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**ON THE OPTIMAL TAXATION OF AN EXHAUSTIBLE
RESOURCE UNDER MONOPOLISTIC EXTRACTION**

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On the Optimal Taxation of an Exhaustible Resource under Monopolistic Extraction[*]

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Abstract

In a simple model of resource depletion (isoelastic demand and constant unit extraction cost), we fully characterize the set of linear efficiency-inducing tax/subsidy schemes. We show that this set is infinite and all the larger as the cost of extraction is low. Depending on the magnitude of the latter, we show that there may exist optimal linear strict taxes, thus allowing the regulator to induce efficiency without subsidizing the mine industry at any date. We illustrate and argue that the exhaustibility constraint the monopolist extractor faces can be exploited by the regulator to relax the standard trade-off between inducing efficiency and raising revenues from the monopoly.

JEL classification: Q30; L12; H21
Keywords: Exhaustible resources; Imperfect competition; Optimal taxation

1 Introduction

Broadly speaking, the literature on regulating a monopoly extracting an exhaustible resource consists of two streams. The first one assumes that the regulator is able to precommit to a future tax/subsidy path. In this context, Bergstrom *et al.* (1981) show that efficiency-inducing taxes linear in resource supply are generally time-dependent. In the case of linear demand functions and costless extraction, they show that if the initial stock of resource is not too large, there exists a tax/subsidy scheme such that the present value of transfers to the monopolist is nil. More recently, in the case of constant extraction cost and isoelastic demand, Im (2002) shows that one optimal linear tax scheme is a constant subsidy to the monopoly. The second stream of this literature focuses on optimal Markov perfect tax/subsidy schemes, thus not requiring an ability of the regulator to precommit to a profile of taxes. Specifically, those schemes are linear time-independent tax functions of the stock of resource still unexploited. Karp and Livernois (1992) is the seminal paper of this strand. In particular, they show that the sets of optimal linear time-dependent tax/subsidy and linear Markov perfect tax/subsidy schemes are identical when the unit cost of extraction is constant and the demand function is isoelastic.

The objective of the present paper is to use a specified model to explicitly and fully characterize the set of efficiency-inducing tax/subsidy paths. From this analysis, we aim at finding a sufficient condition for the existence of optimal strict taxes in order to show that there may exist taxation policies through which a regulator can correct the distortion from market power while raising revenues.

More precisely, we use the model of Im (2002): ability of the regulator to precommit, constant unit cost of extraction and isoelastic demand function[1]. We show that the constant subsidy he proposes is a particular policy of a continuum of efficiency-inducing linear tax policies. We fully characterize the set of applicable optimal tax/subsidy schemes. We find that some of these schemes are surprising: increasing strict unit subsidies to the monopolist can induce her to fasten the depletion compared to the laissez-faire outcome. Next, we show that the lower the cost of extraction, the larger the set of optimal tax schemes and we derive a simple condition ensuring the existence of optimal strict tax policies, thus implying that inducing efficiency does not generally prevent from raising tax revenues. Eventually, we reinterpret the

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[1]The assumption of precommitment ability appears to be not decisive because of the equivalence of optimal schemes under constancy of unit cost and isoelasticity of demand demonstrated by Karp and Livernois (1992). However, this assumption makes the results more intuitive and simplifies a good deal the analysis.

result of Im (2002) and argue that besides the classical way of regulating a monopoly through taxes, the exhaustibility constraint a monopolist extractor faces can be exploited by the regulator to solve the apparent trade-off between inducing efficiency and raising tax revenues.

The paper is organized as follows. Section 2 introduces the general model and exposes the core of the problem. Section 3 studies the isoelastic demand and constant unit cost case. Here are presented our main results. Section 4 concludes.

2   The general problem

2.1   Basics

At each date $t \geq 0$, the flow of extraction in units of resource is $R(t) \geq 0$. Let $S(t)$ be the size in units of resource of the reserves remaining at date t. Then:

$$S(t) = S(0) - \int_0^t R(s) \, ds, \ S(t) \geq 0, \ S(0) = S_0 \text{ given.} \qquad (2.1)$$

The cost of extracting R is given by the cost function $C(R)$ which is assumed to be increasing, convex and such that $C(0) = 0$[2].

There is a representative household taking the price as given. Her inverse-demand function is $P(R)$, assumed to be continuous, strictly decreasing and of the class of functions such that $U(R) = \int_0^R P(x) \, dx$ is finite. Moreover, we assume that $\lim_{R \to 0} P(R) = +\infty$[3].

The social discount rate is denoted by $r \geq 0$.

2.2   Resource extraction under perfect competition and monopoly

There is no uncertainty and all agents perfectly foresee the future. The extraction industry will be alternatively considered to be competitive or monopolistic.

Due to the necessity of the resource as formalized above, we know that $R(t) > 0$ for all $t \geq 0$.

The competitive extraction sector takes the price of the resource as given when maximizing the discounted sum of its instantaneous profits subject to the exhaustibility constraint (2.1). The associated Hamiltonian is $H(S, \lambda^*, R, t) =$

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[2]Of course, a realistic modeling should consider other arguments to the cost function. We restrict them to the flow of extraction, R, for the ease of the presentation.

[3]In what follows, we shall refer to this assumption as the necessity of the resource.

$\big(p(t)R - C(R)\big)e^{-rt} - \lambda^* R$ and the extraction path under perfect competition satisfies:

$$\Big(P(R^*(t)) - C'\big(R^*(t)\big)\Big)e^{-rt} = \lambda^*, \qquad (2.2)$$

where $p(t)$ is the price for the resource, λ^* is the positive and constant costate variable and superscript * is used to mean competitive[4].

Before examining how the introduction of a monopoly in the extraction will alter condition (2.2), let us define the tax/subsidy scheme the regulator sets to correct the distortion that arises due to market power. Let $\{\theta(t)\}_{t \geq 0}$ be an *ad valorem* producer tax so that the producer price is $p(t)\tau(t) = p(t)\big(1 - \theta(t)\big)$[5]. Assume that $\theta(t) < 1$ so that $\tau(t) > 0$ and let us restrict to tax profiles differentiable with respect to time[6]. Consistently with the assumption of perfect foresight, suppose that the regulator is able to announce credibly $\{\theta(t)\}_{t \geq 0}$ from date 0.

The monopolist extractor maximizes the discounted stream of her spot profits subject to (2.1). Strategically, she internalizes the demand for the resource, $P(R)$. The associated Hamiltonian is $H^M(S, R, \lambda^M, t) = \big(\tau(t)P(R)R - C(R)\big)e^{-rt} - \lambda^M R$ and, assuming the concavity of the gross revenue $P(R)R$, the extraction path under monopoly satisfies:

$$\Big\{\tau(t)\Big(P(R^M(t)) + P'(R^M(t))R^M(t)\Big) - C'\big(R^M(t)\big)\Big\}e^{-rt} = \lambda^M, \qquad (2.3)$$

where λ^M is the positive and constant costate variable and superscript ^{M} is used for monopolistic.

Under perfect competition as well as under a monopoly subject to any tax/subsidy scheme, the resource supply is always positive as a result of the necessity assumption: $R^*(t) > 0$, $R^M(t) > 0$, for all $t \geq 0$. This implies that the discounted marginal rents must always be strictly positive, $\lambda^*, \lambda^M > 0$, and that the resource is asymptotically depleted:

$$\int_0^{+\infty} R^*(t) = \int_0^{+\infty} R^M(t) = S_0. \qquad (2.4)$$

The differentiation of condition (2.2) leads to the standard Hotelling rule. This gives a differential equation whose solution under the boundary condition (2.4) is unique. The optimal extraction path $\{R^*(t)\}_{t \geq 0}$ is determined that way. Given a certain tax profile $\{\tau(t)\}_{t \geq 0}$, conditions (2.3) and (2.4) determine in the same way a modified Hotelling rule and the extraction path chosen by the monopoly.

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[4]Here, the perfect competition outcome is the first-best equilibrium.

[5]For the sake of notational simplicity, we shall prefer to use the multiplicative tax denoted by τ instead of the *ad valorem* tax denoted by θ.

[6]This assumption is made for simplicity. One can show that all the optimal tax profiles are indeed differentiable with respect to time.

2.3 Core of the problem

The objective is to design a tax/subsidy scheme $\{\tau^*(t)\}_{t\geq0}$ that induces the monopoly to reproduce the first-best extraction path. We are thus looking for all tax profiles such that the solution to (2.3), for any positive λ^{M7}, under (2.4), is $\{R^*(t)\}_{t\geq0}$, *i.e.* all positive functions $\tau^*(t)$ that satisfy:

$$\left\{\tau^*(t)\Big(P(R^*(t)) + P'(R^*(t))R^*(t)\Big) - C'\big(R^*(t)\big)\right\}e^{-rt} = \lambda^M, \qquad (2.5)$$

where λ^M is any positive constant.

In a more general framework, where $C(.)$ and $P(.)$ are allowed to depend directly on time, Bergstrom *et al.* (1981) study the differential equation resulting from (2.5). For the regulator can set the present-value marginal net return to the monopoly, λ^M, to any positive constant, they obtain that there exists a family of solutions $\tau^*(t)$. Under stationary isoelastic demand and constant marginal cost of extraction, Im (2002) shows that a constant subsidy τ^* satisfies equation (2.5).

In the latter model, we fully characterize the set of efficiency-inducing tax/subsidy schemes. We find that there exists a continuum of solutions to this problem and show that the subsidy proposed by Im (2002) is a particular solution.

2.4 About the cost of regulation

The regulator may not be indifferent on the split of the social surplus resulting from the taxation policy and may thus consider the cost of regulating which is the discounted value of the transfers to the monopoly.

Bergstrom *et al.* (1981) show that there may exist optimal policies at no cost, *i.e.* efficiency-inducing tax/subsidy schemes under which the net discounted transfers are zero. We aim at going further and derive conditions for the existence of optimal policies consisting of strict taxes at all dates.

For raising funds through the tax policy, the constraint the regulator faces is the participation constraint, $\lambda^M > 0$: without leaving a positive marginal rent to the monopoly, she won't be willing to reproduce the optimal extraction path. Bringing this costate variable near to zero allows to extract more rent from the extraction industry.

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[7]The fact that the costate variable of the monopoly must be positive for the tax to be optimal can be easily interpreted. λ^M is the present-value marginal profit of the monopoly at all dates. If we want the monopoly to choose the first-best extraction path, we have to ensure that this marginal profit is non negative. This is a participation constraint the tax policy must satisfy.

Looking at equation (2.5) under a marginal cost of extraction, it may seem intuitive that the lower this unit cost, the larger the set of feasible policies and thus the easier the regulation through strict taxes. However, we shall see that because a change in the unit cost parameter alters the whole first-best extraction path as well as the reaction of the monopoly to a given tax-profile, this is not so obvious.

3   The isoelastic case with constant unit cost of extraction

In order to get explicit analytical results, we choose to work with special functional forms of the demand and the extraction cost functions. The following specifications are the same as those made in Im (2002) and are consistent with the assumptions of the previous section.

The demand function is supposed to be isoelastic: $P(R) = R^{-1/\alpha}$, where $\alpha > 0$ is the price-elasticity of demand. Furthermore, in order to ensure that the monopolist program has a solution, we assume that $\alpha > 1$. The average cost of extraction is assumed to be constant[8]: $C(R) = cR$, where $c \geq 0$ is the unit extraction cost.

Let us characterize the optimal extraction path. From condition (2.2), the first-best extraction path is the solution of differential equation[9]:

$$g_R^*(t) = -\alpha r \big(1 - cR^*(t)^{1/\alpha}\big) \tag{3.1}$$

which satisfies the boundary condition (2.4), *i.e.*:

$$R^*(t) = e^{-\alpha r t}\big(R_0^*(S_0)^{-1/\alpha} + c(e^{-rt} - 1)\big)^{-\alpha}, \tag{3.2}$$

where $R^*(0) = R_0^*(S_0)$ is increasing.

3.1   Set of optimal linear tax/subsidy paths

Let us now characterize the set of optimal tax/subsidy schemes as defined in section 2. Differentiating condition (2.5) and substituting $\dot{R}^*$ from equation

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[8]There are many reasons to think that a realistic modeling must be much more sophisticated. For a discussion, see Dasgupta and Heal (1979), chapter 6. Assuming that the average cost of extraction is constant is a way to simplify a good deal in order to analyze the behavior of the economy.

[9]The derivative with respect to time of any variable X is denoted by $\dot{X}$. Its rate of growth is denoted by $g_X = \dot{X}/X$.

(3.1), one finds that the efficiency-inducing tax schemes are the solutions of differential equation:

$$\dot{\tau}^*(t) = rcR^*(t)^{1/\alpha}\big(\tau^*(t) - \frac{\alpha}{\alpha - 1}\big) \qquad (3.3)$$

which ensure $\lambda^M > 0$, *i.e.* the set of positive functions:

$$\tau^*(t) = \big(\tau^*(0) - \frac{\alpha}{\alpha - 1}\big)e^{rc \int_0^t R^*(s)^{1/\alpha} ds} + \frac{\alpha}{\alpha - 1} \qquad (3.4)$$

which satisfy:

$$\lambda^M = \frac{\alpha - 1}{\alpha}\tau^*(0)R_0^*(S_0)^{-1/\alpha} - c > 0. \qquad (3.5)$$

We shall denote this set by Θ^*

Proposition 1 *There exists an infinite family of efficiency-inducing producer tax/subsidy paths:*
$\Theta^* = \big\{\{\tau^*(t)\}_{t \geq 0} : (3.4), \tau^*(0) > \frac{\alpha}{\alpha - 1}\max\{cR_0^*(S_0)^{1/\alpha}, 1 - e^{-rc \int_0^{+\infty} R^*(t)^{1/\alpha} dt}\}\big\}.$

Proof of proposition 1 □ *Among the set of solutions $\{\tau(t)\}_{t \geq 0}$ which satisfy (3.4), we have to select the optimal ones by eliminating ı) those being negative or zero at some $t \geq 0$ (for taxes to be well-defined) and ıı) those not satisfying condition (3.5) (for participation constraint).*
(3.5) is equivalent to $\tau(0) > \tau_{PC} \equiv cR_0^*(S_0)^{1/\alpha}\alpha/(\alpha - 1)$.
Positivity is ensured for tax schemes (3.4) such that $\tau(0) \geq \alpha/(\alpha - 1)$ since, from (3.3), they are increasing or constant and initially positive. Tax schemes (3.4) such that $\tau(0) < \alpha/(\alpha - 1)$ are decreasing. Ensuring positivity everywhere is thus equivalent to ensuring asymptotic positivity, i.e.: $\min_{t \geq 0}\{\tau(t) : (3.4), \tau(0) < \alpha/(\alpha - 1)\} = \lim_{t \mapsto +\infty} \tau(t) = \big(\tau(0) - \alpha/(\alpha - 1)\big)\exp\{rc \int_0^{+\infty} R^*(t)^{1/\alpha} dt\} + \alpha/(\alpha - 1) > 0$. This condition amounts to $\tau(0) > \tau_{WD} \equiv \big(1 - \exp\{-rc \int_0^{+\infty} R^*(t)^{1/\alpha} dt\}\big)\alpha/(\alpha - 1)$.
Optimality then requires $\tau^*(0) > \tau_{PC}$ and $\tau^*(0) > \tau_{WD}$, that is $\tau^*(0) > \bar{\tau} \equiv \max\{\tau_{PC}, \tau_{WD}\} = \frac{\alpha}{\alpha - 1}\max\{cR_0^*(S_0)^{1/\alpha}, 1 - e^{-rc \int_0^{+\infty} R^*(t)^{1/\alpha} dt}\}$.
One can see that $\tau_{WD} < \alpha/(\alpha - 1)$. From (2.2) at date 0, and $\lambda^* > 0$, we know that $cR_0^*(S_0)^{1/\alpha} < 1$. Hence, $\frac{\alpha}{\alpha - 1}\max\{cR_0^*(S_0)^{1/\alpha}, 1 - e^{-rc \int_0^{+\infty} R^*(t)^{1/\alpha} dt}\} < \alpha/(\alpha - 1)$. ■

In the family Θ^* of optimal tax/subsidy schemes, one can see that some functions are rising strict subsidies, one is a constant one, other tax/subsidy profiles may be falling and there may also exist falling strict taxes (See figure 1).
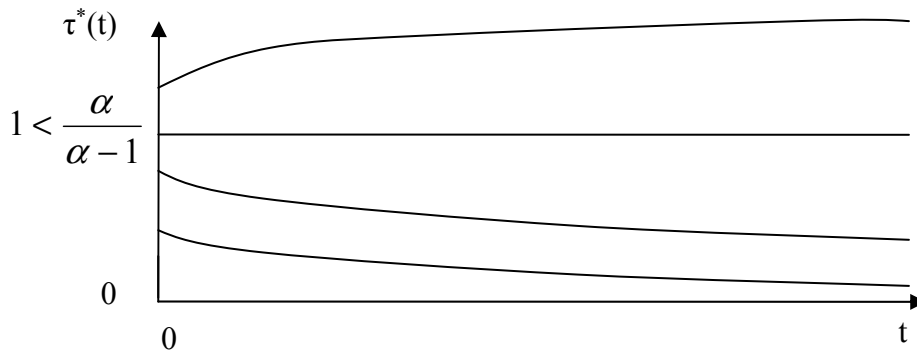
Figure: Optimal tax/subsidy time-profiles

In particular, there is the constant subsidy proposed in Im (2002). However, this scheme appears to one element of an infinite set of taxation policies. One notes from (2.2) and (2.3) that this instrument equalizes the marginal revenue to the price so that these conditions become equivalent. In the same model without exhaustibility constraint, this subsidy would actually be the unique linear tax to correct the distortion arising from market power. This illustrates that the exhaustibility of the resource offers other ways to regulate her[10].

Some others optimal policies can seem counter-intuitive. Stiglitz (1976) shows that, if the unit cost of extraction is constant and the price-elasticity of demand is constant and larger than unity, the monopoly under *laissez-faire* is more conservative than a competitive extractor. Hence, every optimal policy aims to induce the monopoly to extract the resource faster. Surprisingly, we note from proposition 1 that, some of these policies are unit subsidies rising over time[11].

In what follows, we shall see that the possibility for the regulator to exploit the exhaustibility constraint the monopoly faces may allow the former to correct market-power through the use of strict taxes.

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[10]Another interesting example that introducing a state variable in the monopoly's program helps to regulate her is Benchekroun and Long (2004). They show that submitting a standard monopolist producer to a subsidy depending on her historical performances results in a family of subsidy rules and allows to reduce the transfers to the producer.

[11]This has much to do with the presence of a positive unit cost of extraction. Indeed, if it is nil, the marginal revenue equals the producer price, thus being proportional to τ. In that case, each rising subsidy induces the extractor to delay extraction.

3.2 Existence of optimal strict tax profiles

In this subsection, we show that the boundary condition restricting Θ^*, $\tau^*(0) > \frac{\alpha}{\alpha-1} \max\{cR_0^*(S_0)^{1/\alpha}, 1 - e^{-rc \int_0^{+\infty} R^*(t)^{1/\alpha} dt}\}$, is all the more relaxed as the unit cost of extraction is low, so that below a certain threshold cost, the set of efficiency-inducing tax schemes includes strict taxes: $\{\theta(t)\}_{t\geq 0}$ such that $\theta(t) > 1, \forall t$.

We shall see that this result is not trivial for the reason that a change in the extraction cost alters the whole first-best extraction path as well as the reaction of the monopoly to a certain taxation scheme. That is why we choose from now on to write all the critical variables and sets as functions of the unit extraction cost, c.

Proposition 2 $\imath$) *The lower the unit extraction cost, the larger the set of efficiency-inducing taxation schemes in the sense that:* $\forall c, c' \geq 0, c < c' \Rightarrow \Theta^*(c) \supset \Theta^*(c')$.
$\imath\imath$) There exists a threshold cost of extraction below which market power can be corrected through strict taxes: $\exists \bar{c} > 0 : $ *if* $0 \leq c < \bar{c}$ *then* $\exists \{\tau^*(t)\}_{t\geq 0} \in \Theta^*(c) : \tau^*(t) \leq 1, \forall t \geq 0.$

Proof of proposition 2 $\square$ *We are going to use the notations* $\bar{\tau}$, τ_{WD} *and* τ_{PC} *introduced in proof of proposition 1. For the sake of being clear, let us write all the variables of the model as functions of parameter c.*

$\imath$) From condition (2.2) under our specifications, $R^*(t,c)^{-1/\alpha} = \lambda^*(c)e^{rt} + c$, *what leads to* $g_R^*(t,c) = -\alpha r \left(1 + e^{-rt} c \lambda^*(c)^{-1}\right)^{-1}$. *Comparing this expression with (3.1), one obtains:* $\left(1 + e^{-rt} c \lambda^*(c)^{-1}\right)^{-1} = 1 - c R^*(t,c)^{1/\alpha}, \forall t \geq 0.$ *Note that if $c\lambda^*(c)^{-1}$ is increasing in c, then $cR^*(t,c)^{1/\alpha}$ is also increasing in c.*

Again from (2.2), we have $R^*(t,c) = \left(c + \lambda^*(c)e^{rt}\right)^{-\alpha}$. *Using the binding constraint (3.4), we obtain:* $S_0 = \int_0^{+\infty} \left(c + \lambda^*(c)e^{rt}\right)^{-\alpha} dt$, *from which we see that $\lambda^*(c)$ is increasing in c, thus proving that $cR^*(t,c)^{1/\alpha}$ is increasing in c for all $t \geq 0$.*

It follows that $cR^*(0,c)^{1/\alpha}$ and $\int_0^{+\infty} cR^*(t,c)^{1/\alpha} dt$ are increasing in c, implying that $\tau_{PC}(c)$, $\tau_{WD}(c)$ and thus $\bar{\tau}(c)$ are increasing in c. This proves first part of proposition 2.

$\imath\imath$) Note moreover that $\bar{\tau}(c)$ is continuous in c since c affects continuously all the variables. Due to the finiteness of S_0, if $c = 0$, one can easily see that $\tau_{PC} = \tau_{WD} = \bar{\tau} = 0$. Hence, by continuity, $\exists \bar{c} > 0 : \bar{\tau}(c) < 1, \forall c \leq \bar{c}$. Since, from (3.3), all paths $\{\tau^*(t)\}_{t\geq 0}$ in Θ^* such that $\tau^*(0) \leq 1 < \alpha/(\alpha - 1)$ are decreasing over time: if $c \leq \bar{c}$, then $\exists\{\tau^*(t)\}_{t\geq 0} \in \Theta^*(c) : \tau^*(t) \leq 1, \forall t \geq 0.$ This is the second part of proposition 2. $\blacksquare$

Bergstrom *et al.* (1981) used the free extraction case as an illustration of their model. The introduction of a positive unit cost illustrates by proposition 2 that the cost of extraction is a critical parameter when designing and evaluating the cost of the cheapest regulation.

In our model, in the polar case of a free extraction, $c = 0$, one can see from (3.5) that the marginal net return to the monopoly can be set to nearly nought with an *ad valorem* tax sufficiently close to 100% ($\tau = 0^+$), thus capturing almost all the rent. In this case, any positive constant tax is optimal since the monopoly cannot exert her market power[12]. Indeed, in the context of a free extraction, any constant linear tax has only distributional effect and does not alter the extraction path.

3.3 Taxing consumers instead of the monopoly

In this subsection, we shall show that a regulator can also use a consumer tax to induce efficiency. The design of the set of optimal consumer tax/subsidy schemes and its main properties is eased by the simple relation between these schemes and the producer tax/subsidy ones.

Let $\{\theta^C(t)\}_{t \geq 0}$ be an *ad valorem* consumer tax so that the consumer price is $p(t)\tau^C(t) = p(t)(1 + \theta^C(t))$, while $p(t)$ is the producer price[13]. Assume that $\theta^C(t) > -1$ so that $\tau^C > 0$ and let us restrict to tax profiles differentiable with respect to time.

In this context, under our functional forms, the demand function becomes $R(t)^{-1/\alpha} = p(t)\tau^C(t)$. The Hamiltonian associated to the monopoly's optimization program is $H^{MC}(S, R, \lambda^{MC}, t) = (\tau^C(t)^{-1}R^{(\alpha-1)/\alpha} - cR)e^{-rt} - \lambda^{MC}R$. An efficiency-inducing consumer tax/subsidy scheme is a positive function $\tau^{C*}(t)$ that satisfies: $\left(\tau^{C*}(t)^{-1}R^*(t)^{-1/\alpha}(\alpha-1)/\alpha - c\right)e^{-rt} = \lambda^{MC}$, where λ^{MC} is any positive constant. We are then looking for the solutions of differential equation:

$$\dot{\tau}^{C*}(t) = rcR^*(t)^{1/\alpha}\left(\frac{\alpha}{\alpha-1} - \tau^{C*}(t)^{-1}\right) \tag{3.6}$$

which ensure $\lambda^{MC} > 0$, *i.e.* the set of positive functions Θ^{C*}:

$$\tau^{C*}(t) = \left\{\left(\tau^{C*}(0)^{-1} - \frac{\alpha}{\alpha-1}\right)e^{rc\int_0^t R^*(s)^{1/\alpha}\,ds} + \frac{\alpha}{\alpha-1}\right\}^{-1} \tag{3.7}$$

which satisfy:

$$\lambda^{MC} = \frac{\alpha-1}{\alpha}\tau^{C*}(0)^{-1}R_0^*(S_0)^{-1/\alpha} - c > 0. \tag{3.8}$$

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[12]On this, see Stiglitz (1976).

[13]Here again, for notational convenience, we shall prefer to refer to the multiplicative tax denoted by τ rather than to the *ad valorem* one.

Comparing the solutions (3.7) bounded by condition (3.8) and the solutions (3.4) bounded by (3.5), the following proposition applies.

Proposition 3 *The reciprocal of any efficiency-inducing producer tax/subsidy function is an efficiency-inducing consumer tax/subsidy function and* vice versa, *i.e.* $\{\tau^*(t)\}_{t \geq 0} \in \Theta^* \Longleftrightarrow \{\tau^*(t)^{-1}\}_{t \geq 0} \in \Theta^{C*}$.

Proof of proposition 3 $\square$ *We are going to use the same notations as in proofs of propositions 1 and 2 with a superscript ^{C} when they concern the consumer taxation case.*

The same way as for proposition 1, the characterization of Θ^{C*} necessitates to eliminate the non-positive solutions (3.7) as well as the solutions which don't satisfy (3.8).

(3.8) is equivalent to $\tau^{C*}(0) < \tau_{PC}^C \equiv (cR_0^*(S_0)^{1/\alpha}\alpha/(\alpha-1))^{-1} = \tau_{PC}^{-1}$.

From (3.6), solutions (3.7) such that $\tau^C(0) \geq (\alpha-1)/\alpha$ are increasing, all others being decreasing. Hence, positivity is ensured for the former ones and we can focus on the latter ones. Ensuring positivity everywhere is thus equivalent to impose $\tau^C(0) > 0$ and $\lim_{t \mapsto +\infty} \tau^C(t) = ((\tau^C(0)^{-1} - \alpha/(\alpha-1))\exp\{rc\int_0^{+\infty} R^*(t)^{1/\alpha}\,dt\} + \alpha/(\alpha-1))^{-1} > 0$. This leads to the condition $0 < \tau^C(0) < \tau_{WD}^C \equiv (1 - \exp\{rc\int_0^{+\infty} R^*(t)^{1/\alpha}\,dt\})^{-1}(\alpha-1)/\alpha = \tau_{WD}^{-1}$.

Eventually, $\Theta^{C*} = \{\{\tau^{C*}(t)\}_{t \geq 0} : (3.7), 0 < \tau^{C*}(0) < \bar{\tau}^C\}$, where $\bar{\tau}^C = \min\{\tau_{PC}^C, \tau_{WD}^C\} = \min\{\tau_{PC}^{-1}, \tau_{WD}^{-1}\} = (\max\{\tau_{PC}, \tau_{WD}\})^{-1} = \bar{\tau}^{-1}$. Noting moreover that the reciprocal of a positive solution (3.4) is a solution (3.7) and vice versa, *one obtains proposition 4.* ∎

Proposition 4 is very convenient for it ensures that the profiles of the optimal consumer tax/subsidy schemes are symmetric to those of the producer ones and that their set Θ^{C*} has the same properties as Θ^*. Hence, propositions 2 and 3 can be easily replicated to the case of consumer taxation.

As noted by Bergstrom *et al.* (1981), this has a practical implication: if consumers live in a different jurisdiction from the monopoly, it is possible to regulate the latter even if the regulator has only legal authority in the region of the formers. Furthermore, from our analysis follows that taxing the consumers may allow to induce efficiency while raising revenues from the rent of the monopolist.

4   Concluding remarks

In a partial equilibrium model with an isoelastic demand and a constant unit cost of extraction, we have fully characterized the set of linear time-dependent efficiency-inducing tax/subsidy policies. We have shown that this

set is infinite and all the larger as the cost of extraction is low. We have proved that, if the latter is sufficiently low, there exist optimal strict tax schemes, thus allowing the regulator to induce-efficiency while raising revenues from the mine industry at each time.

In the same model, Im (2002) finds that giving a particular constant *ad valorem* subsidy to the monopoly is an optimal taxation policy. The issue of the trade-off between inducing efficiency and raising revenues from a monopoly has thus been extended to the case of resource depletion. We have revisited this apparent trade-off by characterizing the full set of optimal linear tax/subsidy policies.

Because the subsidy proposed by Im (2002) is the only linear tax correcting market-power in the same model without exhaustibility, and since this policy appears to be one particular optimal scheme after introducing this ingredient, our paper illustrates that the exhaustibility constraint the monopolist extractor faces offers new ways of regulating her through taxes.

Generally speaking, we argue that there are actually two possible approaches of the regulation through taxes of a monopolist extractor of an exhaustible resource. The first one is standard. It consists in making the marginal revenue equal to the price so that the market incentives of the monopoly are the same as those of a competitive industry. This can be done through a constant subsidy if the demand for the resource is isoelastic. Under more general specifications, a per unit subsidy equal to the reciprocal of the price-elasticity of demand estimated at the optimal quantity restores the optimum allocation. This is the way to proceed in a static world. Applying this method to the regulation of a monopolist extractor defines a unique *ad valorem* tax scheme. The second approach is peculiar to resource depletion. It consists in providing the monopoly with different incentives to extract at each time by taxing at different rates the quantities extracted at different dates. This approach uses the fact that when supplying an exhaustible resource, a monopoly is constrained on the asymptotic cumulative quantity. Using this peculiarity of depletable resources is more flexible in the sense that it allows to choose among a multiplicity of optimal policies and may in particular allow to use strict taxes instead of standard subsidies.

Applying this second method to induce an efficient behavior is not trivial. Indeed, we have noted that it can lead to surprising results on the optimal tax-profiles and their effects on the monopolist's intertemporal behavior. As an example, we have shown that a rising unit subsidy to the monopoly can lead her to deplete the resource faster.

We know from Karp and Livernois (1992) that, in our model, each optimal tax/subsidy scheme corresponds to a linear Markov perfect tax function of the current remaining stock. However, extending our analysis with stan-

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dard functional forms to a model without precommitment ability could be of interest to examine the profiles of the resulting optimal Markov perfect schemes. Another natural next step would be to introduce asymmetry of information in our simple model.

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