Hedging Exposure to Electricity Price Risk in a Value at Risk Framework

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HEDGING EXPOSURE TO ELECTRICITY PRICE RISK IN A VALUE AT RISK FRAMEWORK

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This paper deals with the question how an electricity end-consumer or distribution company should structure its portfolio with energy forward contracts. This paper introduces a one period framework to determine optimal positions in peak and off-peak contracts in order to purchase future consumption volume. In this framework, the end-consumer or distribution company is assumed to minimize expected costs of purchasing respecting an ex-ante risk limit defined in terms of Value at Risk. Based on prices from the German EEX market, it is shown that a risk-loving agent is able to obtain lower expected costs than for a risk-averse agent.

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1 INTRODUCTION

Worldwide electricity markets are being deregulated. Electricity producers, distribution companies and consumers are free to trade electricity contracts bilaterally amongst each other. As a result, electricity markets have been established to facilitate trading in spot and derivative contracts that involve intra-day, day-ahead or longer-term ahead delivery.

This paper focuses on the purchasing decision of consumers and distribution companies. Overlooking their expected future consumption (or the future consumption of a batch of clients in the case of a distribution company), consumers have to decide when and which contracts to purchase in order to deal with the price risk they face. This price risk origins from the variation in prices on day-ahead markets (or intra-day markets in some countries), as these most closely resemble spot markets. If consumers have not contracted their consumption on beforehand, they have to purchase on these spot markets. Prices on day-ahead markets are extremely volatile and exhibit frequent price jumps[1]. In order to manage the price risk consumers face, they can trade in many derivative contracts.

Popular contracts are the base-load and peak-load forward contracts that involve delivery in a future period of time, for example the calendar year of 2008 or the second quarter of 2007. A typical base-load contract involves the delivery of 1MW of electricity in all hours in the delivery period. A typical peak-load contract involves the delivery of 1MW of electricity in the peak hours in the delivery period, where the definition of peak hours may differ over markets. For instance, the German EEX market defines the peak between 8 am and 8 pm on working days and the Dutch APX market defines the peak between 8 am and 23 pm. These contracts are traded over the counter or as futures on many exchanges. On the German EEX market, one can trade calendar year contracts up to 6 years ahead although the more recent contracts are more liquid.

Given these set of forward and futures contracts that can be traded every day, the consumer has to question herself frequently how to contract her future consumption: to wait and purchase on the day-ahead market or to (partially) hedge this risk by purchasing some of these forward contracts. If she decides to hedge, she actually manages a portfolio of electricity forward and futures contracts that cover (partly) the delivery of her expected future consumption. As being a portfolio manager, her goal in the hedging decision process is to obtain minimum expected costs of consumption conditional on her risk appetite.

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[1] See Bunn and Karakatsani (2003) and Huisman, Huurman and Mahieu (2007) among others for reference on the behaviour of day-ahead electricity prices.

This paper deals with the optimal number of forward contracts to hold in the portfolio such that expected costs are minimized respecting a risk appetite expressed in terms of Value at Risk. Value at Risk is here defined as the maximum costs she is willing to pay for the consumption in a certain delivery period with a given level of confidence. To do so, a one-period model is introduced to calculate the optimal hedge amounts for peak-load and off-peak load contracts.

This portfolio construction problem is in line with the problem formulation of the famous Markowitz (1952) model, where efficient investment portfolios are constructed based on the investors goal to maximize expected future returns given a certain level of risk. In this model, risk is measured by the standard deviation of the portfolio returns. Campbell et al. (2001) introduced a similar portfolio allocation model in which risk is defined in terms of Value at Risk. In the electricity literature, some have followed the Markowitz methodology to address the forward hedging issue, particularly Nasakkala and Keppo (2005) and Woo, Horowitz, Hori and Karimov (2004) who study the interaction between stochastic consumption volumes and electricity prices (day ahead and forward) and propose a mean-variance type model to determine optimal hedging strategies. Vehvilainen and Keppo (2004) take the viewpoint of a generation company and optimize hedging strategies using VaR as a risk measure.

To analyze both stochastic consumption and prices is meaningful, yet complex. Some cases need Monte Carlo simulations in order to provide results. This paper takes a step back and assumes that consumption is deterministic[2]. In a one-period framework, it focuses on the decision how much to hedge in peak-load and off-peak load hours. This paper provides analytical formulas that make it possible to analyze the relation between the quoted forward prices and the risks faced on the day-ahead market in isolation of other stochastic variables.

It is shown that the Markowitz (1952) concept of efficient frontiers also applies to electricity. Applying the formulas obtained from the model using data from the German EEX market, this paper shows that consumers can obtain lower expected costs when they are willing to take more risk.

[2] In practice, load is not deterministic but the variability of consumption volume is much smaller than the variability of day-ahead electricity prices.

2 THE MODEL

Consider an electricity consumer (or a distribution company that purchases electricity for a batch of consumers) who decides today, date t, which contracts to enter in order to purchase the electricity consumption for a later day T. Today, she can trade two forward contracts that deliver on day T: a peak contract that delivers 1MW of electricity in all peak hours of day T and an off-peak contract that delivers 1MW in all off-peak hours[3]. If, before T, she does not enter in any forward contract, she has to purchase her electricity needs on the day-ahead market at time T-1. Therefore, the decision on the number of forward contracts to buy or sell depends on her expectations of the future day-ahead prices in relation to the current forward prices.

The following model aims to determine the optimal number of contracts to purchase. Let P be the set of peak hours and let O be the set of off-peak hours in day T. Let s(h) be the price on the day-ahead market at T-1 for delivery of 1MW in hour h on day T. The volume needed in hour h on day T equals v(h)[4]. Furthermore, let θ_p be the number of peak load forward contracts and let f_p the market peak forward price at time t for delivery at time T. Equivalently, θ_o and f_o represent those values for off-peak hours. The expected costs $E_t\{TC\}$ for the time T electricity consumption equal the sum, over each hour, of the costs of the volume purchased with forwards plus the expected costs of the amount that has to be purchased on the day-ahead market :

$$(1)\quad E_t\{TC\} = \Sigma_{h\in P}\,[\theta_p f_p + (v(h) - \theta_p)\, E_t\{s(h)\}] + \Sigma_{h\in O}\,[\theta_o f_o + (v(h) - \theta_o)\, E_t\{s(h)\}],$$

where E_t represents the expectation conditional on the information available at time t. A rational end consumer wants to minimize the expected costs, but as she faces risk from the open position on the day-ahead market, she does so respecting an ex-ante risk limit. Risk is expressed in terms of a Value-at-Risk type measure and reflects the fact that she would limit the probability that the total costs for consumption on day T exceeds the threshold TC^*.

$$(2)\quad \Pr\{TC \geq TC^*\} = 1\text{-}c,$$

[3] In most power markets, off-peak contracts do not exist and only baseload (involving delivery a constant load of electricity in each hour of the day) and peak contracts are traded. The off-peak contract can be constructed synthetically by buying a baseload contract and selling a peakload contract.

[4] The model assumes that the volume is certain.

where c is a confidence level (typically between 95% and 99% in practice). Substituting the total costs (1) in the VaR constraint (2) yields:

(3) $\Pr\{\Sigma_{h\in P} [(v(h) - \theta_p) s(h)] + \Sigma_{h\in O} [(v(h) - \theta_o) s(h)] \geq TC^* - n_p\theta_p f_p - n_o\theta_o f_o\} = 1\text{-}c$,

Where n_o and n_p equal the number of off-peak and peak hours respectively in the delivery day. Focus on the equation within the probability brackets. The left hand side equals the open position on the day-ahead market. The right hand side equals the difference between the maximum she is willing to pay and the total costs of purchasing her volume using forward contracts. The equation states that the size of her open position on the day-ahead market is determined by the amount that she is willing to pay in excess of the total costs of hedging using the forward contracts.

Following Campbell et al. (2001), the VaR constraint is rewritten in the following way. Let X be the stochastic open position on the day-ahead market in (3) and let D be the probability distribution function of X. Thus,

(4) $X = \Sigma_{h\in P} [(v(h) - \theta_p) s(h)] + \Sigma_{h\in O} [(v_o - \theta_o) s(h)] \sim D(\mu(\theta_p, \theta_o), \sigma^2(\theta_p, \theta_o))$.

Note that the average and variance of D depend on the amounts hedged in the peak and off-peak hours. Normalizing (4) yields:

(5) $Z = \{X - \mu(\theta_p, \theta_o)\} / \sigma(\theta_p, \theta_o) \sim D(0, 1)$.

The c^{th} quantile of D(0,1), ϕ_c, is the number such that $\Pr\{Z \leq \phi_c\} = c$. Substituting this in (5) yields

(6) $\Pr\{X \leq \mu(\theta_p, \theta_o) + \phi_c\, \sigma(\theta_p, \theta_o)\} = c$.

From (3) and (6), it can be shown that

(7) $TC^* - n_p\theta_p f_p - n_o\theta_o f_o = \mu(\theta_p, \theta_o) + \phi_c\, \sigma(\theta_p, \theta_o)$.

Note that the quantile ϕ_c does not depend on θ_p and θ_o when the third and higher moments of distribution function D are the same for peak and off-peak hours. In reality, this is not the case as price spikes in day-ahead markets mostly occur in peak hours. This implies different third and fourth moments. Then, equation (7) still holds, but ϕ_c is then a function of θ_p and θ_o, $\phi_c(\theta_p, \theta_o)$. In the following, it is assumed that the third and higher moments of D are the same for peak and off-peak and, therefore, that ϕ_c is a constant.

In order to formulate the expressions for the mean and variance of D, let $\mu(h)$ be the expected day-ahead price for delivery in hour h of day T conditional on the information available at time t: $\mu(h) = E_t\{s(h)\}$. In addition, define the (24 x 1) vector **V** that contains in row h the value $(v(h) - \theta_p)$ if h is a peak hour or $(v(h) - \theta_o)$ if h is an off-peak hour and the (24x1) vector **M** that contains $\mu(h)$ in row h. Furthermore, let **Ω** be the (24 x 24) covariance matrix that contains in cell (i,j) the covariance between the day-ahead price in hour i and j for delivery on day T.

(8) $\mu(\theta_p, \theta_o) = \mathbf{V}^T \mathbf{M}$

(9) $\sigma^2(\theta_p, \theta_o) = \mathbf{V}^T \boldsymbol{\Omega}\, \mathbf{V}$

The optimization problem then becomes to minimize total expected costs given in (1) subject to the VaR constraint given in (7) with respect to the volumes in the peak hour θ_p and in the off-peak hour θ_o. The optimal hedge numbers can be derived using the Lagrange multiplier method. The Lagrangian *L* is:

(10) $L = n_p\theta_p f_p + n_o\theta_o f_o + \mathbf{V}^T \mathbf{M} + \lambda[\mathbf{V}^T \mathbf{M} + \phi_c (\mathbf{V}^T \boldsymbol{\Omega}\, \mathbf{V})^{½} - TC^* + n_p\theta_p f_p + n_o\theta_o f_o]$.

The optimal hedge numbers can be found by minimizing the Lagrangian subject to the parameters θ_p, θ_o, and the Lagrange multiplier λ.

In order to provide some intuition, the following assumptions apply below in order to provide more insight in the solutions. Assume that the delivery day consist of only one peak and one off-peak hour. Although this assumption takes away 22 hours from the delivery day, the simplification deviates not too far from reality. Huisman, Huurman and Mahieu (2006) showed in their panel model for hourly day-ahead prices that the hourly covariance matrix shows a clear block structure of high correlations among the peak hours and among the off-peak hours and near zero correlations between peak and off-peak hours. In essence, hourly day-ahead prices can be seen to behave in separate peak and off-peak blocks. The simplification into one peak and one off-peak hour therefore reflects the two blocks structure.

The expected total costs function (1) in the two hours framework becomes:

(11) $E_t\{TC\} = \theta_p f_p + (v_p - \theta_p)\, E_t\{s_p\} + \theta_o f_o + (v_o - \theta_o)\, E_t\{s_o\}$

The VaR constraint becomes:

(12) $TC^* - \theta_p f_p - \theta_o f_o = \mu(\theta_p, \theta_o) + \phi_c\, \sigma(\theta_p, \theta_o)$

The expressions for the mean and variance of the distribution function D are:

(13) $\mu(\theta_p, \theta_o) = (v_p - \theta_p)\, \mu_p + (v_o - \theta_o)\, \mu_o$

(14) $\sigma^2(\theta_p, \theta_o) = (v_p - \theta_p)^2\, \sigma^2_p + (v_o - \theta_o)^2\, \sigma^2_o + 2\, \rho\, (v_p - \theta_p)\, (v_o - \theta_o)\, \sigma_p\, \sigma_{o,}$

where μ_p is the expected day-ahead price in the peak, μ_o is the expected day-ahead price in the off-peak, and ρ is the correlation between the prices in the peak and off-peak hour. The Lagrangian *L* then equals:

(15) $L=\theta_p f_p + (v_p - \theta_p)\, \mu_p + \theta_o f_o + (v_o - \theta_o)\, \mu_o + \lambda\, [\mu(\theta_p,\theta_o) + \phi_c \sigma(\theta_p,\theta_o) - TC^* + \theta_p f_p + \theta_o f_o]$

Minimizing L with respect to θ_p, θ_o, and λ yields the following first order conditions:

(16) $\delta L/\delta\theta_p = 0 \Leftrightarrow f_p - \mu_p + \lambda f_p - \lambda\mu_p + \lambda\, \phi_c\, \delta\sigma/\delta\theta_p = 0$

(17) $\delta L/\delta\theta_o = 0 \Leftrightarrow f_o - \mu_o + \lambda f_o - \lambda\mu_o + \lambda\, \phi_c\, \delta\sigma/\delta\theta_o = 0$

(18) $\delta L/\delta\lambda = 0 \Leftrightarrow \mu(\theta_p, \theta_o) + \phi_c\, \sigma(\theta_p, \theta_o) - TC^* + \theta_p f_p + \theta_o f_o = 0$

Rearranging equations (17) and (18) and using the expression for the mean (13) and variance (14) yields the following expression:

$$(19)\qquad \frac{(v_p - \theta_p)}{(v_o - \theta_o)} = \frac{\{(f_p - \mu_p) + \rho \frac{\sigma_p}{\sigma_o}(f_o - \mu_o)\}/\sigma_p^2}{\{(f_o - \mu_o) + \rho \frac{\sigma_o}{\sigma_p}(f_p - \mu_p)\}/\sigma_o^2}$$

In order to interpret this solution, define s_p and s_o

$$(20)\qquad s_p = \frac{\{(f_p - \mu_p) + \rho \frac{\sigma_p}{\sigma_o}(f_o - \mu_o)\}}{\sigma_p^2}, s_o = \frac{\{(f_o - \mu_o) + \rho \frac{\sigma_o}{\sigma_p}(f_p - \mu_p)\}}{\sigma_o^2}$$

such that expression (19) becomes:

$$(21)\qquad \frac{(v_p - \theta_p)}{(v_o - \theta_o)} = \frac{s_p}{s_o}.$$

Equation (21) relates the ratio of the open position in the peak hour over the open position in the off-peak hour is equal to the ratio of s_p over s_o. Thus, the amount of risk the end-consumer wants to take in each hour depends solely on s_p over s_o. To interpret this result, think of s_p and s_o as being efficiency measures. When the correlation between the peak and off-peak prices is zero, s_P in expression (20) equals the ratio of the risk premium in the peak hour over the variance of the peak prices. Otherwise stated, s_P is the risk premium paid for giving up one unit or risk (in terms of variance) in the peak hour. The lower the efficiency level, the least costly it is to hedge in that hour and the lower the open position in that hour will be. Both s_p over s_o thus reflect the costs of hedging per unit of risk in their specific hours.

It is interesting to notice the resemblance with the Markowitz mean-variance framework for constructing efficient investment portfolio's. Within this framework, Sharpe (1966) defined the Sharpe ratio as return premium that an investor receives for taking one unit of risk (measures in terms of volatility). Let $E(r_p)$ be the expected return on the portfolio, let r_f be risk free interest rate and let σ_p be the volatility of the portfolio return. Then, the Sharpe ratio is:

$$(22) \qquad \text{Sharpe ratio} = \frac{E(r_p) - r_f}{\sigma_p}.$$

From a managerial perspective, the Sharpe ratio assesses the efficiency of an investment or a portfolio.

The efficiency measures are crucial for determining optimal hedging strategies. Suppose that s_p and s_o are equal. This implies that hedging in both hours cost the same per unit of risk. Then, equation (21) shows that the optimal open positions in both the peak (v_p - θ_p) and off-peak (v_o - θ_o) are equal. The open positions in peak and off-peak hours are equal when the cost of hedging per unit of risk in the peak hour is equal to the cost of hedging per unit of risk in the off-peak hour.

Suppose that s_p is higher than s_o, the cost of hedging per unit of risk is higher in the peak hour than in the off-peak hour. Equation (21) shows that, in this case, the open position in the off-peak hour is smaller than in the peak hour. This makes sense. As hedging is relatively more expensive in the peak hour, one prefers to hedge more in the off-peak and less in the peak hour. The hedge in the off-peak forms a substitute for a hedge in the peak hour. The open position is smallest in the hour with the lowest cost of hedging per unit of risk and is substituted for a bigger open position in the hour, more expensive, hour.

Hedge policies therefore depend completely on the efficiency measures s_p and s_o. Monitoring these numbers provides clear information on optimal hedging positions. Expression (20) yields that the amount to hedge in the peak is a function of the amount hedged in the off-peak and vice versa. Observe that in this expression, the quantile ϕ_c plays no role. That implies that, if someone decides on the amount to hedge in the off-peak hour, it only depends on the market risk premiums and the day-ahead price volatilities and correlation what the amount in the peak should be. The risk attitude of the end-consumer, expressed in terms of her Value at Risk and confidence level she desires, does not influence the relation between the peak and off-peak hedge. The ratio of open the position in the peak over the open position in the off-peak does not depend on risk aversion and is therefore the same for each consumer.

The optimal hedge amount for the off-peak hour is obtained from substituting equation (21) into (12).

$$\text{(23)} \qquad \theta_o = v_o + \frac{TC^* - v_o f_o - v_p f_p}{(f_o - \mu_o) + (f_p - \mu_p)\dfrac{s_p}{s_o} - \varphi\tilde{\sigma}},$$

where

$$\text{(24)} \qquad \tilde{\sigma}^2 = \sigma_o^2 + \sigma_p^2 \frac{s_p^2}{s_o^2} + 2\rho\rho_p\sigma_o \frac{s_p}{s_o}.$$

The optimal hedge for the peak can be obtained from substituting (23) into equation (21).

3 Managerial Implications

This section focuses on some managerial issues that can be addressed using the outcomes of the above model.

3.1 How to hedge a baseload demand profile?

Some end-consumers have a baseload demand profile. They consume the same amount of electricity in each hour of the day and therefore v(h) = v. Examples of these companies are chemical plants and industrial companies that work 24 hours per day or supermarkets that are open whole day. As baseload forwards, involving delivery in each hour of the delivery period, can be traded directly in worldwide energy markets, it seems at first sight straightforward to hedge the baseload profile with a baseload forward contract. However, from equation (21) it can be seen that this is not, per definition, an optimal strategy. When the volumes of the peak and the off-peak hours are equal, equation (21) becomes:

$$\text{(25)} \qquad \theta_p = v(1 - \frac{s_p}{s_o}) - \theta_o \frac{s_p}{s_o}.$$

The optimal hedges in the peak and off-peak are only equal, such that a baseload contract can be used, when their efficiency levels are the same, $s_p = s_o$. In all other cases, the hedge amounts in the peak and off-peak differ and the end-consumer should hedge with a combination of a peak and off-peak contract. An end-consumer who hedges using a baseload contract in a world where the efficiencies are not equal, could obtain lower expected costs level by applying a different hedging strategy.

3.2 Can a risk loving end-consumer obtain lower expected costs than a risk averse end-consumer?

The model presented above defines risk in terms of Value at Risk. As this risk measure reflects the maximum costs an end-consumer is willing to pay at a certain confidence level, a risk loving end-consumer is someone who applies, *ceteris paribus*, a bigger VaR or a lower confidence level. Therefore, the question whether a risk loving end-consumer obtains lower expected costs than a risk averse end-consumer can be answered by examining the optimal expected cost levels for different VaRs and confidence levels. To do so, the optimal hedge amounts (24) and the expected costs (11) are calculated for the one peak and off-peak hour case based on empirically obtained parameter values.

For each day in December 2006, daily peak and off-peak prices were calculated as the average price in peak hours and off-peak hours in the German EEX market. The average over the daily peak prices equals €50.59 per MWh and the average over the daily off-peak prices was €36.14 per MWh. The standard deviation over the daily peak prices was €16.87 per MWh and over the daily off-peak prices was €10.05 per MWh. The correlation between the daily average prices was 0.23. On the 8th of January 2007, the EEX closing prices for the CAL08 baseload contract was 52.79 and for the peak-load contract was 80.08. The implied off-peak price for CAL08 was 37.58. The average day-ahead prices closed at 37.88 for baseload hours, 52.23 for peak load hours and 23.53 for off-peak hours. Under the assumption that the day-ahead prices observed at January 8 are a proxy of the average price on the day-ahead market and using the standard deviations and correlation from the observed prices in December 2006, the optimal hedge ratios can be calculated.

The following example uses the observed values from the EEX market, as presented above, in the one period framework. Consider an end-user who expects to consume 1MW in the off-peak hour and 2MW in the peak hour. If she would purchase her volume using one off-peak forward contract, priced at 37.58, and two peak forward contracts, priced at 80.08, her total costs would equal €197.74. Suppose that her risk is measured as a 99% VaR and that she expressed a risk appetite equal to €200. That is, she is willing to pay more than €200 in only one out of a hundred days. It is assumed that the distribution function D() is normal.

The efficiency ratios s_p equals 0.0975 and s_o equals 0.0348. As the efficiency measure in the off-peak hour is lower than in the peak, it is more efficient to hedge in the off-peak hour and she will keep a smaller open position in the off-peak than in the peak. The optimal hedge amounts are 0.943 in the off-peak hour and 1.842 in the peak hour. As she is willing to take some risk, she under-hedges in both hours and speculates on the day-ahead market. The open position in the peak hour (equal to 0.158) is bigger than the open position in the off-peak hour (0.057). So, she speculates more in the peak hour as the peak forward contract is relatively expenses per unit of risk she faces in the peak day-ahead market. Given these outcomes, her expected costs are €193.35. If she would take no risk and fully hedge her position with one off-peak forward contract and two peak forward contracts, her costs would be €197.74. By taking risk (recall her risk appetite is €200) she is able to reduce her expected costs. She does so by speculating in the day-ahead market and more aggressively in the peak hour as the forward price for that hour is relatively more expensive per unit of risk than in the off-peak hour. Taking risk is rewarded by lower expected costs.

The following figure sheds more light on the risk – expected costs relation. The x-axis shows different risk appetites (starting from 200) and the y-axis shows the expected costs from the optimal forward positions.

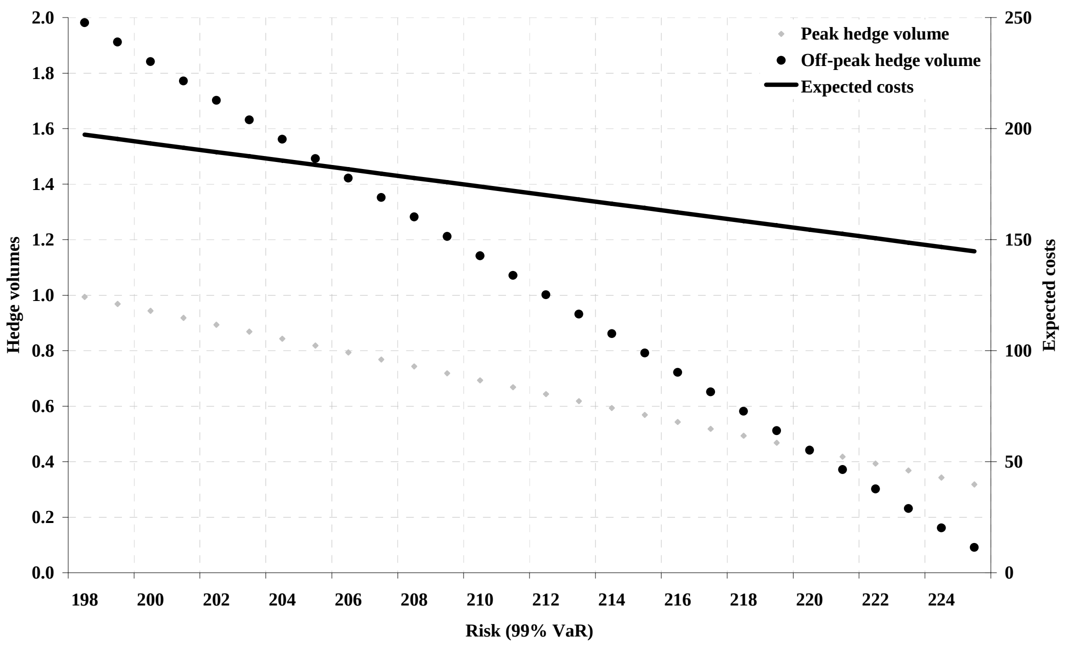

Figure 1. The risk – expected costs relationship for different VaR levels (from 198 through 225) while the confidence level is constant at 99%.

Figure 1 shows that when the end-consumer would take more risk, by increasing the VaR she accepts, she would pay lower expected costs for her electricity consumption. The black expected costs line is the efficient frontier reflecting the lowest costs one can obtain for each level of risk. A risk-loving consumer or distribution company has lower expected costs than a risk-averse consumer or distribution company.

In figure 1, the level of VaR varied to show the relation between risk and expected costs. In a Value at Risk environment, one can express the risk appetite also in terms of confidence level. A risk-loving agent is willing to deal with less confidence than a risk-averse agent. Figure 2 shows the efficient frontier when the risk appetite is varied in terms of confidence level, starting at 99% (risk-averse) down to 95% (risk-loving).

In figure 2 shows a non-linear relation between risk and return, but the conclusions are in line with figure 1. The risk-loving consumer (one who accepts a low level of confidence) obtains lower expected costs than a risk-averse consumer (high level of confidence).

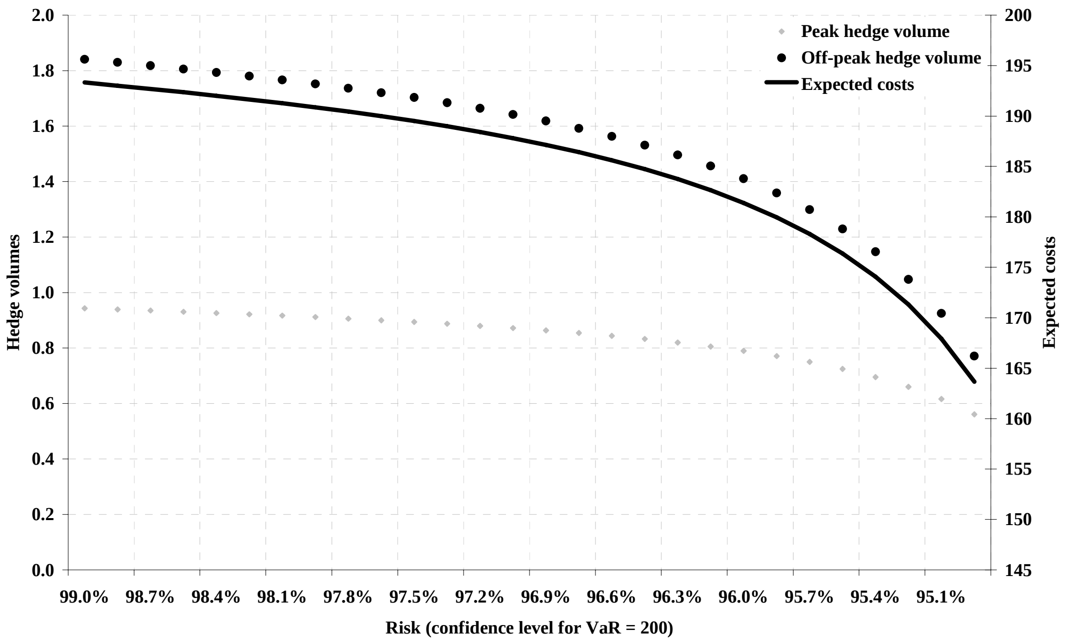

Figure 2. The risk – expected costs relationship for different confidence levels (from 99% through 95%) while the VaR level is constant at 200.

Both figures also reveal the difference between the hedge volumes for different risk levels. In both cases, the consumer reduces the hedge volume more aggressively in the off-peak hour than in the peak hour as she takes more risk. This can be seen that the line corresponding with the off-peak hedge volumes is steeper than the line for the peak hedge volume in both figures. This can be explained that the variance risk in the peak hour is bigger than in the off-peak.

The conclusion from this exercise is that, for parameter values obtained from the EEX market, that willingness to take risk results in lower expected costs.

4 Conclusions

This paper deals with the construction of an electricity portfolio for an electricity consumer or distribution company who needs to purchase electricity for consumption at a future date. A one-period model is presented and optimal hedge amounts are derived in the case of one peak and one off-peak hour. It is shown that a risk-loving consumer can obtain lower expected-costs than a risk-averse consumer. Risk taking behaviour is rewarded by lower expected costs by profiting from the differences in efficiency of hedging using baseload and peakload contracts.

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