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Capital, Labor, and Income in Manufacturing

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Introduction

The beginning student of general equilibrium economics soon reaches the saddening conclusion that "everything depends on everything else." Later on, with advanced study and maturity, comes the more sophisticated realization that "everything depends on everything else in two ways." Thus, for instance, income and output are produced with inputs of labor and capital goods and this provides one way in which they are interdependent. Reciprocally, (final) output must eventually be sold to recipients of wages or profits and this provides a second source of interdependence. How, then, on reading the title of this paper did you know which of the two relations it was going to be about? I think the probability is high that you assumed (correctly) that it would be about production relations and the reason is that papers on income distribution almost always are. Why?

The neoclassical theory of distribution is usually pigeonholed under the heading "marginal productivity" and marginal productivities derive from production functions. This is true enough; but the subset of Walrasian equilibrium equations having to do with marginal productivity does not constitute a complete theory of income distribution. This has been obscured by the custom of fitting aggregate production functions. To see the point it is only necessary to think of a two-product economy in which each of the products has a production function of Cobb-Douglas type, but with different elasticities. Then the relative shares of wages and profits in the net output of each industry are indeed determined solely by the marginal productivity conditions; but the macroeconomic distribution of income depends on the size of the two industries, hence ultimately on the demand of different classes of income-receivers for the two products. Therefore, when modern economists like Nicholas Kaldor and Joan Robinson claim that neoclassical distribution theory is all wrong because even in equilibrium relative shares depend on the rate of investment and propensities to save, they demonstrate only an imperfect under-

NOTE: I must acknowledge my debt to Henry Y. Wan, Jr., of M.I.T. for his careful and alert assistance.

standing of general equilibrium theory and the maxim that "everything depends on everything else in two ways."[1]

I say all this to make it quite clear that this paper does not pretend to offer a complete neoclassical account of the determination of distributive shares in the economy as a whole. In the first place, that is a general equilibrium problem, while this paper is limited to a few two-digit manufacturing industries. In the second place, I shall be concerned only with the production function half of the determinants of distributive shares; my object is to estimate production functions, elasticities of substitution between labor and capital, rates of technical progress, and perhaps some other interesting parameters descriptive of technical conditions in some branches of manufacturing. Naturally, I believe that information of this kind is vital to an understanding of distribution (and of other things), but I do not believe it to be the whole story.[2]

Fitting production functions for manufacturing industries together or separately is old stuff. The methods I shall use, however, are fairly new. The are put together from some recent papers of mine and my colleagues.[3] The main differences from standard practice are the following. First is the choice of a production function. About the only classes of production functions used in empirical work are the Leontief and Cobb-Douglas type. Thus elasticities of substitution are from the very beginning assumed to be either zero or one. Recently Arrow, Chenery, Minhas and I introduced a broader class of production functions characterized by a constant elasticity of substitution but permitting that elasticity to have any value. We discussed the fitting of such

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[1] See Nicholas Kaldor: "Alternative Theories of Distribution," *Review of Economic Statistics*, Vol. XXIII(2), 1955–56, pp. 83–100, reprinted in *Essays in Value and Distribution*, London, 1960; and Joan Robinson: "Letter to the Editor," *Econometrica*, Vol. 27(3), 1959, p. 490. Also R. Solow: "Notes Toward a Wicksellian Model of Distributive Shares," in *The Theory of Capital*, ed. Lutz and Hague, London, 1961; and R. Findlay: "Economic Growth and the Distributive Shares," *Review of Economic Studies*, Vol. XXVII(3), 1960, pp. 167–178.

[2] Perhaps I should make a detailed apology for violating my neoclassical boy scout's oath and talking about rough indexes of "capital" instead of about precisely defined vectors of capital goods. But that would take me away from my real business; and besides, a clear statement of a view to which I subscribe is to be found in my friend Paul Samuelson's contribution to the Festschrift for Gustav Akerman. I intend to meet that problem directly in a later paper.

[3] Particularly R. Solow: "Investment and Technical Progress," in *Mathematical Methods in the Social Sciences, 1959*. Stanford, 1960, and K. Arrow, H. Chenery, B. Minhas and R. Solow: "Capital-Labor Substitution and Economic Efficiency," *Review of Economics and Statistics*, Aug. 1961, pp. 225–50.

production functions to several mixtures of data. A second novelty in this paper is that I use a cross-section analysis (in which observations represent different regions of the United States in 1956) mainly for the purpose of estimating the elasticities of substitution; time-series analyses of the whole of each United States industry for 1949–58 then yield estimates of the other parameters. In the time-series analysis—this is a third difference from the usual approach—I allow for technological progress of a kind which requires up-to-date capital to be adopted. In fact the kind of technological change admitted is a pure and simple improvement in the efficiency of capital goods (which is *not* the same thing as capital-saving innovations). I confess that my motivation for sticking to this kind of technical change is about 60 per cent convenience and 40 per cent curiosity about the consequences of such an assumption.

I shall begin by setting out the theoretical foundations of the analysis; then comes a straight report of the empirical results, together with some brief interpretive notes.

The Representation of Technical Change

Consider a constant-returns-to-scale production function $Q = F(K, L)$. The most general way of indicating that technological limitations are shifting with the acquisition of new knowledge is simply to recognize that the production function shifts arbitrarily through time; we could write $Q = F(K, L; t)$ with the function homogeneous of degree one in K and L. A very restrictive (but the most commonly made) specification is that the effects of technical progress are neutral (or uniform, to choose a word not already encrusted with old meanings) in the sense that marginal rates of substitution do not change when innovation occurs; in that case the shifting production function can be written $Q = A(t)F(K, L)$. An intermediate assumption—more general than uniformity but less than perfectly general—is that technical advance takes the form of making labor and capital goods more productive in the precise sense that anything one man-hour or one machine-hour could do last year can now be done by 0.9 man-hour or 0.8 machine-hours. Formally, one can write $Q = F(\mu(t)K, \lambda(t)L)$.[4] If μ and λ are simply proportional, this is

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[4] It should be obvious that, given constant returns to scale, nothing further is added by putting an extra multiplicative uniform factor out front.

nothing but uniform technical change all over again. Otherwise, homogeneity implies that

$$F[\mu(t)K,\ \lambda(t)L] = \lambda(t)F\left[\frac{\mu(t)}{\lambda(t)}K,\ L\right],$$

which is the same thing as uniform technical progress like $\lambda(t)$ accompanied by a change in the productivity of capital goods like $\mu(t)/\lambda(t)$ and no change in the specific productivity of labor.

Now suppose $\lambda(t)$ is constant (that is, equals 1, with no loss of generality) so that the only technical change is an increased productivity of capital goods. Notice that this kind of technical progress is not necessarily capital saving (in the usual sense of decreasing the marginal productivity of capital goods relative to that of labor at any specified K/L ratio). It is in fact capital saving, uniform, or labor saving according as the elasticity of substitution between K and L is less than, equal to, or greater than one. To see this, just write down the expressions involving marginal products for the share of rentals divided by the share of wages; an increase in μ works exactly like an increase in K; hence if the elasticity of substitution is less than one, an increase in μ lowers the share of rentals relative to wages; but since the K/L ratio is really unchanged, this must mean that the rental rate has fallen relative to the wage, and the change is capital saving; etc. Note also that the argument does not require λ to be constant; one simply thinks about μ/λ. In the empirical work, however, I shall mostly make the strong assumption that $\lambda = 1$.

The nice thing about this assumption is that it gives clear effect to the belief that technical progress is indissolubly bound up with investment and permits this case[5] to be handled in a particularly simple way. Suppose that, *with capital of vintage v* (i.e., produced at time v), output can be produced according to the production function:

$$Q_v(t) = F[\mu(v)K_v(t),\ L_v(t)] = L_v(t)F[\mu(v)K_v(t)/L_v(t),\ 1] \quad (1)$$

where

$Q_v(t)$ is output at time t produced with capital of vintage v;

$K_v(t)$ is the stock of vintage v capital surviving at time t;

$L_v(t)$ is the labor force assigned to operate it.

Of course $\mu(v)$ is the productivity factor attached to vintage v

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[5] Analyzed from a less general point of view in my "Investment and Technical Progress."

capital and the whole point is that it is fixed forever once the act of investment takes place. The market will shuffle the homogeneous labor force $L(t)$ over the existing stock of capital of various vintages in such a way that

$$L(t) = \sum_{v=-\infty}^{t} L_v(t)$$

and the real wage (in terms of the product of this industry) is equal to the marginal product of labor which will have to be the same for all vintages of capital. Letting $W(t)$ be the product wage in this industry at time t, this means that

$$W(t) = F_2[\mu(v)K_v(t), L_v(t)] = F_2[\mu(v)K_v(t)/L_v(t), 1] \quad (2)$$

since partial derivatives of F are homogeneous of degree zero. It follows that $\mu(v)K_v(t)/L_v(t)$ depends *only* on the product wage and may be written $\varphi[W(t)]$. Now returning to (1) we see

$$\begin{aligned} Q(t) &= \sum_{v=-\infty}^{t} Q_v(t) = \sum_{v} L_v(t)F(\varphi[W(t)], 1) \\ &= F(\varphi[W(t)], 1) \sum_{v} L_v(t) \qquad (3) \\ &= F(\varphi[W(t)], 1)L(t) \\ &= F(\varphi L, L) \end{aligned}$$

But since $\varphi[W(t)] = \mu(v)K_v(t)/L_v(t)$, it follows that

$$L(t) = \sum_{v} L_v(t) = \frac{1}{\varphi}\sum_{v} \mu(v)K_v(t) = \frac{1}{\varphi}J(t) \quad (4)$$

where $J(t) = \sum_{v=-\infty}^{t} \mu(v)K_v(t)$ is a kind of "productivity-corrected" stock of capital at time t. Combining (3) and (4), we have

$$Q(t) = F(\varphi L, L) = F[J(t), L(t)] = F\Big[\sum_{v=-\infty}^{t} \mu(v)K_v(t), L(t)\Big], \quad (5)$$

which says that we can read off total output by inserting in the single-vintage production function the total input of labor and a suitably weighted sum of surviving capital inputs of various vintages.[6] (The weights are just the productivity factors.)

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[6] This simple result depends crucially on the assumption that the full substitution possibilities of capital remain unimpaired even after it has been built. I propose to take up the opposite case in a later paper.

The CES Production Function

For statistical purposes it is necessary to specialize the production function to some particular functional form. The three-parameter family of production functions with constant but arbitrary elasticity of substitution can be written

$$Q = \gamma[\delta K^{-\rho} + (1 - \delta)L^{-\rho}]^{-1/\rho} \tag{6}$$

We have dubbed this the class of CES production functions. The constants γ, δ, and ρ may be called the efficiency, distribution and substitution parameters, respectively. As has been shown, the elasticity of substitution between capital and labor along the production functions (6) is given by $\sigma = (1 + \rho)^{-1}$.[7] Thus the Cobb-Douglas function is the special case with $\rho = 0$, in which case the exponents are δ and $(1 - \delta)$; this accounts for calling δ the distribution parameter.

Since (6) is nonlinear in its parameters, direct estimation from time series of Q, K and L, would be a complicated job. If, however, (6) is written

$$Q^{-\rho} = \gamma^{-\rho}[\delta K^{-\rho} + (1 - \delta)L^{-\rho}],$$

it is seen that an outside estimate of ρ will permit the other two parameters to be estimated by straightforward methods. The problem then is to find a convenient estimate of the elasticity of substitution. Now it has been shown[8] that for observations along any production function, the elasticity of Q/L with respect to W at any point will be equal to σ, provided the observations were generated by profit maximization in competitive labor and product markets, and provided the "price" of value-added is the same in all markets—which implies that the cost of capital varies, since the wage does. Since σ is constant along (6), it follows that an estimate of σ can be obtained as the slope of a regression of $\log Q/L$ on $\log W$ from observations generated under the circumstances described.

In fact, putting the product-wage W equal to the marginal productivity of labor from (6), one easily calculates

$$\log \frac{Q}{L} = \frac{1}{1+\rho} \log W + \log [\gamma\rho(1 - \delta)^{-1}]^{\frac{1}{1+\rho}} \tag{7}$$

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[7] See K. Arrow *et al.* in *RES*, August 1961.

[8] *Ibid.*

In our article on capital-labor substitution,[9] we took the leap of applying (7) to observations arising from the same manufacturing industry at about the same time in different countries. We were forced to conclude, on the basis of other data not used in (7) that the same industry in different countries could not honestly be treated as operating along the same production function, but that it is a fair working hypothesis that international differences in production functions are concentrated in the (uniform!) efficiency parameter γ. In this paper I propose to estimate (7) from observations pertaining to the same two-digit manufacturing industry in the same year in the nine different census regions of the United States. Since I do not have the additional information (mainly regional stocks of capital) necessary to test the hypothesis that observations thus generated come from a common production function, I am simply assuming that they do. This is risky, of course; but I hope I am justified in assuming that technological conditions among regions of the United States are substantially more homogeneous than among countries of the world in widely different phases of economic development.

It is to be noted that, because of the reduction (5), I am entitled to ignore the differing vintage structures of the capital stock in a given industry across regions at a single point of time.

I can hope, then, from the interregional cross-section analysis of a given industry, to estimate the parameter $\sigma = 1/(1 + \rho)$, and therefore $\rho = 1 - (\sigma/\sigma)$, along with the constant $[\gamma\rho(1 - \delta)^{-1}]^{\frac{1}{1+\rho}}$. Next, in another fit of recklessness, I propose to use the estimate of ρ thus obtained in a time series analysis of data for the same industry, aggregated across regions for the whole United States.

Specializing (5) to the CES production function (6), we have

$$Q(t) = \gamma[\delta J(t)^{-\rho} + (1 - \delta)L(t)^{-\rho}]^{-1/\rho} \qquad (8)$$

whence, as before,

$$Q(t)^{-\rho} = \gamma^{-\rho}[\delta J(t)^{-\rho} + (1 - \delta)L(t)^{-\rho}]. \qquad (9)$$

Here $J(t) = \sum_{v=-\infty}^{t} \mu(v)K_v(t)$ as in (5). From (9) we note:

$$Q(t)^{-\rho} - \gamma^{-\rho}(1 - \delta)L(t)^{-\rho} = \gamma^{-\rho}\delta J(t)^{-\rho}. \qquad (10)$$

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[9] *Ibid.*

Call the left-hand side of (10) $R(t)$. The critical thing is that, given time series for $Q(t)$ and $L(t)$, we can "produce" a time series for $R(t)$. This is because the cross-section analysis has provided an estimate of ρ, so we can generate $Q^{-\rho}$ and $L^{-\rho}$ from Q and L. In addition, from an estimate of the constant term in (7), together with an estimate of ρ we can compute an estimate of $\gamma^{-\rho}(1 - \delta)$. Thus if we rewrite (10)

$$R(t) = \gamma^{-\rho}\delta J(t)^{-\rho} \tag{10'}$$

we can create an estimated time series for $R(t)$. With $J(t)$ we are not so lucky, since it depends on the unknown productivity-increase function μ. But a final burst of assumptions will do the trick.[10]

Write out (10′) in detail:

$$R(t) = \gamma^{-\rho}\delta \left[\sum_{v=-\infty}^{t} \mu(v)K_v(t) \right]^{-\rho} \tag{10''}$$

From (10″) we have

$$R(t)^{-1/\rho} = \gamma\delta^{-1/\rho} \sum_{v=-\infty}^{t} \mu(v)K_v(t) \tag{10'''}$$

and put the left-hand side, a time series which we can also estimate, equal to $Z(t)$. It will be recalled that $K_v(t)$ stands for the stock of capital of vintage v surviving in year t. Suppose that the survival curve for capital in this industry is exponential so that

$$K_v(t) = (1 - \theta)^{t-v}K_v(v) = (1 - \theta)^{t-v}I(v)$$

where $I(v)$ is gross investment in year v. This amounts to the assumption of a constant rate of mortality θ for capital, so that the average length of life[11] is $1/\theta$. Assume also that $\mu(v) = (1 + \mu)^v$, so that the productivity of capital increases geometrically at rate 100 μ per cent per year. Inserting all these assumptions in (10′′) yields:

$$Z(t) = \gamma\delta^{-1/\rho}(1 - \theta)^t \sum_{v=-\infty}^{t} \left(\frac{1+\mu}{1-\theta}\right)^v I(v)$$

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[10] From here on the reasoning is very much like that in my "Investment and Technical Progress."

[11] It is a weakness that the durability of capital is assumed to be technically fixed and unaffected by the process of technical change.

Finally

$$\Delta Z(t) = Z(t) - Z(t-1) = \gamma\delta^{-1/\rho}(1-\theta)^{-1}(1+\mu)^t I(t) - \frac{\theta}{1-\theta} Z(t) \quad (11)$$

whence

$$\frac{\Delta Z(t) + \frac{\theta}{1-\theta} Z(t)}{I(t)} = \gamma\delta^{-1/\rho}(1-\theta)^{-1}(1+\mu)^t$$

or

$$\frac{Z(t) - (1-\theta)Z(t-1)}{I(t)} = \gamma\delta^{-1/\rho}(1+\mu)^t \quad (12)$$

Once again, given a trial value of θ, the left-hand side of (12), say $Y(t)$, can be estimated as a time series from what has gone before and a time series of gross investment in the industry. A linear regression of the logarithm of $Y(t)$ on t should then yield:

$$\log Y(t) = \log(\gamma\delta^{-1/\rho}) + t \log(1+\mu), \quad (13)$$

and the estimated constants of this line (together with the assumed value of θ and all the previous by-products) make it possible to wind up with estimates of γ, δ and μ which are the only outstanding parameters.[12]

Cross-Section Analysis: Data

For the cross-section estimate of equation (7) I have used data extracted from the 1956 *Annual Survey of Manufactures* (see Table 1). The year 1956 was chosen as a year of relatively high employment with little excess capacity; because it followed a similar year, one might hope the data would not represent a period of rapid transition. Of course (7) is an equilibrium relation and one can never hope to have observed an economy in full equilibrium. Relative tranquillity is the most one can ask for. As a measure of output for each industry, I have used value added. The labor input figures represent the total number of employees. These are not corrected for hours worked, since this information is avail-

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[12] In principle one could repeat this process for different trial values of θ, and the best-fitting version of (13) taken as giving final estimates of the other parameters *and* θ. My assistant Henry Wan is of the opinion that θ ought to be estimated exogenously, since it is not integral to the model being studied and should not be used to absorb error. This seems to me to be a strong argument and, were data available, worth following up.

TABLE 1
VALUE ADDED AND TOTAL PAYROLLS PER EMPLOYEE, BY INDUSTRY AND REGION, 1956

Industry (Two-digit Census Categories)			Census Regions[a]								
			NE	MA	ENC	WNC	SA	ESC	WSC	M	P
20	Food	A.	8,081	9,444	9,844	9,567	7,722	9,752	8,425	8,772	10,798
		B.	3,814	4,294	4,403	4,348	3,253	3,557	3,418	3,858	4,448
21	Tobacco	A.	5,802	6,318	10,221		14,732	16,399	2,999		
		B.	2,198	2,638	2,981		3,110	3,463	1,459		
22	Textile	A.	5,562	5,920	6,960	6,012	4,786	4,568	4,188		7,424
		B.	3,475	3,620	3,835	3,242	2,895	2,773	2,632		3,953
23	Apparel	A.	4,508	5,102	4,739	4,301	3,832	3,495	3,792	4,592	5,430
		B.	2,615	3,042	3,117	2,742	2,406	2,147	2,352	2,618	3,190
24	Lumber	A.	4,484	5,175	5,534	5,417	3,690	3,880	4,064	6,839	7,979
		B.	3,122	3,300	3,397	3,276	2,193	2,215	2,456	3,879	4,670
25	Furn. and fixtures	A.	6,273	7,127	7,380	6,903	5,732	5,022	5,616		7,432
		B.	3,595	3,957	4,204	4,019	3,185	3,079	3,197		4,054
26	Paper	A.	8,815	8,639	9,399	10,572	11,343	10,067	11,940	12,696	12,547
		B.	4,528	4,541	4,824	4,488	4,418	4,615	4,749	4,978	5,011
27	Printing, publ.	A.	7,700	9,977	8,773	7,634	8,039	7,744	7,481	7,120	8,479
		B.	4,519	5,117	5,087	4,274	4,611	4,361	3,992	4,223	4,939
28	Chemicals	A.	12,945	16,025	15,370	15,231	15,174	13,289	23,501	12,086	14,870
		B.	4,863	5,181	5,335	4,501	4,794	4,904	5,358	5,762	5,391

29	Petroleum	A.	12,817	14,249	20,616	17,386	10,709	16,846	19,938	18,110	16,486
		B.	5,492	5,836	6,134	5,403	4,770	4,875	6,003	5,543	5,946
30	Rubber	A.	7,933	9,389	8,621	12,456	7,803	13,274	13,721	13,719	9,450
		B.	4,405	4,631	5,000	4,668	3,803	5,000	4,609	4,965	4,863
31	Leather	A.	4,717	4,868	5,908	5,530	4,643	5,725	4,305	8,807	6,457
		B.	3,168	3,123	3,480	2,828	2,901	2,644	2,393	4,551	3,329
32	Stone, clay, glass	A.	8,053	8,986	9,688	10,140	7,645	8,449	10,608	10,311	8,610
		B.	4,427	4,565	4,656	4,326	3,714	3,571	3,953	4,149	4,419
33	Primary metals	A.	9,545	9,817	10,584	9,384	10,101	10,674	11,697	17,160	11,749
		B.	5,110	5,131	5,409	4,760	5,158	4,824	4,807	5,370	5,277
34	Fabricated metals	A.	7,759	7,967	8,866	8,925	8,258	7,489	7,818		9,181
		B.	4,409	4,518	4,923	4,549	4,104	4,027	4,448		4,838
35	Machinery, nonelec.	A.	8,641	9,083	9,730	8,705	7,775	9,590	10,617	9,491	9,080
		B.	4,906	5,169	5,424	4,667	4,449	4,510	4,974	4,856	5,126
36	Elec. machinery	A.	7,699	8,647	8,576	7,854	8,365	9,286	8,453	5,796	8,230
		B.	4,038	4,658	4,679	4,356	4,351	3,638	4,132	3,868	4,734
37	Transportation equip.	A.	8,740	8,880	10,015	9,155	9,383	9,115	6,156	8,301	9,050
		B.	5,290	5,468	5,374	5,073	5,139	4,592	5,178	5,399	5,796
38	Instruments	A.	7,814	9,599	8,821	9,568	5,596	6,690		6,069	9,141
		B.	4,319	5,212	4,774	4,935	3,718	4,199		3,832	5,152

SOURCE: *Annual Survey of Manufacturers*, 1956.

[a] The regions are: New England (NE), Middle Atlantic (MA), East North Central (ENC), West North Central (WNC), South Atlantic (SA), East South Central (ESC), West South Central (WSC), Mountain (M), and Pacific (P).

A = Value added per employee.

B = Payroll per employee.

able only for production workers; it would have been possible to use the production-workers manhours and payrolls, but the more inclusive data seemed to give a better fit. Finally, as a wage indicator I have used total payrolls per employee. Thus for each census region in each industry, the first row gives value added per employee and the second gives total payrolls per employee. Most of the industries have observations for each of the nine census regions. In a few cases one or more regions are missing because not all two-digit manufacturing industries are sufficiently dispersed.[13]

Cross-Section Analysis: Results

We can rewrite (7) simply as

$$\log \frac{Q}{L} = a \log W + b; \qquad (7')$$

a and *b* were estimated for each industry by a regression of value added per employee on payrolls per employee across regions. Table 2 summarizes the results.

In reading Table 2 one must remember that each regression line is based on nine or fewer observations so the usual "two-standard-error" rule does not hold. To give some idea of the sampling properties of the statistics, it is enough to know that the 95 and 97.5 percentiles of the t distribution with 7 degrees of freedom are 1.895 and 2.365 respectively. Thus, for instance, in testing a hypothesis on the slope of one of the regressions the acceptance region is about 1.895 or 2.365 standard errors on either side of the hypothetical regression slope, depending on whether the level of significance is 10 per cent or 5 per cent. Another useful critical value to keep in mind is this: for 9 observations, the 95 and 97.5 percentiles of the sampling distribution of the correlation coefficient (when the true parent correlation is zero) are .582 and .666. To reject the hypothesis that value added per employee and payrolls per employee are uncorrelated takes a sample correlation coefficient of .582 or .666 at the 10 per cent and 5 per cent levels, if the test is against a symmetrical alternative. In the present context, where the only plausible alternative to zero correlation would

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[13] In many respects it would have been better to conduct the analysis on a state-by-state basis. I chose the larger geographical unit both because I was hurried and wished to save hand computation time and because on a state basis there would presumably be more variation among observations in product-mix within the broad two-digit industries.

TABLE 2

REGRESSIONS: VALUE ADDED PER EMPLOYEE ON PAYROLLS PER EMPLOYEE[a]

	Industry	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
20	Food	7	.6905	.2221	1.2680	.5799	.0702	0.4482	0.1594	.1118
21	Tobacco	4	1.9633	.2950	.2514	.9172	.1889	−0.4906	0.8799	.2865
22	Textile	6	1.2668	.1466	.2180	.9256	.0556	−0.2106	0.8419	.1434
23	Apparel	7	1.0075	.1308	.4877	.8944	.0476	−0.0074	0.6163	.1285
24	Lumber	7	.9928	.0930	.5070	.9421	.0633	0.0073	0.6001	.2404
25	Furniture & fixtures	6	1.1216	.1050	.4049	.9500	.0329	−0.1084	0.6970	.1183
26	Paper	7	1.7736	1.0058	−.3790	.3076	.1232	−0.4362	1.2383	.0433
27	Printing, publ.	7	1.0237	.2061	.5359	.7789	.0482	−0.0232	0.5924	.0827
28	Chemicals	7	.1410	.9476	2.4862	.0032	.1914	6.0922	0.0000	.0714
29	Petroleum	7	1.4536	.7121	.2876	.3731	.1698	−0.3121	0.8205	.0843
30	Rubber	7	1.4773	.8769	.0767	.2885	.2030	−0.3231	0.9494	.0819
31	Leather	7	.8911	.2657	.6708	.6164	.1362	0.1222	0.4710	.1813
32	Stone, clay, glass	7	.3200	.4592	1.7516	.0649	.1136	2.1250	0.0043	.0875
33	Primary metals	7	1.8697	1.2475	−.6537	.2429	.1611	−0.4652	1.4185	.0457
34	Fabricated metals	6	.8009	.2950	.9130	.5512	.0512	0.2486	0.3198	.0656
35	Machinery, nonelectrical	7	.6348	.4492	1.2074	.2219	.0782	0.5753	0.1493	.0615
36	Machinery, electrical	7	.3735	.5371	1.5434	.0646	.1310	1.6774	0.0160	.0862
37	Transportation equipment	7	.0586	.8217	2.0646	.0007	.1398	16.0649	0.0000	.0602
38	Instruments	6	1.5945	.1452	−.3437	.9526	.0471	−0.3728	8.6322	.1227

SOURCE: Calculated from Table I.

[a] Col. 1 equals the number of degrees of freedom (two less than the number of regions present); col. 2, the estimated value of a; col. 3, its standard error; col. 4, the estimated value of b; col. 5, the squared correlation coefficient; col. 6, the standard error of estimate; col. 7, the value of ρ corresponding to the estimated a; col. 8, the value of $\gamma^{-\rho}(1 - \delta)$ corresponding to the estimated a and b (see equation (10)); and col. 9, the sample standard deviation of the independent variable, in this case the annual wage or payroll per employee. The reason for presenting this last statistic is made clear in the text.

seem to be positive correlation, the same critical values for r give 5 per cent and 2.5 per cent levels of significance instead.

With these benchmarks before us, we can look at the goodness of fit of the cross-section regressions. Remember that column (5) of Table 2 gives r^2, not r. Of the nineteen two-digit industries, we can classify six (Nos. 21–25, and 38) as giving excellent fits. Another four (Nos. 20, 27, 31, and 34) fit less well, but with correlations which are clearly statistically significant. There are four complete failures (Nos. 28, 32, 36, 37). This leaves five industries (Nos. 29, 26, 30, 33, 35 in descending order of goodness of fit) with correlations like .6108, .5546, .5371, .4929, .4711, which are just on the borderline of statistical significance. With due skepticism of the results, it seems worthwhile to carry this last batch of industries on through the next stage of the analysis. So for the later time-series calculations there will be fifteen interesting industries left.

It is in general bad policy to seek good excuses for poor results. The hardy econometrician must learn to take his lickings in the conscious realization that "you can't win 'em all." That is surely the proper attitude in the present case. The model is after all a pretty bold simplification. Two-digit manufacturing industries are heterogeneous enough so that interregional differences in product-mix may still be important. I am fitting an equilibrium relation to data generated in the midst of an investment boom. There are very few observations. And besides, the results are not so bad. Still I am tempted to produce one possible excuse for those industries which give low correlations.

Note that my whole method depends on the existence of interregional differences in wage levels. If wage rates were approximately the same in all regions, then the theory maintains that value added per employee ought also to be the same in all regions. There would surely be minor differences in value added per head, if only on account of weather, product-mix and error of measurement. But if there were no wage differentials (or very small ones), I would get approximately zero correlations. It is interesting, then, that the industries with low correlations are almost uniformly those for which the sample standard deviation of the (log) wage variable is lowest. The eight industries for which that standard deviation is greater than .1000 are all among the satisfactory cases; the eight worst-fitting industries have with one or two exceptions the least variation in wages. I am curious about what distinguishes the industries with little interregional wage variation from those with much. One's first inclination is to check the prevalence of industry-wide collective bargaining, or especially mobile labor forces. It is true that the little-wage-variation group does include some industries with strong national trade unions (such as primary metals, transportation equipment, nonelectrical machinery, electrical machinery, and rubber). But this is a subject for more than casual observation.[14]

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[14] I note that a simple rank correlation of the interregional standard deviation of wages with the fraction of production workers organized in each industry yields a rank correlation coefficient of —.75, clearly significant.

Cross-Section Analysis: Interpretation

If the cross-section data of Table 1 are generated under ideal conditions—that is, if they represent an approximation to competitive profit-maximizing equilibrium along a common production function—then the regression slopes given in column 2 of Table 2 ought to be estimates of the elasticity of substitution between labor and (efficiency-corrected) capital in each industry. That is quite an "if," and I would not wish this investigation to be thought of as more than a reconnaissance (or maybe a raid!). I must warn the reader that even a very good fit in Table 2 is not powerful evidence in favor of the intended interpretation. I have already mentioned one reason: even if the interpretation were right, the absence of interregional wage variation would make the statistical analysis meaningless. There is a more obvious reason: my argument runs in terms of the observed figures for net output per head being a response to a market-given wage, but a strong positive correlation might alternatively be interpreted as a wage differential which arises because of intrinsic interregional productivity differentials. The source of the productivity differentials might be almost anything—differences in effective production functions, differences in product-mix, differences in the age, sex or educational composition of the labor force. The only way in which I could hope to clinch my (hoped-for) interpretation would be to produce some kind of regional data on stocks of real capital and to show that the output, labor, and capital observations actually fall along a production function with the appropriate elasticity of substitution. Of course, no such capital data exist; and even if the usual sort of stock data were available, they might not measure the productivity-corrected concept I have used at the end of Section 2 of this paper.

Nor is it easy to test the estimated elasticities of substitution for consistency with the observed pattern of distributive shares in each industry over time. In theory, the more rapidly growing input ought to be imputed a decreasing or increasing relative share of the net output of an industry according as the elasticity of substitution between inputs is less than or greater than one. The case of unit elasticity of substitution is the famous Cobb-Douglas case of constant relative shares. The ratio of total payrolls to value added is easily obtainable for each industry in the period 1947, 1949–58, from the Censuses and Annual Surveys of Manufactures, and these are reproduced in Table 3. It may not be certain in every case which was the rapidly growing input—labor services or the services of a productivity-corrected stock of capital. Still it is probably a safe bet that over this decade of investment boom the capital factor increased more rapidly than the labor factor; this would be true without the correction for technological progress and even more strongly with it. Even so, it is not all clear sailing. Given the intrinsic sluggishness of a time series of relative shares and the fact that its business-cycle variation is likely to be as great as or even greater than the long-term component of its decade movement, I would hesitate to read anything out of Table 3.

It is interesting that the two industries for which Table 2 makes the strongest claim of a greater-than-unit elasticity of substitution, namely tobacco and instruments, do turn out in Table 3 to give evidence of a *declining* wage share. On the face of it, this is in conformity with the interpretation of the cross-section slopes as elasticities of substitution. The situation with less-than-unit estimates for the elasticity of substitution is, if possible, even less definite, mainly because columns 2 and 3 of Table 2 do not exhibit any slopes which are significantly less than unity. But I must strongly emphasize that I am not arguing, nor do I believe, that short-run distributive phenomena are to be explained in terms of long-run equilibrium behavior along neoclassical lines. I conclude then that Table 3 can provide no strong evidence either for or against the neoclassical interpretation of the cross-section results.

If, for the sake of the argument, we interpret the cross-section regression slopes as estimates of elasticities of substitution, they turn out sharply to contradict the empirical results obtained in our earlier study.[15] In that paper most of the estimates proved to be significantly less than unity, and none of the elasticities of substitution gave any indication of being greater than one. Also, the statistical fits (to approximately contemporaneous census data from eleven to nineteen different countries) were all very good. By contrast, in Table 2 at most ten of the nineteen regressions have a good fit. Even more surprisingly, the interregional cross-sections are not characterized by uniformly less-than-unit elasticities of substitution. Of the ten good fits in Table 2, three give point estimates of the elasticity of substitution which are less than one,

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[15] Arrow *et al.* in *RES,* August 1961.

TABLE 3
Wages as Percentage of Value Added

Industry	1947	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958
20 Food	41.98	44.54	43.70	45.55	44.96	44.11	46.28	45.56	45.30	44.58	44.70
21 Tobacco	32.09	26.69	26.36	26.88	27.74	25.67	26.29	25.11	23.73	23.04	20.85
22 Textile	53.10	62.71	59.61	63.42	63.59	63.84	63.86	61.25	62.19	61.44	59.14
23 Apparel	56.89	64.00	66.21	62.89	63.50	62.02	62.21	62.27	62.20	61.38	59.93
24 Lumber	53.56	62.24	55.21	57.20	60.06	59.61	60.64	59.91	62.26	64.02	64.08
25 Furniture and fixtures	59.81	61.07	60.76	59.70	58.96	61.53	60.89	59.41	57.78	58.05	59.85
26 Paper	44.55	51.12	46.76	43.60	48.35	48.85	48.41	47.86	47.72	48.46	49.24
27 Printing and publishing	53.34	58.90	59.28	58.01	57.72	57.24	57.87	56.67	55.73	55.68	57.16
28 Chemicals	35.61	35.78	32.36	34.10	36.50	36.48	36.08	32.90	33.73	33.76	32.93
29 Petroleum[a]											
30 Rubber	60.13	59.71	52.49	55.66	59.75	56.40	55.65	55.65	54.92	55.03	52.92
31 Leather	56.99	64.31	63.33	64.02	63.58	64.22	62.74	62.43	62.11	61.87	61.16
32 Stone, clay. glass	52.49	53.99	48.74	51.33	52.15	51.93	50.71	48.05	48.02	48.95	46.83
33 Primary metals	62.35	60.67	52.29	52.59	53.38	54.54	53.82	50.20	49.58	53.73	54.92
34 Fabricated metals	57.56	59.65	54.81	55.87	57.53	58.52	57.89	57.87	57.38	57.70	57.86
35 Machinery, nonelec.	61.50	59.30	57.77	59.98	57.62	58.86	58.28	59.49	59.47	58.60	57.52
36 Machinery, elec.	58.32	54.87	52.60	55.50	54.56	56.18	53.37	55.74	56.42	54.62	54.53
37 Transportation equipment	53.37	58.10	54.75	61.98	61.64	61.83	59.57	56.56	59.99	57.50	58.70
38 Instruments	61.59	60.82	58.50	62.66	59.15	56.81	56.39	56.79	57.34	57.64	55.31

[a] Not calculated.

three or four, are greater than one, and three or four are essentially equal to one. Only three of the estimated elasticities are significantly different from one at the 90 per cent level (two-sided test) and all three of these are on the greater-than-one side.

I cannot now give a completely satisfactory reconciliation of the two sets of empirical results, but I can suggest lines along which I think an explanation might be found. In the first place, so far as goodness of fit is concerned, the earlier observations come from a list of countries which included the United States and Canada at the high-wage end and Ceylon, India and Iraq at the low-wage end. Within any one industry the range of wage rates was always very wide, the highest running at least ten or twenty times the lowest. Within my interregional samples, the wage variation is much smaller; never in any industry is the highest wage as much as twice the lowest and almost always the range is much narrower. I have already mentioned that the lowest-wage-differential industries tend to be the ones for which the regression results are poorest. The point of analyzing interregional cross-sections is the chance that technology is much more homogeneous across such regions than across countries at widely different levels of development.[16]

Another difference between the earlier and the present results may cast some light on the size of the estimated elasticities. In the "capital-labor substitution" paper, we analyzed selected three-digit manufacturing industries (in the International Standard Industrial Classification); I have dealt with two-digit industries here. It seems plausible that, in general, elasticities of substitution should be smaller the more narrowly defined the industrial classification, and larger the higher the degree of aggregation. The reason is fairly obvious: one can imagine subindustries each of which has zero elasticity of substitution but which when aggregated exhibit highly variable factor proportions because the mix of subindustries is different in different regions. One would expect to find highly labor-intensive (low value added per employee) subindustries concentrated in low-wage regions, and conversely. Another way to

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[16] It probably bears repeating that in ACMS it turned out that the data could be explained on the hypothesis of neutral differences in technology among countries. I would not even go so far as to believe that census regions of the U.S. share an identical technology within any industry. But I believe it is important to know how much of observed interregional differences are differences in technology and how much are adaptation to differing price structures.

see this point is to recognize that shifting the product mix is one way in which substitutions between labor and capital takes place; hence the more aggregated the industry the more plentiful the opportunities for substitution. Since the industrial classification in the earlier paper was finer than the one I have used here, it is perhaps natural that the elasticities of substitutions should have been systematically smaller.

This proposition could be checked in the obvious way: by replicating the cross-section regressions for each of the three-digit subindustries within a two-digit industry. It should turn out that the two-digit elasticity of substitution exceeds an average of the three-digit components. I have not had the time to perform such a check; but inspection of a few scatter diagrams suggests that it may indeed work that way. Further research is required before one can say how consistent the present results are with the earlier ones, but I think they might actually make a sensible pattern.

Here is perhaps the place to mention two other directions in which the analysis might conceivably be pushed—neither one led to anything in preliminary trials. First, the cross-section analysis could be done on a state-by-state basis. This would have two advantages: it would increase the number of observations in each regression, and it would increase the variability of the independent variable, the annual wage. There is a disadvantage too: the differences between one state and another are more likely to reflect simple product-mix differences than those between one census region and another. In any case, a few scatter diagrams were drawn up on the state basis; there is only a slight increase in interregional wage variability (not surprising, since census regions are fairly homogeneous blocks of states) and the promised improvement in fit was not great enough to justify rerunning the regressions.

Secondly, one of the serious weaknesses in the theoretical justification of this method is the assumption that observations represent situations of equilibrium. In the hope of dodging this difficulty, the previous year's wage was introduced as a second independent variable in each regression. But the intercorrelation between 1955 and 1956 wages across regions was in nearly all cases so high that the results were uninterpretable. There were cases in which using the 1955 wage (either alone or in combination with 1956) gave a noticeably higher fit than when only the 1956 wage was used. It seems entirely too *ad hoc* to do anything under the circumstances but discard the results using 1955 wages. Still, a more careful attack on this problem ought to consider some of the standard ways of formulating a disequilibrium model.[17]

Time Series Analysis: Data

To carry out the time series analysis suggested in equations (8) to (13), I need statistical counterparts of $Q(t)$, $L(t)$ and $I(t)$ for each industry. At first I intended to use the Commerce series for income originating, equivalent full-time employees, and capital expenditures, with appropriate deflation where necessary. But the published figures for capital expenditures group together a number of the two-digit industries into an other durables and an other nondurables category, and I was refused access to the disaggregated breakdowns. So I returned to Census data. They have the advantage of being on an establishment basis, which is probably best for an attempt to estimate production functions. But continuous series are available only for the decade 1949–58 and this is a very short period. Production functions for industries cannot be expected to describe short-run behavior faithfully, because year-to-year variations may be dominated by cyclical considerations—shortness of time, idle capacity, unexpected changes in output and other disequilibrium phenomena. So long as technological change is given an explicit place in the analysis, it would seem that long time series are more likely to give sensible results. This analysis of a single postwar decade must be thought of as a pilot study.

For $Q(t)$ I have taken, from the Annual Surveys and Censuses of Manufactures, value added deflated by an index prepared by Charles Schultze.[18] The Schultze index is designed for deflating income originating rather than value added, but I did not attempt to make the necessary adjustments.

For $L(t)$, corresponding to the cross-section analyses, I have

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[17] There is another aspect of the situation which has become clearer to me since this paper was written. As mentioned earlier, the model assumes that price is uniform across regions while wage and capital costs vary. For some commodities it may be more plausible to assume that capital costs are the same, while both price and wage vary. Then the results require a quite different interpretation, and it may be necessary to deflate by an interregional price index.

[18] See Charles L. Schultze and J. Tryon: *Prices and Costs in Manufacturing Industries*, Study Paper 17, U.S. Congress, Joint Economic Committee, Washington, 1960. For present purposes the index was shifted to 1954 = 100.

used the Census figure for total employees. This series is no doubt deficient for several reasons. Presumably it does not correctly reflect inputs of labor services because of short-run fluctuations in hours worked. And it is further distorted by the grouping together of production workers and supervisory workers, subject as these classes are to sharply divergent trends. I use total employees rather than production worker man-hours for two reasons: greater inclusiveness, because to omit nonproduction workers altogether would be to omit an important input and component of value added; and comparability with the cross-section analyses.

Finally, a statistical counterpart of $I(t)$ has been put together out of the annually reported estimates of expenditures on plant and equipment in the Annual Surveys and Censuses of Manufactures. The plant expenditures and equipment expenditures were deflated separately, using the corresponding implicit deflators from the GNP accounts, and the deflated figures were added together to give $I(t)$.[19]

These three time series are reproduced for each two-digit industry in Table 4. For carrying out the analysis described in the third section of this paper, the only other inputs required are estimated values of the parameters ρ, $\gamma^{-\rho}$ $(1 - \delta)$, and θ. The first two of these are taken from the cross-section analysis and have already been recorded in columns 7 and 8 of Table 2. As for θ, the mortality rate on plant and equipment combined, trials were made with $\theta = .10$ and $.05$; that is, with average lifetimes of ten and twenty years for capital equipment.

Time Series Analysis: Results

Figure 1 shows scatter diagrams of log Y_t against time (see equation (13)) for fifteen of the nineteen industries. The remaining four comprise two in which computational difficulties could not be overcome, and two in which the results were negative. In principle, the slope of the regression line should be for each industry an estimate of $\log_e (1 + \mu)$. (And since μ is a small number, the slope should be approximately μ or a bit higher.)

One major difficulty appears immediately: in every instance, the

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[19] The capital expenditures for 1958 are not yet available in the two-category breakdown. For that year, the total was deflated by an index combined of the two implicit deflators with weights obtained from interpolation between 1957 and 1959.

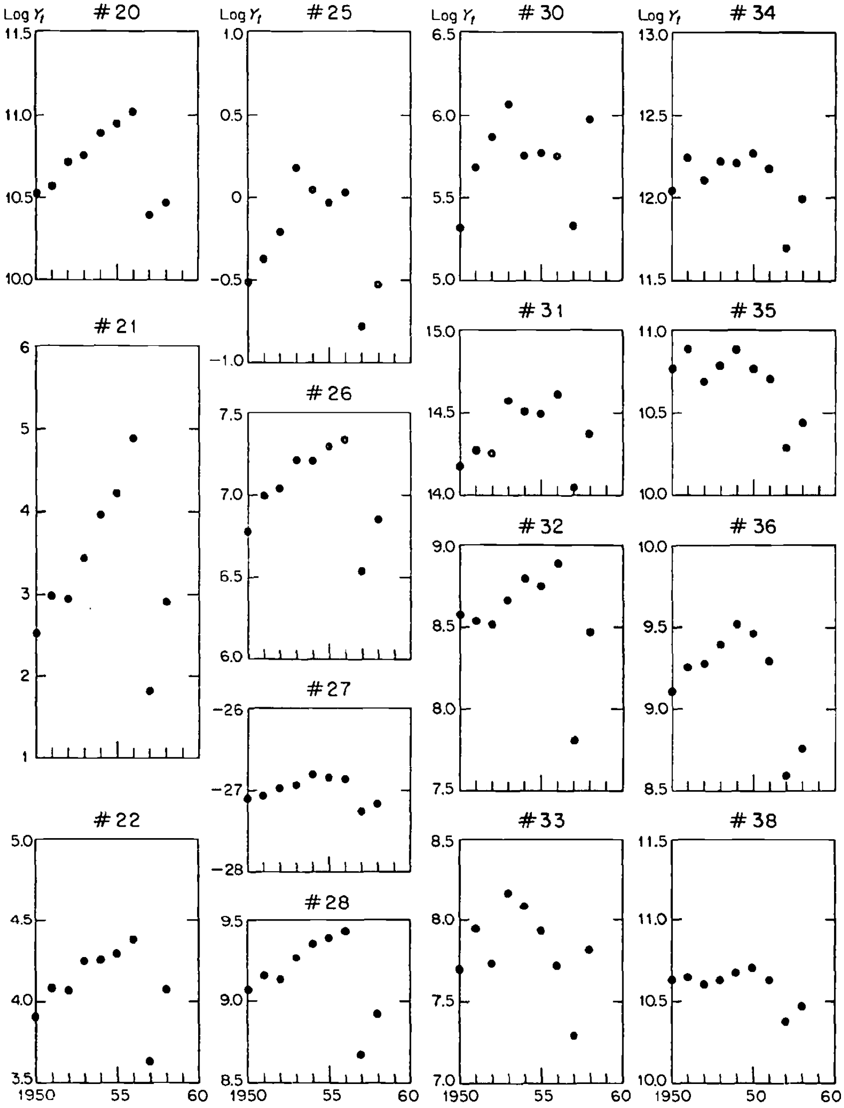

FIGURE 1.

last two points fall far below the rest of the scatter and completely spoil the picture. The reason for this is easy to find. The last two observations cover the period 1957–58. In most manufacturing industries these were recession years in which output increased only weakly (1957) or decreased (1958), while investment remained strong (1957) or at least positive (1958). The kind of model I am using can only explain this as a kind of sudden negative technological change. Of course we know better. We know that there is such a thing as idle capital. But the model, built on the assumption of free substitutability between hired labor and concrete capital goods, has no room for idle capital. The model thinks that labor is the variable factor, capital is an overhead, and let the quasi rents fall where they may. Much current speculation has it the other way: labor in fact being converted to an overhead while capital lies idle.

This is a difficulty faced by all models using a neoclassical production function, but it is troublesome only in the short run. If I had thirty or forty years of data, the business cycle dips would appear less substantial. It seems possible to construct a simple model which faces this problem squarely and is yet in the neoclassical spirit and I hope to describe how in a subsequent paper.[20]

Within the framework of this essay there are two possible ways to handle the situation. It would probably be preferable to try to adjust the last two (or indeed all) outputs in each time series to measure capacity rather than current output, and then to recompute. Lacking the time and facilities to do that, I must simply ignore the last two observations. And, rather than calculate linear regressions based on five degrees of freedom, I report in Table 5 the slope of a line drawn through the first and seventh observation in each scatter. These numbers are estimates of the annual rate of technological progress in each industry. They tend to be larger than we normally expect, but that is because they are not rates of increase of "output per unit of input." They are rates of "purely capital-augmenting" technical change, and would have to be divided in half or thirds to be converted into an approximate rate of uniform technical change.

It would be possible, from an estimate of the intercept of each line, to produce estimates of the other parameters of the production functions, γ and δ. Then, in turn, one could go back to the distributive facts and compare observed relative shares with those read off from the estimated production function. But it would be misleading to attempt this deeper analysis and interpretation without much longer time series, and I shall not try.

I report one puzzling fact. The whole procedure seems to be

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[20] Since published as "Substitution and Fixed Coefficients in the Theory of Capital," *Review of Economic Studies,* June 1962.

TABLE 4

Deflated Value Added, Number of Employees, and Expenditures on Plant and Equipment, by Industry

	Industry[a]	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958
20	Food										
Q		10,544	11,094	11,341	11,431	11,627	13,400	13,791	14,974	15,445	15,977
L		1,463	1,493	1,474	1,480	1,455	1,647	1,675	1,707	1,688	1,667
I		860	736	729	544	551	789	776	811	799	779
21	Tobacco										
Q		919	958	985	919	996	988	1,077	1,156	1,180	1,373
L		101	92	94	93	95	95	96	93	88	84
I		27	20	19	23	30	28	26	43	36	32
22	Textile										
Q		4,305	5,275	4,326	4,667	5,060	4,749	5,111	5,080	4,999	4,962
L		1,170	1,245	1,195	1,135	1,158	1,037	1,059	1,044	989	919
I		502	490	432	334	258	226	254	270	250	186
23	Apparel										
Q		4,589	4,694	4,643	4,902	5,382	5,147	5,632	5,650	5,802	5,798
L		1,161	1,151	1,123	1,143	1,227	1,190	1,248	1,271	1,264	1,168
I		64	73	74	56	61	77	83	80	92	90
24	Lumber										
Q		2,391	3,150	3,100	3,173	3,395	3,188	3,438	3,308	3,144	3,056
L		649	750	770	743	720	646	693	698	646	583
I		176	223	254	184	185	217	293	267	177	276
25	Furniture and fixtures										
Q		1,464	1,739	1,631	1,701	1,895	1,966	2,421	2,318	2,247	2,238
L		310	346	336	332	361	341	366	376	375	358
I		49	67	59	57	60	62	77	86	75	74

26 Paper										
Q	3,269	3,933	4,046	3,825	4,499	4,581	5,112	5,162	5,411	5,507
L	447	478	495	482	533	530	549	565	566	556
I	378	348	414	383	402	533	539	684	663	522
27 Printing and publ.										
Q	5,381	5,618	5,780	5,807	5,845	6,265	6,510	6,985	6,958	6,911
L	756	763	765	773	760	804	823	854	867	861
I	304	284	259	195	197	237	247	264	283	351
28 Chemicals										
Q	5,947	7,635	8,018	8,855	9,628	9,443	10,744	11,713	12,122	12,608
L	612	642	703	739	768	739	747	760	764	730
I	714	701	1,043	1,053	954	927	739	987	1,094	971
29 Petroleum	Not calculated									
30 Rubber										
Q	1,361	1,983	1,496	1,451	1,797	1,904	2,196	1,852	1,849	1,741
L	222	239	253	255	270	247	265	265	260	232
I	76	93	121	131	128	128	123	139	129	110
31 Leather										
Q	1,538	1,797	1,380	1,594	1,699	1,637	1,850	1,848	1,832	1,864
L	375	385	354	361	375	357	366	367	362	342
I	35	30	24	19	25	28	36	34	27	26
32 Stone, clay, glass										
Q	2,196	3,697	4,045	4,015	3,922	3,822	4,292	4,653	4,512	4,946
L	453	491	529	510	506	492	525	536	526	552
I	228	259	343	259	282	301	447	662	567	376
33 Primary metals										
Q	7,400	10,058	10,589	9,974	11,477	9,747	12,837	12,492	11,352	9,360
L	1,016	1,129	1,244	1,240	1,288	1,150	1,204	1,319	1,272	1,073
I	678	637	1,195	1,655	1,184	910	949	1,507	1,988	1,171

(continued)

TABLE 4 (concluded)

Industry[a]	1949	1950	1951	1952	1953	1954	1955	1956	1957	1958
34 Fabricated metals										
Q	5,772	7,105	7,374	7,348	8,535	7,596	8,337	8,291	8,359	7,939
L	872	989	1,035	1,008	1,118	1,019	1,093	1,102	1,114	1,038
I	276	368	376	338	428	432	444	454	456	391
35 Machinery, nonelectrical										
Q	9,226	10,272	11,847	13,281	14,251	12,339	13,217	14,354	13,589	12,485
L	1,295	1,368	1,604	1,651	1,691	1,542	1,605	1,718	1,707	1,549
I	418	392	669	637	691	714	634	810	858	645
36 Machinery, electrical										
Q	4,264	4,984	5,437	6,734	7,766	7,403	8,041	8,873	8,656	7,675
L	663	766	877	957	1,096	959	1,001	1,080	1,084	1,005
I	223	227	314	392	374	341	325	434	453	354
37 Transportation equipment										
Q	8,007	8,812	10,210	11,946	14,955	13,926	15,860	16,466	18,217	14,650
L	1,140	1,218	1,469	1,650	1,912	1,705	1,812	1,792	1,900	1,480
I	316	400	636	696	673	925	823	1,311	988	494
38 Instruments										
Q	1,325	1,529	1,571	2,091	2,234	2,129	2,264	2,403	2,516	2,492
L	205	226	253	279	285	273	283	297	307	291
I	69	75	90	74	91	94	103	132	126	105

SOURCE: Calculated from Annual Surveys of Manufacture and Censuses of Manufactures.

[a] Q = deflated value added.
L = number of employees.
I = deflated expenditures on plant and equipment.

TABLE 5
Rough Estimate of Rate of Technological Progress, by Industry

Industry Number		Crude Estimate of $\log_e (1 + \mu)$
20	Food	.0823
21	Tobacco	.3921
22	Textile	.0787
24	Lumber	essentially zero[a]
25	Furniture and fixtures	.0905
26	Paper	.0831
27	Printing and publ.	.0434
28	Chemicals	.0589
30	Rubber	.0736 (but see scatter)
31	Leather	.0730
32	Stone, clay, and glass	.0521
33	Primary metals	too irregular
34	Fabricated metals	.0325 (sixth observation, see scatter)
35	Machinery, nonelectrical	essentially zero
36	Machinery, electrical	.0389 (sixth observation, see scatter)
38	Instruments	essentially zero

[a] Omitted from Figure 1 since scatter is practically horizontal.

extremely insensitive to changes in the hypothetical rate of depreciation. The calculations were carried out on the alternative assumptions $\theta = .05$ and $\theta = .10$. The resulting figures for $\log_e Y_t$ differed only by one or two digits in the fourth decimal place.

Conclusion

I think it would be unwise to overinterpret these statistical results. It will take further research—and, most especially, longer time series—before one can know whether the figures in column 2 of Table 2 can be thought of as estimates of elasticities of substitution between labor and capital, and whether the figures in Table 5 have any merit at all as estimated rates of capital-augmenting technological progress. On the theoretical side, it would be interesting to see whether different and more satisfactory results could be obtained if it were not assumed that prices are uniform across regions. I would also like to state my complete agreement with a point raised several times in the discussion of this paper: that interregional differences reflect to an unknown extent differences in the quality of labor and the input of "social capital."

Despite these qualifications, all of which seem to be remediable, I believe that the method illustrated here holds some promise of elucidating the characteristics of production relations and their (partial) implications for distribution.

COMMENT

ROBERT EISNER, Northwestern University

At one point in this paper Mr. Solow suggests that his investigation be thought of as no "more than a reconnaissance (or maybe a raid!)." In either event it is a heroic effort; Solow is an able, as well as an intrepid scout, and the remarks that follow are not meant to suggest that had I been on the mission I would have fared better. But with the advantage of hindsight, we may be able to raise some useful points with regard to the role of the mission as well as the limitations of the intelligence it has brought back.

Solow is appropriately modest. He recognizes that "everything depends upon everything else," but he hopes, with data of "a few two-digit manufacturing industries" and a particular kind of production function, to estimate parameters which will indicate "elasticities of substitution between labor and capital, rates of technical progress, real rates of return on capital, and perhaps some other interesting parameters descriptive of technical conditions in some branches of manufacturing." "*Naturally*," Solow writes (the underscoring is mine), "I believe that information of this kind is vital to an understanding of distribution (and of other things) but I do not believe it to be the whole story." Not the "whole story" but, Solow suggests, it relates to the "production function *half* of the determinants of distributive shares." (Italics mine.)

I seem to recall that Marx used to inveigh against the view, attributed to "bourgeois economists," that the laws of economics which they discovered (and the laws of distribution in particular) were "natural." There is after all nothing in nature that suggests that individuals (or corporations) should "own" or have title to the services of factors of production and should then receive returns equal to the marginal product of these factor services times the quantity of such services that they supply. I indeed have some very serious doubts that there is much in modern capitalism, as we know it, to determine distributive shares on this basis.

NOTE: I am indebted to my colleague, Robert H. Strotz, for helpful reactions to a draft of this comment.

It is easy to develop an illusion (as a scout or an economist) that a function that looms large in a particular market, or under specific assumptions including certain parametric assumptions, explains something as well in a wider or more general equilibrium or under the particular parametric assumptions which apply to actual behavior. Most of us recall, from histories of economic thought if not personal experience, the fate of seemingly irrefutable views that the amount saved depended positively on the rate of interest or the propensity of individuals to abstain from consumption. The Keynesian revolution led to the conclusion not only that this depends on the "everything else" encompassed by the marginal efficiency of investment schedule and liquidity preference, but that for certain not unreasonable parameters of these latter functions saving might not depend at all upon abstinence or interest rates—or might even depend on them only in perverse fashion. With a cynicism born of these confusions—of another era, of course—I would not, as does Solow, so quickly dismiss Kaldor and Mrs. Robinson for ignorance of elementary economics in denigrating or denying the role of the production function in the determination of relative shares. Everything depends on everything else in the sense that the world must mesh, but perhaps it is production functions themselves, quantities and values of inputs, and the product-mix that adjust so that the distribution of output conforms to a system fully determined without the information with which Solow is concerned—just as our abstinence has been found to adjust to income as determined by investment demand and a common floor to all marginal efficiencies.

My own hunch is that much more will be learned about distributive shares by investigations that deal with indifference surfaces and "production functions" involving *probability distributions* of outputs and returns, so that equilibria refer to combinations of expected returns and magnitudes of risk. Our economic and political institutions would determine in large part the risks facing individuals in their choices of actions leading to income; preference functions, themselves perhaps socially conditioned, would then determine, jointly with the supply conditions of risk and expected income, the equilibrium shares of final output. Such investigations would leave full scope to roles for monopolistic imperfections, lack of perfect information, disequilibria made chronic by immobilities (bred in part by uncertainty and ignorance) of specific and heterogeneous factors in a world of change. But if these are the critical elements—in nations, regions and industries—technologies and inputs of factors are among the determined rather than the determinants of distributive shares.

But all this is to question the priorities to be established to this kind of reconnaissance. It will perhaps also constrain us sharply in our evaluation of the results, but to see these constraints we shall have to get down to particulars.

Solow has on another occasion referred aptly to the role of "crucial" assumptions. It will be sobering to note the assumptions underlying the present analysis.

1. Constant returns to scale (production function homogeneous of degree one).
2. Constant elasticity of substitution.
3. Neutral or "uniform" technological advance.
4. Homogeneous labor, equally adapted to all vintages of capital. (There is no technological unemployment in this model!)
5. The rate of technological advance varying only with the rate of addition of new capital.
6. The productivity of new capital independent of the quantities of old capital ($Q = \Sigma Qv(t) = \Sigma F_v[u(v)K(t), Lv(t)]$).
7. The same production function within each two digit manufacturing industry for all vintages of capital and for all regions.
8. Perfect competition in labor and product markets.
9. Observations representing points of equilibrium (points of profit maximization consistent with perfect competition).
10. A mortality rate for capital that is constant, in percentage terms, and the same for all vintages and all regions.
11. The productivity of capital increasing from vintage to vintage at a constant percentage rate, the same for all regions.
12. Arguments of the production function restricted to privately owned capital stocks, of various vintages, and labor.

Not all of these assumptions are crucial, in the sense that their relaxation would complicate the analysis but not necessarily make it impossible. In a number of cases the fact that the empirical investigation has been conducted as if assumptions were true, while in fact they are not, will not seriously bias the estimates that Solow obtains. But in a number of cases they may introduce serious bias and it would be useful, in a more searching critique, to examine them one by one, to note first the effect that various degrees of inaccuracy of the assumptions would imply for estimates of critical parameters and second the extent to which such inaccuracies exist.

One should, in any event, distinguish carefully between assumption and empirical finding. For example, constant elasticity of substitution, as long as such elasticity is of the usual sign, implies that the marginal product of capital will never be negative (in fact, it will be positive), regardless of the capital-labor ratio. This may not be unreasonable in regard to the *gross* magnitude, value added, which is the dependent variable in much of Solow's current analysis. But it would be pure assumption to argue, as Solow has done elsewhere,[1] that variable factor proportions will then imply that any saving ratio is consistent with any rate of growth *and* positive net returns to capital. Estimating an elasticity assumed to be constant tells us little about whether the elasticity is in fact constant, particularly for ranges of factor proportions not included in the estimation.

Similarly, an estimate of the rate of productivity increase associated with technological advance assumed to be dependent only upon the application of new private capital to a homogeneous labor supply tells us nothing about the productivity that may be accruing or might accrue from education, training and redistribution of a heterogeneous and changing labor supply.

However, let us focus on just a few of the assumptions that may prove relevant to the completed estimates of this admittedly unfinished inquiry. For I shall now concentrate on the estimates of the elasticity of substitution between capital and labor which Solow offers on the basis of cross-sections of regional census data with regard to value added, payrolls, and labor force for each of the manufacturing industries which he considers. He notes that these estimates prove larger than estimates secured on the basis of international, "three-digit" cross sections reported upon in the paper by Arrow, Chenery, Minhas and Solow. I shall argue that the current estimates are in fact biased upward because of basic misspecifications and lack of identification of the underlying relations being measured. I shall argue in particular, that the inaccuracy of the assumptions of perfect competition and of equilibrium, the abstraction from the problems of risk and the insufficiency of arguments in the production function, all contribute to

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[1] "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics*, February 1956, pp. 65–94.

overestimates of the constant elasticity of substitution which Solow assumes.

The basis for the empirical analysis which Solow has completed, and to which much of the rest of this comment will be addressed, is the relation

$$\log \frac{Q}{L} = a \log W + b \tag{7'}$$

where, dealing with 1956 data for each of nineteen "two-digit" manufacturing industries, Q/L is the ratio of value added to the number of employees, and W is the ratio of payrolls to the number of employees for each of the nine census regions in the United States. Solow points out that, as demonstrated in the Arrow-Chenery-Minhas-Solow paper, on the basis of the assumptions of profit maximization in competitive labor and product markets, which implies that the product wage is equal to the marginal product of labor, the estimate of a is an estimate of the elasticity of substitution, σ. Solow's procedure then implies further that the various values of the wage in the various regions, more exactly, the ratios of payrolls to the number of employees, generate various factor ratios which in turn imply various ratios of value added to the number of employees, Q/L. This means that for each industry, the payroll-to-employee ratio determines the capital-employee ratio and consequently Q/L.

First, let us accept Solow's assumption that value added is related to the wage rate on the basis of a production function involving "capital" owned within the industry and "labor." But let us now add as arguments of this function a number of other capital and labor "factors," F_i, which might include capital *external* to the industry (railroads, highways, schools and general public services) and specially skilled workers (including managers and entrepreneurs). In a Solow-type CES equation, this would imply

$$Q = \gamma \left(\sum_{i=1}^{n} \delta_i F_i^{-\rho} \right)^{-\frac{1}{\rho}} \tag{E.1}$$

where $i = 1$ and $i = 2$ would relate to Solow's "capital" and "labor" respectively and $\Sigma\delta_i = 1$ (as in Solow's two-argument function). It will be noted that in Solow's CES function, as well as in many other "reasonable" production functions, this will mean that the more of any factor the higher will be the marginal product of other factors. (In mathematical terms, the cross partial derivatives will be positive.) But then regions with high endowments of factors not specified by Solow (F_i, for $i > 2$), would tend to have high marginal products of both labor and capital within the industry and hence high values of both wage and nonwage income.

If the cross partial derivatives of the function in logarithmic form were equal, that is, if increased endowments of these other factors increased the marginal products of Solow's "labor" and "capital" in proportion, the estimate of a, which Solow takes to be the elasticity of substitution, would be biased toward unity. (The estimate of a would tend to be greater or less than unity to the extent that the cross partials of other factors with "capital" were greater or less than the cross partials of other factors with labor.) This would occur even if, as seems more likely to me, the true elasticity of substitution between capital and labor is much closer to zero than Solow likes to think.

But now let us introduce profit maximization to a world with changing demand and entrepreneurs acting on the basis of probability distributions of expected returns. In such a world, with neither factor markets nor product markets perfectly competitive, decision makers adjust their payrolls partly to current demand and partly to the expectation of permanent demand. Salaried employees are not fired and do not even suffer reductions in pay when demand and output fall. Borrowing obviously from another field of economic inquiry, we may suggest that output and payrolls may both be viewed usefully as related separately to permanent and transitory components of a fluctuating demand. However, payrolls will be relatively more sluggish than output in reacting to changes in demand. Thus, letting w = the logarithm of the ratio of payrolls to the number of employees, and letting v = the logarithm of the ratio of value added to the number of employees, we conjecture that the true relations underlying Solow's observations may better be written:

$$w_P = k_P v_P + c_P, \tag{7.1}$$

$$w_T = k_T v_T + c_T, \text{ where } 0 < k_T < k_P \leq 1 \tag{7.2}$$

and, with $w = w_P + w_T$ and $v = v_P + v_T$, the "reduced form,"

$$w = kv + c. \tag{7.3}$$

This is to say that long run changes in output per man, whether based on productivity or increases in the value of output related to changing demand (or changing degrees of monopoly), may (if $k_P = 1$) reflect themselves in proportionate changes in wage and salary payments per employee but short-run fluctuations in output result clearly in less than proportionate changes in payrolls. Thus, if one were to ignore the distinction between v_P and v_T but instead estimate k in (7.3), the estimate obtained would be less than unity. The greater the discrepancy between k_P and k_T and the greater the value of the ratio

$$T = \frac{\text{variance of } v_T}{\text{variance of } v},$$

the smaller the estimate would be.

Now it may be noted that the k we would be estimating would be the reciprocal of Solow's a, which is his estimate of the elasticity of substitution. Hence the lower the value of k the greater the value of Solow's estimate of the elasticity of substitution.

To the extent that the regional observations used by Solow in making his estimates include a transitory variance of demand and hence of output, he will overestimate—even if other necessary assumptions are met—the elasticity of substitution. This leads us immediately to two conclusions: 1. Since in all cases some of the variance of output must be transitory in nature, there will be a general overestimate by Solow of elasticities of substitution. 2. The extent of this overestimate can be predicted on the basis of estimates of the proportion of variance of output which is accounted for by its transitory component. In fact, the relation between the "true" elasticity of substitution, which Solow would obtain if there were no transitory variance, and the estimate which he actually obtains as a result of transitory variance and other disturbances (as we shall see), may be written as

$$\frac{\sigma}{\sigma_b} = 1 - T\left(1 - \frac{k_T}{k_P}\right), \qquad \text{(E.2)}$$

where σ = the "true" elasticity of substitution

σ_b = Solow's biased estimate of the elasticity of substitution

T = the proportion of the variance of the logarithm of output accounted for by its transitory component

and k_T and k_P = the coefficients of transitory and permanent output as indicated in (7.2) and (7.1).

The underlying reasoning is that in response to temporary declines in demand there are substantial declines in v and lesser drops in w. Thus one will find both transitory components, of v and w, negative. The observed or measured w below the mean of w will tend to be observations with w_T negative but relatively close to zero, and v_T also negative but of relatively great absolute value. One can hence infer that on the average, for every observation (v_i, w_i), there is a vector (v_{P_i}, w_{P_i}) such that, since the means of v_T and w_T can be assumed to be zero, for v_i and w_i less than their means, $v_{P_i} > v_i$ and $w_{P_i} > w_i$. The reverse would be true for v_i and w_i greater than their means but what is crucial is that the regression coefficient of v_T on w_T would be greater than the regression coefficient of v_P on w_P. Perhaps all this can be seen best by writing the regression equations in the direction in which Solow estimated them. He in fact estimated a in

$$v = aw + b. \tag{7}$$

I say that underlying this relation is the equilibrium or "permanent" relation in which Solow is really interested,

$$v_P = a_P w_P + b_P, \tag{E.3}$$

and the transitory relation (or rather, relation involving the transitory components),

$$v_T = a_T w_T + b_T, \tag{E.4}$$

where $a_T > a_P$. This last inequality, for which one can offer good justification both in terms of theory and data, will imply that $a > a_P$, with the ratio of a to a_P also depending, of course, on the ratios of variances of transitory and permanent components.[2]

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[2] A rigorous treatment of the relation between coefficients of measured variables and coefficients involving their permanent components may be found in the writer's "The Permanent Income Hypothesis: Comment," *The American Economic Review*, December, 1958, especially, pp. 985–990. It is to be noted, however, that the argument there is focussed on the special case highlighted by Milton Friedman in which the transitory and permanent components are uncorrelated for each variable and the regression coefficient involving the transitory components is equal to zero. It should be noted that in the present case transitory and permanent components of each variable are judged to be positively correlated (both being influenced in the same direction, but in different magnitudes, by changes in demand) and the regression coefficient of the transitory components, taking the relation in the same direction as did Solow, is considered not only nonzero but larger in magnitude than the regression coefficient for the permanent components. Our argument may be put diagrammatically, to illustrate

Solow observes that the elasticities he estimates in the current investigation are higher than those obtained in earlier empirical work with the same model but with three-digit industries. This he explains by the greater scope for indirect substitution in two-digit industries. Our current argument, however, suggests a quite different explanation. Demand conditions are likely to vary less between regions, at a given time, than between products. If the demand for radios is down in one region it is likely to be down in all. But the demand for radios may be down while the demand for turbine-generators, also in the two-digit category (Number 36), may be up. Turbine-generators may be made largely in Schenectady and radios elsewhere so that the interproduct difference will show up as a regional variation as the product classes become larger.

While we have focussed on a "permanent output" hypothesis as an explanation of the discrepancy between Solow's current and other estimates of the elasticity of substitution, our argument can be generalized to include all factors that result in interregional differences in value added which are less than proportionately related to payrolls. We should single out, in particular, differences in degrees of monopoly in product markets. We may concede that there is some tendency for monopoly profits to be passed on to employees but casual empiricism would suggest that the pass-on is less than proportionate. Again it may be argued that product markets in manufactured goods are likely to be national in character and that differences in degrees of monopoly would not show up in a cross section of analyses of identical products, but will

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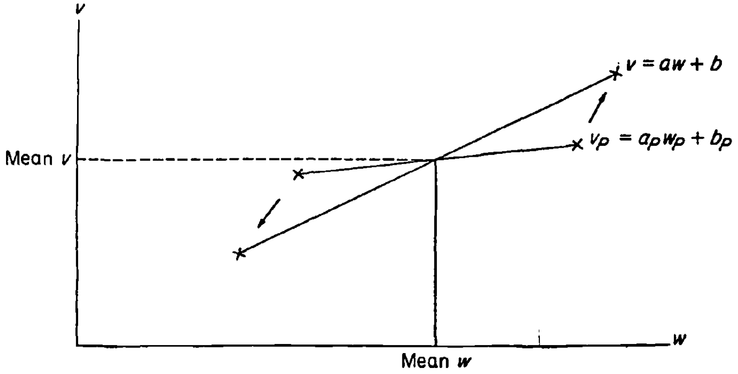
the shift from the "permanent" relation Solow requires to the relation, including transitory and permanent components, which he actually measures in the following figure:



show up as a result of interregional differences in product mix in broader product classes. It may be added that product markets are not, of course, fully national in character, so that differences in degrees of monopoly may manifest themselves even in cross sections of more narrowly defined product groups, thus contributing to overestimates of elasticity of substitution by Solow in such cases as well. However, the overestimate is likely to be larger the larger the product classes, and hence the larger the interregional variation in degrees of monopoly related to interregional differences in product mix.

The general point of this critique should by now be clear. As in virtually all serious econometric inquiry, the crucial problems are those of specification and identification. One may concede that there is an underlying relation tying the share of capital to a production function and capital-labor ratios as posited by Solow. But there is clearly also a set of underlying relations involving the value of current product and current payrolls which reflect degrees of monopoly, distinctions between transitory and permanent elements, exposure to risk, and other factors. My own hypothesis is that Solow has measured an unspecified linear combination of these various relations. He has no more right to say that his estimates apply to the elasticity of substitution than his less sophisticated precursors, observing prices and quantities of commodities sold or rates of interest and saving, could properly use them to estimate elasticities of demand or interest elasticities of consumption or saving. I personally would surmise that the elasticity of substitution between capital and labor is both very low (nearer to zero than one) and variable, getting lower as the capital-labor ratio is pushed to the point where the marginal *net* product of capital would become zero and then negative. I base this hypothesis on the fact that a production function with these characteristics would be consistent with the chronic and related problems involving growth and stability which we observe and I consider it important to explain. But this leads back to an older controversy that I have enjoyed with Solow. I am happy to confine the present argument to issues closer at hand.

NESTOR E. TERLECKYJ, Office of Management and Organization, Bureau of the Budget

Professor Solow suggests a very interesting new analysis ingeniously and impressively formulated, and demonstrates its application to a body of data. The empirical results are tentative, as the data apparently are not sufficient to bear the full burden of analysis.

Solow proposes a new way to estimate the production functions of industries and their shifts through time. The procedure is designed to yield estimates of elasticities of substitution of technological progress and of the rates of return. These results are obtained through a combination of cross-section and time series analyses.

The production function used by Solow is homogeneous of degree one in labor and capital, and the time shift representing technical progress appears in the present exposition exclusively as the trend in productivity of new capital goods produced in successive periods. Aside from its conceptual restrictiveness, this assumption seems to place a very heavy burden on the capital investment data in empirical application.

The elasticity of substitution for any given industry is assumed to be constant over the whole function and through time, but can have any value. This appears preferable to fixing its value in advance, particularly since, as Solow suggests, the actual estimate may in a large measure depend on the level of industry definitions used. Estimation of the elasticity of substitution rests on the assumption of competitive equilibrium.

In Solow's model, productivity of capital produced in any given year remains constant throughout the life of the asset. As time goes on, individual items of old capital are retired from use. Thus, Solow's concept of capital consumption involves scrapping of assets operating at their original efficiency, rather than deterioration of capital goods through use and the passage of time. I have no quarrel with this formulation on pragmatic grounds, particularly as I think it permits us to account for obsolescence where it is important. Nevertheless, it abstracts from quality deterioration on the one hand, and on the other, from improvements involving relatively minor investment, as well as from "external economies" generated by growth of experience over time. These abstractions, to be sure, may be more of a limitation on applications at the microeconomic level than for analyses cast within a broader framework.

Capital stock, then, at any point in time consists of items of different vintages and each vintage has its own constant productivity. Solow assumes a homogeneous labor force which is assigned to capital of different vintages so as to equalize the marginal productivity of labor. Since the model requires that the unit productivity of capital of any given vintage remain constant at all times, it implies flexibility in the capital-labor ratios for all the various vintages of capital.

Through a series of substitutions in the formulas, Solow expresses output as a function of the wage rate and the labor input alone, netting out capital. This allows him later on to estimate the elasticity of substitution from regional cross-section data when the regional age composition of capital is unknown. The estimate of the elasticity of substitution, after a simple transformation, later serves in the time series analysis to estimate the remaining parameters of the production function. In the present paper Solow assumes elasticities of substitution to be constant through time. It may be pointed out, however, that the technique proposed by Solow may be used to generate a whole time series of the estimates of the elasticities of substitution. Such estimates would provide a test for the important assumption of no time trends in elasticities of substitution. Estimates of other constants in the production function, notably the rate of technical change, depend on the value of elasticity of substitution.

In the present application of his analysis, Solow makes what appears to be a very strong assumption that the average life of capital is constant across industries and over time. This assumption, however, is not inherent in his approach. The interindustry variation in the life of capital may be taken into account with the help of outside data, and the time variation may be allowed for by building a time trend in the life of capital into the model (or by limiting the time period of analysis to subperiods of relatively constant average lives of durable assets).

The Solow model includes an exponential growth of productivity, which here is the same as the rate of growth in productivity of new capital. This, I believe, defines the length of the period over which Solow's approach may be expected to give most useful results. Obviously, the period cannot be too short, otherwise the cycle and other short-run changes would introduce too many disturbances into estimates of productivity trends. On the other hand, the period of analysis cannot be too long, because then the long waves and secular accelerations in productivity growth which came to light in John Kendrick's work might be expected to vitiate the exponential fit.

The empirical results, on the whole, are tentative, and they are offered in the spirit of a tentative report on a pilot project.[1] The obvious problem seems to be the narrowness of the information base. At most only nine observations are available for the cross-section analysis, and again the period employed in the time series estimates covers only a decade. Until the analysis is tested on a stronger body of data, it seems impossible to evaluate the firmness of the results it yields. Further testing would also indicate whether any assumptions need to be modified and how.

The analysis formulated by Professor Solow certainly deserves careful study. Although it has many facets, each interesting in its own right, I find its greatest appeal in the promise it holds—in conjunction with complementary research—to indicate answers to some very important questions about productivity growth.

As the next step, it appears to be worthwhile to test Solow's model on a large body of data, together with other models, such as production functions which allow imputation of productivity gains to labor input and functions containing a "disembodied" productivity trend. The various estimates of productivity change would then be usefully compared with each other and with independent estimates, such as those yielded by Kendrick's study. Such an exercise obviously would involve a major research project but the mechanics of computation can probably be handled easily by electronic computers.

JOHN W. KENDRICK, George Washington University

My admiration was aroused by Professor Solow's ingenious attempt to estimate elasticities of substitution indirectly from regional data. Since I happen to have made estimates of both real inputs and prices of the two major factor groupings, labor and property, I thought it would be interesting directly to compute elasticities of substitution for the same manufacturing industry

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[1] I would like to raise one point concerning the interpretation of time series results. It is suggested in the paper that the very sharp and sudden drop in productivity from 1956 to 1957 which appears on the scatter diagrams is to be attributed to the 1957–58 recession. One would expect the 1953–54 recession to produce a similar result since it was not too different from the 1957–58 decline either in depth or seasonal timing, and investment held up quite well. The scatter diagrams, however, do not indicate any effect of the earlier recession. This seems to argue strongly against a purely cyclical explanation of the break in productivity trends. Also, seemingly inconsistent with the cyclical explanation is the fact that in all cases productivity in the trough year 1958 appears higher than in 1957, while one would be inclined to expect the opposite on the basis of an idle-capacity explanation.

TABLE 1
Manufacturing Groups: Factor Inputs and Prices, 1957 and Elasticity of Substitution, 1953–57

	INDEX NUMBERS, 1957 (1953 = 100)				
	Factor Input		*Factor Price*		Elasticity of Substitution[a]
	Labor	Capital	Labor	Capital	
Total Manufacturing	95.1	112.2	123.5	94.6	.62
Beverages	78.0	83.9	121.7	103.5	.45
Food	100.4	105.0	121.0	101.1	.25
Tobacco	86.4	104.2	134.8	109.1	.88
Textile mill products	83.7	93.3	114.0	94.8	.59
Apparel	95.0	98.8	117.4	75.4	.09
Lumber and wood products	85.0	113.2	120.9	57.6	.40
Furniture and fixtures	99.6	116.1	118.2	108.9	1.86
Paper, pulp, and products	103.0	133.0	124.4	77.9	.55
Printing and publishing	106.2	108.0	121.1	110.6	.18
Chemicals	104.9	116.0	125.5	107.5	.65
Petroleum and coal products	94.5	125.6	129.0	72.6	.51
Rubber products	98.9	113.7	119.5	79.8	.35
Leather and products	95.3	104.0	116.8	96.9	.47
Stone, clay, and glass	100.5	132.9	124.0	90.4	.89
Primary metals	93.9	117.0	129.8	98.9	.81
Fabricated metal products	88.5	117.1	123.7	86.1	.78
Machinery (exc. electrical)	96.5	113.3	121.9	88.5	.50
Electrical machinery	100.1	94.8	119.2	127.6	.80
Transportation equipment	96.6	116.0	120.6	90.9	.65
Miscellaneous, incl. instruments	97.6	95.9	124.8	110.3	−.14

Source: Estimates by John W. Kendrick, assisted by Maude R. Pech, based on concepts, sources, and methods described in *Productivity Trends in the United States*, Princeton University Press for NBER, 1961.

[a] Ratio computed by arc formula.

groups used by Solow (see Table 1). These estimates are based on relative prices and quantities for the years 1953 and 1957, which span the year to which Solow's estimates relate.

The elasticities shown in my table must, therefore, be interpreted in a dynamic context. To be taken as indicative of elasticities under static, equilibrium conditions, not only must all the assumptions underlying Solow's model be made, but also the assumption that the shapes of the relevant production functions did not change between the two periods. From the dynamic viewpoint, the "historical" elasticities of substitution are useful summary descriptions of what happened over a given period to the interrelation of changes in relative prices and in relative input quantities. This interrelationship is, of course, the chief element in explaining changes in income shares.

Now for a word about the concepts underlying the several variables used in the elasticity calculations. Labor input is man-hours worked in each of the twenty major groups, but in the manufacturing segment as a whole it is man-hours worked in each group weighted by base-period average hourly labor compensation in each. Property input is assumed to move proportionately to real capital stocks (plant, equipment, and working capital) in each group, and in the manufacturing segment the real stock in each group is weighted by base-period rates of return. This is the reason that cycle-peak years were chosen for the comparisons.

The price of each factor class is calculated as the factor cost of each (compensation of labor and of capital) divided by the input of each. Thus, the "price" of labor is its average hourly compensation in each industry group—an average influenced by inter-industry shifts within each group, but not among groups as far as the segment is concerned. The average hourly earnings of employees is imputed to the relatively small number of proprietors and unpaid family workers in the various groups. The price of capital in each group is, in effect, the product of the average price of the underlying capital goods and the average rate of return (capital compensation—interest, rent, and profit before tax—divided by the value of the capital assets in current replacement values). The rates of return are affected by interindustry shifts within each of the manufacturing groups, but not among the groups in the segment as a whole. These concepts are discussed in more detail in my National Bureau study, *Productivity Trends in the United States* (1961).

The elasticities of substitution computed from my estimates differ widely from those obtained in Solow's regression equations, and the former appear to be more plausible. All but one are less than unity, and all but one indicate a negative relation between the rates of change in relative prices and in relative quantities. I believe that further progress in the estimation of elasticities of substitution and analysis of changing factor shares lies along the road of refining concepts and measures of capital inputs and their prices.