Introducing the indirect addilog system in a computable general equilibrium model: a case study for Palestine

Paul de Boer* and Marco Missaglia

Econometric Institute Report EI 2005-25

Abstract

A popular functional form for modeling the consumption block of a computable general equilibrium model (CGE) is the Linear Expenditure System (LES) for which the Engel curves are straight lines. To allow for more general shapes two other systems have been proposed in recent literature: An Implicitly Directly Additive Demand System (AIDADS, a generalization of LES) and the Specialized Constant Differences of Elasticities (CDE) system. To calibrate the parameters outside information on all income elasticities and all own price elasticities is needed, whereas LES only requires information on income elasticities and the Frisch parameter. In this paper we consider a special case of CDE, the Indirect Addilog System (IAS) that allows for non-straight Engel curves, whereas its outside data requirement is the same as for LES. The only disadvantage is that all cross price elasticities of a particular price are the same. In many developing countries there is hardly any information on price responses so that the AIDADS and CDE cannot be used. We propose the use of IAS rather than LES. In the empirical part we use IAS in a CGE model for Palestine and show that predictions of macro-economic indicators are remarkably close to those of IMF.

Keywords: CGE model, consumption block, functional forms, indirect addilog system, Palestine

*
This paper is dedicated to Professor W.H. Somermeyer, Director of the Econometric Institute from 1966 until his untimely death on 31 May 1982. Wim Somermeyer was friend and guide to author Paul de Boer, as well as supervisor of his Ph.D. Thesis.

Paul de Boer, Department of Econometrics, Erasmus University Rotterdam, P.O. Box 1738, NL-3000 DR Rotterdam, the Netherlands, e-mail: pmdeboer@few.eur.nl, Marco Missaglia, Dipartemento di Economia Pubblica e Territoriale, Università degli Studi di Pavia, Corso Strada Nuova 65, IT-27100 Pavia, Italy, e-mail: missagli@unipv.it

June 2005

1. Introduction

In production and consumer theory the mostly used production function and utility function is presumably the one due to Cobb and Douglas (1928). A major shortcoming of the Cobb-Douglas utility function is that preferences are homothetic implying unitary income elasticities so that Engel curves are straight lines through the origin. Moreover, the budget shares are constant.

Tinbergen (1942) proposed to generalize the Cobb-Douglas production function by introducing positive minimum amounts of capital and labor. Shortly after the war this idea was introduced in the theory of consumption in a series of articles: Klein and Rubin (1948-1949), Samuelson (1948), Geary (1949-1950) and Stone (1954). This function is known as the Stone-Geary utility function and the ensuing demand model as the Linear Expenditure System (LES).

In case of the LES function the marginal budget shares are constant; the income elasticities are not unitary, but the Engel curves are still straight lines, though not through the origin.

In the early fifties of the last century this shortcoming of the LES function was recognized at Statistics Netherlands by Somermeyer and Wit who wanted to compare the income elasticities in the Netherlands in the pre-war period based on the last pre-war budget survey (1935/1936) with those of the post-war period for which the first budget survey was conducted in 1951 (Wit, 1957). In this period of shortage of data, one had to rely upon simplifying assumptions in order to be able to estimate the values of these elasticities. Somermeyer and Wit (1956) introduced a budget allocation model that has the same data requirements as the LES function, but that does not have the limitation of constancy of marginal budget shares. It was discovered later that this model had already been introduced by Leser (1941); a fact of which Somermeyer and Wit were unaware of.

They published their results in Dutch so that this model was not known in the outside world and Houthakker (1960), who was unaware of the Somermeyer-Wit (1956) contribution, derived this model departing from an implicitly indirectly additive utility function. It is due to Houthakker that this model is named “indirect addilog system”. After Houthakker’s discovery Wit (1960) published the English translation of the 1956 and 1957 articles.

As time went on, more and more data became available and the indirect addilog system was abandoned in favor of more general models, like the almost ideal demand system, introduced by Deaton and Muellbauer (1980). A drawback of this model is that the fitted budget shares do not necessarily lie in the unit interval (see point v below) and that negativity (see point vi below) cannot be imposed.

In the world of CGE-modeling, where there is usually is shortage of data, the most used model for consumer behavior is the LES model; see for instance Blonigen c.s. (1997) for an excellent treatment. It is shown therein that, if one disposes of a Social Accounting Matrix (SAM) and if one disposes of outside information on the income elasticities and on the value of the Frisch parameter, a value is assigned to each of the parameters of the LES functional form (in the terminology of CGE modeling: the parameters are *calibrated*).

In 1975 Hanoch published an article in which Houthakker's implicit additivity was generalized to what Hertel c.s. (1991) named "specialized CDE functional forms". The use of this functional form in CGE modeling requires the specification of all income- and own price elasticities as outside information (2n, where n stands for the number of commodities) for the calibration of the parameters, which is (much) more demanding than for the LES for which the income elasticities and the Frisch parameter (or one own price elasticity) are required (n+1).

In a series of articles, the so-called AIDADS (An Implicitly Directly Additive Demand System), due to Rimmer and Powell (1996), is advocated (Cranfield c.s., 2000, 2003, Reimer and Hertel, 2004). This model is a generalization of the LES that allows for non-linear Engel curves. Like the specialized CDE functional form the drawback is that it requires more outside information than LES.

Reimer and Hertel (2004) give a list of five theoretical properties which are met by the AIDADS model:

i. The utility function must be non-homothetic

ii. The demand should be well-behaved across the tremendous variation in incomes

iii. The fitted budget shares should be in the [0,1] (= unit) interval

iv. The demand system should have a parsimonious parameterization

v. The demand system should incorporate the economic restrictions of adding-up, homogeneity and symmetry.

In addition to v the demand system should ideally incorporate the restriction of negativity, i.e. the utility function must be strictly quasi-concave so that optimization subject to the budget constraint yields a maximum (equivalently, the indirect utility function must be strictly quasi-convex or the Slutsky matrix must be negative semi-definite of rank n-1). Consequently, we add to this list:

vi. The demand system should incorporate the economic restriction of negativity.

In many developing countries, Palestine being a prominent example, data scarcity is such that the implementation of the AIDADS or the Specialized CDE functional form (not to mention the almost ideal demand system) is not feasible so that one has to resort to a simplification. In case of the AIDADS the simplification would lead to the LES, with the disadvantage of linearity of the Engel curves, whereas in case of the specialized CDE the simplification is the indirect addilog system.

In this paper we introduce into CGE modeling the use of the indirect addilog system, which exhibits non-linear Engel curves and which satisfies the five requirements of Reimer and Hertel mentioned above, as well as the theoretical restriction of negativity (point vi above).

It is shown that it has exactly the same data requirements as the LES, while its theoretical properties are more attractive than those of the LES. The theoretical conclusion of this paper is that, if one wishes to construct a CGE model for a country that is faced with scarcity of data, the indirect addilog model should be preferred over the LES.

The organization of the paper is as follows. In section 2 we give the specialized CDE model that has been used by Hertel c.s. (1991) and show that the indirect addilog system is a special case. Consequently, requirement iv (parsimony) is met with. Section 3 is devoted to a discussion of the addilog system and it is given under which parameter restrictions this demand system satisfies the theoretical requirements iii, v and vi mentioned above. The indirect utility function and mathematical derivations are given in appendix A. In section 4 we pay attention to the income elasticities and the shapes of the Engel curves implied by the indirect addilog system. It follows from this section that three types of non-linear Engel curves are allowed for and that the demand is well-behaved across a tremendous variation in incomes so that the system also meets the requirements i and ii mentioned above. In section 5 we deal with the price and substitution elasticities. We show that that all cross elasticities of a particular price are the same and that the differences of the elasticities of substitution are constant (CDE). Section 6 is devoted to the calibration of the parameters and we show that we need exactly the same outside information that is used to calibrate the LES parameters, namely, the income elasticities and the Frisch parameter.

Section 7 contains our empirical application: we apply the indirect addilog model in the framework of the CGE model for Palestine that has been used by Missaglia and de Boer (2004) in their contribution to the "Food-for-Work versus Cash-for-Work" debate: should workers participating in an employment program be paid in food and other essentials or in cash? It is shown that the Social Accounting Matrix (SAM) for 2002 simulated on the basis of the so-called intifada-shock applied to the 1998 benchmark SAM leads to macro figures that are remarkably close to those predicted by the IMF which followed an entirely different approach.

Since in the CGE model used we take account of the demand of leisure in order to model the labor market, we devote the appendix B to a treatment of the extended indirect addilog system that introduces leisure into the indirect utility function. Moreover we show how to calculate the equivalent and compensating variation.

The empirical conclusion of this paper is that the use of the indirect addilog system in a CGE model leads to meaningful results.

2. The specialized CDE functional form applied to consumer demand

Hertel c.s. (1991) summarizes the results of Hanoch (1975) of the specialized CDE functional form applied to *technology* in their Figure 1. In this approach returns to scale play a role so that the indirect addilog model is presented as a special case of the Explicit Indirect CDE (called "Explicit additivity" by Hertel c.s., 1991). However, in the framework of consumer demand the restriction on the parameter g , to be defined below, ($g = -1$) plays no role since *preferences* are ordinal. It is easily seen below that, from the mathematical point of view, the demand functions following from the explicit Indirect CDE do not depend on g . Since according to Hertel c.s. the explicit indirect CDE exhibits a parsimonious parameterization, the indirect addilog model is parsimonious as well (point iv of the list above, which will be abbreviated to "the list" in the sequel).

Let x_i denote the quantity that a consumer demands from commodity i (=1,…,n) and p_i the corresponding price. We assume that a consumer desires to attain at least a utility level of u and that he minimizes the cost. Let m denote the minimum cost of attaining utility level u and let the normalized prices be $z_i = p_i / m$.

Following Hertel c.s. (1991) we write the specialized CDE functional form as follows:

$$G(z,u) = \sum_{i=1}^{n} B_i u^{e_i b_i} z_i^{b_i} \equiv 1 \tag{1}$$

Using Roy's identity it follows from (1) that the demand for commodity i is:

$$x_i = \frac{B_i b_i u^{e_i b_i} z_i^{b_i - 1}}{\sum_{k=1}^{n} B_k b_k u^{e_k b_k} z_k^{b_k}} \tag{2}$$

(Hertel c.s.,1991, equation (4)).

In order to apply (2) in a CGE model we first have to calibrate the parameters e_i and b_i. Thereto we need to know all the income elasticities and all the own price elasticities. Then, the parameters B_i are easily calibrated using data from a SAM.

A special case of the specialized CDE functional form is the Explicit Indirect CDE which follows from the restriction that:

$$e_i b_i = g \tag{3}$$

In this case the demand for commodity i boils down to:

$$x_i = \frac{B_i b_i z_i^{b_i - 1}}{\sum_{k=1}^{n} B_k b_k z_k^{b_k}}$$

Using the reparametrization $c_i = B_i b_i$ and $\alpha_i = b_i$ we arrive at:

$$x_i = \frac{c_i z_i^{a_i - 1}}{\sum_{k=1}^{n} c_k z_k^{a_k}} \tag{4}$$

This functional form was introduced by Leser (1941), by Somermeyer and Wit (1956), and by Houthakker (1960) to whom this system owes its name of *indirect addilog*.

Defining the budget share $w_i = z_i x_1$ (4) can, alternatively, be written as:

$$w_i = \frac{c_i z_i^{\alpha_i}}{\sum_{k=1}^{n} c_k z_k^{\alpha_k}} \quad (5)$$

If we impose the restriction:

$$\alpha_i = 1 - \sigma$$

where σ denotes the elasticity of substitution, we obtain the demand relations following from the CES utility function, of which the Cobb-Douglas utility function ($\sigma = 1$) and Leontief ($\sigma = 0$) are special cases (see de Boer, 1997, who dealt with the indirect addilog counterpart in the theory of production, i.e. with Hanoch's HCDES production function).

3. The indirect addilog model of consumer demand

In section 2 we have supplied the demand equations of the specialized CDE functional form and have shown that a special case gives rise to the following equations for the budget shares:

$$w_i = \frac{c_i (p_i / m)^{\alpha_i}}{\sum_{k=1}^{n} c_k (p_k / m)^{\alpha_k}} \quad (6)$$

where we replaced in (5) the normalized prices, i.e. z_i, by p_i / m.

In appendix A (c.f. (A7)) we show that the restrictions on the parameters are:

$$c_i > 0 \text{ and } \alpha_i \leq 1 \quad (7)$$

where the equality sign may apply for at most one value of i.

For the interpretation of the parameters we quote from Somermeyer and Langhout (1972): "The c_k - with indeterminate level – may be interpreted as "preference coefficients" and the α_k as "reaction parameters"; the higher the value of α_k (i.e. the closer it is to 1), the more "urgent" the consumption of k may be considered to be, at least at lower income levels".

The preference coefficients c_i are indeterminate, that is to say: if we multiply each of them by the same factor, the equations (6) do not change. Therefore we impose the identifying restriction that the preference coefficients sum up to one:

$$\sum_{i=1}^{n} c_i = 1 \quad (8)$$

Since (6) adds up to 1, it is easily seen that the indirect addilog system satisfies the theoretical restriction of adding-up, whereas multiplication of all prices and expenditure by the same factor leaves the normalized prices unaffected. Consequently, the system satisfies the theoretical restriction of homogeneity as well (see point v of the list).

It is clear that (7) implies that all shares are positive and, since they add up to one, that they are smaller than 1. Consequently, the fitted budget shares are in the [0, 1] (= unit) interval (point iii of the list).

In the appendix A (c.f. (A12)) it is proved that the off-diagonal elements of the Slutsky matrix are:

$$s_{ij} = \frac{\partial x_i}{\partial p_j} + \frac{\partial x_i}{\partial m} x_j = x_i x_j m^{-1}(1 - \alpha_i - \alpha_j + \overline{\alpha}) = s_{ji}$$

where

$$\overline{\alpha} = \sum_{j=1}^{n} w_j \alpha_j \qquad (9)$$

Consequently, the Slutsky matrix is symmetric, as it should be (see point v of the list).

It should be noted that, in view of (7), the off-diagonal elements of the Slutsky matrix may be negative, zero or positive so that the indirect addilog system allows for complementarities, indifference and substitutability between commodities.

Van Driel (1974) has shown that the Slutsky matrix is negative semi-definite with rank (n-1) if and only if (7) holds true. Consequently, the addilog system satisfies requirement vi (negativity).

4. Income elasticities and the shapes of Engel curves

Income elasticities

In this section we summarize the findings of Somermeyer and Langhout (1972).

For ease of exposition we order the commodities such that:

$$\alpha_1 = \min_j \alpha_j \quad \text{and} \quad \alpha_n = \max_j \alpha_j$$

It is proved in appendix A (c.f. (A13)) that the income elasticities are equal to:

$$E(x_i, m) = 1 + \overline{\alpha} - \alpha_i \qquad (10)$$

It should be noted that since the income elasticities are not equal to one, preferences are non-homothetic (point i of the list).

It follows from (10) that commodity i is necessary when $a_i > \overline{a}$ and luxury when $a_i < \overline{a}$.

The income elasticities are lower-bounded as well as upper-bounded:

$$\alpha_1 < 1 + \alpha_1 - \alpha_i \leq E(x_i, m) \leq 1 + \alpha_n - \alpha_i$$

They are approaching their lower and upper limits according as m tends to infinity and to zero respectively. In view of (7) the lower bound is at most one, while the upper bound is at least equal to one. The lower bound is zero or negative, allowing for inferior commodities if:

$$\alpha_i \geq 1 + \alpha_1 \tag{11}$$

Engel curves

Let m_i denote the expenditure on commodity i: $m_i = p_i x_i$

It is shown in Somermeyer and Langhout (1972) that:

$$\lim_{m \to 0} m_i = p_i \lim_{m \to 0} x_i = 0 \tag{12}$$

and

$$\begin{aligned} \lim_{m \to \infty} m_i = p_i \lim_{m \to \infty} x_i &= \infty \quad \text{if } \alpha_i < 1 + \alpha_1 \\ &= \text{finite} \quad \text{if } \alpha_i = 1 + \alpha_1 \\ &= 0 \quad \text{if } \alpha_i > 1 + \alpha_1 \end{aligned} \tag{13}$$

As shown in (11) the criterion $1 + \alpha_1$ also rules the sign of the lower bound of the income elasticity. Equation (12) means that the Engel curve arises from the origin; while equations (13) imply the possibility of three main types of Engel curves, viz.:

(1) unlimited monotonic increase if $\alpha_i < 1 + \alpha_1$

(2) monotonic increase to a maximum (saturation) level if $\alpha_i = 1 + \alpha_1$, and

(3) decrease towards zero after having reached a maximum level if $\alpha_i > 1 + \alpha_1$.
.
For more details, as well as an application to the Netherlands, we refer to Somermeyer and Langhout (1972).

It should be noted that types (2) and (3) cannot occur if $\alpha_1 > 0$. In practice, however, this case is not likely to occur, since this implies that all budget items are of a fairly urgent nature.

It follows from this section that three types of (non-linear) Engel curves are allowed for and that the demand is well-behaved across a tremendous variation in incomes (point ii of the list).

5. Price and substitution elasticities

Price elasticities

From (A14) in appendix A it follows that the own price elasticities read:

$$E(x_i, p_i) = (1 - w_i)\alpha_i - 1 < 0 \tag{14}$$

so that Giffen goods are excluded, whereas the cross price elasticities are:

$$E(x_i, p_j) = -w_j \alpha_j \tag{15}$$

which means that all cross elasticities of a particular price are the same.

The price response implied by (15) is the following. If the price increase of p_j refers to a necessary commodity, i.e. α_j positive and rather close to one, then the expenditure on all other commodities will *decrease* with a given percentage $-w_j\alpha_j$. If the price increase of p_j refers to a luxury commodity, i.e. α_j negative, and of considerable magnitude, then all other expenditures will *increase* with $-w_j\alpha_j$ (>0!). It follows that luxuries are price elastic and necessities inelastic. Thus both positive and negative cross price effects are distributed neutrally over all other commodities. In many circumstances such proportional effects do not seem to be an unreasonable price response in the framework of a CGE model.

Admittedly, CDE and AIDADS allow for more flexibility with respect to cross price responses. Constancy of the cross price elasticities is the price that has to be paid for a parameterization of the indirect addilog system that is more parsimonious than CDE or AIDADS.

Elasticities of substitution

Having derived the income and price elasticities, the Allen partial elasticities of substitution, σ_{ij}, easily follow from the well-known relationship:

$$E(x_i, p_j) = w_j[\sigma_{ij} - E(x_i, m)]$$

and, for the own elasticities, read:

$$\sigma_{ii} = -\frac{1}{w_i}(1 - \alpha_i) + (1 + \overline{\alpha})$$

while for the cross-elasticities they are:

$$\sigma_{ij} = -\alpha_i - \alpha_j + (1 + \overline{\alpha}) \tag{16}$$

The well-known property that the *differences* of the cross-elasticities are constant easily follows from (16). It should be noted that Chung (1994, p.44, (3.71)) incorrectly states that they are equal to zero. This is caused by the fact that Chung incorrectly uses the indirect utility function in the definition of the Allen partial elasticity of substitution instead of the cost function.

6. Calibration of the parameters

In order to calibrate the parameters α_i we need to have outside information on the income elasticities and on the Frisch parameter, the elasticity of marginal utility with respect to total expenditure m, denoted by f_m.

It is derived in appendix A (c.f. (A1 7)) that for the addilog system the Frisch parameter is equal to:

$$f_m = -(1 + \bar{a}) \qquad (17)$$

Consequently, it follows from (10) and (17) that the calibrated values of α_i are:

$$a_i = -[f_m + E(x_i, m)] \qquad (18)$$

We assume that a Social Accounting Matrix (SAM) is available. We put all prices equal to 1, we denote the household expenditure in the SAM by m^0 and the budget shares by w_i^0. Then, taking the identifying restriction (8) into account, we use (6) to calibrate c_i:

$$c_i = w_i^0 [m^0]^{a_i} / \sum_{j=1}^{n} w_j^0 [m^0]^{a_j} \qquad (19)$$

7. An application of the indirect addilog system: a CGE model for Palestine

In this section we show a concrete Computable General Equilibrium (CGE) application of the addilog demand system we have previously illustrated. The application is related to our previous works on the Palestinian economy (Missaglia and de Boer, 2004; de Boer and Missaglia, 2005) and, most notably, it refers to the building of a counterfactual Social Accounting Matrix (SAM) for the Palestinian economy in 2002. The last available official SAM for the Palestinian economy dates back to 1998 and the need to build a new, counterfactual SAM comes from the tremendous shock suffered by the Palestinian economy after the outbreak of the second intifada, which is likely to have dramatically changed its size and composition over the last years.

The counterfactual SAM has been obtained by giving our CGE model for the Palestinian economy a major so-called "intifada-shock". Before illustrating our results and comparing them with the aggregate figures estimated by the International Monetary Fund (IMF), it is worth giving a short description of both the CGE model and the intifada-shock.
The CGE model (for a detailed description see Missaglia and de Boer, 2004) is an "almost" standard, static model where, compared with a fully standard model, two elements are to be emphasized. First, the household (in the model there is just one representative consumer) consumption behavior is based on the addilog demand system we have described in-depth in the previous sections of the paper. Second, in the

model we use the unemployment theory delineated in the migration literature by Harris and Todaro (1970) to describe the wage gap between rural and urban jobs. The core of the theory is described by the following arbitrage condition (acting as a wage curve):

$$PL = \frac{LF}{LF + UNEMP} b.PLF$$

The wage rate paid by Palestinians firms to Palestinian workers, PL, must be equal, in equilibrium, to the expected wage rate of the Palestinian workers employed in Israel or in the settlements. The latter is equal to the wage rate prevailing in Israel or in the settlements, PLF, multiplied by the probability of getting a job over there and the factor b. The probability of getting a job in Israel or in the settlements is simply given by the ratio of the Palestinians actually employed there (LF) to the workers who look for a job there: those who manage (LF) and those who do not (UNEMP). The factor b may be interpreted as the inverse of the probability of getting a job in Palestine, so that the arbitrage condition states nothing but the equality between two expected wages:

P (Job in Palestine).PL = P (Job in Israel or in the settlements).PLF

The “intifada-shock” we gave to the 1998 benchmark to build our counterfactual 2002 SAM is a set of several shocks (again, for a fully detailed description of the intifada-shock see Missaglia and de Boer, 2004; and de Boer and Missaglia, 2005): a reduction in the capital stock, due to the physical damages provoked by the conflict; a fall in the level of labour income earned in Israel and the settlements, due to closures imposed by the Israeli authorities; an increase in donors’ disbursements; a reduction in the household’s propensity to save, a natural reaction during hard times; a reduction in the PA savings, due to the withholding of clearance revenues by Israel and the use of an increasing amount of money to pay for public employees’ wages; an increase in the labor force; finally, an increase in the parameter b, i.e. a reduction in the probability of getting a job in Palestine.
From this major intifada-shock we derive a counterfactual SAM for 2002 and, by aggregation, we can thus derive the figures for the components of GNI. In table 1 our figures (DBM) are presented together with those derived by the IMF (2003) through the help of a purely macro, income-expenditure model.

Table 1. Comparison between DBM and the IMF

	1998 (million US$)			2002 (prices 1998, million US$)		
	DBM	IMF	Ratio	DBM	IMF	Ratio
Private Consumption	3,977	4,245	.937	3,630	3,956	.917
Public Consumption	976	954	1.023	1,158	1,041	1.112
Total fixed investment	1,675	1,494	1.121	998	661	1.509
Exports	729	886	.823	470	426	1.103
Imports	3,053	3,321	.919	2890	2,896	.997
GDP	4,304	4,258	1.011	3366	3,188	1.055
NFI	779	903	.863	390	465	0.838
GNI	5,083	5,161	.985	3,756	3,653	1.028

* The ratio is the figure DBM divided by the one of the IMF

The main difference between our (DBM) results and the IMF results concerns total fixed investment, larger in DBM, and, symmetrically, private consumption, lower in DBM. The sum of the two items, however, is remarkably close to each other. One possible explanation for this difference can be found in the role played in our model by the "Construction" sector. Indeed, "in the Palestinian economy more than half of total investment is concentrated into unproductive investment, such as residential building..." (Astrup and Dessus, 2002, p.18). A part of this "investment", its annual equivalent, should be assimilated to consumption, something that does not add anything to the productive capacity of the economy. However, in the SAM we used to calibrate the model the output of the "Construction" sector is classified as investment, which may explain the origin of the observed difference.

It must also be noted that, as expected, the figures (DBM) we obtain for 2002 are close to the figures we obtained by imposing the same intifada-shock to a previous version of the model where we used a Linear Expenditure System (LES) instead of an addilog system (see De Boer and Missaglia, 2005). But a word of caution is needed: the *aggregate* figures (those reported in Table 1) are very close, but the sectoral composition of consumption is different. This is not surprising. Indeed, in a LES framework the marginal budget shares are constant, but this kind of limitation disappears in an addilog framework: faced with the same negative shock, households will reduce more (compared with what they would do in a LES framework) the consumption of some goods and will reduce less the consumption of some other goods.

References

Astrup, C. and S. Dessus (2002), Exporting labor or goods: long term implications for the Palestinian economy, MENA Working Papers Series # 29 (The World Bank).

Blonigen, B.A, J.E. Flynn and K.A. Reinert (1997), Sector-focused general equilibrium modeling, in: J.F. Francois and K.A. Reinert, *Applied methods of trade policy analysis: a handbook*, Cambridge University Press, Cambridge, UK, pp. 189-230.

Cobb, C.W. and P.H. Douglas (1928), A theory of production, *American Economic Review*, Papers and Proceedings, XVIII, pp. 139-165.

Chung, J.W. (1994), *Utility and production functions*, Blackwell, Oxford.

Cranfield, J.A.L., P.V. Preckel, J.S. Eales and T.W. Hertel (2000), On the estimation of 'an implicitly additive demand system', *Applied Economics*, 32, pp. 1907-1915.

Cranfield, J.A.L., J.S. Eales, T.W. Hertel and P.V. Preckel (2003), Model selection when estimating and predicting consumer demands using international, cross section data, *Empirical Economics*, 28, pp. 353-364.

Deaton, A.S. and J. Muellbauer (1980), An almost ideal demand system, *American Economic Review*, 70, pp. 312-326.

De Boer, P.M.C. (1997), On the relationship between input-output coefficients and Hanoch's linear homogeneous constant differences of elasticities of substitution production function, *Economic Systems Research*, 9, pp. 259-264.

De Boer, P.M.C. and M. Missaglia (2005), Economic consequences of intifada , Report EI-2005-21 of the Econometric Institute, Erasmus University Rotterdam, the Netherlands.

Geary, R.C. (1949-1950), A note on 'A constant utility index of the cost of living', *Review of Economic Studies*, 18, pp. 65-66.

Hanoch, G. (1975), Production and demand models with direct and indirect implicit additivity, *Econometrica*, 43, pp. 395-419.

Harris, J. and M. Todaro (1970), Migration, unemployment and development: a two-sector analysis, *American Economic Review*, 60, pp.126-142.

Hertel, T.W., P.V. Preckel, M.E. Tsigas, E.B. Peterson and Y. Surry (1991), Implicit additivity as a strategy for restricting the parameter space in computable general equilibrium models, *Economic and Financial Computing*, 1, pp. 265-289.

Houthakker, H.S. (1960), Additive preferences, *Econometrica*, 28, pp. 244-257.

IMF (2003), West Bank and Gaza: economic performance and reform under conflict conditions, (IMF,September).

Klein, L.R. and H. Rubin (1948-1949), A constant –utility index of the cost of living, *Review of Economic Studies*, 15, pp. 84-87.

Leser, C.E.V. (1941), Family budget data and price-elasticities of demand, *Review of Economic Studies*, 9, pp. 40-57.

Missaglia, M. and P.M.C. de Boer (2004), Food-for-work versus cash-for-work: emergency assistance in Palestine, *Economic Systems Research*, 16, pp. 367-390.

Reiner, J.J. and T.W. Hertel (2004), Estimation of international demand behaviour for use with input-output based data, *Economic Systems Research*, 16, pp. 347-366.

Rimmer, M.T. and A.A. Powell (1996), An implicitly additive demand system, *Applied Economics*, 28, pp. 1613-1622.

Samuelson, P.A. (1948), Some implications of linearity, *Review of Economic Studies*, 30, pp. 88-90.

Somermeyer, W.H. and J.W.W.A. Wit (1956), Een verdeelmodel (an allocation model), Report M14, Statistics Netherlands, Voorburg.

Somermeyer, W.H. and A. Langhout (1972), Shapes of Engel curves and demand curves: implications of the expenditure model, applied to Dutch data, *European Economic Review*, 3, pp. 351-386.

Stone, R. (1954), Linear expenditure system and demand analysis: an application to the British pattern of demand, *Economic Journal*, 64, pp. 511-532.

Tinbergen, J. (1942), Professor Douglas' production function, *Revue de l'Institut International de Statistique*, 10, p. 37-48.

Van Daal, J. (1983), On the utility functions of the indirect addilog budget allocation model, Report 8304, Econometric Institute, Erasmus University Rotterdam, the Netherlands.

Van Driel, J. (1974), Consumer expenditure allocation models, Report 7405, Econometric Institute, Erasmus University Rotterdam, the Netherlands.

Varian, H.R. (1992), *Microeconomic analysis*, third edition, Norton, New York.

Wit, J.W.W.A. (1957), Inkomenselasticiteiten in 1935/'36 en 1951 voor Nederland: toepassing van een model van inkomensbesteding, *Statistische en Econometrische Onderzoekingen*, Statistics Netherlands, Voorburg, pp. 27-41.

Wit, J.W.W.A. (1960), Income elasticities in 1935/'36 and 1951 in the Netherlands: an application of a model for income spending, *Statistical Studies*, Vol. 10, Statistics Netherlands, Voorburg.

Appendix A The indirect utility function and some derivations

The indirect utility function of the indirect addilog system reads:

$$v(p,m) = -\sum_{j=1}^{n} c_j (p_j / m)^{(\alpha_j)} \qquad \text{(A1)}$$

where:

$$(p_j / m)^{(\alpha_j)} = \frac{(p_j / m)^{\alpha_j} - 1}{\alpha_j} \quad \text{for } \alpha_j \neq 0 \qquad \text{(A2)}$$

$$= \log(p_j / m) \quad \text{for } \alpha_j = 0$$

(the so-called Box-Cox transformation). Since from an empirical point of view the special case $\alpha_i = 0$ is not interesting, we shall disregard it in the sequel and we shall only consider the first line of (A2) for which it will be shown below that it gives rise to the budget share equations (6).

Derivation of the indirect utility function with respect to m leads to:

$$\frac{\partial v(p,m)}{\partial m} = \sum_j c_j p_j^{a_j} m^{-a_j - 1} \qquad \text{(A3)}$$

which is only increasing in m for all $p_j > 0$, when:

$$c_j > 0 \qquad \text{for } j = 1,\ldots,n \qquad \text{(A4)}$$

Secondly, we derive from (A1) that under (A4):

$$\frac{\partial v(p,m)}{\partial p_i} = -c_i p_i^{\alpha_i - 1} m_i^{-\alpha_i} < 0 \qquad \text{(A5)}$$

hence the indirect utility function is decreasing in prices, as it should be.

From (A5) we derive:

$$\frac{\partial^2 v}{\partial p_i \partial p_j} = (1 - a_i) c_i p_i^{a_i - 2} m^{-a_i} \qquad \text{for } i = j$$

$$= 0 \qquad \text{for } i \neq j$$

The indirect utility function is strictly convex, and consequently strictly quasi-convex, whenever:

$$\alpha_i < 1 \qquad \text{for } i = 1,\dots,n \qquad \text{(A6)}$$

Van Daal (1983) has shown that the indirect utility function of the indirect addilog model is strictly quasi-convex if and only if:

$$c_i > 0 \text{ and } \alpha_i \leq 1 \qquad \text{(A7)}$$

where the equality sign may apply for at most one value of i.

Application of Roy's identity, i.c.:

$x_i = -\dfrac{\partial v(p,m)/\partial p_i}{\partial v(p,m)/\partial m}$ to the first line of (A2), using (A3) and (A5), leads to:

$$x_i = \frac{c_i (p_i / m)^{\alpha_i - 1}}{\sum_{k=1}^{n} c_k (p_k / m)^{\alpha_k}} \qquad \text{(A8)}$$

which, after pre-multiplication with p_i / m, gives the budget share equations (6).

From (A8) we derive:

$$\frac{\partial x_i}{\partial m} = x_i m^{-1} (1 - \alpha_i + \overline{\alpha}) \qquad \text{(A9)}$$

where

$$\overline{\alpha} = \sum_{j=1}^{n} w_j \alpha_j \qquad \text{(A10)}$$

and:

$$\frac{\partial x_i}{\partial p_j} = -\alpha_j x_i x_j m^{-1} - (1-\alpha_i) p_i^{-1} x_i \delta_{ij} \tag{A11}$$

where $\delta_{ij} = 1$ for $i = j$,and
$= 0$ for $i \neq j$

(the so-called Kronecker delta).

Consequently, using (A9) and (A11), the typical element s_{ij} of the Slutsky matrix:

$$s_{ij} = \frac{\partial x_i}{\partial p_j} + \frac{\partial x_i}{\partial m} x_j = x_i x_j m^{-1}(1 - \alpha_i - \alpha_j + \overline{\alpha}) = s_{ji} \text{ for } i \neq j \tag{A12}$$

Van Driel (1974) has shown that the Slutsky matrix is negative semi-definite with rank (n-1) if and only if (A7) holds true.

From (A9) the income elasticities easily follow:

$$E(x_i, m) = 1 + \overline{\alpha} - \alpha_i \tag{A13}$$

whereas the price elasticities follow from (A11):

$$E(x_i, p_j) = -w_j a_j - (1 - a_i) d_{ij} \tag{A14}$$

The Frisch parameter, the elasticity of marginal utility with respect to total expenditure m is defined as:

$$f_m = \frac{m}{?} . \frac{\partial ?}{\partial m} \tag{A15}$$

The relationship between the indirect utility function and the marginal utility λ (see for instance equation (7.10) in Varian (1992), page 108) is: $? = \frac{\partial v(p, m)}{\partial m}$, so that it follows from (A3) that for the indirect addilog system we obtain:

$$? = \sum_j c_j p_j^{a_j} m^{-a_j - 1} \tag{A16}$$

From (A15) and (A16) it follows that:

$$f_m = -(1 + \overline{a}) \tag{A17}$$

Appendix B The indirect addilog system with leisure

Model and calibration

We assume that the household has an exogenously given time endowment, denoted by TS, that it allocates over labor supply, denoted by LS, and *leisure*, denoted by x_{n+1}, i.e.

$$TS = LS + x_{n+1} \tag{B1}$$

We take account of the consumption of leisure, valued at the wage rate p_{n+1}, define the *extended* household expenditure:

$$em = m + p_{n+1} x_{n+1} \tag{B2}$$

and assume that the household maximizes the *extended* indirect addilog utility function:

$$v(p, em) = -\sum_{j=1}^{n+1} c_j \frac{(p_j / em)^{\alpha_j} - 1}{\alpha_j} \tag{B3}$$

subject to the *extended* household expenditure (B2). (Note that we abstract from the special case that one or more α_j is equal to zero).

It follows straightforwardly that the optimal shares in the *extended* budget are:

$$w_i = \frac{c_i (p_i / em)^{a_i}}{\sum_{k=1}^{n+1} c_k (p_k / em)^{a_k}} \qquad i = 1, \ldots, n+1 \tag{B4}$$

As before, we need to have outside information on the expenditure elasticities and on the Frisch parameter in order to calibrate the parameters α_i and c_i (i running from 1 to n for the commodities, while i is equal to $n+1$ for *leisure*). In this framework we need to have values of these elasticities (and of the Frisch parameter) with respect to the *extended* household budget, em (including leisure), but in practice they are usually supplied with respect to the budget, m (excluding leisure). Moreover, in practice a value of the elasticity of labor supply with respect to the budget m is specified, rather than the expenditure elasticity of *leisure* with respect to the *extended* budget.

First, we consider the case that i runs from 1 to n:

$$E(x_i, em) = \frac{\partial x_i}{\partial m} . \frac{m}{x_i} \frac{\partial m}{\partial em} . \frac{em}{m} = E(x_i, m) . \frac{em}{m} \tag{B5}$$

The expenditure elasticities with respect to the extended budget are equal to those with respect to the budget (excluding leisure) multiplied by em/m, i.e. the inverse of the share of the budget (excluding leisure) in the extended budget (including leisure). Secondly, we consider leisure (x_{n+1}). Using (B1) we derive:

$$E(LS, m) = -\frac{\partial x_{n+1}}{\partial m} \cdot \frac{m}{x_{n+1}} \cdot \frac{x_{n+1}}{LS}$$

Using (B5) and replacing x_{n+1} by $(TS - LS)$ according to (B1), we arrive at:

$$E(x_{n+1}, em) = -E(LS, m) \cdot \frac{LS}{(TS - LS)} \cdot \frac{em}{m} \quad \text{(B6)}$$

Finally, for the Frisch parameter with respect to the extended budget we derive:

$$\phi_{em} = \frac{em}{\lambda} \cdot \frac{\partial \lambda}{\partial em} = \frac{em}{m} \cdot \phi_m \quad \text{(B7)}$$

From the outside information on the income elasticities, $E(x_i, m)$ for $i = 1,..., n$, the elasticity of labor supply, $E(LS, m)$, the Frisch parameter ϕ_m, and on TS we derive (B5) - (B7) that are used in the same way as before for the calibration of the parameters α_i $(i = 1,..., n+1)$.

The parameters c_i are calibrated in the same way as in (19), em replacing m, and $n+1$ replacing n:

$$c_i = w_i^0 [em^0]^{a_i} / \sum_{j=1}^{n+1} w_j^0 [em^0]^{a_j} \quad \text{(B8)}$$

Equivalent and compensating variation

Suppose that we have two different policy regimes: the "benchmark equilibrium", and the "proposed change". Under the "benchmark equilibrium" the consumer faces prices and (extended) expenditure (p^0, em^0), and under the "proposed change" he faces (p^1, em^1).

The equivalent variation (EV) measures the expenditure change at current prices (p^0) that would be equivalent to the proposed change in terms of its impact on utility.

Let em^{ev} denote the expenditure that at current prices (p^0) would yield utility level $V(p^1, em^1)$. Then, the equivalent variation is defined as:

$$EV = em^{ev} - em^0$$

It follows from the indirect utility function (B3) that em^{v} has to be solved numerically from:

$$\sum_{j=1}^{n+1} \frac{c_j[(p_j^1 / em^1)^{\alpha_j} - (p_j^0 / em^{ev})^{\alpha_j}]}{\alpha_j} = 0 \qquad \text{(B9)}$$

The compensating variation (CV) measures the income change that would be necessary to compensate the consumer for the price change, induced by the "proposed change". Let em^{cv} denote the expenditure that at prices p^1 would yield the utility level $V(p^0, em^0)$. Then, the compensating variation is equal to:

$$CV = em^1 - em^{cv}$$

where em^{cv} has to be solved numerically from:

$$\sum_{j=1}^{n+1} \frac{c_j[(p_j^1 / em^{cv})^{\alpha_j} - (p_j^0 / em^0)^{\alpha_j}]}{\alpha_j} = 0 \qquad \text{(B10)}$$