \prod_{k=1}^a \lambda_k \bar{L}_t^{\phi} \prod_{k=1}^a \lambda_k^{\phi} \left(\varrho_{Lm} w_{t+a}^m + l_{t+a}^m + \phi l_{t+a} \right) \right]}{E_t \sum_{a=0}^{\infty} (\beta \varphi_m^W)^a \left[\left(\bar{W}_t^m \prod_{k=1}^a \lambda_k \right)^{\varrho_{Lm}} \bar{L}_t^m \prod_{k=1}^a \lambda_k \bar{L}_t^{\phi} \prod_{k=1}^a \lambda_k^{\phi} \right]} - \frac{E_t \sum_{a=0}^{\infty} (\beta \varphi_m^W)^a \left[\left(\bar{W}_t^m \prod_{k=1}^a \lambda_k \right)^{\varrho_{Lm}} \bar{L}_t^m \prod_{k=1}^a \lambda_k \bar{\Gamma}_t(i) \prod_{k=1}^a \lambda_k \left(\varrho_{Lm} w_{t+a}^m + l_{t+a}^m + \gamma_{t+a}(i) \right) \right]}{E_t \sum_{a=0}^{\infty} (\beta \varphi_m^W)^a \left[\left(\bar{W}_t^m \prod_{k=1}^a \lambda_k \right)^{\varrho_{Lm}} \bar{L}_t^m \prod_{k=1}^a \lambda_k \bar{\Gamma}_t(i) \prod_{k=1}^a \lambda_k \right]}. \tag{176}$$

Simplifying:

$$\breve{w}_t^m(i) = \frac{E_t \sum_{a=0}^{\infty} (\beta \varphi_m^W)^a \left[\prod_{k=1}^a \lambda_k^{1+\phi+\varrho_{Lm}} \left(\varrho_{Lm} w_{t+a}^m + l_{t+a}^m + \phi l_{t+a} \right) \right]}{E_t \sum_{a=0}^{\infty} (\beta \varphi_m^W)^a \left[\prod_{k=1}^a \lambda_k^{1+\phi+\varrho_{Lm}} \right]} - \frac{E_t \sum_{a=0}^{\infty} (\beta \varphi_m^W)^a \left[\prod_{k=1}^a \lambda_k^{2+\varrho_{Lm}} \left(\varrho_{Lm} w_{t+a}^m + l_{t+a}^m + \gamma_{t+a}(i) \right) \right]}{E_t \sum_{a=0}^{\infty} (\beta \varphi_m^W)^a \left[\prod_{k=1}^a \lambda_k^{2+\varrho_{Lm}} \right]}. \tag{177}$$

K.3 Constant trend

Assuming constant trend $\lambda_t = \lambda$ we can rewrite approximation (177) as:

$$\breve{w}_t^m(i) = (1 - \beta \varphi_m^W \lambda^{1+\phi+\varrho_{Lm}}) E_t \sum_{a=0}^{\infty} (\beta \varphi_m^W \lambda^{1+\phi+\varrho_{Lm}})^a \left[\varrho_{Lm} w_{t+a}^m + l_{t+a}^m + \phi l_{t+a} \right] - (1 - \beta \varphi_m^W \lambda^{2+\varrho_{Lm}}) E_t \sum_{a=0}^{\infty} (\beta \varphi_m^W \lambda^{2+\varrho_{Lm}})^a \left[\varrho_{Lm} w_{t+a}^m + l_{t+a}^m + \gamma_{t+a}(i) \right]. \tag{178}$$

K.4 Steady state

To obtain log-linearization around steady state we substitute $\lambda = 1$ into (178) which yields:

$$\breve{w}_t^m(i) = (1 - \beta \varphi_m^W) E_t \sum_{a=0}^{\infty} (\beta \varphi_m^W)^a \left[\varrho_{Lm} w_{t+a}^m + l_{t+a}^m + \underbrace{\phi l_{t+a}}_{=\gamma_{t+a}(i)} \right],$$

and simplifying:

$$\breve{p}_{h,t}^n(j) \simeq (1 - \varphi_n b) \, E_t \sum_{a=0}^{\infty} (\varphi_n b)^a \left[\phi l_{t+a} - \gamma_{t+a}(i) \right]. \tag{179}$$

L Optimality condition in Taylor-wage setting

Taking into account Taylor assumptions and maximizing (35) with respect to wage rates W_t^m, s.t. relevant demand for labor (103) and budget constraint (7) for every sector $m = FT, FN, VT, VN$, is derived in the similar manner as for Calvo-wage setting in Appendix K, but again with two differences. First, $\varphi_n = 1$ in every expression, and second, instead of infinite sum $\sum_{a=0}^{\infty}$ in every expression we have $\sum_{a=t}^{t+N_m-1}$. Changing these two elements in the derivation of (78) yields formula (81) in the main text.

The log-linearized version of the wage index of equation (82) around $\theta_m = 1$ we obtain:

$$\ln W_t^m(i) \simeq \sum_{a=t}^{t+N_m^W-1} \ln \breve{W}_{t-a}^m(i), \tag{180}$$

which also holds in natural values, so that subtracting this natural value expression from (180) we obtain the log-linearization of (82) in deviations from their natural values (see, for instance, Collard and Dellas (2003)):

$$w_t^m(i) \simeq \sum_{a=t}^{t+N_m^W-1} \breve{w}_{t-a}^m(i). \tag{181}$$

M Calibration

Table 1
Calibrated parameters and proposed densities from the literature

Parameters	Our prior			Levin *et al.* (2005)	Ortega& Rebei(2006)	Ratto *et al* (2005b)
	Type[bounds]	Mean(SD)	Based on	Type;Mean(SD)	Type;Mean(SD)	Type;Mean(SD)
$\alpha_{CT}, \alpha_{CT}^*, \alpha_{Ch}, \alpha_{Cf}^*$	B [0.45;0.55]	0.5(0.05)	O&R06		B; 0.5(0.05)	
β, β^*		0.99	L06			
χ, χ^*		2/3				
$\chi_{FN}^*, \chi_{FN,f}$		7	AD&M			
$\chi_{FT}^*, \chi_{FT,f}$		7	AD&M			
$\chi_{FN}^N, \chi_{FN,f}^N, \chi_{FT}^N, \chi_{FT,f}^N$		5				
$\chi_{FN}^T, \chi_{FN,f}^T, \chi_{FT}^T, \chi_{FT,f}^T$		5				
$\eta_{CT}, \eta_{CT}^*, \eta_{IT}, \eta_{IT}^*$		3				
$\eta_{Ch}, \eta_{Cf}^*, \eta_{Ih}, \eta_{If}^*$		5				
$\gamma_{FN}, \gamma_{FN}^*, \gamma_{FT}, \gamma_{FT}^*$		5				
$\gamma_{VN}, \gamma_{VN}^*, \gamma_{VT}, \gamma_{VT}^*$		5				
κ, κ^*	N [0.15;0.8]	0.5(0.125)				
ϕ, ϕ^*	B [0.16;0.6]	0.3(0.1)	Mo00	N; 1.2(0.5)		B; 0.45(0.18)
σ, σ^*	N [1.4;3]	2(0.5)	O&R06	N; 2(0.5)		
$\rho_{A_{NT}}, \rho_{A_{NT}^*}$	B [0.70;0.99]	0.85(0.1)	O&R06	B; 0.5(0.25)	B; 0.85(0.1)	B; 0.5(0.2)
$\rho_{A_T}, \rho_{A_T^*}$	B [0.70;0.99]	0.85(0.1)	O&R06	B; 0.5(0.25)	B; 0.85(0.1)	B; 0.5(0.2)
$\rho_{A_{VN}}, \rho_{A_{VN}^*}$	B [0.70;0.99]	0.85(0.1)	O&R06			B; 0.5(0.2)
$\rho_{A_{VT}}, \rho_{A_{VT}^*}$	B [0.70;0.99]	0.85(0.1)	O&R06			B; 0.5(0.2)
ρ_i, ρ_i^*	B [0.6;0.9]	0.75(0.05)	Mo00	N; 1(0.15)	B; 0.85(0.1)	B; 0.8(0.1)
$\vartheta_1, \vartheta_1^*$	IG [0.02;2.5]	1.5(0.1)	T93		IG; 1.5(0.2)	B; 1.25(0.1)
$\vartheta_2, \vartheta_2^*$	B [0.02;0.3]	0.25(0.1)	T93		N; 0.2(0.1)	
$\vartheta_3, \vartheta_3^*$	B [0.02;0.3]	0.25(0.1)	T93			
$\vartheta_4, \vartheta_4^*$	B [0.02;0.3]	0.25(0.1)	T93			
$\vartheta_5, \vartheta_5^*$	B [0.02;0.3]	0.25(0.1)	T93			
$\upsilon_{LFT}, \upsilon_{LFT}^*$		0.36	O&R06		B; 0.36(0.05)	
$\upsilon_{LFN}, \upsilon_{LFN}^*$		0.36	O&R06		B; 0.34(0.05)	
$\varphi_{n_{FT}}, \varphi_{n_{FT}}^*$	B [0.35;0.8]	0.5(0.2)			B; 0.67(0.05)	B; 0.5(0.2)
$\varphi_{n_{FN}}, \varphi_{n_{FN}}^*$	B [0.35;0.8]	0.5(0.2)			B; 0.67(0.05)	B; 0.5(0.2)
$\varphi_{n_{VT}}, \varphi_{n_{VT}}^*$	B [0.15;0.8]	0.375(0.1)				
$\varphi_{n_{VN}}, \varphi_{n_{VN}}^*$	B [0.15;0.8]	0.375(0.1)				
$\varphi_{n_{WT}}, \varphi_{n_{WT}}^*$	B [0.35;0.8]	0.5(0.2)			B; 0.67(0.05)	B; 0.5(0.2)
$\varphi_{n_{WN}}, \varphi_{n_{WN}}^*$	B [0.35;0.8]	0.5(0.2)			B; 0.67(0.05)	B; 0.5(0.2)
$\varpi_{FT}, \varpi_{FT}^*, \varpi_{FN}, \varpi_{FN}^*$		1				
$\varpi_{VT}, \varpi_{VT}^*, \varpi_{VN}, \varpi_{VN}^*$		1				
$\omega_{FT}, \omega_{FT}^*, \omega_{FN}, \omega_{FN}^*$	IG [0.001;2]	0.02(2)	R05	IG; 0.6(0.6)	IG; 1.5(2)	B; 0.1(2)
$\omega_{VT}, \omega_{VT}^*, \omega_{VN}, \omega_{VN}^*$	IG [0.001;2]	0.02(2)	R05			

Note: O&R06 refers to Ortega and Rebei (2006), L06 to Levin *et al.* (2006), D05 to Dellas (2005), T93 to Taylor (1993), R05 to Ratto *et al.* (2005b), and Mo00 to Monacelli (2000). The probability density types beta, normal and inverted gamma are abbreviated as B, N and IG, respectively.

The calibration used by Erceg *et al.* (2005) assumes an annualized interest rate of 3%, a risk aversion parameter of 2, a habit formation parameter $\theta = 0.7$ but measured in the form $C_t - \theta C_{t-1}$, the Frisch elasticity of labor of 1/5.[52] Wage and price markups are calibrated at 0.2, a Calvo probability parameter of 0.75, and an implied annual inflation of 4 %.

N Impulse response functions

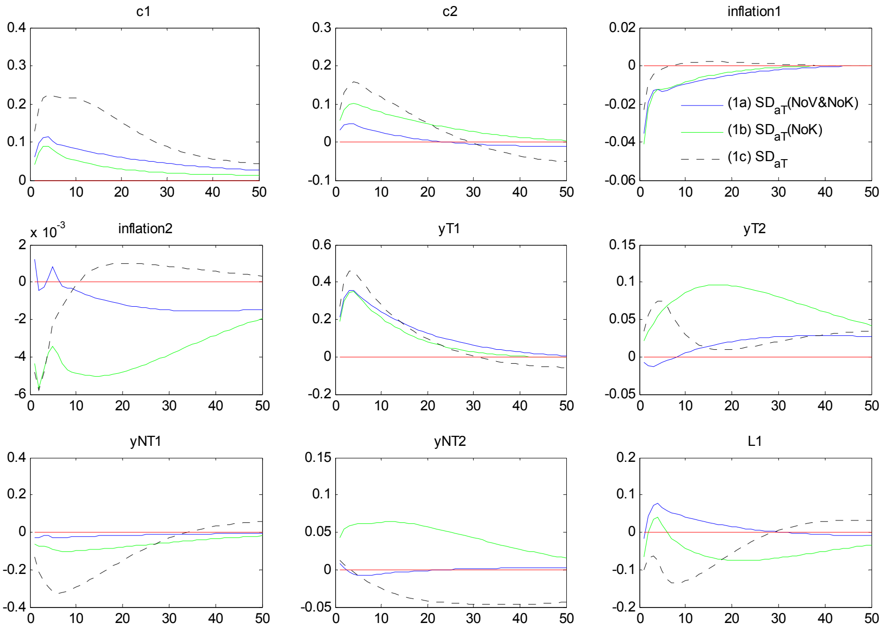

Figure 1. Technology shock in home tradable final goods sector

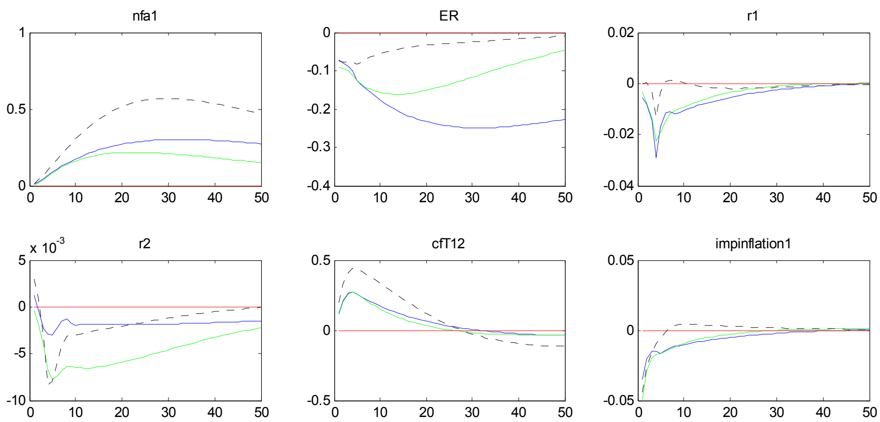

Figure 2. Technology shock in home tradable final goods sector (continuation).

[52] It is defined as the elasticity of the labor supply with respect to wage, leaving the marginal utility of consumption constant.

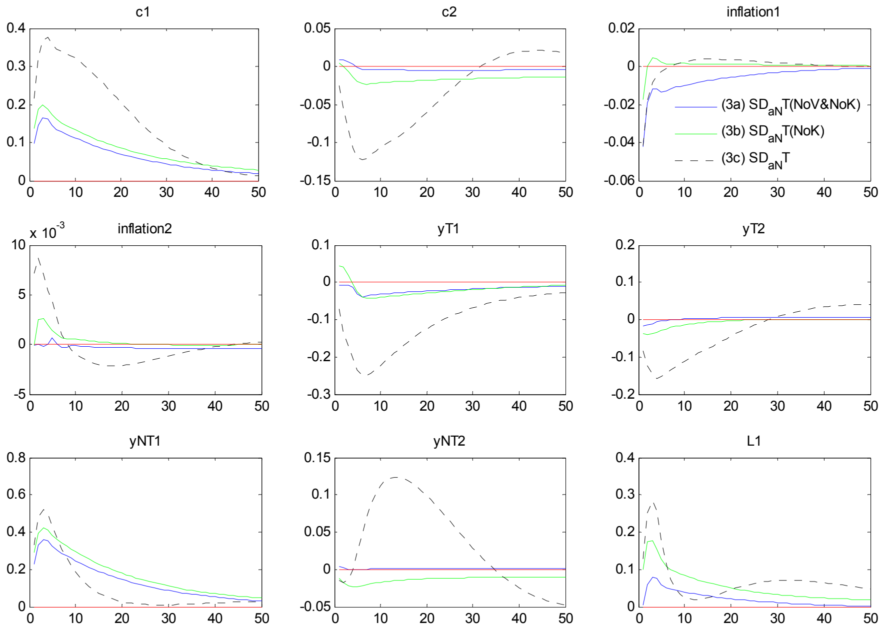

Figure 3. Technology shock in home non-tradable final goods sector.

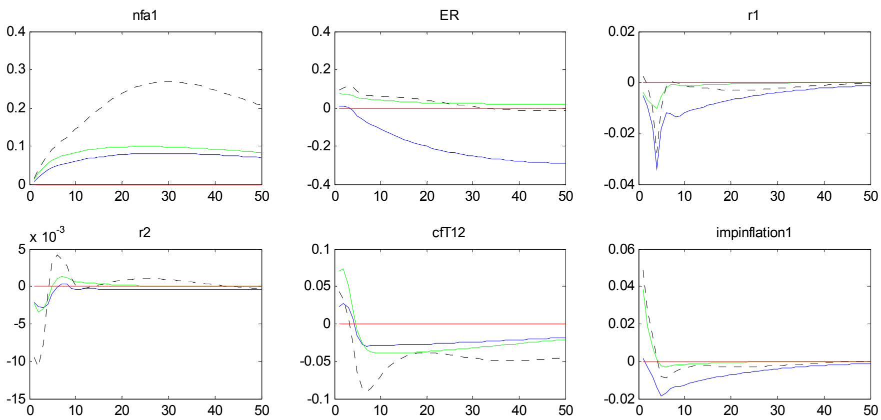

Figure 4. Technology shock in home non-tradable final goods sector (continuation).

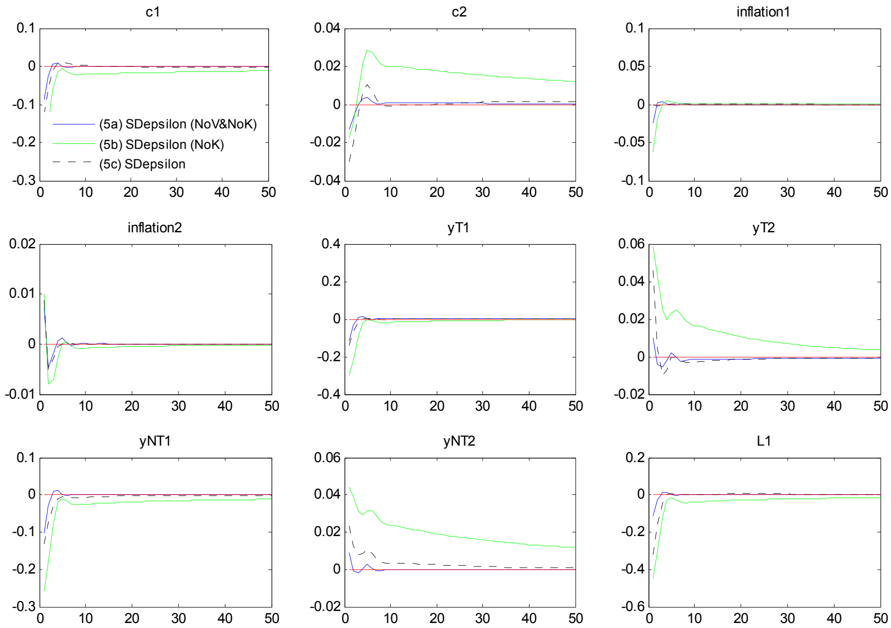

Figure 5. Shock in home money demand.

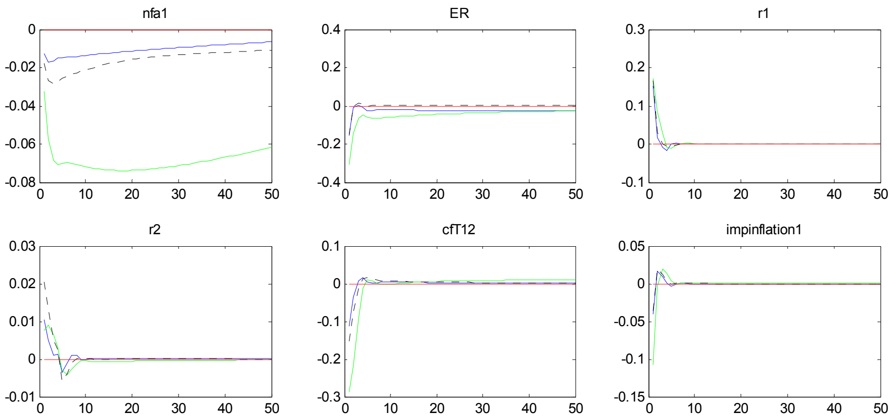

Figure 6. Shock in home money demand sector (continuation).

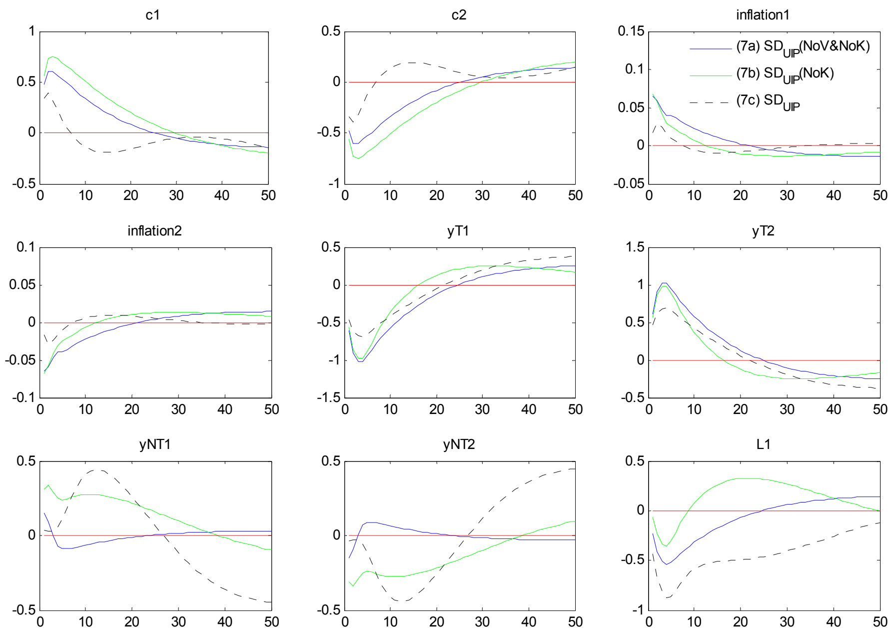

Figure 7. Shock in exchange rate (EUR/$).

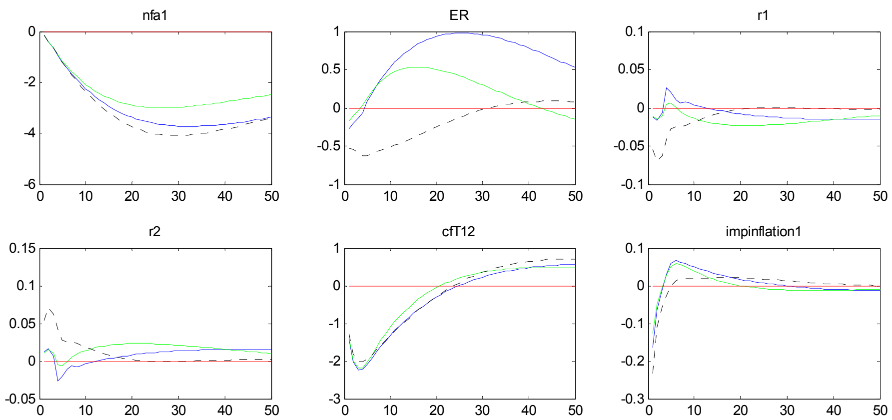

Figure 8. Shock in exchange rate (EUR/$)(continuation).

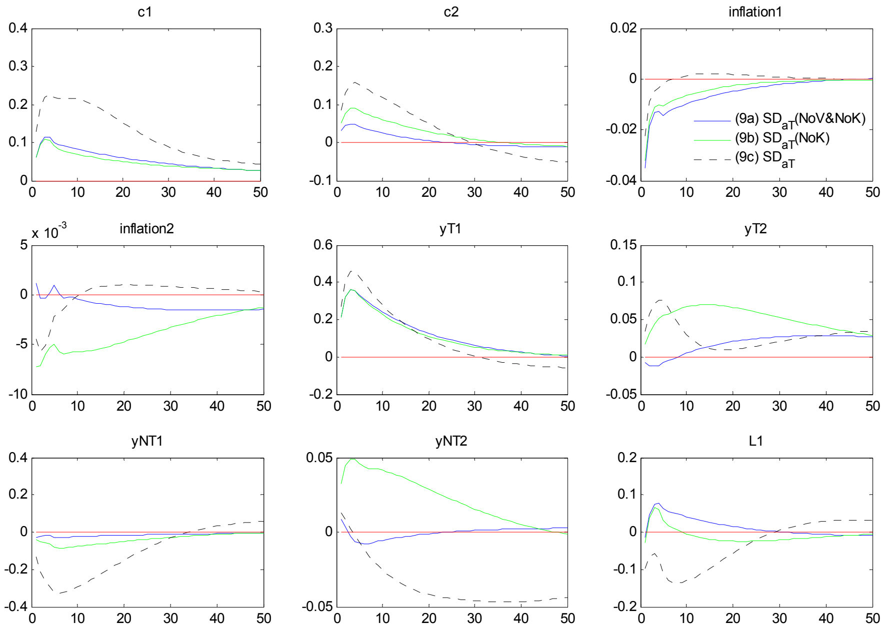

Figure 9. Shock idem to Figure 1 and no interest rate smoothing (Rule I).

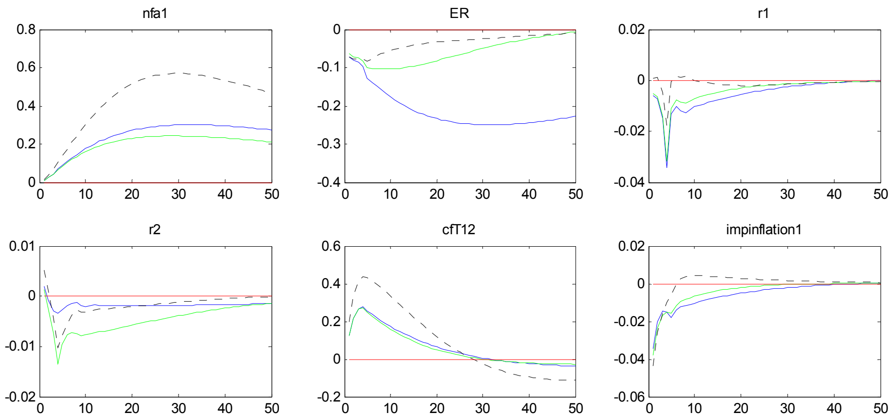

Figure 10. Shock idem to Figure 1 and no interest rate smoothing (Rule I) (continuation).

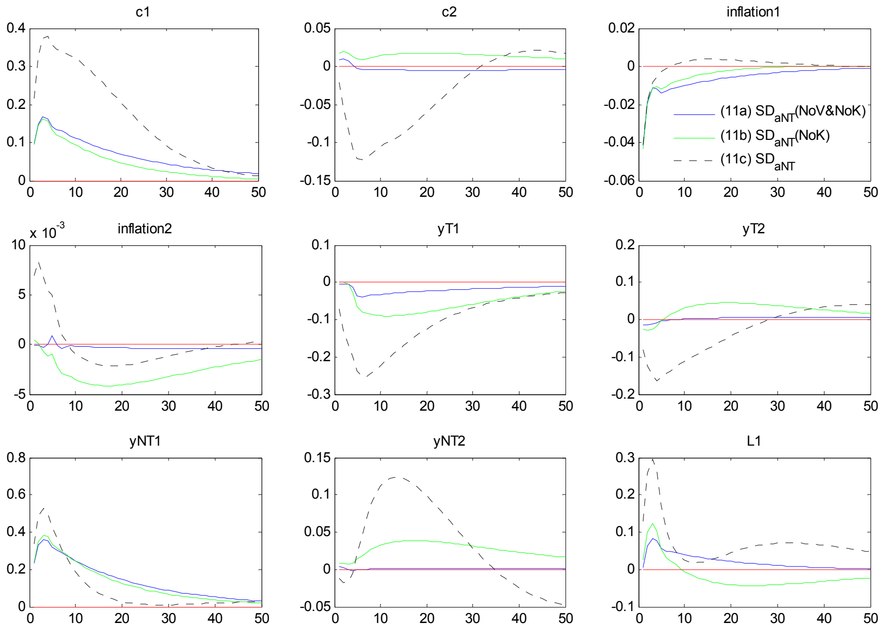

Figure 11. Shock idem to Figure 3 and no interest rate smoothing (Rule I).

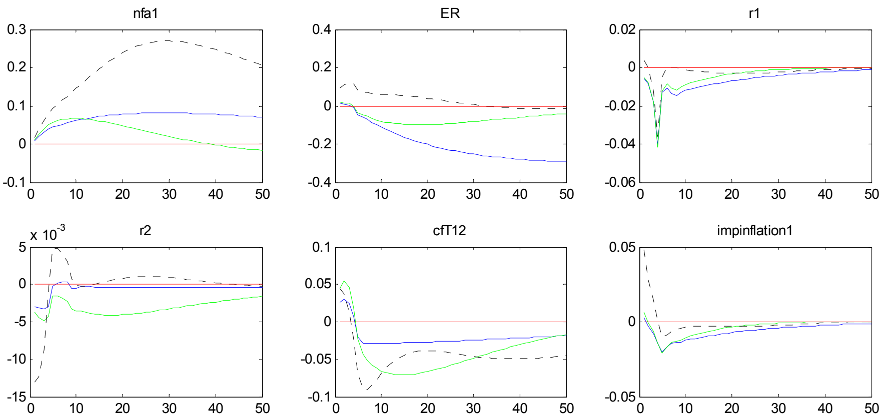

Figure 12. Shock idem to Figure 3 and no interest rate smoothing (Rule I) (continuation).

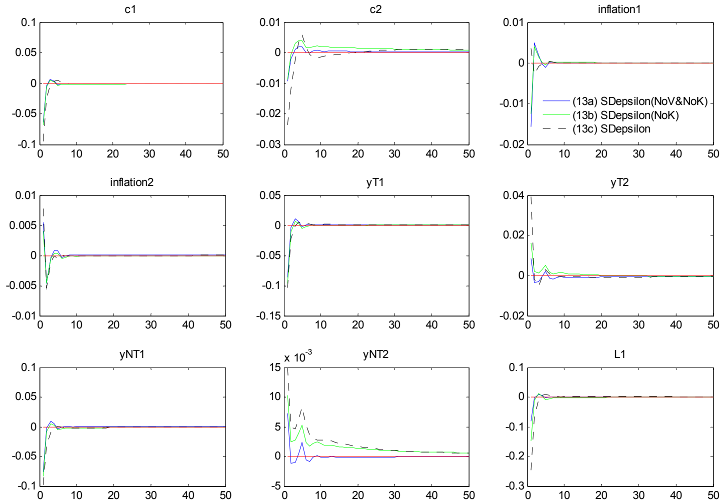

Figure 13. Shock idem to Figure 5 and no interest rate smoothing (Rule I).

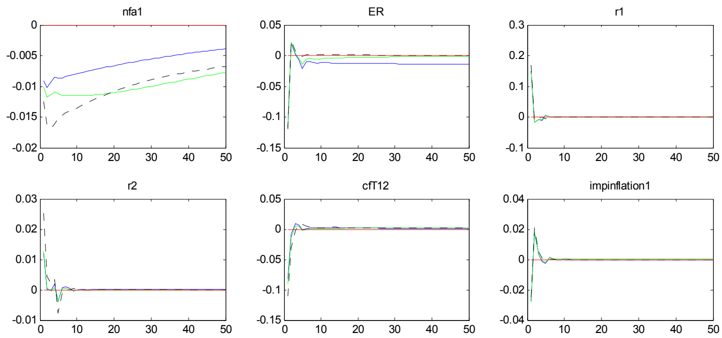

Figure 14. Shock idem to Figure 5 and no interest rate smoothing (Rule I) (continuation).

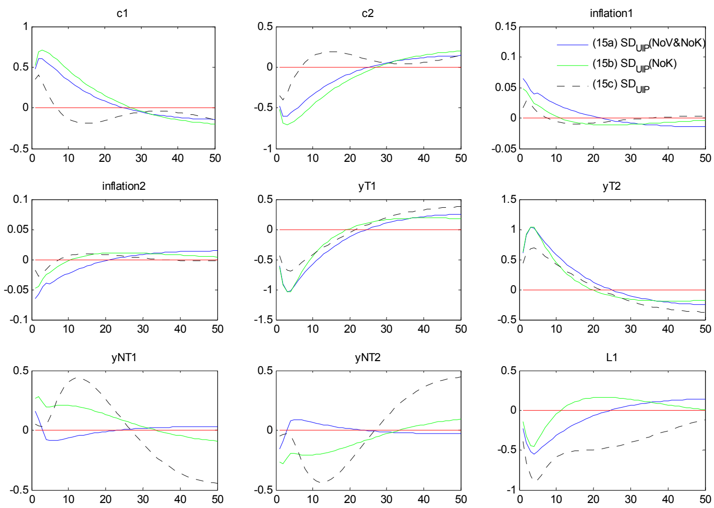

Figure 15. Shock idem to Figure 7 and no interest rate smoothing (Rule I).

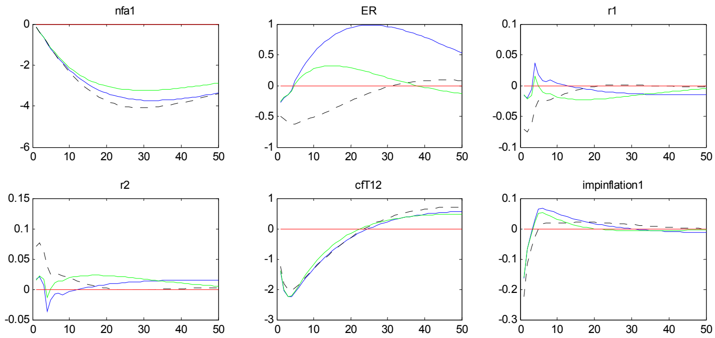

Figure 16. Shock idem to Figure 7 and no interest rate smoothing (Rule I) (continuation).

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