Block Structure Multivariate Stochastic Volatility Models

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Abstract

Most multivariate variance models suffer from a common problem, the “curse of dimensionality”. For this reason, most are fitted under strong parametric restrictions that reduce the interpretation and flexibility of the models. Recently, the literature has focused on multivariate models with milder restrictions, whose purpose was to combine the need for interpretability and efficiency faced by model users with the computational problems that may emerge when the number of assets is quite large. We contribute to this strand of the literature proposing a block-type parameterization for multivariate stochastic volatility models.

Keywords: block structures; multivariate stochastic volatility; curse of dimensionality

JEL classifications: C32, C51, C10.

1. Introduction

Classical portfolio allocation and management strategies are based on the assumption that risky returns series are characterized by time invariant moments. However, the econometric literature of the last few decades demonstrated the existence of dynamic behaviour in the variances of financial returns series. The introduction of such empirical evidence may constitute an additional source of performance for portfolio managers, as evidenced by Fleming, Kirby and Ostdiek (2001), or may be relevant for improving the market risk measurement and monitoring activities (see, for example, Hull and White (1998) and Lehar et al. (2002)). Two families of models emerged in the literature, namely GARCH-type specifications (see Engle (2002)), and Stochastic Volatility models (see Taylor (1986) and Andersen (1994)).

However, portfolio management strategies often involve a large number of assets requiring the use of multivariate specifications. Among the possible alternative models, we cite the contributions of Bollerslev (1990), Engle and Kroner (1995), Ling and McAleer (2003), Asai and McAleer (2006, 2009), and the surveys in McAleer (2005), Bauwens, Laurent and Rombouts (2006), and Asai, McAleer and Yu (2006). Most models, if not all, suffer from a common problem, the well-known “curse of dimensionality”, whereby models become empirically infeasible if fitted to a number of series of moderate size (in some cases, the models may become computationally intractable with even 5 or 6 assets). In order to match the need of introducing time-varying variances with practical computational problems, several restricted models are generally used: the diagonal

VECH specifications suggested by Bollerslev et al. (1988), the scalar VECH and BEKK models proposed by Ding and Engle (2001), the CCC model of Bollerslev (1990), and the dynamic conditional correlation models of Engle (2002) and Tse and Tsui (2002). However, the introduction of significant and strong restrictions reduces the interpretation and flexibility of the models, possibly affecting the purportedly improved performance they may provide and/or the appropriateness of the analysis based on their results.

Recently, the literature has focused on multivariate models with milder restrictions, whose purpose was to combine the need for interpretability and efficiency faced by model users with the computational problems that may emerge when the number of assets is quite large. Among the contributions in this direction, we follow the approach of Billio, Caporin and Gobbo (2005). They proposed specifying the parameter matrices of a general multivariate correlation model in a block form, where the blocks are associated with assets sharing some common feature, such as the economic sector. Our purpose is to adopt this block-type parameterization and adapt it to multivariate stochastic volatility models.

In general terms, Multivariate Stochastic Volatility (MSV) models have a parameter number of order $O(n^2)$, where n is the number of assets. With the introduction of block parameter matrices, we may control the number of parameters and obtain a model specification which is feasible, even for a very large number of assets. Furthermore, as in the contribution of Billio et al. (2005), the models we propose follow the spirit of sectoral-based asset allocation strategies since they will presume the existence of common dynamic behaviour within assets or financial instruments belonging to the same economic sector. This assumption is not as strong as postulating the existence of a unique factor driving all the variances and covariances, since the financial theory may suggest the existence of sector-specific risk factors (sectoral asset allocation is often followed by portfolio managers and characterized by a number of managed financial instruments).

As distinct from an extremely restricted model, we also recover part of the spillover effect between variances, which allows monitoring of the interdependence between groups of assets, an additional element which may be relevant. Within our modeling approach, the coefficients may be interpreted as sectoral specific, while the assets will be in any case characterized by a specific long term variance through the introduction of unrestricted constants in the variance equations.

Clearly, the restrictions proposed may not necessarily be accepted by the data, as more ‘complete’ models will, in general, provide better results. We will show that the introduction of such restrictions provides limited losses, while yielding a significant improvement over the more restricted specifications. We will evaluate and compare the alternative models following the Monte Carlo likelihood (MCL) estimation approach for both univariate and multivariate SV models, as in Sandmann and Koopman (1998) and Asai and McAleer (2006).

The plan of the remainder of the paper is as follows. Section 2 presents the block-structure modelling approach within a general MSV framework, and also compares the model to Factor SV specifications, and addresses some estimation issues. Section 3 presents an empirical example based on US stock market data for selected firms. Section 4 gives some concluding comments.

2. MSV Models

The block-structure model we present can be considered as a restricted specification of a general MSV model. In fact we will show how the modelling approach consists in defining a set of parametric restrictions that makes the model feasible, but without losing the interpretation of coefficients.

Consider a general MSV model that will be used as reference model. Let R_t be the return series on an asset, and define $y_t = R_t - E\left(R_t \middle| \Im_{t-1}\right)$ as the mean-adjusted return. Then, the basic SV model is defined as

$$\begin{aligned} & y_t = \sigma\varepsilon_t \exp\left(0.5h_t\right), \quad \varepsilon_t \sim N\left(0,1\right), \\ & h_{t+1} = \phi h_t + \eta_t, \quad \eta_t \sim N\left(0, \sigma_\eta^2\right), \end{aligned} \tag{1}$$

where $E\left(\varepsilon_t \eta_s\right) = 0$ for all t and s. By setting $g_t = h_t - \mu$, where $\mu = 2\ln\sigma$, we have an alternative representation, namely $y_t = \varepsilon_t \exp\left(0.5g_t\right)$ and $g_{t+1} = \mu + \phi\left(g_t - \mu\right) + \eta_t$.

For the *M*-dimensional stochastic vector, the MSV model of Harvey, Ruiz and Shephard (1994) is defined by

$$
\begin{aligned}
& y_t = \bar{D} D_t \varepsilon_t , \\
& \varepsilon_t \sim N(0, P), \\
& \bar{D} = \mathrm{diag}\{\sigma\}, \\
& D_t = \mathrm{diag}\{\exp(0.5 h_t)\} = diag\{\exp(0.5 h_{1,t}), \exp(0.5 h_{2,t}), \ldots \exp(0.5 h_{M,t})\},
\end{aligned}
\tag{2}
$$

where P is the correlation matrix, σ is the M-vector of standard deviation parameters, and h_t is M-vector of the stochastic components, which follows a VAR(1) process given as

$$
\begin{aligned}
& h_{t+1} = \phi \circ h_t + \eta_t , \\
& \eta_t \sim N(0, \Sigma_\eta),
\end{aligned}
\tag{3}
$$

where the operator $\circ$ denotes the Hadamard (or element-by-element) product, ϕ is an M-dimensional coefficient vector and Σ_η is the covariance matrix. For convenience, we call this type of MSV model the 'basic MSV' model.

In the following, we present a closely related specification, the Factor MSV model, and then we introduce the Block-Structure MSV model.

2.1. Factor MSV model

An alternative class of MSV models was first introduced by Harvey et al. (1994), and then extended by Jacquier et al. (1995, 1999), and Chib et al. (2006), among others. The basic model has the following structure:

$$
\begin{aligned}
& y_t = Df_t + \varepsilon_t, \\
& \varepsilon_t \sim N\left(0, \operatorname{diag}\left\{\sigma_1^2, \sigma_2^2, \ldots \sigma_M^2\right\}\right), \\
& f_{i,t} = \exp\left(0.5 h_{i,t}\right) \xi_{i,t}, \quad i = 1, 2, \ldots k \\
& \xi_{i,t} \sim N\left(0,1\right), \\
& h_{i,t+1} = \phi_i h_{i,t} + \eta_{i,t}, \\
& \eta_t \sim N\left(0, I_k \sigma_\eta^2\right),
\end{aligned}
\tag{4}
$$

where D is the $m \times k$ matrix of factor loadings, f_t is the k-dimensional vector of factors, which follow univariate SV models, and all the innovation terms are mutually uncorrelated. This model has a limited number of parameters, but also has some drawbacks. In fact, as shown in Asai et al. (2008), the conditions imposed on the mean innovations, ε_t, that is, homoscedasticity and diagonality of the covariance matrix, are too restrictive and not consistent with the empirical evidence. This is particularly evident when the number of factors, k, is much smaller than the number of assets, M. In this case, if the assets are used to create a portfolio, there must exist at least one vector of weights providing a homoskedastic portfolio.

2.2. Block Structure Model

We now assume that the M assets are divided into B groups, with the i-th group containing m_i assets ($M = m_1 + m_2 + \cdots + m_B$). We define a block structure for the volatility by assuming that each group of assets is characterized by a common parametric behaviour in the volatility equation. Consider the variance dynamics of the Harvey et al. (1994) model in equations (1)-(3):

$$
\begin{aligned}
&h_{t+1} = \Phi h_t + \eta_t, \\
&\eta_t \sim N\left(0, \Sigma_\eta\right),
\end{aligned}
\tag{5}
$$

where the matrix of parameters of persistence, Φ , and the covariance matrix of log-volatility, Σ_η, will have constraints given by block structures. We define

$$
\begin{aligned}
&\Phi = \begin{pmatrix} \phi_1 I_{m_1} & O & O & O \\ O & \phi_2 I_{m_2} & O & O \\ O & O & \ddots & O \\ O & O & O & \phi_B I_{m_B} \end{pmatrix}, \\
&\Sigma_\eta = \begin{pmatrix} \sigma_{u,11} I_{m_1} & \sigma_{u,21} L_{m_1,m_2} & \cdots & \sigma_{u,B1} L_{m_1,m_B} \\ \sigma_{u,21} L_{m_2,m_1} & \sigma_{u,22} I_{m_2} & \cdots & \sigma_{u,B2} L_{m_2,m_B} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{u,B1} L_{m_B,m_1} & \sigma_{u,B2} L_{m_B,m_2} & \cdots & \sigma_{u,BB} I_{m_B} \end{pmatrix},
\end{aligned}
\tag{6}
$$

where $L_{m,n}$ is an $m \times n$ matrix with diagonal elements of ones and off-diagonal elements of zeros. If the number of assets in each block is the same, namely $m_i = M/B$, we have $\Phi = \mathrm{diag}\{\phi_1, \ldots, \phi_B\} \otimes I_M$ and $\Sigma_\eta = \Sigma_u \otimes I_M$, where $\Sigma_u = \{\sigma_{u,ij}\}$, and $\otimes$ is the Kronecker product. By construction, the vector of volatilities has a block-structure given that the factors affecting the overall volatilities are sector or block specific. Hereafter, we refer to the model in equations (2), (5) and (6) as the Volatility Block Structure (VBS) MSV model.

For convenience, we will show the structure of each block in detail. Denote the vector of filtered returns of the *i*-th group as $y_{it} = \left(y_{1t}^{(i)}, y_{2t}^{(i)}, \ldots, y_{m_i t}^{(i)}\right)'$, where $y_{jt}^{(i)}$ is the filtered return of the *j*-th asset in the *i*-th group. Similarly, denote $h_{it} = \left(h_{1t}^{(i)}, h_{2t}^{(i)}, \ldots, h_{m_i t}^{(i)}\right)'$. For the mean-adjusted returns of the *i*-group, we have the following structure:

$$\begin{aligned} y_{it} &= \bar{D}_i D_{it} \varepsilon_t, \quad \varepsilon_t \sim N\left(0, P_i\right), \\ \bar{D}_i &= \mathrm{diag}\{\sigma_i\}, \\ D_{it} &= \mathrm{diag}\left\{\exp\left(0.5 h_{it}\right)\right\} \\ h_{i,t+1} &= \phi_i h_{it} + u_{it}, \quad u_{it} \sim N\left(0, \sigma_{u,ii} I_{m_i}\right), \end{aligned} \tag{7}$$

where P_i is the correlation matrix, $\sigma_i = \left(\sigma_1^{(i)}, \ldots, \sigma_{m_i}^{(i)}\right)'$, and $\sigma_j^{(i)}$, ϕ_i and $\sigma_{u,ii}$ are scalar parameters. We refer to this as the 'one-block SV' model.

Now we turn to the vector of volatilities of the *i*-th block, which is defined as

$$v_{it} = \sigma_i \mathrm{diag}\left\{\exp\left(0.5 h_{it}\right)\right\}. \tag{8}$$

Thus, the vector of volatilities for all assets is given by $v_t = \left(v'_{1t}, \ldots, v'_{Bt}\right)'$. We specify the mean equation of $y_t = \left(y'_{1t}, \ldots, y'_{Bt}\right)'$ as

$$y_t = V_t \varepsilon_t, \quad \varepsilon_t \sim N\left(0, P\right), \tag{9}$$

where $V_t = \mathrm{diag}\left(v_t\right)$, and P is the correlation matrix.

The numbers of parameters in $\sigma = \left(\sigma'_1, \ldots, \sigma'_B\right)'$ and P are M and $M\left(M-1\right)/2$, respectively. For the equation of the main components, the numbers of parameters in ϕ and Σ_u are B and $B\left(B+1\right)/2$, respectively. When $M = 12$ and $B = 4$ ($M = 50$ and $B = 5$), the number of parameters in the BS-MSV model is 92 (1295). For the MSV model of Harvey, Ruiz and Shephard (1994), the number of parameters for the case $M = 12$ ($M = 50$) is 168 (2600). Thus, the BS-MSV model is parsimonious in terms of the number of parameters.

The BS-MSV model still suffers from the number of parameters, which increases with the speed of M^2. Thus, we propose the Complete BS (CBS) model, which consists of equations (2), (5) and (6), subject to the restriction:

$$P=\begin{pmatrix} P_1 & q_{12}J_{12} & \cdots & q_{1B}J_{1B} \\ q_{12}J_{12} & P_2 & \cdots & q_{2B}J_{2B} \\ \vdots & \vdots & \ddots & \vdots \\ q_{1B}J_{1B} & q_{2B}J_{2B} & \cdots & P_B \end{pmatrix} \tag{10}$$

where $P_i=\left\{p_{jk}^{(i)}\right\}$ and $Q=\left\{q_{lm}\right\}$ are the $m_i \times m_i$ and $B \times B$ correlation matrices, respectively, and J_{ij} is an $m_i \times m_j$ matrix of ones. In this model, Q captures the correlations between the blocks. As the number of parameters in P is given by $B\left(B-1\right)/2+\sum_{i=1}^{B} m_i\left(m_i-1\right)/2$, the CBS-MSV model has $M+B+B^2+\sum_{i=1}^{B} m_i\left(m_i-1\right)/2$ parameters. In the limiting case of blocks characterized by the same number of assets, when $M=12$ and $B=4$ ($M=50$ and $B=5$), the number of parameters in the CBS-MSV model is 56 (305). Therefore, the CBS model drastically reduces the number of parameters. With the further restriction that $p_{jk}^{(i)}=p^{(i)}$, the CBS-MSV model has $M+2B+B^2$ parameters. In this case, assuming again that each block includes M/B series, $M=12$ and $B=4$ ($M=50$ and $B=5$) yields 36 (85) parameters.

Note that similar block structures could be used for the specification of the factor loading matrix, *D*, of the model in (4). In this alternative representation, the latent factors could be associated with the specific blocks created with the assets. Alternatively, making the *D* matrix unrestricted, block specifications could be used to generalize the model in (4) by introducing spillovers across the factor variances and by removing the diagonality assumption over the innovation covariance matrices.

2.3 Model estimation

For the estimation of the MSV models, we use the Monte Carlo likelihood (MCL) approach proposed by Durbin and Koopman (1997). Sandmann and Koopman (1998) applied the MCL method to the univariate SV model, while Asai and McAleer (2006) adapted it for the MSV model. These two papers rely on the logarithmic transformation of squared returns, as in Harvey, Ruiz and Shephard (1994), allowing a state-space form with non-Gaussian measurement errors. In the MCL method, the likelihood function can be approximated arbitrarily by decomposing it into a Gaussian part, which is constructed by the Kalman filter, and a remainder function, for which the expectation is evaluated through simulation.

3. Empirical analysis

Three groups of three assets from three different sectors (B=3 and M=9) are used, namely Chemical, General Financials, and Oil and Gas Producers. Table 1 reports the selected stocks and a descriptive analysis of their returns. These assets have been selected from among a small list of the largest companies between each sector on the basis of the correlations between the squared returns. All the selected stocks belong to the large cap segment of the NYSE, and enter the S&P 500 index. Given the approach followed in the asset selection, intuitively there possibly exist common patterns in the variances. We chose such a selection approach in order to provide an example where the proposed modelling approach may be useful. We believe that the block structur MSV model may be of little interest if the assets under study all belong to different sectors or if they are characterized by low correlations.

The series considered are total return indices, collected in the sample period 2 January 2002 to 10 April 2007, giving T=1375 observations. Note that the period covered excludes the effects of the technology market drawdown, while it may be influenced by the wars in Afghanistan and Iraq and by the increasing trend in oil prices. Furthermore, we exclude the global financial crisis period.

Table 1 reports a preliminary descriptive analysis of the 9 stocks, showing that in the period considered the average returns are positive (the stock market was characterized by an upward trend in prices), and very close between assets of the same sector, while there is a slight difference between sectors: the chemical sector has lower returns than the general financial sector which is, in turn, dominated by the oil and gas producer sector. This is a somewhat expected result, given the relevant increase in the oil prices in the later years of the sample. The standard deviations of the oil sectors are the smallest, while the General Financial sector has the highest risk. The Chemical sector has the most leptokurtic densities; the Oil and Gas Produces stocks are negatively skewed, while the others are all positively skewed, a fact that is also reflected in the median returns.

The correlations within each sector are quite high, around 0.65 for the Chemical sector, about 0.8 for the General Financial stocks, and close to 0.75 for the Oil and Gas Producers firms. Between sectors, the correlations are lower and vary between 0.32 and 0.54. Notably, the correlations between assets of different sectors have a block-like structure: the correlations between a chemical firm and a general financial stock are close to 0.5, higher than the correlations between a chemical and an oil producer firm, at around 0.4. Finally, the correlations between a general financial firm and an oil producer firm are again close to 0.4.

In order to develop the conditional mean for each return, we used the following data sets; a set of interest rates (US Treasury bond 3 months, 6 months, 9 months, 1-3 years, 3-5 years, 5-7 years), oil prices, and two dummies (January and Monday). Interest rates are in the form of bond indices. Following Ait-Sahalia and Brandt (2001) and Pesaran and Timmerman (1995, 2000), we fit the conditional mean returns with the constant term, the lagged return, the contemporaneous dummies, the lagged Oil returns, and the deviations between the returns of the rates (the following differences between bond indices returns: 6 months minus 3 months, 1-3 years minus 6 months, and so on), giving 10 explanatory variables, as follows:

$$E\left(R_t\right) = \beta_1 + \beta_2 R_{t-1} + \beta_3 D_t^{Jan} + \beta_4 D_t^{Mon} + \beta_5 R_t^{Oil} + \beta_6 V_{1t} + \cdots + \beta_{10} V_{5t} .$$

The deviations between the rates, V_{it}, can be considered as a proxy for the curvature of the yield curve, and hence may be useful in predicting stock movements.

In the proposed equation, the curvature of the yield curve and the oil returns are contemporaneous. Clearly, the model may suffer from simultaneity problems, given that the explanatory variables may be predictable, and we are not including an appropriate equation for their behaviour. In order to validate the returns model, we run a set of causality tests, specifically we consider both the Granger causality test and a test for bidirectional causality. In the first case, we run standard causality tests based on a lagged relation between variables for all pairs of stock returns and explanatory variables. In all cases we have limited evidence of causality (few tests have p-values below 0.2, and none is below 0.05).

In order to implement the bidirectional causality test, we estimated two simultaneous systems, the restricted version without contemporaneous feedback from the explanatory variables to the stocks. In this case (we have 54 restrictions, 6 restrictions on each equation, the 5 variables related to the bonds and the oil price, for the 9 stocks), the restricted likelihood is 97571.22, the full likelihood is 97605.34, and the LR test statistic is 68.24. Assuming an asymptotic density following a chi-square distribution with 54 degrees of freedom, we have a p-value of 0.092. We interpret this result as a rejection of the bidirectional causality (even if the decision was not extremely clear). Given the outcome of the tests, we can safely run the analysis on the equation with a contemporaneous relationship.

Table 2 gives the results for the conditional mean equation. The number in the first column denotes the corresponding explanatory variable. For instance, #1 is the constant term, and # 2 is the AR(1) coefficient. The heteroskedasticity consistent standard errors are given in parentheses. Although most of the parameter estimates are insignificant, there are some exceptions, such as R_t and V_{2t} for AIR PRDS. & CHEMS.

For the volatility equation, we first estimated the univariate SV models defined in equation (1). Table 3 shows the MCL estimates for the univariate models. Although the estimates of ϕ for the financial sector are relatively low, the estimates are typical of those available in the literature for SV models. Furthermore, we note a similarity between the volatility constants, σ, associated with the stocks belonging to the same sector.

We also estimated the basic trivariate SV models (2) and **Error! Reference source not found.** for the three sectors, and Table 4 presents the results. By introducing the off-diagonal elements of Σ_η and *P*, the estimates of ϕ are smaller than the corresponding estimates in Table 3 for all the sectors. The correlation coefficients based on Σ_η are very high and replicate the ordering of Table 1, with the Chemical sector characterized by lower correlations between assets. As the estimate for Σ_η is very close to a singular matrix for the chemical sector, the standard errors are unreliable, and hence are not reported. As the values of ϕ are close to each other in each sector, and since the estimates of Σ_η indicate the existence of common movements in η_t, we need to consider common structures by using factor models and/or the BS models.

Table 5 shows the log-likelihood, AIC and BIC for the trivariate SV model. Comparing these values with those in Table 3, we conduct an LR test for the off-diagonal elements of Σ_η and *P*. The test statistic has a $\chi^2(6)$ asymptotic density, and we are able to reject the null hypothesis that all the off-diagonal elements of Σ_η and *P* are equal to zero for all three sectors.

As the first step of our BS approach, we estimated the one-block trivariate SV model for the three sectors. Table 6 shows the MCL results for the one-block model. Due to the BS approach, the estimates of ϕ are less than the smallest values of the corresponding sector in Table 3. Table 7 gives the log-likelihood, AIC and BIC for the one-block model. The one-block model for the chemical sector has smaller AIC and BIC values than the basic MSV model in (2) and **Error! Reference source not found.**. For the remaining two sectors, BIC favours the one-block model, while AIC chooses the basic MSV model. As a result, we find that the BS approach would be a good candidate for effectively reducing the number of parameters for high dimensional models.

Next, we consider the 6-variate SV model with 2 blocks. Tables 8-10 show the MCL estimates for the combination of sectors {(General Financials, Oil and Gas Producers), (General Financials, Chemicals) and (Oil and Gas Producers, Chemicals)}. With respect to Tables 9 and 10, the estimates of ϕ became relatively low by including the chemical sector. Finally, we report in Table 11 the estimates of the full 9-variate model with three blocks associated with the three economic sectors. The parameter estimates are in line with those reported in Tables 8-10.

A direct comparison of the BS specifications with the full model estimate is not directly available due to the computation complexity of the 6-variate and 9-variate full models. Only three-variate specifications are available, both in their full and BS specifications, and standard likelihood ratio tests clearly favour the full models. However, when the cross-sectional dimensions increase, the BS specifications remain feasible, while the full SV models are not. This is a particularly strong advantage of the model presented in the paper, which also maintains the parameter interpretation, and also allows for correlated innovations as the matrices P and Σ_η are not restricted to be diagonal or block-diagonal.

A more detailed comparison of the full and BS specifications for stochastic volatility models is left to future research.

4. Conclusion

In this paper we presented a class of multivariate stochastic volatility models which is nested in the model of Harvey et al. (1994). A distinctive feature of our model is that, contrary to fully parameterized MSV models, it remains feasible in moderate to large cross-sectional dimensions. This result is achieved by imposing a block structure on the model parameter matrices. The variables could be grouped by using some economic or financial criteria, or could follow data-driven classifications. In addition, by the introduction of blocks, if these have an economic interpretation, the model proposed preserves the interpretation of the coefficients, a feature which is generally lost in feasible MSV models.

We also presented an empirical application where the proposed model was estimated for a set of US equities, showing its feasibility. A more advanced comparison between the BS specification and alternative MSV models is left for future research.

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Table 1: Descriptive Statistics, and Covariance and Correlation Matrices

Panel a: Descriptive Statistics

	100xMean	100xMedian	10xSt.dev.	Min.	Max.	Asymmetry	Kurtosis
CHEMICALS							
Air Products & Chemicals	0.042	<0.001	0.145	-0.062	0.071	0.024	5.119
Rohm & Haas	0.038	<0.001	0.159	-0.058	0.082	0.422	5.443
Eastman Chemicals	0.051	<0.001	0.163	-0.093	0.127	0.254	9.957
GENERAL FINANCIALS							
Goldman Sachs Group	0.062	<0.001	0.159	-0.069	0.070	0.097	4.811
Lehman Brothers Holding	0.059	<0.001	0.177	-0.071	0.081	0.158	4.653
Merrill Lynch & Co.	0.042	<0.001	0.172	-0.082	0.085	0.019	5.604
OIL AND GAS PRODUCERS							
Chevron	0.052	0.039	0.132	-0.069	0.053	-0.395	4.891
Exxon Mobil	0.058	0.085	0.137	-0.088	0.093	-0.295	7.208
Conocophillips	0.070	0.059	0.154	-0.064	0.056	-0.283	3.951

Panel b: Covariance and Correlation Matrices

	1	2	3	4	5	6	7	8	9
1	0.00021	**0.73269**	**0.61095**	0.53726	0.51572	0.52548	0.40045	0.46694	0.35719
2	0.00017	0.00025	**0.67517**	0.53321	0.51677	0.52087	0.39271	0.46088	0.32575
3	0.00014	0.00017	0.00026	0.46894	0.45004	0.46141	0.38057	0.42790	0.32588
4	0.00012	0.00014	0.00012	0.00025	**0.82244**	**0.80014**	0.37420	0.42386	0.33972
5	0.00013	0.00015	0.00013	0.00023	0.00031	**0.79181**	0.38655	0.42824	0.34614
6	0.00013	0.00014	0.00013	0.00022	0.00024	0.00030	0.39852	0.44392	0.34378
7	0.00008	0.00008	0.00008	0.00008	0.00009	0.00009	0.00017	**0.81550**	**0.76500**
8	0.00009	0.00010	0.00010	0.00009	0.00010	0.00010	0.00015	0.00019	**0.74794**
9	0.00008	0.00008	0.00008	0.00008	0.00009	0.00009	0.00015	0.00016	0.00024

Note: The numbers in the first column and first row identify the assets following the asset order included in the first panel. The main diagonal contains the variances, the lower triangular portion of the matrix contains the covariances, and the upper part contains the correlations. Entries in **bold** identify correlations between assets belonging to the same group.

Table 2: OLS Estimates for AR(1)+X Filter

$$E\left(R_t\right) = \beta_1 + \beta_2 R_{t-1} + \beta_3 D_t^{Jan} + \beta_4 D_t^{Mon} + \beta_5 R_t^{Oil} + \beta_6 V_{1t} + \cdots + \beta_{10} V_{5t}$$

	AIR PRDS.& CHEMS.	ROHM & HAAS	EASTMAN CHEMICALS	GOLDMAN SACHS GP.	LEHMAN BROS.HDG.	MERRILL LYNCH & CO.
1	0.052257 (0.046047)	0.084552 (0.050201)	0.067398 (0.050442)	0.054578 (0.050055)	0.039213 (0.056137)	0.080864 (0.052324)
2	**-0.077430** (0.037420)	-0.069550 (0.035631)	-0.018482 (0.038766)	-0.024159 (0.032569)	-0.016149 (0.031604)	0.021568 (0.032707)
3	-0.074751 (0.13259)	-0.14643 (0.14241)	-0.31699 (0.16288)	0.010587 (0.12788)	0.090702 (0.13867)	-0.13165 (0.14648)
4	0.014976 (0.090221)	-0.11898 (0.099846)	0.084124 (0.10393)	0.088226 (0.10812)	0.065721 (0.12071)	-0.095373 (0.11541)
5	0.013533 (0.024869)	-0.028756 (0.025734)	-0.027399 (0.028098)	-0.033285 (0.026247)	-0.018488 (0.029564)	-0.046207 (0.028043)
6	3.1120 (6.4079)	2.8912 (7.2347)	8.4777 (7.3226)	-0.38731 (7.0470)	6.3135 (7.8102)	0.78673 (7.5957)
7	**-3.2335** (1.4022)	-3.1210] (1.5698)	-2.5573 (1.5909)	-3.9432 (1.6209)	-5.6362 (1.7475)	-4.2337 (1.7772)
8	0.38499 (1.0361)	-0.13320 (1.1541)	-0.14864 (1.1445)	1.1068 (1.2072)	1.9268 (1.3164)	0.33607 (1.3198)
9	-2.6849 (1.4334)	-2.7242 (1.6277)	-3.0337 (1.5659)	-4.7312 (1.6841)	-3.6699 (1.9407)	-4.3171 (1.8594)
10	-0.22876 (0.30836)	0.047840 (0.31261)	0.21566 (0.27479)	0.35943 (0.51958)	0.019772 (0.56653)	0.18090 (0.42547)

Note: The explanatory variables are explained in the text. The heteroskedasticity consistent standard errors are given in parentheses.

Table 2 (Cont.): OLS Estimates for AR(1)+X Filter

$$E\left(R_t\right)=\beta_1+\beta_2 R_{t-1}+\beta_3 D_t^{Jan}+\beta_4 D_t^{Mon}+\beta_5 R_t^{Oil}+\beta_6 V_{1t}+\cdots+\beta_{10} V_{5t}$$

	CHEVRON	EXXON MOBIL	CONOCOPHILLIPS
1	0.059555 (0.040970)	0.070209 (0.043159)	0.063182 (0.048373)
2	-0.048486 (0.037402)	-0.068125 (0.038850)	0.011006 (0.033751)
3	-0.093713 (0.12285)	-0.041243 (0.12445)	-0.030855 (0.15012)
4	0.042643 (0.092067)	-0.011393 (0.087914)	0.058970 (0.10621)
5	-0.00074654 (0.022122)	0.012146 (0.022738)	-0.019895 (0.027144)
6	-4.6020 (6.5236)	-2.7178 (6.4400)	1.4510 (7.5031)
7	-1.2256 (1.2764)	-2.9594 (1.3506)	-2.1886 (1.3993)
8	0.42700 (0.94531)	1.3214 (1.0640)	1.1725 (0.97668)
9	-2.3026 (1.3921)	-2.5810 (1.4706)	-2.5616 (1.4800)
10	-0.13796 (0.43368)	-0.47793 (0.36712)	-0.13105 (0.37475)

Note: The explanatory variables are explained in the text. The heteroskedasticity consistent standard errors are given in parentheses.

Table 3: MCL Estimates for Univariate SV Models

$$y_t = \sigma\varepsilon_t \exp(0.5h_t), \quad \varepsilon_t \sim N(0,1),$$
$$h_{t+1} = \phi h_t + \eta_t, \quad \eta_t \sim N(0,\sigma_\eta^2)$$

	ϕ	σ_η	σ	LogLik	AIC	BIC
AIR PRDS.& CHEMS.	0.9221	0.2418	1.2607	-3002.03	6010.06	6025.72
	(0.0346)	(0.0670)	(0.0598)			
ROHM & HAAS	0.7944	0.4132	1.3517	-3054.91	6115.82	6131.48
	(0.0732)	(0.0854)	(0.0486)			
EASTMAN CHEMICALS	0.8234	0.4563	1.2933	-3029.88	6065.76	6081.41
	(0.0477)	(0.0628)	(0.0546)			
GOLDMAN SACHS GP.	0.9626	0.1494	1.4251	-3015.25	6036.51	6052.17
	(0.0199)	(0.0450)	(0.0819)			
LEHMAN BROS.HDG.	0.9806	0.1197	1.5830	-2968.64	5943.27	5958.93
	(0.0095)	(0.0299)	(0.1335)			
MERRILL LYNCH & CO.	0.9929	0.0740	1.5414	-2964.94	5935.88	5951.54
	(0.0046)	(0.0193)	(0.2060)			
CHEVRON	0.9575	0.1543	1.1952	-2903.96	5813.92	5829.58
	(0.0158)	(0.0308)	(0.0629)			
EXXON MOBIL	0.9717	0.1365	1.1995	-2925.08	5856.16	5871.82
	(0.0125)	(0.0309)	(0.0803)			
CONOCOPHILLIPS	0.9858	0.0935	1.3946	-2926.95	5859.90	5875.56
	(0.0074)	(0.0226)	(0.1231)			

Table 4: MCL Estimates for Basic Trivariate SV Models

$$y_t = \bar{D} D_t \varepsilon_t, \quad \varepsilon_t \sim N\left(0, P\right),$$
$$\bar{D} = \mathrm{diag}\left\{\sigma\right\}, \quad D_t = \mathrm{diag}\left\{\exp\left(0.5 h_t\right)\right\},$$
$$h_{t+1} = \phi \circ h_t + \eta_t, \quad \eta_t \sim N\left(0, \Sigma_\eta\right)$$

	ϕ	Σ_η			σ	P	
AIR PRDS.&	0.4346	0.6335			1.1261	1	
CHEMS.	(NA)	(NA)			(NA)		
ROHM & HAAS	0.4334	0.6381	0.6433		1.2429	0.6344	1
	(NA)	(NA)	(NA)		(NA)	(NA)	
EASTMAN	0.4726	0.6081	0.6057	0.6488	1.2360	0.5175	0.5957
CHEMICALS	(NA)	(NA)	(NA)	(NA)	(NA)	(NA)	(NA)
GOLDMAN	0.9251	0.0539			1.2873	1	
SACHS GP.	(0.0265)	(0.0229)			(0.0616)		
LEHMAN	0.9643	0.0351	0.0253		1.4920	0.70279	1
BROS.HDG.	(0.0128)	(0.0137)	(0.0112)		(0.0963)	(0.0167)	
MERRILL	0.9729	0.0292	0.0208	0.0198	1.4754	0.69111	0.76335
LYNCH & CO.	(0.0102)	(0.0115)	(0.0087)	(0.0089)	(0.1087)	(0.0204)	(0.0128)
CHEVRON	0.8712	0.1130			1.0705	1	
	(0.0518)	(0.0539)			(0.0428)		
EXXON MOBIL	0.8974	0.0946	0.0824		1.1358	0.7010	1
	(0.0389)	(0.0415)	(0.0365)		(0.0478)	(0.0179)	
CONOCOPHILL	0.9479	0.0525	0.0429	0.0328	1.3907	0.6800	0.6767
IPS	(0.0213)	(0.0233)	(0.0187)	(0.0155)	(0.0695)	(0.0218)	(0.0196)

Note: Standard errors are given in parentheses. Given that Σ_η is close to singular, the standard errors are unreliable and hence are not reported.

Table 5: AIC and BIC for Basic Trivariate SV Models

	Chemicals	General Financials	Oil and Gas Producers
LogLike	-8777.7	-8508.6	-8406.5
AIC	17585.4	17047.2	16842.9
BIC	17663.7	17125.5	16921.2

Table 6: MCL Estimates for One-Block Trivariate SV Models

$$y_{it} = \bar{D}_i D_{it} \varepsilon_t, \quad \varepsilon_t \sim N(0, P_i),$$
$$\bar{D}_i = \mathrm{diag}\{\sigma_i\}, \quad D_{it} = \mathrm{diag}\{\exp(0.5 h_{it})\}$$
$$h_{i,t+1} = \phi_i h_{it} + u_{it}, \quad u_{it} \sim N(0, \sigma_{u,ii} I_{m_i}),$$

	ϕ_i	$\sigma_{u,ii}$	σ_i	P_i	
AIR PRDS.& CHEMS.			1.2154 (0.0411)	1	
ROHM & HAAS	0.8109 (0.0409)	0.1337 (0.0316)	1.3524 (0.0448)	-0.7204 (0.0180)	1
EASTMAN CHEMICALS			1.3245 (0.0464)	-0.6400 (0.0251)	-0.6894 (0.0204)
GOLDMAN SACHS GP.			1.3472 (0.1462)	1	
LEHMAN BROS.HDG.	0.9935 (0.0276)	0.0031 (0.0012)	1.5592 (0.1702)	0.7080 (0.0135)	1
MERRILL LYNCH & CO.			1.5482 (0.1694)	0.6869 (0.0160)	0.7706 (0.0119)
CHEVRON			1.1061 (0.0681)	1	
EXXON MOBIL	0.9739 (0.0073)	0.0128 (0.0037)	1.1644 (0.0717)	0.7173 (0.0153)	1
CONOCOPHILLIPS			1.4059 (0.0864)	0.6730 (0.0186)	0.6804 (0.0171)

Note: Standard errors are given in parentheses.

Table 7: AIC and BIC for One-Component Trivariate SV Models

	Chemicals	General Financials	Oil and Gas Producers
LogLike	-8875.86	-8545.33	-8454.49
AIC	17767.7	17106.7	16925.0
BIC	17809.5	17148.4	16966.7

Table 8: MCL Estimates for BS-MSV Models
(General Financials, Oil and Gas Producers)

$$y_t = \bar{D} D_t \varepsilon_t, \quad \varepsilon_t \sim N(0, P),$$
$$\bar{D} = \mathrm{diag}\{\sigma\}, \quad D_t = \mathrm{diag}\{\exp(0.5 h_t)\},$$
$$h_{t+1} = \Phi h_t + \eta_t, \quad \eta_t \sim N(0, \Sigma_\eta),$$
$$\Phi = \mathrm{diag}\{\phi_1, \dots, \phi_B\} \otimes I_M, \quad \Sigma_\eta = \Sigma_u \otimes I_M.$$

	ϕ	Σ_u	
GOLDMAN SACHS GP. LEHMAN BROS.HDG. MERRILL LYNCH & CO.	0.9961 (0.0021)	0.0020 (0.0009)	
CHEVRON EXXON MOBIL CONOCOPHILLIPS	0.9760 (0.0067)	0.0019 (0.0009)	0.0115 (0.0033)

	σ	P					
GOLDMAN SACHS GP.	1.3506 (0.1971)	1					
LEHMAN BROS.HDG.	1.5623 (0.2286)	0.7081 (0.0135)	1				
MERRILL LYNCH & CO.	1.5797 (0.2338)	0.6796 (0.0163)	0.7690 (0.0120)	1			
CHEVRON	1.1179 (0.0708)	-0.4462 (0.0412)	-0.4553 (0.0399)	-0.4012 (0.0471)	1		
EXXON MOBIL	1.1799 (0.0748)	-0.4161 (0.0429)	-0.4280 (0.0419)	-0.4194 (0.0371)	0.7163 (0.0152)	1	
CONOCOPHILLIPS	1.4300 (0.0921)	-0.4271 (0.0428)	-0.4476 (0.0400)	-0.4282 (0.0424)	0.6697 (0.0185)	0.6778 (0.0170)	1

LogLike	AIC	BIC
-16939.6	33931.2	34066.9

Note: Standard errors are given in parentheses.

Table 9: MCL Estimates for BS-MSV Models (General Financials, Chemicals)

$$
\begin{aligned}
&y_t = \bar{D} D_t \varepsilon_t, \quad \varepsilon_t \sim N(0, P), \\
&\bar{D} = \operatorname{diag}\{\sigma\}, \quad D_t = \operatorname{diag}\{\exp(0.5 h_t)\}, \\
&h_{t+1} = \Phi h_t + \eta_t, \quad \eta_t \sim N(0, \Sigma_\eta), \\
&\Phi = \operatorname{diag}\{\phi_1, \ldots, \phi_B\} \otimes I_M, \quad \Sigma_\eta = \Sigma_u \otimes I_M.
\end{aligned}
$$

	ϕ	Σ_u	
GOLDMAN SACHS GP. LEHMAN BROS.HDG. MERRILL LYNCH & CO.	0.9743 (0.0068)	0.0107 (0.0037)	
AIR PRDS.& CHEMS. ROHM & HAAS EASTMAN CHEMICALS	0.8817 (0.0334)	0.0204 (0.0056)	0.0867 (0.0258)

	σ	P					
GOLDMAN SACHS GP.	1.3086 (0.1789)	1					
LEHMAN BROS.HDG.	1.5036 (0.0954)	0.7076 (0.0158)	1				
MERRILL LYNCH & CO.	1.4683 (0.2354)	0.6871 (0.0182)	0.7639 (0.0162)	1			
AIR PRDS.& CHEMS.	1.2232 (0.1017)	-0.5114 (0.0343)	-0.5390 (0.0361)	-0.5463 (0.0335)	1		
ROHM & HAAS	1.3680 (0.0519)	-0.4351 (0.0433)	-0.4602 (0.0632)	-0.5118 (0.0492)	-0.7109 (0.0172)	1	
EASTMAN CHEMICALS	1.3464 (0.0512)	-0.4236 (0.0495)	-0.4675 (0.0770)	-0.3970 (0.0551)	-0.6219 (0.0243)	-0.6810 (0.0208)	1

LogLike	AIC	BIC
-17311.2	34674.5	34810.2

Note: Standard errors are given in parentheses.

Table 10: MCL Estimates for BS-MSV Models
(Oil and Gas Producers, Chemicals)

$$y_t = \bar{D} D_t \varepsilon_t, \quad \varepsilon_t \sim N(0, P),$$
$$\bar{D} = \mathrm{diag}\{\sigma\}, \quad D_t = \mathrm{diag}\{\exp(0.5 h_t)\},$$
$$h_{t+1} = \Phi h_t + \eta_t, \quad \eta_t \sim N(0, \Sigma_\eta),$$
$$\Phi = \mathrm{diag}\{\phi_1, \ldots, \phi_B\} \otimes I_M, \quad \Sigma_\eta = \Sigma_u \otimes I_M.$$

	ϕ	Σ_u	
CHEVRON EXXON MOBIL CONOCOPHILLIPS	0.9597 (0.0101)	0.0178 (0.0051)	
AIR PRDS.& CHEMS. ROHM & HAAS EASTMAN CHEMICALS	0.8413 (0.0353)	0.0150 (0.0054)	0.1080 (0.0261)

	σ	P					
CHEVRON	1.1108 (0.0555)	1					
EXXON MOBIL	1.1688 (0.0595)	0.7245 (0.0154)	1				
CONOCOPHILLIPS	1.4075 (0.0734)	0.6777 (0.0191)	0.6883 (0.0174)	1			
AIR PRDS.& CHEMS.	1.2169 (0.0436)	-0.3623 (0.0687)	-0.4741 (0.0409)	-0.4139 (0.0478)	1		
ROHM & HAAS	1.3572 (0.0190)	-0.5088 (0.0365)	-0.4591 (0.0386)	-0.4648 (0.0375)	-0.7127 (0.0177)	1	
EASTMAN CHEMICALS	1.3285 (0.0477)	-0.3989 (0.0520)	-0.4789 (0.0396)	-0.3859 (0.0551)	-0.6237 (0.0252)	-0.6810 (0.0207)	1

LogLike	AIC	BIC
-17260.4	34572.7	34708.4

Note: Standard errors are given in parentheses.

Table 11: MCL Estimates for BS-MSV Models

(General Financials, Oil and Gas Producers, Chemicals)

$$
\begin{aligned}
&y_t = \bar{D} D_t \varepsilon_t, \quad \varepsilon_t \sim N(0, P),\\
&\bar{D} = \mathrm{diag}\{\sigma\}, \quad D_t = \mathrm{diag}\{\exp(0.5 h_t)\},\\
&h_{t+1} = \Phi h_t + \eta_t, \quad \eta_t \sim N(0, \Sigma_\eta),\\
&\Phi = \mathrm{diag}\{\phi_1, \ldots, \phi_B\} \otimes I_M, \quad \Sigma_\eta = \Sigma_u \otimes I_M.
\end{aligned}
$$

	ϕ	Σ_u		
GOLDMAN SACHS GP. LEHMAN BROS.HDG. MERRILL LYNCH & CO.	0.9873 (0.0058)	0.0043 (0.0012)		
CHEVRON EXXON MOBIL CONOCOPHILLIPS	0.9618 (0.0094)	0.0019 (0.0012)	0.0171 (0.0049)	
AIR PRDS.& CHEMS. ROHM & HAAS EASTMAN CHEMICALS	0.8598 (0.0489)	0.0096 (0.0040)	0.0143 (0.0052)	0.0902 (0.0326)

LogLike	AIC	BIC
-25660.0	51428.0	51709.8

Note: Standard errors are given in parentheses. Estimates for σ and P are omitted.