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Chapter Author: V. Eldon Ball, Rolf Fare, Shawna Grosskopf, Richard Nehring

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Productivity of the U.S. Agricultural Sector: The Case of Undesirable Outputs

V. Eldon Ball, Rolf Färe, Shawna Grosskopf, and Richard Nehring

13.1 Introduction

The purpose of this paper is two-fold: (a) to show how to model the joint production of desirable (marketed) and undesirable (nonmarketed, or "bad") outputs in a way that is useful for productivity analysis, and (b) to apply that model to data on the U.S. agricultural sector using activity analysis techniques.

To put our work in perspective with the productivity literature, we note that we follow Solow (1957) in the sense that we are modeling a production technology in order to identify productivity growth and technical change. Our approach differs significantly from Solow's in several ways. Instead of a single output, we include a vector of good outputs as well as a vector of undesirable outputs in our model. Thus, instead of a single output production function, we use distance functions as our representation of technology. These allow us to model joint production of goods and bads. Instead of specifying a parametric form of the technology, we use activity analysis

V. Eldon Ball is leader, agricultural productivity program, Economic Research Service, U.S. Department of Agriculture. Rolf Färe is professor of economics and agricultural economics and resource economics at Oregon State University. Shawna Grosskopf is professor of economics at Oregon State University. Richard Nehring is agricultural economist, Economic Research Service, U.S. Department of Agriculture.

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to construct a nonparametric representation of technology that also allows us to identify the production frontier. That, in turn, permits us explicitly to identify deviations from frontier performance and shifts in the frontier itself. Following Caves, Christensen, and Diewert (1982) we employ a discrete approach rather than the differential approach used by Solow. Finally, our approach does not require information on input and output prices or shares, which are used to aggregate inputs and outputs in the growth accounting/Solow approaches. Clearly, that is particularly useful in the case in which one wishes to include joint production of desirable and undesirable outputs, since the latter are typically nonmarketed.

The paper begins with a discussion of how we model the joint production of desirable and undesirable outputs both conceptually and empirically. Next we turn to a discussion of the Malmquist productivity index and how it may be computed. Since the Malmquist productivity index is defined in terms of output distance functions, it seeks the greatest feasible expansion of all outputs, both good and bad. Since the expansion of bad outputs may be undesirable (due to regulations, for example), we turn to a modified version of that index, which we refer to as the Malmquist-Luenberger productivity index in section 13.4. This index allows for contractions of undesirable outputs and expansions of "good," or desirable, outputs.

We apply our methods to a panel of state-level data recently made available by the U.S. Department of Agriculture's Economic Research Service (ERS). This data set includes variables that proxy effects of pesticides and fertilizer on groundwater and surface-water quality for the 1972–93 time frame. Although we consider our results to be preliminary, we find—as expected—that measured productivity differs when undesirable outputs are accounted for.[1] Our preferred model—the Malmquist-Luenberger index—generally reports higher productivity growth for states with declining trends in water contamination resulting from the use of pesticides and chemical fertilizers.

13.2 Modeling Technologies with Good and Bad Outputs

The production of desirable outputs is often accompanied by the simultaneous or joint production of undesirable outputs. Examples include the paper and pulp industry, electricity generation, and agriculture, among many others.

If we wish to measure productivity when both desirable and undesirable outputs are produced, we should obviously account explicitly for their

1. At the moment we cannot say whether the differences we observe are significant. Future research plans include application of bootstrapping methods to allow us to pursue such hypothesis testing.

joint production. If we denote desirable outputs by $y \in \Re^M_+$, undesirable outputs by $b \in \Re^I_+$, and inputs by $x \in \Re^N_+$, then the technology may be written as

$$(1) \qquad T = [(x, y, b) : x \text{ can produce } (y, b)].$$

The technology consists of all feasible input and output quantities; that is, it consists of all desirable and undesirable outputs that can be produced by the given input vectors.

To model the joint production of the good and bad outputs, it is convenient to model the technology in terms of the output sets, that is,

$$(2) \qquad P(x) = [(y, b) : (x, y, b) \in T].$$

Clearly, T can be recovered from $P(x)$ as

$$(3) \qquad T = [(x, y, b) : (y, b) \in P(x), x \in \Re^M_+].$$

Thus the technology is equivalently represented by either its output sets $P(x)$, $x \in \Re^N_+$ or its technology set T.

For the case in which a single desirable output is produced, the technology is often modeled by means of a production function $F(x)$. This function is defined on the output set $P(x)$ as

$$(4) \qquad F(x) = \max[y : y \in P(x)].$$

As before, the output sets and hence the technology may also be recovered from this representation of technology, namely as

$$(5) \qquad P(x) = [y : F(x) \geq y].$$

We model the idea that reduction of bad outputs is costly by imposing what we call "weak disposability of outputs," that is,

$$(6) \qquad (y, b) \in P(x) \text{ and } 0 \leq \theta \leq 1 \text{ imply } (\theta y, \theta b) \in P(x).$$ [2]

In words, this states that reduction of undesirable outputs is feasible only if good outputs are also reduced, given fixed input levels. Hence it may be infeasible to reduce the undesirable outputs only, that is, if (y, b) is feasible and $b' < b$ then it may be impossible to produce (y, b') using x, that is, $(y, b) \in P(x)$ and $(y, b') \notin P(x)$. Clearly, if undesirable outputs could be disposed of costlessly (freely), then this problem would not arise.

With respect to the good outputs, we assume that they are freely or strongly disposable, that is,

$$(7) \qquad \text{if } (y, b) \in P(x) \text{ and } y' \leq y \text{ imply } (y', b) \in P(x).$$

2. Shephard (1970) introduced the notion of weak disposability of outputs.

The reason for distinguishing between desirable and undesirable outputs in terms of their disposability is that the former typically have a positive price, whereas the latter are typically not marketed and, therefore, do not have readily observable prices.

The notion that desirable and undesirable outputs are jointly produced is modeled by what Shephard and Färe (1974) call "null-jointness." In words this means that if no bad outputs are produced, then there can be no production of good outputs. Alternatively, if one wishes to produce some good outputs then there will be undesirable byproducts of production. More formally, we have

$$(8) \qquad (y, b) \in P(x) \text{ and } b = 0 \text{ then } y = 0,$$

that is, if (y, b) is a feasible output vector consisting of desirable outputs y and undesirable outputs b, then if no undesirable outputs are produced $(b = 0)$ then by null-jointness, production of positive desirable outputs is not feasible, so $y = 0$.

In order to develop a framework for the empirical measurement of productivity with good and bad outputs, we need to formulate an explicit reference technology. Here we assume that at each time period $t = 1, \ldots, \bar{t}$ there are $k = 1, \ldots, K$ observations of inputs and outputs,

$$(9) \qquad (x^{t,k}, y^{t,k}, b^{t,k}), \qquad k = 1, \ldots, K, \qquad t = 1, \ldots, \bar{t}.$$

Following Färe, Grosskopf, and Lovell (1994) we define the output sets from the data as an activity analysis or data envelopment analysis (DEA) model,[3] namely

$$(10) \qquad P^t(x^t) = [(y^t, b^t) : \begin{array}{ll} \sum_{k=1}^{K} z_k^t y_{km}^t \geq y_m^t, & m = 1, \ldots, M, \\ \sum_{k=1}^{K} z_k^t b_{ki}^t = b_i^t, & i = 1, \ldots, I, \\ \sum_{k=1}^{K} z_k^t x_{kn}^t \leq x_n^t, & n = 1, \ldots, N \\ z_k^t \geq 0, & k = 1, \ldots, K], \end{array}$$

where z_k^t are the intensity variables, which serve to form the technology from convex combinations of the data.

To illustrate the model in equation (10), we assume that there are two firms $k = 1,2$ producing one desirable and one undesirable output with the data in table 13.1.

The data in table 13.1 and the corresponding output set are illustrated in figure 13.1. The two observations are labeled $k = 1$ and $k = 2$ in the figure. Each uses one unit of input to produce their good and bad outputs.

3. Charnes, Cooper, and Rhodes (1978) first coined the terminology "DEA."

Table 13.1 Hypothetical Data Set

Observation (k)	Good (y)	Bad (b)	Input (x)
1	1	(1/2)	1
2	2	2	1

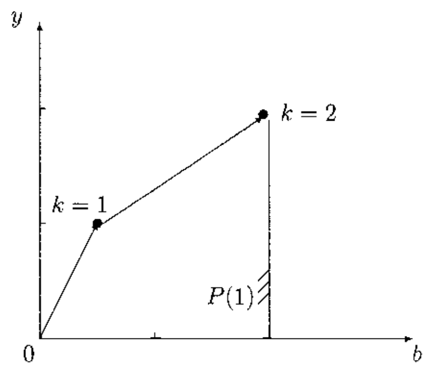

Fig. 13.1 Output set with weak disposability and null-jointness of bads

The output set $P(1)$ is constructed from these two observations such that outputs are weakly disposable and the good output y is strongly disposable. Moreover, the figure shows that if $b = 0$ then $y = 0$; thus the two outputs are jointly produced or null-joint in the terminology of Shephard and Färe.

In general one can show that equation (10) satisfies equations (6) and (7) in addition to satisfying constant returns to scale, that is,

$$P(\lambda x) = \lambda P(x), \qquad \lambda > 0. \tag{11}$$

In words, constant returns to scale means that proportional scaling of the input vector x yields proportional scaling of the output set $P(x)$.

Moreover, one can also show that inputs are strongly disposable in the following sense:

$$P(x') \subseteq P(x) \text{ for } x \geq x'. \tag{12}$$

For the good and bad outputs to satisfy null-jointness at each period t, we need to assume that the bad outputs satisfy the following two conditions:

$$\sum_{k=1}^{K} b_{ki}^{t} > 0, \qquad i = 1, \ldots, I,$$

$$\sum_{i=1}^{I} b_{ki}^{t} > 0, \qquad k = 1, \ldots, K.$$

The first inequality says that each bad is produced by at least one firm. The second states that each firm produces at least one bad. Now, referring back to the activity analysis formulation of technology in equation (10), suppose that the right-hand side of the constraints on the bad outputs are such that $b_i^t = 0$, $i = 1, \ldots, I$. If we have null-jointness that means that we should also have $y_m^t = 0, m = 1, \ldots, M$. The inequalities above guarantee that this is so, since together they require that each intensity variable is multiplied by at least one nonzero value of b_{ki}^t. Thus the only way to have $\Sigma_{k=1}^{K} z_k^t b_{km}^t = 0$ when these constraints hold is to have $z_k^t = 0$ for all k, which would imply that $y_m^t = 0$, $m = 1, \ldots, M$ as required for null-jointness of y and b.[4]

Next we show that the model in equation (10) has the property that decreases in the production of bads require that inputs be increased. We demonstrate this property using the data in table 13.1. Based on our data, if we wish to produce $(y, b) = (2, 2)$, then we must employ one unit of input. Now if we wish to produce $(y, b) = (2, 1)$; that is, if we wish to reduce the bad output by one unit, then we must increase our input usage to $x = 2$.[5] This shows that resources may be required to "clean up" the bad outputs. For additional properties of the production model in equation (10), see Färe and Grosskopf (1996).

13.3 The Malmquist Productivity Index

In this section we discuss the Malmquist productivity index as proposed by Färe et al. (1989). Their index is based on Shephard's output distance function and is the geometric mean of two of the Malmquist indexes introduced by Caves, Christensen, and Diewert (1982).

The output distance function, introduced into economics by Shephard (1970), is given by

$$(13) \qquad D_o(x, y, b) = \inf[\theta : (y/\theta, b/\theta) \in P(x)]$$
$$= \inf[\theta : (x, y/\theta, b/\theta) \in T]$$

where the last equality holds, since $(x, y, b) \in T$ if and only if $(y, b) \in P(x)$; see equations (2) and (3). The output distance function is the largest feasible expansion of the output vector (y, b), and it has the property of being a complete representation of the technology, that is,

$$(14) \qquad D_o(x, y, b) \leq 1 \quad \text{if and only if} \quad (y, b) \in P(x).$$

4. Note that it is the strict equality on the bad output constraints that drives this result; thus null-jointness and weak disposability are intimately related in the activity analysis model.

5. Scaling observation $k = 1$ by a factor of 2 yields the desired result.

Thus the output distance function assigns a value to the input-output vector (x, y, b) of less than or equal to 1 for feasible input-output vectors only.

As a representation of the technology, the distance function inherits the properties assumed for the technology; see Färe (1988) or Shephard (1970) for details. In addition to the inherited properties it is homogeneous of degree +1 in good and bad outputs (y, b).

In the simple case in which a single (good) output is produced, there is the following direct relationship between the production function $F(x)$ and the output distance function $D_o(x, y)$.

$$D_o(x, y) = \frac{y}{F(x)} \tag{15}$$

To see this we note that since $D_o(x, y)$ is a complete representation of the technology, we have

$$P(x) = [y : D_o(x, y) \leq 1]. \tag{16}$$

Now, if we apply the definition of a production function to equation (16) and use the fact that $D_o(x, y)$ is homogeneous of degree +1 in outputs, we have

$$\begin{aligned} F(x) &= \max[y : y \in P(x)] = \max[y : D_o(x, y) \leq 1] \\ &= \max[y : y \cdot D_o(x, 1) \leq 1] \\ &= \frac{1}{D_o(x, 1)}. \end{aligned} \tag{17}$$

Now $D_o(x, 1) \cdot y = D_o(x, y) = y/[F(x)]$.

Färe, Grosskopf, Lindgren, and Roos (1989, hereafter FGLR) define the Malmquist productivity indexes for adjacent periods as

$$M_t^{t+1} = \left[\frac{D_o^{t+1}(x^{t+1}, y^{t+1}, b^{t+1})}{D_o^{t+1}(x^t, y^t, b^t)} \frac{D_o^t(x^{t+1}, y^{t+1}, b^{t+1})}{D_o^t(x^t, y^t, b^t)}\right]^{1/2}. \tag{18}$$

This index is the geometric mean of the two output-oriented Malmquist indexes introduced by Caves, Christensen, and Diewert (1982), namely

$$\frac{D_o^{t+1}(x^{t+1}, y^{t+1}, b^{t+1})}{D_o^{t+1}(x^t, y^t, b^t)} \quad \text{and} \quad \frac{D_o^t(x^{t+1}, y^{t+1}, b^{t+1})}{D_o^t(x^t, y^t, b^t)}. \tag{19}$$

It is of interest to compare the FGLR formulation with Robert Solow's (1957) index, noting of course that the Malmquist indexes are all in discrete time.

Solow assumes that the technology can be represented by an aggregate production function

(20) $$y^t = F(x^t, t) = A(t)(x_1^t)^{\alpha(t)}(x_2^t)^{1-\alpha(t)},$$

where the residual $A(t)$ captures technical change and $\alpha(t)$ is input x_1's output share in period t. Here, we simplify by setting $\alpha(t) = \alpha$ for all t. Then, using the equivalences derived above between production functions and distance functions in the scalar output case, we can insert Solow's production function into the Malmquist index in equation (18), which yields

(21) $$M_t^{t+1} = \left\{\frac{[A(t+1)(x_1^t)^{\alpha}(x_2^t)^{1-\alpha}]/y^t}{[A(t+1)(x_1^{t+1})^{\alpha}(x_2^{t+1})^{1-\alpha}]/y^{t+1}}\frac{[A(t)(x_1^t)^{\alpha}(x_2^t)^{1-\alpha}]/y^t}{[A(t)(x_1^{t+1})^{\alpha}(x_2^{t+1})^{1-\alpha}]/y^{t+1}}\right\}^{1/2}$$
$$= \frac{[(x_1^t)^{\alpha}(x_2^t)^{1-\alpha}]/y^t}{[(x_1^{t+1})^{\alpha}(x_2^{t+1})^{1-\alpha}]/y^{t+1}} = \frac{A(t+1)}{A(t)}.$$

This shows that the Malmquist index proposed by FGLR is the discrete analog of the Solow residual where $\alpha(t) = \alpha$.

FGLR also show that their Malmquist index decomposes into two component measures, one accounting for efficiency change (MEFFCH_t^{t+1}) and one measuring technical change (MTECH_t^{t+1}). These are

(22) $$\text{MEFFCH}_t^{t+1} = \frac{D_o^{t+1}(x^{t+1}, y^{t+1}, b^{t+1})}{D_o^t(x^t, y^t, b^t)}$$

and

(23) $$\text{MTECH}_t^{t+1} = \left[\frac{D_o^t(x^{t+1}, y^{t+1}, b^{t+1})}{D_o^{t+1}(x^{t+1}, y^{t+1}, b^{t+1})}\frac{D_o^t(x^t, y^t, b^t)}{D_o^{t+1}(x^t, y^t, b^t)}\right]^{1/2}$$

where

(24) $$M_t^{t+1} = \text{MEFFCH}_t^{t+1} \cdot \text{MTECH}_t^{t+1}.$$

In the Solow formulation we would have

(25) $$\text{MEFFCH}_t^{t+1} = 1$$

and

(26) $$\text{MTECH}_t^{t+1} = \frac{A(t+1)}{A(t)},$$

implying that there is no efficiency change in the Solow formulation (production is implicitly assumed to be technically efficient) and therefore productivity change is due solely to technical change.

To illustrate the Malmquist index we simplify our model and assume that one input x is used to produce one output y.

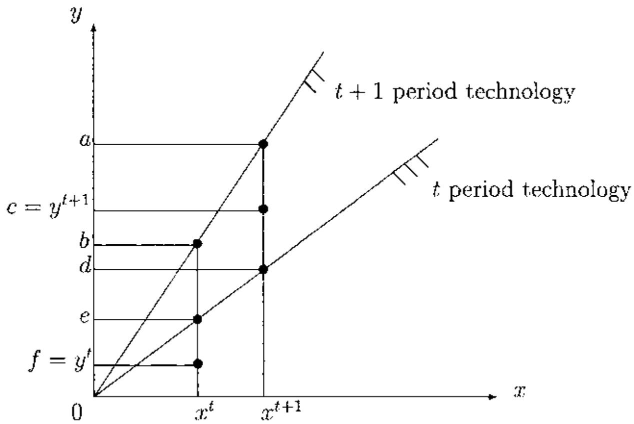

Fig. 13.2 Output-oriented Malmquist productivity index

Two technologies T^t and T^{T+1} are included in figure 13.2 along, with two observations, (x^t, y^t) and (x^{t+1}, y^{t+1}). Notice that these observations are not technically efficient; rather, production occurs "below" the boundary of the associated total product set in both periods. The Malmquist index as measured along the y-axis equals

(27) $$M_t^{t+1} = \left(\frac{0c/0a}{0f/0b}\frac{0c/0d}{0f/0e}\right)^{1/2}.$$

Under constant returns to scale (as in our figure), this is equivalent to the ratio of the average products in the two periods, which has clearly increased over time. Thus the overall index will be greater than one indicating an improvement in productivity between period t and $t + 1$.

One of the nice features of the Malmquist index is the fact that we can identify the two component measures defined above (see equations [22] and [23]), which allows us to identify sources of productivity change over time. In our figure, the efficiency change component is

(28) $$\mathrm{MEFFCH}_t^{t+1} = \left(\frac{0c}{0a}\frac{0e}{0f}\right),$$

and the technical change component is

(29) $$\mathrm{MTECH}_t^{t+1} = \left(\frac{0a}{0d}\frac{0b}{0e}\right)^{1/2}.$$

The efficiency change term MEFFCH_t^{t+1} tells us whether an observation is getting closer or farther from the frontier over time; that is, it tells us whether an observation is catching up to the frontier. In our example, the observation is actually falling farther behind the frontier over time; that is, it was closer to the frontier in period t than in period $t + 1$. Thus the value of this term would be less than 1, indicating a decline in technical efficiency over time.

On the other hand, our figure illustrates that technical progress has occurred between t and $t + 1$. This is captured in our technical change component MTECH_t^{t+1}, which is greater than 1 for our observation. MTECH_t^{t+1} tells us how much the frontier has shifted over time evaluated at the observed inputs in periods t and $t + 1$ (the index takes the geometric mean of the shifts in the frontier at these two input levels).

In our example there has been a decline in efficiency over time, but an improvement in terms of technical change. The product of these two yields the overall productivity change, which in this example is greater than 1, signalling an overall improvement in productivity. This means that technical change accounted for the observed improvement in productivity in this case.

To calculate the productivity index and its component measures we estimate the component distance functions using linear programming techniques. These are discussed in some detail in Färe, Grosskopf, Norris, et al. (1994) for the interested reader.

13.4 The Malmquist-Luenberger Productivity Index

We now turn to the Malmquist-Luenberger productivity index, which—unlike the Malmquist index described earlier that treats all outputs the same—allows us to credit observations for increases in good outputs yet "debit" them for increases in undesirable outputs. This index was introduced by Chung (1996) and Chung, Färe, and Grosskopf (1997), and it is based on the output-oriented directional distance function. This distance function differs from the Shephard output distance function in that it does not necessarily change outputs (y, b) proportionally, but rather changes them in a preassigned direction. This new distance function is a special case of the directional technology distance function (see Chambers, Chung, and Färe 1998, where the latter is essentially the shortage function due to Luenberger; see, e.g., Luenberger 1992, 1995).

Consider a direction vector $(g_y, -g_b) \neq 0$, where $g_y \in \Re_+^M$. Then the output-oriented directional distance function is defined as

$$(30) \quad \mathbf{D}_o(x, y, b;\, g_y, -g_b) = \sup[\beta : (y + \beta g_y, b - \beta g_b) \in P(x)].$$

This function is defined by adding the direction vector to the observed vector and scaling that point by simultaneously increasing good outputs and decreasing bad outputs. Figure 13.3 illustrates.

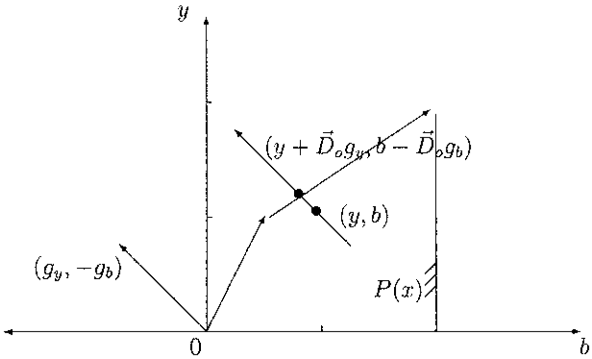

Fig. 13.3 Output-oriented directional distance function

In this figure the output set is denoted by $P(x)$ and the output vector (y, b) is an element of that set. The direction vector is $(g_y, -g_b)$ and the distance function expands the output vector as much as is feasible along the direction vector. It ends up at $(y + \mathbf{D}_o g_y, b - \mathbf{D}_o g_b)$, where $\mathbf{D}_o = \mathbf{D}_o(x, y, b; g_y, -g_b)$.

In order to see the relation between the directional and the Shephard output distance functions, suppose we change the direction slightly (eliminate the negative sign on the bad outputs) and choose $g_y = y$ and $g_b = b$, then

$$(31)\quad \mathbf{D}_o(x, y, b; y, b) = \sup[\beta : (y + \beta y, b + \beta b) \in P(x)]$$
$$= \sup\{\beta : [y(1 + \beta), b(1 + \beta) \in P(x)]\}$$
$$= \sup\{1 - 1 + \beta : [y(1 + \beta), b(1 + \beta) \in P(x)]\}$$
$$= -1 + \sup\{(1 + \beta) : [y(1 + \beta), b(1 + \beta) \in P(x)]\}$$
$$= \frac{1}{D_o(x, y, b) - 1}.$$

Thus, if we choose the directions $g_y = y$ and $g_b = b$, we find that the directional distance function is essentially Shephard's output distance function. In our figure, the observed point would be projected to the frontier in this case by scaling to the Northeast in our figure, seeking the largest feasible increase in both good *and* bad outputs. This would mean that observations with relatively more bads, ceteris paribus, would be deemed more efficient, which is inconsistent with the notion that the bads are undesirable. If we ignore the undesirable outputs and scale only on the good outputs (as in the traditional output distance function and Malmquist index), we would go due North to the frontier. To sum up,

(32) $$\mathbf{D}_o(x, y, b; y, b) = \left[\frac{1}{D_o(x, y, b)}\right] - 1.$$

or

(33) $$D_o(x, y, b) = \frac{1}{1 + \mathbf{D}_o(x, y, b: y, b)}.$$

The Malmquist-Luenberger index used here is defined by choosing the direction as the observed vector of good and bad outputs (y, $-b$) and making use of the Malmquist index in equation (18) and the idea of equation (33).

(34) $$\mathrm{ML}_t^{t+1} = \left[\frac{1 + \mathbf{D}_o^t(x^t, y^t, b^t; y^t, -b^t)}{1 + \mathbf{D}_o^t(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})} \cdot \frac{1 + \mathbf{D}_o^{t+1}(x^t, y^t, b^t; y^t, -b^t)}{1 + \mathbf{D}_o^{t+1}(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})}\right]^{1/2}.$$

The definition is such that if the direction had been (y, b) instead of (y, $-b$), it would coincide with the Malmquist index in equation (18). As in that case, improvements in productivity are signalled by values of the index greater than unity.

Like the Malmquist index, the Malmquist-Luenberger index can be decomposed into components, namely an efficiency change and a technical change component,[6]

(35) $$\mathrm{MLEFFCH}_t^{t+1} = \frac{1 + \mathbf{D}_o^t(x^t, y^t, b^t; y^t, -b^t)}{1 + \mathbf{D}_o^{t+1}(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})}$$

(36) $$\mathrm{MLTC}_t^{t+1} = \left[\frac{1 + \mathbf{D}_o^{t+1}(x^t, y^t, b^t; y^t, -b^t)}{1 + \mathbf{D}_o^t(x^t, y^t, b^t; y^t, -b^t)} \cdot \frac{1 + \mathbf{D}_o^{t+1}(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})}{1 + \mathbf{D}_o^t(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})}\right]^{1/2}.$$

As for the Malmquist index, the product of these two components equals the Malmquist-Luenberger index ML_t^{t+1} and the components have similar interpretations. Values greater than 1 indicate improvements, and values less than 1 indicate declines in performance.

The directional distance functions, like the Shephard distance functions,

6. One may further decompose both of these components, analogous to the Malmquist index decomposition developed in Färe, Grosskopf, and Lovell (1994) or Färe, Grosskopf, Norris, et al. (1994).

can be estimated as solutions to linear programming problems. As an example, let us consider the $(k', t+1)$ observation of data relative to the period t reference technology, that is,

$$\text{(37)} \qquad \mathbf{D}_o^t(x^{t+1,k'}, y^{t+1,k'}, b^{t+1,k'}; y^{t+1,k'}, -b^{t+1,k'}) = \max \beta$$

subject to

$$\sum_{k=1}^{K} z_k^t y_{km}^t \geq (1+\beta) y_{k'm}^{t+1}, \qquad m = 1, \ldots, M,$$

$$\sum_{k=1}^{K} z_k^t b_{ki}^t = (1-\beta) b_{k'i}^{t+1}, \qquad i = 1, \ldots, I,$$

$$\sum_{k=1}^{K} z_k^t x_{kn}^t \leq x_{k'n}^{t+1}, \qquad n = 1, \ldots, N$$

$$z_k^t \geq 0, \qquad k = 1, \ldots, K.$$

Our empirical illustration includes computations of both the Malmquist (goods only) and Malmquist-Luenberger productivity indexes. One may also compute what we call a Luenberger productivity indicator, which is also based on the directional distance functions described above. This index has an additive rather than multiplicative structure and is described briefly in appendix A.

Next, we turn to our empirical illustration.

13.5 Empirical Illustration: The Case of U.S. Agriculture

In this section we provide an empirical illustration of the measurement of productivity in the presence of undesirable outputs using a unique data set developed by the U.S. Department of Agriculture's (USDA's) Economic Research Service (ERS), in cooperation with the USDA's Natural Resources Conservation Service (NRCS). As part of our illustration, and paralleling the discussion of the theoretical models, we include results for: (a) productivity based on goods alone, using the Malmquist productivity index, and (b) productivity including both goods and bads using the Malmquist-Luenberger index (our preferred model). Before turning to these results, we first turn to a discussion of the data.

The data used to construct our productivity indexes based on desirable outputs and inputs alone are described in Ball et al. (1999). The inputs include services of capital, land, labor, and intermediate goods. The desirable outputs are crops and livestock. The data are available for forty-eight states over the period 1960–93 and are used to construct a state-by-year panel.

As a first step, we construct longitudinal indexes of outputs and inputs. An index of relative real output (alternatively, real input) between two states is obtained by dividing the nominal output (input) value ratio for the two states by the corresponding output (input) price index. We construct multilateral price indexes using a method proposed independently by Eltetö and Köves (1964) and Szulc (1964) (henceforth EKS). The "EKS" index is based on the idea that the most appropriate index to use in a comparison between two states is the Fisher index, which is expressed as the geometric mean of the Laspeyres and Paasche indexes.

When the number (K) of states exceeds two (i.e., $K > 2$), the application of the Fisher index number procedure to the $[K(K-1)]/2$ possible pairs of states yields a matrix of bilateral price indexes that does not satisfy Fisher's circularity test. The problem is to obtain results that satisfy the circularity test and that deviate the least from the bilateral Fisher indexes.

If P^{ij}_{EKS} and P^{ij}_{F} represent the EKS and Fisher indexes between states i and j, the EKS index is formed by minimizing the following distance criterion:

$$\Delta = \sum_{i=1}^{K}\sum_{j=1}^{K}[\ln P^{ij}_{\text{EKS}} - \ln P^{ij}_{\text{F}}(p^j, p^i, y^j, y^i)]^2$$

From the first-order conditions for a minimum, it can be shown that the multilateral index with the minimum distance is given by

$$(38) \qquad P^{ij}_{\text{EKS}} = \left[\prod_{k=1}^{K}\frac{P_{\text{F}}(p^j, p^i, y^j, y^i)}{P_{\text{F}}(p^i, p^k, y^i, y^k)}\right]^{1/K}.$$

The EKS index between states i and j is, therefore, a geometric mean of K (the number of states) ratios of bilateral Fisher indexes. The multilateral EKS indexes defined by equation (38) satisfy transitivity while minimizing the deviations from the bilateral Fisher indexes.

Comparisons of levels of capital, land, labor, and intermediate inputs require relative input prices. Relative price levels of capital inputs among states are obtained via relative investment-goods prices, taking into account the flow of capital services per unit of capital stock.

Differences in the relative efficiency of land prevents the direct comparison of observed prices. We construct price indexes for land based on an application of the hedonic regression technique. This approach assumes that the price of a good is a function of its characteristics, and it estimates the imputed prices of such characteristics by regressing the prices of goods on their differing quantities of their characteristics.

For our cross-section of states, we estimate the following equation by ordinary least squares (OLS):

$$(39) \qquad \ln w^j_k = \sum_{k=1}^{K}\delta_k \text{D}_\text{k} + \sum_{m=1}^{M}\beta_m x^j_{km} + \varepsilon_{kj}, \qquad k = 1,\ldots,K,$$

where w_k^j is the price of the jth parcel of land in state k, x_{km}^j is the mth characteristic of the jth parcel of land in state k, and D_k is a dummy variable equal to unity for the corresponding state and zero otherwise. When the log of price is related to linear-state dummy variables as above, a hedonic price index can be calculated from the antilogs of the δ_k coefficients.

In constructing indexes of relative labor input, we assume that the relative efficiency of an hour worked is the same for a given type of labor in all forty-eight states. Hours worked and average hourly compensation are cross-classified by sex, age, education, and employment class (employee versus self-employed and unpaid family workers). Since average hourly compensation data are not available for self-employed and unpaid family workers, each self-employed worker is imputed the mean wage of hired workers with the same demographic characteristics.

Finally, all our calculations are base-state invariant, but they are not base-year invariant. We use 1987 as the base year for all our time series indexes. The reason for this is that the detailed price comparisons are carried out only for 1987, which means that we construct price indexes for the remaining years by chain linking them to 1987. Thus we have a "true" panel of data with both time and spatial consistency and comparability.

The data for the undesirable outputs were constructed in collaboration with ERS, EPA, and NCRS; details are included in appendix B and are based on Kellogg, Nehring, and Grube (1999), Kellogg and Nehring (1997), and Kellogg, Nehring, Grube, Plotkin, et al. (1999). These undesirable outputs were intended to capture the effects of the agricultural use of chemical pesticides and fertilizers on groundwater and surface water quality. There are essentially two sets of undesirable outputs. The first set includes separate indexes of nitrogen and pesticide leaching into groundwater and runoff into surface water. This first set of undesirable outputs does not attempt to adjust for toxicity or risks from exposure. To summarize, the first set of undesirable outputs includes the following:

nitrogen leaching (from fertilizer)
nitrogen runoff (from fertilizer)
pesticide leaching
pesticide runoff

In contrast, the second set of indexes of undesirable outputs does explicitly account for toxicity and, hence, environmental risk. However, these risk-adjusted indexes are available only for pesticides. In this case we have risk-adjusted indexes for both pesticide runoff and leaching. We also have two types of risk: that associated with exposure to humans and that associated with exposure to fish. To sum up, we have the following for the risk-adjusted case:

human risk–adjusted effect of pesticide leaching
human risk–adjusted effect of pesticide runoff

fish risk–adjusted effect of pesticide leaching
fish risk–adjusted effect of pesticide runoff

As with the data on good output and input quantities, we normalize on 1987.

In our empirical illustration we begin by computing Malmquist productivity and its components based solely on the data on good outputs and inputs. This provides us with a benchmark for the traditional approach that does not explicitly account for byproducts of agricultural production. Next we compute Malmquist-Luenberger productivity indexes with a number of alternative good and bad specifications.

As a reference point, we first compute productivity with the Malmquist index for the traditional model in which only good outputs are included. Table 13.2 includes a summary of average annual productivity changes for this model for each of the forty-eight states in our sample, including the extended decomposition of productivity into technical change, efficiency change, and change in scale efficiency (as in Färe, Grosskopf, Norris, et al. 1994). Recalling that values above 1 signal improvements and those below 1 declines in performance,[7] these results suggest that there has been widespread improvement in productivity in the agricultural sector, due largely to technical change. Another noteworthy feature is the high level of technical efficiency across the board.

Turning now to the analysis of productivity in which we explicitly account for potential detrimental effects of pesticide and fertilizer use, we begin with a brief overview of the trends in production of undesirable outputs. Table 13.3 displays average annual growth rates in the non–risk adjusted measures of undesirable outputs by state. The first two columns proxy the effects of excess pesticides and nitrogen fertilizers on ground water quality; the second two proxy their effects on surface water quality. The average annual increase for all the states in our sample is on the order of 1.5 percent in groundwater contamination and between 2.7 and 3.7 percent in surface water contamination. A quick glance at the individual state averages reveals considerable variation. One interesting pattern we observe is that if we average over the major corn-producing states (Illinois, Indiana, Iowa, Michigan, Minnesota, Missouri, Nebraska, Ohio, Wisconsin, and South Dakota) they have average increases above the national average in pesticide runoff and leaching, particularly in the earlier part of our time period 1972–81. In the later part of the period 1981–93, growth rates declined but were still positive. In contrast, if we average across the large cotton-producing states (Alabama, Arizona, Arkansas, California, Georgia, Louisiana, Mississippi, North Carolina, Tennessee, and Texas) we observe below-average pesticide runoff and leaching, with a decline in leaching and increase in runoff from the earlier part of the period to the later

7. Subtracting 1 from the value in the table gives the average annual percentage increase or decrease in the associated index.

Table 13.2 **Traditional Malmquist Model: No Bads**

	MALMQ	EFFCHG	TECHCH	SCALECH	VRSEFFCH
Alabama	1.0097	1.0011	1.0086	1.0001	1.0010
Arizona	1.0090	0.9986	1.0104	1.0000	0.9986
Arkansas	1.0330	1.0069	1.0260	0.9914	1.0157
California	1.0200	1.0000	1.0200	1.0000	1.0000
Colorado	1.0119	0.9901	1.0220	0.9901	1.0000
Connecticut	1.0227	1.0000	1.0227	1.0000	1.0000
Delaware	1.0406	1.0000	1.0406	1.0000	1.0000
Florida	1.0234	1.0000	1.0234	1.0000	1.0000
Georgia	1.0234	0.9997	1.0237	0.9983	1.0014
Idaho	1.0240	0.9974	1.0267	0.9997	0.9977
Illinois	1.0281	1.0000	1.0281	1.0000	1.0000
Iowa	1.0043	0.9776	1.0273	0.9776	1.0000
Indiana	1.0223	0.9932	1.0293	0.9997	0.9935
Kansas	1.0164	0.9865	1.0302	0.9927	0.9938
Kentucky	1.0059	0.9988	1.0072	1.0016	0.9971
Louisiana	1.0161	0.9902	1.0262	0.9999	0.9903
Maine	1.0037	0.9873	1.0166	1.0000	0.9873
Maryland	1.0063	0.9925	1.0140	1.0012	0.9913
Massachusetts	1.0180	1.0000	1.0180	1.0000	1.0000
Michigan	1.0091	0.9934	1.0158	1.0020	0.9914
Minnesota	1.0031	0.9799	1.0237	0.9857	0.9940
Mississippi	1.0233	0.9959	1.0275	0.9965	0.9994
Missouri	0.9950	0.9870	1.0081	0.9996	0.9874
Montana	1.0085	0.9955	1.0130	1.0037	0.9918
North Carolina	1.0354	1.0047	1.0305	1.0035	1.0012
North Dakota	1.0132	0.9876	1.0259	1.0003	0.9873
Nebraska	1.0244	0.9960	1.0285	0.9940	1.0020
New Hampshire	1.0137	1.0045	1.0092	1.0045	1.0000
New Jersey	1.0102	1.0049	1.0052	0.9995	1.0054
New Mexico	1.0072	0.9998	1.0075	0.9998	1.0000
Nevada	0.9992	0.9903	1.0090	1.0000	0.9903
New York	1.0134	0.9979	1.0155	1.0002	0.9978
Ohio	1.0039	0.9955	1.0085	1.0012	0.9943
Oklahoma	0.9953	0.9904	1.0049	1.0018	0.9886
Oregon	1.0058	0.9990	1.0068	1.0000	0.9990
Pennsylvania	1.0056	0.9962	1.0094	1.0016	0.9946
Rhode Island	1.0112	1.0019	1.0093	1.0019	1.0000
South Carolina	1.0115	0.9980	1.0135	1.0002	0.9977
South Dakota	1.0094	0.9850	1.0248	1.0015	0.9835
Tennessee	1.0093	0.9998	1.0095	1.0020	0.9977
Texas	1.0128	0.9998	1.0130	0.9998	1.0000
Utah	1.0095	0.9989	1.0107	1.0022	0.9967
Vermont	1.0125	1.0000	1.0125	1.0000	1.0000
Virginia	1.0090	0.9969	1.0121	1.0011	0.9958
Washington	1.0330	1.0000	1.0330	1.0000	1.0000
West Virginia	1.0035	0.9927	1.0109	0.9991	0.9936
Wisconsin	1.0116	0.9923	1.0195	0.9923	1.0000
Wyoming	1.0112	0.9964	1.0149	1.0031	0.9933
Grand mean	1.0135	0.9958	1.0177	0.9989	0.9969

Notes: Average annual productivity change (geometric mean) 1972–1993. MALMQ = Malmquist. EFFCHG = efficiency change. TECHCH = technology change. SCALECH = scale efficiency change. VRSEFFCH = change in efficiency under variable returns to scale (VRS).

Table 13.3 **Growth Rates of Undesirable Outputs, 1972–93**

	Pesticide Leaching	Nitrogen Leaching	Pesticide Runoff	Nitrogen Runoff
U.S. Growth	1.457975	1.605701	2.720772	3.742445
Alabama	−0.66178	−1.86044	0.158949	0.347739
Arizona	−4.30669	0.916722	2.436606	3.300701
Arkansas	−1.06582	5.730058	1.126583	6.048018
California	−4.17287	−0.31007	3.456654	2.886361
Colorado	3.345479	−0.20643	4.996925	2.71688
Delaware	3.359957	2.147653	4.197571	3.300701
Florida	−1.54628	−3.3083	1.398584	0.41434
Georgia	−2.71806	−1.90331	−0.49902	0.224131
Idaho	7.377519	1.75107	7.314736	n.a.
Illinois	4.4356	1.660584	4.313539	1.903807
Indiana	4.456804	−1.12226	4.548081	−0.56724
Iowa	3.499369	5.640201	3.94283	6.778445
Kansas	4.754962	2.30858	4.968092	4.028983
Kentucky	7.069839	−0.36536	9.034774	2.624652
Louisiana	1.125453	3.579004	3.035036	5.306479
Maryland	3.78025	1.948008	4.713547	2.739829
Michigan	4.618252	2.493359	5.89248	4.306935
Minnesota	3.862868	5.672146	4.832284	7.276444
Mississippi	−1.76854	0.856069	0.372424	1.971661
Missouri	0.825782	1.66205	3.510189	2.838578
Montana	6.271978	7.162273	8.527898	n.a.
Nebraska	4.784035	2.174189	4.648753	2.708397
Nevada	−12.214	n.a.	n.a.	n.a.
New Jersey	0.030331	0.988759	2.943945	3.300701
New Mexico	−1.55457	0.61349	3.799125	6.601402
New York	4.419186	−0.05891	8.909959	4.034752
North Carolina	0.228167	0.009149	0.402215	1.539176
North Dakota	6.379878	22.63614	6.55775	23.59918
Ohio	4.738903	4.12331	4.840595	5.412055
Oklahoma	1.091972	4.995701	2.390846	5.991969
Oregon	5.804647	−2.97216	5.881847	n.a.
Pennsylvania	3.816409	0.367492	6.010053	2.271067
South Carolina	−4.32401	0.989152	−4.94202	1.698452
South Dakota	3.79704	10.19079	5.526619	12.30475
Tennessee	1.808472	0.930429	2.213043	3.36549
Texas	1.983041	2.23293	1.440772	3.657867
Utah	3.129082	−1.12566	4.249905	n.a.
Virginia	1.680214	1.396692	4.965877	4.72977
Washington	6.299346	−3.54085	8.745588	n.a.
West Virginia	1.343964	0.272183	4.896003	3.300701
Wisconsin	3.943217	3.907079	5.88467	7.66399
Wyoming	3.195111	0.207072	6.247087	n.a.
Corn states				
1972–81	5.94335	7.54430	6.52824	7.34857
1981–93	3.38477	−1.88265	3.70539	1.22602
1972–93	3.99776	2.42642	4.38584	3.67482

Table 13.3 (continued)

	Pesticide Leaching	Nitrogen Leaching	Pesticide Runoff	Nitrogen Runoff
Cotton states				
1972–81	1.19993	4.48643	1.21995	4.11132
1981–93	−3.47005	−1.99648	1.67742	3.67881
1972–93	−0.97291	1.06712	1.24173	3.33962

Note: Pesticide data are in acre treatment terms. n.a. = not available.

part. Corn states have higher growth rates of nitrogen leaching and runoff than cotton states as well for the full time period, but this is due to very fast growth in the earlier years and dramatic relative declines (and reductions in nitrogen leaching) in the later part of the time period.

When we turn to the human risk– and fish risk–adjusted measures of pesticide leaching and runoff that are summarized in table 13.4, the patterns over the time period are even more dramatic, reflecting the changes in chemical use over our time period (which are accounted for in our risk adjustment).[8] Here we see average reductions in groundwater contamination of almost 3 percent and surface water contamination of almost 5 percent when adjusted for risk to humans. When adjusted for risk to the fish population, we observe a decrease of more than 4 percent in groundwater contamination and a 5 percent *increase* in surface water contamination. Again, a glance at the state-by-state results reveals very wide variation.

Although the non–risk adjusted patterns are more clear-cut, we do see differences when we compare corn- and cotton-producing states: On average, corn-producing states show declines in fish risk–adjusted effects of pesticides, whereas, on average, the major cotton-producing states show increases over the entire time period, although both cotton and corn states show declines in the later years compared to the earlier years. In terms of the human risk–adjusted trends, both cotton and corn states showed increases in all but human risk–adjusted runoff over the earlier part of our period (1972–81), but have reduced both leaching and runoff over the later part (1981–93) of our time period. Recall that on average the major corn- and cotton-producing states exhibited positive (but falling) growth in non–risk adjusted leaching and runoff, with rates for corn-producing states exceeding those of cotton-producing states on average. In contrast, based on patterns for risk adjusted water pollution, corn-producing states have reduced their pesticide damages, especially relative to cotton-producing states. Thus, other things being equal, we would expect that the Malmquist-Luenberger model would signal lower productivity growth, es-

8. If there is strong variation from year to year, then these patterns may be obscured when we are looking at average annual changes, as we do here.

Table 13.4 **Growth Rates: Human– and Fish–Risk Adjusted Leaching and Runoff, Average Annual Growth Rates 1972–93**

	Human Risk		Fish Risk	
	Leaching	Runoff	Leaching	Runoff
Alabama	−5.08470	−1.06678	−0.20990	6.09136
Arizona	n.a.	−10.83268	n.a.	17.80097
Arkansas	−1.27859	3.40360	−4.09960	5.20793
California	3.24825	5.80737	−3.73905	17.19049
Colorado	−1.16701	−12.73851	−6.14268	−4.54310
Delaware	−2.05306	−6.86531	10.87384	−4.47528
Florida	−10.59873	−19.54001	−0.02943	7.87374
Georgia	−4.99913	−13.72472	3.41988	1.54338
Idaho	−9.78556	−13.78041	n.a.	−20.54799
Illinois	−1.20208	−5.39991	−16.98420	−4.12020
Indiana	−1.65013	−8.41646	8.37802	−3.38601
Iowa	−4.93930	−4.57139	−16.34977	−3.18199
Kansas	0.91712	−3.86589	−1.49433	1.23912
Kentucky	2.06691	1.34536	1.03638	1.61903
Louisiana	−1.17057	4.74926	−1.64338	11.82326
Maryland	−2.65318	−4.94753	1.17823	−0.06571
Michigan	−1.69295	−1.93151	−14.94175	0.38559
Minnesota	−7.97353	−1.74013	−21.99322	−0.44378
Mississippi	−4.89603	1.58262	8.43648	14.43836
Missouri	−1.80773	−11.42821	2.65703	−1.57733
Montana	3.42449	−3.64715	n.a.	−1.90672
Nebraska	−0.55471	−3.09422	−16.65064	2.17545
Nevada	n.a.	n.a.	n.a.	n.a.
New Jersey	−6.35944	9.83772	1.23985	−0.49122
New Mexico	−4.70814	−8.79969	−9.30233	−6.00275
New York	−4.35752	−0.85828	−4.13647	2.71222
North Carolina	−4.08713	−12.08633	4.46486	−0.34217
North Dakota	−12.60767	−4.53576	n.a.	3.42076
Ohio	−1.28696	−8.31534	−20.45495	−0.17986
Oklahoma	−5.13906	−6.80717	3.45326	−0.42510
Oregon	0.83694	4.42137	8.11467	−2.70753
Pennsylvania	−4.09475	2.86996	−6.62414	3.64236
South Carolina	−3.60931	−8.14836	1.32843	−0.42646
South Dakota	−12.84851	−3.35518	−24.51314	−3.90894
Tennessee	−2.32381	0.83153	1.89718	8.54155
Texas	−0.98647	−5.75428	−0.31741	4.98538
Utah	n.a.	−29.62295	n.a.	−40.13550
Virginia	−2.62515	−11.10284	8.18294	3.49778
Washington	−4.96490	−2.82736	−22.56023	−8.96142
West Virginia	−5.01294	−8.97672	n.a.	−6.41974
Wisconsin	−7.56621	−0.85298	−14.81612	1.11831
Wyoming	−8.16177	−11.15268	n.a.	−14.15974
United States	−2.69915	−4.83464	−4.29376	5.17032
Corn states				
1972–81	4.31171	−3.89088	3.69083	4.20984
1981–93	−7.78590	−6.16768	−26.36327	−6.55263
1972–93	−2.60121	−5.19191	−13.48294	−1.94014

Table 13.4 (continued)

	Human Risk		Fish Risk	
	Leaching	Runoff	Leaching	Runoff
Cotton states				
1972–81	1.77296	−2.60857	13.20231	9.02206
1981–93	−6.89547	−3.13453	−9.02549	8.98561
1972–93	−3.18043	−2.90912	0.50071	9.00123

Note: n.a. = not available.

pecially in the earlier years (relative to productivity without the undesirable outputs), in corn-producing states when we include indexes of pesticide runoff and leaching that are not adjusted for risk; and productivity improvements (relative to the goods-only case) when we include indexes of pesticide runoff and leaching that are adjusted for human and fish risk.

We include our disaggregated results in appendix C.[9] Here we focus on selected states and begin with our results using non–risk adjusted measures of water contamination; see table 13.5. We display annual average productivity growth rates in our data for two subperiods: 1972–81 and 1981–93. These were chosen to capture the observed trends in our measures of water contamination. The first two columns of data summarize the Malmquist productivity growth for the case in which we ignore bads; that is, we include only marketable agricultural outputs in our model. Generally, productivity increases on average between the two time periods. If we include the undesirable outputs in our model and penalize states for increases in water contamination (see the columns labeled "Malmquist-Luenberg"), the average growth rates are typically different, as expected.

To get a sense of whether the difference accords with intuition, we include in the last two columns an indicator of whether water contamination increased or decreased between the two time periods. For example, if we look at the pattern for Colorado under heading A, we see that productivity increased between the two time periods when we look at goods only. When we include nitrogen leaching and runoff, the productivity growth in each period is lower; that is, there is a positive gap between the goods-only index and the good and bad index (Malmquist-Luenberger), and that gap increases over time, which is consistent with the observed increase in this

9. We note that for the individual states, we encountered cases in which there were no solutions to what we call the "mixed period problems," that is, those in which data in one period are compared to the frontier in a different period. This occurred in both the standard Malmquist case and the Malmquist-Luenberger case. The number of such instances is recorded under the 0's columns in the appendix tables. We conjecture that data smoothing would reduce the incidence of nonsolutions, since we encountered very few problems when we constructed averages over the major cotton- and corn-producing states. We also suspect that the "bads" data—which are not generated in the same way as the rest of the data set—may not be consistent with the production theoretic model that underlies our analysis.

Table 13.5 **Average Change in Productivity in Selected States: 1972–81 versus 1981–93**

	Malmquist (goods only)		Malmquist-Luenberger (goods and bads)		Gap			Water Contamination	
	1972–81 (a)	1981–93 (b)	1972–81 (c)	1981–93 (d)	1972–81 (e)	1981–93 (f)	Change (g)	Up	Down
				A. Nitrogen Leaching and Runoff					
Colorado	0.9909	1.0393	0.9881	1.0340	0.0028	0.0053	0.0025	x	
Maryland	1.0008	1.0062	1.0019	1.0140	−0.0011	−0.00781	−0.0066		x
Michigan	1.0045	1.0162	0.9930	1.0021	0.0115	−0.0059	−0.0174		x
Nebraska	1.0144	1.0410	1.0343	1.0439	−0.0199	−0.0029	0.0170	x	
New York	1.0154	1.0182	1.0027	0.9945	0.0127	0.0237	0.0110	x	
Ohio	0.9993	1.0151	1.0159	1.0297	−0.0165	−0.0145	0.0020	x	
Pennsylvania	1.0071	1.0066	1.0076	1.0100	−0.0005	−0.0033	−0.0029		x
Texas	1.0146	1.0147	1.0166	1.0288	−0.0019	−0.0141	−0.0121		x
Wisconsin	1.0075	1.0171	0.9873	1.0507	0.0202	−0.0335	−0.0537		x
				B. Pesticide Leaching and Runoff					
Colorado	0.9909	1.0393	0.9851	1.0415	0.0058	−0.0022	−0.0081		x
Maryland	1.0009	1.0062	1.0026	1.0108	−0.0019	−0.0046	−0.0028		x
Michigan	1.0045	1.0162	0.9956	1.0163	0.0088	−0.0001	−0.0089		x
Nebraska	1.0160	1.0355	1.0127	1.0335	0.0033	0.0020	−0.0013		x
New York	1.0154	1.0182	1.0030	1.0020	0.0124	0.0162	0.0039	x	
Ohio	0.9993	1.0151	0.9947	1.0157	0.0046	−0.0006	−0.0051		x
Pennsylvania	1.0071	1.0066	1.0084	1.0084	−0.0013	−0.0018	−0.0005		x
Texas	1.0146	1.0147	1.0046	1.0274	0.0100	−0.0126	−0.0226		x
Wisconsin	1.0075	1.0171	1.0004	1.0099	0.0071	0.0072	0.0001	x	

Note: Values in column e = a − c; in column f, b − d; in column g, f − e.

type of pollution in Colorado over the two time periods. Generally, we would expect to see a positive value in the gap-change column when pollution increases, and a negative gap change when pollution declines.

The data under heading B confirm this pattern for our other non–risk adjusted measures of pollution: pesticide leaching and runoff. For example, the decline in this measure of pollution for Colorado results in a higher measured average growth in productivity in the 1981–93 period using the Malmquist-Luenberger index (which accounts for bads) than in the simple Malmquist measure (which includes only good outputs).

Table 13.6 includes a sample of our results when we use the risk adjusted measures of pollution. We note that, on average, trends in these adjusted measures of pollution—with the exception of fish risk–adjusted runoff—decline over the 1972–93 time period.[10] Thus we would expect to see Malmquist-Luenberger indexes (that adjust for pollution) that are higher than their unadjusted counterparts, particularly in the 1981–93 period for states that realize declines in these types of pollution. Comparing the growth rates in the second and fourth columns (or the difference in column f) confirms this result for both the case of fish risk– and human risk–adjusted measures of pollution.

As an even more general summary of these results, we calculate productivity growth for average cotton- and corn-growing states; the results are displayed in table 13.7. By partitioning the time period into the two subperiods 1972–81 and 1981–93, we see a pattern of falling pollution for all but the fish risk–adjusted measure of pollution in the cotton states. As expected, this yields average productivity growth in the latter period that is higher when we account for both goods and bads (Malmquist-Luenberger in 1981–93 column), than when we ignore them (Malmquist 1981–93 column).

Thus, for many states, we find that if we account for risk adjusted water contamination caused by the use of agricultural chemicals, agricultural productivity growth—especially in the latter part of the time period studied here—is higher than the traditional growth measures that ignore these byproducts. This is consistent with the general pattern of falling human– and fish risk–adjusted runoff and human risk–adjusted measures of leaching we observe in the raw data.

10. For example, we can trace the dramatic shifts in the use and composition of pesticides over the period between 1960 and 1993, which involved a major increase in the use of herbicides relative to insecticides and the substitution of more environmentally benign pesticides for highly toxic pesticides. In the early 1960s, concern about the environmental consequences was minimal. Rising concern in the mid-1960s ultimately resulted in the EPA ban on the agricultural use of many chemicals in the 1970s and 1980s, including DDT (1972) and toxaphene (1983), which had been widely used in cotton production. The banning of aldrin, chlordane, and heptachlor had similar effects for corn producers. Nevertheless, there are major corn herbicides, such as atrazine, that have not been banned, and that constitute major components of our indexes of pesticide pollution, especially for corn-producing states.

Table 13.6 **Average Change in Productivity in Selected States: 1972–81 versus 1981–93**

	Malmquist (goods only)		Malmquist-Luenberger (goods and bads)		Gap			Water Contamination	
	1972–81 (a)	1981–93 (b)	1972–81 (c)	1981–93 (d)	1972–81 (e)	1981–93 (f)	Change (g)	Up	Down
				A. Fish Risk–Adjusted Leaching and Runoff					
Indiana	1.0269	1.0432	0.9929	1.0528	0.0340	−0.0096	−0.0436		x
Illinois	1.0381	1.0682	1.0395	1.0728	−0.0014	−0.0045	−0.0031		x
Missouri	0.9967	0.9815	0.9854	0.9853	0.0113	−0.0037	−0.0150		x
Ohio	0.9993	1.0151	0.9927	1.0302	0.0066	−0.0151	−0.0217		x
Wisconsin	1.0075	1.0171	1.0097	1.0143	−0.0022	0.0028	0.0050	x	
				B. Human Risk–Adjusted Leaching and Runoff					
Arkansas	1.0360	1.0329	1.0515	1.0251	−0.0154	0.0078	0.0232	x	
Louisiana	1.0184	1.0218	1.0295	0.9827	−0.0111	0.0391	0.0502	x	
Nebraska	1.0160	1.0355	1.0156	1.0719	0.0003	−0.03632	−0.0366		x
Ohio	0.9993	1.0151	0.9832	1.0247	0.0162	−0.0096	−0.0257		x
Texas	1.0146	1.0147	1.0065	1.0206	0.0081	−0.0059	−0.0140		x

Table 13.7 **Average Change in Productivity in Hypothetical Corn and Cotton States: 1972–81 versus 1981–93**

	Malmquist (goods only)		Malmquist-Luenberger (goods and bads)		Gap			Water Contamination	
	1972–81 (a)	1981–93 (b)	1972–81 (c)	1981–93 (d)	1972–81 (e)	1981–93 (f)	Change (g)	Up	Down
Corn state									
Nitrogen	0.9967	0.9870	1.0061	0.9990	−0.0094	−0.0129	−0.0026		x
Pesticides	0.9967	0.9870	0.9986	1.1018	−0.0019	−0.0237	−0.0219		x
Human risk	0.9967	0.9870	0.9930	1.0046	0.0037	−0.0175	−0.0212		x
Fish risk	0.9967	0.9870	1.0060	1.0252	−0.0093	−0.0382	−0.0289		x
Cotton state									
Nitrogen	0.9919	0.9874	1.0126	1.0205	−0.0206	−0.0331	−0.0125		x
Pesticides	0.9919	0.9874	1.0108	1.0159	−0.0188	−0.0285	−0.0096		x
Human risk	0.9919	0.9874	1.0124	1.0131	−0.0205	−0.0257	−0.0052		x
Fish risk	0.9919	0.9874	1.0262	1.0252	−0.0343	−0.0336	0.0007	x	

Notes: Hypothetical corn state is averaged over Illinois, Indiana, Iowa, Michigan, Minnesota, Missouri, Nebraska, Ohio, Wisconsin, and South Dakota. Hypothetical cotton state is averaged over Alabama, Arizona, Arkansas, California, Georgia, Louisiana, Mississippi, North Carolina, Tennessee, and Texas.

13.6 Summary

In this paper we provide an overview of some approaches to modeling and measuring productivity in the presence of joint production of desirable and undesirable outputs. These have in common an axiomatic production theoretic framework, in which joint production is explicitly modeled using the notion of null-jointness proposed by Shephard and Färe; and weak disposability of outputs is imposed to model the fact that reduction of bad outputs may be costly.

In measuring productivity in the presence of undesirable outputs, traditional growth-accounting and index number approaches face the problem that prices of the undesirable outputs typically do not exist, since such outputs are generally not marketed. An alternative that does not require price information is the Malmquist productivity index, which is based on ratios of Shephard-type distance functions. These do not require information on prices, which suggests that they would be an appropriate methodological tool. Although an improvement over ignoring undesirable outputs, the Malmquist index computed with bads may not have well-defined solutions, and it effectively registers increases in the bads (as in the goods), ceteris paribus, as improvements in productivity.

In order to address these problems we introduce an alternative productivity index based on a generalization of the Shephard distance functions, namely, what we call directional distance functions. Not only are these distance functions generally computable in the presence of undesirable outputs, but they also allow us to credit firms for *reductions* in undesirable outputs while crediting them for *increases* in good outputs. The Malmquist-Luenberger index is constructed from directional distance functions but maintains the Malmquist multiplicative structure, allowing us to compare our results with the traditional Malmquist productivity index, which credits only for increases in good outputs. Although not included here, the Luenberger productivity indicator is another model that is based on directional distance functions but that has an additive structure.[11] All of these productivity indexes are computable using linear programming techniques very similar to traditional data envelopment analysis. Further attention should be paid, however, to dealing with mixed period problems with no solutions, perhaps through data-smoothing techniques.

We apply our methods to state level data recently made available by ERS, which include variables that proxy effects of pesticides and nitrates (found in fertilizers) on groundwater and surface water. As expected, we

11. The directional distance function, of course, combines both additive and multiplicative features. The additivity, and ability simultaneously to adjust inputs and outputs, while not exploited here, may be used to establish the duality of the directional distance function (which scales on outputs and inputs) to profits. See Chambers, Chung, and Färe (1998).

find that measured productivity differs when we account for bads—declines in water contamination were associated with improvements in the Malmquist-Luenberger index as expected, ceteris paribus. Future research plans include pursuit of hypothesis testing, using (for example) bootstrapping techniques.

Another potentially fruitful avenue of research in this area would be to compute the shadow prices of the undesirable outputs to provide a benchmark for the opportunity cost of reducing undesirable outputs from the production side, as proposed by Färe and Grosskopf (1998). We would also like to generalize our production model better to model the roles of the environment and of consumers who evaluate the risks imposed by changes in the quality of groundwater and surface water. Along these lines, Färe and Grosskopf also proposed development of a network model to include the interaction of the environment with bads and consumers; this could be employed to provide a benchmark for computing shadow prices that reflect consumer evaluation of reductions in agricultural by-products.

Appendix A

The Luenberger Productivity Indicator

Both productivity indexes discussed in sections 13.3 and 13.4 are multiplicative in nature. Here we introduce a productivity measure, which is additive. We follow W. E. Diewert (1993) and refer to the additive measure as an indicator. The indicator introduced here is an output-oriented version of the Luenberger productivity indicator introduced by Chambers (1996). It is based on the output-oriented directional distance function discussed in section 13.4 above. We define the index as

$$(38)\quad L_t^{t+1} = 1/2[\mathbf{D}_o^{t+1}(x^t, y^t, b^t; y^t, -b^t) - \mathbf{D}_o^{t+1}(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}) + \mathbf{D}_o^t(x^t, y^t, b^t; y^t, -b^t) - \mathbf{D}_o^t(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})].$$

As we did in section 13.4, we take the direction $(g_y, -g_b)$ to be the observed values of the good y and bad b outputs. Following the idea of Chambers, Färe, and Grosskopf (1996), the Luenberger index can be additively decomposed into an efficiency change and a technical change component,

$$(39)\quad \mathrm{LEFFCH}_t^{t+1} = \mathbf{D}_o^t(x^t, y^t, b^t; y^t, -b^t) - \mathbf{D}_o^{t+1}(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})$$

and

$$(40)\quad \mathrm{LTECH}_t^{t+1} = \frac{1}{2}[(\mathbf{D}_o^{t+1}(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}) - \mathbf{D}_o^{t}(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}) + \mathbf{D}_o^{t+1}(x^{t}, y^{t}, b^{t}; y^{t}, -b^{t}) - \mathbf{D}_o^{t}(x^{t}, y^{t}, b^{t}; y^{t}, -b^{t})],$$

respectively. The sum of these two components equals the Luenberger index. This index can be computed using the same programming problems described in the discussion of the Malmquist-Luenberger index.

In passing we note that one may also define a Luenberger productivity indicator based on a directional distance function, which, in addition to scaling on good outputs, also scales on the input vector. This feature implies that one cannot transform it into multiplicative form as we have done with the Malmquist-Luenberger index. It has the advantage, however, of being dual to the profit function, which implies that it is a natural component of profit efficiency. This type of Luenberger productivity indicator was employed by Chambers, Färe, and Grosskopf (1996).

Appendix B

Environmental Indicators of Nitrogen and Pesticide Leaching and Runoff from Farm Fields

Indicators of groundwater and surface water contamination from chemicals used in agricultural production, and trends over regions and over time in factors that are known to be important determinants of chemical leaching and runoff, have been used to calculate indexes for environmental contamination. The determinants include the intrinsic leaching potential of soils; cropping patterns; chemical use; chemical toxicity; and annual rainfall and its relationship to surface runoff and to percolation through the soil. Consequently, the indexes of undesirable outputs that have been estimated represent changes over time and over regions (states) in the potential for agricultural contamination of water resources. The changes are assumed to be useful proxies for actual contamination.

Eight indexes of undesirable outputs have been compiled for the 1972–93 period, and can be matched with recently completed series of conventional inputs and outputs to create a 21×48 panel of inputs and both desirable marketed and undesirable unmarketed outputs of agricultural production activities in the United States. The eight indexes include four that are not adjusted for risk of exposure to toxic chemicals, and four that include a risk adjustment. The four undesirable outputs that have been

This appendix was written by Richard Nehring (ERS) and Robert Kellogg (NRCS) based on several working papers, including those cited in the bibliography.

compiled with adjustments for environmental weights relating to weather and soil/chemical characteristics, but not adjusting for risk, are

1. nitrates in groundwater, measured as adjusted pounds of excess nitrogen,
2. nitrates in surface water, measured as adjusted pounds of excess nitrogen,
3. pesticides in groundwater, measured as adjusted acre treatments of pesticides, and
4. pesticides in surface water, measured as adjusted acre treatments of pesticides.

The new indexing approaches incorporate the diversity of soil and climatic conditions across the United States into base-year environmental weights by estimating intrinsic vulnerability factors for each of the 3,041 counties in the United States. These environmental weights are converted to indexes of pesticide contamination using county-level crop production statistics and the best available pesticide use estimates by crop and by region. Indexes of nitrate contamination are constructed by multiplying county-level estimates of excess nitrogen from crop production by the county-level environmental weights. The non–risk adjusted pesticide leaching index was derived by adapting the field-level screening procedure used by USDA's National Resource Conservation Service (NRCS) to help farmers evaluate the potential for pesticide loss from a field, and extending the procedure to the national level. All indexes represent chemical loss at the edge of the field for runoff and at the bottom of the root zone for leaching.

The four undesirable outputs that have been compiled with adjustments for environmental weights relating to climate and soil/chemical characteristics, including risk, are

1. pesticides in groundwater, measured as adjusted pounds of pesticide based on chronic human exposure in drinking water;
2. pesticides in groundwater, measured as adjusted pounds of pesticide based on chronic safe levels for fish;
3. pesticides in surface water, measured as adjusted pounds of pesticide based on chronic human exposure in drinking water; and
4. pesticides in surface water, measured as adjusted pounds of pesticide based on chronic safe levels for fish.

The pesticide and nitrogen indexes reflect land-use soil characteristics of about 160,000 sample points for 1982 and 1992 and are based on USDA's 1992 National Resources Inventory (NRI). The NRI was used to determine the percent composition of soil types in each resource subregion by crop. The percent composition for 1982 was applied to 1972–86, and the percent composition for 1992 was applied to 1987–93.

Estimates of the eight undesirable indicators are based on major crop production that accounts for the bulk of chemical use in most states analyzed. For pesticide indexes adjusted for risk we used county data on acres planted for seven crops—barley, corn, cotton, rice, sorghum, soybeans and wheat. For nitrogen indexes and pesticide indexes not adjusted for risk, we used county data on acres planted and yields for the same seven crops. County data on acres planted and yields are available from the National Agriculture Statistics Service (NASS) for 1972 to the present.

The pesticide-use time series was derived from two sources, USDA and Doane's. The Doane Pesticide Profile Study provided a database of application rates and percent acres treated by chemical, crop, and year for 1987–93 for the United States as a whole and broken down into seven agricultural production regions. For 1972–86, the Doane pesticide use data and NASS chemical use surveys for selected years were used to generate similar estimates. A total of approximately 200 pesticides was included with somewhat greater coverage in more recent years (i.e., 1986–93 compared to 1972–86), and with greater coverage in the acre treatment formulation than in the risk adjusted index. Pesticide use parameters for all years are made for each of seven Doane reporting regions. Application rates and estimates of percent of acres treated for each chemical used in these seven regions were imputed to the 2,200 resource polygons for each crop.

State-level nitrogen fertilizer application rates were obtained from NASS and ERS survey data on commercial fertilizer applications. Annual data were available for ten major corn states, six major cotton states, and sixteen major wheat states. For other states, application rates were estimated based on other survey data rates used in neighboring states. All nitrogen application rates used represent average rates for the state. Excess nitrogen is defined as the difference between the amount of nitrogen applied from all sources (chemical fertilizers plus soybean and legume credits) and the amount of nitrogen removed in the crop production process (see Kellogg, Maizell, and Goss 1992). During 1972–93, residual nitrogen from barley, corn, cotton, rice, sorghum, soybeans, and wheat accounted for the bulk of residual nitrogen in most of the states analyzed.

In addition to the soil and chemical environmental weights as previously described, the risk adjusted pesticide indicators involve estimation of environmental risk. Environmental risk was estimated using threshold exceedence units (TEUs). Threshold concentrations used for each chemical correspond to the maximum safe level for human chronic exposure in drinking water. Where available, water quality standards were used. For other pesticides, estimates of the maximum safe level were made from published toxicity data. For each chemical used on each crop and soil type in each resource substate area, the per-acre pesticide loss concentration was calculated and then divided by the threshold concentration. Where the threshold concentration was exceeded, the ratio was multiplied by the acres treated to obtain estimates of TEUs. TEUs per substate area were obtained by summing TEUs over chemicals, crops, soil type, and resource polygons in each substate area. TEUs per state were obtained by converting substate estimates to state estimates using conversion factors derived from the National Resource Inventory. This procedure was repeated for each year in the time series to produce a spatial-temporal environmental indicator. Separate indicators were constructed for pesticides in leachate and pesticides dissolved in runoff. Irrigated acres were included in the total acres, but were not treated differently than nonirrigated acres with respect to the potential for pesticide loss. The fish-related indicators were calculated using threshold concentrations based on chronic "safe" levels for fish.

Other measures of outputs and inputs needed to estimate TFP growth are calculated only as state aggregates, so the eight undesirable outputs need to be aggregated to the state level. Since changes in fertilizer and pesticide use, environmental loadings from these chemicals, and computed environmental weight vary dramatically by state and county, this aggregation is the important last step in the index construction, making possible an accounting of the geographic diversity of the potential for water contamination. Some summary information by state is displayed in table 13B.1.

Table 13B.1 Pesticide Leaching and Runoff Scores as Percent of U.S. Total, 1984 versus 1993

	Leaching Score		Runoff Score	
	1984	1993	1984	1993
Colorado	1.06	1.07	0.26	0.28
Illinois	7.48	9.36	11.63	10.04
Indiana	5.87	7.37	5.79	5.40
Kansas	2.48	4.19	3.83	4.51
Kentucky	2.68	2.71	1.54	1.59
Maryland	1.30	1.17	0.38	0.29
Michigan	3.09	3.32	0.94	0.96
Minnesota	2.48	2.83	4.22	4.05
Montana	0.30	0.41	0.23	0.40
Nebraska	6.03	8.79	3.30	3.19
New York	1.14	1.07	0.22	0.38
Pennsylvania	2.06	1.81	0.85	0.80
South Dakota	0.81	1.13	1.28	1.58
Texas	1.52	2.18	3.15	3.93
Virginia	2.05	1.26	0.33	0.24
West Virginia	0.07	0.05	0.02	0.02
Wisconsin	2.65	2.79	0.88	1.04
Subtotal	43.07	51.51	38.85	38.70
Major corn states	38.17	46.98	47.12	43.57
Major cotton states	38.37	31.44	42.22	45.01
Other states	23.46	21.58	10.66	11.42
Total	100.99	100.00	100.00	100.00

Appendix C
State-Specific Results

	Average Annual Change in Productivity, 1972–93								
		Pesticides				Nitrates			
	Malmquist (goods)	Malmquist	0's	Malmquist-Luenberg	0's	Malmquist	0's	Malmquist-Luenberg	0's
Alabama	1.0097	0.9985		n.s.		1.0021		1.0187	15
Arizona	1.0090	0.9292	3	0.9509	20	0.9731	15	n.s.	
Arkansas	1.0033	0.9403	14	n.s.		1.0621		n.s.	
California	1.0200	1.0366	7	1.0172		1.1040	14	1.0620	15
Colorado	1.0119	1.0028		1.0075		1.0112	2	1.0051	
Delaware	1.0406	1.0322		n.s.		1.0062		n.s.	
Florida	1.0234	0.9690		0.9811	18	1.0223		1.0055	16
Georgia	1.0234	0.9838		n.s.		1.0054		n.s.	
Idaho	1.0240	1.0173		1.0203	6	1.1215	19	1.0508	15
Illinois	1.0281	1.0284		1.0289	4	1.0491		n.s.	
Indiana	1.0223	1.0201		1.0242	2	1.0139		1.0043	15
Iowa	1.0043	0.9997		1.0051		1.0070		1.0600	17
Kansas	1.0164	1.0190		1.0143		1.0323		1.0182	10
Kentucky	1.0059	1.0074		1.0040		1.0078		1.0016	5
Louisiana	1.0161	1.0089	11	n.s.		1.0221		1.0001	9
Maryland	1.0063	1.0081		1.0004		1.0058		1.0016	

Michigan	1.0091	1.0067		1.0051		1.0059		1.0070	
Minnesota	1.0031	0.9965		0.9982		1.0175	7	0.7967	20
Mississippi	1.0233	1.0201	11	n.s.		1.0276		n.s.	
Missouri	0.9950	1.0002		1.0020		1.0185		1.0010	8
Montana	1.0085	0.9920		1.0079		1.1133	17	0.9528	
Nebraska	1.0244	1.0266		1.0221		1.0350		1.0456	3
Nevada	0.9992	n.s.		n.s.		n.s.		1.0325	5
New Jersey	1.0102	1.0012		1.0164	6	1.0034		1.0017	
New Mexico	1.0072	0.9984		0.9829		1.0261	2	1.0034	
New York	1.0134	0.9982		0.9995		0.9990	3	0.9941	1
North Carolina	1.0354	1.0362		1.0435	11	1.0326		n.s.	
North Dakota	1.0263	1.0108	8	0.9971		0.9467	17	0.9981	7
Ohio	1.0039	1.0057		1.0041		1.0147		1.0137	3
Oklahoma	0.9953	0.9793		0.9974		1.0074	11	n.s.	
Oregon	1.0058	n.s.		0.9959		n.s.		n.s.	
Pennsylvania	1.0056	1.0084		1.0069		1.0020	1	1.0007	
South Carolina	1.0115	0.9800		n.s.		1.0016		n.s.	
South Dakota	1.0094	0.9895		0.9972		0.7654	19	0.9249	13
Tennessee	1.0093	1.0066		1.0270	5	1.0088		1.0199	5
Texas	1.0127	1.0268	1	1.0157		1.0139		1.0179	2
Utah	1.0095	n.s.		1.0129	1	1.0424	16	1.0028	16
Virginia	1.0090	1.0050		1.0079		1.0055		1.0100	10
Washington	1.0030	n.s.		n.s.		n.s.		n.s.	
West Virginia	1.0035	1.0169		1.0135		0.9999		1.0048	
Wisconsin	1.0116	0.9925		1.0053		0.8611	11	1.0112	
Wyoming	1.0112	0.9593		0.9961	1	0.9914	15	1.0063	1

(*continued*)

Appendix C. (continued)

		Human Risk–Adjusted				Fish Risk–Adjusted			
	Malmquist (goods)	Malmquist	0's	Malmquist-Luenberg	0's	Malmquist	0's	Malmquist-Luenberg	0's
Alabama	1.0097	1.0139	1	1.0136	3			n.s.	
Arizona	1.0090	n.s.		1.0183	1	n.s.		1.0263	17
Arkansas	1.0033	1.0294		1.0340	4	1.1319		1.0401	20
California	1.0200	0.9561	11	1.0010	5	1.2457	2	1.0156	1
Colorado	1.0119	1.0297		1.0172		1.0343		1.1309	0
Delaware	1.0406	1.0868	5	1.0001	15	1.0374	3	1.0963	18
Florida	1.0234	n.s.		n.s.		1.1310	4	1.0488	15
Georgia	1.0234	1.1407	4	n.s.		0.9555		n.s.	
Idaho	1.0240	n.s.		1.0389		n.s.		0.9961	9
Illinois	1.0281	0.9959		n.s.		1.0585	2	1.0687	2
Indiana	1.0223	0.9981		1.0218	5	1.0382		1.0202	2
Iowa	1.0043	0.9592		n.s.		1.0651		1.0687	16
Kansas	1.0164	1.0263		1.0166		1.0348		1.0194	15
Kentucky	1.0059	1.0047		1.0016		1.0112		0.9967	1
Louisiana	1.0161	1.0181		0.9983	3	1.1414		1.0185	16
Maryland	1.0063	1.0055	1	1.0168		1.0282	4	0.9778	13
Michigan	1.0091	0.9836		1.0137	5	1.0147		1.0278	2
Minnesota	1.0031	0.9697		n.s.		1.0420		1.0246	5

Mississippi	1.0233	1.0104		1.0283	2	1.1493		n.s.	
Missouri	0.9950	0.9860		0.9849	4	1.0552		0.9937	2
Montana	1.0085	n.s.		1.0015		n.s.		0.9736	8
Nebraska	1.0244	1.0255		1.0445	6	1.1066		1.0541	13
Nevada	0.9992	n.s.		n.s.		n.s.		1.0068	7
New Jersey	1.0102	1.0867	1	1.0081	2	0.5782	15	0.9508	17
New Mexico	1.0072	1.0097		1.0193		1.1456	3	1.0248	
New York	1.0134	1.0179		1.0079		0.9129	1	1.0144	2
North Carolina	1.0354	0.9692	1	1.0307	3	0.9680		1.0336	8
North Dakota	1.0263	1.0625	15	1.0513	3	1.1250		1.0263	9
Ohio	1.0039	0.9873		0.9978	2	1.0132		1.0101	1
Oklahoma	0.9953	1.0000		1.0083		1.1751	1	1.0002	8
Oregon	1.0058	n.s.		1.0093	2	1.0933	19	1.0216	6
Pennsylvania	1.0056	1.0073		1.0068		.08604	5	0.9811	16
South Carolina	1.0115	1.0164	2	1.0214	1	0.9473		0.8964	14
South Dakota	1.0094	1.0606	2	n.s.		1.0189		1.0135	9
Tennessee	1.0093	1.0052		1.0113		1.0786		1.0165	15
Texas	1.0127	1.0139		1.0116		1.0424	2	1.0243	14
Utah	1.0095	n.s.		n.s.		n.s.		0.9767	20
Virginia	1.0090	0.9724	1	1.0093	1	0.9533		0.9984	1
Washington	1.0030	n.s.		1.0163	3	1.1747		0.9891	10
West Virginia	1.0035	1.0051	1	1.0051		0.9882	2	1.1229	19
Wisconsin	1.0116	0.9951		1.0052	13	1.1167		1.0080	2
Wyoming	1.0112	1.0577	15	1.0055	2	0.9880	14	1.0078	8

Notes: n.s. = no solution. 0's columns contain number of infeasibilities.

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Comment Robin C. Sickles

General Comments

This paper is a new installment in a series of excellent papers by combinations of these authors and their colleagues dealing with the use of Malmquist indexes. Here the Malmquist index is used to decompose productivity growth into technology change and efficiency change (along the lines of Färe et al. 1994) while allowing for the presence of freely nondisposable byproducts—in particular, indexes of leaching and runoff from nitrogen fertilizer and pesticides and indexes of human and fish risk-adjusted effects of such. Ball, Färe, Grosskopf, and Nehring's data set is quite unique and is measured at the state level during the period 1972–93. If the "bads" are excluded, then their analysis suggests widespread productivity growth. When controlling for the effects of positively trending pesticide and fertilizer use, productivity growth falls. The results appeal to intuition and point to an effective tool for productivity index construction when negative externalities are not freely disposable. This work is cutting edge, makes a wonderful stand-alone empirical contribution, and has a modeling framework that can be ported to many other applications. A student and I, for example, are utilizing it in revising China's growth prospects in light of a proper valuation of its environmental pollution as its economy rapidly develops (Jeon and Sickles 2000).

The paper provides, among other things, an answer to the question " . . . how do we modify the standard productivity index to reflect the relative value to the producer (consumer) of outputs (services and attributes) when there are no market prices to serve this role?" The empirical

Robin C. Sickles is professor of economics and statistics at Rice University.

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setting for this work is in agriculture, and the authors do a yeoman's job in developing the empirical instruments and theoretical structure to provide answers to this question, as well as to many others. The paper also delivers something else (on which I will focus some of my remarks below), and that something is a way of dealing with the generic problem of the estimation of productivity for service industries and industries in which not only are attributes improperly given positive values, but outputs often are not given negative value. This problem has been explored extensively in the productivity literature, but the authors are quiet on this and the closely related literature on hedonics. A nice starting point is the edited volume by Griliches (1992), which the authors should integrate into their paper.

Clearly, in this type of index number construction, the standard role of statistics and inference is lost; I will address this issue below as well. However, let me point out at this juncture one problem with not having standard errors. Since contiguous states are not independent in agricultural pollution, bootstrapping exercises to attach standard errors to estimates need some form of dependency (Flachaire et al. 2000). This is a relatively new area of research in applied statistics and one that could benefit from a close perusal by the authors. Moreover, the innovation of the directional distance function in this analysis is not clear to me. What is the role of a preassigned direction for changing outputs instead of the proportional changes utilized in the standard Malmquist index?

Pollution produced in the course of agricultural production is called a "negative externality" in economics. Spillovers from publicly produced infrastructures, R&D, and so on have been a topic of serious research interest for some time. Might this literature and the strand promoted in this paper benefit from some cross-fertilization?

If one maintains a constant-returns-to-scale assumption throughout, then the decomposition of the Malmquist index into the technical and efficiency change components is accurate. However, under a variable-returns-to-scale assumption, their Malmquist index remains accurate, but their decomposition may not be completely accurate (Ray and Desli 1997; Grifell-Tatjé and Lovell 1998).

For those of us who question the nonstatistical nature of these indexes, one can link them in a very direct way to more conventional, production-based models and see to what extent it may be robust across methods. I will highlight these links below in a way that hopefully provides comfort to regression-based productivity analysts. Before I do, however, I will try to provide a statistical interpretation to the directional-distance measures that are introduced and analyzed in Ball, Färe, Grosskopf, and Nehring (chapter 13 in this volume; hereafter BFGN), based on a bootstrapping approach recently introduced by Simar and Wilson (2000a, b).

Construction of Stochastic Productive Efficiency Measures Using Programming and Bootstrapping

The Malmquist-Luenberger productivity index used in BFGN is based on the output-oriented directional distance function (Chung, Färe, and Grosskopf 1997). This is different from the Malmquist index, which changes the desirable outputs and undesirable outputs proportionally since one chooses the direction to be $g = (y^t, -b^t)$, more good outputs and less bad outputs. The rationale of this kind of directional choice is that there might be institutional regulations limiting an increase in bad outputs, in particular pollutant emission. To accomplish this the production technology is defined in terms of the output sets, that is, $P(x^t) = (y^t, b^t)|(x^t, y^t, b^t) \in F^t$ and the directional distance function then is defined as $\mathbf{D}_0^t(x^t, y^t, b^t; g) = \sup[\beta|(y^t + \beta g_y, b^t - \beta g_b) \in P(x^t)]$ where g_y and g_b are subvectors for y^t and b^t of the direction vector g.

The Malmquist-Luenberger productivity index is then defined as

$$\mathrm{ML}_0^{t,t+1} = \frac{1 + \mathbf{D}_0^t(x^t, y^t, b^t; y^t, -b^t)}{1 + \mathbf{D}_0^t(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})} \cdot \frac{1 + \mathbf{D}_0^{t+1}(x^t, y^t, b^t; y^t, -b^t)}{[1 + \mathbf{D}_0^{t+1}(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})]^{1/2}},$$

which easily can be decomposed into the product of Malmquist-Luenberger technical change and efficiency change indexes. Solutions based on solving a set of linear programming problems may be infeasible if the direction vector g passing through the output set at $t + 1$ is not producible for technologies existing at time t.

The index numbers outlined above provide us with only point estimates of productivity growth rates and the decompositions into their technical and efficiency components. Clearly there is sampling variability and thus statistical uncertainty about this estimate. In order to address this issue, we begin by assuming a data generating process (DGP) where production units randomly deviate from the underlying true frontier. These random deviations from the contemporaneous frontier at time t, measured by the distance function, are further assumed to result from inefficiency. Using the Simar and Wilson (2000a, b) bootstrapping method, we can provide a statistical interpretation to the Malmquist or Malmquist-Luenberger index.

The following assumptions serve to characterize the DGP.

1. $[(x_i, y_i, b_i), i = 1, \ldots n]$ are independently and identically distributed (i.i.d.) random variables on the convex production set.

2. Outputs y and b possess a density $f(\cdot)$ whose bounded support $D \subseteq R_+^q$ is compact where q is the numbers of outputs.

3. For all (x, y, b), there exist constant $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that $f(\mathbf{D}_0(x, y, b; y, -b)|x, y, b) \geq \varepsilon_1$ for all $\mathbf{D}_0 \in [0, \varepsilon_2]$.
4. For all (x_i, y_i, b_i), $\mathbf{D}_0(x, y, b; y, -b)$ has a conditional probability density function $f(\mathbf{D}_0|x, y, b)$.
5. The distance function $\mathbf{D}_0$ is differentiable in its argument.

Under the those assumptions, $\hat{\mathbf{D}}_0$ is a consistent estimator of $\mathbf{D}_0$, but the rate of convergence is slow. The random sample $\chi = [(x_i, y_i, b_i), i = 1, \ldots, n]$ is obtained by the DGP defined by assumptions 1–4, and bootstrapping involves replicating this DGP. It generates an appropriately large number B of pseudo-samples $\chi^* = [(x_i^*, y_i^*, b_i^*), i = 1, \ldots, B]$ and applies the original estimators to these pseudo-samples. For each bootstrap replication $b = 1, \ldots, B$, we measure the distance from each observation in the original sample χ to the frontiers estimated for either period from the pseudo data in χ^*. This is obtained by solving

$$\hat{\mathbf{D}}_0^{t*}[x^{t+1}(k'), y^{t+1}(k'), b^{t+1}(k'); y^{t+1}(k'), -b^{t+1}(k')] = \max \beta,$$

subject to

$$(1 + \beta)y_m^{t+1}(k') \leq \sum_{k=1}^{K} z^t(k) y_m^{t*}(k) \qquad m = 1, \ldots, M$$

$$\sum_{n=1}^{N} z^t(k) b_n^{t*}(k) = (1 - \beta) b_n^{t+1}(k') \qquad n = 1, \ldots, N$$

$$\sum_{l=1}^{L} z^t(k) x_l^{t*}(k) \leq x_l^{t+1}(k') \qquad l = 1, \ldots, L$$

$$z^t(k) \geq 0 \qquad k = 1, \ldots, K.$$

For two time periods, this yields bootstrap estimates $[\hat{\mathbf{D}}_0^{t*|t}(b), \hat{\mathbf{D}}_0^{t*|t+1}(b), \hat{\mathbf{D}}_0^{t+1*|t}(b), \hat{\mathbf{D}}_0^{t+1*|t+1}(b)]$ for each decision-making unit (DMU). These estimates can then be used to construct bootstrap estimates $\widehat{\mathrm{ML}}_0(b)$, $\widehat{\mathrm{MLECH}}_0(b)$ and $\widehat{\mathrm{MLTCH}}_0(b)$. The bootstrap method introduced by Efron (1979) is based on the idea that if the $\widehat{\mathrm{DGP}}$ is a consistent estimator of DGP, the bootstrap distribution of $\sqrt{n}Q\,[\hat{\mathbf{D}}_0^*(b), \hat{\mathbf{D}}_0]$ given $\hat{\mathbf{D}}_0$ is asymptotically equivalent to the sampling distribution of $\sqrt{n}Q(\hat{\mathbf{D}}_0, \hat{\mathbf{D}}_0)$ given the true probability distribution $\mathbf{D}_0$ where $Q(\cdot,\cdot)$ is a reasonable function. The confidence interval of the estimator then can be computed by noting that the bootstrap approximates the unknown distribution of $(\widehat{\mathrm{ML}}_0^{t,t+1} - \mathrm{ML}_0^{t,t+1})$ by the distribution of $[\widehat{\mathrm{ML}}_0^{t,t+1}(b) - \widehat{\mathrm{ML}}_0^{t,t+1}]$ conditioned on the original data set. Therefore, we can find critical values of the distribution, a_α, b_α by simply sorting the value $[\widehat{\mathrm{ML}}_0^{t,t+1}(b) - \widehat{\mathrm{ML}}_0^{t,t+1}]$ $b = 1, \ldots, B$ and then find $(\alpha/2)$ percentile and $[100 - (\alpha/2)]$ percentile values. We can also correct finite-sample bias in the original estimators of the indexes using the bootstrap estimates. The bootstrap bias estimate for original estimator $\widehat{\mathrm{ML}}_0^{t,t+1}$ is

$$\widehat{\mathrm{bias}}_B(\widehat{\mathrm{ML}}_0^{t,t+1}) = \frac{1}{B}\sum_{b=1}^{B}\widehat{\mathrm{ML}}_0^{t,t+1}(b) - \widehat{\mathrm{ML}}_0^{t,t+1}.$$

Therefore, the bias corrected estimate of ML_0^{t+1} is computed as

$$\widehat{\widehat{\mathrm{ML}}}_0^{t,t+1} = \widehat{\mathrm{ML}}_0^{t,t+1} - \widehat{\mathrm{bias}}_B\,[\widehat{\mathrm{ML}}_0^{t,t+1}] = 2\widehat{\mathrm{ML}}_0^{t,t+1} - \frac{1}{B}\sum_{b=1}^{B}\widehat{\mathrm{ML}}_0^{t,t+1}(b).$$

The variance of bias-corrected estimator will be $4\mathrm{var}(\widehat{\mathrm{ML}}_0^{t,t+1})$ as $B \to \infty$. The bias-corrected estimator can have higher mean square error than the original estimator. So, we have to compare $4\mathrm{var}(\widehat{\mathrm{ML}}_0^{t,t+1})$, the mean squared error of $\widehat{\widehat{\mathrm{ML}}}_0^{t,t+1}$ with the $\mathrm{var}(\widehat{\widehat{\mathrm{ML}}}_0^{t,t+1}) + [(\widehat{\mathrm{bias}}_B(\widehat{\mathrm{ML}}_0^{t,t+1})]^2$, the mean squared error of the original estimator $\widehat{\mathrm{ML}}_0^{t,t+1}$. $\mathrm{Var}(\widehat{\mathrm{ML}}_0^{t,t+1})$ can be estimated as the sample variance of the bootstrap estimators $[\widehat{\mathrm{ML}}_0^{t,t+1}(b)]_{b=1}^{\mathrm{B}}$. The bias-corrected estimator will have higher mean squared error if $\mathrm{var}[\widehat{\mathrm{ML}}_0^{t,t+1}(b)] > 1/3[\widehat{\mathrm{bias}}_B(\widehat{\mathrm{ML}}_0^{t,t+1})]^2$. An eleven-step bootstrapping algorithm suggested in Simar and Wilson (2000a, b), which replicates the DGP but which assumes i.i.d. errors recently has been implemented for this model by Jeon and Sickles (2000).

Construction of Productive Efficiency Measures Using Regression-Based Procedures

The radial measures of technical efficiency the authors consider in this paper are based on the output distance function. The goal of parametric, semiparametric, and fully nonparametric (as well as nonstatistical) linear programming approaches is to identify the distance function and hence relative technical efficiencies. The output distance function is expressed as $D(X, Y) \leq 1$, where Y is the vector of outputs and X is the vector of inputs. The output distance function provides a natural radial measure of technical efficiency that describes the fraction of possible aggregated outputs produced, given chosen inputs. For a J-output, K-input technology, the deterministic distance function can be approximated by $[(\Pi_1^J Y_j^{\gamma_j})/(\Pi_1^K X_k^{\beta_k})] \leq 1$ where the coefficients are weights that describe the technology of the firm. When a firm is producing efficiently, the value of the distance function equals 1 and it is not possible to increase the index of total output without either decreasing an output or increasing an input. Random error and firm effects could enter the output distance function in any number of ways. If we shift the output distance function by an exponential function of these terms (in much the way technical change is treated in traditional, single-output production functions), then, following Lovell and colleagues (1994) by multiplying through by the denominator, taking logarithms of outputs and inputs, and imposing the required linear homogeneity of the distance function in outputs, the distance function can be rewritten as

$$-\ln y_J = \sum_{j=1}^{J-1} \ln y^*_{jit}\gamma_j - \sum_{k=1}^{K} \ln x_{kit}\beta_k + \alpha_{it} + \varepsilon_{it},$$

where y_J is the normalizing output, $y^*_{jit} = (y^*_{jit}/y_J)$; and where α_{it} are one-sided (negative in this formulation) efficiency effects, and ε_{it} are random errors. The panel stochastic distance frontier thus can be viewed as a generic panel data model where the effects are interpreted as firm efficiencies and which fits into the class of frontier models developed and extended by Aigner, Lovell, and Schmidt (1977), Meeusen and van den Broeck (1977), Schmidt and Sickles (1984), and Cornwell, Schmidt, and Sickles (1990, hereafter CSS).

Parametric estimation can be carried out by conventional least squares or instrumental variables. Assuming that technological changes diffuse to all firms in the industry, firm-specific efficiencies can be distinguished from technology change, and the total of these, productivity change, can be estimated. Alternative mle estimators that rely on parametric specifications of the composed error are also available.

Semiparametric estimation can also be carried out in several ways. One can utilize a Robinson-type estimator for the mean of the stochastic distance frontier or use kernel-based procedures to model certain dependency structures between the random effects (α_i) and selected regressors, such as the right-hand side y's.

Park, Sickles, and Simar (1998) develop a framework for estimating the sort of model in which we are interested, namely, a panel model in which the stochastic efficiency effects are allowed to be correlated with selected regressors (in particular the y's), thus ensuring the endogenous treatment of multiple outputs in this regression-based distance function specification. Derivation of the semiparametric efficient estimator for the slope coefficients and the corresponding estimator for the boundary function that leads naturally to the construction of a relative efficiency measure in terms of the distance function are found in Park, Sickles, and Simar (1998). In the empirical implementations, one can use the "within" estimator of CSS as the initial consistent estimator and the bootstrap method for selecting the bandwidth in constructing the multivariate kernel-density estimates.

Given the efficient estimator $\hat{\theta}_{N,T}$, α_i are predicted by

$$\hat{\alpha}_i = \overline{S}_i(\hat{\theta}_{N,T}).$$

Under the assumptions of the model above, Park, Sickles, and Simar (1998) prove that as T and $T\sigma^2_{N,T}$ go to infinity:

$$L_P = \left[\sqrt{T}(\hat{\alpha}_i - \alpha_i) \rightarrow N(0,\sigma^2)\right].$$

Relative technical inefficiencies of the ith firm with respect to the jth firm can be predicted by $\hat{\alpha}_i - \hat{\alpha}_j$. We are most interested in firm-relative efficiencies with respect to the most efficient firm: $\max_{j=1,\ldots,N}(\hat{\alpha}_j)$.

Although DEA models the technology flexibly, it does not allow for random error. This is a shortcoming that the semiparametric methods overcome while still allowing for flexibility in functional form. Semiparametric estimators based on kernel methods such as Nadaraya-Watson have not been extensively applied in the efficiency literature, especially for multioutput firms. The parametric output distance function can be modified in two ways. First, we can allow efficiencies to be time varying. Second, we can start by making minimal functional form assumptions on the inputs. The distance function can be rewritten as

$$Y_{it} = f(X_{it}) + Y^*_{it}\gamma + \alpha_{it} + \varepsilon_{it}.$$

We can include additional assumptions on the time-varying properties of the technical efficiencies. We apply a specification that is the same as in CSS. Several other authors have allowed efficiencies to change over time. CSS model efficiencies as a quadratic function, while Kumbhakar (1990) models efficiencies as an exponential function of time. Others include Battese and Coelli (1992) and Lee and Schmidt (1993), who allow for other model specifications. Semiparametric estimation proceeds in the following manner. Assuming that the inputs are not correlated with the effects, the conditional expectation for the distance frontier function is

$$E[Y_{it}|X_{it}] = f(X_{it}) + E[Y^*_{it}|X_{it}],$$

where the means of the random effects, α_{it}, are also uncorrelated with the inputs. Subtracting this conditional expectation from the distance function provides us with the model to be estimated,

$$Y_{it} - E[Y_{it}|X_{it}] = Y^*_{it} - E[Y^*_{it}|X_{it}]\gamma + \alpha_{it} + \varepsilon_{it},$$

where

$$f(x_{it}) = E[Y_{it}|X_{it}] - E[Y^*_{it}|X_{it}]\gamma.$$

The model is estimated in two steps. First, the conditional expectations are estimated. To estimate the conditional mean we can use a kernel-based nonparametric regression. Next, the transformed model can be estimated by the CSS estimator. The residuals are then used to estimate the parameters in the time-varying model.

These methods hopefully have demonstrated the isophorphism of regression-based alternatives to the programming-based methods employed by BFGN, and will the provide the productivity researcher with a framework for analysis that gives a more intuitive and familiar look to their methods. I trust that the index number constructions employed by them continue to be adopted and refined by researchers and practitioners in business and in the government.

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