Catching-up with the "locomotive": a simple theory*

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Abstract

This paper extends the standard neoclassical model by considering a technology sector through which an economy with limited human capital attempts to catch up with a given "locomotive" pushing exogenously technical progress. In periods of technological stagnation, economies close enough to the frontier may find it optimal to not catch up, which reinforces worldwide technological sclerosis. Under sustainable technological growth, all the other economies will sooner or later engage in imitation. Such a phase of technology adoption may be delayed depending on certain deep characteristics of the followers.

1 Introduction

The neoclassical theory of economic growth (Solow, 1957) entails the idea of an economic locomotive driving worldwide technological progress through the assumption of exogenous technological progress. Related concepts like convergence and catching-up have been developed along the way (see one of the numerous textbooks in economic growth, like Barro and Sala-i-Martin, 1995). However, in this benchmark, convergence refers to the level of income per capita being equalized in the long-run under certain assumptions, and in particular assuming that all countries have immediate access to the same technology at no cost. It is now widely

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admitted that such an assumption is heroic, under-development principally originates in the difficulty of transferring top technologies to countries with very limited technological absorption capacity due to the scarcity of human capital. This note revisits the neoclassical growth model by introducing a technology adoption variable into the benchmark and bringing the limited human capital resources into the analysis. To unburden the analysis, we shall assume that human capital resources are fixed for ever. These resources should be allocated between technology adoption (or imitation of a given economic growth locomotive in the neighborhood of the economy) and production of the final good. Technological evolution follows a catching-up equation pretty much in line with Nelson and Phelps (1966), as recently resumed by Benhabib and Spiegel (1994) and Boucekkine et al. (2006). The paper derives optimal adoption policies and their implications for the long-term technological gap between the locomotive and the follower depending on the strength of the locomotive's economic growth. We show how the latter aspect dramatically shape the outcomes of the model (nonzero or zero long-run technological gap, adoption delays, etc...). In periods with no significant technological progress (i.e, pre-industrial periods), economies close enough to the technological frontier may find it optimal to not catch up with the locomotive, which reinforces worldwide technological sclerosis. When the locomotive economy experiences sustainable technological growth, then all the other economies will sooner or later engage in imitation. Such a phase of technology adoption may be delayed depending on certain deep characteristics of the followers. Our model delivers therefore a simple non-strategic theory of adoption delays.

2 The model

Suppose that the economy is populated by infinitely lived agents with a strictly increasing and concave consumption-based utility function $U(C(t))$. For simplicity, we normalize population size to 1. We also model normalize total human capital available in this economy to 1 to reflect scarcity as outlined above. A benevolent planner aims at maximizing the welfare of the economy by choosing the optimal human capital allocation across the final good and the technology sectors, and the optimal consumption plan over time:

$$\max_{u(t),l(t),C(t)} \int_0^\infty e^{-\rho t} U(C(t))dt, \tag{1}$$

with ρ time preference, subject to:

$$Y(t) = A(t)K(t)^{\alpha}l(t)^{1-\alpha}, \tag{2}$$

$$\dot{A} = \phi(t)u(t)\left(A^0(t) - A(t)\right), \tag{3}$$

$$\dot{K} = I(t) - \delta K(t) = Y(t) - C(t) - \delta K(t), \tag{4}$$

and the resource constraint

$$u(t) + l(t) = 1, 0 \leq u(t), 0 \leq l(t). \tag{5}$$

Equation (2) gives the production function which is the usual neoclassical one: $K(t)$ is the stock of capital at t, $l(t)$ is production human capital, and α is capital share. $A(t)$ is the level of technology, and in contrast to the standard neoclassical model, we assume that it follows the dynamic law of motion (3), which features a kind of output of a technology sector. The economy imitates the technology of the regional locomotive, here $A^0(t)$, $u(t)$ is human capital in the technology sector, and $\phi(t)(< 1)$ is an exogenous variable indexing the learning (or absorbing capacity) of this economy. Just like in standard endogenous growth theory (see Romer, 1990), we assume constant return to human capital in the technology sector. We also assume that $A^0(t)$ grows at constant exogenous rate $\gamma \geq 0$: $A^0(t) = A^0 e^{\gamma t}$, $\forall t \geq 0$. Initially, $K(0)$ and $A(0) < A^0(0)$ are given: the technology of our economy is lagged with respect to the locomotive.

The optimal control problem above (with control) constraints is standard, and can be handled using the Lagrangean:

$$\begin{aligned}\mathcal{L} \; &= U(C(t)) + \mu_1(t)\left(A(t)K^{\alpha}l^{1-\alpha} - C - \delta K\right) + \mu_2(t)\phi(t)\, u\left(A^0(t) - A(t)\right)\\ &+\xi(t)u(t) + \eta(t)l(t) + w(t)(1 - u(t) - l(t)),\end{aligned}$$

where $\mu_1(t)$ and $\mu_2(t)$ are the costate variables associated with capital and technology stocks respectively, $w(t)$ is the Lagrange multiplier associated with human capital resource constraint (therefore representing the shadow wage or shadow remuneration of human capital), and ξ and $\eta(t)$ are the Kuhn-Tucker multipliers for nonnegative constraints on the allocation of human capital across the two sectors. The first-order necessary (and sufficient) conditions are:

$$\mu_1(t) = U'(C), \tag{6}$$

$$\dot{\mu_1}(t) = \rho\mu_1 - \mu_1(\alpha A K^{\alpha-1} l^{1-\alpha} - \delta), \tag{7}$$

$$\dot{\mu_2}(t) = \rho\mu_2 - \left[\mu_1 K^{\alpha} l^{1-\alpha} - \mu_2(t)\phi(t)u\right], \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial u} = \mu_2(t)\phi(t)\left(A^0(t) - A(t)\right) + \xi(t) - w(t) \leq 0, \;\; \xi_t \geq 0, \;\; \xi_t u_t = 0, \tag{9}$$

with equality if $u_t > 0$,

$$\frac{\partial \mathcal{L}}{\partial l} = \mu_1(t)(1-\alpha)A(t)K^{\alpha} l^{-\alpha} + \eta(t) - w(t) \leq 0, \;\; \eta_t \geq 0, \;\; \eta_t l_t = 0, \tag{10}$$

with equality if $l_t > 0$, and the transversality conditions $\lim_{t\to\infty} e^{-\rho t}\mu_1 K(t) = 0$, and $\lim_{t\to\infty} e^{-\rho t}\mu_2 A(t) = 0$. The conditions are standard. In particular, (9) and (10) are the optimality conditions with respect human capital shares in the technology and final good sectors respectively. The both account for scarcity of human capital and for the non-negativity constraints (in the standard way). Notice that by (10) the corner solution $l(t) = 0$ is impossible (since $\alpha < 1$). However, both the corner and interior solutions are possible for $u(t)$ resulting in different technological development paths, as explained below. From now on, we assume a logarithmic utility function and $\phi(t) = \phi > 0$ for simplicity.

3 Main results

Define the balanced growth paths (BGP) as particular solutions to the optimality conditions and feasibility constraints listed in Section 2 such that human capital allocation across sectors is time-independent and variables $K(t)$, $C(t)$ and $A(t)$ grow at non-negative rates. We denote the growth rate of variable X as g_X. We are able to gather the following two main results.

3.1 Catching-up in periods of technological stagnation

This pre-industrial case corresponds to $\gamma = 0$ in our model. This case is characterized by the following proposition.

Proposition 1 *Suppose* $\gamma = 0$ *and* $A^0(t) = A^0(0)$.

(i) $u(t) = 0$*, for all* $t \geq 0$*, is an optimal choice if*

$$\frac{A^0(0)}{A(0)} \leq 1 + \frac{\rho(1-\alpha)}{\phi}. \tag{11}$$

(ii) Otherwise, if $\frac{A^0(0)}{A(0)} > 1 + \frac{\rho(1-\alpha)}{\phi}$*, there exists* $\underline{t} > 0$*, such that, catching up in technology happens at* $\underline{t}$ *where* $A(t) = A^0(0), \forall t \geq \underline{t}$ *and* $u(t) = \begin{cases} > 0, \forall\, 0 < t < \underline{t}, \\ = 0, \forall\, t \geq \underline{t}. \end{cases}$

Condition (11) puts an upper bound on the initial technological gap: if it is low enough, that is if the technological frontier is close enough, then the economy may not devote any effort to technological catching-up: the productivity gain in doing so does not compensate for the loss in consumption (and therefore in welfare) coming from diverting human capital from the final good sector to the technology sector. Notice that the upper bound in (11) is increasing in the discount rate, ρ, and decreasing in the learning parameter, ϕ: the lower the weight assigned to the future and the less efficient is learning in technology, the wider the range of initial situations that can potentially induce this case of technological sclerosis. In contrast, when the economy is far enough the frontier, investing human capital in technological upgrading makes sense, and the economy eventually catches up the stagnant locomotive at finite time. But since leapfrogging is not optimal in our model, this case also yields technological sclerosis at finite time. Interestingly enough, transitory periods of technological progress only occur in the most lagged countries (relative to the locomotive) in such a stagnant environment featuring the pre-industrial era.

3.2 Catching-up with a growing "locomotive"

We now consider the more contemporaneous case of locomotives growing at strictly positive growth rates. The induced technological evolution in the follower countries is then drastically different from the one featured for the stagnation case above.

Proposition 2 *Let* $\gamma > 0$*.*

(i) The necessary condition for $u = 0$ *to be optimal is* $\frac{A^0(0)}{A(0)} \leq 1 + \frac{\rho(1-\alpha)}{\phi}$*. Furthermore,* $u = 0$ *can only be optimal for a finite time adjustment period, that is when* $t \leq \frac{1}{\gamma} \ln\left(\frac{[\rho(1-\alpha)+\phi]A(0)}{\phi A^0(0)}\right)$*.*

(ii) Along the BGP, all the variables, except u*, grow at constant rates* $g_A = \gamma, \quad g_C = g_K = g_Y = \frac{\gamma}{1-\alpha}$*. The human capital share in the technology sector is the unique positive solution of* $\frac{\gamma(1-u)}{(1-\alpha)u} = \rho + \gamma + \phi u^{\theta}$*. It is increasing in the rate of technological progress carried out by the leader,* $\frac{d\overline{u}}{d\gamma} > 0$*.*

The proposition has several interesting results. In contrast to periods of technological stagnation, having a growing locomotive always prevents from getting into technological sclerosis in the long-run. As in the standard neoclassical growth model, BGPs exist with the growth rate of technology equal to the locomotive's. Moreover, the faster the locomotive, the more human capital the economy will put in the technology sector (although $A(t)$ will not converge in level to $A^0(t)$). Nonetheless, the economy can choose to experience a period of zero technological progress, exactly as in the stagnation case studied above and under the same condition on the initial technological gap. However, this period is transitory, and it features a kind of **adoption delay**. Adoption is delayed because the planner finds it optimal to put all the human capital resources in the final good sector in the short-run (given the weight assigned to present consumption) and to take advantage of the technological push of the locomotive in later stages (when the weight assigned to consumption is low enough). In this sense, this paper uncovers a new case for delaying adoption (see Boucekkine et al., 2004, for more insight into the literature of adoption delays). The lower the initial technological gap, the longer the adoption delay. The delay is also decreasing in the exogenous pace of technological progress, γ, and learning, ϕ, while it grows with the discount rate, ρ.

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Appendix (not intended for publication)

A. Lemma

Before the main result, we have the following lemma which is used in the proof of the proposition.

Lemma 1 *If there is a balanced growth path and* $0 < u(t) < 1$*, then along the BGP,* $u(t) = \overline{u} \in (0,1)$ *is a constant.*

Proof If there is BGP, then along the BGP, the market clearing condition (5) is $\overline{u}e^{g_u t} + \bar{l}e^{g_l t} = 1$, where constants $\overline{u}$ and $\bar{l}$ are labors in different sectors when the BGP starts.

Due to the fact that labor force is finite, the growth rates g_u and g_l can not be both positive and from market clearing condition can not be both negative. As mentioned previously $l = 0$ cannot be optimal, hence we must have $g_l = 0$. For the case $u > 0$, we must also have $g_u = 0$.

Therefore $g_u = g_l = 0$ and along the BGP both u and l are positive constants with $\overline{u} + \bar{l} = 1$. We finish the proof.

B. Proof Proposition 1: $\gamma = 0$ and $A^0(t) = A^0(0)$

Suppose the optimal balanced growth path starts from some time $t = \widetilde{t} \geq 0$ and along the BGP $u(t) = 0$ for $t \geq \widetilde{t}$. Then from labor market clearing condition, we have $l(t) = 1$. Obviously, it is possible that $\widetilde{t} = 0$, where there is no imitation at all; or $\widetilde{t} > 0$, which reads there may be some imitation during some period until $\widetilde{t}$.

In the case of no imitation, $\widetilde{t} = 0$, the state equations and the first order conditions are

$$\begin{cases} \dot{K} = A(t)K^{\alpha} - C(t) - \delta K(t), \\ \dot{A} = 0, \\ \frac{\dot{C}}{C} = \alpha A(t) K^{\alpha-1} - (\rho + \delta), \\ \dot{\mu_2}(t) = \rho\mu_2 - \dfrac{K^{\alpha}}{C}, \\ \mu_2(t)\phi\left(A^0(t) - A(t)\right) + \xi(t) \leq \dfrac{(1-\alpha)A(t)K^{\alpha}}{C}(= w(t)), \\ \xi \geq 0. \end{cases} \tag{12}$$

The 2nd equations reads $A(t) = A(0) < A^0(t)$ for all $t > 0$. Hence the 3rd equations shows that $g_C + (\rho + \delta) = \alpha A(0)K(t)^{\alpha-1}$ which holds if and only if $K(t) = \underline{K}$ is a constant. And the 1st equations presents $g_C = 0$. As by products, we also have $\underline{K} = \left(\frac{\rho+\delta}{\alpha A(0)}\right)^{\frac{1}{\alpha-1}}$, $\underline{C} = \underline{K}^\alpha(A(0) - \delta \underline{K}^{1-\alpha})$. Therefore, consumption-capital ratio is $\frac{\underline{C}}{\underline{K}} = \frac{\rho + (1-\alpha)\delta}{\alpha}$.

The 4th equations in (12) leads to $\frac{\dot{\mu_2}}{\mu_2} = g_{\mu_2} = \rho - \frac{\underline{K}^\alpha}{\underline{C}\mu_2}$ which is a constant if and only if $\mu_2(t) = \underline{\mu_2}$ is a constant (or $g_{\mu_2} = 0$). Moreover, $\underline{\mu_2} = \frac{1}{\rho}\frac{\underline{K}}{\underline{C}}\underline{K}^{\alpha-1} = \frac{\rho+\delta}{\rho(\rho+\delta(1-\alpha))A(0)}$.

Combining the last two inequalities in (12), it yields

$$
\begin{aligned}
0 \leq \xi \quad & \leq (1-\alpha)A(0)\underline{K}^{\alpha-1}\tfrac{\underline{K}}{\underline{C}} - \phi\underline{\mu_2}(A^0(0)e^{\gamma t} - A(0)) \\
& = \frac{\rho+\delta}{\rho(\rho+\delta(1-\alpha))}\,\frac{[\rho(1-\alpha)+\phi)]A(0) - \phi A^0(0)e^{\gamma t}}{A(0)}.
\end{aligned} \tag{13}
$$

With $\gamma = 0$, the necessary condition for the above inequality holds is $[\rho(1-\alpha)+\phi]A(0) - \phi A^0(0) \geq 0$ or equivalently $\frac{A^0(0)}{A(0)} \leq \frac{\rho(1-\alpha)+\phi}{\phi} = 1 + \frac{\rho(1-\alpha)}{\phi}$.

When $\widetilde{t} > 0$ and $u > 0$ for at least $0 < t < \widetilde{t}$, the state equations and the first order conditions are

$$
\begin{cases}
\mu_2(t)\phi\left(A^0(t) - A(t)\right) = \frac{(1-\alpha)A(t)K^\alpha(1-u(t))^{-\alpha}}{C}(= w(t)), \\
\dot{K} = A(t)K^\alpha(1-u(t))^{1-\alpha} - C(t) - \delta K(t), \\
\dot{A} = \phi u(A^0(t) - A(t)), \\
\frac{\dot{C}}{C} = \alpha A(t)K^{\alpha-1}(1-u(t))^{1-\alpha} - (\rho+\delta), \\
\dot{\mu_2}(t) = (\rho + \phi u)\mu_2(t) - \frac{K^\alpha(1-u(t))^{1-\alpha}}{C}.
\end{cases} \tag{14}
$$

Substituting the 3rd equation into the 1st one, it yields $\mu_2\frac{\dot{A}}{A} = \mu_2 g_A = \frac{(1-\alpha)uK^\alpha(1-u(t))^{-\alpha}}{C}$. Dividing the 3rd equation by $A(t)$ leads to $g_A = \phi\overline{u}\left(\frac{A^0(0)}{\overline{A}}e^{(\gamma-g_A)t} - 1\right)$, which is a constant if and only if $g_A = \gamma$ where $\overline{A}$ is the technology level along BGP. The situation of γ is zero or not, the results are different.

With $\gamma = 0$, it must follow, from the 3rd equation, $A(t) = A^0(t) = \widehat{A^0}$ which is the catching-up steady state.

If the leading technology reaches its steady state while the underdeveloped one keeps imitation, sooner or later, the underdeveloped one will catch up to the steady state level of the leader. Furthermore, since along the BGP, u is constant and there is nothing more to learn, hence, we must have along the BGP, $u = 0$.

We denote this special time $\widetilde{t} = \underline{t}$. Hence, we can conclude that before $\underline{t}$, there is imitation and $u > 0$. From $t = \underline{t}$, there is catch-up in technology and no need for imitation, $u = 0, \forall\, t > \underline{t}$. That finishes the proof.

C. Proof Proposition 2: $\gamma > 0$

Part (i) Notice, $u = 0$ is one of the solutions and is sufficient for the optimization problem due to Kuhn-Tucker condition. Then, repeating the proof of Proposition 1 until (13), with $\gamma > 0$, the necessary condition for the last inequality in (13) is

$$\frac{[\rho(1-\alpha)+\phi]A(0)}{\phi A^0(0)} \geq e^{\gamma t} \quad \text{or} \quad t \leq \frac{1}{\gamma}\ln\left(\frac{[\rho(1-\alpha)+\phi]A(0)}{\phi A^0(0)}\right)$$

which makes sense if

$$[\rho(1-\alpha)+\phi]A(0) \geq \phi A^0(0).$$

Therefore, the necessary condition for $u = 0$ to be an optimal solution is $\dfrac{A^0(0)}{A(0)} \leq \dfrac{\rho(1-\alpha)+\phi}{\phi} = 1 + \dfrac{\rho(1-\alpha)}{\phi}$ and maximum time is $t \leq \dfrac{1}{\gamma}\ln\left(\dfrac{[\rho(1-\alpha)+\phi]A(0)}{\phi A^0(0)}\right)$.

Part (ii) With $u > 0$ and slackness condition in (9), we must have $\xi = 0$. Hence, the state equations and the first order conditions are given by (14).

Suppose there is BGP, and by Lemma 1, it follows $u(t) = \overline{u}$.

Substituting the 3rd equation of (14) into the 1st one, it yields

$$\mu_2 \frac{\dot{A}}{A} = \mu_2 g_A = \frac{(1-\alpha)uK^\alpha(1-u(t))^{-\alpha}}{C} \tag{15}$$

where $\overline{A}$ is the technology level along BGP.

Dividing the 3rd equation by $A(t)$, it yields $g_A = \phi\overline{u}\left(\dfrac{A^0(0)}{\overline{A}}e^{(\gamma - g_A)t} - 1\right)$ which is a constant if and only if $g_A = \gamma$. With $\gamma > 0$, along the BGP, the 4th equation reads $g_C = \alpha\overline{A}e^{\gamma t}(1 -$

$\overline{u})^{1-\alpha}\overline{K}^{\alpha-1}e^{g_K(\alpha-1)t}-(\rho+\delta)$, or

$$g_K=\frac{\gamma}{1-\alpha}g_C+\rho+\delta=\alpha(1-\overline{u})^{1-\alpha}\overline{AK}^{\alpha-1}. \tag{16}$$

The 5th equation is equivalent to $\frac{\dot{\mu_2}}{\mu_2}=\rho+\phi\overline{u}-\frac{(1-\overline{u})^{1-\alpha}K^{\alpha}}{\mu_2 C(t)}$. Substituting (15) into it, we have

$$g_{\mu_2}=\rho+\phi\overline{u}-\frac{(1-\overline{u})\gamma}{(1-\alpha)\overline{u}}. \tag{17}$$

Dividing the 2nd equation by $K(t)$ and rearranging the term, il yields $g_K=\overline{AK}^{\alpha-1}(1-\overline{u})^{1-\alpha}e^{(g_A-(1-\alpha)g_K)t}-\delta-\frac{C}{K}e^{(g_C-g_K)t}$. Since $g_A=\gamma=(1-\alpha)g_K$, it must follow

$$g_C=g_K=\frac{\gamma}{1-\alpha},\quad g_K=\overline{AK}^{\alpha-1}(1-\overline{u})^{1-\alpha}-\delta-\frac{C}{K}. \tag{18}$$

Combining (18) and (16), we obtain $\frac{\gamma}{1-\alpha}+\rho+\delta=\alpha(1-\overline{u})^{1-\alpha}\overline{AK}^{\alpha-1}=\alpha\left(\frac{\gamma}{1-\alpha}+\delta+\frac{\overline{C}}{\overline{\overline{K}}}\right)$. Hence, consumption-capital ratio along the BGP is

$$\frac{\overline{C}}{\overline{\overline{K}}}=\frac{\gamma+(1-\alpha)\delta+\rho}{\alpha}. \tag{19}$$

Taking derivative of (15), diving by μ_2, and rearranging terms, it follows

$$g_{mu_2}=\alpha g_K-g_C=-\gamma. \tag{20}$$

Combining (20) and (17), il yields

$$\frac{\gamma(1-\overline{u})}{(1-\alpha)\overline{u}}=\rho+\gamma+\phi\overline{u},\quad u>0. \tag{21}$$

Define $f(\overline{u})=\rho+\gamma+\phi\overline{u}$, $g(\overline{u})=\frac{\gamma(1-\overline{u})}{(1-\alpha)\overline{u}}$. It is easy to check $f(0)=\rho+\gamma>0,\quad f'(\overline{u})>0,\quad f(1)=\rho+\gamma$ and $\lim_{\overline{u}\to 0}g(\overline{u})=+\infty,\quad g'(\overline{u})=-\frac{\gamma}{(1-\alpha)\overline{u}^2}<0,\quad g(1)=0$. Therefore, (21) gives one and only one solution of $\overline{u}$, such that $0<\overline{u}<1$.

Effect of γ. Let us now prove that $\overline{u}$ is increasing with respect with γ.

Taking total differential with respect to γ and $\overline{u}$ in equation (21) and rearranging terms, il follows

$$[\gamma(2-\gamma)+\rho(1-\alpha)+2(1-\alpha)\overline{u}]d\overline{u}=(1-(2-\alpha)\overline{u})d\gamma.$$

Hence,

$$\frac{d\overline{u}}{d\gamma} = \frac{1-(2-\alpha)\overline{u}}{B(\gamma,\overline{u})},$$

where $B(\gamma,\overline{u}) = \gamma(2-\gamma) + \rho(1-\alpha) + 2(1-\alpha)\overline{u}$ is always positive, as long as $\gamma \leq 2$. Therefore, the sign of $\frac{d\overline{u}}{d\gamma}$ will only depend on $(1-(2-\alpha)\overline{u})$.

Denote $X(u) = 1-(2-\alpha)u$. It is easy to check $X(0) = 1, \quad X(1) = -(1-\alpha) < 0$ and $X'(u) = -(2-\alpha) < 0$, that is, $X(u)$ is decreasing in term of $u \in (0,1)$. And at $u^* = \frac{1}{2-\alpha}$, $X(u^*) = 0$.

Furthermore, $f(u^*) = \rho + \gamma + \phi(u^*)$ and $g(u^*) = \gamma < f(u^*)$, which states that $\overline{u} < u^*$. And hence at $\overline{u}$, $X(\overline{u}) > 0$. As a result, it yields $\frac{d\overline{u}}{d\gamma} > 0$. We finish the proof.