AVERAGE COSTS VERSUS NET PRESENT VALUE: A COMPARISON FOR MULTI-SOURCE INVENTORY MODELS

ERWIN VAN DER LAAN AND RUUD TEUNTER

Bibliographic data and classifications of all the ERIM reports are also available on the ERIM website: www.erim.eur.nl

ERASMUS **R**ESEARCH **I**NSTITUTE OF **M**ANAGEMENT

REPORT SERIES *RESEARCH IN MANAGEMENT*

Average Costs versus Net Present Value: a comparison for multi-source inventory models

Erwin van der Laan

Rotterdam School of Management, Erasmus University Rotterdam,P.O. Box 1738, NL-3000 DR Rotterdam, The Netherlands

Rund Tennter

Faculty of Economics and Management, Otto-von-Guericke University Magdeburg,PO Box 4120, 39016 D-Magdeburg, Germany

Abstract

While the net present value (NPV) approach is widely accepted as the right framework for studying production and inventory control systems, average cost (AC) models are more widely used. For the well known EOQ model it can be verified that (under certain conditions) the AC approach gives near optimal results, but does this also hold for more complex systems? In this paper it is argued that for more complex systems, like multi-source systems, one has to be extremely careful in applying the AC approach on intuition alone, even when these systems are deterministic. Special attention is given to a two-source inventory system with manufacturing, remanufacturing, and disposal, and it is shown that for this type of models there is a considerable gap between the AC approach and the NPV aprroach.

Keywords: Net present value, average costs, inventory control, manufacturing, remanufacturing, disposal, holding costs.

1 Introduction

Several authors (e.g. Hadley, 1964; Trippi, 1974; Thompson, 1975; Hofmann, 1998; Klein Haneveld and Teunter, 1998) have argued that for the EOQ model the average cost (AC) framework as an approximation to the superior net present value (NPV) framework leads to near optimal results under the following conditions:

- Products are not moving too slow,
- Interest rates are not too high,
- The customer payment structure does not depend on the inventory policy.

The first two conditions have to guarantee that compounded interest does not effect the results too much. That the latter condition is crucial was first put forward by Beranek (1966), who's concern was confirmed later by Grubbström (1980) and Kim *et al.* (1984) .

The main objections against the average cost approach, as it is usually applied as an approximation to the net present value approach, are threefold:

O1 The time value of money is not explicitly taken into account,

O2 There is no distinction between out-of-pocket holding costs and opportunity costs due to inventory investment, while other sources of opportunity costs/yields (fixed ordering costs, product sales) are not taken into account at all.

O3 Initial conditions are not taken into account

Yet, the net present value approach is often rather complicated, so an approximation may still be preferred.

Several authors have tried to deal with the above problems by showing that a certain transformation of the holding cost parameters in EOQ-type models gives near optimal results from an NPV perspective. This, however, shifts the problem to finding the right transformation. Up to now only ad hoc solutions have been given that are often very counter-intuitive (see e.g. Beranek, 1966; Corbey et al., 1999). No general principle has been developed to solve the transformation problem.

This paper intends to systematically analyze the differences between the AC and NPV approach and its consequences for modeling inventory systems. To that end we will analyze a number of deterministic models with increasing complexity, starting with the standard EOQ model and moving towards multi-echelon and multi-source models. It is shown that there are basically two classes of systems: 1. systems for which a transformation of model parameters exists that is independent of decision variables, such that the AC approach and the NPV approach are approximately equal, and 2. systems for which such a transformation does not exist.

This paper is further organized as follows: In the next section we propose a general principle that allows us to handle the NPV approach for deterministic systems in a very simple way. Moreover, with this principle we can easily compare the AC approach with the NPV approach. We then show how the NPV approach compares to the AC approach for the EOQ model (Section 3), multi-echelon systems (Section 4), and multi-source systems (Section 5). The theoretical results are further illustrated by a small numerical study in Section 6. We end with a summary and discussion of the main results in Section 7.

2 A general principle for the NPV approach in deterministic models

We define the *Net Present Value* (NPV) as the total discounted cash-flow over an infinite horizon. Additional to the NPV we define the Annuity Stream (AS) as

 $AS = r\{NPV\}.$

where r denotes the discount rate. The annuity stream is the transformation of a set of discrete and/or continuous cash flows to one continuous stream of cash-flows, such that the latter has the same net present value as the original set of cash-flows. The notion of an annuity stream is useful, since it can be directly compared with average costs.

Figure 2.1 Comparison between the average cash-flow per time unit (C/T) , the anuity stream $\overline{(AS)}$, and its linearisation (AS) for $T = 4$, $C = 1$, and $r = 0.2$.

If T denotes the cycling time of a discrete cash-flow C, with first occurrence time T_1 , then the annuity stream is given by

$$
AS = rC \sum_{n=0}^{\infty} e^{-r(T_1 + nT)} = \frac{rCe^{-rT_1}}{1 - e^{-rT}},
$$
\n(1)

This can be written as the McLaurin expansion in r

$$
AS = \frac{rCe^{-rT_1}}{1 - e^{-rT}} = \frac{C}{T} + C\left[r\left(\frac{1}{2} - \frac{T_1}{T}\right) + O(r^2 \max\{T, T_1\})\right],
$$

so that we have the following linearisation in r of the annuity stream:

$$
\overline{AS} = \frac{C}{T} + rC\left(\frac{1}{2} - \frac{T_1}{T}\right). \tag{2}
$$

Note that in most practical applications r is small and $0 \leq T_1 \leq T$, so that the above approximation is quite reasonable.

The first term of (2), C/T , denotes the average cash-flow per time unit, as it would follow from a standard AC calculation. The second term may be viewed as a first order correction term to account for the time value of money. This is graphically shown in Figure 2.1. Approximately, the AC approach underestimates the interest component of the annuity stream if $T_1 \leq T/2$ and overestimates otherwise. The results of both approaches are the same if $T_1 \approx T/2$.

The above only holds for discrete cash-flows, but we can do a similar analysis for continuous cash-flows. Suppose a continuous cash-flow p with rate λ that starts at time T_1 , then the annuity stream is given by

$$
AS = rp\lambda \int_{T_1}^{\infty} e^{-rt} dt
$$

= $rp\lambda e^{-rT_1}$
= $p\lambda [1 - rT_1 + O(r^2T_1^2)]$

Figure 3.1 Relevant cash-flows for the EOQ model with continuous demand.

$$
\approx p\lambda[1-rT_1]. \tag{3}
$$

The way that the AC approach usually deals with the *underestimation* of the interest component for cash-flows related to variable production costs is to add a certain factor to the out-of-pocket holding cost parameter. This factor is usually taken as the interest rate r times the 'value' of the stocked item. This approach has a number of disadvantages. First, it assumes that the overestimation is proportional with average inventory. We will show that this does not need to be the case. In fact, size and timing of cash-flows are dependent on cycle times rather than the existence of physical stocks. Second, it only deals with underestimation of the interest component and not with overestimation, since the value of a stocked item is usually taken to be positive. Third, this approach only considers the interest components of variable production costs, while interest components of all other cash flows (fixed costs, sales, etc.) are not taken into account. Finally, it is unclear what is meant by the 'value' of a stocked item, since this depends on the type of decision that has to be made.¹

3 From NPV to AC with the EOQ model

First consider the basic EOQ model in an NPV framework (Figure 3.1). Demand for a product with selling price p is continuous with rate λ , generating a continuous cash inflow of λp per time unit. Every T periods a batch of Q products is produced against variable cost c per product and fixed cost K per batch (zero lead time) starting at time $t = 0$. To keep the analysis simple and transparent we will not consider out-of-pocket holding costs. Note that in the AC framework holding costs appear as an approximation to the annuity stream to account for interest components. We will refer to this holding cost parameter as the `opportunity cost rate of inventory investment'.

The total annuity stream for this deterministic system consists of the annuity stream due to a)

 1 Depending on the type of decision that has to be made one could say that a product return has value zero if it has been obtained for free. At the same time one could say it has value $c_p - c_r$, since after remanufacturing against cost c_r it can be sold for c_p . As a third option one could say that its value is $c_m - c_r$, since this is the difference between manufacturing against cost c_m and remanufacturing against cost c_r .

the variable revenues and production costs (AS_v)

$$
AS_v = r \left(p \lambda \int_0^\infty e^{-rt} dt - cQ \sum_{n=0}^\infty e^{-rn} \right)
$$

= $p \lambda - \frac{rcQ}{1 - e^{-rT}}$
 $\approx (p - c)\lambda - rcQ/2,$ (4)

and b) the annuity stream due to fixed set-up costs (AS_f)

$$
AS_f = -r \sum_{n=0}^{\infty} K e^{-rnT}
$$

=
$$
-\frac{rK}{1 - e^{-rT}}
$$

$$
\approx -K\lambda/Q - rK/2,
$$
 (5)

where we have used linearizations (2) and (3) with $T_1 = 0$. Combining (4)–(5) we arrive at the approximated total annuity stream function

$$
\overline{AS} = (p - c)\lambda - K\lambda/Q - rcQ/2 - rK/2 \tag{6}
$$

The first term in (6) denotes marginal net profits per time unit, and the second term denotes the average set-up costs per time unit. The other terms are interest components.

The standard AC approach calculates the average profit (AP) function as

$$
AP = (p - c)\lambda - hQ/2 - K\lambda/Q,\tag{7}
$$

where h is the holding cost rate to account for the opportunity costs of inventory investment. Optimizing (7) leads to the well-known EOQ formula, but it is not immediately clear what the value of h should be. However, if we want that optimizing AP gives the same order size as optimizing \overline{AS} we should choose $h = rc$. Although this value will appeal to most people's intuition it is important to note that more complicated models, as the ones encountered in the remainder of the paper, call for more complicated holding cost rates for which an intuitive explanation is often hard to give.

4 Multi-echelon systems

Consider a two-echelon system consisting of processes i, $i \in \{1,2\}$, with lead time L_i , and processing cost c_i (Figure 4.1). Here, S is a stocking point for serviceable inventory. A production batch of size Q is initiated every T time units starting at time $T_1 = 0$. As soon as process 1 finishes process 2 starts. As soon as process 2 finishes, the batch enters serviceable inventory (Figure 4.2). Note that production costs are incurred at the beginning of each process and that product sales only start after the first production batch has entered the serviceable inventory, i.e., at time $L_1 + L_2$.

Figure 4.1 Relevant cash-flows for the two-echelon system.

Figure 4.2 The inventory processes of a two-echelon system.

A formal deduction of AS_v gives

$$
AS_v = r \left(\frac{p \lambda e^{-r(L_1 + L_2)}}{r} - \frac{Qc_1 + Qc_2 e^{-rL_1}}{1 - e^{-rT}} \right)
$$

$$
\approx (p - c_1 - c_2) \lambda - r ((c_1 + c_2)Q/2 - c_2 L_1 \lambda) - rp(L_1 + L_2) \lambda.
$$
 (8)

The first term in (8) is just the marginal net profits per time unit, whereas the second term denotes the opportunity costs of inventory investment. The last term represents the opportunity costs of delayed product sales.

The traditional average cost approach would calculate the average profit function as the average net marginal prots per time unit minus the average holding costs per time unit,

$$
AP_v = (p - c_1 - c_2)\lambda - h_1 L_1 \lambda - h_2 L_2 \lambda - h_s Q/2, \tag{9}
$$

where the second term is the average work in process inventory of process 1 charged with opportunity holding cost rate h_1 , the third term is the average work in process inventory of process 2, charged with rate h_2 , and the fourth term is the average serviceable inventory charged with rate h_s . Equation (9) corresponds to (8) if we employ the following transformation of cost parameters:

$$
h_1 \rightarrow r(p - c_2)
$$

\n
$$
h_2 \rightarrow rp
$$

\n
$$
h_s \rightarrow r(c_1 + c_2)
$$

The parameter h_s can be interpreted intuitively as the interest rate times the total marginal production costs. The other holding cost rates are less intuitive, but that is not really a problem since for any value of these parameters the difference between NPV and AC will merely be a constant.

The annuity stream due to fixed set-up costs is

$$
AS_f = -r\left(\frac{K_1 + K_2 e^{-rL_1}}{1 - e^{-rT}}\right)
$$

$$
\approx -\frac{(K_1 + K_2)\lambda}{Q} - r\left(\frac{K_1 + K_2}{2} - \frac{K_2 L_1 \lambda}{Q}\right).
$$

In the traditional average cost approach opportunity costs of set-ups are never explicitly taken into account (compare to the EOQ model, where opportunity costs of set-ups are a constant and can be left out). Here, however, we see that the opportunity costs do depend on the order size Q and can no longer be discarded. Again, we can map (up to a constant) the average cost approach to the linearization of the annuity stream by the transformation

$$
K_1 \rightarrow K_1
$$

$$
K_2 \rightarrow K_2(1 - rL_1)
$$

Figure 5.1 Schematic representation of a manufacturing/remanufacturing system.

Summarizing, we can say that the traditional average cost approach is still applicable for multiechelon structures, as long as the right transformations of model parameters are used. This however results in a paradoxical situation: Using the average cost approach in order to avoid an NPV analysis, requires an NPV analysis to find the correct transformations. It is comforting though that for this class of models the traditional average cost models can still be applied.

5 Multi-source systems

Until now we only considered situations in which inventories consist of products that all have generated the same cash-flows. Additional problems may arise if inventories consist of products that have been produced in different ways against different costs. This is the case with products that can be both newly manufactured and remanufactured from old products. Remanufactured products have the same functionality and quality as newly produced products and can therefore be sold at the same market for the same price. In this sense they are indistinguishable and can be put in the same inventory. However, the cash flows generated by manufactured products are different from remanufactured products, since they follow from different processes with different costs. In this section we show how this affects the difference between NPV and AC.

5.1 A system with manufacturing and remanufacturing

Consider a two source system (Figure 5.1), where product demand can be fulfilled both by manufactured products, with marginal cost c_m and fixed set-up cost K_m , and remanufactured products, with marginal cost c_r and fixed set-up cost K_r . Manufactured and remanufactured products have the same quality standards and are sold on the same market against the same price p. The main difference between the manufacturing process and remanufacturing process is that the latter depends on the flow of product returns, which for now is assumed to be deterministic with rate γ , $0 < \gamma < \lambda$.

This system was first proposed by Schrady (1967) and further analyzed by Richter (1996) and Teunter (1998). In the above-mentioned papers the system is controlled by subsequently producing N manufacturing batches and M remanufacturing batches. For ease of explanation we assume here that $N = M = 1$ so that the system is controlled by repeatedly producing one manufacturing batch of size Q_m , succeeded by one remanufacturing batch of size Q_r .² We as-

 2 The following analysis is easily extended to arbitrary N and M , but this would only lengthen the mathematical

Figure 5.2 Relevant cash-flows for the two-source system without disposal.

Figure 5.3 The inventory processes of a two-source system.

sume that at time 0 we start with zero inventory of both serviceables and remanufacturables. Thus, to start up the system and to guarantee a monotonous ordering strategy at the same time, we have to start with a manufacturing batch of size Q_r . The first manufacturing batch of size Q_m then occurs at time $T_r = Q_r/\lambda$ and the first remanufacturing batch occurs at time $T = (Q_m + Q_r)/\lambda$. Continuing this way, manufacturing batches and remanufacturing batches occur every T time units. Leadtimes are assumed to be zero. Note that, since all returns are used for remanufacturing, we have $Q_r = \gamma T$, $Q_m = (\lambda - \gamma)T$ and $Q_r = \frac{1}{\lambda - \gamma}Q_m$. The timing of all relevant cash-flows is visualized in Figure 5.2.

The AS_v for this system reads

$$
AS_v = p\lambda - r \left(Q_r c_m + \frac{Q_m c_m e^{-rT_r} + Q_r c_r e^{-rT}}{1 - e^{-rT}} \right)
$$

$$
\approx (p\lambda - c_m(\lambda - \gamma) - c_r \gamma) - r c_m Q_m \left(\frac{1}{2} - \frac{\gamma}{\lambda} \right) - r (c_m - c_r/2) Q_r.
$$
 (10)

Again, the first term denotes the total marginal profits and the last two terms denote the total opportunity costs of inventory investment.

Let's now compare the above expression with the corresponding average profit function. Figure 5.3 depicts the inventory process of serviceables and remanufacturables, from which we derive

expressions without gaining additional insight.

that the long run average inventory of remanufactured products equals $Q_rT_r/(2T) = (\gamma/\lambda)Q_r/2$, the long run average inventory of manufactured products equals $Q_m(T - T_r)/(2T) = (1 \gamma/\lambda$) $Q_m/2$, and the average inventory of remanufacturable products equals $Q_r/2$. This leads to the following average prot function:

$$
AP_v = (p\lambda - c_m(\lambda - \gamma) - c_r\gamma) - h_m(1 - \frac{\gamma}{\lambda})Q_m/2 - h_r(\frac{\gamma}{\lambda})Q_r/2 - h_nQ_r/2.
$$
 (11)

Clearly, both opportunity costs and average inventory are linear in Q_m and Q_r so that h_m , h_r , and h_n can be chosen such that (11) is equivalent to (10). Since the average inventory of remanufactured products and the average inventory of remanufacturables are both linear in Q_r , either h_r or h_n is redundant. Naturally we choose $h_n = 0$ because there are no investments in remanufacturable inventory and thus no associated opportunity cost exist. This gives

$$
h_m \rightarrow r c_m \left(\frac{\lambda}{\lambda - \gamma}\right)
$$

\n
$$
h_r \rightarrow r c_r \left(2 - \frac{\lambda}{\gamma}\right)
$$

\n
$$
h_n \rightarrow 0.
$$

So, setting $h_n = 0$ leads to different holding cost rates for manufactured and remanufactured products. This is rather counter-intuitive and may lead to the (false) conclusion that in fullling product demands priority should be given to either manufactured or remanufactured products, whichever generates more opportunity costs. That this conclusion is false can be clearly seen when we look at it from an NPV perspective. The financial consequences of selling either a manufactured item or a remanufactured item are exactly the same, since they generate the same cash inflow at the same time.

These counter-intuitive results can be avoided by choosing the same holding cost rates for manufactured and remanufactured products. This gives

$$
h_m \rightarrow r c_m
$$

\n
$$
h_r \rightarrow r c_m
$$

\n
$$
h_n \rightarrow r (c_m - c_r).
$$

What about the set-up costs? The AS_f is derived as

$$
AS_f = -r\left(K_m + \frac{K_m e^{-rT_r} + K_r e^{-rT}}{1 - e^{-rT}}\right)
$$

$$
\approx -(\lambda - \gamma)K_m/Q_m - \gamma K_r/Q_r - rK_m\left(\frac{3}{2} - \frac{\gamma}{\lambda}\right) + rK_r/2,
$$
 (12)

and we observe that the opportunity costs of set-ups do not depend on the policy parameters. This however will change in the next section.

Our framework shows that the only way to in
uence the opportunity costs of holding inventories is to somehow change the *timing* of the investments c_m and c_r , for instance by using pull and push type policies (see van der Laan et al., 1998), or to somehow change the fraction of (re)manufactured products by using a disposal policy (see e.g. Inderfurth, 1997; van der Laan and Salomon, 1997). In the next paragraph we extend the manufacturing/remanufacturing model with the option to dispose product returns.

5.2 A system with manufacturing, remanufacturing, and disposal

A number of authors have considered disposal strategies in a manufacturing/remanufacturing environment in order to optimize total system costs (e.g. Heyman, 1977; Inderfurth, 1997; Richter,1996; Simpson, 1978; van der Laan, 1997). However, doing so, care should be taken in the modeling process. Including the disposal option enables to in
uence the throughput of the manufacturing and remanufacturing process. From Section 3.1 we have learned that when the throughput of the system depends on policy parameters, the traditional average costs approach may not be appropriate.

Consider the example of Section 5.1, but instead of remanufacturing allproduct returns we decide to use only a fraction $U, 0 \leq U \leq 1$, and continuously dispose a fraction $1-U$. The unit 'cost' related to disposal, c_d can be positive (for instance if products contain hazardous materials, which need to be processed in an environmental friendly manner), or negative (for instance if product returns have a positive salvage value and can be sold to a third party). Define the decision variable $\Gamma = U\gamma$, then the total amount of disposals during a production cycle of length T equals $(\gamma - \Gamma)T$.

The AS_v for the situation with continuous disposals is

$$
AS_v = p\lambda - (\gamma - \Gamma)c_d - r\left(Q_r c_m + \frac{Q_m c_m e^{-rT_r} + Q_r c_r e^{-rT}}{1 - e^{-rT}}\right)
$$

\n
$$
\approx (p\lambda - c_m(\lambda - \Gamma) - c_r \Gamma - c_d(\gamma - \Gamma))
$$

\n
$$
-r c_m Q_m \left(\frac{1}{2} - \frac{\Gamma}{\lambda}\right) - r\left(c_m - c_r/2\right) Q_r.
$$

The parameter c_d only appears in the marginal cost term so there are no opportunity costs associated with product disposal.

If $c_d > 0$, it is more efficient from a financial point of view to dispose as late as possible. One could choose to dispose a batch of remanufacturables whenever a certain capacity limit has been reached. Here, we choose to accumulate the products to be disposed and dispose them all at once whenever a remanufacturing batch is initiated, i.e. at time T_m , $T + T_m$, and so on. The amount disposed at the end of each cycle equals $(\gamma - \Gamma)T$. The AS_v for batch-disposals thus reads

$$
AS_v = p\lambda - r \left(Q_r c_m + \frac{Q_m c_m e^{-rT_r} + (Q_r c_r + (\gamma - \Gamma)T c_d)e^{-rT}}{1 - e^{-rT}} \right)
$$

\n
$$
\approx (p\lambda - c_m(\lambda - \Gamma) - c_r \Gamma - c_d(\gamma - \Gamma))
$$

\n
$$
-r c_m Q_m \left(\frac{1}{2} - \frac{\Gamma}{\lambda}\right) - r \left(c_m - c_r/2\right) Q_r + r c_d (Q_m + Q_r) \left(\frac{\gamma - \Gamma}{2\lambda}\right) .
$$
\n(13)

Note the opportunity cost/yield related to disposal, $rc_d(Q_m+Q_r)\left(\frac{\gamma-\Gamma}{2\lambda}\right)$.

The annuity stream due to (re)manufacturing set-ups is derived as

$$
AS_f = -r\left(K_m + \frac{K_m e^{-rT_r} + K_r e^{-rT}}{1 - e^{-rT}}\right)
$$

$$
\approx -(\lambda - \Gamma)K_m/Q_m - \Gamma K_r/Q_r - rK_m\left(\frac{3}{2} - \frac{\Gamma}{\lambda}\right) + rK_r/2.
$$
 (14)

We observe that the opportunity costs of set-ups depend on the policy parameter Γ , and can no longer be ignored (Compare with (12)).

Combining (13) and (14) we find that for $0 < \Gamma < \lambda$ the total annuity stream is given by

$$
AS = p\lambda - r\left(K_m + Q_r c_m + \frac{(K_m + Q_m c_m)e^{-rT_r} + (K_r + Q_r c_r + (\gamma - \Gamma)T c_d)e^{-rT}}{1 - e^{-rT}}\right), \quad (15)
$$

which can be approximated by the function

$$
\overline{AS} = p\lambda - c_m(\lambda - \Gamma) - c_r \Gamma - c_d(\gamma - \Gamma)
$$

$$
-(\lambda - \Gamma)K_m/Q_m - \Gamma K_r/Q_r
$$

$$
-rK_m - r(K_m + c_m Q_m) \left(\frac{1}{2} - \frac{\Gamma}{\lambda}\right) + rK_r/2 - r(c_m - c_r/2)Q_r
$$

$$
+ rc_d(Q_m + Q_r) \left(\frac{\gamma - \Gamma}{2\lambda}\right).
$$
\n(16)

The traditional AC approach calculates the total average prot function as the total marginal profits, set-up costs, and inventory costs as

$$
AP = p\lambda - c_m(\lambda - \Gamma) - c_r\Gamma - c_d(\gamma - \Gamma)
$$

-(\lambda - \Gamma)K_m/Q_m - \Gamma K_r/Q_r
-h_m(1 - \frac{\Gamma}{\lambda})Q_m/2 - h_r(\frac{\Gamma}{\lambda})Q_r/2 - h_n(\frac{\gamma}{\lambda}) (Q_m + Q_r)/2. (17)

Using the relation $Q_r = \frac{1}{\lambda - \Gamma} Q_m$ it is easily verified that we can transform AP into AS (up to a constant), by using the following transformations of c_r , h_m , h_r , and h_n :

$$
c_m \rightarrow c_m + rK_m/\lambda
$$

\n
$$
h_m \rightarrow rc_m
$$

\n
$$
h_r \rightarrow rc_m
$$

\n
$$
h_n \rightarrow r(c_m - c_r - (1 - \Gamma/\gamma)c_d)
$$
\n(18)

Clearly, this is a non-linear transformation in the decision variable Γ , which indicates the considerable gap between the traditional average cost approach and the linearization of the annuity stream. This is further illustrated by some analytical and numerical results in the next section.

6 Analytical and numerical comparison of alternative transformations

Consider the inventory system with manufacturing, remanufacturing, and batch-disposal of Section 5.2. In this section we investigate how the average cost approach performs with respect to the linearization of the annuity stream approach, when forcing a linear transformation of the cost parameters that does not depend on decision variables.

Since demand is either fullled by manufacturing or remanufacturing and the number of (re) manufacturing batches within production cycle T is fixed to one, we have $Q_r = \frac{1}{\lambda - \Gamma} Q_m$. Hence, for $0 < \Gamma \leq \gamma < \lambda$ expressions (15) – (17) can be transformed into functions of Q_m and Γ only. For the special case $\Gamma = 0$ similar expressions are derived in the appendix. For all the numerical examples in this section we use the base-case scenario of Table 1, unless specied otherwise.

parameter	λ	γ	p	c_m	c_r	c_d	K_m	K_r	r
value	20	10	20	10	5	5	10	10	0.10

Table 1 Base case scenario

As a performance measure for batch size Q we define the relative difference

$$
R(Q) = \left[1 - \frac{\tilde{AS}(Q)}{\tilde{AS}(Q^{\rm AS})}\right] \times 100\%,
$$

where $A\beta(.) = A\beta(.) - (p\lambda - c_m(\lambda - \gamma) - c_r\gamma)$ is the relevant annuity stream and Q^{++} is the batch-size that maximizes $AS(.)$.

In our analysis we consider two transformations.

Transformation A

An intuitive, though rather naive, transformation is the following:

$$
h_m \rightarrow r c_m
$$

\n
$$
h_r \rightarrow r c_r
$$

\n
$$
h_n \rightarrow 0
$$

The above choice follows from the (false) intuition that opportunity costs of inventory investment are (approximately) equal to the interest rate times the average inventory investment. Parameter c_r is chosen according to (18)) to take the opportunity cost of remanufacturing batches into account:

$$
c_m \to \begin{cases} c_m + rK_m/\lambda, & \text{if } \Gamma > 0\\ c_m, & \text{otherwise} \end{cases}
$$
 (19)

Transformation B

A seemingly more sophisticated transformation of h_m , h_r , and h_n was proposed by Inderfurth and Teunter (1998) on the basis of a heuristic argument: "The money tied up in a non-serviceable item is $-cd$, since that could have been 'earned' by disposing of it. Hence, $h_n = r(-c_d)$ (...). The money tied up in a remanufactured item is that tied up in a non-serviceable item plus the cost c_r of remanufacturing the item. Hence, $h_r = r(c_r - c_d)$ (...). The money tied up in a manufactured item is simply the cost c_m of manufacturing an item. Hence, $h_m = rc_m$." Summarizing:

$$
h_m \rightarrow rc_m
$$

\n
$$
h_r \rightarrow r(c_r - c_d)
$$

\n
$$
h_n \rightarrow -rc_d
$$

Parameter c_r is chosen according to (19) to take the opportunity cost of remanufacturing batches into account. Note that for $c_d = 0$ Transformation A and Transformation B are equivalent.

To compare the various approaches, we consider two cases.

Case 1: $\Gamma = 0$

If $\Gamma = 0$ the difference between \overline{AS} and AP is given by

$$
\overline{AS} - AP = \left[h_m - rc_m + (h_n + rc_d) \left(\frac{\gamma}{\lambda} \right) \right] Q_m/2 - rK_m/2 \tag{20}
$$

where the last term is just a constant but the first term depends on Q_m . If Transformation B is applied the first term vanishes, hence the two approaches are equal up to a constant. For Transformation A however the righthand side of (20) equals

$$
r c_d \left(\frac{\gamma}{\lambda}\right) Q_m/2 - r K_m/2
$$

Thus under *Transformation* A the two approaches will differ significantly for large enough $|c_d|$ (see Table 2).

					Transform. A		Transform. B	
c_d	$Q_m^{\rm AS}$	$\tilde{AS}(Q_m^{\textrm{AS}})$	$Q_m^{\overline{\rm AS}}$		$Q_m^{\rm AP}$	QAP	$Q_m^{\rm AP}$	
-15	15.1	73.01	15.1	0.0	20.0	1.5	15.1	0.0
-10	16.3	24.94	16.3	0.0	20.0	2.1	16.3	0.0
-5	17.7	-22.97	17.9	0.0	20.0	-0.7	17.9	0.0
$\boldsymbol{0}$	19.7	-70.67	20.0	0.0	20.0	0.0	20.0	0.0
$\overline{5}$	22.4	-118.10	23.1	0.0	20.0	-0.1	23.1	0.0
10	26.6	-165.12	28.3	0.0	20.0	-0.4	28.3	0.0
15	33.8	-211.49	40.0	-0.1	20.0	-0.9	40.0	-0.1

Table 2 Performance of Q_m and Q_m under the base-case scenario for $I = 0$ and various values of c_d .

Case 2: $\Gamma = \gamma$

If $\Gamma = \gamma$ the difference between \overline{AS} and AP is given by

$$
\overline{AS} - AP = [h_m (1 - \frac{\gamma}{\lambda}) - rc_m (\frac{1}{2} - \frac{\gamma}{\lambda})] Q_m/2
$$

$$
+\left[h_r\left(\frac{\gamma}{\lambda}\right)-r(2c_m-c_r)+h_n\right]Q_r/2-rK_m\left(\frac{3}{2}-\frac{\gamma}{\lambda}\right)+rK_r/2.
$$
\n(21)

Under Transformation A the righthand side of (21) reduces to

$$
r(c_r - c_m) \left(\frac{\gamma}{\lambda}\right) \left(\frac{\lambda + \gamma}{\lambda - \gamma}\right) Q_m/2 - rK_m \left(\frac{3}{2} - \frac{\gamma}{\lambda}\right) + rK_r/2.
$$

The difference will be significant for large enough $|c_m - c_r|$ and/or γ (see Table 3 and 4).

					Transform. A		Transform. B	
c_r	$Q_m^{\rm AS}$	$\tilde{AS}(Q_m^{\rm AS})$	$Q_m^{\overline{\rm AS}}$	$R(Q_m^{\overline{\rm AS}})$	$Q_m^{\rm AP}$	$R(Q_m^{\rm AP})$	$Q_m^{\rm AP}$	$R(Q_m^{\rm AP})$
θ	14.2	-28.71 14.1		0.0	28.3	-23.8	∞^{α}	$-\infty$
5°	16.3	-25.00	16.3	0.0	23.1	-6.0	∞^{α}	$-\infty$
10	19.7	-20.67	20.0	0.0	20.0	0.0	40.0	-26.7
15	26.1	-15.27	28.3	-0.3	17.9	-7.5	28.3	-0.3
20	43.1	$-7.47 \mid \infty^{\alpha}$		$-\infty$	16.3	-75.5	23.1	-31.6

Table 3 Performance of Q_m and Q_m under the base-case scenario for $I = \gamma$ and various values of c_r . a) The objective function is an increasing function in Q_m .

					Transform. A		Transform. B		
	$Q_m^{\rm AS}$	$\tilde{AS}(Q_m^{\rm AS})$	$Q_m^{\overline{\rm AS}}$	$R(Q_m^{\overline{\rm AS}})$	$Q_m^{\rm AP}$	$\langle Q_m^{\rm AP} \rangle$ R($Q_m^{\rm AP}$	$R(Q_m^{\rm AP})$	
	19.7	-20.67	20.0	$0.0\,$	20.0	0.0	20.0	0.0	
5	24.5	-25.27	24.5	$0.0\,$	27.5	-0.6	32.1	-3.6	
10	16.3	-25.00	16.3	$0.0\,$	23.1	-6.0	∞^{α}	$-\infty$	
15	7.0	-28.66	7.1	0.0	12.1	-15.4	∞^{α}	$-\infty$	
19	1.2	-33.42	1.2	$0.0\,$	2.1	-16.6	∞^{α}	$-\infty$	

Table 4 Performance of Q_m^{max} and Q_m^{max} under the base-case scenario for $1 = \gamma$ and various values of γ . \sim 1 ne objective function is an increasing function in $Q_m.$

Under Transformation B the righthand side of (21) reduces to

$$
r(c_r - c_m - c_d) \left(\frac{\gamma}{\lambda}\right) \left(\frac{\lambda + \gamma}{\lambda - \gamma}\right) Q_m/2 - rK_m \left(\frac{3}{2} - \frac{\gamma}{\lambda}\right) + rK_r/2.
$$

The difference will be significant for large enough $|c_m + c_d - c_r|$, γ , and/or $|c_d|$ (see Table 3,4, and 5).

7 Discussion

Although the net present value approach is the more appropriate framework, average cost models are dominating the field of inventory control and production planning. In this paper we have shown that the traditional average cost approach, which does not make a distinction between opportunity costs of holding inventories and physical inventory costs, leads to reasonable results

					Transform. A		Transform. B	
c_d	$Q_m^{\rm AS}$	$\tilde{AS}(Q_m^{\rm AS})$	$Q_m^{\overline{\rm AS}}$	$\overline{O^{AS}}$	$Q_m^{\rm AP}$	$R(Q_m^{\text{AP}})$	$Q_m^{\rm AP}$	$R(Q_m^{\rm AP})$
-15	16.3	-25.00	16.3	0.0	23.1	-6.0	11.5	-6.1
-10	16.3	-25.00	16.3	0.0	23.1	-6.0	13.3	-2.1
-5	16.3	-25.00	16.3	0.0	23.1	-6.0	16.3	0.0
$\boldsymbol{0}$	16.3	-25.00	16.3	0.0	23.1	-6.0	23.1	-6.0
$\overline{5}$	16.3	-25.00	16.3	0.0	23.1	-6.0	∞^{α}	$-\infty$
10	16.3	-25.00	16.3	0.0	23.1	-6.0	∞^{α}	$-\infty$
15	16.3	-25.00	16.3	0.0	23.1	-6.0	∞^{α}	$-\infty$

Table 5 Performance of Q_m^{\dagger} and Q_m^{\dagger} under the base-case scenario for $1 = \gamma$ and various values of c_d . \sim 1 ne objective function is an increasing function in $Q_m.$

for single-source systems, but not necessarily for multi-source systems. The NPV approach does make a clear distinction between physical inventory costs and opportunity costs, since the two are not directly related. The latter does not depend on physical stocks at all, but only on the amount and timing of the investments.

The traditional approach only takes the opportunity costs of *holding inventories* into account, but this should not be a general rule. All cash-flows generate opportunity costs or yields that cannot be disregarded if the cash-flows depend on decision parameters. For example, in a manufacturing/remanufacturing system with disposal the throughput of the (re)manufacturing process is controlled by a decision variable. In that case also opportunity costs of set-ups and disposals should be taken into account. Clearly, these opportunity costs have got little to do with physical inventories.

Main conclusion of this paper is that basically there are two classes of models: a class for which a holding cost transformation exists that does not depend on decision variables, such that NPV coincides with AC (up to a constant), and a class for which such a transformation does not exist. A typical example of the latter class is a system with manufacturing, remanufacturing, and batch disposal.

References

W. Beranek (1966). Financial implications of lot-size inventory models. Management Science 13(8):401-408.

R.W. Grubbström (1980). A principle for determining the correct capital costs of work-inprogress and inventory. International Journal of Production Research 18(2):259-271.

G. Hadley (1964). A comparison of order quantities computed using the average annual cost and the discounted cost. Management Science 10(3):472-476.

D.P. Heymann (1977). Optimal disposal policies for a single-item inventory system with returns. Naval Research Logistics Quarterly 24:385-405.

C. Hofmann (1998). Investments in modern production technology and the cash flow-oriented EPQ model. International Journal of Production Economics 54:193-206.

K. Inderfurth (1997). Simple optimal replenishment and disposal policies for a product recovery system with leadtimes. OR Spektrum 19:111-122.

Inderfurth K. and R. Teunter (1998). The 'right' holding cost rates in average cost inventory models with reverse logistics", Preprint nr. 28/98, Fakultät für Wirtschaftswissenschaft, Otto von Guericke Universitat, Magdeburg, Germany. Magdeburg, Germany.

Y.H. Kim, K.H. Chung and W.R. Wood (1984) A net present value framework for inventory analysis. International Journal of Physical Distribution & Materials Management $14(6):68-76.$

W.K. Klein Haneveld and R.H. Teunter (1998). Effects of discounting and demand rate variability on the EOQ, International Journal of Production Economics, 54:173-192.

K. Richter (1996). The EOQ repair and waste disposal model with variable setup numbers. European Journal of Operational Research, 95:313-324.

D.A. Schrady (1967). A deterministic inventory model for repairable items. Naval Research $Logistics$ Quarterly, 14:391-398.

V.P. Simpson (1978). Optimum solution structure for a repairable inventory problem. Operations Research $26:270-281$.

R. Teunter (1998). Economic ordering quantities for remanufacturable item inventory systems. Preprint nr. 31/98, Fakultat fur Wirtschaftswissenschaft, Otto von Guericke Universitat, Magdeburg, Germany.

H. E. Thompson (1975). Inventory Management and capital budgeting: a pedagogical note, Decision Sciences 6:383-398.

R. R. Trippi and D. E. Lewin (1974). A present value formulation of the classical EOQ problem, Decision Sciences 5:30-35.

E. van der Laan and M. Salomon (1997). Production planning and inventory control with remanufacturing and disposal. European Journal of Operational Research 102:264-278.

E. van der Laan, M. Salomon, R. Dekker and L. Van wassenhove (1998). Inventory control in hybrid systems with remanufacturing, Management Science 45(5):733-747.

Acknowledgement: The research presented in this paper makes up part of the research on re-use in the context of the EU sponsored TMR project REVersed LOGistics (ERB 4061 PL 97-5650) in which take part the Otto-von-Guericke Universitaet Magdeburg (D), the Erasmus University Rotterdam (NL), Eindhoven University of Technology (NL), the Aristoteles University of Thessaloniki (GR), the University of Piraeus (GR), and INSEAD (F).

Part of the research has been done during the first author's stay at the department of Technology Management of INSEAD, Fontainebleau, France, for which INSEAD is greatly acknowledged. The first author also acknowledges the financial support provided by the Dutch Organization for Scientific Research, NWO.

Appendix

If $\Gamma = 0$ there are no cash-flows related to remanufacturing operations. Hence, expressions (15) $-$ (17) are given as

$$
AS = p\lambda - r \left(\frac{K_m + Q_m c_m + (\frac{\gamma}{\lambda}) Q_m c_d e^{-rQ_m/\lambda}}{1 - e^{-rQ_m/\lambda}} \right),
$$

$$
\overline{AS} = p\lambda - c_m \lambda - c_d \gamma - \lambda K_m / Q_m - r(K_m + c_m Q_m) / 2 + rc_d \left(\frac{\gamma}{\lambda} \right) Q_m / 2,
$$

and

$$
AP = p\lambda - c_m\lambda - c_d\gamma - \lambda K_m/Q_m - h_mQ_m/2 - h_n\left(\frac{\gamma}{\lambda}\right)Q_m/2.
$$

ERASMUS **R**ESEARCH **I**NSTITUTE OF **M**ANAGEMENT

REPORT SERIES *RESEARCH IN MANAGEMENT*

Publications in the Report Series Research^{*} in Management

Impact of the Employee Communication and Perceived External Prestige on Organizational Identification Ale Smidts, Cees B.M. van Riel & Ad Th.H. Pruyn ERS-2000-01-MKT

Critical Complexities, from marginal paradigms to learning networks Slawomir Magala ERS-2000-02-ORG

Forecasting Market Shares from Models for Sales Dennis Fok & Philip Hans Franses ERS-2000-03-MKT

A Greedy Heuristic for a Three-Level Multi-Period Single-Sourcing Problem H. Edwin Romeijn & Dolores Romero Morales ERS-2000-04-LIS

Integer Constraints for Train Series Connections Rob A. Zuidwijk & Leo G. Kroon ERS-2000-05-LIS

Competitive Exception Learning Using Fuzzy Frequency Distribution W-M. van den Bergh & J. van den Berg ERS-2000-06-LIS

Start-Up Capital: Differences Between Male and Female Entrepreneurs, 'Does Gender Matter?' Ingrid Verheul & Roy Thurik ERS-2000-07-STR

The Effect of Relational Constructs on Relationship Performance: Does Duration Matter? Peter C. Verhoef, Philip Hans Franses & Janny C. Hoekstra ERS-2000-08-MKT

Marketing Cooperatives and Financial Structure: a Transaction Costs Economics Analysis George W.J. Hendrikse & Cees P. Veerman ERS-2000-09-ORG

∗ ERIM Research Programs:

l

- LIS Business Processes, Logistics and Information Systems
- ORG Organizing for Performance
- MKT Decision Making in Marketing Management
- F&A Financial Decision Making and Accounting
- STR Strategic Renewal and the Dynamics of Firms, Networks and Industries

A Marketing Co-operative as a System of Attributes: A case study of VTN/The Greenery International BV, Jos Bijman, George Hendrikse & Cees Veerman ERS-2000-10-ORG

Evaluating Style Analysis Frans A. De Roon, Theo E. Nijman & Jenke R. Ter Horst ERS-2000-11-F&A

From Skews to a Skewed-t: Modelling option-implied returns by a skewed Student-t Cyriel de Jong & Ronald Huisman ERS-2000-12-F&A

Marketing Co-operatives: An Incomplete Contracting Perspective George W.J. Hendrikse & Cees P. Veerman ERS-2000-13– ORG

Models and Algorithms for Integration of Vehicle and Crew Scheduling Richard Freling, Dennis Huisman & Albert P.M. Wagelmans ERS-2000-14-LIS

Ownership Structure in Agrifood Chains: The Marketing Cooperative George W.J. Hendrikse & W.J.J. (Jos) Bijman ERS-2000-15-ORG

Managing Knowledge in a Distributed Decision Making Context: The Way Forward for Decision Support Systems Sajda Qureshi & Vlatka Hlupic ERS-2000-16-LIS

Organizational Change and Vested Interests George W.J. Hendrikse ERS-2000-17-ORG

Strategies, Uncertainty and Performance of Small Business Startups Marco van Gelderen, Michael Frese & Roy Thurik ERS-2000-18-STR

Creation of Managerial Capabilities through Managerial Knowledge Integration: a Competence-Based Perspective Frans A.J. van den Bosch & Raymond van Wijk ERS-2000-19-STR

Adaptiveness in Virtual Teams: Organisational Challenges and Research Direction Sajda Qureshi & Doug Vogel ERS-2000-20-LIS

Currency Hedging for International Stock Portfolios: A General Approach Frans A. de Roon, Theo E. Nijman & Bas J.M. Werker ERS-2000-21-F&A

Transition Processes towards Internal Networks: Differential Paces of Change and Effects on Knowledge Flows at Rabobank Group Raymond A. van Wijk & Frans A.J. van den Bosch ERS-2000-22-STR

Assessment of Sustainable Development: a Novel Approach using Fuzzy Set Theory A.M.G. Cornelissen, J. van den Berg, W.J. Koops, M. Grossman & H.M.J. Udo ERS-2000-23-LIS

Creating the N-Form Corporation as a Managerial Competence Raymond vanWijk & Frans A.J. van den Bosch ERS-2000-24-STR

Competition and Market Dynamics on the Russian Deposits Market Piet-Hein Admiraal & Martin A. Carree ERS-2000-25-STR

Interest and Hazard Rates of Russian Saving Banks Martin A. Carree ERS-2000-26-STR

The Evolution of the Russian Saving Bank Sector during the Transition Era Martin A. Carree ERS-2000-27-STR

Is Polder-Type Governance Good for You? Laissez-Faire Intervention, Wage Restraint, And Dutch Steel Hans Schenk ERS-2000-28-ORG

Foundations of a Theory of Social Forms László Pólos, Michael T. Hannan & Glenn R. Carroll ERS-2000-29-ORG

Reasoning with partial Knowledge László Pólos & Michael T. Hannan ERS-2000-30-ORG

Applying an Integrated Approach to Vehicle and Crew Scheduling in Practice Richard Freling, Dennis Huisman & Albert P.M. Wagelmans ERS-2000-31-LIS

Informants in Organizational Marketing Research: How Many, Who, and How to Aggregate Response? Gerrit H. van Bruggen, Gary L. Lilien & Manish Kacker ERS-2000-32-MKT

The Powerful Triangle of Marketing Data, Managerial Judgment, and Marketing Management Support Systems Gerrit H. van Bruggen, Ale Smidts & Berend Wierenga ERS-2000-33-MKT

The Strawberry Growth Underneath the Nettle: The Emergence of Entrepreneurs in China Barbara Krug & Lászlo Pólós ERS-2000-34-ORG

Consumer Perception and Evaluation of Waiting Time: A Field Experiment Gerrit Antonides, Peter C. Verhoef & Marcel van Aalst ERS-2000-35-MKT

Trading Virtual Legacies Slawomir Magala ERS-2000-36-ORG

Broker Positions in Task-Specific Knowledge Networks: Effects on Perceived Performance and Role Stressors in an Account Management System David Dekker, Frans Stokman & Philip Hans Franses ERS-2000-37-MKT

An NPV and AC analysis of a stochastic inventory system with joint manufacturing and remanufacturing Erwin van der Laan ERS-2000-38-LIS

Generalizing Refinement Operators to Learn Prenex Conjunctive Normal Forms Shan-Hwei Nienhuys-Cheng, Wim Van Laer, Jan N Ramon & Luc De Raedt ERS-2000-39-LIS

Classification and Target Group Selection bases upon Frequent Patterns Wim Pijls & Rob Potharst ERS-2000-40-LIS

New Entrants versus Incumbents in the Emerging On-Line Financial Services Complex Manuel Hensmans, Frans A.J. van den Bosch & Henk W. Volberda ERS-2000-41-STR

Modeling Unobserved Consideration Sets for Household Panel Data Erjen van Nierop, Richard Paap, Bart Bronnenberg, Philip Hans Franses & Michel Wedel ERS-2000-42-MKT

The Interdependence between Political and Economic Entrepeneurship ERS-2000-43-ORG Barbara Krug

Ties that bind: The Emergence of Entrepreneurs in China Barbara Krug ERS-2000-44-ORG

What's New about the New Economy? Sources of Growth in the Managed and Entrepreneurial Economies David B. Audretsch and A. Roy Thurik ERS-2000-45-STR

Human Resource Management and Performance: Lessons from the Netherlands Paul Boselie, Jaap Paauwe & Paul Jansen ERS-2000-46-ORG