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The Rebound Effect with Energy Production: A Partial Equilibrium Analysis

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Abstract: Rebound is the extent to which improvements in energy efficiency fail to translate fully into reductions in energy use because of the implicit fall in the price of energy, when measured in efficiency units. This paper discusses aspects of the rebound effect that are introduced once energy is considered as a domestically produced commodity. A partial equilibrium approach is adopted in order to incorporate both energy use and production in a conceptually tractable way. The paper explores analytically two interesting results revealed in previous numerical simulations. The first is the possibility that energy use could fall by more than the implied improvement in efficiency. This corresponds to negative rebound. The second is the finding that the short-run rebound value can be greater than the corresponding long-run value.

Key words: Energy demand, energy efficiency, rebound, partial equilibrium
JEL classification: Q41, Q43, Q55, Q56

1. Introduction

This paper discusses aspects of the rebound effect that are introduced once energy is considered as a domestically produced commodity. Rebound is the extent to which improvements in energy efficiency fail to translate fully into reductions in energy use because of the implicit fall in the price of energy, when measured in efficiency units (Brookes, 1978; Jevons, 1865; Khazzoom, 1980). Our previous work has concentrated on analysing this phenomenon in a general equilibrium setting, using numerical simulation (Allan *et al.*, 2006; Hanley *et al.* 2007, Turner, 2009). The present paper explores in greater depth two interesting results revealed in these simulations. The first is the possibility that energy use could fall by more than the implied improvement in efficiency. This corresponds to negative rebound. The second is the finding that the short-run rebound value can be greater than the corresponding long-run value. This simulation finding contradicts previous theoretical work (Saunders, 2008; Wei, 2007).

A partial equilibrium approach is adopted in order to incorporate both energy use and production in a conceptually tractable way. This facilitates discussion of a key aspect of the determinants of energy use: the fact that energy is an important intermediate input in its own production. A partial equilibrium analysis also allows a diagrammatic representation of key results.[i] The paper is organised in the following way. Section 2 outlines the partial equilibrium framework. Sections 3 and 4 analyse the impact of an efficiency improvement in energy efficiency in final demand and intermediate use respectively and Section 5 is a short conclusion.

2. Partial Equilibrium Framework

The partial equilibrium set up is as follows. There is a unified market for energy, which is wholly domestically supplied. Domestic demand for energy is made up of two elements, which we label final and intermediate demand. Final demand for energy includes not only consumption and export demand, but also the demand for energy as an input in other, non-energy, industries. Therefore in this paper the term "intermediate demand" refers solely to the use of energy by the energy sector itself.[ii]

Improvements in energy efficiency can occur either in the use of energy in final or intermediate demands and these two types of improvement are treated separately here.

Final demand for energy, measured in natural units, is a function of the energy price, P, energy efficiency, Γ, and a vector Z of other variables, so that:

(1) $$E_D^F = E_D^F(P, \Gamma^F, Z_D^F)$$

where E is the quantity of energy, the D subscript identifies demand and the F superscript final demand.[iii] Demand for energy for intermediate use is given as:

(2) $$E_D^I = E_D^I(E_D^T, \Gamma^I, Z_D^I)$$

where the I superscript here represents intermediate demand. Recall that this is energy used in the production of energy. Energy as an intermediate good is a derived demand and therefore dependant on the total demand for energy, E_D^T. Further, there is no energy price in equation (2) because the price of energy as an input is fixed relative to the price of energy as an output. However, as will be clear later in this section, a change in the efficiency parameter, Γ^I, changes the price of the energy input measured in efficiency units. This does affect the intermediate demand for energy. Total energy demand is then the sum of the final and intermediate demands, so that:

(3) $$E_D^T = E_D^F + E_D^I$$

where the T superscript identifies total.

The total domestic supply of energy is given as a function of energy price and the efficiency of energy use in the production of energy and again a vector of other variables. This is expressed here as an inverse supply function where:

(4) $$P = P(E_S^T, \Gamma^I, Z^P)$$

and in equilibrium the total domestic energy demand is met by total domestic supply.

(5) $$E^T = E_S^T = E_D^T$$

Essentially we have six equations to determine the six endogenous variables: $E_D^F, E_D^I, E_S^T, E_D^T, E^T$ and *P*. In this paper we are primarily concerned with the impact of exogenous changes in energy efficiency, Γ^F and Γ^I, on total energy output, E^T. All the other exogenous variables represented by the vectors Z_D^F, Z_D^I and Z^P are held constant.

We begin by presenting equations (1) to (5) in proportionate terms.

(6) $$e_D^F = -\eta_P^F p + \eta_\Gamma^F \gamma^F$$

(7) $$e_D^I = e_D^T + \eta_\Gamma^I \gamma^I$$

(8) $$e_D^T = \frac{e_D^F}{1+\alpha} + \frac{\alpha e_D^I}{1+\alpha}$$

(9) $$p = \frac{e_S^T}{\beta_P^T} + \lambda_\Gamma^I \gamma^I$$

(10) $$e_S^T = e_D^T = e^T$$

In equations (6) to (10), the lower case letters represent the proportionate changes in the corresponding upper case variables shown in equations (1) to (5) and the sub and superscripts retain the same meaning. Therefore, for example, the proportionate change in energy final demand is:

$$e_D^F = \frac{dE_D^F}{E_D^F}.$$

The proportionate changes in energy efficiency are represented by γ. Other parameters are as follows: η represents elasticity of demand, β elasticity of supply and λ the elasticity of supply price with respect to the energy efficiency in production. The parameter α is the initial share of energy intermediate demand to final demand. The following restrictions apply to the parameter values:

$$\beta_P^T, \eta_P^F \geq 0, \lambda_\Gamma^I < 0 \text{ and } 1 > \alpha > 0.$$

Without further information, we cannot sign the two key energy efficiency demand elasticities, $\eta_\Gamma^F, \eta_\Gamma^I$. In equation (6) we have imposed the requirement that the production function be linear homogeneous, so that implicitly $\eta_S^I = 1$. Finally, the other exogenous variables that affect energy demand and price are assumed constant so that: $z_D^F, z_D^I, z^P = 0$

Simultaneously solving equations (6) to (10) generates the result:

$$e^T = \left[\frac{\beta_P^T}{\beta_P^T + \eta_P^F}\right]\left[\eta_\Gamma^F \gamma^F + (\alpha\eta_\Gamma^I - \lambda_\Gamma^I \eta_P^F)\gamma^I\right] \tag{11}$$

In the remaining sections of the paper we separately consider the impact on domestic energy output of efficiency improvements in energy final demand and intermediate use. Both algebraic and diagrammatic approaches are used. But before this more detailed treatment, it is useful to consider some general points about equation (11). Given the restrictions on parameter values, the first term on the RHS lies between zero and one. That is to say:

$$1 > \frac{\beta_P^T}{\beta_P^T + \eta_P^F} > 0$$

This has two implications. First, the direction of the change in total energy output is determined by the sign of the second term on the RHS of equation (11). This depends crucially on the sign of the direct energy efficiency elasticities, $\eta_\Gamma^F, \eta_\Gamma^I$ and λ_Γ^I, which have yet to be determined. Second, the term $\left[\frac{\beta_P^T}{\beta_P^T + \eta_P^F}\right]$ represents the operation of the energy market which limits the variation in energy use around zero. That is to say, the increase in price as energy demand increases - and the reduction in energy price as energy demand falls - restricts the size of the deviation in total energy output that results from any efficiency change.

For this analysis it is useful to make a distinction between energy as measured in natural units, E, and energy measured in efficiency units, E^E. The energy supply sector delivers energy in natural units and concern over the level of energy use, either in terms of sustainability or the impact on global warming, generally relates to use measured in these units. However, measuring energy in efficiency units better reflects the useful work that energy performs. Therefore in looking more closely at the demand for energy this will prove to be more easily expressed in terms of a demand for efficiency units. These different measures are linked through the efficiency parameter Γ, so that energy in efficiency units is given by:

(12) $$E^E = \Gamma E$$

Similarly the price of energy in efficiency units, P^E, is given as:

(13) $$P^E = \frac{P}{\Gamma}$$

Initially the efficiency parameter is taken to equals unity, so that $E = E^E$.

3. An improvement in efficiency in final demand energy use: $\gamma^F > 0$, $\gamma^I = 0$.

3.1 Impact on domestic energy production

Setting $\gamma^I = 0$ in equation (11), the key parameter in determining the change in energy production is the elasticity of final demand for energy with respect to changes in energy efficiency in final demand, η_Γ^F. An efficiency improvement changes energy demand, measured in efficiency units, through the change in energy price (measured in the same units). Specifically:

(14) $$e_D^{F,E} = -\eta_P^F p^E$$

Equation (14) can be restated in terms of prices and quantities measured in natural units, using equations (12) and (13) expressed in proportionate terms. These expressions are:

(15) $$e_D^{F,E} = e_D^F + \gamma^F$$

and

(16) $$p^E = p - \gamma^F$$

Substituting equations (15) and (16) into equation (14) and imposing a zero change in the price of energy measured in natural units gives:

(17) $$e_D^F = (\eta_P^F - 1)\gamma^F$$

So that:

(18) $$\eta_\Gamma^F = \frac{\partial e_D^F}{\partial \gamma^F} = (\eta_P^F - 1)$$

Substituting equation (18) in equation (11), and imposing $\gamma^I = 0$, produces:

(19) $$e^T = \left[\frac{\beta_P^T}{\beta_P^T + \eta_P^F}\right](\eta_P^F - 1)\gamma^F$$

where

$$\frac{\partial e^T}{\partial \eta_P^F} = \frac{\beta_P^T(\beta_P^T + 1)\gamma^F}{(\beta_P^T + \eta_P^F)^2} > 0, \quad \frac{\partial e^T}{\partial \beta_P^T} = \frac{\eta_P^F(\eta_P^F - 1)\gamma^F}{(\beta_P^T + \eta_P^F)^2} > 0 \; iff \; \eta_P^F > 1$$

Equation (19) gives the proportionate change in total domestic energy output resulting from an improvement in energy efficiency in final demand. This is shown to depend solely on the price elasticity of final energy demand and the price elasticity of supply.

The proportionate change in energy production is positively related to the final demand price elasticity. More specifically, where $\eta_P^F = 0$, so that energy demand is completely price inelastic, domestic energy production falls by the full amount of the efficiency change: $e^T = -\gamma^F$. Where final energy demand has unitary elasticity, so that $\eta_P^F = 1$, domestic energy production remains unchanged following increased energy efficiency. Where energy final demand is price elastic, with $\eta_P^F > 1$, energy production rises in line with energy efficiency in final demand use.

The impact of changing the price elasticity of supply is a little more complex. Specifically, where energy demand is elastic, raising the elasticity of supply increases energy output and reduces the increase in energy price. On the other hand, where energy demand is price inelastic, making the energy supply more elastic increases the fall in domestic energy output and reduces the fall in energy price.

3.2 Diagrammatic representation

It is of pedagogic interest to represent these results diagrammatically. Combining equations (6), (7), (8), (10) and (18) and imposing $\gamma^I = 0$ gives the total energy demand function in this case as:

(20) $$e_D^T = e_D^F = -\eta_P^F p + (\eta_P^F - 1)\gamma^F$$

Note first that the proportionate change in total energy demand is equal to the proportionate change in final energy demand. This is because there is no change in the ratio between final and intermediate demand for energy: there is no change in the efficiency of energy use as an intermediate input in the production of energy.

Equation (20) indicates that introducing an energy efficiency improvement produces a parallel shift in the energy demand curve. However, the extent (and even the direction) of this shift depends on the price elasticity of demand, that is that the slope of the demand curve.

The situation is illustrated diagrammatically in Figure 1. This shows the energy demand curves calibrated as proportionate changes in quantity demanded and price from the original equilibrium (0,0). Two initial demand curves, D_L^1 and D_H^1 are shown, with low and high price elasticities respectively. After the efficiency shock, the new demand curves must pass through the point ($-\gamma^F$, γ^F). That is to say, from equation (16), if the price of energy in natural units increases by a proportionate amount γ^F, the price in efficiency units remains unchanged. In these circumstances, the demand for energy in efficiency units stays constant. From equation (15) this implies that the demand in natural units falls by the proportion γ^F. This point ($-\gamma^F$, γ^F) is on the negatively sloped 45° line passing through the initial equilibrium, (0,0).

Where energy demand is inelastic, so that $\eta_P^F < 1$, the slope of the total energy demand curve is steeper than 45°. This applies to demand curve D_L^1 here. For the new energy demand curve to pass through the point ($-\gamma^F$, γ^F), it shifts inwards, to the left. This is represented by the curve D_L^2 in Figure 1. On the other hand, if energy demand is elastic, so that the slope is greater than 45°, the new energy demand curve still goes through the point ($-\gamma^F$, γ^F), but this now represents a shift outwards, to the right. This is illustrated by curve D_H^2 in Figure 1.

Figure 2 shows the energy market equilibria for improvements in the efficiency of energy use in final demand. The supply relationship is given by equation (9), imposing $\gamma^I = 0$ so that:

(21) $$e_S^T = \beta_P^T p$$

D_L^2 and D_H^2 are taken from Figure 1 and represent price inelastic and elastic energy demand curves. For the inelastic demand curve, D_L^2, the proportionate change in energy use and price (e_L and p_L) are both negative, whilst with the elastic demand, D_H^2, both e_H and p_H are positive. Figure 3 illustrates the impact of increasing the supply elasticity where energy demand is price inelastic. The supply curve S_H is more price elastic than the curve S_L. As the supply elasticity increases, the change in energy output falls from e_L to e_H and the proportionate price fall is reduced from p_L to p_H.

3.3. Rebound

Rebound measures the extent to which the change in energy output falls short of the improvement in energy efficiency. Rebound is driven by the reduction in the price of energy, measured in efficiency units, generated by the efficiency improvement. One important consideration is the fact that in the case under consideration at present, the efficiency improvement only applies to a subset of energy uses; that is, final demand. The degree of rebound, R^F, is therefore defined as:

(22)
$$R^F = \frac{\frac{dE^T}{E_D^F} + \gamma^F}{\gamma^F} = \frac{(1+\alpha)e^T}{\gamma^F} + 1$$

Where the reduction in energy production, expressed as a proportion of the energy initially used in final demand, is equal to the efficiency change, so that $\frac{dE^T}{E_D^F} = -\gamma^F$, there is no rebound, $R^F = 0$. If there is no change in energy output, so the $\frac{dE^T}{E_D^F} = 0$, $R^F = 1$. If $\frac{dE^T}{E_D^F} > 0$, $R^F > 1$ and "backfire" occurs.

Substituting equation (19) into equation (22) gives:

(23)
$$R^F = \left[\frac{(1+\alpha)\beta_P^T(\eta_P^F - 1)}{\beta_P^T + \eta_P^F}\right] + 1$$

Where

$$\frac{\partial R^F}{\partial \eta_P^F} = \frac{(\beta_P^T(1+\alpha)(\beta_P^T+1)}{(\beta_P^T+\eta_P^F)^2} > 0, \quad \frac{\partial R^F}{\partial \beta_P^T} = \frac{\eta_P^F(1+\alpha)(\eta_P^F-1)}{(\beta_P^T+\eta_P^F)^2} > 0 \; iff \, \eta_P^F > 1$$ [iv]

Where $\eta_P^F = 0$, so that the energy final demand function is totally inelastic, $R^F = -\alpha$: that is to say, there is negative rebound. The fall in energy output, expressed as a proportion of the energy use subject to the efficiency improvement, is greater than the proportionate change in efficiency. This is because of the reduction in derived intermediate demand for energy that accompanies the reduction in final demand.[v] Where $\eta_P^F = 1$, so that the energy final demand function has unitary elasticity, rebound equals 1. There is no change in energy production as a result of the improvement in energy efficiency in final demand use. Where $\eta_P^F > 1$, backfire occurs.

From equation (23), for rebound to equal zero requires:

(24) $$(1+\alpha)\beta_P^T(\eta_P^F-1) = -\beta_P^T - \eta_P^F$$

which can be expressed as:

(25) $$\eta_P^F = \frac{\alpha\beta_P^T}{1+(1+\alpha)\beta_P^T}$$

where

$$\frac{\partial \eta_P^F}{\partial \beta_P^T} = \frac{\alpha}{\left[1+(1+\alpha)\beta_P^T\right]^2} > 0, \frac{\partial^2 \eta_P^F}{(\partial \beta_P^T)^2} = \frac{-2\alpha}{\left[1+(1+\alpha)\beta_P^T\right]^3} < 0 .$$

and

$$\eta_P^F \to \frac{\alpha}{1+\alpha} \; as \; \beta_P^T \to \infty$$

Figure 4 is constructed in η_P^F, β_P^T space. It shows the combinations of parameter values η_P^F, β_P^T that generate negative rebound, positive rebound and backfire effects for an efficiency improvement in energy final demand. The horizontal line along the η_P^F axis, where $\beta_P^T = 0$, and the vertical line, where $\eta_P^F = 1$, show the combinations of parameter values where R^F = *1*. There is 100% rebound. Where $\eta_P^F > 1$ and $\beta_P^T > 0, R^F > 1$, there is backfire. The parameter space defined by $\beta_P^T > 0$ and $1 > \eta_P^F > \frac{\alpha\beta_P^T}{1+(1+\alpha)\beta_P^T}$, generates positive rebound: $1 > R^F > 0$. Finally, for the parameter values $\beta_P^T > 0$ and $\eta_P^F < \frac{\alpha\beta_P^T}{1+(1+\alpha)\beta_P^T}$, then rebound is negative so that $R^F < 0$.

3.4 Rebound adjustment over time

If we introduce an efficiency shock, then the temporal adjustment depends on how the elasticities of demand and supply change over time. In general we expect demand and supply elasticities to be greater in the long run than in the short run. This is because it is possible to adjust more fully to the change in price. From equation (23) it is clear that the more elastic final energy demand becomes, the larger the degree of rebound and it is perhaps this result that implicitly drives the belief that the rebound value is greater in the long run than the short run. However, where energy is domestically produced, the elasticity of supply is also expected to increase over time, typically through adjustments in capacity. But equation (23) shows that the impact on rebound for changes in the supply elasticity parameter is ambiguous. This result result differs from that of Saunders (2008) and Wei (2007). Under some circumstances changes in the supply elasticity over time can potentially reverse the expected time path of the rebound effect.

For changes over time:

(26) $$\frac{\partial R^F}{\partial t} = \frac{\partial R^F}{\partial \eta_P^F}\frac{\partial \eta_P^F}{\partial t} + \frac{\partial R}{\partial \beta_P^T}\frac{\partial \beta_P^T}{\partial t}$$

As argued above, we expect that $\frac{\partial \eta_P^F}{\partial t}, \frac{\partial \beta_P^T}{\partial t} > 0$. From equation (23), this implies that if $\eta_P^F > 1$, then $\frac{\partial R^F}{\partial t} > 0$: we definitely get the expected qualitative result. Rebound (in this case backfire) will increase over time. However, if $\eta_P^F < 1$, then the result is uncertain. Expressing the price elasticity changes as proportionate changes, then $\frac{\partial R^F}{\partial t} > 0$ iff:

(27) $$\frac{\partial R^F}{\partial \eta_P^F}\frac{\partial \eta_P^F}{\partial t} > -\frac{\partial R^F}{\partial \beta_P^T}\frac{\partial \beta_P^T}{\partial t} \rightarrow \frac{\partial \eta_P^F}{\partial t}\frac{1}{\eta_P^F} > \left[\frac{1-\eta_P^F}{\beta_P^T+1}\right]\frac{\partial \beta_P^T}{\partial t}\frac{1}{\beta_P^T}$$

Given the restrictions on the parameter values, equation (27) shows that $\eta_P^F < 1$ is a necessary, but not sufficient condition for the short-run rebound to be greater than the long-run value. The proportionate increase in the supply elasticity also needs to be larger, and potentially a lot larger, than the proportionate increase in the demand elasticity. This second requirement might be thought to imply that rebound would still be expected to increase from the short- to the long-run time interval.

However, in conventional partial equilibrium analysis, the long-run supply elasticity can be much higher than the short run. In fact, long-run supply is often characterised as perfectly elastic. Indicating short- and long-run values with the superscripts S and L, this implies that $\beta_P^{T,L} \rightarrow \infty$ (Sraffa, 1926). Under these conditions, using equation (23):

(28) $$R^L = (1+\alpha)\eta_P^{F,L} - \alpha$$

Therefore using equation (23) and (28) for values of $\eta_P^{F,L} < 1$, the short-run and long-run rebound effects will be the same if:

(29) $$\eta_P^{F,S} = \frac{\beta_P^{T,S}\eta_P^{F,L}}{\beta_P^{T,S} + 1 - \eta_P^{F,L}}$$

where $\frac{\partial \eta_P^{F,S}}{\partial \eta_P^{F,L}}, \frac{\partial \eta_P^{F,S}}{\partial \beta_P^{T,S}} > 0, \frac{\partial^2 \eta_P^{F,S}}{\left(\partial \eta_P^{F,L}\right)^2} > 0, \frac{\partial^2 \eta_P^{F,S}}{(\partial \beta_P^{T,S})^2} < 0$ [vi]

Given the discussion around equations (23), (28) and (29), it is possible to identify combinations of parameter values for which the short-run rebound value is greater than the corresponding long-run value. First, it important to note that the relevant range for this relationship is $\eta_P^{F,L} \in (0,1)$ so that the upper bound value for the long-run demand elasticity is set at unity, producing:

(30) $$\bar{\eta}_P^{F,L} = 1$$

Second, the short-run demand elasticity must lie between an upper and lower bound: $\eta_P^{F,S} \in (\underline{\eta}_P^{F,S}, \bar{\eta}_P^{F,S})$. Given that $\frac{\partial \eta_P^F}{\partial t}$ is taken to be non-negative, the upper bound value for the short-run demand elasticity is the given by the actual long-run elasticity demand, so that:

(31) $$\bar{\eta}_P^{F,S} = \eta_P^{F,L}$$

From equation (29), the lower bound value for the short-run demand elasticity is given by:

(32) $$\underline{\eta}_P^{F,S} = \frac{\beta_P^{T,S}\eta_P^{F,L}}{\beta_P^{T,S} + 1 - \eta_P^{F,L}}$$

Combining equations (30), (31) and (32) implies that the short-run elasticity of demand must meet the following restrictions:

(33) $$1 > \eta_P^{F,L} > \eta_P^{F,S} > \frac{\beta_P^{T,S}\eta_P^{F,L}}{\beta_P^{T,S} + 1 - \eta_P^{F,L}}$$

Figures 5 and 6 show combinations of parameter values that satisfy inequality (33) and therefore produce short-run rebound effects that are greater than the corresponding long-run values. Figure 5 gives combinations of $\eta_P^{F,L}$ and $\eta_P^{F,S}$ for which this is true, for a given a value of $\beta_P^{T,S}$. Figure 6 shows similar combinations of $\beta_P^{T,S}$ and $\eta_P^{F,S}$, for a given value of $\eta_P^{F,L}$.

In Figure 5, equation (32) is used to generate the two end points and the midpoint of the lower bound short-run elasticity curve. The end points are given by the results: where $\eta_P^{F,L} = 0, \underline{\eta}_P^{F,S} = 0$ and where $\eta_P^{F,L} = 1, \underline{\eta}_P^{F,S} = 1$. Also, if $\eta_P^{F,L} = \frac{1}{2}, \underline{\eta}_P^{F,S} = \frac{\beta_P^{T,S}}{2\beta_P^{T,S} + 1}$. The upper bound value is given by equation (30) and is a 45° line through the origin. The set of parameter combinations that produce higher short-run rebound are indicated by the area between the two curves. Where the value of $\beta_P^{T,S}$ is zero, the whole area under the 45° line is shaded. In this case, the short-run rebound value takes the value 1 independently of the short-run demand elasticity, and this is always greater than the long-run rebound value (as long as $\eta_P^{F,L} < 0$). As $\beta_P^{T,S}$ increases, the lower-bound short-run elasticity curve gets closer and closer to the 45° line.

For a given value of $\eta_P^{F,L}$ (< 1), Figure 6 shows the combinations of values for $\eta_P^{F,S}$ and $\beta_P^{T,S}$ that provide a higher short-run than long-run rebound value. In this case the upper-bound value is given by the horizontal line where $\eta_P^{F,S} = \eta_P^{F,L}$. The lower-bound relationship is again determined from equation (32). Where $\beta_P^{T,S}$ is zero, as argued above, the lower-bound value for $\eta_P^{F,S}$ is also zero. As $\beta_P^{T,S} \to \infty, \underline{\eta}_P^{F,S} \to \eta_P^{F,L}$, and where $\beta_P^{T,S} = 1, \underline{\eta}_P^{F,S} = \frac{\eta_P^{F,L}}{2 - \eta_P^{F,L}}$. Again the area between the two curves in Figure 6 shows the combination of parameter values that give a higher short-run rebound.

4. An improvement in energy efficiency as an intermediate input: $\gamma^F=0, \gamma^I > 0$.

4.1Impact on domestic energy production

The paper now focuses on the impact of an efficiency improvement in the use of energy as an intermediate input. Recall this refers solely to the use of energy in the production of energy. Setting $\gamma^F=0, \gamma^I > 0$ in equation (11) produces:

$$e^T = \left[\frac{\beta_P^T}{\beta_P^T + \eta_P^F} \right] (\alpha \eta_\Gamma^I - \lambda_\Gamma^I \eta_P^F) \gamma^I \qquad (34)$$

To analyse fully the impact of this efficiency disturbance we need to investigate more thoroughly the determination of the elasticities η_Γ^I and λ_Γ^I. We again adopt both an algebraic and diagrammatic approach.

Begin with the intermediate energy demand, given generically in equation (2). Equation (16) demonstrates that increased energy efficiency in a particular use reduces the price of energy in that use, as measured in efficiency units. When energy is used as an intermediate input, this reduction in price increases the cost-minimising use of energy, measured in efficiency units, as against other inputs. However, equation (15), appropriately adjusted to refer to intermediate demand, indicates that with an increase in efficiency, a given energy use in efficiency units translates into a lower use in natural units.

Applying equation (16) in this case implies that the introduction of the efficiency improvement, γ^I, reduces the energy input prices, in efficiency units, relative to the energy output, in natural units, by γ^I. The application of a side relation of the Constant Elasticity of Substitution (CES) production function (Heathfield and Wibe, 1987) in this case produces the result:

(35) $$e_D^T - e_D^{I,E} = \sigma(p^E - p) = -\sigma\gamma^I$$

where σ is the elasticity of substitution, given a positive sign. Our concern is with the intermediate demand in natural units, so rearranging equation (35) and using equation (16) produces:

(36) $$e_D^T + \gamma^I \sigma = e_D^{I,E} = e_D^I + \gamma^I$$

Further rearranging equation (36) gives:

(37) $$e_D^I = e_D^T + (\sigma - 1)\gamma^I$$

which implies:

(38) $$\eta_\Gamma^I = \sigma - 1\,.$$

Further, combining equation (37) with equations (6), (8) and (10) and setting $\gamma^F = 0$ gives the energy demand curve, where γ^F=0, $\gamma^I > 0$, as:

(39) $$e_D^T = -\eta_P^F p + \alpha(\sigma - 1)\gamma^I$$

To analyse the impact on energy supply, note first that the increase in energy efficiency in production reduces the price of energy, measured in natural units. Using:

(40) $$p = -\alpha\gamma^I$$

The level of the non-energy intermediate input, N^I, drives the energy supply function. It is instructive to find the change in output resulting from the efficiency improvement, where the use of the non-energy input is at the initial level. For this to be the case the price of the non-energy input must be unchanged. Adopting and adapting the CES side relation introduced in equation (35):

(41) $$e_S^T - n_S^I = \sigma(p^N - p)$$

Substituting equation (40) into equation (41) and imposing $n_S^I, p^N = 0$ gives:

(42) $$e_S^T = \alpha\sigma\gamma^I$$

If the changes in energy supply price and quantity given in equations (40) and (42) are used to fit a new inverse supply function, this has the form:

(43) $$p = \frac{e_S^T}{\beta_P^T} - \alpha\left[\frac{\sigma + \beta_P^T}{\beta_P^T}\right]\gamma^I$$

so that:

(44) $$\lambda_T^I = -\alpha\left[\frac{\sigma + \beta_P^T}{\beta_P^T}\right].$$

Substituting equations (39) and (44) into equation (34) produces:

(45) $$e^T = \left[\frac{\alpha\left[(\beta_P^T + \eta_P^F)\sigma + (\eta_P^F - 1)\beta_P^T\right]}{\beta_P^T + \eta_P^F}\right]\gamma^I$$

Alternatively:

(46) $$e^T = \left[\frac{\alpha\left[\beta_P^T(\sigma - 1) + \eta_P^F(\beta_P^T + \sigma\right]}{\beta_P^T + \eta_P^F}\right]\gamma^I$$

Equations (45) and (46) show clearly two sufficient conditions for energy use to rise in response to an increase in efficiency of energy as an intermediate input (that is $e^T > 0$). These are that $\sigma > 1$ or $\eta_P^F > 1$.

In this case, the responsiveness of energy use to changes in the final demand and supply elasticities is given by:

$$\frac{\partial e^T}{\partial \eta_P^F} = \frac{\alpha\gamma^I \beta_P^T(\beta_P^T+1)}{(\beta_P^T+\eta_P^F)^2} > 0 \text{ and } \frac{\partial e^T}{\partial \beta_P^T} = \frac{\alpha\gamma^I \eta_P^F(\eta_P^F-1)}{(\beta_P^T+\eta_P^F)^2} > 0 \; iff\, \eta_P^F > 1$$

As with improvements in the energy efficiency in final demand, (equation 19), increases in the price elasticity of final demand for energy generate higher levels of domestic energy output. However, domestic energy output increases with the price elasticity of supply only where the final energy demand is price elastic.

Giving the final energy demand and supply elasticities extreme values produces the following results. For variations in the elasticity of supply, where $B_P^T = 0, e^T = \alpha\sigma\gamma^I$ and where $B_P^T \to \infty, e^T \to \alpha(\sigma - 1 + \eta_P^F)\gamma^I$. For the elasticity of demand, where $\eta_P^F = 0, e^T = \alpha(\sigma-1)\gamma^I$ and where $\eta_P^F \to \infty, e^T \to \alpha(B_P^T + \sigma)\gamma^I$

4.2 Diagrammatic representation

Again, comprehension is improved through the diagrammatic representation of the results generated algebraically. To begin, equation (39) gives the total energy demand expressed as proportionate changes. With no alteration in the efficiency of intermediate energy use, the total energy demand curve passes through the origin, with a negative slope equal to the price elasticity of energy final demand. This is shown as DD in Figure 7.

An improvement in energy efficiency in intermediate use produces a parallel shift in the total energy demand curve. The direction of this shift depends on the value of the elasticity of substitution in the production of energy, σ. Where this elasticity is greater than unity, the total energy demand curve shifts outwards by an amount equal to $\alpha(\sigma_H - 1)\gamma^I$. This term is the increase in energy used as an intermediate input, expressed as a proportion of the total energy demand. In Figure 7, $D_H D_H$ is the new total energy demand curve. Where the elasticity of substitution is less than unity, the total energy demand curve makes a parallel shifts inwards by an amount equal to $\alpha(1 - \sigma_L)\gamma^I$. This is represented by the curve D_LD_L. In this case, although the improvement in energy efficiency increases the use of energy as an intermediate input when measured in efficiency units, its use falls when measured in natural units.

Where total energy demand curves of different elasticities are compared, then these pass through the same point on the horizontal total energy output axis. This implies that in Figure 7, where the total energy demand curve shifts from DD to D_LD_L for example, then variations in the demand elasticity would be represented initially by curves having differing slopes which all pass through the origin. After the introduction of the efficiency improvement, these curves would all now pass through the point $(-\alpha(1-\sigma_L)\gamma^I, 0)$.

Where there is an improvement in energy efficiency in production, this is also accompanied by a shift of the energy supply function. The inverse supply curve, in which the supply price is given as a function of the level of output, is given by equation (43). Where there is no improvement in energy efficiency, the supply curve, SS in Figure 8, passes through the origin, with a positive slope equal to $\frac{1}{\beta_P^T}$. An improvement in energy efficiency in intermediate use produces a downward parallel shift of this function. As argued in Section 4.1, the new supply curve must pass through the point $(\alpha\sigma\gamma^I, -\alpha\gamma^I)$. The supply curve therefore has an intercept with the vertical price axis at the point $-\alpha\left[\frac{\sigma+\beta_P^T}{\beta_P^T}\right]\gamma^I$.

Where the supply elasticity is varied, changing the slope of the supply curve shows this. These initially all pass through the origin. After the introduction of the efficiency improvement in intermediate use, the supply curves then all pass through the point $(\alpha\sigma\gamma^I, -\alpha\gamma^I)$.

The adjustment to domestic energy output is found where the new supply and demand curves intersect. The first point that is clear is that where the elasticity of substitution in energy production is greater than unity, domestic energy output rises. Essentially, even where the output to meet final demand is constant, the substitution of energy for other intermediates used in the production of energy will mean that total energy demand and output will increase.

The more interesting case is where the elasticity of substitution is less than unity. This case is represented in Figure 9. Three energy demand curves are shown - D_LD_L, D_{UU} and D_HD_H – which represent energy final demand price elasticities that have, respectively, low, unitary and high values. There are also two energy supply elasticities, low and high, represented by S_LS_L and S_HS_H.

First note that with either supply curve, the domestic energy output rises as the final energy demand elasticity increases: $e_2^T > \alpha\sigma\gamma^I > e_3^T$ and $e_1^T > \alpha\sigma\gamma^I > e_4^T$. Second, as the supply elasticity varies, the impact on domestic energy production depends on the price elasticity of final energy demand. First, where demand elasticity is unity, the energy efficiency improvement in intermediate use generates the new equilibrium at the point ($\alpha\sigma\gamma^I$, $-\alpha\gamma^I$). Given that all the supply elasticities pass through this point, this equilibrium is invariant to changes in the elasticity of supply. However, if the final energy demand is price elastic, D_HD_H, an increase in the supply elasticity will increase domestic energy output: $e_1^T > e_2^T$. On the other hand, if the final demand energy demand is price inelastic, D_LD_L, then increasing the supply elasticity reduces the energy output: $e_4^T < e_3^T$. Where the demand and supply elasticities take extreme values, they are represented by vertical (for inelastic) and horizontal (for elastic) functions. It is therefore straightforward to verify the algebraic results surrounding equation (46).

Finally Figure 9 illustrates a key difference between an improvement in the final and intermediate demand energy efficiency. In the case of final demand efficiency improvements, an increase in the elasticity of supply never reduces energy output where backfire occurs. That is to say, with increased energy efficiency in final demand use, where energy output rises, an increase in the energy supply elasticity reinforces that increase.

However, with improvements in the efficiency of energy use as an intermediate input, this situation no longer holds. The case in Figure 8, where final energy demand is inelastic, is a situation where although backfire occurs, increasing the elasticity of supply reduces domestic energy output. Figure 9 presents a situation where the long-run supply curve, S_LS_L, is taken to be infinitely elastic whilst the short-run curve, S_SS_S is more inelastic. Note that, as compared to the initial position, in this case in the short-run energy output increases as a result of the efficiency improvement but in the long-run it falls: $e_S^T > 0, e_L^T < 0$.

4.3 Rebound

The rebound expression in this case is given by:

(47)
$$R^I = \frac{\frac{dE^T}{E_D^I} + \gamma^I}{\gamma^I} = \frac{(1+\alpha)e^T}{\alpha\gamma^I} + 1$$

Substituting equation (45)

(48)
$$R^I = \left[\frac{(1+\alpha)\left[(\beta_P^T + \eta_P^F)\sigma + (\eta_P^F - 1)\beta_P^T \right]}{\beta_P^T + \eta_P^F} \right] + 1$$

In Figure 11, we use equation (48) to segment the parameter space η_P^F, σ to show those sets of parameter values that generate negative rebound, positive rebound and backfire.

To begin, for backfire $R^I > 1$: energy use rises as a result of the efficiency improvement. From equation (48) this implies:

(49)
$$(\beta_P^T + \eta_P^F)\sigma + (\eta_P^F - 1)\beta_P^T > 0$$

Stating expression (49) as an equality and rearranging produces the locus of points where $R^I = 1$. These are where:

(50) $$\eta_P^F = \frac{\beta_P^T(1-\sigma)}{\beta_P^T+\sigma}$$

with $\frac{\partial \eta_P^F}{\partial \sigma} = -\frac{\beta_P^T(\beta_P^T+1)}{(\beta_P^T+\sigma)^2} < 0$, and $\frac{\partial^2 \eta_P^F}{\partial \sigma^2} = \frac{2\beta_P^T(\beta_P^T+1)}{(\beta_P^T+\sigma)^3} > 0$. Also where $\sigma = 0, \eta_P^F = 1$ and where $\eta_P^F = 0, \sigma = 1$.

To identify the locus of parameter values which generate zero rebound, set $R^I = 0$. This marks the border between situations of negative and positive rebound. Using equation (48), this implies:

(51) $$\frac{(1+\alpha)\left[(\beta_P^T+\eta_P^F)\sigma+(\eta_P^F-1)\beta_P^T\right]}{\beta_P^T+\eta_P^F} = -1$$

Rearranging equation (51) produces:

(52) $$\eta_P^F = \frac{\beta_P^T(\alpha-\sigma(1+\alpha))}{(\sigma+\beta_P^T)(1+\alpha)+1}$$

where

$$\frac{\partial \eta_P^F}{\partial \sigma} = \frac{-\beta_P^T(\beta_P^T+1)(1+\alpha)^2}{\left[(\sigma+\beta_P^T)(1+\alpha)+1\right]^2} < 0,$$

$$\frac{\partial \eta_P^F}{\partial \beta_P^T} = \frac{(\alpha-\sigma(1+\alpha))(\sigma(1+\alpha)+1)}{\left[(\sigma+\beta_P^T)(1+\alpha)+1\right]^2} > 0$$

$$\frac{\partial^2 \eta_P^F}{\partial \sigma^2} = \frac{2\beta_P^T(\beta_P^T+1)(1+\alpha)^3}{\left[(\sigma+\beta_P^T)(1+\alpha)+1\right]^3} > 0$$

and

$$\frac{\partial^2 \eta_P^F}{(\partial \beta_P^T)^2} = \frac{-2(1+\alpha)\left[(\alpha - \sigma(1+\alpha))(\sigma(1+\alpha)+1)\right]}{\left[(\sigma + \beta_P^T)(1+\alpha)+1\right]^3} < 0$$

Further, if $\sigma = 0, \eta_P^F = \frac{\beta_P^T \alpha}{\beta_P^T(1+\alpha)+1}$, if $\eta_P^F = 0, \sigma = \frac{\alpha}{1+\alpha}$, if $\beta_P^T = 0, \eta_P^F = 0$ and if $\beta_P^T \to \infty, \eta_P^F \to \frac{\alpha}{1+\alpha} - \sigma$

Figure 11 shows that where the energy efficiency improvement is in the use of energy as an intermediate input, the range of values for which there backfire or positive occurs is greater than where the efficiency gain is in the use of energy in final demand. Where the efficiency gain is to the use of energy in final demand, backfire is ruled out by inelastic final demand. However, where the efficiency gain is to intermediate demand, then the substitution of energy in the production of energy augments any final demand effects. As can be seen from Figure 11, an elasticity of substitution in production of greater than unity generates backfire, independently of the level of the elasticity of energy final demand. Again for negative rebound, the final demand elasticity that rules out negative rebound is lower where the energy efficiency improvement applies to the use of energy as an intermediate input.

4.4 Rebound adjustment over time

To determine the temporal adjustment associated with the efficiency shock to the intermediate use of energy, again adopt equation (26). That is to say, the change in the rebound effect over time is identified as the sum of the impact of changes in the price elasticities in final demand and total supply. Differentiating equation (26) by parts and expressing for an increase in energy efficiency in intermediate use gives:

(53) $$\frac{\partial R^I}{\partial t} = \frac{\partial R^I}{\partial e^T}\left[\frac{\partial e^T}{\partial \eta_P^F}\frac{\partial \eta_P^F}{\partial t} + \frac{\partial e^T}{\partial \beta_P^T}\frac{\partial \beta_P^T}{\partial t}\right]$$

Begin by differentiating expression (47), which gives the rebound associated with an increase in energy efficiency as an intermediate input, with respect to total energy.

(54) $$\frac{\partial R^I}{\partial e^T} = \frac{(1+\alpha)}{\alpha\gamma^I} > 0$$

Following the discussion in Section 3.4, the final energy demand and total energy supply elasticities are taken to increase over time, so that $\frac{\partial \eta_P^F}{\partial t}, \frac{\partial \beta_P^T}{\partial t} > 0$. From the discussion following equation (46), we know that as the energy final demand elasticity increases, so does the total energy change associated with an improvement in energy efficiency as an intermediate input. Therefore for the rebound to fall over time in this case, the effect of an increase in the total energy supply elasticity on total energy must be negative, and this effect must outweight the positive final demand effect. This implies that:

(55) $$-\frac{\partial e^T}{\partial \beta_P^T}\frac{\partial \beta_P^T}{\partial t} > \frac{\partial e^T}{\partial \eta_P^F}\frac{\partial \eta_P^F}{\partial t}.$$

Substituting the expressions for $\frac{\partial e^T}{\partial \eta_P^F}$ and $\frac{\partial e^T}{\partial \beta_P^T}$, as derived from equation (46), into equation (55) gives:

(56) $$\frac{\partial R^I}{\partial t} < 0 \; iff \; \frac{\partial \beta_P^T}{\partial t}\frac{1}{\beta_P^T} > \left[\frac{\beta_P^T + 1}{1-\eta_P^F}\right]\frac{\partial \eta_P^F}{\partial t}\frac{1}{\eta_P^F}$$

Surprisingly, this is the same condition that holds for the efficiency improvement in energy use in final demand. Whilst the rebound values are typically quite different, the determinants of the time path of the rebound effects are precisely the same. Again for the rebound value to fall over time, the price elasticity of energy final demand must be less than unity and the proportionate change in the supply elasticity over time must be greater than the change in the demand elasticity to the extent given in expression (56).

Again where the long-run total energy supply is characterised as being perfectly elastic, so that $\beta_P^{T,L} \rightarrow \infty$, the parameter values that generate a short-run rebound that is greater than the corresponding long-run value is identical to that in expression (29), discussed in Section 3.4. That is to say, the analysis comparing short-run and long-run rebound effects for improvements in energy use in final demand is the same as the analysis where the energy improvements apply to intermediate use. Similarly the Figures 5 and 6 are equally applicable.

The reason for the similarity in the dynamic results can be understood through comparing Figures 2 and Figure 9. These are the figures that show the energy change for the two types of energy improvement. The figures have different total energy demand and supply shifts. However, the relative positions of the two key points for the dynamic analysis are the same in both cases. These are the points through which the new total energy demand and supply curves pivot when the elasticities change. In both figures these points are on a negatively sloped 45º line with the demand point above the supply. This configuration generates the identical dynamic results.

5. Conclusion

This paper presents a partial equilibrium analysis of the impacts of energy efficiency improvements on energy output, where energy is a domestically produced commodity. The paper considers improvements in the efficiency of energy use in both intermediate and final demand and explains the basic factors underlying the size of rebound effects under these circumstances. There are three main findings.

First, where energy enters as an intermediate input in the production of energy itself, negative rebound effects can occur. That is to say, the reduction in total energy use is greater than proportionate increase in energy efficiency multiplied by the initial energy use that receives the efficiency improvement. This effect does not occur as the result of some exotic production function but rather through Input-Output type multiplier effects. Put most straightforwardly, if there are improvements in the efficiency of the use of energy as final demand, and this leads to a reduction of energy use in final demand, this will be accompanying by a corresponding reduction in the use of energy as an intermediate input. This potential negative feedback can occur even when the initial efficiency improvement is in the use of energy as an intermediate. Figures 4 and 11 show the parameter values for which negative rebound effects occur with the two types of energy efficiency improvement. Typically this requires the energy final demand elasticity to be very low and the supply elasticity to be high.

Second, the rebound effect is greater for improvements that occur in the use of energy as an intermediate input in its own production than in the use of energy in final demand. This is because in the case of efficiency improvements in the use of energy in production there is a stimulus to additional output that occurs both through the substitution in production but also through the stimulus in final demand through the reduced price of energy, in natural units. Here it is important to reiterate that we are considering only the use of energy in its own production: where it is used as an intermediate in the production of other commodities, in a partial equilibrium analysis this is treated as final demand. However, in Input-Output accounts, energy is typically identified as a major intermediate input in its own production.

Third, the adjustment that is made in supply of energy over time means that the short-run rebound effect can be greater than the corresponding long-run value. The partial equilibrium analysis implies that this outcome can occur both for improvements in the efficiency of energy in both final and intermediate demand uses. Moreover the conditions for this outcome to occur are identical in the two cases. It depends upon a number of key parameters. First, the long-run elasticity of energy final demand must be less than unity. Second, the increase in elasticity of total energy supply between the short run and the long run must be large, relative to the change in the elasticity of energy final demand. Essentially where energy final demand is price inelastic, capacity in domestic energy production will adjust downwards in the long run. Disinvestment will take place. The short run situation of low domestic energy prices due to excess capacity can maintain relatively high levels of energy production. However, adjustment over time implies energy prices rising over time as excess capacity is removed and a subsequent potential reduction in demand and output.

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Footnotes

[i] The primary general equilibrium impact on energy production and use ignored here is the effect on the overall level of economic activity and demand that would come through any rise in factor prices that occurs through the increased efficiency of the economy.

[ii] The term intermediate demand in a general equilibrium (for example Input-Output) system would refer to energy demand for all intermediate use.

[iii] The distinction between energy measured in natural and efficiency units is discussed in Section 3.1. Where energy measured in efficiency units is used in the analysis this is identified by an appropriate superscript.

[iv] The partial derivatives for the rebound expression given in equation (23) reflect those from total energy expression in equation (19), given that:

$$\frac{\partial R^F}{\partial \eta_P^F} = \frac{\partial R^F}{\partial e^T}\frac{\partial e^T}{\partial \eta_P^F}, \frac{\partial R^F}{\partial \beta_P^T} = \frac{\partial R^F}{\partial e^T}\frac{\partial e^T}{\partial \beta_P^T}$$

and from equation (23):

$$\frac{\partial R^F}{\partial e^T} = \frac{1+\alpha}{\gamma^F} > 0$$

[v] This is a standard Input-Output multiplier effect (Miller and Blaire, 2009)

[vi] The precise values are given as

$$\frac{\partial \eta_P^{F,S}}{\partial \eta_P^{F,L}} = \frac{\beta_P^T(\beta_P^T+1)}{(\beta_P^T+1-\eta_P^{F,L})^2} > 0$$

$$\frac{\partial^2 \eta_P^{F,S}}{(\partial \eta_P^{F,L})^2} = \frac{2\beta_P^T(\beta_P^T+1)}{(\beta_P^T+1-\eta_P^{F,L})^3} > 0$$

$$\frac{\partial \eta_P^{F,S}}{\partial \beta_P^T} = \frac{\eta_P^{F,L}(1-\eta_P^{F,L})}{(\beta_P^T+1-\eta_P^{F,L})^2} > 0$$

and

$$\frac{\partial^2 \eta_P^{F,S}}{(\partial \beta_P^T)^2} = \frac{-2\eta_P^{F,L}(1-\eta_P^{F,L})}{(\beta_P^T+1-\eta_P^{F,L})^3} < 0$$

Figure 1-Shifts in the energy demand function following improvements in energy efficiency in final demand use

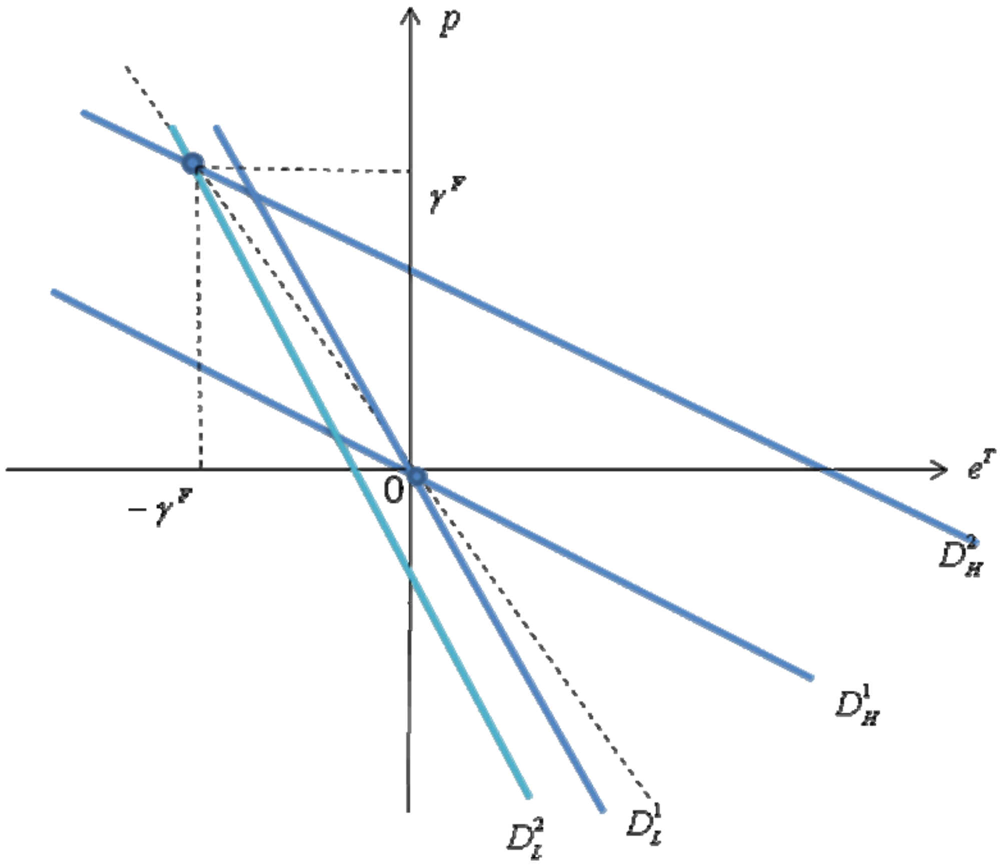

Figure 2-Equilibria following improvements in energy efficiency in final demand use with high and low elasticity final energy demand

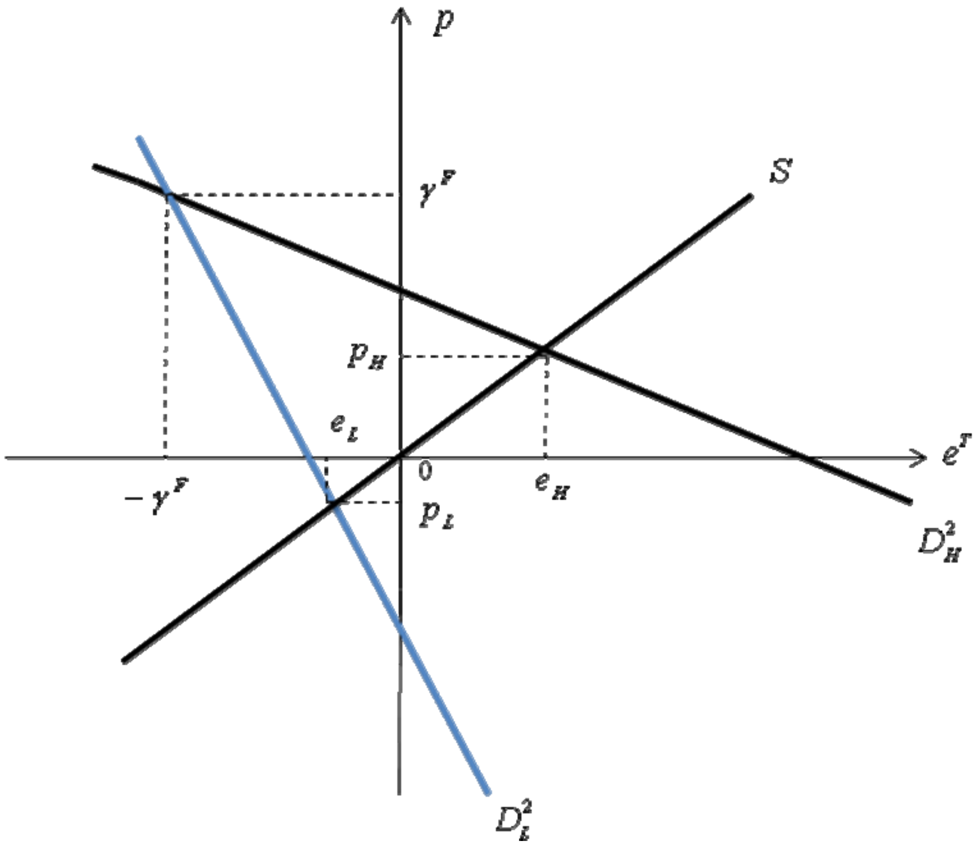

Figure 3-Equilibria following improvements in energy efficiency in final demand use with high or low energy supply elasticities

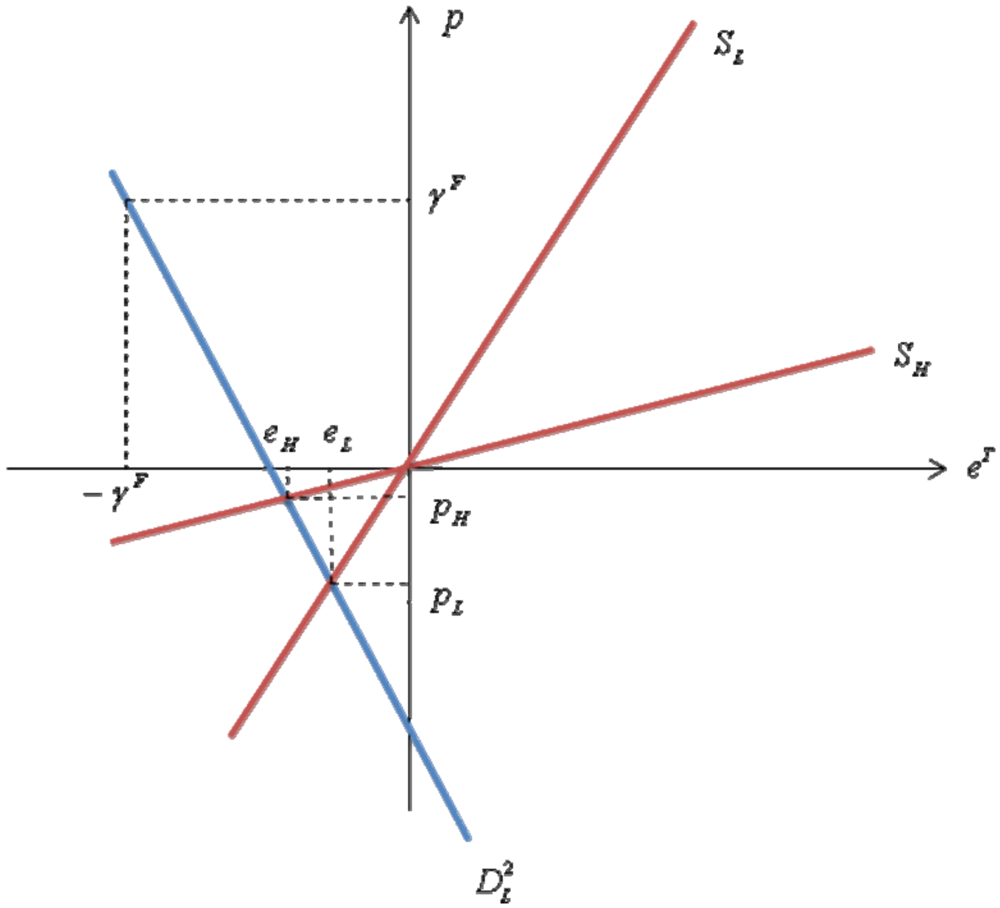

Figure 4- Parameter values producing negative rebound positive rebound and backfire responses following improvements in energy efficiency in final demand use

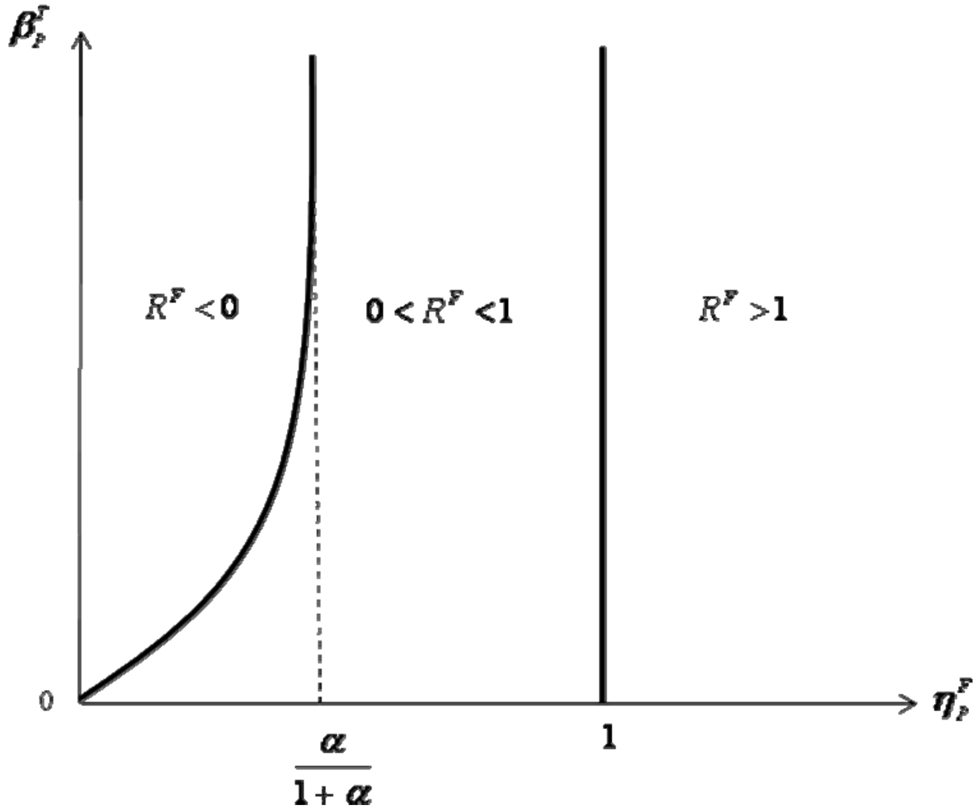

Figure 5- Combinations of the parameters $\eta_p^{F,L}$ and $\eta_p^{F,S}$ where the short run rebound effect is greater than the long run rebound effect following an improvement in energy efficiency.

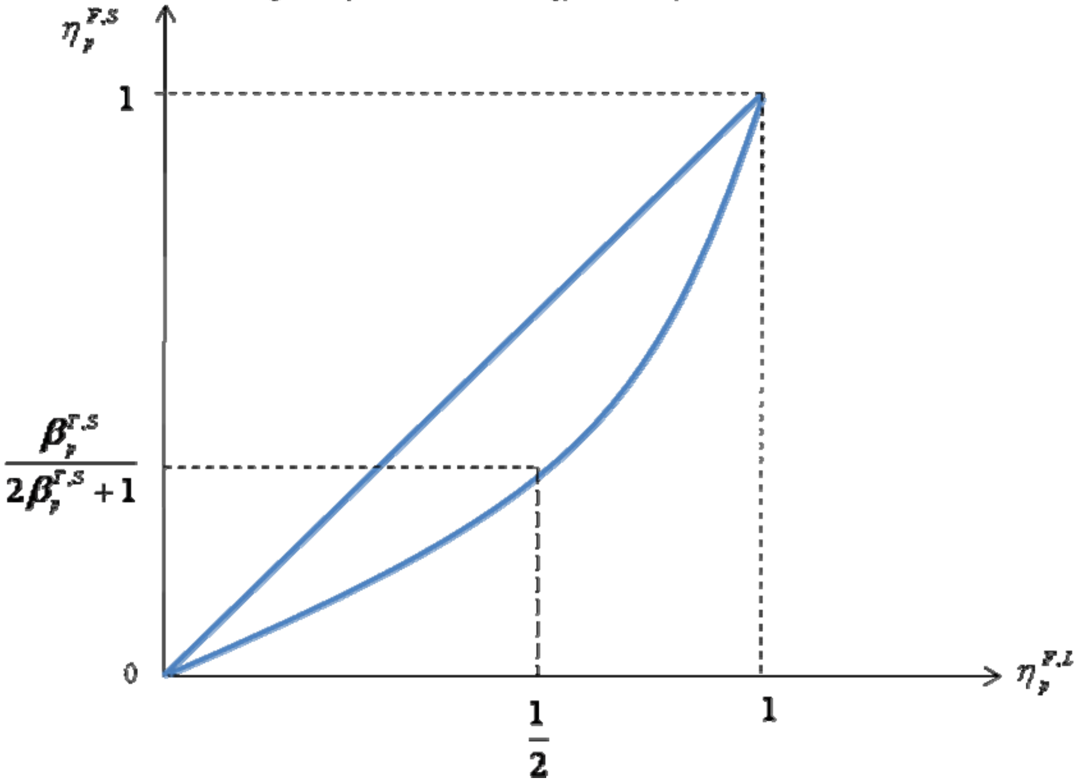

Figure 6- Combinations of the parameters $\eta_p^{F,S}$ and $\beta_p^{T,S}$ where the short run rebound effect is greater than the long run effect following an improvement in energy efficiency in final demand.

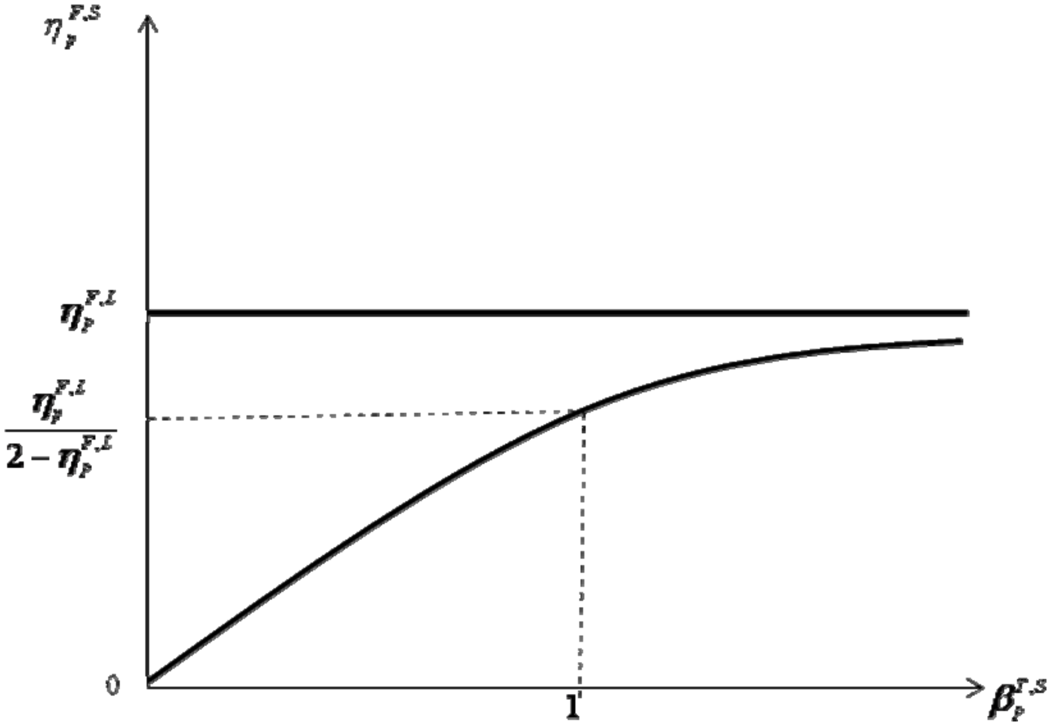

Figure 7-Shifts in the total energy demand function following improvements in energy efficiency in intermediate use.

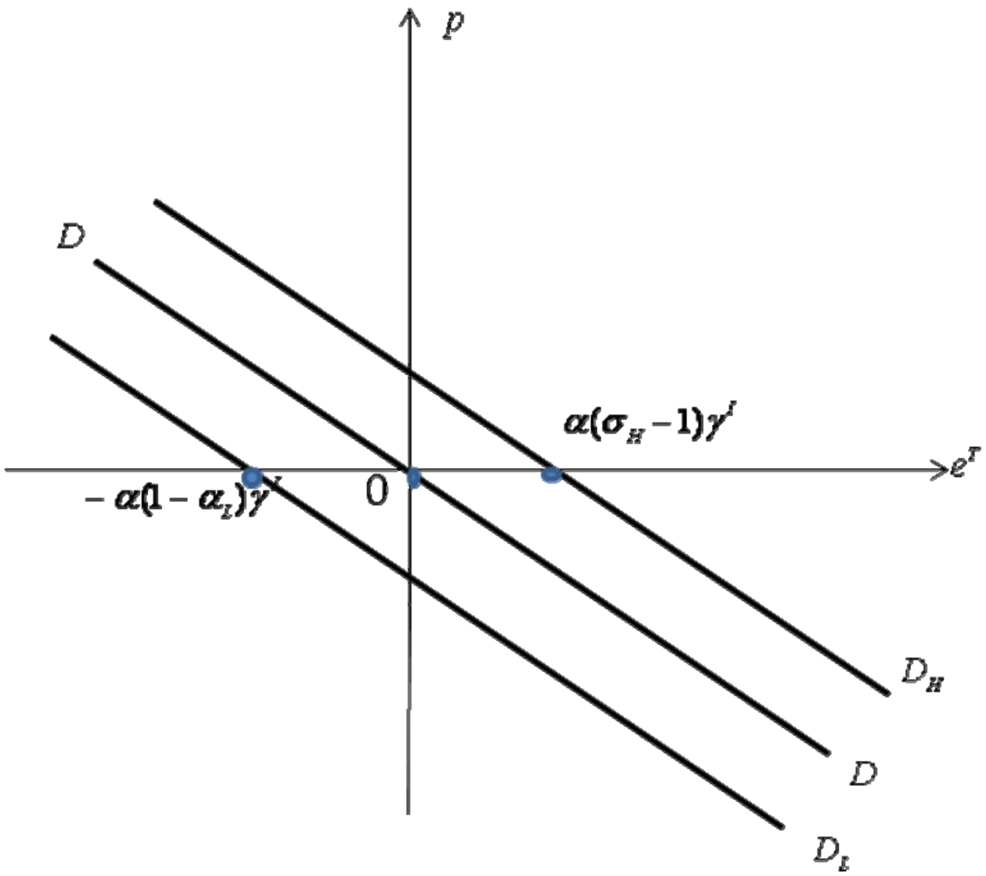

Figure 3- Shifts in the total energy supply function following improvements in energy efficiency in intermediate use.

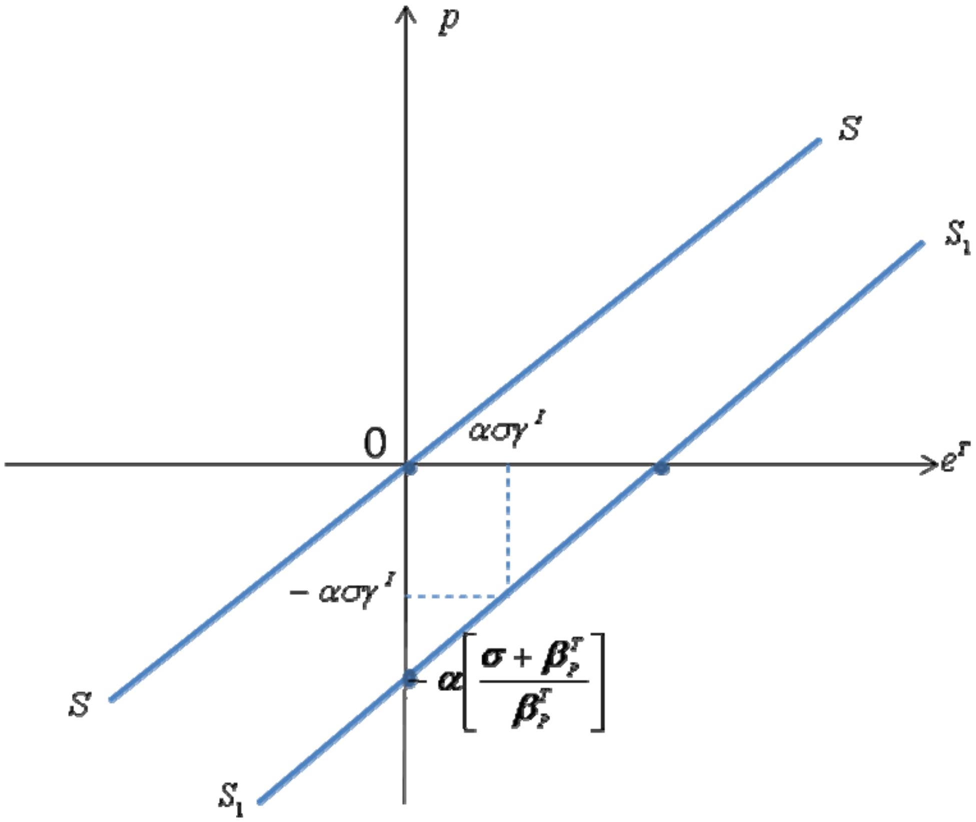

Figure 9- Equilibria with high and low total energy demand and supply elasticities following an improvement in energy efficiency in intermediate use.

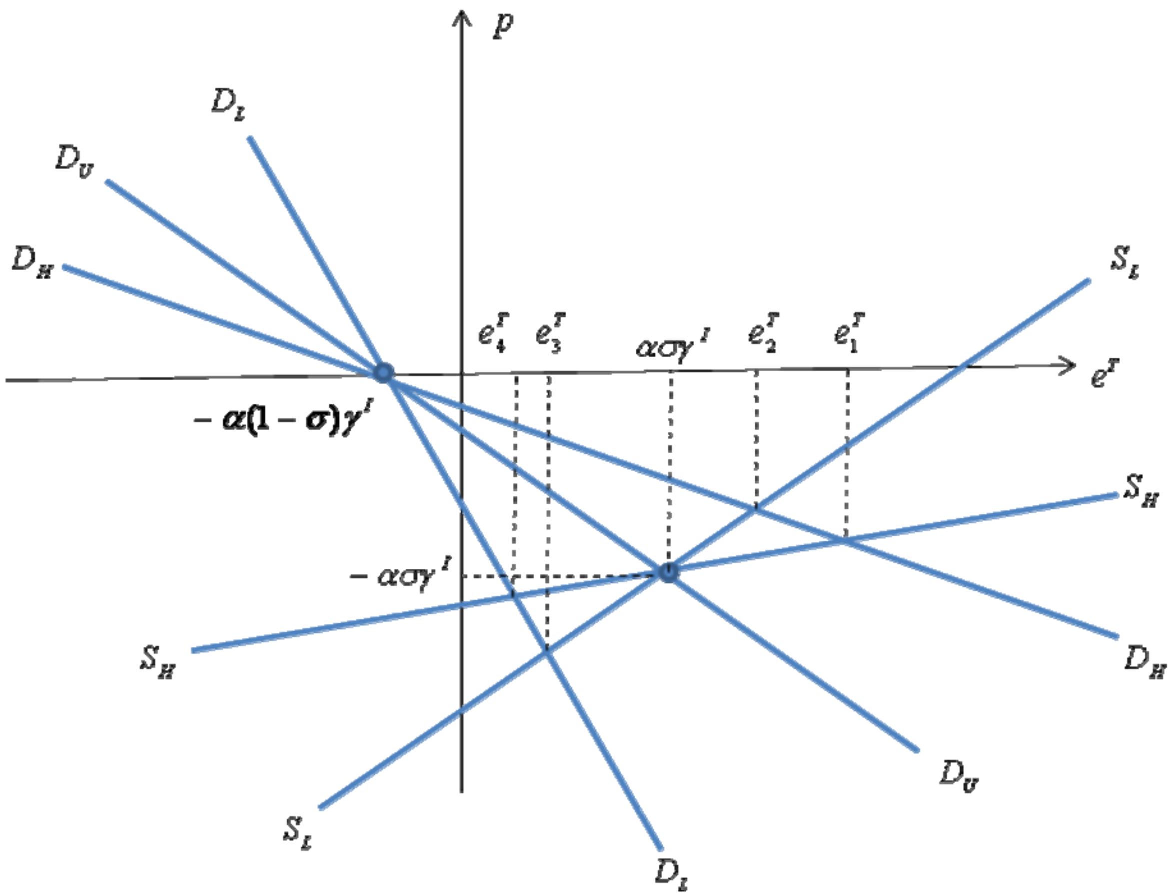

Figure 10- Short and long run equilibria in total energy output following improvements in energy efficiency in intermediate use.

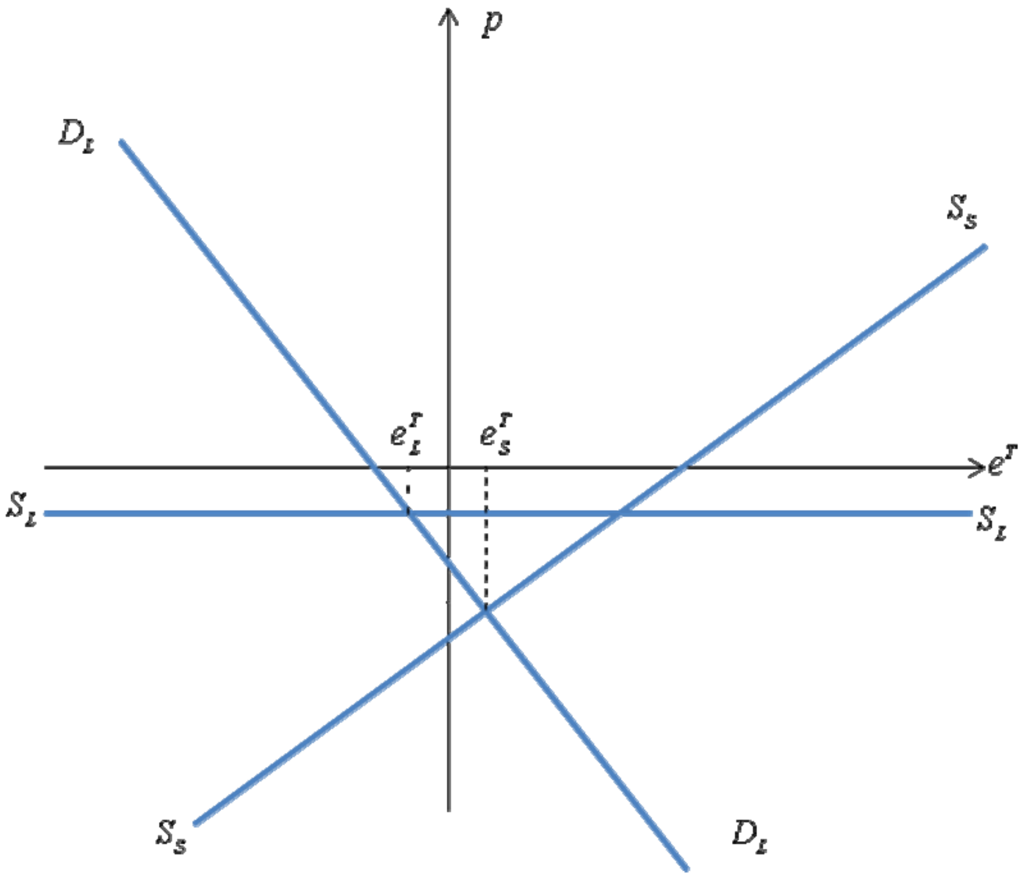

Figure 11- Parameter Values producing negative rebound, positive rebound and backfire responses following improvements in energy efficiency in intermediate use.