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REMARKS ON REAL VALUE ADDED

BY CHRISTOPHER A. SIMS

In a recent article in this journal, Stefano Fenoaltea [1976] discussed measurement of real value added. The article concludes that a certain class of measures is superior to all others: "The current-price values added of different industries are. . .to be deflated by the *same* price index."[1] As part of an extensive discussion of the criteria for a good measure of real value added, on which the article bases its choice of a class of indices, the article asserts that the assumption of separability of the production function, on which earlier discussions by Arrow [1974] and myself [1969] has been based, is reasonable only "on a literal but unusual definition of 'real value added' as a thing in its own right; and it has little to do with the meaning of 'real value added' in the context which coined the phrase in the first place."[2]

Though I did once write [1969] that separability is required in order for "the notion of real value added" to "make any sense", I agree with Fenoaltea that it may sometimes be useful to compute "real value added" for industries with non-separable production functions, just as it may sometimes be useful to compute "real income" for groups of consumers with different utility functions. Nonetheless, it must be expected that any good measure of real value added will sometimes misbehave, according to some intuitively natural criteria, when industry production functions are non-separable.

Likewise I agree with Fenoaltea that his type of index is a reasonable one in some applications—I have used an index of this type in some empirical work of my own [1968]. Nonetheless he is wrong to claim that his class of indexes is better than the other types of index to which he compares his own. In any situation where there is more than one primary input, Fenoaltea's index is capable of producing anomalous results, even in some situations where the alternatives to which he compares his index behave reasonably. The simulations reported in Fenoaltea's article never make his index misbehave, because they all deal only with the case of a single primary input.

Section 1 below presents examples of undesirable behavior by Fenoaltea's type of index. Section 2 argues that Fenoaltea's own criteria for a good real value added measure point to the central importance of a separability assumption. Section 3 summarizes conclusions.

[1] Fenoaltea [1976], p. 121, emphasis in original.
[2] Fenoaltea [1976], pp. 119–120.

I. Anomalies Arising From Assuming Value Added Has Only One Price

For an industry which produces its output directly from primary factors, without any purchases of intermediate inputs, most economists would agree that measurement of real value added presents no difficulties: real value added should in this case be identical with real output. In fact, all of the indexes Fenoaltea considers, except his own, agree in this case in measuring real value added as real output. The indexes Fenoaltea considers are: gross output; David's index; the standard index of activity; and the double-deflated index. For the case of one output, two primary inputs (K and L) and one intermediate input (M), these measure the percentage change in real value added as, respectively,

Gross output:

$$\frac{dQ}{Q} \tag{1}$$

David index:

$$\frac{\left(\frac{dQ}{Q} - \frac{s_M dM}{M}\right)}{s_V} + \left(\frac{s_M}{s_V}\right)\left(\frac{dP}{P} - \frac{dT}{T}\right)^3 \tag{2}$$

Activity:

$$\frac{\frac{s_K dK}{K} + \frac{s_L dL}{L}}{s_V} \tag{3}$$

Double-deflation:

$$\frac{\frac{dQ}{Q} - \frac{s_M dM}{M}}{s_V}, \tag{4}$$

where Q is output, K capital, L labor, M materials, V value added, P the price of output, T the price of materials, and subscripted s's are shares of output. Fenoaltea's own indexes measure the percentage change in real value added as

Fenoaltea indexes:

$$\left(\frac{dQ}{Q} + \frac{dP}{P} - s_M \frac{\left(\frac{dM}{M} + \frac{dT}{T}\right)}{s_V}\right) - \frac{dP_V}{P_V} \tag{5}$$

[3]This complicated formula may not make apparent the computational appeal of David's index: The index is current value added deflated by the price of output.

where P_V is a price index for value added. Clearly if there are no materials inputs, so that $s_M = 0$, $s_V = 1$, indexes (1), (2), and (4) all reduce to dQ/Q, as does (3) under competitive assumptions with $Q = F(K, L)$, F homogeneous of degree one. (5) is dQ/Q in this case only if $dP_V/P_V = dP/P$. Thus for two industries in which relative price has changed (5) does not give the natural answer, regardless of how P_V is measured.

For example, if demand for the output of each of two industries is unit-elastic and neither uses intermediate inputs, Fenoaltea's indexes will never show change in the industries' relative real value added, even though (1)-(4) could all agree on large changes in relative real value added.

Another class of examples, in which intermediate inputs are allowed, arises when we consider two industries with fixed-coefficients technologies and inelastic demands. All inputs and outputs in these industries remain fixed while relative prices change. Most economists would agree that a reasonable definition of real value added would not vary as prices change in this situation. Obviously indexes (1), (3), and (4) behave reasonably here, while (2) and (5) do not.

2. Is Separability Irrelevant?

Fenoaltea claims, "A proper value measure of industrial production ...measures both the value of activity and the value of its results."[4] Though he does not give a precise explanation of what he means by "results" and "activity", he does give synonyms for activity (industry, inputs) and for results (industrial production, net output).[5] I agree that it is essential to most interpretations of real value added statistics that they measure both "real primary inputs" and "real net output". It seems to me reasonable to require that when a given industry uses exactly the same vector of primary inputs at two different times or in two different places, then any measure of "real primary inputs" should be the same in those two times or two places. This implies that "real primary inputs" is some function a of the vector of primary inputs L. Correspondingly, "real net output" ought to be a function r of the vector y of industry outputs and intermediate goods inputs. If there is to be any single number which always measures both "real primary input" and "real net output", then we must always have $r(y) = a(L)$, for any y and L consistent with the industry's technology.

Now we have that if a point (y, L) is technologically efficient it must satisfy $r(y) = a(L)$. If conversely any point (y, L) such that $r(y) =$

[4] [1976], p. 118.
[5] [1976], p. 112.

$a(L)$ is efficient, then $r(y) = a(L)$ defines the technology, and the technology is separable in y and L.[6]

When the technology is smoothly differentiable and convex, one can expect that the set of efficient points will be an $N - 1$ dimensional surface, where N is the total number of inputs and outputs, and the set of points $r(y) = a(L)$ will also be such a surface. Since the latter surface contains the former, and both have the same dimension, they will be at least locally identical. This completes a rough sketch of a proof that a smooth technology which admits an unambiguous measure of real value added must be separable.[7]

3. Conclusions

If one admits that the real value added makes most sense when "net output" is some function of primary inputs, then separability of the technology is important to the notion of real value added. Under perfect competition, the technology is linear homogeneous and separable, indexes (3) and (4) the Divisia index of primary inputs and the "double-deflated" index of net output are both locally exact. The gross output index (1) is locally exact only when intermediate inputs are absent or more in proportion to total output. The David index is locally exact only if intermediate inputs are absent or the price of intermediate input moves in proportion to the output price. The Fenoaltea index is exact only if $(dP/P - s_M dT/T)/s_V$ is the same in all industries and dP_V/P_V is chosen equal to this quantity.

Now it may sometimes happen that even though the separable-technology assumption is reasonable, the data necessary to compute (3) or (4) are unavailable, the technology is not homogeneous, or competitive assumptions are not satisfied. Then one of the other indexes may be preferable, and it would therefore be worthwhile to explore in more detail exactly what assumptions beyond separability are required to justify each of the other indexes.[8]

[6] A technology is separable in y and L if it can be expressed as $g(h_1(y), h_2(L)) = 0$, where h_1 and h_2 are each one-dimensional.

[7] I readily admit that this geometric argument is so sketchy as to be irritating to the skeptic. A more complete argument might develop interesting insight into special cases where real value added exists in non-separable technologies. Clearly fixed coefficient technologies in which the efficient set is the intersection of $r(y) = a(L)$ with some additional restrictions provide one class of such special cases. Note that when there is only one primary input, as in all Fenoaltea's examples, the technology is automatically separable.

[8] E.G., the David index works when all prices move in proportion, while the Fenoaltea index in principle works under slightly less restrictive assumptions. But it may be that in a closed economy with separable technology in each industry a fixed price of value added across industries implies fixed relative prices.

Also, it is not too hard to imagine situations where "net output" or "total input" is in some sense meaningful, even though the two notions cannot be treated as identical. For example, it may be reasonable to compute a net output measure as "value added at international prices" in an economy with tariff barriers. The fact that there is not in general any way to identify this "net output" with any one-dimensional measure of "total input" need not, in some applications, affect the usefulness of the net output measure. However, in such applications, where separability is not claimed, a distinction between net output and total input is unavoidable one cannot pretend that "real value added" is both things at once.[9]

University of Minnesota

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[9] An important new paper on this topic which appeared after this note was in proofs is Sato [1976].