A Range-Based Multivariate Model for Exchange Rate Volatility

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A Range-Based Multivariate Model for Exchange Rate Volatility

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Abstract

In this paper we present a parsimonious multivariate model for exchange rate volatilities based on logarithmic high-low ranges of daily exchange rates. The multivariate stochastic volatility model divides the log range of each exchange rate into two independent latent factors, which are interpreted as the underlying currency specific components. Due to the normality of logarithmic volatilities the model can be estimated conveniently with standard Kalman filter techniques. Our results show that our model fits the exchange rate data quite well. Exchange rate news seems to be very currency-specific and allows us to identify which currency contributes most to both exchange rate levels and exchange rate volatilities.

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1 Introduction

In order to estimate the (latent) volatility of financial time series, like stock returns or exchange rates, it is quite common in the mainstream financial economics literature to use squared or absolute returns.[1] For example, in the popular ARCH-type volatility models, the deterministic volatility process is based on these assumptions. Typically, the error distribution is symmetric. When moving to stochastic volatility models the distributional assumptions on the errors can lead to models that are more difficult to estimate. This is caused by the fact that the joint distribution of the returns and the latent volatility is highly dimensional and non-normal. Many different approaches have been undertaken to estimate stochastic volatility models. See Shephard (1993), Harvey, Ruiz, and Shephard (1994), Jacquier, Polson, and Rossi (1994), Andersen and Sørensen (1996), and Mahieu and Schotman (1998), among others. Many of these papers focus on the univariate case, whereas for practical financial applications multivariate models are more appropriate.

In this paper the logarithmic range is used as the dependent variable in a multivariate stochastic volatility model. The main advantage of the log range is that its distribution is approximately normal. See Alizadeh, Brandt, and Diebold (2002) for a discussion on the properties of the log range. Together with the fact that the log range is an accurate proxy for volatility, the normality of the log range can be exploited for estimating stochastic volatility models. More precisely, we can apply standard Kalman filter techniques to estimate the parameters of stochastic volatility models very efficiently.

This paper extends the analysis in Alizadeh, Brandt, and Diebold (2002) along the lines set out in Mahieu and Schotman (1994). In the latter paper a parsimonious multivariate volatility model for exchange rates is presented that is built around the assumption that exchange rate returns can be decomposed into independent currency-specific factors. As a result the variance of the exchange rate returns is the sum of the two currency-specific variances. In this paper we also split the log range of several exchange rates into two latent factors which will be interpreted as currency-specific effects. These latent factors are assumed to be independent

[1] see for example Andersen (1992), and Bai, Russell, and Tiao (2001) for an exposition on measuring volatilities.

and follow AR(1) processes. The model is estimated using the EM-algorithm. Due to the distinct properties of the log range compared with other volatility approximations we are able to estimate a parsimonious multivariate stochastic volatility model.

The outline of this paper is as follows. In Section 2 some history and a motivation of the range as an approximation of the volatility will be given. In Section 3 the multivariate stochastic volatility model is presented. The exchange rate data is discussed in Section 4. Section 5 documents the results of our empirical studies and Section 6 presents the news decomposition results. Section 7 concludes.

2 Using the range as a measure for volatility

Many econometric models are built around having symmetry in the data-generating process. Especially for multivariate models of volatility we would like to impose a symmetric distribution. The disadvantage of traditional return-based volatility proxies is that in general a non-zero skewness is imposed on the volatility measure (Bai, Russell, and Tiao (2001)). This hampers efficient estimation of, especially, stochastic volatility models. Moreover, a higher excess kurtosis w.r.t. a normal distribution occurs as well. This could lead to inefficient parameter estimates if there are normality restrictions imposed on the model, as is the case for using quasi maximum likelihood methods, like in Harvey, Ruiz, and Shephard (1994). New ways to approximate volatility has emanated from recent developments in empirical finance. These approaches focus on the inputs needed to measure volatility. The most promising volatility measures are presented in the papers of Andersen, Bollerslev, Diebold, and Labys (2001) and Andersen, Bollerslev, Diebold, and Ebens (2001) and Alizadeh, Brandt, and Diebold (2002).

Andersen, Bollerslev, Diebold, and Labys (2001) and Andersen, Bollerslev, Diebold, and Ebens (2001) uses realized volatility, which deploys sums of squares and cross-products of intra-day high-frequency returns to tackle the volatility estimation problem. Alizadeh, Brandt, and Diebold (2002) make use of the range as a proxy for volatility. The range is defined by the difference between the log high and log low of the security for a certain time period. The main difference between these two methods is the availability of the data. While it can be quite difficult to obtain intra-day information of e.g. stocks, exchange rates etc. needed to compute the realized volatility measure, it is relatively easy to acquire the high and low. Alizadeh, Brandt, and Diebold (2002) mentions that

> "*...Despite the fact that the range is a less efficient volatility proxy than realized volatility under ideal conditions, it may nevertheless prove superior in real-world situations in which market microstructure biases contaminate high-frequency prices and returns. ...*"

One of these market microstructure biases is the bid-ask bounce which is the movement in the price not due to news but merely caused by a non-zero spread of the security such that the buys and the sells do not occur at the same price. Bai, Russell, and Tiao (2001) perform an analysis of the benefits of using realized volatility as an indicator for measuring daily volatility. They conclude that, even after accounting for some microstructural features of high-frequency data, the benefits may be limited when the underlying returns are autocorrelated. Furthermore, when using realized volatility the kurtosis may be large, thereby lowering the precision with which the volatility can be estimated.

Given the aforementioned reasons we choose to use the logarithmic range, as suggested by Alizadeh, Brandt, and Diebold (2002) as a proxy for volatility. Let S_t be the exchange rate, and let S_t^{high} and S_t^{low} be the high and low respectively. Consequently, the volatility measure can be defined as

$$y_t \equiv \ln\left(\ln(S_t^{\text{high}}) - \ln(S_t^{\text{low}})\right). \tag{1}$$

Note that this measure is always positive, when high and low prices are not equal. Interestingly, the log-range (1) turns out to have a distribution that is very close to a normal distribution. This feature allows us to construct a multivariate stochastic volatility model for exchange rates that does not have the drawbacks of traditional stochastic volatility models, based purely on squared or absolute returns. Another advantage of using the logarithmic range as a volatility estimate is that the measurement error of the estimate is much lower. See, for example, Andersen and Bollerslev (1998), and the discussion in Alizadeh, Brandt, and Diebold (2002).

3 Multivariate stochastic volatility model

In this section a new multivariate stochastic volatility model for exchange rates is presented. We impose a similar factor structure on the exchange rates as in Mahieu and Schotman (1994). If $s_{ij}(t)$ is the exchange rate return between currencies i and j, we assume that it can be decomposed into two currency-specific components

$$s_{ij}(t) = e_i(t) - e_j(t), \tag{2}$$

with $e_i(t)$ and $e_j(t)$ the news components of currencies i and j, respectively. We assume that the news components are independent. If we define $\lambda_i(t)$ as the variance of the news factor $e_i(t)$ ($\lambda_i(t) \equiv var(e_i(t))$) then the variance of the exchange rate can be written as

$$var\,(s_{ij}(t)) = var\,(e_i(t)) + var\,(e_j(t)) = \lambda_i(t) + \lambda_j(t). \tag{3}$$

We use this setup as the basis for our multivariate model. The range-based volatility measure that we introduced in Section 2 applies to the logarithmic volatility. Let $y_{ij,t}$ be the logarithmic range for the exchange rate between currencies i and j, as defined in (1). We assume that we can decompose the logarithmic range into two independent factors that relate to the two currencies. Consequently, we assume that

$$y_{ij,t} = \alpha_{it} + \alpha_{jt}, \tag{4}$$

with α_{it} and α_{jt} two latent factors. In our empirical application we focus on 4 currencies. This implies that we can construct 6 exchange rates for which we can compute the logarithmic ranges. The model is very flexible in the sense that the number of factors can be extended to encompass different effects, e.g. a world effect, a region effect, and extreme events. Also a larger number of currencies can be accommodated straightforwardly.

The model with only the currency-specific factors is presented below where the log range of all possible exchange rates of the 4 countries under consideration are used. For notational purposes we collect the log range at time t for all exchange rates in the vector y_t. The model that we estimate is given by the following equations.

$$y_t = c + Z\alpha_t + \varepsilon_t, \qquad \varepsilon_t \sim N(0, H), \qquad t = 1, \ldots, n \tag{5}$$

$$\alpha_{t+1} = T\alpha_t + \eta_t, \qquad \eta_t \sim N(0, Q), \qquad t = 1, \ldots, n \tag{6}$$

$$\alpha_1 \sim N(0, I) \tag{7}$$

where

$$y_t = \begin{pmatrix} y_{USD,GBP_t} \\ y_{USD,JPY_t} \\ y_{USD,EUR_t} \\ y_{GBP,JPY_t} \\ y_{GBP,EUR_t} \\ y_{JPY,EUR_t} \end{pmatrix},$$

and

$$Z = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix},$$

$$\alpha_t = \begin{pmatrix} \alpha_{USD_t} \\ \alpha_{GBP_t} \\ \alpha_{JPY_t} \\ \alpha_{EUR_t} \end{pmatrix}.$$

The idea behind the model is to split the log range of each exchange rate into a constant, the corresponding currency-specific factors (using Z as selection matrix) and an error term. There are no restrictions on H, the covariance matrix of the measurement errors. The factors evolve over time according to 4 univariate autoregressions of order 1. Consequently, both T (the autoregressive parameter matrix) and Q (covariance matrix of the errors) in the transition equations are diagonal. The part $\alpha_1 \sim N(0, I)$ is needed to initialize the Kalman filter. Because the distribution of the log range is closer to the normal distribution than the distribution of the log absolute returns, standard Kalman filter techniques can be used to estimate the parameters.

It is not straightforward to estimate traditional stochastic volatility model (see for example Jacquier, Polson, and Rossi (1994)). A problem with a standard Maximum Likelihood (ML) approach lies in the fact that the factors are latent processes. This means that the likelihood function is not tractable and therefore can not be optimized directly. One solution is to integrate the latent factors out of the log likelihood by using simulation techniques.[2]

We apply the Expectation-Maximization (EM) algorithm (Dempster, Laird, and Rubin (1977) to obtain the parameters of the model (5)-(7). This method splits the estimation of the parameters and the latent factors α_t into two steps: an expectation and a maximization step. By iterating these steps Dempster, Laird, and Rubin (1977) have shown that the likelihood function increases. This iterative procedure can be continued until convergence takes place. We restrict the discussion here to the essentials of the EM-algorithm. In the Appendix we present a detailed derivation.

The expectation step consists of finding the expectation of the log likelihood function conditional on the data and the parameters. In other words, given the data y, where $y = \{y_1, ..., y_n\}$, and given a value for the parameter set of the model, which is denoted by $\psi = \{c, H, T, Q\}$, the expression $E(-2lnL|y, \psi)$ is calculated. An estimate of the latent factors α_t, $E(\alpha_t|y)$, is obtained by the standard Kalman filter and smoother given the parameters (see for example Shumway and Stoffer (2000) and Durbin and Koopman (2001)). These filter and smoother recursions are also given in the Appendix. Also an estimate of the expected cross-products of the latent factors, $E(\alpha_t \alpha_t'|y)$, which appears in the expected log likelihood, can be expressed using the output of the Kalman filter and smoother.

The maximization step consists of maximizing the expected log likelihood given the approximation of the latent factors and the cross-products. This maximization can be done analytically instead of numerically because the expressions for the optimal parameters given an expectation of the factors are straightforward. The derivation of the analytical solutions follow Shumway and Stoffer (2000). Our model differs slightly from their exposition in the sense that our multivariate volatility model prescribes that T and Q are diagonal. All deriva-

[2] See, for example, Jacquier, Polson, and Rossi (1994) and Shephard (1993).

tions are presented in the Appendix. The estimates of the parameters are given by

$$c = n^{-1} \sum_{t=1}^{n} \{y_t - Z\hat{\alpha}_t\} \tag{8}$$

$$H = n^{-1} \sum_{t=1}^{n} \left\{(y_t - c - Z\hat{\alpha}_t)(y_t - c - Z\hat{\alpha}_t)' + Zcov(\alpha_t|y)Z'\right\} \tag{9}$$

$$T_{kk} = (S_{10})_{kk}/(S_{00})_{kk} \text{for } k = 1, ..., 4 \tag{10}$$

$$Q_{kk} = n^{-1} \left[(S_{11})_{kk} - (S_{10})^2_{kk}/(S_{00})_{kk}\right] \text{ for } k = 1, ..., 4 \tag{11}$$

where

$$S_{11} = \sum_{t=1}^{n} \left\{\hat{\alpha}_{t+1}\hat{\alpha}'_{t+1} + P_{t+1}(I_m - N_t P_{t+1})\right\}$$

$$S_{10} = \sum_{t=1}^{n} \left\{\hat{\alpha}_{t+1}\hat{\alpha}'_t + P_t L'_t (I_m - N_t P_{t+1)}\right\}$$

$$S_{00} = \sum_{t=1}^{n} \left\{\hat{\alpha}_t\hat{\alpha}'_t + P_t(I_m - N_{t-1}P_t)\right\}$$

The algorithm can be started by setting the following initial values:

$$c_0 = \bar{y}_t$$

$$H_0 = cov(y_t)$$

$$T_0 = 0_{mxm}$$

$$Q_0 = \frac{1}{m} \sum_{k=1}^{m} H_{0,kk}$$

In Section 5 we present the results for the multivariate volatility model on a set of exchange rate data.

4 Exchange rate data

The daily high, low and close prices of six exchange rates were collected from Moneyline Telerate. The high and low values are computed over a 24-hour period which starts at 10pm GMT, 5pm US Eastern Time. The exchange rate set consists of all possible combinations of the following currencies: US Dollar (USD), UK Sterling (GBP), Japanese yen (JPY) and euro (EUR). The sample runs from September 1, 1989 until July 22, 2002. The observations where the high is lower than the low are considered to be typos and these values are swapped. This occurs 2 times for the USD/GBP, and 9 times for the JPY/EUR exchange rate and it does not occur for the other rates. Furthermore, the missing values are replaced with data from the previous (working) day. The USD/EUR and JPY/EUR have a total of 7 missing values which are on the same dates. The USD/GBP, GBP/JPY and GBP/EUR have a total of 6 missing values, also on the same dates, which form a subset of the dates of the aforementioned 7 missing values. All these days can be linked to holidays. Finally, the USD/JPY has no missing values. After deleting the 11 observations for which the high equals the low (for these observations the log range is not defined) for at least one exchange rate a data set consisting of a total of 3351 observations is obtained. Then the log range for each exchange rate is constructed according to the definition. These time series are presented in Figure 1 and some descriptive statistics are reported in Table 1. The covariance and correlation matrices appear in Table 2. As was already noticed in Mahieu and Schotman (1994) the covariance structure of exchange rates expressed in a common numeraire has a very distinct feature. The covariances of all dollar exchange rate returns are all in the same order of magnitude. This motivates Mahieu and Schotman (1994) to present a multivariate exchange rate model that exploits this feature. In this paper we follow this approach again.

From Figure 1 it seems that the log range shows mean-reverting behavior and is quite volatile. Also substantial correlation between the several log range series seems to be present. Furthermore, the skewness and kurtosis is rather high for some exchange rates. A QQ-plot is constructed to see whether the log range is close to the normal distribution. Figure 2 shows the results. The data look quite normal except for the tails where some outliers can be detected.[3]

Alizadeh, Brandt, and Diebold (2002) use prices of currency futures contracts and for these series the distributions for the logarithmic range are closer to the Gaussian distribution than the ones computed on the logarithm of the absolute returns. To check this for our data set, the logarithmic absolute return of the exchange rates is calculated using the closing price. The days for which the return is zero are thrown out of the sample. In Table 3 some descriptive statistics are reported. The QQ-plots are also made for these series and are shown in Figure 3. It is clear that the log absolute returns deviate rather substantially from normality. The logarithmic range performs better and therefore this volatility proxy is used to estimate our multivariate stochastic volatility model.

5 Results

In this section the results of the model will be presented and evaluated.[4] The optimal parameters and the value of the log likelihood are given in Table 4. The estimated currency components are displayed in Figure 4. The elements of $\hat{H}$ are smaller than the corresponding elements in the covariance matrix of the data (Table 2) as is expected because part of the variability in the data is absorbed by the latent state variables. The correlation matrix constructed from $\hat{H}$ in Table 4 shows that, on average, the correlations have decreased with respect to the original data. The $\hat{T}$ matrix shows a rather high persistence in the latent log volatilities, which is a well-known feature in daily financial time series.

Looking at the estimated latent factors in Figure 4 it is clear that the Yen component is the most volatile of all. For the Pound component it seems that after a sudden increase in log volatility there is a period in which it decreases again until a new shock arrives. Furthermore, for some periods the movements in the components are comparable to each other which shows that the states might be dependent.

In tables 5 and 6 some diagnostics for the estimated measurement errors, transition errors

[3] Our empirical results do not change considerably after removal of some of these outliers. These results are available from the authors upon request.

[4] All calculations needed for construction of the graphs and tables are done with the software package Matlab, version 6.

and currency-specific components are given. It is clear that for the transition errors and latent factors there is considerable autocorrelation present while this is less severe for the measurement errors.

5.1 Adding a world factor

In this section a fifth factor, a so-called world component, is added to the model. Therefore the log range of each exchange rate is now split into two currency-specific components and one world component which is the same across all exchange rates. The new selection matrix Z and the state vector α_t can then be written as

$$Z = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix},$$

$$\alpha_t = \begin{pmatrix} \alpha_{USD_t} \\ \alpha_{GBP_t} \\ \alpha_{JPY_t} \\ \alpha_{DEM_t} \\ \alpha_{WORLD_t} \end{pmatrix}$$

The rest of the model remains unchanged. The model is initialized with the optimal values of the four state model and $T_{0,55} = 0$ and $Q_{0,55} = \sum_{k=1}^{4} Q_{0,kk}$. The optimal parameters and the value of the log likelihood is given in Table 7. The estimated currency components are displayed in Figure 5. Note that the states do not seem to alter much graphically.

The elements of $\hat{H}$ have decreased somewhat compared to the former results. Also the correlation between the errors is lower than before. There is a bit more persistence in the transition equation because of the higher values on the diagonal of $\hat{T}$ and the variances of the currency-specific components (in $\hat{Q}$) are lower, while the world component variance is quite high compared to the aforementioned variances.

Because the four-state model is a restricted version of the five state model, a Likelihood

Ratio (LR) test can be performed to evaluate whether the former model can be rejected in favor of the extended model. There are 2 restrictions on the unrestricted model (the fifth element of both T and Q are set to 0) therefore the LR test statistic has an asymptotic distribution of χ^2_2. The test statistic is equal to LR $= 2(\log L_{unrestricted}$-$\log L_{restricted}) =$ 2(-8499.492-(-8567.818)) $= 136.65 \geq 9.21 = \chi^2_{2,0.99}$ so the four state model is too restrictive compared to the five state model.

6 News factors

In this section news factors for the several countries are extracted from the multivariate stochastic volatility model including a world component. Remember that every exchange rate can be decomposed into two currency specific factors as stated in Equation (2). Mahieu and Schotman (1994) have shown that such a currency-specific news factor is a weighted average over the exchange rates returns containing that currency. The weights are constructed from the (conditional) variances of the news factors. The intuition behind this is that the currency specific news can be extracted from all exchange rates that can be constructed with the same currency. The higher the variance of the exchange rate, the less informative and therefore the corresponding exchange rate has less weight in the news factor. Following Mahieu and Schotman (1994), the news factor at time t, $\hat{e}(t)$, for currency i with $i \in I = \{USD, GBP, JPY, EUR\}$ can be written as

$$\hat{e}_i(t) = \frac{\sum_{j \in I, j \neq i} \lambda_j(t)^{-1} s_{ij}(t)}{\sum_{j \in I} \lambda_j(t)^{-1}} \tag{12}$$

where $s_{ij}(t)$ is the log return of the exchange rate.[5] Also note that $s_{ji}(t) = -s_{ij}(t)$. To apply this we again note that in our model the logarithmic volatility is modelled instead of the variance itself. So to obtain the equivalent quantity of the $\lambda(t)$'s in the above equation we

[5]Note that the formulation of the news factors (12) is the same for both the models with and without world component and that there is no separate world news factor. The news factor is calculated using a numeraire currency. Therefore it is impossible to identify a world news component which has the same impact on each exchange rate because it is indistinguishable with the numeraire currency. However, adding a world component has an impact on the estimated parameters and latent currency components and therefore also has influence on the news factors for the several currencies.

have to transform the time-varying $\hat{\alpha}_t$ that we obtain from the multivariate volatility model. Note that $\hat{\alpha}_t = E(\alpha_t|y)$. Then using the fact that when $X \sim N(\mu, \sigma^2)$ it holds that the moment generating function equals $E(\exp(tX)) = \exp(t\mu + t^2\sigma^2/2)$. Consequently we obtain

$$\lambda_j(t) = E(\exp(\alpha_{j,t})^2|y) = \exp(2\hat{\alpha}_{j,t} + 2V_{j,t}) \tag{13}$$

where

$$V_{j,t} = Var(\alpha_{j,t}|y)$$

The smoother which is used in the optimization process (see also the Appendix) delivers $V_{j,t}$. It is the variance of factor j conditional on all available information. The graphs of the $\lambda(t)$'s for the model including the world component are shown in Figure 6.[6] To investigate the influence of the news factors on the underlying exchange rate, an indexed exchange rate is constructed from each of the news series $\hat{e}_i(t)$. The news indices are defined by

$$I_{i,news}(t) = 100(1 + \sum_{i=1}^{t} \hat{e}_i(t))$$

Figures 7 show the indexed exchange rates together with the accompanying news indices for the model including the world factor. From the figure it can be seen that shocks to the exchange rates can be assigned to specific currencies in several periods. Nevertheless, many shocks to the exchange rate occur simultaneously in both currencies.

7 Concluding remarks

In this paper we have presented a new approach to estimate multivariate stochastic volatility models for exchange rate volatilities. The model exploits the convenient distributional characteristics of logarithmic high-low ranges. Also, the logarithmic range has been shown to be a very efficient estimate of volatility compared to return-based volatility proxies, like

[6] The latent variances of the model excluding the world factor are very similar to the variance presented in the figure. They are available upon request.

the absolute or squared return. Furthermore, the model draws upon the decomposition of exchange rates into currency-specific factors. Taken together these characteristics allow us to estimate a parsimonious multivariate model for exchange rate volatility in a very efficient way.

Our results show that the model can be estimated efficiently through standard Kalman filter techniques within an EM-algorithm. We find that the currency-specific volatilities are substantially different from each other. Adding a world factor to the model does not change the values of the news factors considerably, although we can reject the initial 4-factor model in favor of the model that adds the world factor.

The analysis in this paper can be extended in several ways. First, the specific factor that we imposed can be investigated. For example, each currency-specific could be split into a persistent and a stationary component.[7] Secondly, a further analysis of the news series could be pursued in order to find relationships with economic variables, like interest rates, and monetary variables. Lastly, the multivariate model could be used for analyzing the prices of options.

[7] See also Alizadeh, Brandt, and Diebold (2002), who perform this analysis for univariate stochastic volatility models.

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Appendix

The Kalman filter and smoother recursions

The Kalman filter and smoother are given for the sake of completeness. These equations are taken from Durbin and Koopman (2001) and the same notation will be used. Because there appears a constant in the measurement equation which is not present in the standard filter, one adjustment to the recursions is made. Note however that the smoother does not need to be changed because the effect of the constant is fully captured by v_t which also appears in the smoother. Furthermore, note that compared to Durbin and Koopman (2001) for the state space model used in this paper Z is constant, H, T, Q are non-time-varying parameter matrices and R_t is equal to the unity matrix. First the model is given again for reference purposes:

$$y_t = c + Z\alpha_t + \varepsilon_t,\ \varepsilon_t \sim N(0, H),\ t = 1, ..., n$$

$$\alpha_{t+1} = T\alpha_t + \eta_t,\ \eta_t \sim N(0, Q),\ t = 1, ..., n$$

$$\alpha_1 \sim N(a_1, P_1)$$

Then the Kalman filter is given as follows: If there are m states, $a_1 = 0_m$ and $P_1 = I_m$ then for $t = 1, ..., n$

$$v_t = y_t - c - Za_t$$

$$F_t = ZP_tZ' + H$$

$$K_t = TP_tZ'F_t^{-1}$$

$$L_t = T - K_tZ$$

$$a_{t+1} = E(\alpha_{t+1}|Y_t) = Ta_t + K_tv_t$$

$$P_{t+1} = cov(\alpha_{t+1}|Y_t) = TP_tL_t' + Q$$

Next the output of the Kalman filter is used in the smoother to construct a proxy for the latent factors. The smoother equations are as follows: If $r_n = 0_m$ and $N_n = 0_{m\times m}$ then for $t = 1, ..., n$

$$r_{t-1} = Z'F_t^{-1}v_t + L_t'r_t$$

$$N_{t-1} = Z'F_t^{-1}Z + L_t'N_tL_t$$

$$\hat{\alpha}_t = E(\alpha_t|y) = a_t + P_tr_{t-1}$$

$$V_t = P_t - P_tN_{t-1}P_t$$

Given the parameters of the model an estimation for the latent factors is then $\hat{\alpha}_t$. Furthermore the disturbance smoother equations from Durbin and Koopman (2001) are used to obtain the estimated errors for the measurement and transition equation, $\hat{\varepsilon}_t$ and $\hat{\eta}_t$.

The disturbance smoother

$$
\begin{aligned}
u_t &= F_t^{-1} v_t - K_t' r_t \\
D_t &= F_t^{-1} + K_t' N_t K_t \\
\hat{\varepsilon}_t &= E(\varepsilon_t | y) = H u_t \\
cov(\varepsilon_t | y) &= H - H D_t H \\
\hat{\eta}_t &= E(\eta_t | y) = Q r_t \\
cov(\eta_t | y) &= Q - Q N_t Q
\end{aligned}
$$

Also in the estimation of the parameters of the model, the (inter temporal) covariance between smoothed states is needed. These expressions (see also Table 4.4 of Durbin and Koopman (2001)) are given by

$$
\begin{aligned}
cov(\alpha_t | y) &= P_t (I_m - N_{t-1} P_t) \\
cov(\alpha_t, \alpha_{t+1} | y) &= P_t L_t' (I_m - N_t P_{t+1)}
\end{aligned}
$$

The EM-algorithm (Dempster, Laird, and Rubin (1977)) consists of an estimation step and a maximization step. We follow the discussion in Shumway and Stoffer (2000) (Chapter 4, paragraph 3).

The Expectation-step of EM

In this step the expectation of the log likelihood is taken given the data and the parameters. The log likelihood in this case is

$$
-2 \ln L = \ln |Q| + \sum_{t=1}^{n} (\alpha_{t+1} - T\alpha_t)' Q^{-1} (\alpha_{t+1} - T\alpha_t) +
$$

$$
+ \ln |H| + \sum_{t=1}^{n} (y_t - c - Z\alpha_t)' H^{-1} (y_t - c - Z\alpha_t)
$$

which is similar to equation (4.69) in Shumway and Stoffer (2000). Taking the expectation of this expression gives something similar to equation (4.71) of this reference.

$$
-2 \ln L = \ln |Q| + trace \left\{ Q^{-1} \left[S_{11} - S_{10} T' - T S_{10}' + T S_{00} T' \right] \right\} +
$$

$$
+ \ln |H| + trace \left\{ H^{-1} \sum_{t=1}^{n} \left\{ (y_t - c - Z\hat{\alpha}_t)(y_t - c - Z\hat{\alpha}_t)' + Z cov(\alpha_t | y) Z' \right\} \right\}
$$

with

$$
S_{11} = \sum_{t=1}^{n} \left\{ \hat{\alpha}_{t+1} \hat{\alpha}_{t+1}' + cov(\alpha_{t+1} | y) \right\} = \sum_{t=1}^{n} \left\{ \hat{\alpha}_{t+1} \hat{\alpha}_{t+1}' + P_{t+1} (I_m - N_t P_{t+1}) \right\}
$$

$$S_{10} = \sum_{t=1}^{n} \left\{ \hat{\alpha}_{t+1}\hat{\alpha}_t' + cov(\alpha_{t+1}, \alpha_t|y) \right\} = \sum_{t=1}^{n} \left\{ \hat{\alpha}_{t+1}\hat{\alpha}_t' + P_t L_t'(I_m - N_t P_{t+1)} \right\}$$

$$S_{00} = \sum_{t=1}^{n} \left\{ \hat{\alpha}_t\hat{\alpha}_t' + cov(\alpha_t|y) \right\} = \sum_{t=1}^{n} \left\{ \hat{\alpha}_t\hat{\alpha}_t' + P_t(I_m - N_{t-1}P_t) \right\}$$

In the above analysis some of the properties of the trace-operator are used. Also in the derivation they use that

$$E(\alpha_t\alpha_t'|y) = \hat{\alpha}_t\hat{\alpha}_t' + cov(\alpha_t|y)$$

$$E(\alpha_{t+1}\alpha_t'|y) = \hat{\alpha}_{t+1}\hat{\alpha}_t' + cov(\alpha_{t+1}, \alpha_t|y)$$

The P_t^n and $P_{t,t-1}^n$ defined in Shumway and Stoffer (2000) are equivalent to respectively $cov(\alpha_t|y)$ and $cov(\alpha_t, \alpha_{t-1}|y)$. The Kalman filter and smoother are applied to obtain $\hat{\alpha}_t$.

The maximization step of EM

Here an estimate of the parameters are given. The log likelihood function after the expectation step is given above. Because T and Q are assumed diagonal this expression can be simplified.

$$\ln|Q| + trace\left\{Q^{-1}\left[S_{11} - S_{10}T' - TS_{10}' + TS_{00}T'\right]\right\} =$$

$$\sum_{k=1}^{m} \ln Q_{kk} + \sum_{k=1}^{m} Q_{kk}^{-1}\left[(S_{11})_{kk} - 2(S_{10})_{kk}T_{kk} + (S_{00})_{kk}T_{kk}^2\right]$$

$$+ \ln|H| + trace\left\{H^{-1}\sum_{t=1}^{n}\left\{(y_t - c - Z\hat{\alpha}_t)(y_t - c - Z\hat{\alpha}_t)' + Zcov(\alpha_t|y)Z'\right\}\right\}$$

where is used that

1. if matrices A and B are diagonal then so is AB,
2. if matrix A is diagonal then $trace(AB) = \sum a_{ii}b_{ii}$,
3. $trace(AB) = trace(BA)$,
4. and $trace(A + B) = trace(A) + trace(B)$.

Then the estimates of the parameters are given by

$$c = n^{-1}\sum_{t=1}^{n}\{y_t - Z\hat{\alpha}_t\}$$

$$H = n^{-1}\sum_{t=1}^{n}\left\{(y_t - c - Z\hat{\alpha}_t)(y_t - c - Z\hat{\alpha}_t)' + Zcov(\alpha_t|y)Z'\right\}$$

$$T_{kk} = (S_{10})_{kk}/(S_{00})_{kk} \text{for } k = 1, ..., 4$$

$$Q_{kk} = n^{-1}\left[(S_{11})_{kk} - (S_{10})_{kk}^2/(S_{00})_{kk}\right] \text{ for } k = 1, ..., 4$$

Ex. Rate	Mean	St. dev.	Skewness	Kurtosis	Min	Max
Dollar/Pound	-5.052	0.610	-0.317	3.814	-8.128	-3.012
Dollar/Yen	-4.764	0.534	0.034	4.074	-8.333	-2.307
Dollar/Euro	-4.743	0.471	0.064	3.223	-6.801	-2.972
Pound/Yen	-4.603	0.473	0.164	3.266	-6.529	-2.497
Pound/Euro	-4.923	0.458	0.080	3.162	-6.599	-3.281
Yen/Euro	-4.682	0.520	-0.471	6.186	-8.348	-2.406

Table 1: Some characteristics, as the mean, standard deviation, skewness, kurtosis, minimum and maximum, of the log range of the exchange rates Dollar/Pound, Dollar/Yen, Dollar/Euro, Pound/Yen, Pound/Euro and Yen/Euro are reported for the period September the 1st, 1989 until July the 22nd, 2002 (3351 observations). See Section 4 for further details.

	covariance matrix					
Dollar/Pound	0.372	0.067	0.176	0.111	0.093	0.095
Dollar/Yen		0.285	0.100	0.170	0.075	0.166
Dollar/Euro			0.222	0.086	0.107	0.124
Pound/Yen				0.224	0.113	0.173
Pound/Euro					0.210	0.106
Yen/Euro						0.270
	correlation matrix					
Dollar/Pound	1	0.205	0.614	0.386	0.333	0.300
Dollar/Yen		1	0.399	0.674	0.309	0.597
Dollar/Euro			1	0.384	0.497	0.507
Pound/Yen				1	0.519	0.701
Pound/Euro					1	0.443
Yen/Euro						1

Table 2: The covariance and correlation matrix of the log range of the exchange rates Dollar/Pound, Dollar/Yen, Dollar/Euro, Pound/Yen, Pound/Euro and Yen/Euro are reported for the period September the 1st, 1989 until July the 22nd, 2002 (3351 observations).

Ex. Rate	Mean	St. dev.	Skewness	Kurtosis	Min	Max
Dollar/Pound	-5.906	1.074	-0.570	3.034	-8.864	-3.155
Dollar/Yen	-5.725	1.100	-0.794	3.661	-9.500	-2.588
Dollar/Euro	-5.722	1.096	-0.866	3.770	-9.748	-3.389
Pound/Yen	-5.633	1.129	-0.976	4.197	-10.176	-2.771
Pound/Euro	-5.962	0.994	-0.567	3.296	-10.241	-3.439
Yen/Euro	-5.621	1.086	-0.853	3.694	-9.741	-2.640

Table 3: Some characteristics, as the mean, standard deviation, skewness, kurtosis, minimum and maximum, of the log absolute return of the exchange rates Dollar/Pound, Dollar/Yen, Dollar/Euro, Pound/Yen, Pound/Euro and Yen/Euro are reported for the period September the 1st, 1989 until July the 22nd, 2002 (3050 observations).

$\hat{c}$	-5.0506					
	-4.7631					
	-4.7416					
	-4.6002					
	-4.9200					
	-4.6800					
$\hat{H}$	0.2669	0.0299	0.1091	0.0845	0.0524	0.0831
		0.1816	0.0586	0.0852	0.0390	0.0726
			0.1571	0.0516	0.0583	0.0781
				0.1345	0.0428	0.0883
					0.1176	0.0289
						0.1731
	Correlation matrix constructed from $\hat{H}$					
	1	0.1359	0.5330	0.4458	0.2960	0.3868
		1	0.3472	0.5452	0.2669	0.4096
			1	0.3547	0.4293	0.4739
				1	0.3405	0.5790
					1	0.2027
						1
$diag(\hat{T})$	0.9628					
	0.9671					
	0.9586					
	0.9428					
$diag(\hat{Q})$	0.0022					
	0.0016					
	0.0051					
	0.0036					
$\log L$	-8567.818					

Table 4: The optimal parameter values together with the value of the log likelihood function after estimating model (5)-(7) are given above. Also the correlation matrix of the measurement errors is constructed using $\hat{H}$. Estimation takes place over the period September the 1st, 1989 until July the 22nd, 2002 (3351 observations).

	Mean	St. dev.	Skewness	Kurtosis	Min	Max
$\hat{\varepsilon}_t$						
Dollar-Pound	0.000	0.505	-0.353	4.278	-3.119	1.636
Dollar-Yen	0.000	0.406	0.052	4.053	-2.757	1.502
Dollar-Euro	0.000	0.380	0.159	3.382	-1.815	1.496
Pound-Yen	0.000	0.347	0.196	3.283	-1.577	1.389
Pound-Euro	0.000	0.326	0.177	3.348	-1.153	1.413
Yen-Euro	0.000	0.395	-0.688	8.249	-3.075	1.635
$\hat{\eta}_t$						
Dollar component	0.000	0.014	0.442	3.577	-0.043	0.064
Pound component	0.000	0.012	0.856	5.789	-0.033	0.084
Yen component	0.000	0.024	0.387	3.708	-0.081	0.126
Euro component	0.000	0.021	0.201	4.414	-0.107	0.114

Table 5: Some characteristics, as the mean, standard deviation, skewness, kurtosis, minimum and maximum, of the estimated errors, $\hat{\varepsilon}_t$ and $\hat{\eta}_t$, of model (5)-(7) are reported. The model is estimated for the period September the 1st, 1989 until July the 22nd, 2002 (3351 observations).

	Autocorrelation	Ljung-Box(30) test	ARCH(1) test	ARCH(10) test
$\hat{\varepsilon}_t$	0.251	3557.759	66.297	85.108
		(0.000)	(0.000)	(0.000)
	0.042	267.713	1.475	14.664
		(0.000)	(0.225)	(0.145)
	0.009	145.543	5.924	28.336
		(0.000)	(0.015)	(0.002)
	0.014	51.681	2.834	16.965
		(0.008)	(0.092)	(0.075)
	0.044	135.840	15.514	40.790
		(0.000)	(0.000)	(0.000)
	0.075	812.786	185.073	348.075
		(0.000)	(0.000)	(0.000)
$\hat{\eta}_t$	0.781	5566.740	1256.347	1241.970
		(0.000)	(0.000)	(0.000)
	0.819	6098.786	2145.762	2152.774
		(0.000)	(0.000)	(0.000)
	0.734	4072.146	1212.641	1228.535
		(0.000)	(0.000)	(0.000)
	0.694	3809.306	1052.801	1101.687
		(0.000)	(0.000)	(0.000)
$\hat{\alpha}_t$	0.996	61275.862	3304.500	3319.003
		(0.000)	(0.000)	(0.000)
	0.996	63439.700	3310.952	3327.527
		(0.000)	(0.000)	(0.000)
	0.995	61099.448	3283.760	3307.060
		(0.000)	(0.000)	(0.000)
	0.992	57449.170	3226.471	3263.822
		(0.000)	(0.000)	(0.000)

Table 6: Some diagnostics of the estimated measurement errors, $\hat{\varepsilon}_t$, transition errors, $\hat{\eta}_t$, and currency components, $\hat{\alpha}_t$, of model (5)-(7) are reported as the first order autocorrelation, the Ljung-Box test on autocorrelation up to order 30 (with χ^2_{30} as asymptotic distribution), and the ARCH LM test to test for first and tenth order ARCH (with respectively χ^2_1 and χ^2_{10} as asymptotic distribution). The model is estimated for the period September the 1st, 1989 until July the 22nd, 2002 (3351 observations).

Parameter						
$\hat{c}$	-5.0510					
	-4.7640					
	-4.7423					
	-4.6006					
	-4.9201					
	-4.6806					
$\hat{H}$	0.2437	0.0067	0.0863	0.0621	0.0304	0.0615
		0.1611	0.0368	0.0644	0.0168	0.0537
			0.1363	0.0297	0.0374	0.0588
				0.1143	0.0214	0.0689
					0.0974	0.0091
						0.1564
	Correlation matrix constructed from $\hat{H}$					
	1	0.0336	0.4735	0.3723	0.1972	0.3151
		1	0.2486	0.4748	0.1340	0.3383
			1	0.2380	0.3350	0.4029
				1	0.2029	0.5154
					1	0.0736
						1
$diag(\hat{T})$	0.9603					
	0.9782					
	0.9680					
	0.9544					
	0.6097					
$diag(\hat{Q})$	0.0022					
	0.0009					
	0.0036					
	0.0026					
	0.0206					
$\log L$	-8498.7213					

Table 7: The optimal parameter values together with the value of the log likelihood function after estimating model (5)-(7) including a world factor are given above. Also the correlation matrix of the measurement errors is constructed using $\hat{H}$. Estimation takes place over the period September the 1st, 1989 until July the 22nd, 2002 (3351 observations).

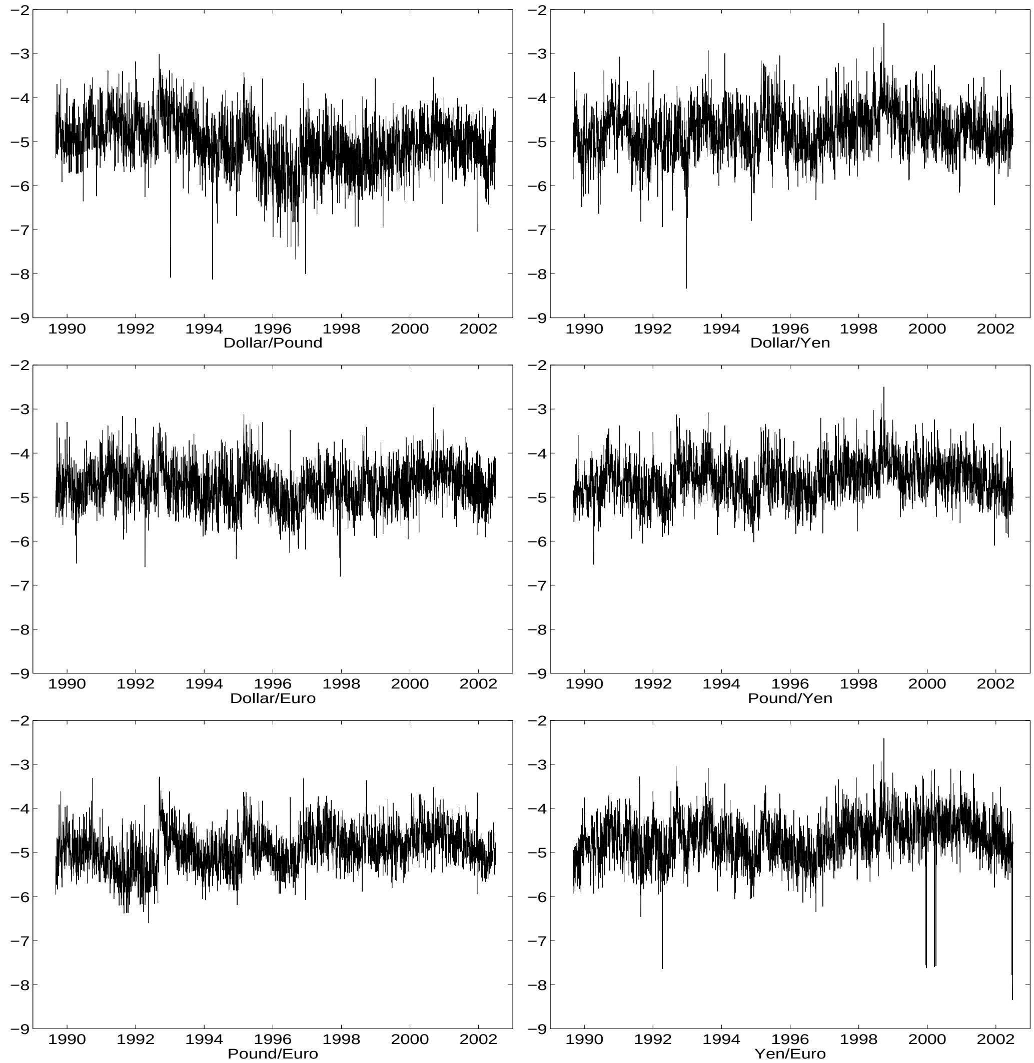

Figure 1: These graphs show the log range of the exchange rates Dollar/Pound, Dollar/Yen, Dollar/Euro, Pound/Yen, Pound/Euro, Yen/Euro for the period September the 1st, 1989 until July the 22nd, 2002 (3351 observations). See Section 4 for further details.

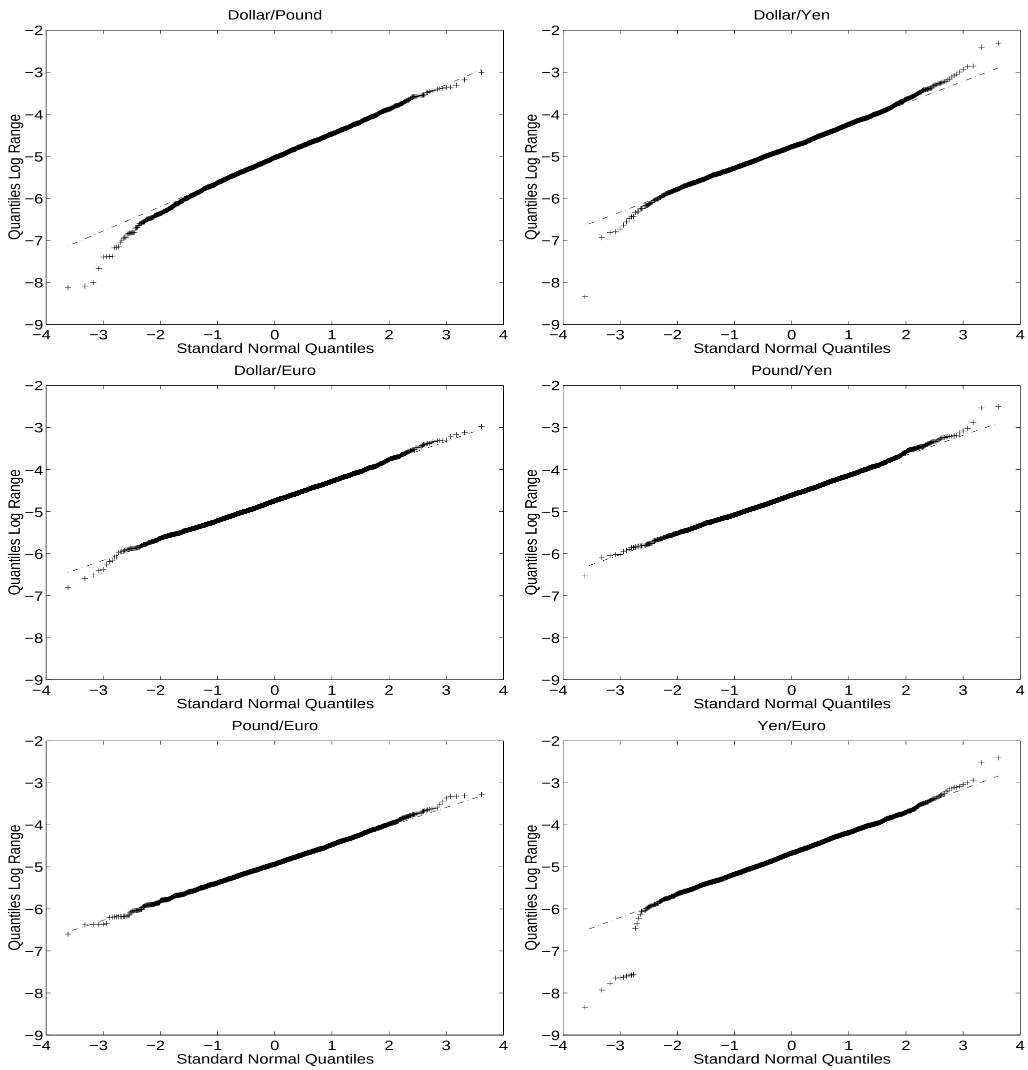

Figure 2: These graphs show the QQ-plot for the log range of the exchange rates Dollar/Pound, Dollar/Yen, Dollar/Euro, Pound/Yen, Pound/Euro, Yen/Euro for the period September the 1st, 1989 until July the 22nd, 2002 (3351 observations).

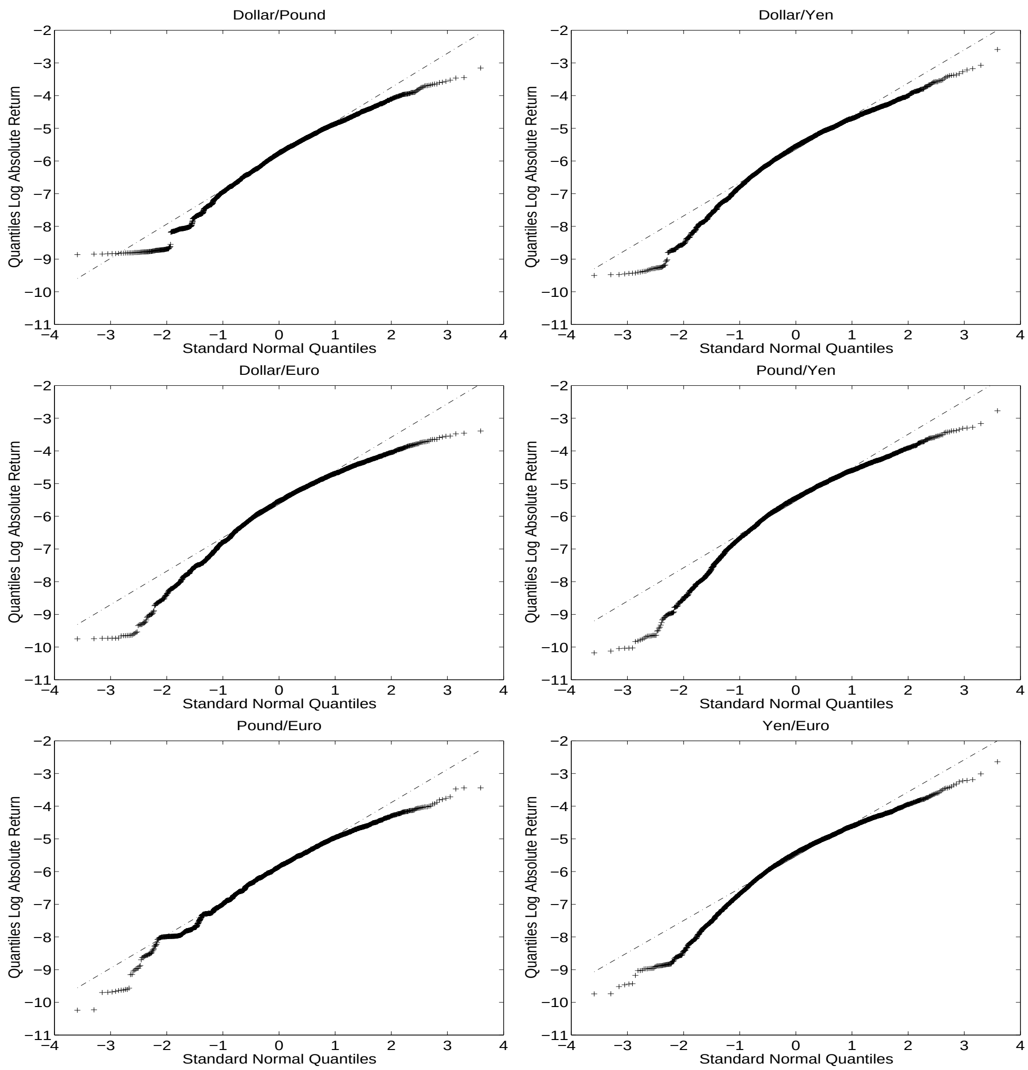

Figure 3: These graphs show the QQ-plot for the log absolute returns of the exchange rates Dollar/Pound, Dollar/Yen, Dollar/Euro, Pound/Yen, Pound/Euro, Yen/Euro for the period September the 1st, 1989 until July the 22nd, 2002 (3050 observations).

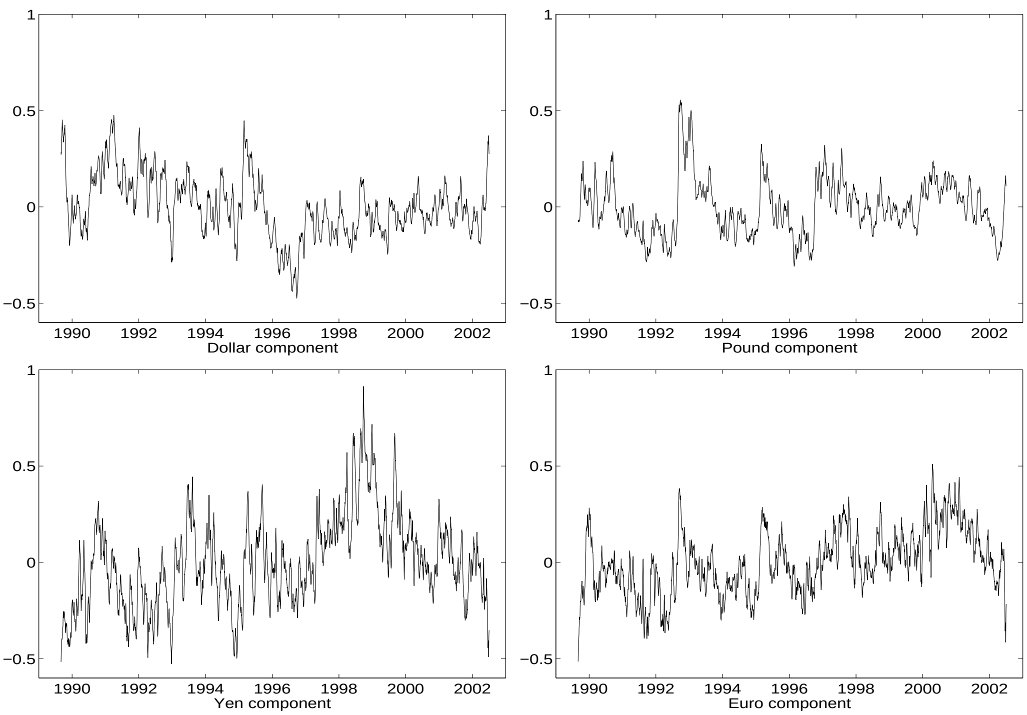

Figure 4: The graphs of the 4 estimated latent factors of model (5)-(7) which are interpreted as the Dollar, Pound, Yen and Euro component respectively. Estimation takes place over the period September the 1st, 1989 until July the 22nd, 2002 (3351 observations).

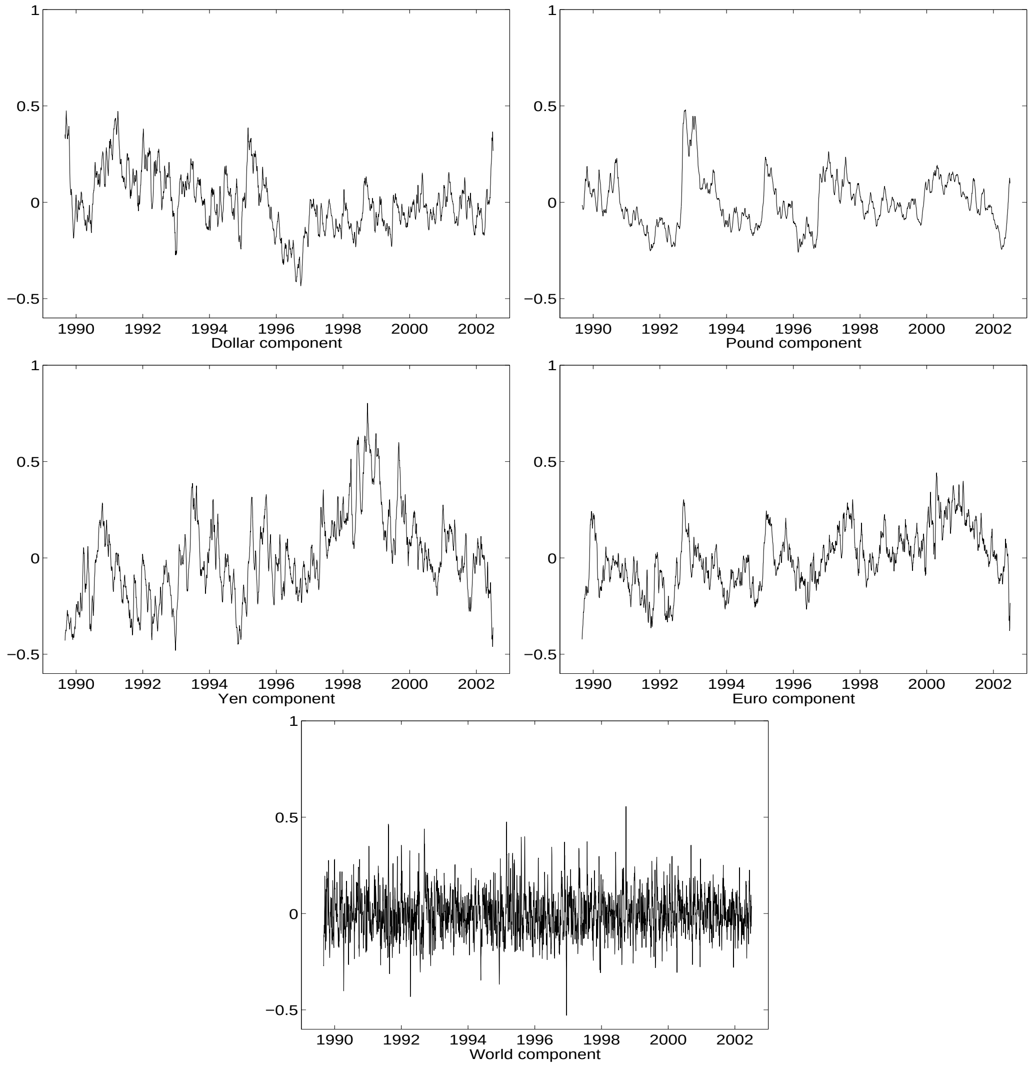

Figure 5: The graphs of the 5 estimated latent factors of model (5)-(7) which are interpreted as the Dollar, Pound, Yen, Euro and World component respectively. Estimation takes place over the period September the 1st, 1989 until July the 22nd, 2002 (3351 observations).

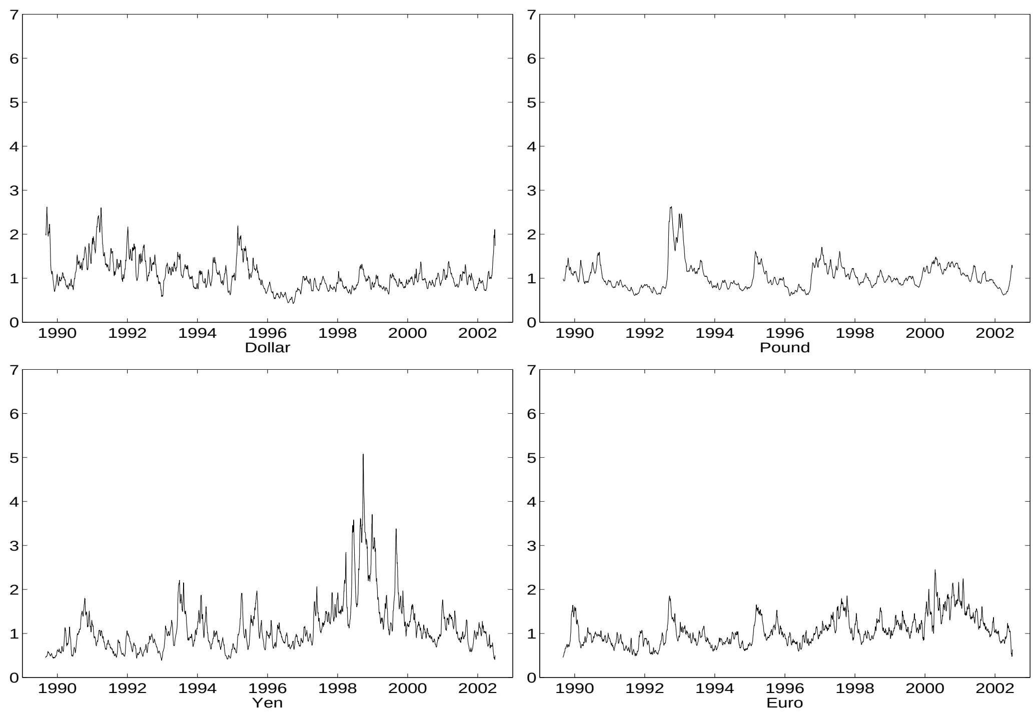

Figure 6: The graphs of the λ's as defined in equation (13) are shown. Estimation of model (5)-(7) including a world factor takes place over the period September the 1st, 1989 until July the 22nd, 2002 (3351 observations).

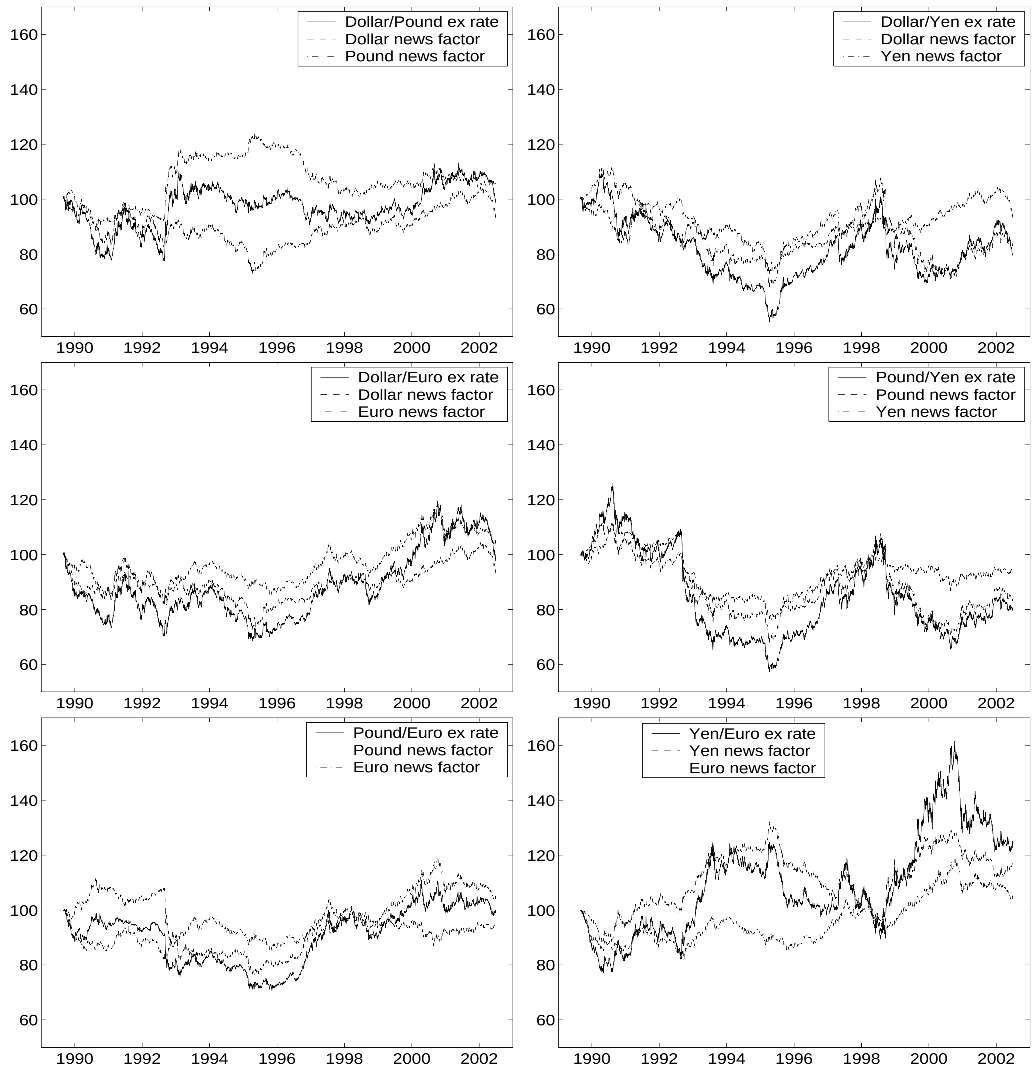

Figure 7: These graphs show the indexed exchange rate together with the accompanying news indices. Estimation of model (5)-(7) takes place over the period September the 1st, 1989 until July the 22nd, 2002 (3351 observations).

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