Electricity Portfolio Management: Optimal Peak / Off-Peak Allocations

Ronald Huisman, Ronald J. Mahieu and Felix Schlichter

ERIM REPORT SERIES *RESEARCH IN MANAGEMENT*	
ERIM Report Series reference number	ERS-2007-089-F&A
Publication	December 2007
Number of pages	21
Persistent paper URL	http://hdl.handle.net/1765/10775
Email address corresponding author	rhuisman@rsm.nl
Address	Erasmus Research Institute of Management (ERIM) RSM Erasmus University / Erasmus School of Economics Erasmus Universiteit Rotterdam P.O.Box 1738 3000 DR Rotterdam, The Netherlands Phone: + 31 10 408 1182 Fax: + 31 10 408 9640 Email: info@erim.eur.nl Internet: www.erim.eur.nl

Bibliographic data and classifications of all the ERIM reports are also available on the ERIM website:
www.erim.eur.nl

ERASMUS RESEARCH INSTITUTE OF MANAGEMENT

REPORT SERIES
RESEARCH IN MANAGEMENT

ABSTRACT AND KEYWORDS	
Abstract	Electricity purchasers manage a portfolio of contracts in order to purchase the expected future electricity consumption profile of a company or a pool of clients. This paper proposes a mean-variance framework to address the concept of structuring the portfolio and focuses on how to allocate optimal positions in peak and off-peak forward contracts. It is shown that the optimal allocations are based on the difference in risk premiums per unit of day-ahead risk as a measure of relative costs of hedging risk in the day-ahead markets. The outcomes of the model are then applied to show 1) whether it is optimal to purchase a baseload consumption profile with a baseload forward contract and 2) that, under reasonable assumptions, risk taking by the purchaser is rewarded by lower expected costs.
Free Keywords	optimal electricity sourcing, hedge ratio, forward risk premiums, electricity portfolio management
Availability	
Classifications	

Electricity Portfolio Management: Optimal Peak / Off-Peak Allocations

Ronald Huisman [a,c,*], Ronald Mahieu [b,c,1] Felix Schlichter [d]

[a] *Erasmus University Rotterdam, Erasmus School of Economics.*

[b] *RSM Erasmus University*

[c] *FinEdge International Group*

[d] *Delta Energy*

Abstract

Electricity purchasers manage a portfolio of contracts in order to purchase the expected future electricity consumption profile of a company or a pool of clients. This paper proposes a mean-variance framework to address the concept of structuting the portfolio and focuses on how to allocate optimal positions in peak and off-peak forward contracts. It is shown that the optimal allocations are based on the difference in risk premiums per unit of day-ahead risk as a measure of relative costs of hedging risk in the day-ahead markets. The outcomes of the model are then applied to show 1) whether it is optimal to purchase a baseload consumption profile with a baseload forward contract and 2) that, under reasonable assumptions, risk taking by the purchaser is rewarded by lower expected costs.

Key words: Optimal electricity sourcing, hedge ratio, forward risk premiums, Electricity portfolio management

---

Preprint submitted to Elsevier *7 December 2007*

1 Introduction

The task of electricity purchasers (end-consumers and distribution companies) is to contract the future expected electricity consumption for the company they work for or for a pool of clients. In liberalized electricity markets, they can do so by managing a portfolio of contracts that involve delivery of electricity in future time periods and/or financially settle the difference between a fixated and variable price. Examples of such contracts are day-ahead contracts, derivatives such as forwards, futures, swaps, variable volume or swing options and direct or indirect investments in energy production facilities (the latter being a power purchasing agreement in which the owner purchases electricity from a power plant and pays according to a formula that relates the price to fuel prices). Proper management of the portfolio involves a continuous assessment of which contracts to buy or sell (instrument selection) and at what moment (timing). The objective of the purchaser is then to obtain the least expected costs for electricity consumption while facing not too much risk from fluctuations in the prices of these contracts over time.

Since the beginning of the liberalization of energy markets, researchers have primarily focused on the price characteristics of different energy commodities and the valuation of derivative contracts. The issue how to structure efficient energy portfolios has received much less attention and is undervalued as poorly constructed portfolios exhibit either too high expected costs at a given risk

* Corresponding author. Address: Erasmus University Rotterdam, Erasmus School of Economics, Applied Economics, P.O. Box 1738, 3000 DR, Rotterdam. Tel: +31 10 4081450.
Email addresses: rhuisman@rsm.nl (Ronald Huisman), rmahieu@rsm.nl (Ronald Mahieu), felixschlichter@gmail.com (Felix Schlichter).
[1] This author acknowledges financial support from NETSPAR (www.netspar.nl).

level or exhibit too much risk for the current level of expected costs. This paper focuses on optimal instrument selection and specifically deals with the question how electricity purchasers should choose between peak and off-peak forward contracts in order to structure their portfolios optimally. To do so, we construct a simple one-period framework and cast the allocation problem in a portfolio framework to find the optimal allocations to the forward contracts and the day-ahead market. Obviously, taking positions in forward contract is basically determine hedge ratios. In this paper however, we prefer to refer these hedge ratio 'allocation' as we assume that the purchasers task is to manage the overall - inter temporal - risk of her portfolio and not to focuses on hedging the risk of a specific delivery period.

The paper is organized as follows. Section 2 discusses literature on energy portfolio management. Section 3 presents the conceptual model. Section 4 highlights some managerial implication of the model and provides answers to the questions how one should purchase a baseload consumption profile and whether taking risk is rewarded by lower expected costs. Section 5 concludes.

2 Portfolio structuring in electricity markets

To facilitate trading of power contracts many countries have established over-the-counter (OTC) and centralized markets. The two most commonly used markets are the day-ahead and forward / futures markets. On the day-ahead market, traders can submit bids and offers for amounts of electricity to be delivered in separate hours in the next day. This market is the closest equivalent to a spot market.[2] Electricity purchasers use day-ahead markets for buying (a part of) their electricity consumption, but the amount of price variation in these markets is substantial. As electricity is not efficiently directly storable, prices are very volatile, and seasonal and price spikes frequently occur.[3]

Instead of taking the risk of price variations in the day-ahead market, purchasers seek protection,depending on their risk appetite, and manage a portfolio of derivative contracts that involve delivery at future dates against fixed prices. Popular contracts are the so-called baseload and peakload forward and futures contracts that can be traded on all OTC markets (forwards) and exchanges (futures).[4] Baseload contracts involve the delivery of 1MW in all hours of the delivery period against a price fixated at the moment at which the transaction occurs. Peakload contracts are defined similarly, but involve delivery only in the peak hours of the delivery period. Delivery periods range from weeks, months, quarters to calendar year sometimes up to six years ahead. By holding a portfolio of these contracts, the purchaser can already lock in the acquisition of (a part of) the expected future consumption long before the actual consumption period against fixed prices and can thereby manage the risk faced from price variations in the day-ahead market.

The prices of baseload and peakload forwards exhibit different characteristics than day-ahead prices. According to the expectation theory[5], forward prices

[2] Many countries also run imbalance markets in which power can be traded that will be delivered on the same day. However, the liquidity of these markets is rather limited and these markets are seen as ultimate balancing markets in order to trade away forecast errors. These markets are left out of the analysis.
[3] We refer to Bunn and Karakatsani (2003) and Huisman et al. (2007), among others, for an overview on (hourly specific) day-ahead price characteristics.
[4] In this paper, we do not differentiate between forwards and futures and only mention forward contracts, although in reality small price differences might occur due to differences in settlement procedures and margining schemes.
[5] We refer to Fama and French (1987) for an overview of forward premiums in

for non-storable commodities reflect the expectation of market participants on the (average) spot price in the delivery period and a risk premium that compensates producers for bearing the uncertainty of committing to sell against fixed prices. In electricity markets, risk premiums can be positive and negative. For instance, Bessembinder and Lemmon (2002) and Bunn and Karakatsani (2005) find negative risk premiums in low-demand off-peak hours due to power producers who are willing to pay a premium for not to cut down production from plants with long ramp-up and ramp-down times in order to be able to produce more in the high-demand and expensive peak hours.

For the portfolio manager, forward contracts make it possible to fixate delivery prices, thereby reducing the exposure to the price fluctuations in the day-ahead market. In effect, purchasing power with forward contracts is hedging the risk faced from day-ahead market, the expected hedging costs being equal to the risk premium embedded in the forward price.

Given a set of forward and futures contracts that can be traded every day, the task of the portfolio manager is to determine the optimal selection of forward contracts to hold for various delivery periods. The optimal selection depends on a risk assessment of the day-ahead market, an expectation regarding the expected price in the day-ahead market in the delivery period, the amount of risk premium she needs to pay and her personal (or the company's) appetite for taking risk. The goal of the portfolio manager is to maintain such a portfolio that yields lowest expected costs for electricity consumption while respecting her risk appetite.

This paper follows the ideas from the Markowitz (1952), who proposes a

commodity markets.

methodology to construct efficient investment portfolios based on the investors goal to maximize expected future returns on their investments given a certain level of risk. The idea to use portfolio theory to construct energy portfolio is not new. Several researchers have followed the Markowitz methodology to address the hedging decision process, particularly Näsäkkälä and Keppo (2005) and Woo et al. (2004). They study the interaction between stochastic consumption volumes and electricity prices (day-ahead and forward contracts) and propose a Markowitz-style mean-variance framework to determine optimal hedging strategies. Vehvilainen and Keppo (2004) take the viewpoint of a generation company and optimize hedging strategies using VaR as a risk measure instead of standard deviation. To analyze both stochastic consumption and prices is extremely meaningful, yet complex. The above mentioned papers need Monte Carlo simulations to provide results. This paper takes a step back and aims to provide analytical insight in the optimal hedging amounts, in order to gain intuition on the interaction between variables such as day-ahead price and risk expectation and the forward risk premium.

3 The purchasing decision in a one period framework

At time t, consider an electricity purchaser who has to decide on how to purchase the consumption for future delivery of electricity at day T ($t < T-1$). Delivery takes place in all hours of day T. The actual consumption volume during hour h on day T equals $v(h,T)$ MWh ($h = 1, \ldots, 24$).

At time t, the purchaser has the opportunity to enter in forward contracts [6] that facilitate delivery on day T at prices fixated at time t. We assume that two

[6] Or futures contracts. We assume that forwards and futures are equivalent.

forwards contracts exist that deliver in day T: a peak contract that delivers 1MW of electricity in all the peak hours of day T and an off-peak contract that delivers 1MW in all the off-peak hours of day T.[7] Let H_p be the set of peak hours in day T and let H_o be the set of off-peak hours. The number of peak and off-peak hours in day T equal N_p and N_o, respectively. The purchaser has to decide the number of peak contracts, θ_p, and off-peak contracts, θ_o, to acquire. We assume that the forward contracts can be traded in any size with perfect liquidity, no short-sale constraints and zero transaction costs. At time t, the market prices for the forward contracts equal $f_o(t, T)$ for the off-peak contract and $f_p(t, T)$ for the peak contract.

In addition to entering in forward contracts, the purchaser can purchase electricity for delivery in day T in the day-ahead market at time $T-1$. We assume that the day-ahead market offers the last opportunity before delivery to purchase electricity. Therefore, the purchaser will use this market to settle the difference between consumption volume and the volume acquired with forward contracts[8]. Let $s(h, T-1)$ be the price in the day-ahead market at $T-1$ for delivery of 1MW in hour h of day T. Given the above assumptions,

[7] In most electricity markets, off-peak contracts cannot be traded directly but can be constructed synthetically by simultaneously buying one baseload contract (that delivers a fixed volume in each hour of the delivery period) and selling one peak contract. The assumption that an off-peak contract can be traded in the market is made for simplicity and does not lead to a loss in generalization.

[8] In real life, imbalance and other intraday markets exist where electricity can be purchased that will be delivered in the same day. However, the liquidity of these markets is thin relative to the day-ahead markets and practitioners use these intraday markets to settle slight changes in volume that occur in the delivery day. Therefore, our assumption that the purchaser uses the day-ahead to settle the differences between consumption volume and previously contracted volume does not deviate too far from reality.

the total costs for electricity consumption at day T, $C(T)$, equals:

$$C(T) = N_o\theta_o f_o(t,T) + N_p\theta_p f_p(t,T) + \sum_{h\in H_o}(v(h,T)-\theta_o)s(h,T-1) + \sum_{h\in H_p}(v(h,T)-\theta_p)s(h,T-1). \tag{1}$$

The total costs function equals the sum of costs from the volumes purchased with peak and off-peak forward contracts at time t and the costs of the remaining purchases in the day-ahead market. At time t, the day-ahead prices and the actual consumption volumes are uncertain. Therefore, the total costs $C(T)$ are uncertain too. The objective of the purchaser is then to construct an optimal allocation over the peak and off-peak forward contracts that yields the lowest expected costs. However in dealing with the uncertainty, we assume that the purchaser is not willing to take more risk than his risk appetite specifies. This objective is in line with the framework initially proposed by Markowitz (1952) for investment portfolios. In this world an investor first decides on the amount of risk he is willing to take and then to find the portfolio that yields the highest expected return at that risk level. In this paper, we assume that the goal of the purchaser is to achieve lowest expected costs at a risk level that meets her risk appetite. The risk appetite is defined in terms of a maximum variance level (or volatility by taking the square root) that the purchaser allows, $\sigma^2_{\max}$. Then, the optimization problem of the purchaser becomes:

$$\begin{aligned} \min_{\theta_o,\theta_p} \quad & E_t\{C(T)\} \\ s.t. \quad & \mathrm{var}_t\{C(T)\} \leq \sigma^2_{\max}, \end{aligned} \tag{2}$$

where $\mathrm{var}_t\{C(T)\}$ is the variance of the total costs anticipated at time t. To further specify the optimization problem and the uncertainty from variations in prices and volumes, we assume that the statistical properties of day-ahead prices and volumes can be described by the first two moments [9]. For convenience, we switch to matrix notation.

Let $\boldsymbol{s}(T-1)$ be a 24x1 vector with the stacked day-ahead prices and is distributed with the 24x1 mean vector $\boldsymbol{\mu}_s$ and the 24x24 hourly covariance matrix $\boldsymbol{\Omega}_s$. Let $\boldsymbol{f}_t$ be a 2x1 vector with the stacked forward prices $(f_o(t,T) \;\; f_p(t,T))'$. Likewise, let $\boldsymbol{\theta}_t$ be the 2x1 vector $(\theta_o \;\; \theta_p)'$ and $\boldsymbol{N}$ the 2x1 vector with $(N_o \;\; N_p)$.

The hourly volumes $v(h)$ are stacked in the 24x1 vector $\boldsymbol{v}(T)$. The stochastic properties of the hourly volumes are represented by the 24x1 vector with hourly means $\boldsymbol{\mu}_v$ and the hourly 24x24 covariance matrix $\boldsymbol{\Omega}_v$. Lastly, we introduce a 24x2 selection matrix $\boldsymbol{B}$ with row h equalling $(1 \;\; 0)$ when h is an off-peak hour and $(0 \;\; 1)$ when h is a peak hour $(h = 1 \ldots 24)$.

In matrix notation, the cost function $C(T)$ from (1) can then be written as

$$C(T) = \boldsymbol{\theta}_t'(\boldsymbol{N} \cdot \boldsymbol{f}_t - \boldsymbol{B}'\boldsymbol{s}(T-1)) + \boldsymbol{v}(T)'\boldsymbol{s}(T-1), \tag{3}$$

with '$\cdot$' the element-wise matrix multiplication operator. In order to proceed to an analytical solution, we assume that the day-ahead prices and the actual consumption volumes are expectation-independent and variance-independent. This implies among others that all covariances between day-ahead prices and volumes are zero, i.e. a deviation in the actual consump-

[9] Electricity prices exhibit strong higher moments characteristics, most noteworthy skewness and strong leptokurtosis. See for example, Huisman and Mahieu (2003) and Bunn and Karakatsani (2003). However, in this paper we concern with finding an optimal allocation in a mean-variance (two moments) framework. As a result the higher moment characteristics of electricity prices do not play a role in our analysis.

tion volume from its expected value does not necessary lead to a change in day-ahead prices. This assumption does not hold in real life when a large increase in volumes would lead to a different marginal fuel in the merit order. However, for relatively small changes and under normal market conditions, the assumption applies. Using the results from Bohrnstedt and Goldberger (1969), we have that $E_t\{\boldsymbol{v}(T)'\boldsymbol{s}(T-1)\} = \boldsymbol{\mu}_v'\boldsymbol{\mu}_s$ and $\mathrm{var}_t\{\boldsymbol{v}(T)'\boldsymbol{s}(T-1)\} = \mathrm{tr}\,(\boldsymbol{\Omega}_s\boldsymbol{\Omega}_v) + \boldsymbol{\mu}_s'\boldsymbol{\Omega}_v\boldsymbol{\mu}_s + \boldsymbol{\mu}_v'\boldsymbol{\Omega}_s\boldsymbol{\mu}_v$, with $\mathrm{tr}(\cdot)$ the matrix trace operator. From (3) and the above statistical properties, we can write the expectation and variance of the total cost, condition on information available at t, as:

$$\mathrm{E}_t\{C(T)\} = \boldsymbol{\theta}_t'(\boldsymbol{N}\cdot\boldsymbol{f}_t - \boldsymbol{B}'\boldsymbol{\mu}) + \boldsymbol{\mu}_v'\boldsymbol{\mu}_s, \tag{4}$$

and

$$\begin{aligned}\mathrm{var}_t\{C(T)\} = {} & \boldsymbol{\theta}_t'\boldsymbol{B}'\boldsymbol{\Omega}_s\boldsymbol{B}\boldsymbol{\theta}_t + \mathrm{tr}\,(\boldsymbol{\Omega}_s\boldsymbol{\Omega}_v) + \boldsymbol{\mu}_s'\boldsymbol{\Omega}_v\boldsymbol{\mu}_s + \boldsymbol{\mu}_v'\boldsymbol{\Omega}_s\boldsymbol{\mu}_v + \\ & 2\boldsymbol{\theta}_t'\left(\boldsymbol{B}'\boldsymbol{\mu}_s\right)\left(\boldsymbol{\mu}_v'\boldsymbol{\mu}_s\right).\end{aligned} \tag{5}$$

Solving the optimization problem (2) for the optimal allocations $\boldsymbol{\theta}_t$, we specify the Lagrangian:

$$\begin{aligned}\mathcal{L} = {} & \boldsymbol{\theta}_t'(\boldsymbol{N}\cdot\boldsymbol{f}_t - \boldsymbol{B}'\boldsymbol{\mu}_s) + \boldsymbol{\mu}_v'\boldsymbol{\mu}_s + \\ & \lambda\Big\{\boldsymbol{\theta}_t'\boldsymbol{B}'\boldsymbol{\Omega}_s\boldsymbol{B}\boldsymbol{\theta}_t + \mathrm{tr}\,(\boldsymbol{\Omega}_s\boldsymbol{\Omega}_v) + \boldsymbol{\mu}_s'\boldsymbol{\Omega}_v\boldsymbol{\mu}_s + \boldsymbol{\mu}_v'\boldsymbol{\Omega}_s\boldsymbol{\mu}_v + \\ & 2\boldsymbol{\theta}_t'\left(\boldsymbol{B}'\boldsymbol{\mu}_s\right)\left(\boldsymbol{\mu}_v'\boldsymbol{\mu}_s\right) - \sigma^2_{\max}\Big\},\end{aligned} \tag{6}$$

with λ a Lagrange multiplier. The first-order Karush-Kuhn-Tucker conditions are

$$\left.\frac{\partial \mathcal{L}}{\partial \boldsymbol{\theta}_t}\right|_{\hat{\boldsymbol{\theta}}_t,\hat{\lambda}} = \boldsymbol{N} \cdot \boldsymbol{f}_t - \boldsymbol{B}'\boldsymbol{\mu} + 2\lambda\boldsymbol{\theta}_t'\boldsymbol{B}'\left(\boldsymbol{\Omega}\boldsymbol{B} + \boldsymbol{\mu}_s\left(\boldsymbol{\mu}_v'\boldsymbol{\mu}_s\right)\right) = 0 \tag{7a}$$

$$\begin{aligned}\left.\frac{\partial \mathcal{L}}{\partial \lambda}\right|_{\hat{\boldsymbol{\theta}}_t,\hat{\lambda}} &= \boldsymbol{\theta}_t'\boldsymbol{B}'\boldsymbol{\Omega}_s\boldsymbol{B}\boldsymbol{\theta}_t + \mathrm{tr}\left(\boldsymbol{\Omega}_s\boldsymbol{\Omega}_v\right) + \boldsymbol{\mu}_s'\boldsymbol{\Omega}_v\boldsymbol{\mu}_s + \boldsymbol{\mu}_v'\boldsymbol{\Omega}_s\boldsymbol{\mu}_v + \\ &\quad 2\boldsymbol{\theta}_t'\left(\boldsymbol{B}'\boldsymbol{\mu}_s\right)\left(\boldsymbol{\mu}_v'\boldsymbol{\mu}_s\right) - \sigma^2_{\max} \leq 0\end{aligned} \tag{7b}$$

$$\begin{aligned}&\Big\{\boldsymbol{\theta}_t'\boldsymbol{B}'\boldsymbol{\Omega}_s\boldsymbol{B}\boldsymbol{\theta}_t + \mathrm{tr}\left(\boldsymbol{\Omega}_s\boldsymbol{\Omega}_v\right) + \boldsymbol{\mu}_s'\boldsymbol{\Omega}_v\boldsymbol{\mu}_s + \boldsymbol{\mu}_v'\boldsymbol{\Omega}_s\boldsymbol{\mu}_v + \\ &2\boldsymbol{\theta}_t'\left(\boldsymbol{B}'\boldsymbol{\mu}_s\right)\left(\boldsymbol{\mu}_v'\boldsymbol{\mu}_s\right) - \sigma^2_{\max}\Big\}\hat{\lambda} = 0\end{aligned} \tag{7c}$$

$$\hat{\lambda} \geq 0 \tag{7d}$$

If the Lagrange multiplier $\hat{\lambda} = 0$ then the variance restriction is not binding. This implies that the risk limit $\sigma^2_{\max}$ cannot be met. In that case the optimal allocation to forward contracts should be found from (7a). Inspection of this condition shows that depending on the sign of the composite forward premia $N_o f_o(t,T) - \sum_{h=1}^{24} \boldsymbol{B}_{h1} s(h, T-1)$ for off-peak hours and $N_p f_p(t,T) - \sum_{h=1}^{24} \boldsymbol{B}_{h2} s(h, T-1)$ for peak hours the positions would become unlimited. Clearly this is not possible. As a result the variance restriction is binding: $\hat{\lambda} > 0$. In that case we can solve for the optimal allocation $\hat{\boldsymbol{\theta}}_t$ from equation (7a) and the equality

$$\boldsymbol{\theta}_t'\boldsymbol{B}'\boldsymbol{\Omega}_s\boldsymbol{B}\boldsymbol{\theta}_t + \mathrm{tr}\left(\boldsymbol{\Omega}_s\boldsymbol{\Omega}_v\right) + \boldsymbol{\mu}_s'\boldsymbol{\Omega}_v\boldsymbol{\mu}_s + \boldsymbol{\mu}_v'\boldsymbol{\Omega}_s\boldsymbol{\mu}_v + 2\boldsymbol{\theta}_t'\left(\boldsymbol{B}'\boldsymbol{\mu}_s\right)\left(\boldsymbol{\mu}_v'\boldsymbol{\mu}_s\right) = \sigma^2_{\max}.$$

An analytical solution could be found for $\boldsymbol{\theta}_t$, but these are not straightforward and do not provide any intuition. [10] In order to provide analytical results that are provide insight, we continue in a more stylized setting.

[10] Note, that one can easily find a numerical solution to the equality.

Huisman et al. (2007) show that the covariance matrix of hourly day-ahead prices exhibits a clear block structure of high correlations among either the peak hours and the off-peak hours and near-zero correlations between peak and off-peak hours. To some extent, hourly day-ahead prices can be seen to behave in independent peak and off-peak blocks. Motivated bu this result, we simplify the above model by bringing the number of hours back to just two: one off-peak and one peak hour. Each hour can then be seen as being a representative hour for all the hours in its block. Assuming two hours in the delivery day and zero correlation between the peak and off-peak hour, we rewrite the model as follows. The expected costs function (4) becomes:

$$\mathrm{E}_t\{C(T)\} = \theta_o f_o(t,T) + \theta_p f_p(t,T) + [\mu_{vo} - \theta_o]\mu_{so} + [\mu_{vp} - \theta_p]\mu_{sp}, \qquad (8)$$

with μ_{vo} (μ_{vp}) the mean of the volumes in the representative off-peak (peak) hour and μ_o (μ_p) the mean of the day-ahead prices that are representative for the off-peak (peak) hour. The variance equation (5) becomes:

$$\begin{aligned}\mathrm{var}_t\{C(T)\} = \mu_{so}^2\sigma_{vo}^2 + (\mu_{vo} - \theta_o)^2\,\sigma_{so}^2 + \sigma_{vo}^2\sigma_{so}^2 + \\ \mu_{sp}^2\sigma_{vp}^2 + (\mu_{vp} - \theta_p)^2\,\sigma_{sp}^2 + \sigma_{vp}^2\sigma_{sp}^2.\end{aligned} \qquad (9)$$

In the above equation, σ_{so} and σ_{sp} represent the standard deviation of the off-peak and peak prices and σ_{vo} and σ_{vp} reflect the standard deviation of the off-peak and peak volumes. The first-order conditions that result from the minimization of the expected cost with respect to the variance restriction equal:

$$\left.\frac{\partial \mathcal{L}}{\partial \theta_o}\right|_{\hat{\boldsymbol{\theta}},\hat{\lambda}} = f_o - \mu_{so} - 2\hat{\lambda}(\mu_{vo} - \hat{\theta}_o)\sigma_{so}^2 = 0, \tag{10a}$$

$$\left.\frac{\partial \mathcal{L}}{\partial \theta_p}\right|_{\hat{\boldsymbol{\theta}},\hat{\lambda}} = f_p - \mu_{sp} - 2\hat{\lambda}(\mu_{vp} - \hat{\theta}_p)\sigma_{sp}^2 = 0, \tag{10b}$$

$$\begin{aligned}\left.\frac{\partial \mathcal{L}}{\partial \lambda}\right|_{\hat{\boldsymbol{\theta}}} = {} & \mu_{so}^2\sigma_{vo}^2 + \left(\mu_{vo} - \hat{\theta}_o\right)^2 \sigma_{so}^2 + \sigma_{vo}^2\sigma_{so}^2 + \\ & \mu_{sp}^2\sigma_{vp}^2 + \left(\mu_{vp} - \hat{\theta}_p\right)^2 \sigma_{sp}^2 + \sigma_{vp}^2\sigma_{sp}^2 - \sigma_{\max}^2 = 0.\end{aligned} \tag{10c}$$

Rearranging (10a) and (10b) yields

$$\frac{(\mu_{vp} - \hat{\theta}_p)}{(\mu_{vo} - \hat{\theta}_o)} = \frac{(f_p - \mu_{sp})/\sigma_{sp}^2}{(f_o - \mu_{so})/\sigma_{so}^2}. \tag{11}$$

Equation (11) reveals that the optimal allocation are such that the ratio of the expected open positions equals the ratio of cost of hedging per unit of risk. The ratio of expected open positions, the left-hand-side of (11), shows how the purchaser will divide her risk appetite over the off-peak and peak hour in the day-ahead market. If this ratio equals one, the purchaser purchases the same volume in the off-peak and peak hour. If the ratio is higher (lower) than one, the purchaser buys more in the peak (off-peak) hour.

The right hand side of equation (11) is the ratio of the hedging costs per unit of (variance) risk in the peak hour over the off-peak hour. The hedging costs per unit of risk equals the forward premium $(f_{\cdot} - \mu_{s\cdot})$ divided by the amount of (variance) risk $\sigma_{s\cdot}^2$. The higher this number is, the purchaser pays for hedging away day-ahead price risk. Equation (11) yields the intuitive result that the ratio of open positions equals the ratio of relative hedging costs: the more expensive it is to hedge in one hour, the bigger the open position in that hour relative to the other hour. For convenience, we let η be the ratio of the hedging costs per unit of (variance) risk in the peak hour over the off-peak hour, that is:

$$\eta = \frac{(f_p - \mu_{sp})/\sigma_{sp}^2}{(f_o - \mu_{so})/\sigma_{so}^2}. \tag{12}$$

Equation (11) reveals another important result. The risk appetite of the purchaser, $\sigma^2_{\max}$ does not affect the ratio of the open positions. That is, every purchaser will structure her optimal portfolio such that equation (11) holds. Therefore, to determine the optimal portfolio, the purchaser follows a two-step approach: firstly, she determines the optimal relative allocations to the peak and off-peak hour and, secondly, she will set the exact allocation levels based on her risk appetite. This result is in line with the *separation pinciple* that results from modern portfolio theory. That is, the optimal investment portfolio is constructed in two succeeding steps. Firstly, the most efficient portfolio is determined (referred to as the market portfolio by Markowitz). Secondly, the investor chooses between investing in the market portfolio and the risk-free interest rate in such a way that her resulting portfolio reflects her risk appetite. Note that the market portfolio is the most efficient one and therefore the same for all investors; it's constituents depend on expected return / risk / correlation efficiency, not on risk appetite. To determine the optimal allocations $\hat{\theta}_o$ and $\hat{\theta}_p$, we first define the excess risk appetite σ^2_{e} being the difference between the risk appetite of the electricity purchaser σ^2_{max} and the risks associated with the variation in the consumption volume that cannot be hedged by taking positions in forward contracts (but could be managed by improving consumption volume forecasting methods). We assume that the purchaser is aware of the volume risk and that she accounts for that in her risk appetite. Therefore, we assume that $\sigma_{\mathrm{e}}^2 \geq 0$. Thus:

$$\sigma_{\mathrm{e}}^2 = \sigma_{max}^2 - \left(\mu_{so}^2 + \sigma_{so}^2\right)\sigma_{vo}^2 - \left(\mu_{sp}^2 + \sigma_{sp}^2\right)\sigma_{vp}^2.$$

Substituting (11) in (10c) and solving for θ_p, we find that the optimal allocation for the peak hour:

$$\hat{\theta}_p = \mu_{vp} - \sqrt{\frac{\sigma_{\mathrm{e}}^2}{\sigma_{so}^2/\eta^2 + \sigma_{sp}^2}}, \tag{13}$$

and the optimal solution for the off-peak hour $\hat{\theta}_o$:

$$\hat{\theta}_o = \mu_{vo} - \sqrt{\frac{\sigma_{\mathrm{e}}^2}{\sigma_{so}^2 + \sigma_{sp}^2\eta^2}}. \tag{14}$$

The optimal hedge ratios θ_o and θ_p depend on the hourly expected consumption volumes, the excess risk appetite, the variances of the hourly prices and the ratio of relative hedging costs η. Solving for the optimal total expected costs, we insert the optimal hedging positions $\hat{\theta}_o$ and $\hat{\theta}_p$ in the expected total cost function (8). We obtain:

$$\mathrm{E}_t\{C(T)\} = \mu_{vo}f_o + \mu_{vp}f_p - \frac{(f_o - \mu_{so})\sigma_{\mathrm{e}}}{\sqrt{\sigma_{so}^2 + \sigma_{sp}^2\eta^2}} - \frac{(f_p - \mu_{sp})\sigma_{\mathrm{e}}}{\sqrt{\sigma_{so}^2/\eta^2 + \sigma_{sp}^2}} \tag{15}$$

4 Managerial implications

In this section, we apply the outcomes of the model from the previous section to answer two questions that real-life purchaser deal with.

4.1 How to use baseload contracts to purchase a baseload profile?

Several electricity consumers have a baseload consumption profile. That is, they consume the same expected amount of electricity in each hour of the day (i.e. $\mu_{vo} = \mu_{vp} = \mu_v$). Examples of such companies are chemical plants and industrial companies that work 24 hours per day or supermarkets that are open all day. Baseload contracts (OTC forwards or futures) deliver in each hour of the delivery period and therefore perfectly match the baseload consumption profile. At first glance, it seems straightforward to purchase the baseload profile directly with a baseload contract. But is this optimal?

It can be seen from (11) that this is not, by definition, an optimal strategy. To see this, we use the results from the model in the previous section to verify whether the optimal solution suggests a baseload contract to use (that is when $\hat{\theta}_p$ equals $\hat{\theta}_o$). For a baseload profile, (11) can be rewritten as:

$$\hat{\theta}_p = \mu_v - (\mu_v - \hat{\theta}_o)\eta, \tag{16}$$

with η as defined in (12). It is obvious that $\hat{\theta}_p$ only equals $\hat{\theta}_o$ when η equals 1. The latter occurs when the relative risk-adjusted costs of hedging is the same for both hours. In all other cases, a baseload contract is not the optimal strategy and the purchaser would choose a combination of peak and off-peak contracts and a position on the day-ahead market instead. A purchaser who only uses baseload contracts in a world where the relative costs of hedging are not equal, could obtain lower expected costs level by applying a different hedging strategy.

4.2 Will taking more risk be rewarded with lower expected costs?

In order to provide insight in this issue, we need the first order partial derivative of the optimal expected costs function (15) with respect to the risk appetite $\sigma^2_{\max}$ of the purchaser. Recall that in the above formulas, we used the risk appetite in excess of the volumetric risk, σ^2_{e}, instead of the risk appetite itself. As the excess risk appetite depends linearly on the risk appetite, we will proceed with the excess risk appetite. From (15), it can be seen that the first order derivative of the optimal total costs with respect to σ_{e} is negative assuming that the expected risk premiums are positive. This implies that taking risk is rewarded by lower expected costs when expected risk premiums are positive.

The following case provides more insight in the risk versus expected costs relation and the hedging decision of the purchaser. On January 8th 2007, a power purchaser is considering how to purchase the expected consumption of 1MW in the off-peak hour and 2MW in the peak hour of a specific delivery day in 2008. We assume that she faces no volumetric risk: $\sigma^2_{vo} = \sigma^2_{vp} = 0$. The purchaser can trade a baseload and peakload calendar year 2008 contract (CAL08), which we assume is representative for the delivery day, or she can wait and purchase the electricity needed in the day-ahead market. On the 8th of January 2007, the EEX closing prices for the CAL08 baseload contract was 52.79 (euros per MWh) and for the peak-load contract the price closed at 80.08. The implied off-peak price for CAL08 was 37.58. If the purchaser would buy the total volume using one off-peak contract and two peak contracts (or, equivalently, one baseload and one peakload contract), her total cost equals 197.74 euros. These cost represent the zero-risk portfolio, given no volumetric risk. In order to examine whether the purchaser can improve upon these expected expenditures, we apply the allocation model from the previous section. Then, we have to determine values for the expected day-ahead prices and the associated standard deviations. For simplicity reasons, we assume that the average day-ahead prices on January 8th (37.88 for baseload hours (euros per MWh), 52.23 for peak load hours and 23.53 for off-peak hours) equal the expected day-ahead prices in the delivery period. For the standard deviations of the prices, we calculated the standard deviations of the daily average off-peak and peak prices over the previous calendar year 2006. These are 16.87 euros per MWh for the peak hours and 10.05 euros per MWh for the off-peak hours. Figure 1 presents the outcomes of the model for different values for the risk appetite. The solid line in figure 1 represents the expected cost. In case the

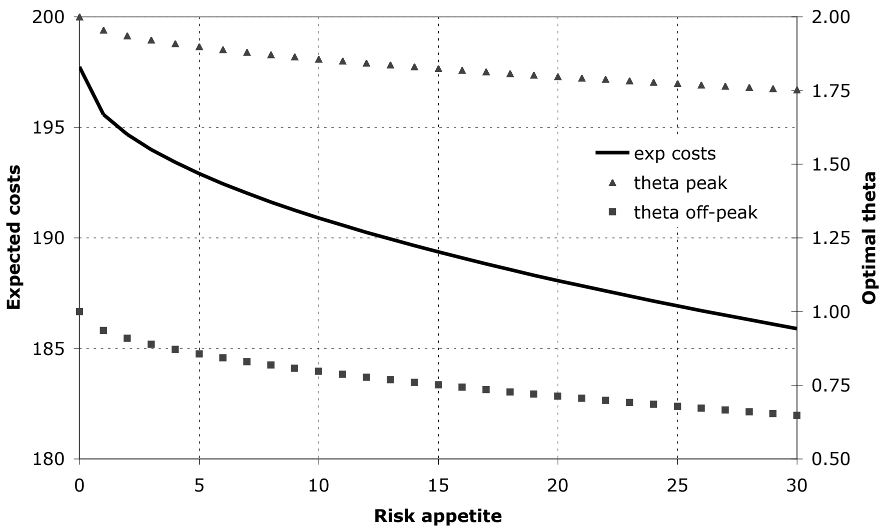

Fig. 1. The relation between risk appetite and expected costs.

risk appetite is zero, $\sigma^2_{\max} = 0$, the purchaser is not willing to take risk, the total expected cost equal 197.74 euros per MWh. In case the purchaser wants to take more risk, the expected costs decline: taking more risk is rewarded by lower expected cost. The lower expected costs are obtained by lowering the number of forward contracts in the portfolio, to profit from the lower expected prices in the day-ahead market. While increasing the risk appetite, the optimal θ_o and θ_p decline but the level in which they decline is different. This depends on the ratio of relative risk premia in (11). In our example, the value of this ratio equals 0.70. This implies that the risk premium in the peak contract is lower per unit of risk than the risk premium for the off-peak hours. Therefore, the purchaser can take more risk by lowering the number of off-peak forward contracts faster than the number of peak contracts, as the risk premium in the off-peak contract is higher per unit of risk.

5 Concluding remarks

In this paper, we have introduced a one period framework to examine the optimal allocations to peak and off-peak forward contracts of an electricity purchaser who wants to hedge both price and volumetric risks. The results show that building an optimal portfolio with electricity forward contracts is a two-step procedure. At first, purchasers find the optimal allocation to peak contracts relative to the off-peak contracts, to profit from differences in the relative hedging costs efficiency over both contracts. These relative positions are the same for everybody as they are not influenced by the individual risk appetite. Secondly, the purchaser chooses the exact allocations to meet her risk appetite. In addition, we apply the model to focus on two empirical issues from purchasers. The first shows that it is only optimal to source a baseload consumption profile with a baseload forward contract when the hedging costs per unit of day-ahead risk is the same for both peak and off-peak contracts. In practice, these marginal hedging costs are not likely to be the same at all times, indicating that a purchaser would be better off in holding a different portfolio with peak and off-peak contracts instead. The second issue reveals that purchasers with more appetite to take risk on the day-ahead market are rewarded with lower expected purchasing costs, provided that expected risk premiums are positive.

References

Bessembinder, H., Lemmon, M., 2002. Equilibrium pricing and optimal hedging in electricity forward markets. The Journal of Finance 57(3), 1347–1382.

Bohrnstedt, G., Goldberger, 1969. On the exact covariance of products of random variables. Journal of The American Statistical Association 64(328), 1439–1442.

Bunn, D., Karakatsani, N., 2003. Forecasting electricity prices. Tech. rep., London Business School Working Paper.

Bunn, D., Karakatsani, N., 2005. Diurnal reversals of electricity forward premia. Tech. rep., London Business School Working Paper.

Fama, E., French, K., 1987. Commodity futures prices: Some evidence on forecast power, premiums, and the theory of storage. Journal of Business 60(1), 55–73.

Huisman, R., Huurman, C., Mahieu, R., 2007. Hourly electricity prices in day-ahead markets. Energy Economics 29(2), 240–248.

Huisman, R., Mahieu, R., 2003. Regime jumps in electricity prices. Energy Economics 25(5), 425–434.

Markowitz, H., 1952. Portfolio selection. The Journal of Finance 7(1), 77–91.

Näsäkkälä, E., Keppo, J., 2005. Electricity load patterns hedging with static forward strategies. Managerial Finance 31(6), 116–137.

Vehvilainen, I., Keppo, J., 2004. Managing electricity market price risk. European Journal of Operational Research 145(1), 136–147.

Woo, C., Horowitz, I., Horii, B., Karimov, R., 2004. The efficient frontier for spot and forward purchases: an application to electricity. The Journal of the Operational Research Society 55(11), 1130–1136.

Publications in the Report Series Research* in Management

ERIM Research Program: "Finance and Accounting"

2007

Revisiting Uncovered Interest Rate Parity: Switching Between UIP and the Random Walk
Ronald Huisman and Ronald Mahieu
ERS-2007-001-F&A
http://hdl.handle.net/1765/8288

Hourly Electricity Prices in Day-Ahead Markets
Ronald Huisman, Christian Huurman and Ronald Mahieu
ERS-2007-002-F&A
http://hdl.handle.net/1765/8289

Do Exchange Rates Move in Line with Uncovered Interest Parity?
Ronald Huisman, Ronald Mahieu and Arjen Mulder
ERS-2007-012-F&A
http://hdl.handle.net/1765/8993

Hedging Exposure to Electricity Price Risk in a Value at Risk Framework
Ronald Huisman, Ronald Mahieu and Felix Schlichter
ERS-2007-013-F&A
http://hdl.handle.net/1765/8995

Corporate Governance and Acquisitions: Acquirer Wealth Effects in the Netherlands
Abe de Jong, Marieke van der Poel and Michiel Wolfswinkel
ERS-2007-016-F&A
http://hdl.handle.net/1765/9403

The Effect of Monetary Policy on Exchange Rates during Currency Crises; The Role of Debt, Institutions and Financial Openness
Sylvester C.W. Eijffinger and Benedikt Goderis
ERS-2007-022-F&A
http://hdl.handle.net/1765/9725

Do Private Equity Investors Take Firms Private for Different Reasons?
Jana P. Fidrmuc, Peter Roosenboom and Dick van Dijk
ERS-2007-028-F&A
http://hdl.handle.net/1765/10070

The Influence of Temperature on Spike Probability in Day-Ahead Power Prices
Ronald Huisman
ERS-2007-039-F&A
http://hdl.handle.net/1765/10179

Costs and Recovery Rates in the Dutch Liquidation-Based Bankruptcy System
Oscar Couwenberg and Abe de Jong
ERS-2007-041-F&A
http://hdl.handle.net/1765/10461

The Volatility Effect: Lower Risk without Lower Return
David C. Blitz and Pim van Vliet
ERS-2007-044-F&A
http://hdl.handle.net/1765/10460

Strategic Debt: Evidence from Bertrand and Cournot Competition
Abe de Jong, Thuy Thu Nguyen and Mathijs A. van Dijk
ERS-2007-057-F&A
http://hdl.handle.net/1765/10504

Capital Structure Around the World: The Roles of Firm- and Country-Specific Determinants
Abe de Jong, Rezaul Kabir and Thuy Thu Nguyen
ERS-2007-058-F&A
http://hdl.handle.net/1765/10517

The Effects of Federal Funds Target Rate Changes on S&P100 Stock Returns, Volatilities, and Correlations
Helena Chulia-Soler, Martin Martens and Dick van Dijk
ERS-2007-066-F&A
http://hdl.handle.net/1765/10610

Industry Valuation Driven Earnings Management
Tao Jiao, Gerard Mertens and Peter Roosenboom
ERS-2007-069-F&A
http://hdl.handle.net/1765/10608

The Impact of Media Attention on the Use of Alternative Earnings Measures
Miriam Koning, Gerard Mertens and Peter Roosenboom
ERS-2007-073-F&A
http://hdl.handle.net/1765/10609

Irving Fisher and the UIP Puzzle: Meeting the Expectations a Century Later
Rachel A. Campbell, Kees G. Koedijk, James R. Lothian and Ronald J. Mahieu
ERS-2007-088-F&A
http://hdl.handle.net/1765/10774

Electricity Portfolio Management: Optimal Peak / Off-Peak Allocations
Ronald Huisman, Ronald J. Mahieu and Felix Schlichter
ERS-2007-089-F&A
http://hdl.handle.net/1765/10775

---

* A complete overview of the ERIM Report Series Research in Management:
https://ep.eur.nl/handle/1765/1

ERIM Research Programs:
LIS Business Processes, Logistics and Information Systems
ORG Organizing for Performance
MKT Marketing
F&A Finance and Accounting
STR Strategy and Entrepreneurship