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CHAPTER III

SOME THEORETICAL AND PRACTICAL DIFFICULTIES OF COMPARING LONG TERM INTEREST RATES AT DIFFERENT AND ESPECIALLY AT WIDELY SEPARATED DATES

A RATE of interest arises out of an exchange of present money for a promise to make one or more future money payments. It is a peculiar inverse function of the price paid for the promise. The promise is personal property—a *chose in action.* It may be bought and sold like any commodity. The problem of comparing the prices—or yields—of such promises at different dates is naturally very similar, in some respects, to the problem of comparing the prices of commodities at different dates. However, because the promise, if it be considered a commodity, is undoubtedly a commodity *sui generis,* the two problems are in some significant ways decidedly different.

Let us first notice an important similarity. Just as there are different commodities, so are there different promises. To talk of *the* price of bonds—or, inversely, of *the* rate of interest—is like talking of *the* commodity price. In any important market at any particular time there are many commodities and many commodity prices, many promises and many rates of interest. And the promises, like the commodities, differ not only as to their nature but also as to their quality. To compare call money rates on the New York Stock Exchange in 1890 with the yield of West Shore 4's in 1936 would be almost as absurd as to compare the price of potatoes in London in the year 1800 with the price of pig tin in the same city in the year 1900. And to compare the yield of a low grade bond at one date with the yield of a high grade bond at another date would be nearly as meaningless. For commodity prices or interest rates at different dates to be significantly

comparable, the prices must be the prices of identical or approximately identical commodities and the rates the returns from identical or approximately identical loans.

But here we strike an important difference between the two problems. It is much easier to decide whether two commodities at different dates are approximately identical than it is to decide whether two loans at different dates are approximately identical, though, even in the commodity price problem, this difficulty is much greater than it at first sight seems to be.

The brute materiality of physical objects tends to obscure the fact that, as economists, we are interested in only certain of their psychic implications. While the feeblest intelligence may grasp that the economic significance of a bond lies in the promise it represents and not in the mere piece of paper, it is not so easy to see that the economic importance of each and every economic good lies in its possibility of satisfying human desires and not in its mere physical presence, and that consequently a particular physical object may have great economic significance at a particular date even though its importance was negligible at an earlier date or will be so at a later date. Even if they were not subject to physical deterioration or change, innumerable manufactured objects would have an economic history more or less like that of a moustache cup, a hoop skirt, a spinning wheel, an hour glass or a prairie schooner. Though the particular physical object be unused and therefore remain physically unchanged, its economic significance (for all other purposes than those of the antiquarian) dribbles away like water out of a leaky pot. Even if a Model T Ford of the vintage of 1916 had been preserved in a vacuum, what would it sell for now? The value attached to particular economic goods changes with the introduction of new means of satisfying desires.

The change with time in the economic significance of physical objects is one of the chief obstacles encountered in the problem of attempting to compare the 'general level' of commodity prices at widely different dates. During the last twenty years there has been a great increase in the comfort and mechanical efficiency of the transportation that the automobile purchaser gets for his dollar. 'Automobile prices' are much lower than they were twenty years ago. But *exactly* how much lower? What can one mean by 'automobile prices'? What commodity can we use in 1936 to compare with a Model T Ford in 1916,

and what can we use in 1916 to compare with any of the standard models of 1936?

An automobile may present a rather extreme illustration of obsolescence, but a similar process goes on with respect to most commodities, and the longer the time elapsed the greater become the difficulties of adequate comparison. To compare the cost of living in Boston in 1938 with its cost in 1838 may not be quite so difficult as to compare the cost of living in a small village in the far north of Canada with the cost of living in a small village in the interior of Brazil, but the sources of at least one of the difficulties are the same. Many of the commodities that are used in the one time or place are quite different from those used in the other time or place. While buggies, candles, spinning wheels and clipper ships may have been important in the economic life of Boston one hundred years ago their present importance in the same city is virtually nil.

In Chapter II we drew attention to the fact that the relative economic importance of a particular type of loan may be much greater in one place than it is in another place at the same time, or at one time than it was at another time in the same place. In this respect the difficulties of comparing interest rates at different dates are somewhat similar to those to which we have just been referring in connection with commodity prices. However, we must now draw the reader's attention to some peculiarities of promises to pay that make interest rate comparisons even more difficult than commodity price comparisons.

Though their relative *economic* importance may change, there are many commodities whose *physical* characteristics are practically the same yesterday, today and forever. A ton of pig iron in 1938 may be physically the same as was a ton of pig iron in 1838. Systems of grading such commodities as wheat, corn or cotton make it possible to quote the prices of virtually the same physical things over long periods. But the problem of grading loans is quite another story.

A loan is not a physical thing. The buyer of a bond does not buy even future money, he buys only a *promise* to make future payments. And that promise may become as unsubstantial as was the grin of the Cheshire cat after the cat itself had vanished. Earnings may disappear and even apparently sound collateral become valueless. The price of a bushel of wheat or a ton of pig iron can be determined in the open market without knowledge of who grew the one or mined the other,

but the value of a bond cannot be even estimated from an analysis of its terms without considering the degree of confidence that should be placed in the promises it contains. The question, what should middling upland spot cotton sell for at the present moment on the New York Cotton Exchange has at least some meaning; but the question, what is a fair price at the present time on the New York Stock Exchange for a 4 per cent bond maturing in twenty years has none. To mean anything the latter question would have to tell us something about the 'grade' of the 4 per cent bond.

But the 'grade' of a bond is the grade of the promise it represents. For the yield of a bond at one date to be as legitimately comparable with the yield of another bond at another date as is the price of a pound of middling upland spot cotton at one date with the price of another pound of middling upland spot cotton at another date, the promise represented by the one bond at the one date should be the same or virtually the same as the promise represented by the other bond at the other date not merely with respect to terms [1] but also with respect to *goodness*.

But with respect to what *kind* of 'goodness' should the promises be equivalent? In the first place, it clearly cannot be that essential and intrinsic goodness—or lack of goodness—that a determinist philosopher might say the bond possessed on the date in question but which would not be made apparent until the date of maturity.[2] For, if we interpret 'goodness' in this manner, we must grade the promises of all bonds that, in fact, made all 'interest' payments and the 'principal' payment on the dates called for in the bonds as *perfect*—exactly one hundred per cent 'good'—during all the time the loans were outstanding. But the possession of this unforeknown and unforeknowable 'perfection' would be a quite fantastic reason for believing in the legitimacy and usefulness of comparing the yield of one such 'perfect' bond at one date with the yield of another such bond at another date. We must remember that the yields of such 'perfect' bonds differ tremendously on the *same* dates. Though two 4 per cent bonds maturing in twenty years may each meet all future obligations on time, the one bond may be selling at the present moment on a 3½ per cent basis and the other on a 6 per cent basis.

[1] For the usual bond, maturity and coupon rate.

[2] In case of default, perhaps not even then. Cf. Aristotle, *De Interpretatione*, Ch. 9.

In grading bonds at any particular date, we are concerned with how good the bonds *were,* not with how good they *turned out to be.* It is, of course, true that, in a metaphysical sense, how good they turned out to be was how good they really were. But prices and yields can be *directly* affected only by *forecasts* of the future, never by the *facts* of the future. It is, therefore, to forecasts that we must restrict our concept of goodness. It is true that to grade bonds on any basis of how good they *seemed* to be, or even *should* have seemed to be, is to lean upon a flimsy reed. But there clearly is nothing else to do.

Having come to this conclusion, we are faced with the question, should the grading be based on actual or on ideal forecasting, on how good the bonds seemed to be or on how good they *should* have seemed to be. At once we notice that any 'should have seemed' grading is tarred with the same stick as grading based on what the future eventually revealed. It is almost always clear, *after the event,* that, though the future was essentially unknowable, a more shrewd and intelligent analysis of the facts that *were* available should have prevented much of the bad forecasting that actually occurred.

And this difficulty cannot be overcome by arbitrarily limiting the range of facts upon which forecasts 'should have' been based. The essential element in any 'should have' system of grading must clearly be that no pertinent and important consideration shall be neglected. To assume, for example, that, for purposes of yield comparisons, bonds can be more correctly graded by some simple mathematical formula whose variables are all derivable from either present or past financial reports of the debtor corporation than they are in fact graded in the open market is to exhibit an optimism that is difficult if not impossible to defend.

The most intelligent bond buyers are of course interested in how their bonds are 'rated' by the better statistical services, but they desire and obtain more information than is yielded by the symbols AA or B1+. Some years ago the executives of a large statistical organization, which had for years been publishing bond 'ratings', finally came to believe so strongly in the reliability of their own ratings that they initiated a 'switching' service for bond owners. The theory upon which the service was based was that, if a client owned bonds that were selling on a definitely *lower* yield basis than the average yield of bonds of the same 'rating', he should sell them and 'switch' into bonds of the

same rating but selling on a *higher* yield basis. The 'service' was soon abandoned.

But, even if the difficulty of deciding how the bonds *should* have been graded were much less than it is, the question would still have to be answered, is this the kind of grading demanded by the problem? When comparisons of even commodity prices at different dates are being made, is not the relevancy of the prices dependent on what the buyers and sellers *thought* they were exchanging rather than on what they actually were buying and selling? Has not the comparative ease of discovering, for so many important commodities, what actually is being bought and sold obscured the fact that this consideration is, after all and paradoxical as it may sound, in some respects of only secondary significance?

Because a seemingly normal and healthy dairy cow that had been sold 'as is' by one farmer to another for one hundred dollars died the following day, the price paid is not described as a price for moribund cattle; though, if the condition of the cow had been apparent at the time of sale, she *should* have been priced on the basis of 'hide and tallow'.[3] If, before the Cripple Creek strike in Colorado, the land on which the mines were later located had been bought and sold as grazing land, no student of prices would now think of describing the early prices per acre as prices per acre of an extremely rich gold field. If a consignment of eggs were sold at a sheriff's sale, without recourse, and if the buyers had understood and believed that the eggs were at least relatively innocuous, the price would not be describable as a price of 'spots and rots'—even though that was the real status of the consignment.

Bonds vary in quality as much as do eggs; and the history of bond

[3] It is, of course, true that unless the farmer wanted the cow in order to supply his own family with milk, he was buying not merely a promise to provide that future commodity but more fundamentally a promise to provide future money income. The purchase of a cow could have turned out to be a poor *investment* not because the cow died but because of a fall in the price of milk. This is, however, completely outside the realm of ordinary commodity price history. The student of the history of commodity prices can and does brush such considerations aside. He presents a table showing dairy cattle prices at different dates without discussing whether the purchasers did or did not act wisely. He is recounting the prices of an economic good that is also a physical object and his definition is in terms of physical characteristics. His is a history of the prices of physical objects as those prices were made by people who believed they were buying things having such and such physical characteristics.

prices demonstrates conclusively that, unlike eggs, bonds are *usually* graded very incorrectly by the market—and the statisticians. Collapse of the credit of a corporation is seldom seen far in advance; on the other hand, innumerable bonds that pay on time all coupons and the face of the bond are graded low throughout their existence. This is, of course, exactly what might be expected. As the future cannot be known, bonds must be graded on a probability basis and, unless they are of an ultra-superior quality, the information available for grading them on such a basis is almost always quite inadequate.

And, though the market's rating must be considered as of a probability type, it undoubtedly is not arrived at by conscious mathematical calculations. Indeed, the attempt to make any definite and simple mathematical assumptions as to the elements of the probabilities involved easily leads to conclusions that run counter to what is commonly assumed to be fact—such conclusions, for example, as that the yields of low grade, high yield bonds should be expected to fluctuate *less* than the yields of high grade, low yield bonds. And the attempt to formulate mathematical assumptions whose development will not lead to results that conflict with facts or apparent facts all too easily leads to obviously *ad hoc* hypotheses.

The simplest of all probability hypotheses is that the probability of payment of each and every promised future payment is always the same, $\frac{9}{10}$, for example. With this assumption, the *price* of the bond would, on any specified date, be 90 per cent [4] of the price on the same date of a bond containing an identical set of promises but rated by the market as 'absolutely secure'. However, unless the bonds were perpetuities, the *yield* of the lower grade bond would fluctuate less than the yield of the 'absolutely secure' bond.[5]

Such an assumption as that the probabilities of payment are all equal is, of course, quite fanciful. It assumes that the successive probabilities

[4] Less in a community that did not care for gambling and possibly more in one that did.

[5] The lower the yield of a particular bond the greater percentage effect on its yield has a specified percentage change in its price. For example, if a 4 per cent bond maturing in eighteen years falls five per cent in price, from 100 to 95, its yield will increase ten and one quarter per cent (from 4.00 per cent to 4.41 per cent); but if it falls five per cent in price, from 120 to 114, its yield will increase fifteen per cent (from 2.60 per cent to 2.99 per cent).

are completely independent of one another and that the question whether a particular payment will or will not be met is unrelated to whether the preceding payments have been met.[6] But similar strange conclusions result from developing much more appealing hypotheses.

The simplest assumption having any appreciable air of reality is that no payment will be met unless all preceding payments have first been met, but that, as soon as one payment has been met, the probability that the next payment will be met is the same as previously had been the probability that the preceding payment would be met. Under this compound assumption, if the probability that the first payment will be met be designated p, the probability (as of the same date) that the second payment will be met will be p^2, and the probability that the nth payment will be met will be p^n. In these circumstances the functional relationship between the lower grade bond and the 'absolutely secure' bond is not (unless the bonds be perpetuities) simply expressible in terms of *prices*, though it is so expressible in terms of *yields*. On any specified date the R [note 7] of the lower grade bond will equal $\frac{1}{p}$ times the R of the 'absolutely secure' bond.[8]

[6] Even 'income' bonds are not exceptions. The payment or non-payment of preceding coupons is at least evidence of ability or lack of ability to pay the next coupon.

[7] R = the *multiplier* corresponding to the yield, e.g., if the bond is on a 4 per cent per annum basis, R = 1.04 (per annum). Under the semi-annual convention of the bond tables R would, of course, equal 1.02 (per half-year) (see Ch. II).

[8] The price of the lower grade bond is obtained by discounting, not its actually promised interest and principal payments, but their mathematical 'expectations' at the yield of the 'absolutely secure' bond. The price of the 'absolutely secure' bond equals

$$\frac{I}{R} + \frac{I}{R^2} + \ldots + \frac{I}{R^n} + \frac{100}{R^n}$$

and the price of the lower grade bond equals

$$\frac{Ip}{R} + \frac{Ip^2}{R^2} + \ldots + \frac{Ip^n}{R^n} + \frac{100\ p^n}{R^n}.$$

But it is immediately apparent that this value for the price of the lower grade bond is the same as would be obtained by substituting $\frac{R}{p}$ for R in the expression for the price of the 'absolutely secure' bond. I and n are, by the hypothesis of identical promises, the same in both bonds.

For definitions of I, n, and R see Ch. II.

The reader should note here, in passing, that, if the lower grade bond be assumed to differ from the higher grade bond not in the rates at which the future promises are discounted but in the valuations placed on those promises (the 'expectations'), it

For example if $p = \frac{99}{100}$ and if, on a particular date, the yield of the 'absolutely secure' bond were 2 per cent per half year, the yield of the lower grade bond would be 3-1/33 per cent per half year.[9] If the yield of the 'secure' bond advanced to 4 per cent per half year, the yield of the lower grade bond would become 5-5/99 per cent per half year—one and two-thirds times instead of double its former rate of 3-1/33 per cent. The yield of the lower grade bond equals a constant term plus a constant multiple of the yield of the 'secure' bond (see note 9). Its percentage fluctuations are therefore *smaller and not larger* than those of the yield of the secure bond.[10]

It would, of course, be possible to construct hypotheses with respect to the market's estimates of the probabilities of the various payments being met that would not be inconsistent with the requirement that low grade, high yield bonds should fluctuate in yield much *more* than high grade, low yield bonds. But any such hypothesis would necessarily be a mere mathematical curiosity.

(Footnote [8] concluded)
will have an *essentially* shorter 'duration', because the size of the 'expectations' would decrease as their distance in the future increased. Now we know that extent of fluctuation in yield with the passage of time is an inverse function of duration. But it is hard to say whether or not we have here any significant clue as to why low grade bonds might tend to fluctuate in yield more than high grade bonds.

[9] The R of the secure bond equals 1.02 and hence the R of the lower grade bond equals $1.02 \times \frac{100}{99}$ (see note 7). From this result we get 3-1/33 per cent per half year as the yield of the lower grade bond.

In general, if r_1 represent the yield of the 'absolutely secure' bond and r_2 the yield of the lower grade bond, r_2 will equal $100 \frac{(1-p)}{(p)} + \frac{r_1}{p}$ or a constant term plus a constant multiple of the yield of the secure bond.

[10] If the yield of the lower grade (higher yield) bond is to fluctuate exactly as the yield of the 'absolutely secure' bond, in other words if $R_2 - 1 = k\,(R_1 - 1)$, where k is greater than unity, then p, or the probability that each payment will be made if the preceding payment has been made, is such a function of the yields of the two bonds that the larger their yields the smaller is this probability of payment of the lower grade bond. This, of course, amounts to a *fluctuation in grade.*

As we have seen, $p = \frac{R_1}{R_2}$, but, because $R_2 - 1 = k\,(R_1 - 1)$,

$\frac{R_1}{R_2} = \frac{1}{k} + \left(\frac{k-1}{k}\right)\frac{1}{R_2}$ which increases and decreases as R_2 (or R_1, as $R_2 = 1 - k + kR_1$) does the contrary.

We saw in the preceding chapter that the yield of a bond was demonstrably an average, a complicated type of average it is true, but essentially an average. The assumption that the rates of discount used during successive future periods are identical was there proven quite unwarranted. Though it is for many purposes a convenient and useful fiction, it is always a fiction. We illustrated this fact by bringing to the surface the different rates for different future periods *implicit* but hidden in the yields of serial bonds, and we described and explained how the 'average' that we term the 'yield' of a bond tells us nothing about the rates of interest of which it is an average. We furthermore showed that there is no such custom of conscious and deliberate calculation as would lead to any significant market pronouncement concerning the various rates assigned to the different future periods. Indeed, to assume the existence of any such pronouncement would be almost as unscientific and indefensible as to indulge in a 'pathetic fallacy'.

We find ourselves, therefore, faced with a dilemma. We can be virtually certain that any mathematical hypothesis based on the assumption that the constituents of the yields are consciously considered would not give even an approximately true picture of how the grading is actually done and the yields actually arrived at. But to shut our eyes to the composite nature of the yields would be quite as disastrous.

And to top all, our hypothesis would be designed to fit not known facts but preconceived fancies. For, startling as it may sound, we do not *know* that lower grade bonds actually do fluctuate in yield appreciably more than do higher grade bonds. Indeed, there are strong reasons for suspecting that the excessive fluctuation in the yields of so-called lower grade bonds is primarily an indication of fluctuation in the *grades* assigned by the market to such bonds rather than an indication that violent fluctuations in yield normally accompany low, *but unchanging,* market grading.[11]

[11] Sometimes, though rarely, the long term movements of the yield of a particular bond of not quite the highest grade are, for many years, so nearly the same as the long term movements of the yields of bonds of the very highest grade as to suggest strongly that the grade of the slightly poorer bond has been relatively stable throughout the period. West Shore 4's of 2361 (Bond No. 70) is such a bond. As may be seen from Chart 2, the long term trends of the yields of that bond (see Line C of chart) from 1885 to 1930 showed no permanent drift away from the index of the yields of bonds of the very highest grade (Line B and Line D—which has the same *trend* as Line B). However, as may be seen from the chart, the intermediate

And averaging the yields presents no logical solution of the problem of comparing the yields of second grade bonds at widely separated dates. One who had not examined the data carefully and considered the problem critically might easily think that bond yields should, not merely practically but also theoretically, be much better adapted to the making of index numbers than are commodity prices. Surely, such an one might reason, bonds have a homogeneity that commodities do not possess. They are concerned with the satisfaction of one great and undifferentiated desire—the desire for money in the future, while the different desires that can be satisfied by different commodities are almost as varied and multiform as are human wants.

Is not the difference between a high grade bond and a low grade bond (of approximately the same duration) like the difference between a high grade and a low grade sample of the same commodity rather than like the difference between two disparate commodities? Is it not the difference between high grade raw cotton and low grade raw cotton or between good potatoes and poor potatoes rather than the difference between cotton and potatoes?

And is even a difference between two bonds that results from a great difference in their durations so fundamental as is the difference between the two commodities? Cannot the one bond be substituted for the other in a way that is quite impossible with the commodities? The proceeds received when a short term bond matures may be reinvested; and both short and long term bonds can always be sold. While an investor is holding a bond, is not the type of good he is receiving from it unaffected by the length of time that the loan has still to run?

And surely, if we were to restrict our choice of bonds to those whose durations were long and not too violently different, we could virtually eliminate the effects upon their yields of fluctuations in their grades by averaging the yields. Why not take a hint from students of changes in the general level of commodity prices? Simply select a representative group of bonds and follow them through from month to month and year to year by means of some kind of averaging.

The complete answer to this engaging but artless suggestion is that all the evidence indicates that the variations in the market's grading of individual bonds with the passage of time are of such a nature that

(Footnote 11 concluded)

and cyclical movements of the yields of West Shore 4's were, during all this long period of forty-five years, usually *less* than those of bonds of the very highest grade.

their effects could *not* be eliminated by any ordinary averaging of the yields of the bonds. The variations in grade, at different times, of individual bonds in a group are not of a chance or accidental type, fluttering back and forth about some constant norm but always showing the same relation to and distribution about that norm. Even in the form of averages, there is no stability to the grades. *There is no 'constant norm'*. The average and the whole distribution of grades undoubtedly drift in great secular swings, immense waves, and even up and down with the movements of the business cycle.

Though fluctuations in the yields of bonds that are not of ultra-superior quality, relatively to the yields of bonds of such quality, are not closely enough related to fluctuations in the earnings of the debtor corporations to make grading upon this basis feasible, they are definitely and unmistakably correlated with such fluctuations. And, because the earnings of nearly all corporations tend to rise and fall with the business cycle and, in any particular industry, to move together over long periods of time, the grades that the market assigns to the bonds inevitably tend to drift as a group and to move up and down together. Over nearly all the period covered by the statistics of this study the steady secular decrease in the 'spreads' of the yields demonstrates unmistakably that the lower grade railroad bonds were improving in grade *as a class*. When they were not doing so, they were declining in grade *as a class*.

It is, of course, true that the lack of confidence engendered by a business depression tends to increase the relative demand for bonds of the highest grade and to decrease the relative demand for bonds of lower grades, and it is also true that the absolute volume of bonds of the highest grade declines and the absolute volume of bonds of lower grades increases (as those that were formerly of the highest grade move into lower classes). And it is further true that the distribution of bonds by grades could be of such a type that the pressure of bonds new to the grade would be an inverse function of the grade. And it is hard to say *how much* of the increase in the 'spread' of the yields of a group of bonds of different grades during a pronounced decline in general business may be the result of any such vicious combination of depressing factors. But we must remember that such factors cannot materialize and become operative unless there first occurs a change in grading, and that, therefore, they cannot be held accountable for more

than a part of the increase in 'spread'. Chart 15 suggests strongly that at least their ultra long term effects are quite negligible.

And, finally, it is perhaps worth noting that the makers of even commodity price index numbers do not attempt to eliminate the effects of variations in *grade* by averaging. As far as possible they quote the prices of identical grades from year to year, and when changes in grade become necessary they adjust for them.

Ordinary index numbers of the yields of second grade bonds are to a greater or less extent like index numbers of stock prices, and such index numbers are, in one respect, almost *sui generis*. At least over long periods of time their chief interest lies in the picture they give of *changes in the nature* of the things whose prices are quoted. When the price of aluminum falls from a large number of dollars per pound to a small number of cents no one suggests that the figures indicate that aluminum was in the early days a distinctly different metal from what it was later. But a long-continued and pronounced rise in the price of the common stock of an aluminum producing corporation would usually [12] be accepted as proof that the market believed that the earning power of the corporation had increased—in other words, that there had been a change in the *nature* of the thing that was being bought and sold.[13]

This consideration does not, of course, even suggest that the economist should be uninterested in the movements of the prices and yields of second grade bonds—any more than that he should be uninterested in the movements of stock prices. But it does suggest in the strongest possible manner that he should consider carefully the meaning of such movements and realize the inherent difficulties of deducing conclusions concerning the movements of 'interest rates' from such material. The economic consequences of a collapse in the second grade bond market may be just as serious if it is primarily caused by fears of declining earnings as it would be if it were primarily caused by a rise in the 'preference for present over future goods'. And long term trends in the quality of second grade bonds may be quite as interesting as long

[12] In the absence of some important technical factor such as a radical change in the capitalization of the corporation.

[13] Of course some of the rise of a stock or second grade bond may result from a decline in the rate at which expected future dividends or interest payments are discounted. The presence of such an influence would usually be shown by a fall in the yields of long term bonds of the very highest grade.

term trends in interest rates as exemplified in the yields of bonds of the highest grade, but it is highly desirable not to confuse the two.

However, if for no other reason than that the problem is presented in a simpler form, the student of interest rates will tend to be primarily concerned with the yields of the very highest grade bonds rather than with the yields of those of lower grade. The difficulty of measuring the market's estimate of the former is distinctly less than of the latter. Though their yields may not properly be described as 'pure interest'—whatever that may be—there are strong reasons for believing that the estimates of their 'probability of payment' undergo no such radical changes as occur with bonds of lower grade. Though individual bonds that have been given the highest possible rating by both the market and the financial statisticians may within a few years fail to pay interest, no such serious variation in grade is found in the market's rating of AAA or A1+ bonds *as such.* Though it may not be the same *nominal* bond, there is always the possibility of using at the later date *some* bond that the market will apparently be rating the same or nearly the same as it rated the earlier bond. This, as we have already seen, is not true of lower grade bonds.

Bonds of the highest grade are bonds than which there are none better. To a very large extent, the market itself tells us which they are. They are, in general, those bonds that have the lowest yields. Yet an index number using each month the yield of the particular bond showing the lowest average yield during that month is not satisfactory, for the movements of the prices and yields of individual bonds of even the highest grade are often influenced by other factors than those affecting such bonds in general. For example, if the market for a particular bond is 'thin', relatively small purchases by a trustee or other insistent purchaser may run the price up beyond what it normally would be. Or again, rumors that the holders of a small 3 per cent issue selling for 85 may be offered par to clear the decks for a reorganization could easily raise the price to a point at which the yield of the bond would have no general economic significance. In Chapter IV we discuss at some length the problem of deciding what is the most representative yield for the very highest grade of bonds in view of the fact that it is inadvisable to trust entirely to the yield of the *individual* bond having the lowest yield.

Although confining one's consideration to bonds of the highest grade

may eliminate most of the difficulties encountered when the attempt is made to use lower grades, it does not remove them all. Rates depend not only on the security or reliability of the promise but also on what is promised. Time is of the essence of the interest problem and we saw in Chapter II that the rates imputable to different intervals of future time are almost never the same. Because of this fact the yields of long and short term bonds of even the highest grade would seldom even theoretically be the same.

And, because all bonds except perpetuities change their 'durations' with the passage of time, mere averaging of the yields of the same *nominal* bonds is open to the same criticism that could be levelled against comparing cattle prices on appreciably different dates by comparing the prices of identical cattle on the appreciably different dates. The age distribution of the cattle should be the same on the two dates but, as they are the *same* cattle, this cannot be true. If they are now each four and a half years old, a comparison with four years previous would give a result entirely different from what it would if they were now all ten or twelve years old. When we compare the prices of cattle at two different dates we do not take the prices of the same cattle. We take the prices of cattle that are not merely approximately the same grade but also of comparable ages. Even bonds of the highest grade are not exempt from this influence. If their maturities be at all close, they change appreciably in 'duration' with the passage of even a short period of time and this change affects definitely the essential nature of the loan.

From a strictly theoretical standpoint, it would seem necessary to find, for each date, bonds of the same coupon rate and maturity. However, the practical investigator will remember that rates depend on 'duration', rather than on 'years to maturity', and that, if the 'duration' of a bond is not too short, increases or decreases of a few years in the 'time to maturity' affect the yield only negligibly. And, of course, yields could (at least theoretically) be corrected by a statistically-derived equation relating yield to duration.[14]

[14] Practically, any such correction is dangerous. Because bonds whose quotations are usable for any particular dates are relatively few, and the scatter of their yields and durations is very considerable, it is difficult to discover whether the empirical (as opposed to logical) relationship between yields and durations is even of the same kind at different dates. Moreover, even if it were always the same and even if we knew exactly what it was, the wideness of the scatter of the data from which it was derived would introduce a high degree of probable error into

After security and duration, the theoretically next most interesting factor influencing the yield of bonds is taxation. A strong case can, of course, be made for considering the return from tax-free investments as theoretically a purer index of the 'preference for present over future goods' than is the return from taxed investments. Both types merit study. But they must not be confused or mixed. Under a graduated income tax law, it is impossible to express one as a function of the other without introducing a third variable. And, if the tax-exemption is only partial, as is usual, the complications of use and comparison are still further increased. With the introduction of the income tax into American finance, the whole status of such bonds of course completely changed.

A minor practical difficulty encountered by the economic historian who attempts to use the yields of tax-exempt or partly tax-exempt bonds is that, aside from Federal bonds and a very few municipal and state bonds, the markets have usually been relatively poor, and quotations (because sales were largely 'over the counter') less easy to obtain and less reliable than quotations for bonds of the larger corporations. In the earlier period covered by this study it was often impossible to identify state or municipal bonds from the printed titles in the price sheets. For example, for a number of years there were quotations on the Philadelphia Stock Exchange for 'Penna. 6's', but for most of the period we were unable to discover what particular bonds were being quoted, there being at all times different 6's of various maturities outstanding. Finally, the reader must remember that a tax-exempt municipal bond is not necessarily of higher grade than a corporation bond because it sells on a lower yield basis. Its 'probability of payment' may be very definitely less.

Other factors affecting the yields of bonds are theoretically less intriguing though many are of great practical importance. But their nature is such that their influence can seldom be allowed for.

Changes in yield resulting from changes in the relative importance of a particular bond in the general economy because of changes in its markets or marketability are practically impossible to measure or over-

(Footnote [14] concluded)
the statistically-determined constants. The artificiality of the resulting theoretical yields would offset any possible increase in their homogeneity.

However, the statistical questions involve interesting problems in multiple correlation and someone may at some time care to follow them further than we have attempted to do.

come. Indefiniteness in the promise itself, such as exists in callable bonds and in bonds payable in two or more currencies at the option of the holder, may under some circumstances not be important; but when it is, it is extremely difficult to correct for, and attempts to do so seem undesirable if bonds with less ambiguous promises are readily available. Convertible bonds and bonds carrying special privileges of any kind, such as 'circulation' privileges, present similar difficulties. The promise to make future money payments is only one of the elements determining their prices and yields. They are mongrels and it is next to impossible to measure the degree of their contamination.

Changes in markets and marketability may have appreciable effects on the yield of a bond. Paradoxical as it may sound, a case may be made for the contention that, if the yield of a bond is to represent a simon-pure long-term interest rate, the bond must have no market whatsoever. The purchaser must buy it because he is willing to exchange present money for certain specified future money payments. If the yield is to be an index of *his* preference for present over future money he must buy the bond because it promises to pay him certain sums of future money *on the dates specified in the bond.* If he buys $100,000 worth of United States Treasury 3's of 1951 with the intention of selling them in five or six months, because he wishes to make a short term investment and calculates that he can obtain a satisfactory return by the transaction, his *personal* valuation of the promises contained in those bonds probably affects little if at all the price he is willing to pay for them. He may own a profitable business and feel certain that, if he is to continue in business, he will, in a few months, need the $100,000—or whatever he may sell the bonds for. Under such circumstances, he might be unwilling to pay more than thirty or forty thousand dollars for the bonds, if he knew that he could not hypothecate or sell them. The fact that he actually pays par or more has little or no relation to any personal valuation of the promises contained in the bonds. The bonds do not mature for 15 years and the only *personal* comparison that he makes between present and future money is a six-month comparison—nothing more.

Though a corporation may sell a long term bond under conditions that make it illegal for the corporation to retire the issue before maturity, or even to buy individual bonds in the open market, few purchasers—especially if they are individuals or 'natural persons' and not

corporations—would be likely to advance present money if the only good they could obtain in return was the privilege of receiving the interest payments and the principal payment when it became due. Natural persons seldom give to the buying of a bond the same kind of consideration that they give to the purchase of an annuity. The price they pay for the annuity is a real measure of the personal importance that they attach to the promised future payments. They seldom buy long term bonds in any such frame of mind. The marketability of a bond is one of the essential factors that determine its price and yield.

Legal restrictions and even mere custom powerfully affect the markets. As soon as a bond gains entrance to the 'legal' group in which trustees may invest, its price rises. And customs change. Forty years ago the prejudice against 'industrial' bonds was wide spread. The recent collapse of both 'guaranteed' and other real estate mortgages may affect all real estate loans, good, bad and indifferent, for some years to come.

Indefiniteness in the promise itself is sometimes so disturbing as to preclude all possibilities of using yield comparisons as indications of interest rate movements. Though callable bonds may, when selling much below their call price, be apparently unaffected by this element of uncertainty, as they approach that price their yields often begin to lose all significance. If there is any strong feeling in the market that they may actually be called, the yield becomes more or less that of a short term loan whose maturity is the callable date. In any case the yield is affected by a factor that can be neither measured nor corrected for except by comparison with another bond that seems strictly the same in all respects save the callable feature. But why, under such circumstances, use the callable bond at all?

Another illustration of indefiniteness in the promise is seen in bonds that are payable in whichever of several currencies the holder may choose to demand. Though such a clause may have little or no effect on the yields in times when all the currencies referred to in the indenture are rigidly bound together by some such tie as gold convertibility, in times of disturbance and chaos in the foreign exchanges its effect on the prices of the bonds may be so pronounced as to make the yield in any particular currency quite misleading if considered as an interest rate.

As we have already said, bonds granting valuable rights and privi-

leges unrelated to the promise to make future money payments are, for the student of interest rates, mere mongrels. The yields of United States 'circulation' bonds were not, in the years before the Federal Reserve system, interest rates in any simple and direct sense of the term. Convertible bonds selling anywhere near their conversion price act as stocks and not as bonds.

Most of these difficulties are, of course, of greater theoretical than practical importance. When attempting to make really significant comparisons of bond yields at different dates, we can easily refuse to consider mongrel bonds whose prices are affected by other factors than their promises to pay money. We do not need to use convertible bonds. Bonds containing indefinite or uncertain promises can likewise be eliminated. We do not need to use callable bonds—certainly not when their price is anywhere near the call price. The problem of taxable or non-taxable bonds can be solved by deciding how we wish to define yield. When the decision is made, we do not need to mix the two types. Much can be done to obtain bonds whose marketability does not undergo any violent change during the period in which the bonds are used.

Of course, the accuracy of interest rate or bond yield comparisons becomes more and more open to question as the difference in time increases. This is true, even if all possible adjustments are made for changing grade, etc. The yield statistics presented in this volume cover seventy-nine years, and it cannot be denied that it would be extremely difficult to describe the characteristics of a bond of the year 1890, let alone 1857, that would be strictly comparable with any particular bond at the present time. If the credit of the borrowing corporation or municipality is such that its bonds are today generally considered 'absolutely' safe in any but a metaphysical sense, the problem would seem easier. But we must remember that, in 1890 or 1857, bonds of this class may have been relatively more or less scarce, in view of the demand for them, than they are now. The yields of even United States government bonds have, at various times, been unmistakably affected by changes in the volume outstanding. This can only very partially be accounted for by any change in the degree of 'security'. An increase or decrease in the relative volume of even the highest grade bonds outstanding will, in the absence of artificial influences, inevitably affect their yield. Though a technically correct comparison might be made,

the *significance* of their yields would be different at different dates.[15]

Over both short and long periods it is easier to obtain an accurate measure of changes in the average yield of the highest grade bonds than it is to obtain an accurate measure of changes in the yield of bonds of any lower grade. The primary reason for this is that we can give at least a rough and ready answer to what we mean by the best bonds. As soon as we leave the relatively secure ground that they are bonds than which there are none better, and that, in general, they are the bonds that have the lowest yields, and discuss bonds of any lower grade we are faced with the problem of defining their grade. The second reason is that the best bonds do not change grade in any such free and easy manner as do other bonds. Their yields are but little affected by the changing fortunes of individual corporations or even by the changing fortunes of an industry as a whole.

If the earnings of a corporation cover the interest charges on a bond thirty times, the market takes little or no notice of a change in conditions such that the charges are covered only twenty times. On the other hand, a change from one and a half times to once only will probably be considered extremely serious. The yield may rise violently. The lower the grade of a bond the more it tends to act like a common stock. It comes to be significantly affected not only by interest rate factors but also by potential earnings. Whether the yields of the highest grade bonds are, at any particular time, of more or less economic importance than the yields of bonds of some specific lower grade, their meaning is simpler and their values are easier to obtain.

Until the World War there was a rather pronounced tendency for the movements of the yields of the highest grade bonds in the different financial centers of the world to become more and more alike. However, this similarity was almost non-existent in the earliest period covered by our statistical studies and was only irregularly noticeable after the disturbances to the various monetary bases that came during the War. The interest rates and bond yields presented in this volume are all from the eastern financial centers of the United States, indeed chiefly from New York City.

For the study of long term as opposed to short term rates, primary

[15] Comparisons of recent and early figures for even call money rates or commercial paper rates are properly subject to considerable questioning. There seems little doubt that the security of each of these classes of loans was much less in the earlier period,

reliance has been, aside from the last few years, placed on the yields of American railroad bonds. Index numbers based on the yields of municipal bonds or bonds of corporations other than railroads have been introduced only for the sake of checks and comparisons. Such a procedure may seem to need some defense. The reader, especially if he is not an American, may wonder why, if we were primarily interested in the highest grade bonds, we did not use United States government bonds. The sufficient reason is that, during most of the period covered by this study, their yields were seriously affected by their circulation privileges. The bonds were intimately tied up with the whole structure of the national banking system. American 'National Banks' were allowed to issue 'National Bank Notes' based on United States government bonds that they had deposited with the Comptroller of the Currency. Consequently the bonds were bought for two reasons: first, because of the interest they paid; second because they could be used as collateral for the issuance of currency. The yields were naturally much lower than if the bonds had been valued for their interest payments alone.

It might be thought that, since the formation of the Federal Reserve system, the yields of Liberty and Treasury Bonds could be considered an adequate index of long time interest rates in the United States. However, it must not be forgotten that they also carry special banking privileges, such as eligibility for use as collateral with the Federal Reserve banks at par. Finally, the investigator who might consider using them is faced with the practical difficulty that the yields of most of them are ambiguous. The maturity dates are not fixed. They are callable bonds.

We have used index numbers based on the yield of New England municipal bonds as a check on the results obtained from the railroad bonds. We did not consider using such indexes as a substitute for the railroad indexes. The market for municipal bonds has never been such a highly developed market as that for railroad bonds. The accuracy and adequacy of the quotations on which our index of the yields of New England municipals is based are not to be compared with the accuracy and adequacy of the railroad quotations. Available quotations were neither very good nor very numerous. Moreover, the fact that the holder of municipal bonds has always had certain tax exemptions, which were sometimes more and sometimes less valuable, made such

bonds poor material for our purposes. We dropped them entirely as soon as the Federal Income Tax Law began to function.

We were faced with the necessity of using bonds from one or more *industries*. We discovered that if bonds are to be used from more than one industry, each industry should be used by itself for a reasonably long period. Switching back and forth or using even the best bonds of two or more industries at the same time may easily lead to undesirable statistical results. Rather than to switch from one to another, it would seem better to use bonds from different industries independently, construct an index from each industry and attempt to arrive at conclusions by comparing the indexes.

When the decision to use only one industry had finally been made, the railroad industry was the inevitable choice. There was no other industry whose securities were of comparable importance in January 1857, the date we knew we could reach by using railroad bonds. Until many years after 1857 good public utility bonds were scarce. Until very recently, the bonds of no other industry have had the high credit rating of railroad bonds. Even in the recent collapse of railroad credit, railroad bonds of the very highest grade sold on a lower yield basis than any other corporation bonds. We constructed some very helpful index numbers based on the yields of public utility bonds in recent years. But, during most of the period studied, not only were such bonds few in number and mostly of rather low grade but also they were seriously affected by ambiguity of yield. Most of them had callable or convertible features, or both.

Accurate daily quotations for the prices of railroad bonds on the New York Stock Exchange are available back to January 1, 1857. We were unable to find any official sheets before that date. We did not attempt to carry the study further back by using quotations from newspapers. Entirely aside from the fact that newspaper quotations were, in the early days, not nearly so accurate as those contained in the official sheets, it did not seem worth while to attempt to go further back than 1857. January 1857 carries the series into the period that preceded the panic of 1857. Only a few years earlier the railroads were in their mere infancy and price quotations were very scarce and the yields unmistakably but irregular representatives of interest rates. Even for the year 1857 we found only thirteen bonds that could be used. Moreover, the scatter of the yields was very great. The best bond we used in January

1857 had a yield of 6.27 per cent while the worst bond we used had a yield of 9.84 per cent. It is apparent that, in order to get even thirteen bonds in 1857, we had to include some decidedly questionable ones.[16]

The original sources from which we calculated the yields for the individual bonds were in all instances price quotations, never yield quotations. Almost all the bonds were listed on the New York Stock Exchange, a very few being obtained from the Philadelphia, Boston or Baltimore Exchanges. No 'over the counter' quotations were used.[17] For those bonds that were listed on the New York Stock Exchange we obtained quotations for the period January 1857 to December 1877 from the New York Stock Exchange official sheets referred to above. From January 1878 to the present time our primary source for quotation for bonds listed on the New York Stock Exchange was the files of the *Commercial and Financial Chronicle,* and the *Financial Review* [18] published by the same journal. In many instances, mathematical calculations that were made on the yields suggested that certain price quotations were wrong. Wherever such a suggestion occurred other sources than the *Commercial and Financial Chronicle* were used as checks.

In the very earliest period we felt compelled to use almost every railroad bond for which we could obtain continuous or nearly continuous quotations (unless the movements of its yield were unmistakably erratic and violently different from the movements of the average of the group), but in the period after the first few years, we chose the bonds with considerable care. In the first place, we discarded all bonds whose yields were so high as to indicate that their credit was decidedly poor. We knew that their prices would move more like stock prices than we desired. We were studying interest rates. When the movements of the yield of a bond showed any great irregularity, we investigated what was happening to the road and in the market at that time. Usually we

[16] But the reader must not assume that we present an average of the yields of these bonds as picturing anything more than the condition of the industry. How these yields were used to obtain the yields of hypothetical bonds of superlative grade is discussed in the next chapter.

[17] Only in constructing the indexes of the yields of New England municipal bonds did we use 'over the counter' quotations.

[18] The *Financial Review* was not used before 1878, because the prices that it quoted were 'bid' prices on Friday of each week. All the prices for railroad bonds used in this book are actual sale prices. Bonds listed on the Philadelphia, Baltimore and Boston Exchanges were obtained from various sources, official sheets, magazines, etc.

found some outside influence that made the bond undesirable. After a large number of such investigations, we discarded bonds without investigation, when the movement of their yields was so violently erratic as to warrant the assumption that an investigation if made would disclose some good reason for elimination.[19]

Few bonds with callable or convertible features were used. The decision whether such a bond could be used was based on two considerations. First, we checked its action with the action of a tentative general index to see whether the callable or convertible features seemed to have any discoverable effect on its yield. Second, we attempted to decide logically whether the bond would or would not, in the near future, be influenced by either the callable or convertible feature. In case of doubt the bond was not used.[20]

As far as possible we used only bonds having fairly continuous price quotations. We considered the absence of quotations for two or more months in succession much more serious than their non-availability for the same number of months not in succession. Interpolations were made on the assumption that the movements of the logarithms of the yields of the particular bond in the interpolation period were the same

[19] Had the technique described in Chapter IV been developed before we began our study, instead of as we progressed, we undoubtedly would have been much less particular about what bonds we included.

[20] Whether a bond will or will not be called depends upon its yield to maturity at the call price *on the call date*. If the particular corporation or government can borrow at the call date (and pay expenses of floating the issue) at a lower rate than the yield of the bond to maturity at the call price and on the call date, the bond will be called, otherwise it will not.

For example, if a 6 per cent railroad bond, having forty years to run, is selling for $114 (to yield 5.17 per cent to maturity), but is callable thirty years from now at $112, there might be no good reason for assuming that it would be called—in spite of the fact that it is not only selling above par but also above the call price. On the call date at a price of $112, with ten years to run, the yield would be 4.50 per cent. It might easily happen that this particular railroad would not call the bond because it could not save money by so doing.

There is one serious exception to the above statements. Sometimes the yield to maturity on the call date and at the call price is a relatively unimportant element in deciding whether the bond will or will not be called. Suppose a bond maturing twenty years from now were callable in ten years at a price that would give a yield of only 2 per cent from the call date to maturity. Such a bond might be called if it were necessary to get that particular bond out of the way to clear the decks for some consolidation or reorganization. Even rumors of such consolidations or reorganizations will powerfully affect the price (and hence the yield) of a bond. However, such cases are rare. We discarded only two bonds on this account.

as the movements of the logarithms of the general index, plus or minus a straight-line trend.[21] An examination of Appendix A, Table 3 will show that interpolations are not numerous. Only in seven years were they in excess of ten per cent of the total number of individual yields. The largest percentage of interpolations (13.63) occurs in the thirteen-month period January 1898 to January 1899. After 1919 almost no interpolations were necessary.

With very few exceptions, no bond was selected unless it could be used in the index numbers for at least six years without coming closer than ten years to maturity. Since 1909 no bond was kept in the indexes after it had less than fourteen years to maturity. The inclusion of relatively short term bonds in the earlier period was unavoidable. There were not enough longer term bonds available at that time.

No attempt was made to attain a geographical distribution of the railroads whose bonds were used. We are concerned with interest rates, not with changing economic conditions in various parts of the United States. It was considered more important to know that the bonds were being bought and sold in the same market than to know the geographical location of the property. Even Canadian railroad bonds were used if they were actively traded in on the New York Stock Exchange.

Because we were primarily concerned with interest rates rather than

[21] The actual arithmetic of the procedure was as follows: suppose the yield of a bond was lacking for June and that in the particular 13 months under discussion 27 bonds were used. Now suppose that we had the yields of 25 bonds for May, June and July. From these 25 bonds index numbers of yields for the three months would be constructed. The difference between the logarithm of the yield of the particular bond in May and the logarithm of the yield of the index in May would then be found. Similarly the difference between the logarithm of the yield of the particular bond in July and the logarithm of the yield of the index in July would be found. If the logarithm of the yield of the particular bond in May, minus the logarithm of the yield of the index number in May be represented by x, and the logarithm of the yield of the particular bond in July, minus the logarithm of the index in July be represented by y, then to the logarithm of the index number in June is added $\frac{x+y}{2}$. The anti-logarithm of the result is then taken as the yield of the particular bond in June.

After the first index had been constructed some new bonds were introduced. However, the existing interpolations were not then recalculated, since it was apparent that so little difference would be made that recalculation would not be worth while. The interpolated yields are therefore not always exactly what they would have been had they been based on the final index number. The effects of this technical inconsistency are, of course, negligible.

with mere security prices, we did not hesitate to use two or more bonds of the same road. Indeed, we sometimes used two bonds of the same road and secured by the same mortgage, but of different issues, where the only difference was in the coupon rates. The reader must remember that even if two bonds are of the same road, have identical maturity, security, etc., if they are not identically the same bonds, they commonly show a considerable degree of variation in their minor erratic fluctuations. Arbitrage in bonds does not entirely eliminate such differences. If an individual has a block of bonds which he desires to sell and places them on the market in rather rapid succession, his selling will depress the price of that particular bond without necessarily affecting to a corresponding degree the price of another bond, though it be of the same road and have the same maturity and an almost identical position in the capital structure.

The prices from which yields were calculated were arithmetic averages of the high and low prices for each bond for each month. It would have been impossible within the time at our disposal to have calculated monthly averages from daily quotations—and few of the bonds were sold every day in the month. The average monthly price obtained from a high and a low was assumed to be the price of the bond at the middle of the month. The yields were first calculated from ordinary bond tables and then checked with a Johnson and Darville bond yield chart.[22] In Appendix A, Table 1 is given a description of each bond used; in Appendix A, Table 2 are given the monthly high and low prices of each bond used from January 1857 to January 1879 inclusive. The latter table was introduced for this particular period to facilitate the work of future students of the subject. We felt that some investigators might be interested in checking up the relations of the price of greenbacks in gold to the movements of these bonds during this period and for that purpose might be interested not only in yields but also in prices. The table is not continued beyond January 1879. In the succeeding gold period the reader is likely to be satisfied with 'yields'. Moreover, while it is difficult to obtain some of these prices of the earlier period, it is relatively easy to obtain the prices of the later period.

[22] The use of this nomograph makes arithmetic interpolation between yields and between dates unnecessary. The possible error in reading is seldom more than one one-hundredth of one per cent. The chart was patented in 1922 by its inventors and is sold by Prentice-Hall Inc., New York City. It is described, with illustrations, in Justin H. Moore's *Handbook of Financial Mathematics*, pp. 501–5.

They are nearly all contained in the files of the *Commercial and Financial Chronicle*. In Appendix A, Table 3 is presented the yield of each individual bond monthly from January 1857 to January 1936, inclusive. Appendix A, Chart 29 shows the period during which each individual bond was used in the construction of index numbers.

Having selected the bonds and calculated their yields we proceeded to consider the construction of index numbers.[23] The first problem that now presented itself was that of adjusting the yields in order to equalize the maturities—or better perhaps say to equalize the 'durations'. As our problem was the measurement of long time interest rates, it seemed desirable to define the term 'long'. If we defined 'long' as 15 or any other specific number of years' 'duration', we were immediately faced with the fact that, in any particular year, perhaps none of our bonds had a duration of exactly 15 years. Moreover, even if each of their durations were exactly 15 years, with each succeeding year those durations would decrease.

We asked ourselves, would it not be possible from the actual yields of bonds with various durations to calculate what the probable yield would have been if each bond had had at all times the same duration—say 15 years. We puzzled over this problem for some time. In some periods we found a quite perceptible degree of correlation between yield and duration. In general when short term rates such as those for call money, time money and commercial paper were high, the bonds with shorter durations tended to show the higher yields, and *vice versa*, when short term rates were low, the bonds with shorter durations tended to show the lower yields—though in almost all cases there were some evidences of 'lag'. However, the scatter was always extremely great and the correlation always small. The bonds differed not only in duration but also in grade. The regression was unmistakably non-linear though the curvature was, of course, distinctly less than if *maturity* had been used instead of 'duration'. We experimented with various functions of the yields; and with partial regression, attempting to introduce the element of *security* into the problem by assuming it to be measurable in terms of the yields themselves. But the scatter invariably

[23] Index numbers intended to show the course of long term interest rates must be based on *yields* and not on *prices*. A 4 per cent bond selling on a 5 per cent basis will increase in price from year to year if it remains on a 5 per cent basis. Similarly a 6 per cent bond selling on a 5 per cent basis will decrease in price from year to year. Neither price movement would have any simple relation to the interest rate problem.

remained so large as to make any equation derived from the data unmistakably worthless. We finally gave up all attempts to correct for 'duration'. The adjusted yields always seemed to smack too strongly of unreality.

We must admit that consequently a disturbing element, for which we have not corrected, remains in all our results. Moreover, as both the average maturity and the average duration of the bonds we used were shorter in the earlier than in the later period, the index numbers naturally refer in the earlier period to shorter term interest rates than they do in the later period.

In constructing any index number of railroad bond yields for such a long period as from 1857 to 1936 it naturally is impossible to carry the same bonds through the index. The bonds mature and disappear and, if they are being used to measure long term interest rates, they must be discarded years before they mature. This condition naturally necessitated many substitutions in the index. Such substitutions were always made in a January. The resulting index number is therefore a 'chain' index number, made up of 79 separate pieces, each extending over 13 months. The direct comparisons are always from one January to the second January, from the second January to the third, and so on, the intervening months falling into their respective places. In other words, there is always a group of bonds that is unchanged from one January to the next, making possible a comparison from that January to the next with an identical group of bonds. The 79 separate index numbers, each covering 13 months, were chained together in the following manner: If the January 1924 average obtained from the yields of the group of bonds used in the 13 months, January 1924 to January 1925, was eleven-tenths of the January 1924 average obtained from the yields of the group of bonds used in the 13 months, January 1923 to January 1924, it would be multiplied by ten-elevenths. If the adjustment were being made backwards the second January average would be multiplied by eleven-tenths.

As the bonds used in the index number are changed, comparisons of distant dates might be thought to become more and more dangerous. On the other hand, we must remember that railroad bonds constitute a relatively homogeneous group. The introduction of even an entirely new group of bonds each January would not necessarily lead to bad results. We seldom changed any large percentage of the bonds in any

one January. Both the number dropped and the number introduced were usually small, but this is probably not very important. Perhaps as many dangers are connected with keeping the same bonds throughout as arise from changing them. We must remember that there is no particular magic in continuing to quote the price of 'the cow with the crumpled horn' until she becomes a museum piece.

No 'weighting' was used in the construction of any of the various index numbers.[24] Each yield has a weight of one. It did not seem advisable to attempt to assign other than equal weights to the individual yields. While it would be possible to construct a set of arbitrary weights based on the number of bonds outstanding, the number traded in, or other similar criteria, it is questionable whether there would be much value in any such procedure. Weighting by number of shares outstanding would seem highly reasonable if we were constructing an index number of stock prices. And weighting by number of bonds outstanding might be defended in the construction of an index of bond yields that was designed not to exhibit the course of interest rates but rather the fortunes of the borrowing corporations and of the investing public. It would seem a distinctly arbitrary and erratic system of weighting were we concerned primarily with interest rates. On the other hand, if we use *number of bonds traded in* as a basis for a set of weights, we shall find ourselves weighting an issue less and less heavily as it becomes seasoned. The heaviest trading occurs immediately after a bond is issued. Some of the best bonds become more and more inactive, until finally they cannot be used at all because months elapse without a single sale. If, however, anything happens to shake the confidence of the investing public in an issuing corporation, its bonds come out of safe deposit boxes and the volume of trading immediately increases.

As our index numbers are—from the nature of the data—necessarily 'chain' numbers, it is extremely important that no type of averaging used shall involve any mathematical 'drift'. We present index numbers based on geometric and arithmetic averaging, *but the averages are averages of the yields themselves.* They are not averages of 'relatives'. Not merely the geometric but also the arithmetic averages are therefore free from mathematical drift.

[24] Except the implicit weighting in the illustrations of the results of constructing chain index numbers from January to January from averages of *relative* prices with either the earlier or the later January as 'base'.

There are drifts in the averages, to be sure, but they are not mathematical drifts. They are economic drifts. They are caused by secular and cyclical changes in the grades of the bonds. Because their outlook is more seriously affected by fluctuations in earnings, lower grade bonds of course change in grade more violently than do bonds of better grade. This explains why the geometric and arithmetic averages show long time tendencies to drift together or drift apart. From 1857 to 1872 and from 1920 to 1930, and indeed in general from 1857 to 1930, indexes based on arithmetic averaging and indexes based on geometric averaging drift together as we move forward in time. During other periods, such as 1911 to 1920 and 1930 to 1932, they move more or less sharply apart.

But the drift is an economic drift. So long as an industry as a whole is, either because of general business conditions or because of conditions peculiar to the industry, in a period of health and growth, the spread between the yields of any two bonds, one low grade and one high grade, will tend to decrease. Similarly, the 'scatter' of the yields of a group of bonds of various grades will tend to decrease and hence the arithmetic and geometric averages of the yields will tend to approach each other. If the industry enters upon a period of declining business and profits, the scatter of the yields and hence the spread between the arithmetic and geometric averages will tend to increase.

But this drifting together and apart of the arithmetic and geometric index numbers is of greater theoretical than practical importance. For all practical purposes (as may be seen from Appendix A, Table 4) the two index numbers move together. But there is a drift in *both* which is of great importance. In January 1925 the figure for the geometric index (4.774 per cent) and the figure for the arithmetic index (4.776 per cent) are respectively the actual geometric average and the actual arithmetic average of all the bonds used in that month. However, the chain index constructed from geometric averages shows a yield of 9.517 per cent in January 1857 although the actual geometric average of all the bonds used in January 1857 was only 7.994 per cent.

Moreover, the above comparison does not present the picture in as strong colors as it should be presented. There is every reason for believing that the bonds which each month throughout the period showed the very lowest individual yields were much more nearly of the same grade throughout the period than was the average of all the bonds. Now the geometric average index number for January 1857 (9.517 per cent) is

152 per cent of the yield of the 'best bond' in the same month (6.27 per cent) though in January 1925 the index (4.774 per cent) is only 106 per cent of the yield of the best bond used in that month (4.50 per cent). The arithmetic differences are even more startling than the ratios. In January 1857 the difference between the geometric index and the yield of the best bond was 3.25 per cent while in January 1925 it was only 0.27 per cent. The relations of the geometric index to the yields of the best bonds and to the actual geometric averages of the bonds used each year may be seen in Chart 1. The evidences of a pronounced downward drift are unmistakable.

The arithmetic average index numbers show a slightly greater downward drift. We have used the geometric average index numbers to illustrate drift because the reader is less likely to wonder whether he is not faced with a problem of *mathematical* drift or, as Professor Fisher terms it, 'bias'.

In the next two chapters we discuss further the nature of this economic drift. We describe not only how it can be eliminated to obtain index numbers of the yields of hypothetical bonds of superlative grade but also how it can be separated out, measured, and used by itself as an index of the degree of difference at any particular time between the movements of the yields of high and low grade bonds. Finally, we describe its relation to the movements of the yields of preferred stocks and the movements of the prices of common stocks and show how, when properly interpreted, it brings together all securities into one great family.