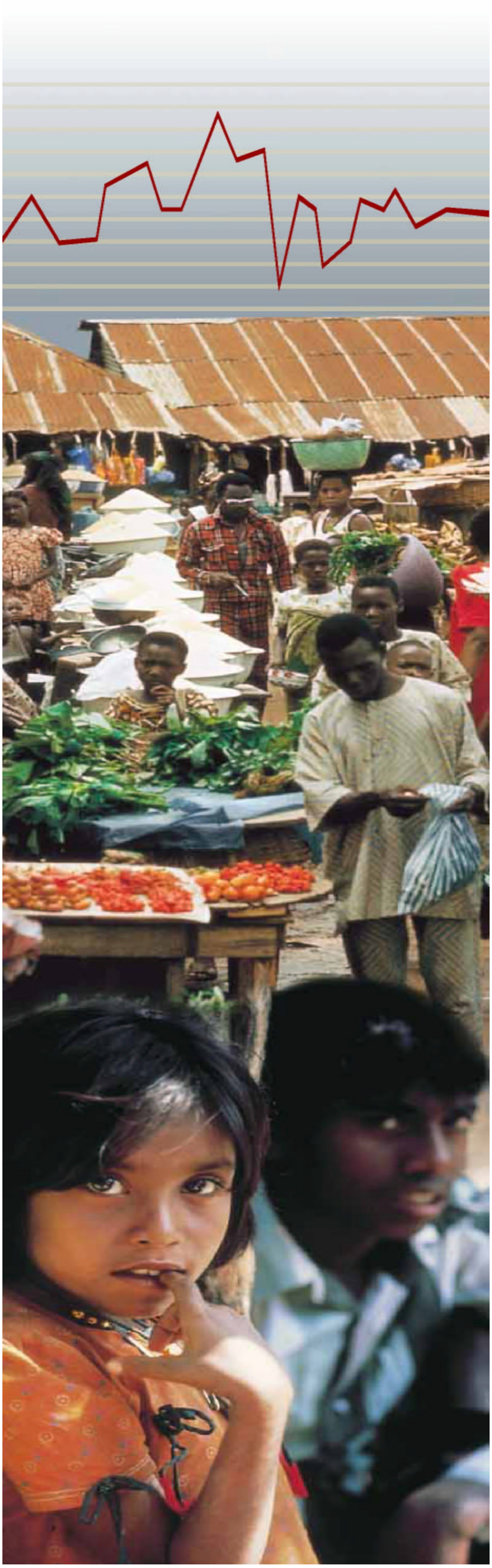

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Poverty, Inequality and Stochastic Dominance, Theory and Practice: The Case of Burkina Faso

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ABSTRACT

In this paper we provide a set of rules that can be used to check poverty or inequality dominance using discrete data. Existing theoretical rules assume continuity in incomes or in percentiles of the population. In reality, with the usual household surveys, this continuity does not exist. However, such a discontinuity can be exploited to test for stochastic dominance. This paper also proposes stochastic dominance conditions that check for the statistical robustness of the inferred rankings. The methodology of this paper is illustrated using Burkina Faso's household surveys for the years of 1994 and 1998.

Key words : Stochastic Dominance, Poverty, Inequality.

JEL Classification : D63, D64.

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1. Introduction

There are several indices used in the literature to measure poverty and inequality. However, disadvantages may arise if these indices are used for comparing distributions. In some instances, the ranking of different distributions may vary depending on the measure of inequality or poverty that is being used[1]. This is essentially explained by the differences in sensitivity of these indices at different parts of the distribution or income level. For some pairs of distributions, the use of stochastic dominance makes it possible to draw more robust conclusions about ordinal comparisons. It should be noted that stochastic dominance at a given social order is not based on a pre-determined functional form for an index, but rather on some desirable properties or axioms that the corresponding class of indices should respect.

The stochastic dominance approach is thus useful in establishing a robust ordinal comparison. Until now however, there exists no theoretical framework with special focus on stochastic dominance with discrete data. This suggests the need to develop fundamental rules for the case of discontinuous distributions. Furthermore, most empirical studies lack statistical tests for stochastic dominance. For this, we suggest conditions concerning the statistical robustness of stochastic dominance rankings.

The paper is organized as follows. In Section 2, we review briefly the basic theoretical approach to check for stochastic dominance in poverty. In Section 3, we develop the general rules to check for stochastic dominance with discrete data. Again, in Section 4, we propose some rules to check for stochastic dominance in inequality using Lorenz curves. We discuss the statistical robustness of stochastic dominance orderings in Section 5. In Section 6, we illustrate findings of this paper by using Burkina Faso surveys for years 1994 and 1998. Conclusions and remarks are made in Section 7.

2. Basic Theoretical Framework

Atkinson (1987) introduced the idea of restricted dominance in poverty. The theoretical poverty dominance conditions have been further and more rigorously established in Foster and Shorrocks (1988 a) and Foster and Shorrocks (1988 b), while bounds to poverty dominance were discussed in Davidson and Duclos (2000). The main aim of using the stochastic dominance approach is to establish a robust ordinal ranking in poverty,

[1] See Araar and Duclos (2005), for instance.

inequality or social welfare based on the adopted social-ethical judgments[2]. The sensitivity of the quantitative indices is not the same at different parts of the distribution. This suggests that ordinal rankings can be reversed using different indices. We note the class of the additive poverty indices that respect the s ethical order by $\Psi^s(z^+)$, where z^+ stands for the upper bound of the range of possible poverty lines.

Additive poverty indices take the general form:

$$P(z)=\int_{y_{min}}^{y_{max}} \upsilon(y;z)dF(y) \qquad (1)$$

where $\upsilon(y;z)$ is the poverty indicator or contribution of household with income y to the poverty index. Suppose that the additive poverty indices respect the focus axiom, then: $\upsilon(y;z)=0$ if $y \geq z$. For the first class of poverty indices noted by $\Psi^1(z^+)$, these indices will be unchanged or will decrease with an increase in income or standard living of the poor household.

$$\Psi^1(z^+)=\left\{ P(z) \middle| \begin{array}{c} \upsilon_y^{(1)}(y;z)\leq 0 \text{ when } y\leq z \\ z \leq z^+, \end{array} \right\} \qquad (2)$$

where $\upsilon_y^{(1)}(y;z)$ is the first derivative of υ in y. The second class of poverty indices $\Psi^2(z^+)$:

- belongs to the first-order class; and
- is convex in living standards or income y. Also, this implies that these indices respect the Pigou-Dalton principle, such that a marginal income transfer from a richer-poor to a poorer-poor reduces poverty.

$$\Psi^2(z^+)=\left\{ P(z) \middle| \begin{array}{c} P(z)\in \Psi^1(z^+), \\ \upsilon_y^{(2)}(y;z)\geq 0 \text{ when } y\leq z, \\ \upsilon(z;z)=0. \end{array} \right\} \qquad (3)$$

The third class of poverty indices concerns indices that:

- belong in the second class; and
- are decreasing in the following composite transfer:

[2] Social-ethical judgements refer here to the sensitivity of the society to the distribution of incomes. In general, these judgements are represented by a given parameter(s) within a functional form for distributive indices. For example, the higher the aversion of society to inequality, the higher the level of the parameter for social aversion to inequality.

- a beneficial Pigou-Dalton transfer within the lower part of the distribution accompanied by an adverse Pigou-Dalton transfer within the upper part of the distribution
- have a non decreasing variance of the distribution

$$\Psi^3(z^+) = \left\{ P(z) \left| \begin{array}{c} P(z) \in \Psi^2(z^+), \\ \upsilon_y^{(3)}(y;z) \le 0 \text{ when } y \le z, \\ \upsilon(z,z) = 0 \text{ and } \upsilon^{(1)}(z,z) = 0. \end{array} \right. \right\} \qquad (4)$$

In general, poverty indices will be members of class $\Psi^s(z^+)$ if $(-1)^s\ \upsilon_y^{(s)}(y;z) \ge 0$ and if $\upsilon^{(i)}(z,z) = 0$ for $i = 0,1,2...,s-2$. As the order s of the class of poverty indices increase, these indices become more and more sensitive to the distribution of income among the poorest. As proposed by Davidson and Duclos (2000), to check the stochastic dominance for the order s, one can compare between dominance curves that take the following form:

$$D^s(z) = \frac{1}{(s-1)!} \int_{y_{min}}^{y_{max}} (z-y)_+^s \, dF(y) \qquad (5)$$

where $(z-y)_+ = (z-y)$ if $z > y$ and zero otherwise. One can remark that this curve is simply a monotonic transformation of the FGT curve. Based on this, one can use the FGT curves directly to check the poverty dominance. The dominance curve can be expressed as follows:

$$D^s(z) = cP(\alpha = s-1, z) \qquad (6)$$

where $c = 1/(s-1)!$ is a constant term. The distribution B dominates in poverty the distribution A for the order $s = \alpha + 1$ if:

$$\Delta^\alpha(z) = P_A(\alpha,z) - P_B(\alpha,z) > 0 \quad \forall z \in [0,\infty] \qquad (7)$$

where $P(\alpha,z)$ is the FGT index. Dominance here refers to the distribution that generates more social welfare or less poverty. Usually, one checks the dominance between two distributions for the following:

- two successive periods for the same country;
- two groups in the same country; or
- two countries.

3. Poverty and Dominance Testing

3.1 The numerical approach

One of the simplest numerical approaches to test for poverty dominance is the *Grid Approach*. The procedure is based on the comparison between two curves for a range of poverty lines z or percentiles p with a fixed step. If the two curves cross, then a simple linear approximation is used to estimate the critical value. However, this approach has the following drawbacks:

a. If there are two successive intersections within the step, the intersections cannot be detected;

b. Using a short step is costly in computations and requires more time; and

c. The linear approximation continues to suffer from some residual error.

3.2 The theoretical approach

With discrete data, we propose to develop the main rules that can be used to consistently check the dominance for the three widely used orders, the first, second and the third. If we note the income for household i, that belongs to distribution D, by y_i^D and its proportion in the population by π_i^D the distribution D is defined as follows:

$$D(Y,\Pi) = \left\{ y_i^D, \pi_i^D \middle| i \in D \right\} \quad (8)$$

Suppose that the two distributions A and B are combined and are sorted by the vector of income Y, to form one data set which takes the following form:

$$S = \{A, B\} = \{ y_i, \pi_i^{A|S}, \pi_i^{B|S} \mid i \in S \} \quad (9)$$

where $\pi_i^{D|S} = \pi_i^D$ if $i \in D$ and zero otherwise. The final step for the treatment of the data is to aggregate them by summing proportions $\pi_i^{D|S}$ according to Y. This procedure ensures that there is only a unique value for each $y_i \in S$. In appendix A, we give an example to explain these steps in clearer way.

Lemma 1

$$P_D(\alpha, z) = P_{D|S}(\alpha, z) \quad (10)$$

where $P_{D|S}(\alpha, z)$ is the FGT index when the distribution $\{ y_i, \pi_i^{D|S} \}$ is used. This lemma indicates that poverty indices do not change with the rearrangement of the data[3].

[3]The rearrangement of the data is required for the theoretical developments that we propose in this paper.

Lemma 2

$$\Delta^{\alpha}(z) > 0 \ \ \forall z \in \left[y^{S}_{min}, z^{+} \right] \Leftrightarrow \Delta^{\alpha}(z) > 0 \ \forall z \in [y_i, y_{i+1}[$$

and $\forall i \in [1, j]$

and $y^{S}_{j-1} < z^{+} \leq y^{S}_{j}$ (11)

y^{S}_{min} is the minimum level of the vector of incomes Y^{S}. This lemma indicates that checking the stochastic dominance condition within the range $\left[y^{S}_{min}, z^{+} \right]$ is equivalent to checking this dominance between ranges, formed by the discrete data until z^{+}.

Theorem 3

Between two successive points y_i and $y_{i+1} \in Y$.

A : If $\Delta^{\alpha=0}(y_i) > 0$, then $\Delta^{\alpha=0}(z) > 0 \ \ \forall z \in [y_i, y_{i+1}[$.

B : If $\Delta^{\alpha=0}(y_i) \neq 0$ and $\Delta^{\alpha=0}(y_i).\Delta^{\alpha=0}(y_{i+1}) < 0$, then the unique intersection is equal to y_{i+1}.

C : If $\Delta^{\alpha=0}(y_i) = 0$, then the range of intersections will be $[y_i, y_{i+1}[$.

Proof

[A] : The curve $\Delta^{\alpha=0}(z)$ takes a horizontal form for the range $z \in [y_i, y_{i+1}[$.

[B] : The sign of $\Delta^{\alpha=0}(z)$ changes after introducing the observation y_{i+1} and the difference increases or decreases vertically at y_{i+1}.

[C] : See [A:].

Theorem 4

Between two successive points y_i and $y_{i+1} \in Y$, the maximum number of intersections between the two curves of dominance for the order $s \geq 2$, is $(s-1)$.

Proof

Since between y_i and y_{i+1} there are no any additional changes except the increase of the poverty line z, the difference between the two curves takes the following polynomial form:

$$\Delta^{\alpha=(s-1)}(z) = \sum_{s=1}^{s} a_s z^{s-1} \qquad (12)$$

where a_s are known parameters. Polynomial with degree s, has exactly s roots, real or complex.

Corollary 5

There is only one intersection between two successive points, y_i and y_{i+1} if:

$$\Delta^{\alpha=1}(y_i).\Delta^{\alpha=1}(y_{i+1}) < 0. \quad (13)$$

Proof

Based on Theorem 4, the function $\Delta^{\alpha=1}(z)$ takes a linear form between y_i, y_{i+1} such that $\Delta^{\alpha=1}(z) = a_1 + a_2 z$. Solving the equation: $a_1 + a_2 z = 0$ gives us a unique root.

Theorem 6

Consider two successive points y_i and $y_{i+1} \in Y$. We have:

$$\Delta^{\alpha=1}(y_i) > 0 \text{ and } \Delta^{\alpha=1}(y_{i+1}) > 0 \Rightarrow \Delta^{\alpha=1}(z) > 0 \; \forall z \in [y_i, y_{i+1}] \quad (14)$$

Proof

Based on theorem 4, the function $\Delta^{\alpha=1}(z)$ takes a linear form between y_i, y_{i+1} such that $\Delta^{\alpha=1}(z) = a_1 + a_2 z$. Since $\Delta^{\alpha=1}(y_i) > 0$ and $\Delta^{\alpha=1}(y_{i+1}) > 0$, the level of the curve $\Delta^{\alpha=1}(z)$ should be higher than zero for all $z \in [y_i, y_{i+1}]$.

Theorem 7

Consider two successive points y_i and $y_{i+1} \in Y$, such that:

$$\Delta^{\alpha=2}(y_i) > 0 \; and \; \Delta^{\alpha=2}(y_{i+1}) > 0 \quad (15)$$

A : $\Delta^{\alpha=1}(y_i) \geq 0 \; and \; \Delta^{\alpha=1}(y_{i+1}) \geq 0$, then $\Delta^{\alpha=2}(z) > 0 \;\; \forall z \in [y_i, y_{i+1}]$

B : $\Delta^{\alpha=1}(y_i) < 0 \; and \; \Delta^{\alpha=1}(y_{i+1}) > 0$, then the maximum number of intersections is two.

Proof

[A] : The level of $\Delta^{\alpha=2}(z)$ for $z \in [y_i, y_{i+1}]$, depends on the initial value $\Delta^{\alpha=2}(y_i)$ and the tangency of $\Delta^{\alpha=2}(z)$ within this interval. If the tangency $\Delta^{\alpha=1}(z) \geq 0 \; \forall z \in [y_i, y_{i+1}]$, the increase of z does not decrease the initial difference. Since it is supposed that the condition: $\Delta^{\alpha=1}(y_i) > 0$ and $\Delta^{\alpha=1}(y_{i+1}) > 0$ is satisfied and based on Theorem 6, the tangency $\Delta^{\alpha=1}(z) \geq 0 \; \forall z \in [y_i, y_{i+1}]$.

[B] : See Theorem 4 for the possible number of intersections.

Theorem 8

Let two successive points y_i and $y_{i+1} \in Y$. If the following conditions are satisfied:

$$\Delta^{\alpha=2}(y_i) > 0 \; and \; \Delta^{\alpha=2}(y_{i+1}) < 0 \tag{16}$$

$$\Delta^{\alpha=1}(y_i) < 0 \; and \; \Delta^{\alpha=1}(y_{i+1}) < 0 \tag{17}$$

then the maximum number of intersections is one.

Proof

Since the tangency $\Delta^{\alpha=1}(z) \leq 0 \; \forall z \in [y_i, y_{i+1}]$, the difference between the two curves continues to decrease between y_i, y_{i+1} and its value is null only for one critical value of z.

This theoretical framework is very useful to design founded procedures to test the stochastic dominance or to estimate all possible critical values accurately. The condition of continuous checking within the interval $[y_i, y_{i+1}]$ is simplified with discrete distributions to bounds of this interval.

3.3 Estimating the critical values

In this context, by critical values we refer to the level of poverty line for which two dominance curves cross. We restrict the discussion here to the three widely used dominance orders; the first, the second, and the third order. Within the interval $[y_i, y_{i+1}]$, the three main cases that one would encounter are:

Case	Difference
A	$\Delta^{\alpha}(y_i) > 0$ *and* $\Delta^{\alpha}(y_{i+1}) < 0$
B	$\Delta^{\alpha}(y_i) < 0$ *and* $\Delta^{\alpha}(y_{i+1}) > 0$
C	$\Delta^{\alpha}(y_i) = 0$

For case **C**, the estimation of the critical value, noted by z^*, is trivial and equals y_i.

3.3.1 Critical values for the first order dominance

For the first order $s = 1$, for the two cases **A** and **B**, the critical value is equal to y_{i+1}. The sign of the difference in headcount $\Delta^{\alpha=0}(z = y_i) = \sum_{j=1}^{i} \pi_j^A - \sum_{j=1}^{i} \pi_j^B$ changes after introducing the observation y_{i+1} and the curve $\Delta^{\alpha=0}(z)$ increases or decreases vertically.

3.3.2 Critical values for the second order dominance

For the two main cases **A** and **B**, we have to solve this simple following equation:

$$\sum_{j=1}^{i} \pi_j^A z - \sum_{j=1}^{i} \pi_j^A y_j = \sum_{j=1}^{i} \pi_j^B z - \sum_{j=1}^{i} \pi_j^B y_j \tag{18}$$

The critical value is equal to:

$$z^* = \frac{H_i^A \mu_i^A - H_i^B \mu_i^B}{H_i^B - H_i^A} \quad (19)$$

where H_i^D and μ_i^D are respectively the headcount and the average income of the poor group when $z = y_i$.

3.3.3 Critical values for the third order dominance

We discuss the possible intersections for case **A** (the discussion of case **B** is similar). Based on Theorems 7 and 8, and where the intersections are possible we have that:

$$\Delta^{\alpha=2}(z^*) = 0 \; and \; z^* \in [y_i, y_{i+1}] \quad (20)$$

To estimate the valid intersections, one should solve the following equation:

$$\sum_{j=1}^{i} \pi_j^A \left(z^2 - 2zy_j + y_j^2 \right) - \sum_{j=1}^{i} \pi_j^B \left(z^2 - 2zy_j + y_j^2 \right) = 0 \quad (21)$$

One can rewrite this equation as follows:

$$a_3 z^2 + a_2 z + a_1 = 0 \quad and \; z \in [y_i, y_{i+1}] \quad (22)$$

where, a_i are known parameters, such that:

$$\begin{aligned} a_1 &= H_i^A \omega_i^A - H_i^B \omega_i^B \\ a_2 &= 2(H_i^B \mu_i^B - H_i^A \mu_i^A) \\ a_3 &= H_i^A - H_i^B \end{aligned} \quad (23)$$

where ω_i^D is the average of square incomes for the group with income below or equal to y_i.

4. Inequality and Stochastic Dominance Test

The widely used approach to test the stochastic dominance in inequality is the comparison between the Lorenz curves. According to the Atkinson's Theorem [4], all indices that respect the Pigou-Dalton principle should indicate that inequality in A is higher than inequality in B when $L_B(p)$ be everywhere above $L_A(p)$. Formally, distribution A dominates in inequality distribution B, with the second order, if [5]:

$$L_A(p) > L_B(p) \; \forall p \in [0,1] \quad (24)$$

[4]See Atkinson (1970).

[5]The distribution dominates in inequality when its level in inequality is the lower. The decrease in

where p refers to the percentile. Here, one can again propose some general rules to check for the stochastic dominance in the presence of discrete data sets. Recall that the Lorenz curve for the percentile p_i can be defined as follows:

$$L(p = p_i) = \frac{1}{\mu}\sum_{j=1}^{i} \pi_j y_j \qquad (25)$$

The main characteristic of the Lorenz curve with discrete data is the straight line that ties $L(p_i)$ and $L(p_{i+1})$. This implies that the difference between two Lorenz curves takes a straight line format between two successive percentiles derived from these two distributions. Hence, the first step is to combine the two percentile vectors of the two distributions. Then we estimate the Lorenz curves for all retained values of this new vector of percentiles (see again the example in Appendix B). We define the difference between two Lorenz curves for the percentile p by $\Delta(p)$.

Theorem 9

Let two successive percentiles p_i, p_{i+1}.

If: $\Delta(p_i) > 0 \ and \ \Delta(p_{i+1}) > 0$, then $\Delta(p) > 0 \ \forall p \in [p_i, p_{i+1}]$

Theorem 10

Between two successive percentiles p_i, p_{i+1}, the maximum number of intersections between the Lorenz curves is one.

Theorem 11

There is only one intersection between two successive percentiles p_i, p_{i+1} if: $\Delta(p_i).\Delta(p_{i+1}) < 0$.

One can generalize this and confirm that these rules are always valid for the comparison between the generalized Lorenz curves, the Lorenz and the concentration curves or between the TIP (Three I's Poverty) curves [6].

5. Statistical Robustness of the Stochastic Dominance

5.1 Stochastic dominance with statistical robustness

Despite the fact that one can estimate the difference between the two distributional curves to check the stochastic dominance for welfare, poverty or inequality, this difference may not be statistically significant. Our interest here is to check if the stochastic dominance

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inequality generates more social welfare.

is statistically robust. One can again estimate the critical values according to the selected statistical significance level.

Proposition 12

The dominance in poverty for the order s is statistically robust if the difference between the two dominance curves is statistically significant for all poverty line $z \in \left[0, z^{+}\right]$.

For the $(\alpha+1)_{th}$ order of dominance, the statistical robustness of the stochastic dominance is satisfied with a critical level of significance θ, if the following null hypothesis H_0 is rejected: $\forall z \in \left[0, z^{+}\right]$.

$$H_0 : \Delta^{\alpha}(z) \leq 0 \;\; against \; H_1 : \Delta^{\alpha}(z) > 0 \qquad (26)$$

The null hypothesis is rejected if:

$$LB_{\Delta^{\alpha}}(z) = \hat{\Delta}^{\alpha}(z) - \hat{\delta}_{\Delta}(z) C_{\theta} > 0 \qquad (27)$$

where $LB_{\Delta^{\alpha}}(z)$ is the lower bound of the confidence interval, $\hat{\delta}_{\Delta}(z)$ is the estimated standard deviation and C_{θ} refers to the cumulative distribution of the estimated parameter $\hat{\Delta}^{\alpha}(z)$, evaluated at the critical significance level θ. Generally, the distribution of the estimate takes the *t-student* form with the smallest number of observations. When the number of observations or the degree of freedom is higher, this distribution converges to the *normal* form. Estimation of the standard errors $\hat{\delta}_{\Delta}(z)$ depends again on the sampling design of the two distributions[7].

Proposition 13

The second order dominance in inequality based on the comparison between Lorenz curves is statistically robust if the difference between the two Lorenz curves is statistically significant $\forall p \in [0,1]$.

5.2 Critical values with statistical robustness

Critical values with statistical robustness refer to the limits of the poverty line z or the percentile p where the statistical robustness of the dominance continues to be checked. Note that the critical values with statistical robustness condition can be different from those based only on basic theoretical conditions of dominance.

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[6]TIP curves simultaneously show the *Incidence*, *Intensity* of poverty and *Inequality* within the poor group. See also Jenkins and Lambert (1998) for the use of the TIP curves.
[7]For this, see Duclos and Araar (2006), chapters 16 and 17.

6. Illustration Using Burkina Faso National Surveys

The two nationally representative Burkina Faso surveys used in this study were carried out in 1994 and 1998. These surveys were made with sample selection using the two-stage stratified random sampling method. The country was stratified into seven in 1994 and ten in 1998. For the survey of 1994, five of these strata were rural and two were urban. Enumeration areas (PSUs, or *zones de dénombrement*) were sampled in the first stage from a census list prepared in 1985. In the first-stage, sampling was done in stratum 7 (Ougadougou-Bobo-Dioulasso) with equal probability and for the other six strata sampling was done with probability proportional to the size of each PSU. Twenty households were then systematically sampled within each of the selected PSUs in a second stage. The survey of 1998 is similar to that of 1994.

The consumption per capita is used to represent the household living standard. For the year 1998, consumption is deflated by the ratio of poverty lines of the two periods. We use the Stata modules that the author has developed based on theoretical findings of this paper to perform the estimations[8].

Figures 1 and 2 show the difference between the FGT curves where the parameter α equals zero. Note that the dominance condition is not satisfied and that the two dominance curves cross, as reported in table 1. One can recall here that the official poverty line in Burkina Faso was 41099 F CFA for the referenced year of 1994. Even if one restricts the range of all possible poverty lines around this official line, intersections are encountered when the poverty line is between 35 000 and 45 000 F CFA.

Figure 3 shows that the condition of dominance is not satisfied for the second order. Intersections between dominance curves are presented in table 2. For a restricted range of poverty line between 35 000 and 45 000 F CFA, intersections are not encountered and the deficit of poverty decreased in 1998. For the severity indices of poverty, one cannot draw a robust conclusion, since intersections are encountered as presented in figure 4 and table 3. Figures 5 and 6 show again the difference between the FGT curves and the lower bound of the estimated difference[9]. One can see that with or without the statistical robustness, conditions of dominance are generally not satisfied. In figure 7 we show that, without the condition of statistical robustness, female headed households dominate male headed households in poverty for the year 1994. By adding the statistical robustness condition, the stochastic dominance is not checked.

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[8]The Stata modules povdom.ado and ineqdom.ado perform the test of dominance and estimate the critical values. These modules are also contained in the DASP Package (Araar (2006))

[9]We have taken into account the sampling design in carrying out the estimation of standard errors and bounds of confident interval. Stata modules that the author has developed for these estimations are

With respect to the dominance in inequality for these two periods, figure 8 and table 4 show that the Lorenz curves cross for two percentiles. Here, one cannot draw again a robust conclusion about the variation in inequality between these two periods.

7. Conclusion

Comparing levels of poverty or inequality between distributions remains a major area of interest to both researchers and policy makers. In the last twenty-five years, several countries have experienced important changes in their economies which have triggered changes in their distribution of income or wealth. In general, cardinal indices have typically been used to assess the evolution of poverty or inequality. This approach can be criticized, since cardinal indices differ in their sensitivities over different parts of the distribution. The stochastic dominance approach allows us in some cases to make a robust ordinal classification of distributions according to their level in poverty or in inequality. Previous theoretical frameworks have, however, been built under the assumption of continuity of incomes. This paper treats the case of discontinuous or discrete distributions. This is justified by the fact that household surveys have a discrete form.

Also, in this paper we propose some conditions for testing the statistical robustness of stochastic dominance orderings. Importantly, such statistical conditions can change our conclusions about dominance relationships. Further, the proposed tools can be useful to design better anti-poverty programs by:

- helping to perform stochastic dominance tests to give more robust ethical results required in the design of anti-poverty programs; and
- providing statistical inference tools useful in empirically designing anti-poverty programs.

For example, if a government plans to target poorer regions, the statistical significance of the estimated differences in poverty between the regions should be validated with statistical tests for stochastic dominance.

The methods of this paper are illustrated with the Burkina Faso household surveys for the years 1994 and 1998. In general, the application shows that one cannot draw a robust conclusion on changes of poverty or inequality between these two periods. This is explained essentially by the statistically insignificant changes in the distribution of living standards between these two periods.

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cdifgt.ado and cdilorenz.ado. These modules can be provided upon request.

Table 1. Intersection between FGT curves ($\alpha = 0$)

#	Critical value z^*	Case
1	16279.44	A
2	16299.83	B
3	16321.05	A
4	16331.54	B
5	16481.85	A
6	16495.57	B
7	16499.07	A
8	16511.96	B
9	16804.51	A
10	16948.54	B
11	16962.91	A
12	17025.70	B
13	17095.26	A
14	17102.50	B
15	17132.36	A
16	17133.75	B
17	17216.77	A
18	17218.41	B
19	17221.39	A
20	17254.08	B
21	17284.45	A
22	17296.76	B
23	17356.22	A
24	17364.65	B
25	17430.86	A
26	17487.57	B
27	17578.69	A
28	38604.25	B
29	38648.23	A
30	38657.02	B
31	334044.38	A
32	421654.78	B
33	430845.78	A
34	1444116.00	B
35	1690216.25	A

Case A: Distribution 1 dominates distribution 2 before the intersection
Case B: Distribution 2 dominates distribution 1 before the intersection

Table 2. Intersection between FGT curves ($\alpha = 1$)

#	Critical value z^*	Case
1	24262.87	A
2	46775.65	B

Case A: Distribution 1 dominates distribution 2 before the intersection
Case B: Distribution 2 dominates distribution 1 before the intersection

Table 3. Intersection between FGT curves ($\alpha = 2$)

#	Critical value z^*	Case
1	32965.56	A
2	56509.56	B

Case A: Distribution 1 dominates distribution 2 before the intersection
Case B: Distribution 2 dominates distribution 1 before the intersection

Figure 1. Difference between FGT Curves: (1998)-(1994) ($\alpha = 0$)

Figure 2. Difference between FGT Curves: (1998)-(1994) ($\alpha = 0$)

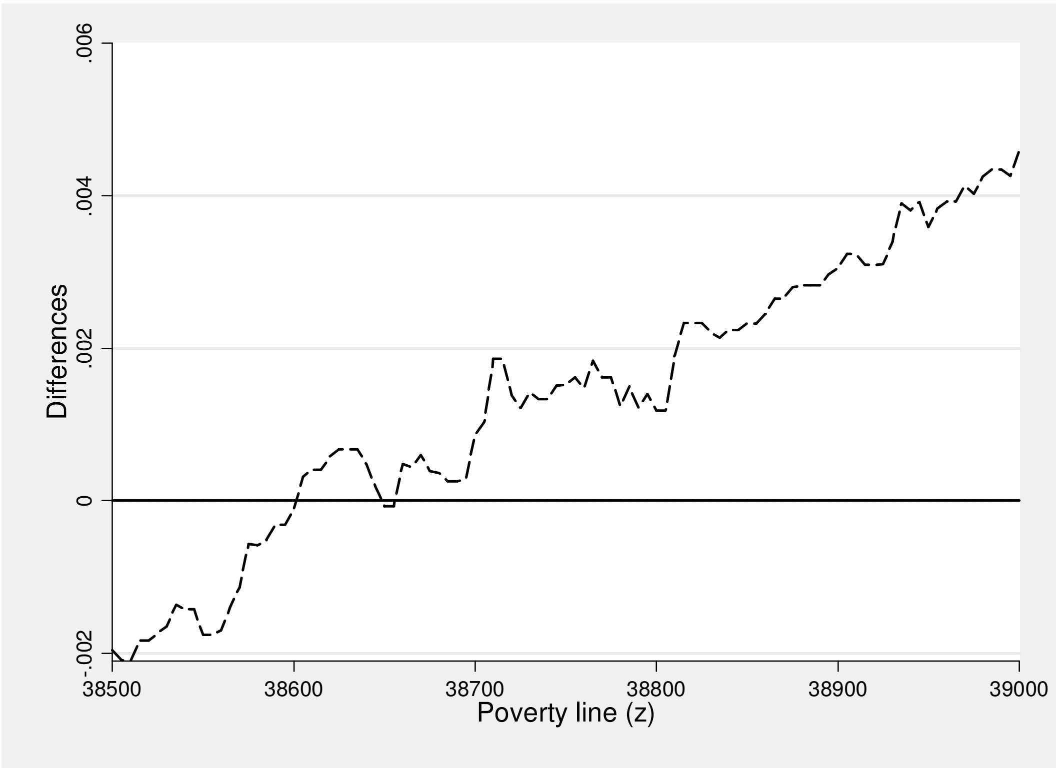

Figure 3. Difference between FGT Curves: (1998)-(1994) ($\alpha = 1$)

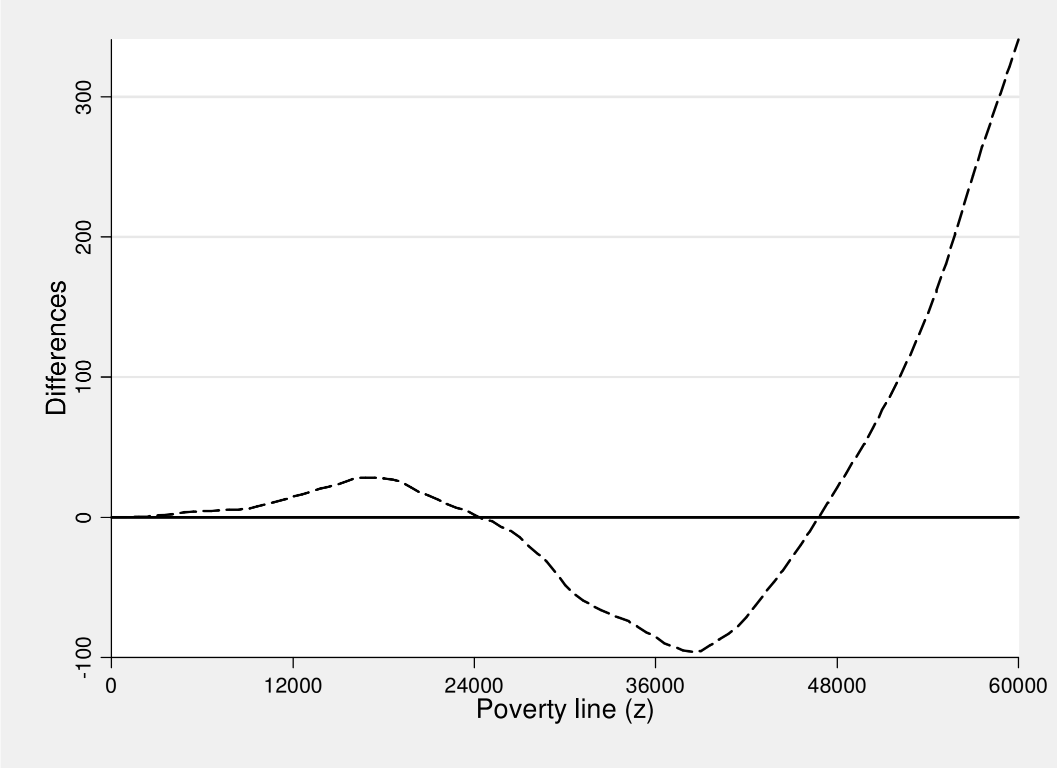

Figure 4. Difference between FGT curves: (1998)-(1994) ($\alpha = 2$)

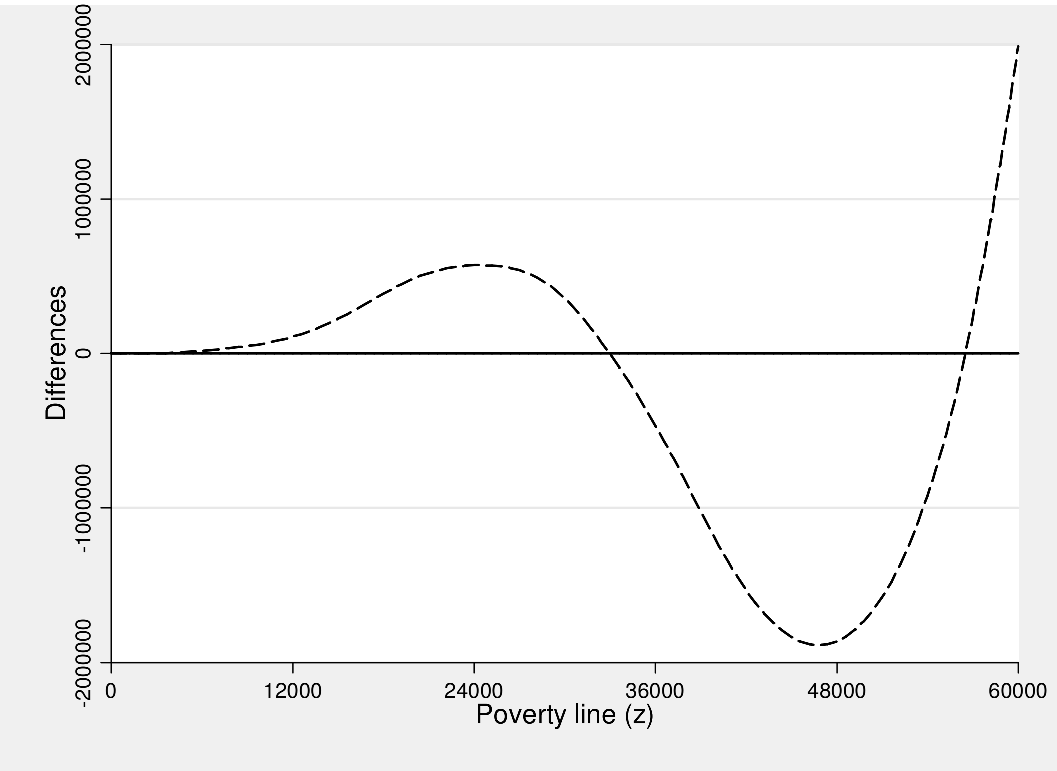

Figure 5. Difference between FGT Curves and the statistical robustness (1998)-(1994): ($\alpha = 0$)

Figure 6. Difference between FGT curves and the statistical robustness (1998)-(1994): ($\alpha = 1$)

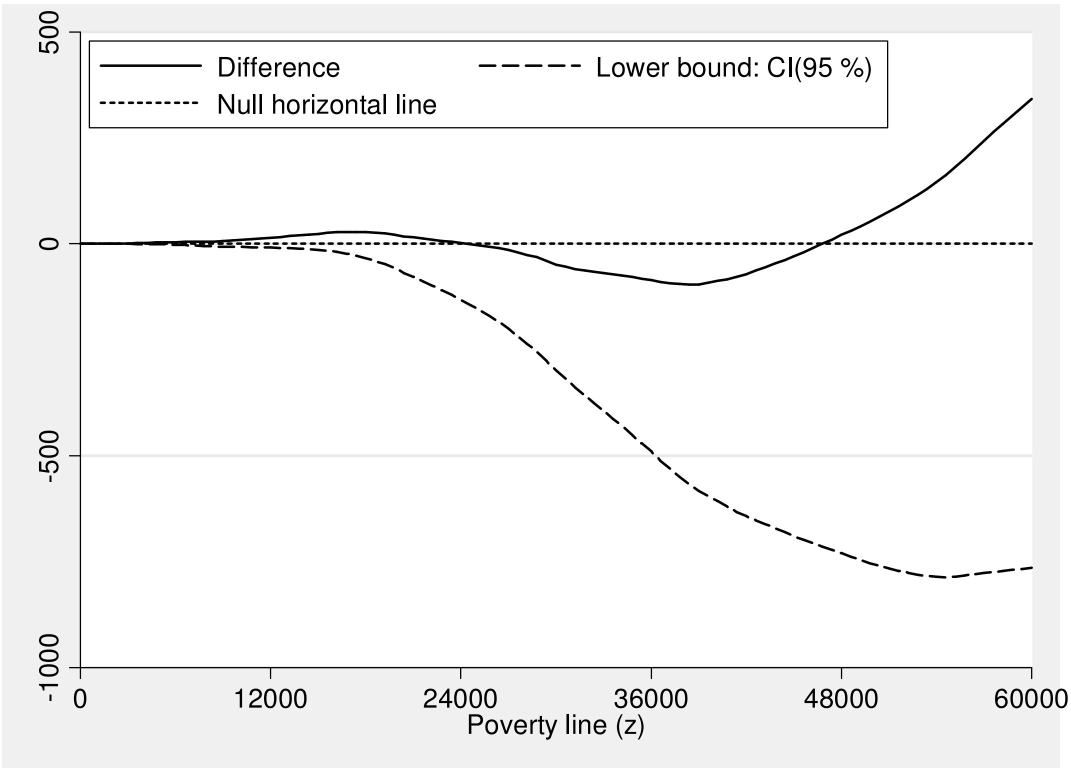

Figure 7. Difference between FGT curves according to the gender of household head: (Male)-(Female): Burkina Faso 1994 (α =0)

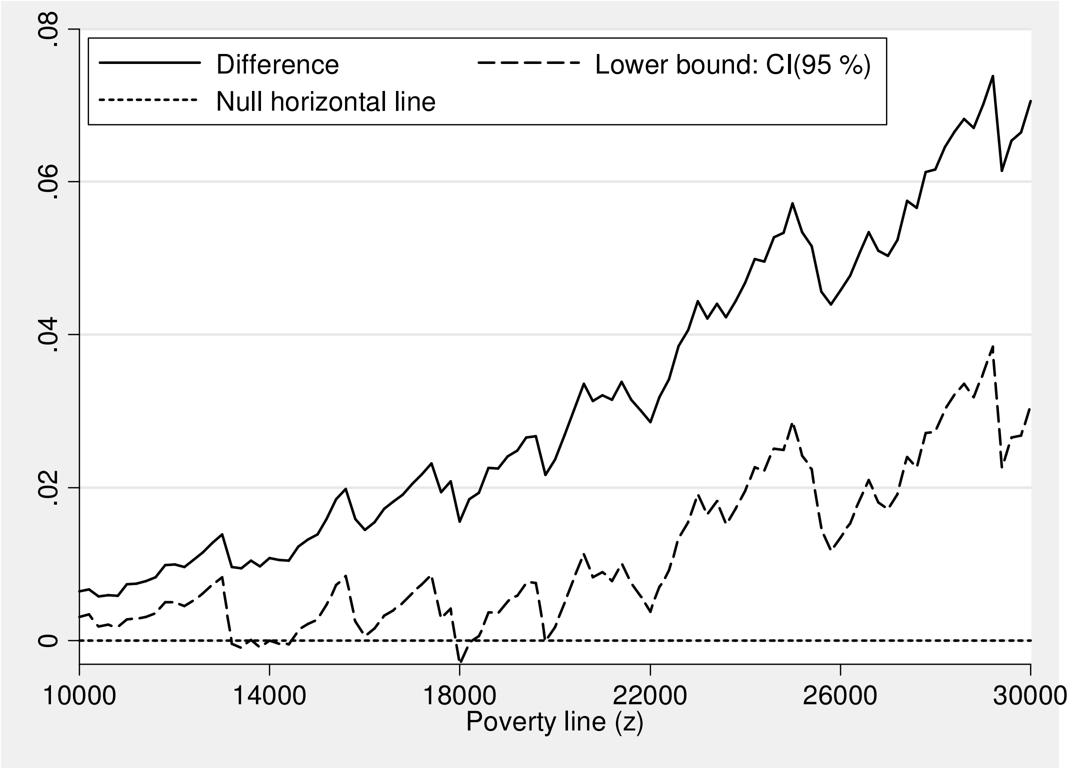

Figure 8. Difference between Lorenz curves (1998)-(1994)

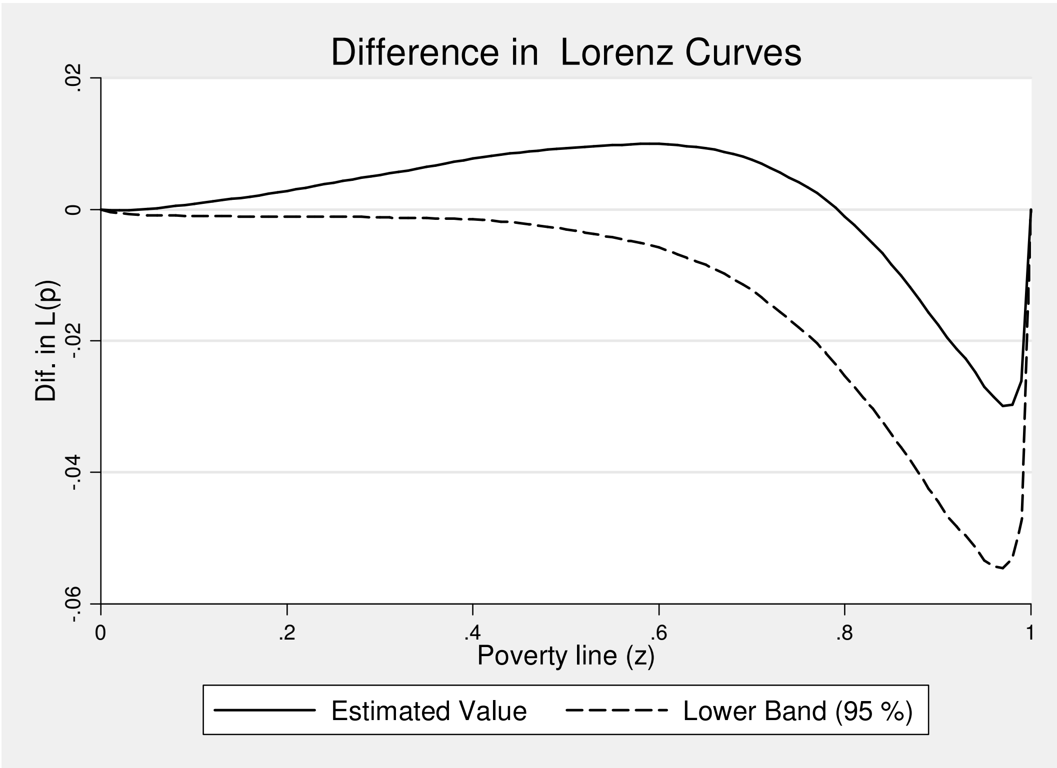

Table 4. Intersection between Lorenz curves: (1994) vs. (1998)

#	Critical value p^*	Case
1	0.048857	B
2	0.791681	A

Case A: Curve_1 is below Curve_2 before the intersection
Case B: Curve_1 is above Curve_2 before the intersection

Appendix A: Illustrative example 1

Data A		**Data** B		**Combined Data:** S		
Y^A	Π^A	Y^B	Π^B	Y	$\Pi^{A\|S}$	$\Pi^{B\|S}$
13	0.4	13	0.3	13	0.4	0.6
15	0.6	13	0.3	15	0.6	0
		30	0.4	30	0	0.4

Appendix B: Illustrative example 2

Data A			**Data** B			**Combined Data:** S		
Y^A	p^A	$L^A(p)$	Y^B	p^B	$L^B(p)$	p	$L^A(p)$	$L^B(p)$
–	0.0	.0000	–	0.0	.0000	0.0	.0000	.0000
3	0.1	.0441	2	0.1	.0408	0.1	.0441	.0408
5	0.4	.2647	3	0.4	.2245	0.4	.2647	.2245
7	0.6	.4706	4	0.5	.3061	0.5	.3676	.3061
9	1.0	1.0000	6	0.8	.6735	0.6	.4706	.4286
			8	1.0	1.0000	0.8	.7353	.6735
						1.0	1.0000	1.0000

References

ARAAR, A. (2006): *DASP: Distributive Analysis Stata Package.*, Laval University, PEP & CIRPEE.

ARAAR, A. AND J.-Y. DUCLOS (2005): "An Atkinson-Gini Family of Social Evaluation Functions: Theory and Illustration using Data from the Luxembourg Income Study," Tech. Rep. LIS-WP: 416.

ATKINSON, A. B. (1970): "On the Measurement of Inequality," *Journal of Economic Theory*, 2, 244–63.

ATKINSON, A. B. (1987): "On the Measurement of Poverty," *Econometrica*, 55, 749–764.

DAVIDSON, R. AND J.-Y. DUCLOS (2000): "Statistical Inference for Stochastic Dominance and the for the Measurement of Poverty and Inequality," *Econometrica*, 68, 1435–1465.

DUCLOS, J. Y. AND A. ARAAR (2006): *Poverty and Equity Measurement, Policy, and Estimation with DAD*, Boston/Dordrecht/London: Springer/Kluwer Academic Publishers.

FOSTER, J. E. AND A. F. SHORROCKS (1988 a): "Poverty Orderings," *Econometrica*, 56, 173–177.

FOSTER, J. E. AND A. F. SHORROCKS (1988 b): "Poverty Orderings and Welfare Dominance," *Social Choice Welfare*, 5, 179–98.

JENKINS, S. P. AND P. J. LAMBERT (1998): ",Three 'I's of Poverty Curves and Poverty Dominance: TIPs for Poverty Analysis," *Research on Economic Inequality*, 8.