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CLUB EFFICIENCY AND LINDAHL EQUILIBRIUM WITH SEMI-PUBLIC GOODS

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Club Efficiency and Lindahl Equilibrium with Semi-Public Goods[*]

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Limit core allocations are the ones that remain in the core of a replicated economy. An equivalent notion for economies with public goods is Schweizer's club efficiency. We extend this notion to economies with goods that have a semi-public nature. The notion encompasses purely private as well as purely public club goods as polar cases. We show that given certain conditions the equivalence of club efficient allocations and Lindahl equilibria holds for a wide range of economies with semi-public club goods. We also show that extension to a more general class of economies seems implausible.

Key words: clubs; club efficiency; Lindahl equilibrium; limit cores.
JEL codes: H41, R51, D71

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1 Club efficiency and Lindahl pricing

Lindahl equilibrium has played a central role in the general equilibrium theory of public goods. This paper shows that Lindahl equilibrium can be assigned a central role in economies with semi-public goods as well. We will provide a competitive basis to Lindahl pricing within a club with a variable membership base which provides its members with semi-public club goods.

The standard methodology in general equilibrium analysis to provide a competitive basis to a price system is through limit core theory. It is well known that in these economies Walrasian equilibrium allocations are the only allocations that cannot be improved upon by any coalition of agents as the economy becomes large. More precisely, Debreu and Scarf (1963) demonstrated that a Walrasian equilibrium allocation remains in the core as the economy becomes larger and that it is the only allocation with this property. In other words, no group of agents can improve upon an equilibrium allocation and equilibrium allocations are the only ones with this property as the economy becomes infinitely large.

This fundamental Debreu-Scarf limit core theorem can be restated for a setting with clubs with a variable membership profile. An allocation is club efficient if there is no club with an arbitrary membership profile that can improve upon it. Schweizer (1983) introduced this notion of club efficiency as a slight strengthening of this limit core concept, allowing for arbitrary real number sizes of clubs. It can be shown that the Debreu-Scarf limit core property, the Walrasian equilibrium concept, and club efficiency are equivalent.[1]

The extension of the core-equivalence theorem to economies with public goods has been widely discussed in the literature. Here we turn our attention to the equivalence of the Lindahl equilibrium concept — an extension of the Walrasian equilibrium notion to economies with public goods — and the limit core property, or equivalently, club efficiency. With purely public goods the main problem is that such goods are not replicated along with the rest of the economy. The resources foregone in their provision are independent of the size of the economy or club. Utility levels can therefore be increased by admitting new members and spreading the burden of the public good. Schweizer (1983) provides Lindahlian price support of a club efficient allocation in an

[1] An indirect proof can be based on noting that Schweizer (1983) showed equivalence of the Walrasian equilibrium concept and club efficiency. Debreu and Scarf (1963) showed equivalence of Walrasian equilibria and the limit core. Hence, club efficiency, the Walrasian equilibrium concept, and the limit core property are the same.

economy with public goods, but must assume that some agents or endowments are given in …xed numbers. The variable agents can escape taxation by the above logic and are, therefore, free riders. The use of the exogenous club endowment for the funding of the public goods amounts to the Henry George Theorem.

If there are no …xed numbers, it is still possible to de…ne the Lindahl equilibrium concept. However, it has a meager competitive basis, for it is just one of many allocations of economies with purely public goods that remain in the core or are club e¢cient (Milleron, 1972).

In this paper we look at the provision of club goods with a semi-public nature. Such goods are de…ned as commodities of an intermediate nature. It is assumed that these commodities are provided through a club, and therefore are principally locally collective. But their rivalry properties might be di¤erent from that of a purely locally public good. In fact we will be able to handle commodities that range in nature from purely public to purely private. For such intermediary club goods, member demands are aggregated like public goods, but the club goods get replicated along with the rest of the economy. Purely private goods and purely public goods are polar cases of such club goods; individual demands are aggregated by summation and maximization, respectively.

Our main theorem is that for certain club goods with a semi-public nature the notions of club e¢ciency and Lindahl equilibrium remain equivalent. For this we extend Schweizer's (1983) equivalence theorem to a model in which the aggregation function for the club goods has a certain speci…cation and certain properties. We also show that it cannot be expected that our Lindahl equivalence result can be extended further to more general speci…cations of the aggregation function.

The second section develops the model, Section 3 states and proves our equivalence result, and Section 4 concludes the paper with a discussion of the result, its relationship to the literature, and its implications.

2 Clubs and club goods

In this section we introduce a model of a club consisting of a certain membership base, an allocation of private goods consumed, and an allocation of so-called club goods, which are provided collectively. In our theory we use a club as a replication device discussed in the previous section.

We consider an economy with a …nite set of consumer types denoted by $t =$

$1, \ldots, T$. A vector $n \in \mathbb{R}^T_+$ represents a profile of a coalition of economic agents, comprising n^t members of type t. A profile $n \in \mathbb{R}^T_+$ forms the membership base of a club. Throughout we assume that agents of the same type are treated equally, i.e., agents of the same type consume the same quantities of private as well as club goods. This assumption enables us to discuss replication properly. In closed models such as the standard model of a replicated pure exchange economy the equal treatment property can be shown as a proposition (Debreu and Scarf, 1963).

We consider a situation with $\ell \in \mathbb{N}$ private goods. Agents of type t are endowed with a commodity bundle $w^t \in \mathbb{R}^\ell_+$. It is assumed that $w^t > 0$ for all t. Private consumption of an agent of type t is now given by $x^t + w^t \in \mathbb{R}^\ell$ where x^t denotes the net consumption of type t. The total net consumption plan is given by $x = \left(x^1, \ldots, x^T \right) \in \mathbb{R}^{\ell T}$.

There are $m \in \mathbb{N}$ club goods. A club good is provided collectively by the club to its members. A club good has a semi-public nature which can range from purely public to purely private. Again assuming equal treatment, an agent of type t now consumes the club goods at levels given by the vector $y^t \in \mathbb{R}^m_+$. The total consumption plan for club goods is now represented by the vector $y = \left(y^1, \ldots, y^T \right) \in \mathbb{R}^{mT}_+$. Total consumption in a club with membership base $n \in \mathbb{R}^T_+$ is now represented by $\bar{y} = \left(n^1 y^1, \ldots, n^T y^T \right) \in \mathbb{R}^{mT}_+$.

The nature of the club goods is introduced through the production technology used for their creation. The production technology is represented by the induced cost function $C : \mathbb{R}^{mT}_+ \to \mathbb{R}^\ell_+$ which for every membership base $n \in \mathbb{R}^T_+$ assigns to every total consumption bundle $\bar{y} = \left(n^1 y^1, \ldots, n^T y^T \right) \in \mathbb{R}^{mT}_+$ a bundle of private goods $C(\bar{y}) \in \mathbb{R}^\ell_+$ that is used to create the club goods at these levels.[2]

We illustrate this definition by some examples. The club goods are purely private if $C(\bar{y}) = \Phi\left(\sum_{t=1}^T n^t y^t \right)$, where the cost function $\Phi : \mathbb{R}^m_+ \to \mathbb{R}^\ell_+$ represents a standard private goods production technology converting the ℓ private good inputs into m private good outputs. (This reduces the model to the standard setting of a pure exchange economy.) The club goods are purely public if $C(\bar{y}) = K \in \mathbb{R}^\ell_+$, where K is some fixed input vector. (This is the case studied by Schweizer, 1983.) Finally, there are many intermediate possibilities, giving the club goods a semi-public nature. For example, if $C(\bar{y}) = \Phi(\max_{t=1,\ldots,T} n^t y^t)$, where $\max \in \mathbb{R}^m_+$ is defined by $\max_i(y^1, y^2) = \max(y^1_i, y^2_i)$ $(i = 1, \ldots, m)$ and, as before, $\Phi : \mathbb{R}^m_+ \to \mathbb{R}^\ell_+$ represents a standard private goods production technology, we can interpret the club goods to be based on a fixed infrastructure such as a network. The capacity of the network has to

[2] We may allow substitution of inputs by generalizing C to a correspondence.

handle the peak demands, which in turn determines the construction costs.

Although these examples span a great variety of natures of club goods, ranging from purely private to purely public, there is an important commonality, namely convexity. In the pure public case, the induced cost function C is constant, which is obviously convex. In the pure private and semi-public cases, C is induced by a private goods cost function $\hat{C}$: If $\hat{C}$ is convex, as is standard in neoclassical production theory (excluding increasing returns to scale in production), then so is C in either case, as the latter is the composition of $\hat{C}$ and either summation (of private goods) or maximization (of semi-public goods). The latter two operations are convex and the composition of convex operations is convex.

A club is now defined as a tuple $(n^t, x^t, y^t)_{t=1,\dots,T}$, where $n = (n^1, \dots, n^T) \in \mathbb{R}_+^T$ is a profile, $x = (x^1, \dots, x^T) \in \mathbb{R}^{\ell T}$ a net consumption plan, and $y = (y^1, \dots, y^T) \in \mathbb{R}_+^{mT}$ is a club good consumption plan. A club $(n^t, x^t, y^t)_{t=1,\dots,T}$ is feasible if

$$\sum_{t=1}^{T} n^t x^t + C\left(n^1 y^1, \dots, n^T y^T\right) \leq 0$$

Net demands for the private goods and the costs for the provision of the club goods sum to zero at most. For simplicity, there is no production of private goods. Its inclusion would be a straightforward extension of the model. Also, club goods are determined in their nature through the cost function introduced, particularly the way it aggregates type demands into club demand. Club demand is fulfilled by a production technology producing private goods only. Indeed, club goods are special by the nature of their consumption rather than the production technology.

A consumer of type t has an extended utility function $U^t : \mathbb{R}^\ell \times \mathbb{R}^m \to \mathbb{R}$ over his total private and club good consumption. However, since his initial endowment w^t is fixed, we may simply write $U^t(x^t, y^t)$. In principle we allow an agent to have short positions in all commodities.

Next we introduce our main efficiency concept. Consider two feasible clubs $(n^t, x^t, y^t)_{t=1,\dots,T}$ and $(n_0^t, x_0^t, y_0^t)_{t=1,\dots,T}$. The club $(n^t, x^t, y^t)_{t=1,\dots,T}$ is an improvement over $(n_0^t, x_0^t, y_0^t)_{t=1,\dots,T}$ if

$$U^t(x^t, y^t) > U^t(x_0^t, y_0^t)$$

whenever $n^t > 0$. (Note that this definition anticipates strong monotonicity of utility, which we will assume in the next section.) If no such improvement exists for a club $(n_0^t; x_0^t; y_0^t)_{t=1;:::;T}$, then $(n_0^t; x_0^t; y_0^t)_{t=1;:::;T}$ is called club efficient, following Schweizer (1983).

A feasible club $(n^t; x^t; y^t)_{t=;:::;T}$ is a Lindahl equilibrium if there exist a private goods price vector $p \in R_+^\ell$ and admission price vectors $p^1; \ldots; p^T \in R_+^m$ with

$$(x_0^t; y_0^t) = \text{argmax } U^t(x^t; y^t) \text{ subject to } px^t + p^t y^t \leqq 0;$$

$$\sum_{t=1}^{T} n_0^t p^t y_0^t = pC\left(n_0^1 y_0^1; \ldots; n_0^T y_0^T\right);$$

and for every alternative club $(n^t; x^t; y^t)_{t=1;:::;T}$

$$\sum_{t=1}^{T} n^t p^t y^t \leqq pC\left(n^1 y^1; \ldots; n^T y^T\right):$$

By the first condition, consumers maximize utility given the market prices for the private goods and the personal fees for the semi-public goods. The fees cover the costs of the provision of the club goods by the second condition. The third condition is not always included: a public administration is in charge of the provision and admission policies, and as such has the objective to maximize its "profits." (This maximal profit is zero by the second condition.)

3 A decentralization result

Relatively little is assumed to arrive at complete decentralization of efficient clubs through appropriate price systems. Following Schweizer (1983), positivity of prices is ensured to render a quasi-equilibrium equilibrium.

Axiom 1 There are two properties that have to be satisfied.

(a) For every type $t = 1; :::; T$ the utility function U^t is assumed to be continuous, quasi-concave, and strongly monotonic.

(b) The club good production technology has to be convex in the sense that the cost function $C : R_+^{mT} \to R_+^\ell$ is convex.

In the context of this assumption we have the following result.

Theorem 1 Any efficient club $(n^t; x^t; y^t)_{t=1;\dots;T}$ with strictly positive endowment, $\sum_{t=1} n^t w^t \gg 0$; can be supported as a Lindahl equilibrium with strictly positive prices.

Proof. Let the club $(n_0^t; x_0^t; y_0^t)_{t=1;\dots;T}$ be efficient.

We construct the following sets. First, for every $t \in T$ we define the prefered set,

$$B^t = \left\{ \left(x^t; 0; \dots; 0; y^t; 0; \dots; 0\right) \,\middle|\, U^t\left(x^t; y^t\right) > U^t\left(x_0^t; y_0^t\right) \right\} \subset \mathbb{R}_+^{\ell+mT}$$

In this definition we let y^t be at location $1 + t$.

Now for any profile $n \in \mathbb{R}_+^T$ we define

$$B_n = \sum_{t=1}^{T} n^t B^t = \left\{ \left(\sum_{t \in T} n^t x^t; n^1 y^1; \dots; n^T y^T \right) \,\middle|\, U^t\left(x^t; y^t\right) > U^t\left(x_0^t; y_0^t\right) \text{ for all } t \right\}$$

Finally, we let

$$B = \cup \left\{ B_n \,\middle|\, n \in \mathbb{R}_+^T \text{ such that } n > 0 \right\} \subset \mathbb{R}_+^{\ell+mT}$$

Second, we introduce the feasible set,

$$D = \left\{ \left(-C\left(n^1 y^1; \dots; n^T y^T\right) - x; n^1 y^1; \dots; n^T y^T \right) \,\middle|\, \begin{array}{l} n > 0;\ x \in \mathbb{R}_+^{\ell}; \\ y^1; \dots; y^T \in \mathbb{R}_+^m \end{array} \right\}$$

We remark that also $D \subset \mathbb{R}^{\ell+mT}$.

B^t is convex by quasi-concavity of U^t for every type t. Consequently, the set B is convex. To show that D is convex, let $(y^1; \dots; y^T; x; n)$ and $(\hat{y}^1; \dots; \hat{y}^T; \hat{x}; \hat{n})$ constitute (but not be) members of D. Define $v = \left(n^1 y^1; \dots; n^T y^T\right)$ and $\hat{v} = \left(\hat{n}^1 \hat{y}^1; \dots; \hat{n}^T \hat{y}^T\right)$. Then $(-C(v) - x; v) \in D$ as well as $(-C(\hat{v}) - \hat{x}; \hat{v}) \in D$.

Now consider $\lambda \in [0; 1]$. We have to show that there exists a tuple $(\tilde{y}^1; \dots; \tilde{y}^T; \tilde{x}; \tilde{n})$ such that $(-C(\tilde{v}) - \tilde{x}; \tilde{v}) \in D$ where $\tilde{v} = \left(\tilde{n}^1 \tilde{y}^1; \dots; \tilde{n}^T \tilde{y}^T\right)$, $\tilde{v} = \lambda v + (1 - \lambda)\hat{v}$, and $C(\tilde{v}) + \tilde{x} = \lambda (C(v) + x) + (1 - \lambda)(C(\hat{v}) + \hat{x})$. This can be accomplished by selecting $\tilde{y}^t = \lambda n^t y^t + (1 - \lambda)\hat{n}^t \hat{y}^t$ for every t; $\tilde{n}^t = 1$; and

$$\tilde{x} = \lambda C(v) + (1 - \lambda) C(\hat{v}) - C(\tilde{v}) + \lambda x + (1 - \lambda)\hat{x}.$$

Now $\tilde{v} = \lambda v + (1 - \lambda)\hat{v}$ and by convexity of the cost function C it follows that

$$\begin{aligned} \tilde{x} &= \lambda C(v) + (1 - \lambda) C(\hat{v}) - C(\tilde{v}) + \lambda x + (1 - \lambda)\hat{x} \\ &\geq C(\lambda v + (1 - \lambda)\hat{v}) - C(\tilde{v}) + \lambda x + (1 - \lambda)\hat{x} \\ &= \lambda x + (1 - \lambda)\hat{x}. \end{aligned}$$

Hence, $\bar{x} \geqq 0$ and thus indeed $(-C(\bar{y}) - \bar{x}; \bar{y}) \in D$, finishing the proof that D is convex.

By efficiency of the club $(n_0^t; x_0^t; y_0^t)_{t=1;\dots;T}$, the intersection of B and D is empty. By the separating hyperplane theorem there exist $p \in \mathbb{R}^{\ell}_{+}$ and $p^1; \dots; p^T \in \mathbb{R}^m_+$ not all equal to zero such that

$$(p; p^1; \dots; p^T)B \geqq (p; p^1; \dots; p^T)D.$$

In fact, since D is a cone,

$$(p; p^1; \dots; p^T)B \geqq 0 \geqq (p; p^1; \dots; p^T)D.$$

Hence, by strong monotonicity of U^t it can be concluded that $p; p^t \geqq 0$. Also, by assumption that the aggregated total endowment is strictly positive, we may conclude that $\sum n_0^t p w^t > 0$. Thus, there is a type t with $n_0^t > 0$ and $pw^t > 0$. For this type t an interior consumption plan is feasible with respect to $px^t + p^t y^t \leqq 0$. Hence, by strong monotonicity and continuity of U^t, using a standard argument, $p \gg 0$ as well as $p^t \gg 0$. Hence, by nonzero endowment assumption, $pw^t > 0$ for all t. By the same argument, all $p^t \gg 0$. We will now prove that these prices constitute a Lindahl equilibrium.

First, the consumer's utility maximization condition. Suppose that the tuple given by $(x^t; 0; \dots; 0; y^t; 0; \dots; 0)$ — with y^t at location $1+t$ — satisfies $U^t(x^t; y^t) > U^t(x_0^t; y_0^t)$. In fact, since $pw^t > 0$ and the utility function is strongly monotonic and continuous, the same holds for a slightly smaller vector. Using strict positivity of prices, it follows that B-member $(x^t; 0; \dots; 0; y^t; 0; \dots; 0)$ fulfills $px^t + p^t y^t > 0$. This proves that $(x_0^t; y_0^t)$ solves the consumer's problem.

Second, we consider the financial balance condition. By strong monotonicity of the U^t's, there is no slack in the feasibility constraint of the efficient club $(x_0^t; y_0^t; n_0^t)_{t=1;\dots;T}$:

$$\sum_{t=1}^{T} n_0^t x_0^t = -C\left(n_0^1 y_0^1; \dots; n_0^T y_0^T\right):$$

Hence, $\left(-C\left(n_0^1 y_0^1; \dots; n_0^T y_0^T\right); n_0^1 y_0^1; \dots; n_0^T y_0^T\right)$ is on the boundary of B, using strong monotonicity and continuity of the U^t's. It also belongs to D. Since prices were shown to separate B and D at zero, the value is zero, yielding financial balance.

Lastly, we consider the problem of the public administration. Note that any other tuple $\left(-C\left(n^1 y^1; \dots; n^T y^T\right); n^1 y^1; \dots; n^T y^T\right)$ also belongs to D. Therefore, it is priced nonpositively, i.e.,

$$\sum_{t=1;\dots;T} n^t p^t y^t - p \cdot C\left(n^1 y^1; \dots; n^T y^T\right) \leqq 0$$

This proves that $(y_0^1; \ldots; y_0^T; n_0)$ indeed solves the public administration's problem.

This completes the proof of the theorem. ∎

With regard to this equivalence theorem we have the following remarks. Club efficiency implies that all individual demands for the club goods are equalized, if the utility functions are strictly quasi-concave. If the population is not replicated, i.e., $n_0 = (1; \ldots; 1)$, the financial balance condition of Lindahl equilibrium can be simplified further to $\sum_{t=1}^{T} p^t y_0 = p y_0$. Hence, if only one club good is supplied and it is designated the numerair, then the admission prices or fees sum to unity.

Also we emphasize that the converse of the theorem is true, implying that it is a true equivalence result. A Lindahl equilibrium is always efficient. The proof is an easy adaptation of Schweizer's (1983) proof of his theorem 2.

Finally, we remark that the implementation of more general club good cost functions is probably very hard. In the next example we consider a cost function that is more general, but fails to lead to equivalence of efficient clubs and the Lindahl equilibria. Although semi-public goods, as we defined them, are quite general, ranging from purely private to purely public clubgoods, they have a distinct structure in that only total consumption by type, $\overline{y} = (n^1 y^1; \ldots; n^T y^T) \in \mathbb{R}_+^{mT}$, affects their provision. In general, a club with profile n and club goods demands y may impose resource requirements in a way that is not separable by type.

Example Consider an economy setting with one private and one club good, i.e., $\ell = m = 1$, and two types of consumers, i.e., $T = 2$, with the following utility functions:

$$\begin{aligned} U^1(x; y) &= \min(2x + 4; y) \\ U^2(x; y) &= \min(2x + 3; 2y) \end{aligned}$$

Now consider a production structure for the club good that does not satisfy the requirements considered in our model. The cost function is given by

$$C\left(n^1; n^2; y^1; y^2\right) = \max n^t \cdot \max y^t$$

This cost function can be interpreted as representing a semi-public good, which provision is based on the maximal consumption capacity requested, where the maximal capacity is $\max n^t$.

Consider the club given by $n_0 = (1; 1)$, $x_0^1 = x_0^2 = -1$, and $y_0^1 = y_0^2 = 2$. This club is efficient, as we demonstrate now.

We show that U^2 cannot be lifted over its club level, 1, whenever $n^2 > 0; U^1 \geqq 2$ (its club level), and feasibility. The latter can be written, invoking linear homogeneity with respect to n_2,

$$n^1x^1 + x^2 + \max(n^1; 1) \cdot \max(y^1; y^2) \leqq 0$$

Hence

$$x^2 \leqq -n^1x^1 - \max(n^1; 1) \cdot y^1$$

Substitute $x^1 \geqq -1$ and $y^1 \geqq 2$ (both from $U^1 \geqq 2$):

$$x^2 \leqq n^1 - 2\max(n^1; 1) \leqq -1$$

Hence $U^2(x^2; y^2) \leqq 1$ indeed, proving club efficiency, and this level is obtained only if $n^1 = 1$ and the feasibility constraint is binding:

$$x_0^1 + x_0^2 + \max(y_0^1; y_0^2) = 0$$

Lindahl pricing by p and substituting the Lindahl break-even constraint for the semi-public goods, the sum of the consumers' budgets is zero. Since each of them is nonpositive, they are all zero. Better clubs must be priced higher, hence positive. But this is not so. Consider any club with n arbitrary, $(x^1; y^1) = (-1; 2)$ again, but $(x^2; y^2) = (-1/2; 1)$. A consumer of type 2 prefers it. This consumption bundle is half the club-efficient bundle, $(x_0^2; y_0^2) = (-1; 2)$; which has zero value, hence it is affordable. The efficient club cannot be supported by a Lindahl equilibrium.

4 Discussion

Our theorem provides price support to allocations that cannot be improved upon by clubs. The prices are linear, unlike Mas-Colell's (1980) personalized price schedules (also used by Gilles and Scotchmer, 1997) or Barham and Wooders' (1998) admission fees or "wages." The theorem and its proof are adaptations of Schweizer's (1983) theorem on club efficient allocations. He obtains the Henry George Theorem for economies with fixed public goods and associated inputs and, if the latter are zero, the welfare and core limit theorems. In the present paper, club goods are not exogenous, but endogenous, namely the outcome of competition among utility maximizers. Moreover, they are not purely public, but semi-public.

After all, it is well known that there is no competitive basis for Lindahl equilibria in pure public goods economies (Milleron, 1972, and Bewley, 1981). Wooders (1978) has conjectured that the core shrinks when there is crowding, but Conley and Wooders (1994) show that the second welfare theorem is generally false. Barham and Wooders (1998) provides useful relationships between optima and competitive equilibria, but all these papers concern economies with only one private and one public good.

An alternative modelling of an economy with multiple public goods such that the Lindahl equilibrium emerges, has been undertaken by Vasil'ev, Weber, and Wiesmeth (1996). That paper uses an alternative core definition, with utility levels of members of blocking coalitions depending on the replica size and the coalition structure. Although our approach to club goods may seem cleaner, the two approaches are closely related, in the sense that the opportunity cost of individual public — or club — goods consumption is not reduced with the size of the economy in either paper. From this perspective the contribution of our paper is the demonstration that Schweizer's theorem encompasses the core limit theorem of Vasil'ev, Weber, and Wiesmeth (1996). The novelty is the simplicity of the analysis; club efficiency is quite a powerful tool. Lindahl equilibria have a competitive basis in economics with semi-public goods.

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