Impulse-Response Analysis of the Market Share Attraction Model

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Abstract

We propose a simulation-based technique to calculate impulse-response functions and their confidence intervals in a market share attraction model [MCI]. As an MCI model implies a reduced form model for the logs of relative market shares, simulation techniques have to be used to obtain the impulse-responses for the levels of the market shares. We apply the technique to an MCI model for a five-brand detergent market. We illustrate how impulse-response functions can help to interpret the estimated model. In particular, the competitive and dynamic structure of the model can be analyzed.

Key words: Market shares; Forecasting; Attraction models; Impulse-Response analysis

*Address for correspondence: D. Fok, Erasmus University Rotterdam, Faculty of Economics, H16-12, P.O. Box 1738, NL-3000 DR Rotterdam, The Netherlands, `e-mail: dfok@few.eur.nl`. A computer program, which was used for all calculations in this paper, can be obtained from the corresponding author. We thank Richard Paap for helpful discussions.

1 Introduction

The market share attraction model (or, multiplicative competitive interaction [MCI] model) is often used to correlate market shares with explanatory variables such as price, promotion, distribution and past market shares, see, for example, Naert and Weverbergh (1981), Leeflang and Reuyl (1984), Brodie and Kluyver (1987), Cooper and Nakanishi (1988), Kumar (1994) and Bronnenberg *et al.* (2000). The model has the important feature that it implies that market shares sum to unity and that they lie within the (0,1) interval. To estimate the model parameters, the model is usually written in a reduced form specification. In this reduced form, the logs of the ratios of market shares to a benchmark market share are correlated with explanatory variables. Hence, the model for I market shares $M_1, \ldots, M_I$ implies a reduced form model for $\log(M_1/M_I)$ to $\log(M_{I-1}/M_I)$, where log denotes the natural logarithm and the market share of brand I is chosen as the benchmark. Several parameters in this reduced form are restricted across equations, while other parameters are functions of unknown parameters. Given all this, it is usually difficult to precisely understand how the model actually correlates the original market shares with explanatory variables, and particularly, how changes in explanatory variables and in innovations (that is, the residuals) affect future observations of the market shares themselves (instead of the log ratios).

In this paper we propose to use impulse-response analysis to help understand the structure of an MCI model. As the reduced form variables differ from the market shares, we should take account of the fact that (1) expected values of logs are not equal to the logs of expected values that is, $\mathrm{E}[\log(M_i/M_I)] \neq \log[\mathrm{E}(M_i/M_I)]$, where E denotes expectation, and (2) that the expected value of a ratio of variables is not equal to the ratio of the relevant expected values that is, $\mathrm{E}[M_i/M_I] \neq \mathrm{E}(M_i)/\mathrm{E}(M_I)$. Therefore, the impulse-response functions for the market shares cannot be obtained from a transformation of the impulse-response function of the reduced form variables and hence we have to rely on a simulation-based method to calculate impulse-response functions.

The outline of our paper is as follows. In Section 2, we briefly discuss representation and estimation issues of the MCI model in its general form. In Section 3, we discuss how one can calculate the impulse-response function and its confidence bounds given an empirically specified MCI model. In Section 4, we apply this to a set of 5 detergent brands, and we illustrate the effect of changes in price level, promotional activities and changes in innovations on future market shares. In Section 5, we conclude our paper with some remarks.

2 Specification and Estimation

The MCI model is usually defined in terms of attractions, typically of I brands of a certain product. The attraction of brand i, $i = 1, \ldots, I$ at time t, $t = 1, \ldots, T$ is defined as

$$A_{i,t} = \exp(\mu_i + \varepsilon_{i,t}) \prod_{j=1}^{I} \prod_{k=1}^{K} x_{k,j,t}^{\beta_{k,j,i}}, \tag{1}$$

where $\varepsilon_t \equiv (\varepsilon_{1,t}, \ldots, \varepsilon_{I,t})' \sim N(0, \Sigma)$ and where $x_{k,j,t}$ denotes the k-th explanatory variable (for example, price or advertising) for brand j at time t and $\beta_{k,j,i}$ is the corresponding coefficient for brand i. The parameter μ_i is a brand-specific constant. The error process (or innovation process) $\varepsilon_{i,t}$ is usually assumed to be only correlated across brands and not over time, that is, $\varepsilon_{i,t}$ is assumed independent of $\varepsilon_{j,t-1}, j = 1, \ldots, I$. Based on the attractions, the market share of brand i at time t is defined as

$$M_{i,t} = \frac{A_{i,t}}{\sum_{j=1}^{I} A_{j,t}}. \tag{2}$$

To capture potential lagged structures in (1), one can include lagged market shares in the specification of the attractions. The most general autoregressive structure follows from the inclusion of lagged market shares of all brands. In that case, when a P-th order autoregressive structure is used, the model becomes

$$A_{i,t} = \exp(\mu_i + \varepsilon_{i,t}) \prod_{j=1}^{I} \prod_{k=1}^{K} x_{k,j,t}^{\beta_{k,j,i}} \prod_{j=1}^{I} \prod_{p=1}^{P} M_{j,t-p}^{\alpha_{p,j,i}}. \tag{3}$$

The combination of (2) with (3) is often called the fully extended multiplicative competitive interaction model [FE-MCI], see also Cooper and Nakanishi (1988).

To estimate the parameters, the model is linearized first by choosing a base brand (say brand I) to which the other brands are related, and then by taking logs of the resulting ratios of variables. Hence, effectively one considers $\log(M_{i,t}/M_{I,t})$. This transformation results in $(I-1)$ equations which are linear in the parameters. These equations form an $(I-1)$-dimensional vector autoregressive model with explanatory variables, to be abbreviated as VARX(P). For the fully extended MCI model, this set of reduced form equations is

$$\begin{aligned}\log M_{i,t} - \log M_{I,t} = (\mu_i - \mu_I) + \sum_{j=1}^{I}\sum_{k=1}^{K}(\beta_{k,j,i} - \beta_{k,j,I}) \log x_{k,j,t} + \\ + \sum_{j=1}^{I}\sum_{p=1}^{P}(\alpha_{p,j,i} - \alpha_{p,j,I}) \log M_{j,t-p} + \varepsilon_{i,t} - \varepsilon_{I,t} \ ,\end{aligned} \tag{4}$$

with $i = 1, \ldots, I-1$.

Note from the expression in (4) that only the differences of parameters, for example $\mu_i - \mu_I$, and the elements in the covariance matrix of $\varepsilon_{i,t} - \varepsilon_{I,t}$ are identified. The notation of the reduced-form MCI model can be simplified by introducing $\mu_i^* = \mu_i - \mu_I$, $\beta_i^* = \beta_i - \beta_I$, $\alpha_i^* = \alpha_i - \alpha_I$ and $\eta_{i,t} = \varepsilon_{i,t} - \varepsilon_{I,t}$, resulting in

$$\log M_{i,t} - \log M_{I,t} = \mu_i^* + \sum_{j=1}^{I}\sum_{k=1}^{K} \beta_{k,j,i}^* \log x_{k,j,t} + \sum_{j=1}^{I}\sum_{p=1}^{P} \alpha_{p,j,i}^* \log M_{j,t-p} + \eta_{i,t}. \tag{5}$$

Given the distributional assumption on ε_t in (1), the $(I-1)$ vector process $(\eta_{i,t}, \ldots, \eta_{I-1,t})'$ is normally distributed with mean zero and covariance matrix $\Sigma^* = L\Sigma L'$, where $L = (\mathbf{I}_{I-1} \vdots - \mathbf{i})$ with $\mathbf{I}_{I-1}$ an $(I-1)$-dimensional identity matrix and $\mathbf{i}$ an $(I-1)$-dimensional unity vector. Note that this general setup imposes no restrictions on the covariance matrix Σ^*.

The parameters in the set of reduced form equations in (5) can be estimated using Generalized Least Squares [GLS]. Given these estimates one may decide to reduce the number of parameters by imposing various parameter restrictions. Some of the restrictions imply that the same parameters appear in all $I-1$ equations, and hence one should resort to GLS with cross-equation parameter restrictions, see Franses and Paap (1999) for further details. In our empirical illustration below, we will indicate how such restrictions can be analyzed.

The key motivation of the present paper lies in the fact that it may not be easy to understand how exactly the explanatory variables affect the levels of the market shares themselves. Indeed, their effect on the logs of the ratios of market shares may in some cases simply be inferred from the sizes and signs of the relevant parameters, but their effect on the market shares themselves is far from trivial. This is caused by the fact that (1) the expected values of a logged variable is not equal to the log of the expected value (and hence simply taking exponentials to get expected relative market shares is not correct) and (2) that the expected value of a ratio is not equal to the ratio of the expectations. So, expected values of the market shares themselves cannot be inferred from the expected values of the reduced form dependent variables. In the next section, we will motivate that it is not possible to calculate these expectations analytically. Therefore, we propose a simulation-based approach to impulse-response analysis to understand the structure of an MCI model. In impulse-response analysis one examines the effects of changes in innovations and changes in explanatory variables on future patterns of the variables to be explained.

3 Impulse-Response Functions

Impulse-response functions [IRF] are usually calculated to obtain insights in the dynamic structure of a model. An impulse-response function gives the expected time path of the dependent variable(s) that will result when a shock is added to a model in steady state. For example, Lütkepohl (1993) discusses impulse-response functions in a multiple time series setting, and it turns out that in that case the impulse-response functions can easily be obtained from the estimated model parameters.

For an MCI model it is slightly more difficult to calculate the IRF, in particular if one wants to calculate it for the market shares themselves, mainly because the expected market share trajectories cannot be directly obtained from the reduced form model. Again, the dependent variables in the reduced-form MCI model are the logs of relative market shares while one usually is interested in the impulse-response functions for the levels of the market shares. A first and obvious method to calculate forecasts, which are the essential components of the IRF, for the market shares is simply to forecast the logs of the relative market shares and next to transform these forecasts to market share forecasts. This method does not necessarily lead to unbiased forecasts as it ignores the two points earlier mentioned regarding expectations, which state that in general $\exp(\mathrm{E}[\log(M_i/M_I)]) \neq \mathrm{E}[M_i]/\mathrm{E}[M_I]$. In this section we therefore propose a simulation-based technique to calculate unbiased forecasts and to generate impulse-response functions for an MCI model. We first outline how one can generate forecasts in Section 3.1, and next how one can calculate the IRF in Section 3.2.

3.1 Forecasting

To forecast the market share of a brand i, we need to specify $M_{i,t}$ in terms of the relative market shares, denoted by $m_{j,t} = M_{j,t}/M_{I,t}$, $j = 1 \ldots I-1$, as these are the variables which are, after a log transformation, modeled in the reduced form MCI model. As $M_{I,t} = 1 - \sum_{j=1}^{I-1} M_{j,t}$, we have

$$M_{I,t} = \frac{1}{1 + \sum_{j}^{I-1} m_{j,t}} \quad \text{and} \quad M_{i,t} = M_{I,t} m_{i,t} = \frac{m_{i,t}}{1 + \sum_{j}^{I-1} m_{j,t}}. \tag{6}$$

As the relative market shares ($m_{i,t}$) are log-normally distributed, see (4), the probability distribution of the market shares themselves is rather complicated as it involves the inverse of the sum of log-normally distributed variables. Moreover, it is difficult to directly calculate the mean of the distribution as there is no simple algebraic expression for this expectation. As correct forecasts should be based on the expected value of the market shares, they themselves are also difficult to calculate analytically.

Given the above, we calculate market share forecasts using a simulation technique. The model specification in (4) is used to generate relative market shares for various synthetic disturbances (η) drawn from a multivariate normal distribution with a covariance matrix, equal to the estimated covariance matrix for η, denoted by $\hat{\Sigma}^*$. In every replication, we calculate the market shares resulting from the generated disturbance vector. The average market share over a number of replications now provides an unbiased estimate of the mean of the market share distribution, and therefore an unbiased forecast of the market shares. Notice that only the parameters of the reduced-form model are required for the simulations.

To be more precise, for one-step ahead forecasting, the relative market shares can be simulated using

$$\begin{aligned} \eta_{t+1}^{(l)} &= (\eta_{1,t+1}^{(l)}, \ldots, \eta_{I-1,t+1}^{(l)})' \text{ from } N(0, \hat{\Sigma}^*) \\ m_{i,t+1}^{(l)} &= \exp(\hat{\mu}_i^* + \eta_{i,t+1}) \prod_{j=1}^{I} \prod_{k=1}^{K} x_{k,j,t+1}^{\hat{\beta}_{k,j,i}^*} \prod_{j=1}^{I} \prod_{p=1}^{P} M_{j,t+1-p}^{\hat{\alpha}_{p,j,i}^*} \ , \end{aligned} \tag{7}$$

where we have rewritten (5) into an attraction format and where the index l denotes the simulation iteration. For every draw of $\eta_{t+1}^{(l)}$, we calculate the corresponding realization of the relative market shares, $m_{i,t+1}^{(l)}$, using the above equation. With a simulated realization of the relative market shares, we can calculate the realization of the market shares using

$$M_{i,t+1}^{(l)} = \frac{m_{i,t+1}^{(l)}}{1+\sum_{j=1}^{I-1} m_{j,t+1}^{(l)}} \quad \text{and} \quad M_{I,t+1}^{(l)} = \frac{1}{1+\sum_{j=1}^{I-1} m_{j,t+1}^{(l)}}. \tag{8}$$

Every vector $(M_{1,t+1}^{(l)}, \ldots, M_{I,t+1}^{(l)})'$ that is generated using this simulation method amounts to a draw from the joint distribution of the market shares. The expected market shares can therefore be estimated by averaging over all simulated market shares that is,

$$\mathrm{E}[M_{i,t+1}|\mathcal{X}_{t+1}, \mathcal{M}_t] = \frac{1}{L}\sum_{l=1}^{L} M_{i,t+1}^{(l)}, \tag{9}$$

where L denotes the number of simulated market share vectors and where $\mathcal{X}_t$ and $\mathcal{M}_t$ contain all information on explanatory variables and on market shares up to and including period t, respectively.

Forecasting $h > 1$ steps ahead is slightly more difficult as the values of the lagged market shares are no longer known. Hence, for these lagged market shares we should use the appropriate simulated values. For example, 2-step ahead forecasts can be calculated by averaging over simulated values $M_{i,t+2}^{(l)}$, based on drawings $\eta_{t+2}^{(l)}$ from $N(0, \hat{\Sigma}^*)$ and on drawings $M_{i,t+1}^{(l)}$, which were already used for the 1-step ahead forecasts. Note that the 2-step ahead forecasts do not need more simulations than the one-step ahead forecasts.

Finally in case $h > P$, that is, all lagged market shares are unknown, the h-step forecasts are calculated using the scheme below. It then holds that $\mathrm{E}[M_{i,t+h}|\mathcal{X}_{t+h}, \mathcal{M}_t] = \frac{1}{L}\sum_{l=1}^{L} M_{i,t+h}^{(l)}$, where

$$\begin{aligned} M_{i,t+h}^{(l)} &= \frac{m_{i,t+h}^{(l)}}{1+\sum_{j=1}^{I-1} m_{j,t+h}^{(l)}} \quad \text{and} \quad M_{I,t+h}^{(l)} = \frac{1}{1+\sum_{j=1}^{I-1} m_{j,t+h}^{(l)}} \\ m_{i,t+h}^{(l)} &= \exp(\hat{\mu}_i^* + \eta_{i,t+h}^{(l)}) \prod_{j=1}^{I}\prod_{k=1}^{K} x_{k,j,t+h}^{\hat{\beta}_{k,j,i}^*} \prod_{j=1}^{I}\prod_{p=1}^{P} M_{j,t+h-p}^{(l)^{\hat{\alpha}_{p,j,i}^*}} \qquad . \\ \eta_{t+h}^{(l)} &= (\eta_{1,t+h}^{(l)}, \ldots, \eta_{I-1,t+h}^{(l)})' \text{ from } N(0, \hat{\Sigma}^*) \end{aligned} \tag{10}$$

When $h \leq P$ some of the lagged market shares are observed, and in that case one can use the observed values instead of the simulated values.

It is also possible to calculate confidence bounds for the forecasted market shares. Actually, the entire distribution function of the market shares can be estimated based on the simulated values. For example, the lower bound of a 75% confidence interval is that value for which it holds that 12.5% of the simulated market shares is smaller than the value.

3.2 Impulse-Response Functions

In this subsection, we describe two types of impulse-response functions. The first impulse-response function is calculated for a shock caused by a temporary change in one of the explanatory variables, for example, by a change in the price of one of the brands. The second IRF captures the effect of a exogeneous shock in one of the residuals (ε_i) in (1), that is, the effects of an innovative shock.

The first IRF is most easy to compute. All explanatory variables are set at their average values, except for possible dummy variables which are set at either 0 or 1. For example, a promotion dummy is set at 0 because no promotion is the "normal" case. Lagged market shares are also set at their average values. For these values of the explanatory variables, the market shares are predicted until a steady state is reached. Notice that in itself these steady states can also be of interest to marketing researchers. This start-up period is only needed when lagged market shares are included in the MCI model. Next, for one period a shock is added to one of the explanatory variables, while for the following periods the average values are used again. The resulting forecasted time-paths of market shares constitute the impulse-response functions.

To analyze the impact of exogeneous shocks via the innovations is slightly more difficult. This is because only the differences of the disturbances (denoted by η, see (5)) are identified, that is,

$$\eta_t = \begin{pmatrix} \varepsilon_{1,t} - \varepsilon_{I,t} \\ \varepsilon_{2,t} - \varepsilon_{I,t} \\ \vdots \\ \varepsilon_{I-1,t} - \varepsilon_{I,t} \end{pmatrix} \tag{11}$$

Clearly, a shock in $\varepsilon_{i,t}$, $i = 1, \ldots, I-1$, will now only have an effect on $\eta_{i,t}$, whereas a shock in $\varepsilon_{I,t}$ will affect all elements of η_t. The impulse-response function for the innovations captures the influence of an exogeneous innovation shock in the attraction of one of the brands. Hence, the interpretation of such a shock is an unexplained large or small attraction of a brand, which leads to a large or small market share of the corresponding brand. The calculation of the impulse-response function is similar to the first case. The explanatory variables are again set at their averages (except for the dummy variables), and the market shares are forecasted until a steady state is reached. For one period, a shock is added to one of the disturbances ($\varepsilon_{i,t+h}$). This can easily be done by adding a shock to the elements of $\eta_{t+h}^{(l)}$ in (10) corresponding to $\varepsilon_{i,t+h}$ for every draw. Again the forecasted time path of market shares form the impulse-response functions.

4 Application

The calculation of the two types of impulse-response functions, discussed in Section 3, is illustrated using an MCI model for weekly market shares of five detergent brands. The sample contains 132 observations. These five brands are assumed to constitute a single product category. The actual market shares are transformed by scaling the market shares with the sum of observed market shares. That is, $M_{i,t}$ is replaced by $M_{i,t}/\sum_{j=1}^{5} M_{j,t}$, at every $t = 1, \ldots, 132$. As explanatory variables we have $P_{i,t}$, the price of brand i in period t relative to all other brands in the market, $D_{i,t}$, which denotes the distribution of brand i in week t, and we have a promotion variable $Pr_{i,t}$ which indicates whether or not there is a promotion (of any type) for brand i in week t. The distribution variable is defined as the fraction of stores in which the brand is available. The fraction is weighted by the size of the store, so large stores have a larger impact on $D_{i,t}$ than small stores. The promotion variable equals 1 if the brand is featured, on display or if there is a price cut. In all other weeks the $Pr_{i,t}$ variable takes the value 0. Table 1 shows a summary of the data. In the model specification we arbitrarily decide to consider brand 5 as the benchmark brand.

Table 1: Data characteristics 5-brand detergent data

	Brand				
	1	2	3	4	5
Average market shares (%)	24.49	22.22	22.62	8.22	22.45
Average relative price	1.12	1.09	1.10	1.09	1.11
Average relative distribution	0.85	0.91	0.93	0.78	0.94
Fraction of promotions*	0.27	0.21	0.39	0.34	0.10

* Fraction of 132 weeks in which the brand is on promotion.

From the attraction specification of the MCI model (4) we see that we cannot use the dummy variables directly as strictly speaking a week without promotion would imply a zero market share. This problem can be solved by transforming the promotion dummy to a variable which is always positive, for example by using $\exp(Pr_{i,t})$ instead of $Pr_{i,t}$.

In case of no promotion $\exp(Pr_{i,t})$ equals 1 and the promotion variable does not add attraction.

We use the model specification strategy outlined in Franses and Paap (1999). To find the optimal lag P we use a combination of the BIC criterion and the LM-test for multivariate serial correlation in the residuals to choose the best lag to use. The BIC criterion indicates that a model without lagged market shares should be preferred. On the other hand, the LM-test clearly indicates the presence of serial correlation in the residuals in a model with $P = 0$. Hence, we decide to consider an extended model with one-period lagged market shares as additional explanatory variables, that is, to set $P = 1$.

Next, a number of restrictions are tested using Likelihood Ratio [LR] tests. The LR-test statistic is defined as: $-2(\log \hat{L}_a - \log \hat{L}_0)$ where $\log \hat{L}_a$ and $\log \hat{L}_0$ denote the log-likelihood evaluated at the estimated parameters under the alternative and the null hypotheses, respectively. Under the null hypothesis, this test statistic is $\chi^2(\nu)$-distributed, where ν equals the number of parameter restrictions.

First of all, the Restricted Covariance Matrix [RCM] assumption is tested. The RCM restriction assumes that $\varepsilon_{i,t}$ is not correlated with $\varepsilon_{j,t}$ for $i \neq j$, and hence that Σ is a diagonal matrix instead of a full matrix. The realization of the test statistic is $-2(186.70 - 190.45) = 7.508$, which is not significant compared with the $\chi^2(5)$ distribution. We therefore accept the RCM restriction.

The second restriction is Restricted Competition [RC], which assumes that (marketing) variables of competitors do not influence own brand attractions. This restriction is rejected for the detergent data as $-2(144.64 - 186.70) = 84.104$ is significant compared with the $\chi^2(45)$-distribution. As there is no further possibility to reduce the model, we give the estimation results for this model in Table 2. Even though some of the parameters can be set equal to zero, we continue with this empirical model. Note that the estimated parameters refer to relative market shares, or equivalently, to the reduced form model in (5), which can also be written in the format of (10).

As the model parameters in Table 2 only give relative effects and because of the inclusion of lagged market shares, it is difficult to see what effect each variable has on the different market shares. Therefore, we use impulse-response functions to analyze the structure of the empirical model. An impulse can be caused by a change in one of the explanatory variables, for example the relative price of brand 1, or it can be caused by an innovation. As an example, Figure 1 shows the impulse-response functions for all brands

Table 2: Estimated parameter values in an empirical MCI model for 5 detergent brands (standard errors in parentheses)

	$\frac{M_{1,t}}{M_{5,t}}$		$\frac{M_{2,t}}{M_{5,t}}$		$\frac{M_{3,t}}{M_{5,t}}$		$\frac{M_{4,t}}{M_{5,t}}$	
Intercept	1.308	(0.69)	2.136	(0.898)	1.121	(0.941)	1.387	(0.894)
$P_{1,t}$	-7.828	(1.139)	-2.220	(1.481)	-0.274	(1.554)	-2.647	(1.476)
$P_{2,t}$	0.054	(0.404)	-3.345	(0.526)	0.003	(0.552)	-0.254	(0.524)
$P_{3,t}$	-0.186	(0.555)	-0.072	(0.722)	-5.241	(0.758)	0.207	(0.72)
$P_{4,t}$	-0.196	(0.796)	2.487	(1.035)	-0.465	(1.085)	-6.682	(1.031)
$P_{5,t}$	7.419	(1.329)	3.987	(1.729)	5.243	(1.813)	8.887	(1.722)
$D_{1,t}$	0.081	(0.288)	0.157	(0.374)	-0.080	(0.392)	-0.428	(0.373)
$D_{2,t}$	0.014	(0.162)	1.124	(0.211)	-0.367	(0.221)	0.104	(0.21)
$D_{3,t}$	0.330	(0.363)	-0.068	(0.472)	1.466	(0.495)	-0.584	(0.47)
$D_{4,t}$	0.027	(0.113)	0.137	(0.147)	-0.084	(0.154)	1.015	(0.146)
$D_{5,t}$	-0.224	(0.402)	-1.535	(0.523)	-1.042	(0.548)	0.174	(0.521)
$\exp(Pr_{1,t})$	0.070	(0.039)	0.080	(0.051)	0.047	(0.054)	-0.064	(0.051)
$\exp(Pr_{2,t})$	-0.012	(0.051)	0.149	(0.067)	-0.047	(0.07)	0.020	(0.066)
$\exp(Pr_{3,t})$	0.037	(0.038)	-0.004	(0.05)	0.094	(0.052)	0.001	(0.05)
$\exp(Pr_{4,t})$	-0.076	(0.044)	0.010	(0.058)	-0.091	(0.06)	-0.007	(0.057)
$\exp(Pr_{5,t})$	-0.012	(0.063)	-0.225	(0.083)	-0.097	(0.087)	-0.014	(0.082)
$M_{1,t-1}$	0.394	(0.113)	0.118	(0.147)	0.143	(0.155)	0.212	(0.147)
$M_{2,t-1}$	0.127	(0.107)	0.558	(0.139)	0.193	(0.146)	0.276	(0.139)
$M_{3,t-1}$	0.180	(0.141)	0.567	(0.183)	0.384	(0.192)	0.621	(0.182)
$M_{4,t-1}$	0.129	(0.061)	0.084	(0.079)	0.091	(0.083)	0.400	(0.079)
$M_{5,t-1}$	-0.192	(0.138)	0.081	(0.179)	-0.103	(0.188)	-0.229	(0.179)

$$\hat{\Sigma}^* = \begin{pmatrix} \hat{\sigma}_1^2 + \hat{\sigma}_5^2 & \hat{\sigma}_5^2 & \hat{\sigma}_5^2 & \hat{\sigma}_5^2 \\ \hat{\sigma}_5^2 & \hat{\sigma}_2^2 + \hat{\sigma}_5^2 & \hat{\sigma}_5^2 & \hat{\sigma}_5^2 \\ \hat{\sigma}_5^2 & \hat{\sigma}_5^2 & \hat{\sigma}_3^2 + \hat{\sigma}_5^2 & \hat{\sigma}_5^2 \\ \hat{\sigma}_5^2 & \hat{\sigma}_5^2 & \hat{\sigma}_5^2 & \hat{\sigma}_4^2 + \hat{\sigma}_5^2 \end{pmatrix} = \begin{pmatrix} 0.024 & 0.015 & 0.015 & 0.015 \\ 0.015 & 0.040 & 0.015 & 0.015 \\ 0.015 & 0.015 & 0.044 & 0.015 \\ 0.015 & 0.015 & 0.015 & 0.039 \end{pmatrix}$$

Note: The restricted covariance matrix implies that $\hat{\Sigma} = \text{diag}(\hat{\sigma}_1^2, \ldots, \hat{\sigma}_I^2)$ in (1), where $\hat{\sigma}_1^2 = 0.009$, $\hat{\sigma}_2^2 = 0.025$, $\hat{\sigma}_3^2 = 0.026$, $\hat{\sigma}_4^2 = 0.025$ and $\hat{\sigma}_5^2 = 0.015$. See Franses and Paap (1999) for details.

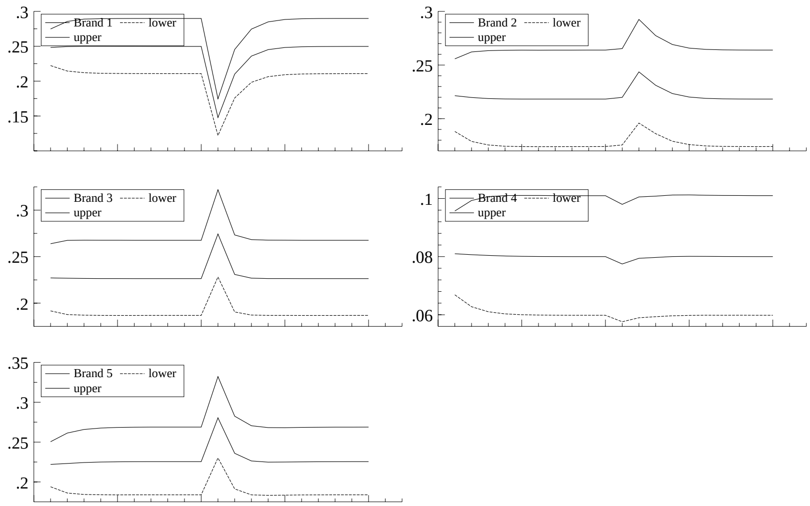

Figure 1: Impulse-response of a 10% increase in the relative price of brand 1, with 75% confidence bounds

for a 10% increase in the relative price of brand 1. The impulse-response functions are calculated using the technique outlined in Section 3, where we use 10,000 replications to calculate the market share forecasts. Approximately halfway the graph the price of brand 1 is temporarily changed. Note that because we have included lagged market shares in the model, there can be a carry-over effect, that is the market shares do not necessarily go back to the normal values directly after the shock. Also note that the promotion variable is also used to indicate a price cut, the impulse-response function for a change in price only considers a "direct" effect. The promotion signal given by a price change is not included.

Figure 1 shows that the market share of brand 1, of course, decreases because of this shock. However, not all other brands benefit from this, as brand 4 also looses market share, albeit only little in absolute sense. Because this brand has a small market share it is difficult to see whether its market share is indeed insensitive to price changes of brand 1. In graphs of relative impulse-response it is easier to make such inference. The relative impulse-response function is based on market share relative to the market share in the steady state. Figure 2 shows such a graph for again a 10% change in the price of brand 1. From this graph we see that indeed brand 4 is relatively insensitive to price changes of brand 1. Next to this, we see that a 10% increase in price leads to a 40% loss in market shares for brand 1. Brand 5 benefits the most, its market share increases with more than

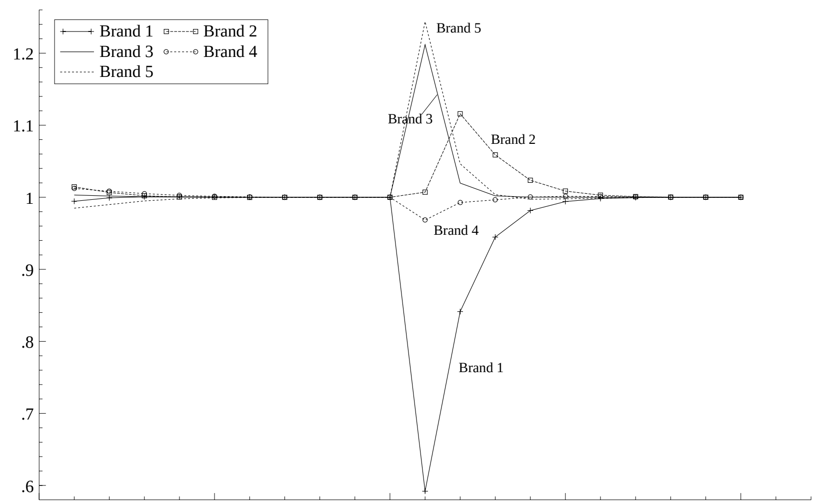

Figure 2: Relative impulse-response of a 10% increase in the relative price of brand 1

20%. Interestingly, brand 2 benefits from a price increase of brand 1 one week later. The same type of graphs can be made for all other brands and variables. We choose to only show the relative impulse-response of a promotion of brand 1, see Figure 3. In contrast to the price-insensitivity of brand 4 to brand 1, brand 4 appears to be quite sensitive to a promotion of brand 1. Next to brand 4, also brand 5 looses substantial market share when brand 1 is on promotion.

The above mentioned shocks to the system are all caused by a change in one of the explanatory variables. The impulse-response functions therefore show the dynamic structure of the model as well as the competitive structure. In an impulse-response function for the innovations however, we can isolate the dynamic structure of the model. The impulse-response function for a shock in the innovation of the attraction of brand 1 is given in Figure 4. The size of the shock is equal to 0.093, the standard deviation of $\varepsilon_{1,t}$. Note that this standard deviation is identified because we imposed the RCM assumption. Of course, the direction of the immediate effect of this shock is clear on beforehand, that is brand 1 will gain market share from all other brands. The relative loss of market share is equal for all competitors, they all loose a bit more than 2% of their market share. The fact that the relative immediate effect of a shock in an innovation is equal for all competitors directly follows from the multiplicative specification of the model in (1). Naturally the effect on future periods is not the same for all competitors. Brand 2 also looses much market share in the next couple of periods, whereas brand 5 recovers rapidly.

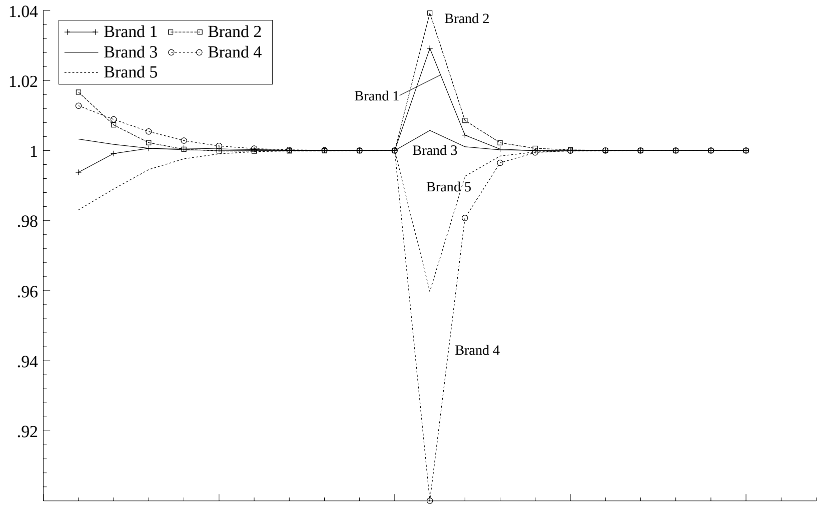

Figure 3: Relative impulse-response of a promotion of brand 1

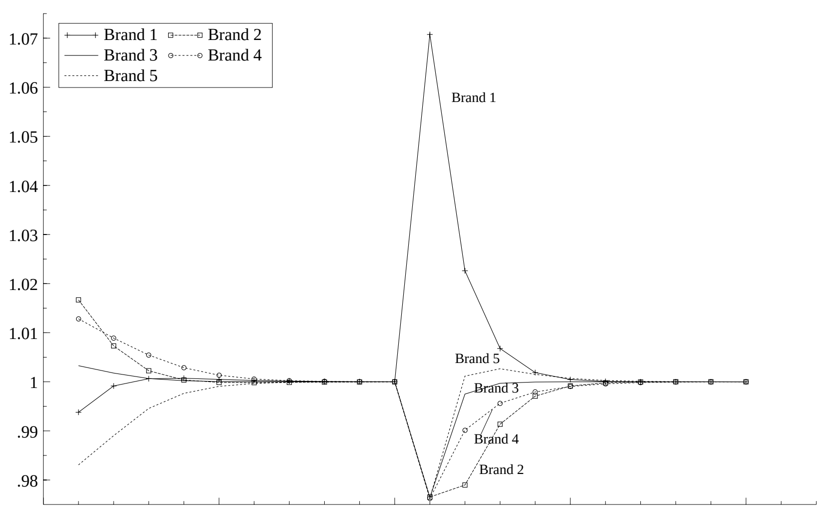

Figure 4: Relative impulse-response of innovation in the attraction of brand 1

5 Conclusion

We proposed a simulation-based technique to calculate impulse-response functions and their confidence intervals in a market share attraction model [MCI]. As an MCI model implies a reduced form model for the logs of relative market shares, simulation techniques have to be used to obtain the impulse-responses for the levels of the market shares. We applied the technique to an MCI model for a five-brand detergent market. We illustrated how impulse-response functions can help to interpret the estimated model. In particular, the competitive and dynamic structure of the model can be analyzed.

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