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IRREVERSIBLE R&D INVESTMENT WITH INTER-FIRM SPILLOVERS

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Irreversible R&D investment with inter-firm spillovers

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Abstract

In our duopoly, an irreversible investment incorporates a significant amount of R&D, so that the improvement it introduces in production processes generates a spillover that lowers the second comer's investment cost. The presence of the inter-firm spillover substantially affects the equilibrium of the dynamic game: for low – and hence realistic – spillover values, the leader delays her investment until the stochastic fundamental has reached a level such that the follower's optimal strategy is to invest as soon as he attains the spillover. This bears several interesting implications. First, because the follower invests upon benefiting from the spillover, in our equilibrium the average time delay between the two investments is short, which is realistic. Second, we show that in case of a major innovation, an optimal public policy requires a substantial intervention in favour of the investment activity, and that an increase in uncertainty – delaying the equilibrium – calls for higher subsidization rates. Third, we find, by means of numerical simulations, that the spillover reduces the difference between the leader's and the follower's maximum value functions. Accordingly, our model can help generate realistic market betas.

Keywords: irreversible investment, knowledge spillover, dynamic oligopoly
JEL classification: C73, L13, O33.

1 Introduction

A substantial body of literature has investigated the importance of irreversibility for investment decisions in stochastic environments. In particular, in the last few years, the attention has focused on duopolistic market structures in which the optimal decision of a firm depends not only on the value of the underlying stochastic fundamental but also on the action undertaken by its competitor. Because large investments rarely come without a significant improvement in production methods, in some recent contributions technical progress plays a significant role.

We complement these streams of literature by analyzing a duopoly in which the lump-sum investment, yielding a reduction in unit production cost, incorporates a relevant amount of R&D. Accordingly, it generates a spillover, which lowers the second comer's investment cost. In our duopoly game, the occurrence of the information leakage from the leader to the follower is stochastic, being governed by a Poisson variable. Because the actual attainment of the informational spillover affects the follower's investment decision, the random process of information leakage also influences the leader's efficiency advantage period.

We find that the presence of the spillover substantially affects the equilibrium of the dynamic game. In fact, in our model, for low – and hence realistic – spillover sizes, the leader delays her investment until the stochastic fundamental is so high that the follower finds it optimal to invest as soon as he benefits from the spillover. This bears several interesting implications.

First, because the follower invests upon benefiting from the spillover, in our equilibrium the average time delay between the two investments is short, which, as we shall argue, is realistic. In contrast, when one calibrates the existing models one finds long time spans separating the leader's and the follower's investments. For example, in the framework proposed by Grenadier (1996) the median time between investments varies from four to eight years when the percentage standard deviation of demand ranges between 0.05 and 0.125. While these values are adequate for the construction sector, to which the model was originally applied by Grenadier, they seem excessive for the manufacturing sector.

Second, we show that in the case of a major innovation an optimal policy requires a substantial public intervention in favour of the investment activity. In the previous literature, a major innovation – inducing the fear of being preempted – triggers a socially premature investment, which calls for some disincentive. The difference in results is to be ascribed to the significant alteration of the equilibrium characteristics,

involved by the presence of a modest spillover.[1] In our framework, an increase in uncertainty – delaying the equilibrium – calls for higher subsidization rates.

Third, the presence of a spillover weakens the dependence of the leader's and follower's maximum value functions from the fundamental. Following Cooper (2006) we can show that the differences in the value functions affect the heterogeneity in the firms' market betas. Accordingly, our model can help to generate market betas that, while different among firms, do not vary excessively. This is interesting because some recent empirical evidence suggests that the market betas show a limited dependence on the book to market ratios, and therefore on the value function (see Ang and Chen (2007) for the U.S. stock markets).

In our framework, the behavior of the follower depends on the information he has about the new technology.

If the spillover has taken place – that is, when the relevant information has already leaked out – the follower's optimal strategy is characterized by a trigger. In fact, when profits are low, the follower finds it optimal to wait; when instead the stochastic variable governing profits is sufficiently high, it is convenient for him to invest as soon as he has obtained the cost-reducing information.

When the spillover has not taken place yet, the follower finds it optimal to invest and pay the full cost, rather than waiting for the uncertain realization of the spillover, only when profits have reached high values. In contrast, when profits are low, it is sensible for the follower to wait in the hope of benefiting from the cost-reducing spillover. Hence, a second threshold exists, determining the value for the fundamental that calls for the follower's investment if the spillover has not materialized.

The innovation leader takes into account such a follower's optimal behaviour; as already highlighted, for realistic spillover values, the leader rationally decides to delay her investment until the stochastic fundamental reaches the threshold that dictates to the follower to invest as soon as he attains the spillover. This result is best understood by considering separately an innovation granting a large unit cost reduction, and one involving a small cost saving.

Consider first the equilibrium prevailing in the previous literature when a major innovation is adopted. In contributions such as Smets (1991), Grenadier (1996), and Nielsen (2002) (who build on Fudenberg and Tirole, 1985), two driving forces characterize the equilibria: the length of the follower's strategic delay, and the intensity of the competitive pressure. These contributions identify a subgame perfect equilibrium, in which the second innovator delays for a long period his decision to invest.

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[1] The analysis developed in Femminis and Martini (2007), who adopt a deterministic environment, leads to similar implications.

This choice is guided by the desire to grasp the increase in profit that is driven by the drift in the stochastic fundamental. The follower's optimal choice implies a long competitive advantage period for the innovation leader, which favors the latter's pay-off at the expense of the follower's one. Hence, to avoid being preempted, the first mover invests "very soon", and the investment is socially premature. The preemption possibility also implies rent equalization. The above contributions suggest that this "early" investment equilibrium is subgame perfect when the size of the innovation is large because the per-period first innovator profits are considerable, which triggers the preemptive behavior. In our model, an increase in the spillover reduces the payoff the leader obtains by investing early. In fact, the spillover makes more convenient for the follower the policy of investing upon information disclosure, thus reducing the corresponding threshold. Such a decline shortens the expected cost advantage period of a leader's early investment, reducing its value. This effect proves to be strong enough to induce the leader to postpone her investment until the fundamental has gone past the trigger prescribing to the follower to invest upon the realization of the spillover. In this equilibrium, the result concerning the social desirability of the investment is overturned, since it, too, is now delayed, which calls for some public incentive.

When the investment does not significantly shrinks the unit production costs, the existing literature – disregarding the possibility of inter-firm spillovers – suggests that both firms invest simultaneously (see Pawlina and Kort, 2006) for a recent and clear exposition). This happens when the per-period profit has become so high that a leader cannot emerge, because the rival would immediately copy her decision. In this case, any innovator – anticipating that there will be no leadership – waits until her investment choice maximizes the joint discounted stream of net profits. The collusive flavour of this equilibrium is apparent, and accordingly it implies underinvestment with respect to the social optimum. The simultaneous investment equilibrium is subgame perfect when the size of the innovation is small because the increase in per-period first innovator profits is not significant, which avoids preemptive behaviours, ruling out the equilibrium in which a leader invests early. When the possibility of spillovers is considered, the simultaneous equilibrium is delayed, since it implies the forsaking of the benefits stemming from the spillover. This reduces the present discounted value of the simultaneous equilibrium; it turns out that such an effect is strong enough that low sizes of the inter-firm spillover are sufficient to rule out this type of equilibrium.

Our contribution is related to several strands of literature.

Smets (1991) and Dixit and Pindyck (1994) use duopoly models to highlight the tension between the option value of waiting – that delays the firms' investment – and the fear of being preempted – that prompts for a quick action. They identify the preemptive equilibrium with rent equalization that we have already discussed. The follower's investment is delayed by the presence of uncertainty, while the leader invests as soon as her payoff is equal to the follower's discounted one. Grenadier (1996) applies this analysis to real estate markets and identifies the possibility of simultaneous entry – which, however, depends on a high initial condition for the fundamental. Bouis *et al* (2009) extend the model to the case of three firms.

Weeds' (2002) considers irreversible investment in competing research projects, in a framework in which profits evolve following a geometric Brownian motion, and the discovery takes place randomly according to a Poisson distribution with constant hazard rate. She finds that, depending on the parameter's values, either the early equilibrium or the simultaneous one is subgame perfect; in the absence of externalities, she suggests that in the early (simultaneous) equilibrium firms over(under)-invest; however, the simultaneous equilibrium is closer to the social optimum than the early one.

Pawlina and Kort (2006) consider an asymmetry between the two firms in the fixed investment cost. Besides identifying the early and the simultaneous equilibria, they find the possibility of a third type of subgame-perfect equilibrium that they label "sequential". When the asymmetry in the investment costs is relevant, the firm bearing the highest cost has no incentive in moving first; rather it is willing to invest only when the stochastic profit has already become high enough. This gives to its opponent, which becomes the leader, the opportunity to invest at the peak of its expected discounted profits. While bearing interesting positive (and normative) implications, this equilibrium still implies long expected delays between the competitors' investment dates.

Nielsen (2002) extends the standard analysis to the case of positive externalities. Under this circumstance, due, e.g., to network effects, the demand for the second comer, and hence its instantaneous profits, are higher than those for the first mover. Hence, the second mover's investment threshold is lower than that of the leaders, and therefore the follower's threshold is reached earlier, and the two firms invest simultaneously there.[2] Moretto (2000) highlights that for high spillovers, in a duopoly characterized by network effect and asymmetric information on the investment cost, a bandwagon strategy is adopted, such that the (joint) adoption may be significantly

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[2]Recently, Moretto (2008) finds that Nielsen's result can be extended to free-entry oligopolistic frameworks.

delayed.

Our contribution differs from those of Nielsen and Moretto in that for us the spillover affects the investment cost, and not the demand side. Hence, it does not apply only to network externalities or to complementary goods sectors. Moreover, our approach leads to sequential entries, which are empirically more relevant than simultaneous ones, provided that the implied delay is not too long.

Armada *et al* (2010) study a duopoly in which the incumbent firms may be taken over by new entrants, which can seize the two slots in the market. They find that the follower, fearing the competition of a potential new entrant, anticipates his entry, while the leader may be induced to delay her investment trigger, because the reduced length of the follower's strategic delay lowers the intensity of the competitive pressure. Despite the reduction in the follower's expected entry lag, the average time between investments is still high.

Among the duopoly games that do not take into account the uncertainty about the fundamental, it is worth mentioning the ones by Stenbacka and Tombak (1994), and Hoppe (2000). Stenbacka and Tombak analyze the role of experience, which implies that the probability of successful implementation of an innovation for a firm is an increasing function of the time distance from its own investment date. However, the probability of success of any firm is not affected by the adoption of the rival, so that there are no spillover effects. Stenbacka and Tombak find that – in the (feedback) market equilibrium – the leader's and the follower's adoption dates are quite dispersed. In Hoppe (2000), firms are uncertain about the profitability of the innovation, which induces an asymmetry between the leader and the follower. The latter observes the leader's outcome and hence becomes aware of the actual profitability characterizing the new technology. When the innovation is likely to be unprofitable, the informational spillover brings about a second-mover advantage, in both the early and the late equilibrium. A late simultaneous adoption prevails when the probability of poor performance for the new technology is particularly high, because this curtails the first mover's expected payoff. When the late equilibrium is subgame perfect, Hoppe finds that an earlier simultaneous adoption would be welfare increasing, while the results are less definite when the early equilibrium prevails. The equilibrium we describe in this paper differs from Hoppe's in that ours, for a large portion of the empirically relevant parameter range, is characterized by a first mover advantage, that leads to rent equalization.

Murto and Keppo (2002) present a model in which several firms compete for a single investment opportunity, which becomes effective only for the first firm that

triggers the investment. When every firm has no information about its rivals' evaluation of the investment opportunity, the investment trigger is located between the monopoly benchmark and the simple Marshallian case. A similar result has been obtained by Lambrecht and Perraudin (2003), in a model in which each firm, observing that the others have not invested, updates its beliefs about the distribution of its competitors' investment costs. Hence, each firm's inaction provides some informational spillover to its rivals. Both papers, in contrast to ours, analyze a strategic interaction that ends as soon as one firm invests.

Our modeling strategy is close to that of Weed (2002), since we model the randomness in profits via a geometric Brownian motion, while the second source of uncertainty – namely, the one stemming from the informational spillover – is dealt with by means of a Poisson distribution with constant hazard rate. From the technical standpoint, another influential contribution is Huisman and Kort (2004). Partly building on Grenadier and Weiss (1997), they incorporate into the duopoly model the possibility that a new technology becomes available at an uncertain future date, which happens according to a constant hazard rate Poisson process. The future availability of a better technique may turn the preemption game into a second mover advantage game. The main result here is that an increase in profit uncertainty tends to delay investment, so that there is an increase in the likelihood that a new technology is introduced before the occurrence of an investment using the existing technique.

The paper proceeds in the standard way. In Section 2, we present our model, and then, in Section 3, we discuss the value functions and the trigger points they imply. In Section 4, we discuss the equilibrium concept adopted in the analysis, and we compute the subgame perfect equilibrium. In Section 5, we spell out the welfare implications of our analysis, computing the optimal subsidization policy that applies in the most interesting case, namely the one of large innovations. Concluding comments in Section 6 end the paper. Three Appendixes present the analytical details, the proofs of the propositions, and the derivations of the profits and social welfare levels for the case of Cournot competition.

2 The Model

Two risk-neutral firms compete in the product market, and have the opportunity to invest in a cost-reducing process to enhance their profit flows. The cost of this irreversible investment is I for the first mover; as for the second firm introducing the innovation, the cost is I if no information has flowed out of the leader firm; otherwise,

the follower's cost is $(1 - \theta)I$, with $\theta \in (0, 1)$ being the parameter capturing the spillover.

Several empirical studies suggest that it takes time to imitate an innovation.[3] Accordingly, in our model, we assume that, from the time of the first investment, the informational spillover takes place randomly according to a Poisson distribution with a constant hazard rate $\lambda > 0$, implying that the expected delay between the leader's investment decision and the time of information leakage is $1/\lambda$. Notice that our modeling assumption implies that $1/\lambda$ is also the *minimum* expected time length of the cost advantage period granted to the leader by the introduction of an improved production process.[4] Notice also that λ, θ, and I are identical for the two firms.

It would have been preferable to consider a disclosure lag characterized by a probability of information diffusion depending not only upon the time elapsed from the introduction of the innovation, but also on the follower's imitation effort:[5] nonetheless, the use of a constant hazard rate – which has been inspired by Weeds (2002), and by Huisman and Kort (2004) – seems to be the optimal compromise between analytical tractability and realism.[6]

The instantaneous profit of each firm is stochastic, but it depends also on the number of firms that have already introduced the innovation. We assume that – when no firm has invested – the profit flow for each firm can be expressed as $\Pi_0 z_t$. Π_0 is the deterministic part of the profit function: the subscript underscores the dependence of this component from the number of firms that have already invested; z_t captures the uncertainty about future profits, and it will be assumed to evolve following a geometric Brownian motion. When one firm has sunk the cost, but the other has not, the first firm's instantaneous profit is $\Pi_1^h z_t$, while the other obtains $\Pi_1^l z_t$. The superscript highlights that – in this case – profit can be *high* (for the firm that has already innovated) or *low* (for the one that has not invested yet). When both firms have innovated, they get $\Pi_2 z_t$. We introduce the following standard assumption:

A1: $\Pi_1^h > \Pi_2 > \Pi_0 > \Pi_1^l$.

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[3] Refer to Mansfield (1985) or Cohen *et al* (2002).

[4] The first mover's cost advantage period is longer than $1/\lambda$ whenever the follower does not find it optimal to invest as soon as he receives the informational spillover.

[5] Both λ and θ should be influenced by the imitation effort. Jin and Troege (2006) suggest that firms can raise the spillover size, paying a convex imitation cost. We preferred not to pursue this development of the model because our framework is already complex: any further extension requires a much heavier use of numerical techniques to select the equilibrium.

[6] Modelling uncertainty by means of Brownian motions precludes what seems even simpler – i.e., the use of a fixed-length disclosure lag, as in Femminis and Martini (2007). In fact, this would add an additional state variable to the model. Grenadier and Weiss (1997) model in a tractable way the arrival rate of a new technology, which is governed by a Brownian motion. However, their profits are deterministic, which avoids the proliferation of the state variables.

$\Pi_2 > \Pi_0$ implies that the new technology is more profitable than the older one; $\Pi_0 > \Pi_1^l$ expresses the fact that the first investment – improving the leader's competitive position – induces a deterioration of the profit for the firm that has not sunk the cost yet; when the firm that is lagging behind undertakes the project, this damages the first mover, so that $\Pi_1^h > \Pi_2$.[7]

The geometric Brownian motion z_t is described by the following expression:

$$dz_t = \alpha z_t dt + \sigma z_t d\omega,$$

where $\alpha \in (0, r)$ is the constant drift parameter measuring the expected growth rate of z_t, $\sigma > 0$ is the instantaneous standard deviation, and $d\omega$ is the increment of a standard Wiener process, where $d\omega \sim N(0, dt)$. The constant riskless interest rate is r. The restriction $\alpha < r$ is necessary to ensure that there is a strictly positive opportunity cost of holding the option to invest, so that it will not be kept forever; this restriction guarantees finite valuations for the discounted streams of expected profits.

3 Value functions and investment thresholds

As is standard, before presenting the equilibrium, we analyze the firms' payoffs. When one firm invests first, it becomes the leader, and so its rival is the follower. Because we focus on the classic case of two ex-ante identical firms, it is not decided beforehand which firm will be leader. Nevertheless, since the firms are ex-ante identical, we can analyze their payoffs as if their roles were pre-determined, as is done, with no loss of generality, by Weeds (2002), Huisman and Kort (2004), and others. Because the follower reacts optimally to the leader's decisions, it is natural to analyze his behavior first. Then, we determine the leader's value of investing.

After discussing the follower's and the leader's value functions, we analyze the case of simultaneous investment.

3.1 The follower's investment problem

Once the leader has invested, the follower's optimal choice depends on the information he has about the new technology.[8] Therefore, we need to characterize his

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[7] The same assumptions are introduced, for example, in Pawlina and Kort (2006).

[8] For ease of exposition, we will continue to refer to the follower as if it were headed by a male CEO, and to the leader as if it were run by a female CEO.

conduct both when he has already benefited from the spillover, and when the relevant information has not yet leaked out.

We start analyzing the follower's optimal choice in the former case. We proceed in this way because the follower's knowledge of the additional information is an absorbing state: once the information has been obtained by the follower, it is not possible to revert to the previous situation. Hence, when the information has been revealed, the follower's optimal behavior cannot be influenced by his optimal choices in the "ignorance" state, while the converse is not true.

If, at time t, the leader has invested, and the follower has obtained the relevant information, he determines his optimal investment rule by solving the stochastic optimal stopping problem:

$$F^d(z_t) = \max_{T^d} E_t \left\{ \int_0^{T^d} \Pi_1^l z_\tau e^{-r\tau} dt + \left[\int_{T^d}^\infty \Pi_2 z_\tau e^{-r\tau} dt - (1-\theta)I \right] e^{-rT^d} \right\}, \quad (1)$$

where E_t denotes expectations conditional on the information available at t, and T^d is the stopping time at which the investment is sunk; the superscript d characterizing the value function highlights that $F^d(z_t)$ is obtained when the relevant information has already been disclosed to the follower.

Essentially, this investment problem can be analyzed by employing the standard real option model presented in Dixit and Pindyck (1994). As shown in Appendix 1, this leads to the following value function:

$$F^d(z_t) = \begin{cases} \frac{\Pi_1^l}{r-\alpha} z_t + \frac{(1-\theta)I}{\gamma-1} \left(\frac{z_t}{\underline{z}} \right)^\gamma & z_t \in (0, \underline{z}) \\ \frac{\Pi_2}{r-\alpha} z_t - (1-\theta)I & z_t \in [\underline{z}, \infty) \end{cases}, \quad (2)$$

where $\gamma = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} > 1$.

The interpretation for $F^d(z_t)$ is standard: for low realizations of the state variable z_t the follower's optimal strategy dictates to wait. In fact, when $z_t < \underline{z}$, the follower finds it optimal to postpone the investment, sinking it at a future date, at which the expected discounted profits will be higher; the second addendum in the first line of (2) captures the follower's option value of waiting until the trigger point $\underline{z}$ is reached. Notice that $(z_t/\underline{z})^\gamma$ can be interpreted as a stochastic discount factor, indicating the expected value of investing at $\underline{z}$ when the fundamental is z_t (refer to Dixit and Pindyck, 1994, and to Bouis *et al*, 2009, for a recent application). When the state variable z_t is sufficiently high ($z_t \geq \underline{z}$), the follower finds that the profit motive is

strong enough to trigger an immediate investment.

In Appendix 1 we show that the threshold level for z_t is given by:

$$z = \frac{\gamma}{\gamma - 1} \frac{r - \alpha}{\Pi_2 - \Pi_1^l} (1 - \theta) I, \tag{3}$$

The comparative statics on z gives sensible results: an increase in σ^2 enlarges the investment trigger, which is reduced by an increase in the investment reward $(\Pi_2 - \Pi_1^l)$, and by an increase in the spillover parameter (which lowers the follower's investment cost). An increase in the effective discount rate $r - \alpha$ induces a larger investment trigger.

We now consider the follower's choice when the information about the new technology is still undisclosed.

In Appendix 1, we solve this optimal stopping problem, showing that the value of the follower is given by

$$F(z_t) = \begin{cases} \frac{\Pi_1^l}{r - \alpha} z_t + \frac{(1-\theta)I}{(\gamma-1)} \left(\frac{z_t}{z}\right)^\gamma + E_3 z_t^{\beta_1} & z_t \in (0, z) \\ -\frac{\lambda}{r+\lambda}(1 - \theta)I + \frac{(r-\alpha)\Pi_1^l + \lambda\Pi_2}{(r+\lambda-\alpha)(r-\alpha)} z_t + E_2 z_t^{\beta_1} + G_2 z_t^{\beta_2} & z_t \in [z, \bar{z}) \\ \frac{\Pi_2}{r-\alpha} z_t - I & z_t \in [\bar{z}, \infty) \end{cases} \tag{4}$$

The absence of a superscript highlights the fact that, at the time of the leader's investment, the follower has no relevant information. Accordingly, this is considered the standard reference case. Notice that $\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}} > 1$, $\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}} < 0$, while the parameters G_2, E_2, and E_3 are:

$$G_2 = \left[\frac{\gamma(r - \alpha)}{(\gamma - 1)(r + \lambda - \alpha)} - \frac{\beta_1 r}{(\beta_1 - 1)(r + \lambda)} \right] \frac{\beta_1 - 1}{\beta_1 - \beta_2} \frac{(1 - \theta)I}{z^{\beta_2}},$$

$$E_2 = \frac{\Pi_2 - \Pi_1^l}{\beta_1(r + \lambda - \alpha)} \bar{z}^{1-\beta_1} - \frac{\beta_2}{\beta_1} G_2 \bar{z}^{\beta_2 - \beta_1}, \tag{5}$$

$$E_3 = E_2 - \frac{r(1 - \theta)I}{(\beta_1 - 1)(r + \lambda)z^{\beta_1}} + \frac{\beta_2 - 1}{\beta_1 - 1} G_2 z^{\beta_2 - \beta_1}.$$

The maximum value function (4), is characterized by two triggers, z, and $\bar{z}$.

When $z_t < z$, the follower would not invest even if the information concerning the new technology was disclosed. When $z \le z_t < \bar{z}$, the follower does not invest while the technological information is undisclosed, but he stands ready to invest upon attainment of the spillover, which grants to the follower a "discount" on his investment costs. Finally, when $z_t \ge \bar{z}$, profits are so high that it is optimal for the

follower to invest, paying the full cost I, instead of waiting for an uncertain spillover.

The value function (4) requires that $\underline{z} \leq \bar{z}$, and accordingly we first provide the relevant details about the trigger $\bar{z}$, and then we comment upon $F(z_t)$. In Appendix 1, we show that $\bar{z}$ is determined by the following nonlinear equation:

$$\left(\frac{\bar{z}}{\underline{z}}\right)^{\beta_2} (1-\theta)I \left[\frac{\gamma(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)} - \frac{\beta_1 r}{(\beta_1-1)(r+\lambda)}\right] = \tag{6}$$
$$= \frac{\Pi_2 - \Pi_1^l}{r+\lambda-\alpha}\bar{z} - \frac{\beta_1(r+\theta\lambda)}{(\beta_1-1)(r+\lambda)}I.$$

Before deriving the results concerning $\bar{z}$, we need to show

Lemma 1 $\left[\frac{\gamma(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)} - \frac{\beta_1 r}{(\beta_1-1)(r+\lambda)}\right] > 0$ *for* $\sigma^2 \in (0,\infty)$, $\lambda \in (0,\infty)$.

Proof. Please refer to Appendix 2. ■

This allows to prove that

Proposition 2 *For any* $\lambda \in (0,\infty)$

   i) The threshold $\bar{z}$ is unique;

   ii) if $\theta = 0$, then $\bar{z} = \underline{z}$;

   iii) $\lim_{\theta \to 1} \bar{z} = \frac{\beta_1(r+\lambda-\alpha)}{(\beta_1-1)(\Pi_2-\Pi_1^l)}I$, *while* $\lim_{\theta \to 1}\underline{z} = 0$;

   iv) $\bar{z} > \underline{z}$ *for* $\theta \in (0,1)$.

Proof. Please refer to Appendix 2. ■

Results $i)$ and $iv)$ are crucial for the logical consistency of the value function (4).

The result in $ii)$ is not surprising: when there is no spillover, the follower's decision boils down to the traditional one, since he only has to decide whether he wants to invest or to keep his option.

When $\theta \to 1$ the technology adoption bears no cost to the follower, and hence it is always optimal for him to upgrade his technique as soon as the relevant – and indeed precious – information leaks out of the leader firm. This explains why $\underline{z} = 0$. Notice that, in this case, $\bar{z}$, which is the threshold triggering investment if no information is revealed, is higher than the trigger that would characterize the follower behavior in a model with no spillover.[9]

Proposition 2 has an interesting implication:

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[9]In this case, the threshold $\bar{z}$ is higher that the follower's trigger in a model with no spillovers $\left(\text{that is } \frac{\gamma}{\gamma-1}\frac{r-\alpha}{\Pi_2-\Pi_1^l}I\right)$. This is guaranteed, for $\lambda \in (0,\infty)$, by Lemma 4, which implies $\frac{\beta_1(r+\lambda-\alpha)}{\beta_1-1} > \frac{\gamma(r-\alpha)}{\gamma-1}$.

Corollary 3 $E_2, G_2 > 0$, and $E_3 < 0$, for $\sigma^2 \in (0, \infty)$, $\lambda \in (0, \infty)$.

Proof. Please refer to Appendix 2. ∎

This Corollary is useful to interpret the maximum value function (4). For $z_t \in (0, \underline{z})$, the value of a firm that has benefited from the spillover must be higher than the value of a firm that has not. The negative term $E_3 z_t^{\beta_1}$ reflects the difference $F^d(t) - F(t)$ (compare the first line in Eq. (4) with the first line in Eq. (2)). When $z_t \in [\underline{z}, \bar{z}]$, the maximum value for a follower that has not enjoyed the spillover is characterized by two option value terms, $E_2 z_t^{\beta_1}$ and $G_2 z_t^{\beta_2}$, that are both positive. In fact, inaction grants two types of advantages to the follower. First, with instantaneous probability λ, he may obtain the spillover; second, because $E_t(dz_t) = \alpha z_t dt > 0$, he expects to move toward the investment threshold $\bar{z}$, which increases his value. The former effect is captured by $G_2 z_t^{\beta_2}$, while the latter boils down to $E_2 z_t^{\beta_1}$. Notice that, if $\lambda = 0$, then $\beta_1 = \gamma$ (refer to Appendix 1) and – in coherence with our interpretation – $G_2 = 0$.

3.2 The leader's investment decision

We now obtain the value of a firm that invests as the leader, given that the follower will act optimally in the future.

Notice that once the leader has invested, she has no further decision to take, and her value is the discounted stream of her future profits. This payoff is negatively affected by the follower's investment, and this effect gets higher with the fundamental, as its increase makes the follower's decision closer. For this reason, the leader's value function need not be monotonic.[10] At the time of investing, the leader is aware that she is about to face an uninformed follower. We already know that the behavior of a follower that has not benefited from the spillover, is different in the three intervals $z_t \in (0, \underline{z})$, $z_t \in [\underline{z}, \bar{z}]$, and $z_t \in [\bar{z}, \infty)$. This difference in the follower's behavior influences the leader's payoff because it affects the length of her cost advantage period. Therefore, the leader's maximum value function has three different shapes in these three intervals.

Collecting the results explained in Appendix 1, we find that the leader's maximum value function is

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[10] Notice also that at $\bar{z}$ there is no optimal choice for a leader that has already invested, and hence the smooth pasting condition does not need to hold (as it happens, e.g., in Weeds, 2002, and in Pawlina and Kort, 2006. In the three-firm model by Bouis *et al*, 2009, the smooth-pasting condition holds everywhere only for the third comer).

13

$$
L(z_t) = \begin{cases} \frac{\Pi_1^h}{r-\alpha} z_t + \frac{\Pi_2 - \Pi_1^h}{r-\alpha} \underline{z} \left(\frac{z_t}{\underline{z}}\right)^\gamma + E_6 z_t^{\beta_1} - I, & z_t \in (0, \underline{z}) \\ \frac{(r-\alpha)\Pi_1^h + \lambda \Pi_2}{(r+\lambda-\alpha)(r-\alpha)} z_t + E_4 z_t^{\beta_1} + G_4 z_t^{\beta_2} - I, & z_t \in [\underline{z}, \bar{z}) \\ \frac{\Pi_2}{r-\alpha} z_t - I, & z_t \in [\bar{z}, \infty), \end{cases} \tag{7}
$$

where E_6, E_4, and G_4 are given by:

$$
\begin{aligned}
G_4 &= \frac{\Pi_1^h - \Pi_2}{\Pi_2 - \Pi_1^l} \frac{(1-\theta)I}{\underline{z}^{\beta_2}} \frac{\gamma}{\beta_1 - \beta_2} \left[1 - \frac{(\beta_1 - 1)(r - \alpha)}{(\gamma - 1)(r + \lambda - \alpha)} \right], \\
E_4 &= -\bar{z}^{(-\beta_1)} \left(G_4 \bar{z}^{\beta_2} + \frac{\Pi_1^h - \Pi_2}{r + \lambda - \alpha} \bar{z} \right), \\
E_6 &= E_4 + \underline{z}^{(-\beta_1)} \left[G_4 \underline{z}^{\beta_2} + \frac{\Pi_1^h - \Pi_2}{r + \lambda - \alpha} \underline{z} \right].
\end{aligned} \tag{8}
$$

For a better understanding of the economic meaning of the above parameters, it is useful to prove the following:

Lemma 4 $\left[1 - \frac{(\beta_1 - 1)(r - \alpha)}{(\gamma - 1)(r + \lambda - \alpha)} \right] > 0$ *for* $\sigma^2 \in (0, \infty), \lambda \in (0, \infty)$.

Proof. Refer to Appendix 2. ∎

Lemma 4 allows us to conclude that $G_4, E_6 > 0$, while $E_4 < 0$, which implies that $L(z_t)$ needs not be monotonic either in the interval $z_t \in (0, \underline{z})$, or in the interval $z_t \in [\underline{z}, \bar{z})$.

Different forces contribute to shaping the leader's maximum value function.

Consider first the interval $z_t \in (0, \underline{z})$. If the leader – investing there – enjoyed forever the instantaneous profit flow of $\Pi_1^h z_t$, her value would be $\Pi_1^h z_t / (r - \alpha)$. The second addendum in the first line of Eq. (7) corrects this value, assuming a follower's investment at $\underline{z}$. But the follower invests at a fundamental higher than $\underline{z}$ whenever the spillover has not materialized at the date at which z_t reaches the threshold $\underline{z}$. This induces a positive effect on the leader's value, which is captured by the third addendum, $E_6 z_t^{\beta_1} > 0$; this effect is the relevant the lower is λ.[11] As depicted in Figure 1, the positive effect tends to dominate when z_t is low, and in fact the lower the fundamental, the more the stochastic discount factor $(z_t / \underline{z})^\gamma$ dampens the negative effects of the second addendum in the first line of Eq. (7).

[Figure 1 about here]

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[11] When $\lambda \to 0$, we have that $\left[1 - \frac{(\beta_1 - 1)(r - \alpha)}{(\gamma - 1)(r + \lambda - \alpha)} \right] = 0$, since $\beta_1 = \gamma$ (compare Eqs. (12) and (15)). Because in this case $G_4 = 0$, $L(z_t) = \frac{\Pi_1^h}{r - \alpha} z_t + \frac{\Pi_2 - \Pi_1^h}{(r - \alpha)} \underline{z} \left(\frac{z_t}{\underline{z}}\right)^\gamma + E_6 z_t^{\beta_1}$ approaches $\frac{\Pi_1^h}{r - \alpha} z_t - \frac{(\Pi_1^h - \Pi_2)}{r - \alpha} \bar{z}^{1-\gamma} z_t^\gamma$, which is the leader's maximum value function in the traditional no-spillover model.

14

When $z_t \in [\underline{z}, \bar{z})$, the interpretation of the maximum value function is straightforward. The expected profit for a leader facing a constant probability of investment on behalf of her competitor is $\frac{(r-\alpha)\Pi_1^h + \lambda\Pi_2}{(r+\lambda-\alpha)(r-\alpha)} z_t$, and it obviously grows with z_t; $E_4 z_t^{\beta_1}$ and $G_4 z_t^{\beta_2}$ are correction terms capturing the fact that an increase in z_t makes closer, on average, the attainment of the threshold $\bar{z}$ that triggers the follower's investment. Accordingly, an increase in the fundamental shortens the leader's cost advantage period, which induces the negative effect summarized by the first correction term. However, an increase in z_t makes less likely the attainment of the spillover, and hence the second correction term is positive (and increasing in λ).

To understand the role of the probability of information disclosure in shaping the leader's maximum value function, consider that, when z_t is close to $\underline{z}$, it takes a long time to reach $\bar{z}$. Accordingly, the average length of the cost advantage period is close to $1/\lambda$, because the probability that $\bar{z}$ is reached before the relevant information is released is negligible. Hence, in the lower part of the interval $[\underline{z}, \bar{z})$, the leader's maximum value function is "almost linear" in z_t because the effect of z_t on profits does not change significantly with z_t itself. In contrast, when z_t is close to $\bar{z}$, the extent of the cost advantage period is affected by the evolution of z_t. Accordingly, in this case an increase in z_t enhances the instantaneous profits for the leader, but reduces the expected duration of her cost-advantage period, which explains the contraction in (the growth of) the leader's maximum value.

Figure 1 confirms this intuition for a realistic value of λ.

Comparing Figure 1 with the pictures portraying the equilibrium for the no-spillover case (see, e.g., Dixit and Pindyck, 1994; Nielsen, 2002; Weeds, 2002) one immediately realizes that the presence of a moderate spillover significantly reduces the difference in the leader's and in the follower's maximum value functions, as well as their dependence on the fundamental. Because the disparities in the value functions affects the heterogeneity in the firms' market betas (as in Cooper, 2006), our model bears the interesting implication of being able to generate betas that, while different between oligopolistic firms, do not vary excessively.

3.3   The simultaneous investment problem

If z_t has reached high values (i.e., for $z_t \in [\bar{z}, \infty)$), the fixed cost is so low in comparison with the expected profits that it is optimal for the second firm to immediately enter upon his rival's investment, without exploiting the inter-firm spillover, as is shown in the third line of (4).

In this case, the first firm is aware that, as soon as she innovates, the second firm will "immediately" follow and invest. Hence, each firm takes its decision anticipat-

ing such a follower's behavior. This leads to a candidate equilibrium where the two firms maximize their joint payoff: knowing that it will be immediately followed, each firm delays its innovation until it can get its maximum discounted sum of profits. In this context, firms remain symmetric, the informational spillover is never exploited, and the maximization of each single firm's payoff coincides with their joint maximization.[12] The two firms determine their optimal investment rule by solving the stochastic optimal stopping problem:

$$S(z_t) = \max_{T_S} E_t \left\{ \int_0^{T_S} \Pi_0 z_\tau e^{-r\tau} dt + \left[\int_{T_S}^\infty \Pi_2 z_\tau e^{-r\tau} dt - I \right] e^{-rT_S} \right\}, \qquad (9)$$

under the constraint that the investment trigger must satisfy the constraint $z_S \geq \bar{z}$. In fact, if $z_S < \bar{z}$, the simultaneous investment cannot be an equilibrium: if a firm invests at z_S, her competitor's best strategy is not to follow immediately. Rather, his best reply, given by $F(z_t)$, is to invest as soon as he benefits from the positive spillover, and to sink the cost at $\bar{z} > z_S$ if no information flows out of the rival.

In Appendix 1, we show that the solution is to invest at $z_S = \max\{z', \bar{z}\}$, where[13]

$$z' = \frac{\gamma}{\gamma - 1} \frac{r - \alpha}{\Pi_2 - \Pi_0} I, \qquad (10)$$

so that the cooperative maximum value function is:

$$S(z_t) = \begin{cases} \frac{\Pi_0}{r-\alpha} z_t + \left(\frac{\Pi_2 - \Pi_0}{r-\alpha} z_S - I \right) \left(\frac{z_t}{z_S} \right)^\gamma & \text{for } z_t \in (0, z_S) \\ \frac{\Pi_2}{r-\alpha} z_t - I & \text{for } z_t \in [z_S, \infty) \end{cases}. \qquad (11)$$

When $z' < \bar{z}$, the value function is continuous, but not differentiable, which is a consequence of the constraint in the maximization process. Notice that the function $S(z_t)$ represents the expected discounted value of investing at z_S conditional upon being in $z_t < z_S$.

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[12] When firms cooperate and side payments are allowed, they may jointly select two different investment triggers (which of course imply different expected profits streams), as in Weeds (2002). If, instead, the two firms can cooperate but they are constrained to invest at the same point, they opt for the trigger we identify in the main text. Hence, our approch is equivalent to allowing for cooperation, excluding the possibility of side payments.

[13] When θ is low, it is possible to show that, $\bar{z} > z'$. The upward-sloping portion of the surface portrayed in Figure 5, Panel (d), actually represents the cases in which $\bar{z} > z'$. It is also possible to prove that $z' > \underline{z}$, always.

16

4 The competitive equilibrium

4.1 The equilibrium concept and the selection procedure

Because in our setup only one investment project is available to the firms, the choice to innovate is an irreversible stopping decision. Our model therefore belongs to the class of symmetric timing games, which can be divided into two sub-classes depending upon which firm (the first mover or the second mover) obtains the higher payoff.

A timing game is of the first mover advantage type if the first mover's highest discounted payoff exceeds the follower's best reply, as is the case in most of the literature. Because the roles of leader and follower are not pre-assigned, if the follower's payoff is lower than that of the leader, the former has an incentive to anticipate the latter's decision, becoming the leader. Accordingly, in these games, the possibility of preemption induces rent equalization. As is common in the literature, we assume away coordination failures – i.e., we assume that the two firms cannot invest together at a rent-dissipation point (like z_L in Figure 1). More precisely, we assume that at a rent-equalization point each firm is randomly selected to become the leader, with probability 1/2, while her opponent becomes the follower, and reacts optimally to the leader's choice.

When the spillover parameter is high, we have a second-mover advantage. An increase in θ directly benefits the follower's payoff by reducing his fixed cost; moreover, a larger spillover makes more convenient to the follower the policy of immediately investing upon information disclosure, and in fact an increase in θ reduces the threshold $\underline{z}$, as shown by Eq. (3). This shortens the leader's expected cost-advantage period for $z_t \in (0, \underline{z})$, reducing her value. In a second mover advantage game, if the task of moving first is exogenously assigned to one of the two firms, this player, behaving optimally, obtains the lower payoff. In this case, as in Huisman and Kort (2004), we assume that each firm is assigned the task of moving first with probability 1/2, so that this firm, which becomes the leader, chooses her highest payoff.[14]

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[14] This assumption (and therefore the equilibrium it implies) may seem arbitrary. In fact, it rules out the mixed-strategies equilibrium often referred to as a war of attrition. In a war of attrition, firms would start randomizing at $\check{z}$, where the leader's discounted payoff reaches a (local) maximum, and they would randomize in $z_t \in [\check{z}, \tilde{z})$. In this interval in fact, $F(z_t) > L(z_t)$, while the discounted value of $L(z_t)$ is decreasing. In such a mixed-strategy equilibrium, firms would obtain, at every point, an expected payoff lower than that of the leader, due to the possibility of getting a low value in case of a simultaneous entry. Nevertheless, our assumption does not twist the selection process in favour of a candidate equilibrium located in the interval $z_t \in [\underline{z}, z)$. In fact, our numerical analysis shows that the second mover advantage games emerge when the spillover parameter is high enough that the simultaneous entry payoff (Eq. (11)) is lower than the value that could be obtained in a war of attrition.

Fudenberg and Tirole (1985) argued that the Pareto-dominant equilibrium is most reasonable. In our case, Pareto ranking implies that all firms prefer the pure-strategy equilibrium involving an advantage for the follower.

We now focus on subgame-perfect equilibria, in which it is not decided beforehand which firm is leader or follower. Hence, we build on the tradition of Fudenberg and Tirole (1985), a tradition that has been followed by, among others, Grenadier (1996), Nielsen (2002), Weeds (2002), Huisman and Kort (2004), and Bouis *et al* (2009).

Subgame perfectness requires that the equilibrium must survive all the possible off-equilibrium deviations. Because our model is highly non-linear, we need first to identify the candidate equilibria and then to select among them using the subgame perfection criterion. Each candidate equilibrium is a couple of strategies that would form a subgame-perfect equilibrium if we restricted our attention to a subset of $z_t \in (0, \infty)$. Once we have identified the candidate equilibria, we select among them using the subgame-perfectness criterion. In doing so, we need to compare the leader's value at any candidate equilibrium, with her payoff at any point lower than the one that is part of the proposed equilibrium. Whenever we can find a point in which the leader's payoff is higher than the discounted value of her payoff at the candidate equilibrium, the leader prefers to invest at that point rather than to wait for the proposed equilibrium, which therefore is not subgame perfect.

The simultaneous investment trigger analyzed in Subsection 3.3 is a candidate equilibrium; in fact, provided that z_t has reached high values (i.e., for $z_t \in [\bar{z}, \infty)$), for each firm it is optimal to invest at z_S if the other does the same. Moreover, we can prove that at least another candidate equilibrium always exists in $z_t \in (0, \bar{z})$.

Theorem 5 *i) For $\theta \in (0, \theta^*]$ there is a candidate (preemptive) equilibrium with the leader investing in $z_t \in (0, \underline{z}) \subseteq (0, \bar{z})$.*

ii) For $\theta \in [\theta^{**}, 1)$ there is a candidate equilibrium with the leader investing in $z_t \in [\underline{z}, \bar{z}) \subseteq (0, \bar{z})$.

Proof. Please refer to Appendix 2. ∎

Unfortunately it is impossible to determine analytically θ^* and θ^{**}, and hence, *a fortiori*, we cannot specify the parameter region in which the candidate equilibria are three (as happens in Figure 1). In fact, in our highly non-linear model, the candidate equilibria can be obtained only by means of numerical techniques. Figure 2 provides an example of the behavior of θ^* and θ^{**} as a function of λ. For the parameter combinations in Area A (i.e., for $\theta < \theta^{**}$) only the preemptive candidate equilibrium exists; in Area B (i.e., for $\theta^{**} \leq \theta \leq \theta^*$) two candidate equilibria exists, while in area C (when $\theta > \theta^*$) only the candidate equilibrium in $z_t \in [\underline{z}, \bar{z})$ exists.

[Figure 2 about here]

Notice that Part i) of Theorem 5 confirms that our analysis is consistent with the standard duopoly model proposed by Smets (1991), Dixit and Pindyck (1994), and others. When the spillover is low, either the equilibrium is preemptive with rent equalization (for $z_t \in (0, \underline{z})$), or the equilibrium that prevails is the joint-profit-maximization simultaneous one ($z_t = z_S$).

As an example of the equilibrium selection procedure, consider Figure 3, which is drawn for a realistic value of the hazard rate λ, for the case of a large innovation, and for a very low value for the spillover parameter ($\theta = 0.02$).[15]

[Figure 3 about here]

From the Figure, it is clear that two candidate equilibria exist. One is the simultaneous equilibrium z_S; the dotted line represents $S(z_t)$, which is the discounted value of investing at z_S when the fundamental is z_t. The other equilibrium prescribes to the follower to wait until the fundamental reaches $\underline{z}$: if, at that time, he has already benefited from the spillover, his optimal strategy is to invest; otherwise, his best strategy is to invest upon disclosure; once the fundamental reaches the trigger $\bar{z}$, he has to invest even if he has not enjoyed the spillover. (Refer to Eq. (4)). The leader's equilibrium response to the follower's strategy described above is to invest at z_L: because the leader's payoff is higher than the follower's for $z_t \in (z_L, \bar{z})$, the possibility of preemption by the follower induces the leader to invest at the rent-equalization point z_L. This follows from the fact that the roles of leader and follower are not pre-assigned: if the follower's payoff is lower than the leader's, the former has an incentive to anticipate the latter's decision, becoming the leader.

To select the subgame-perfect equilibrium for the case depicted in Figure 3, notice that, for some $z_t \in [z_L, \bar{z})$, $L(z_t)$ is higher than $S(z_t)$. Accordingly, the leader prefers to sink the investment cost in z_t, rather than to wait until z_S is reached. This is sufficient to make the simultaneous investment at z_S not an equilibrium. In such a case, because $L(z_t) > F(z_t)$ for $z_t \in (z_L, \bar{z})$, it is in the follower's interest to preempt the leader by investing at $z_t - dz_t$. Hence, by backward induction, we conclude that the equilibrium strategy for the first innovator is to invest when the leader's payoff is equal to the follower's (i.e., at z_L). Accordingly, one firm, which becomes the leader, invests at z_L, while the other follows the strategy prescribed by (4). Notice that we have rent equalization in the equilibrium, due to the possibility of preemption in this first-mover advantage game. Notice also that $S(z_L) > L(z_L) = F(z_L)$, but the leader

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[15]The other relevant parameters values are $\alpha = 0.01$, $r = 0.04$, and $\sigma = 0.03$.

cannot decide to wait, because, for some $z_t > z_L$, it is in the follower's interest to preempt the leader, which makes the outcome of investing simultaneously at z_S not subgame perfect.

In our set-up, the leader maximum value function may cross the follower's more than once. As already underscored, an increase in θ benefits the follower, while damaging the leader. These two effects induce $L(z_t) < F(z_t)$ for some $z_t \in [z_L, \bar{z})$, and the two value functions cross more than once, as is the case in Figure 1, which is drawn for $\theta = 0.07$. As before, to select the subgame-perfect equilibrium, we need to proceed backward, starting from z_S, which – with the parameter values used for Figure 1 – coincides with $\bar{z}$. If $S(z_t) > L(z_t)$ for $z_t \in [z_L, \bar{z})$, the subgame-perfect equilibrium in the interval $[z_L, \bar{z})$ prescribes simultaneous investment at z_S. If not, as is the case, we need to check if the first rent-equalization point that we find moving backward toward z_L is the equilibrium. Call $\tilde{z}$ the first rent-equalization point at the left of $\bar{z}$. To verify whether this point actually represents a subgame-perfect equilibrium, one must check whether the discounted expected value of investing at $\tilde{z}$ conditional upon being in $z_t \in [z_L, \tilde{z})$, is higher than $L(z_t)$. Because with $\theta = 0.07$ this is the case, the leader prefers to wait until $\tilde{z}$ is reached, rather than to sink the investment cost in z_t. Hence, $\tilde{z}$ is the subgame-perfect equilibrium. If this had not been the case, we would have needed to move to the left to the next candidate equilibrium, z_L. The fact that $L(z_t) < F(z_t)$ for some $z_t \in [z_L, \bar{z}]$ does not imply that the game we are considering is of the second-mover advantage type: in fact, the highest discount payoff for the leader (at $z_t \cong 1.31$) is higher than that of the follower.

However, when the spillover parameter is high, we do have a second-mover advantage, as is the case in Figure 4, which is drawn for $\theta = 0.3$. In this case, the best strategy for the firm that is drawn to move first is to invest at $\check{z}$. This strategy grants her the highest current value. From the Figure, it is clear that the leader's payoff at $\check{z}$ is lower than the follower's, but it exceeds the leader's expected discounted value of investing at $\tilde{z}$ (or later).

[Figure 4 about here]

4.2 Numerical selection

Because the equilibrium cannot be identified analytically, we now present some numerical results.[16] We first depict the candidate equilibria as a function of the spillover

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[16] Our routine has been written in Matlab, and it is based on a discretization of the space $[\theta \times \lambda]$, for $\theta \in [0.01, 0.40]$ and $\lambda \in [0.20, 0.67]$. We have used 72.000 gridpoints; however, our results do not

and of the hazard rate, and then we highlight the portion of the parameter space in which the subgame-perfect equilibrium investment trigger for the leader is higher than z. This equilibrium is of particular interest because, when it prevails, the follower's best strategy is to invest as soon as he gets the spillover, so that the average time distance between the leader's and the follower's investment dates is close to $1/\lambda$, implying realistic investment lags for the follower.

To limit the range of relevant values for λ, consider that, in his classic study, Mansfield (1985) reports that in 41% of cases it takes less than twelve months for the innovator's rival to obtain the relevant information. More recently, Cohen *et al* (2002) compute that the average adoption lag for unpatented process innovation is 2.03 and 3.37 years in Japan and the U.S., respectively. These contributions lead us to think that realistic adoption lags are contained between 1.5 and 5 years, so that – recasting the innovation lag in our terms – we simulate the model for $\lambda \in [0.20, 0.67]$.

In our analysis, we fix the discount rate r at 0.04, which is consistent with computing calendar time in years. Then, we notice that the level of the irreversible investment does not play any substantial role: the effect of an higher I is to postpone all of the candidate equilibria without changing their relative convenience. Hence, we choose $I = 100$ with no loss of generality. As for α, we fix it at 0.02 simply because we have verified that, moving it in the interval [0.01, 0.03], does not appreciably modify our results. The role of uncertainty is much more relevant. As we shall detail later, an higher uncertainty increases the investment triggers, and the value of waiting, and hence it plays a role in the equilibrium selection process. Hence, we shall present the result for $\sigma \in \{0.03, 0.1\}$. While the second value may seem high, it has been adopted in various studies to stylize the role of sector-specific uncertainty (see, e.g., Grenadier, 1996; Pawlina and Kort, 2006). The first value has been chosen to portray the polar case of a relatively stable sector.[17]

Another key element is given by the post-investment profit levels. In fact, a significant profit increase for the leader – in the absence of spillovers – favours the preemptive equilibrium, as originally suggested by Fudenberg and Tirole (1985) and verified in the stochastic settings by many contributions (see in particular Nielsen, 2002; Weeds, 2002). In contrast, with no spillover, an investment yielding only a modest profit increase to the front runner tends to induce the selection of a simulta-

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appreciably change for any number of evaluation points larger than 4.500. This routine is available upon request from the authors.

[17]The choice of the value for the low-variance sector has been influenced by Guiso and Parigi (1999), who, using a panel of Italian firms, find a coefficient of variation of one-year ahead expected demand as low as 0.023.

neous equilibrium (see Pawlina and Kort, 2006; or, again, Weeds, 2002). Accordingly, we analyze two different scenarios.

We first consider a major innovation, which is the introduction of a new production technique yielding a significant cost reduction. In this case, the leader – when she is the unique innovator – grasps large profits in comparison with those obtained by the follower. In fact, the cost advantage she enjoys induces her to significantly increase her market share. To simulate this case, we normalize Π_0 to unity, and we assume: $\Pi_1^h = 4$, $\Pi_1^l = 0.25$, and $\Pi_2 = 2.25$.[18] We first compute the three candidate equilibria in the case of a major innovation, when $\sigma = 0.03$, as a function of θ and λ.

[Figure 5 about here]

When the leader invests at $z_L \in (0, \underline{z})$, the follower's optimal strategy is to invest at $\underline{z}$ if he has already benefited from the spillover, and to invest upon attainment of the spillover if at $\underline{z}$ the relevant information is still undisclosed. Accordingly, the investment trigger for the second mover is not significantly influenced by λ: for sensible parameter values, the leader's investment at z_L makes "almost sure" the attainment of the spillover before z_t. Hence, the follower "almost always" invests at $\underline{z}$, which, being the investment trigger for a second mover who has already benefited from the spillover, cannot be influenced by λ (refer to Eq. (3)). The virtual independence of the follower's expected threshold on λ is portrayed in Panel (a) of Figure 5. Panel (b) depicts z_L. Because the second mover's trigger is almost independent from the hazard rate, the leader's cost-advantage period is also not significantly influenced by λ. Hence, the leader's investment threshold is also almost imperceptibly influenced by λ. It is apparent that an increase in θ raises the leader's investment threshold, but lowers that of the follower. In fact, a higher θ – benefiting the follower's payoff by reducing his fixed cost, and shortening the leader's expected cost-advantage period – reduces the incentives to be first, while increasing the follower's value. Notice that Panels (a) and (b) show that the preemptive equilibrium does not exist for high θ, an effect that can be easily interpreted by referring to Figure 4 and that is consistent with Figure 2.

Panel (c) portrays the leader's investment trigger in the candidate equilibrium in the region $[\underline{z}, \bar{z})$. When this equilibrium is of the first-mover advantage type (as $\tilde{z}$ in Figure 1), it is represented by a steeply-upward sloping surface, while the second mover advantage equilibrium (as $\check{z}$ in Figure 4) generates a mildly upward-sloping surface. Accordingly, the impact of θ and λ on the first mover investment trigger is

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[18]Appendix 3 shows that these values are coherent with Cournot competition in the final product market when the innovation size, denoted by x, is 0.50 of the market dimension.

different in the two cases. This happens because the follower significantly benefits from an increase in θ or in λ. Such an increase in payoffs implies a lower incentive to preempt the leader when there is a first mover advantage. The increase in θ and λ harms the leader, because her cost-advantage period shrinks. In a first mover advantage equilibrium, the effects on both the follower's and the leader's value functions are operational, while in a second mover advantage equilibrium, the latter effect only is at work.[19] Panel (c) shows that the equilibrium in which the leader invests after $\underline{z}$ does not exist when θ is low, which is consistent with Figures 2 and 3.

The simultaneous-investment trigger is portrayed in Panel (d). When θ is large, if one firm invests, her competitor's optimal policy is to wait and sink the fixed cost upon realization of the spillover even when z_t and hence the duopoly profits are high. Moreover, an increase in θ raises $\bar{z}$, which is the threshold below which the above policy is optimal. Accordingly, when θ is high, the simultaneous investment trigger is $\bar{z}$, and it is increasing in both of the relevant parameters. Conversely, when θ is low, the presence of the spillover becomes irrelevant, and $z_S = z'$. In this case, the simultaneous investment trigger is constant, as confirmed by Eq. (10).

Having illustrated the candidate equilibria, we now compute the portion of the parameter space in which the leader's subgame-perfect equilibrium-investment trigger is higher than $\underline{z}$. We define as $\theta(\lambda)$ the value for θ such that – given λ – the spillover parameter is high enough that the leader's subgame-perfect equilibrium investment trigger exceeds $\underline{z}$, and, in Figure 6, we portray the threshold $\theta(\lambda)$ for $\sigma \in \{0.03, 0.1\}$.

[Figure 6 about here]

For $\theta < \theta(\lambda)$, the preemptive equilibrium with the leader investing at z_L prevails. To understand the switch from one type of equilibrium to the other, notice that, for a given λ, an increase in θ reduces the leader's payoff. In fact, an increase in θ makes more convenient to the follower the policy of immediately investing upon information disclosure, reducing the threshold $\underline{z}$. This shortens the leader's expected cost-advantage period for $z_t \in [z_L, \underline{z})$, reducing her value. This first limits, and then eliminates, the range for $z_t \in (0, \underline{z})$ such that $L(z_t) > F(z_t)$, ruling out the possibility of an equilibrium in which the leader invests at $z_L < \underline{z}$ (i.e., of a first mover advantage "early" equilibrium). Hence, an increase in θ, for a given λ, favours the selection, as the subgame perfect equilibrium, of a leader's investment trigger higher than $\underline{z}$.

The effects of a larger λ are subtler, but they need to be scrutinized to understand why the threshold $\theta(\lambda)$ is decreasing in λ. A larger probability of information spillover

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[19]To save space, we do not present a panel depicting the follower's expected threshold in this candidate equilibrium. Its qualitative behavior closely mimics that of the leader.

reduces the value of a leader investing in $z_t \in [\underline{z}, \bar{z})$, while obviously benefiting the follower's expected profits. Accordingly, the two payoff functions meet at a later $\tilde{z}$, which implies an higher current value for the equilibrium. (Refer again to Figure 1, and consider that the process we are describing shifts upward $F(z_t)$, and downward $L(z_t)$). This higher value twists the equilibrium selection process toward leader's investment triggers, which are higher than $\underline{z}$. Accordingly, a higher λ requires a lower θ for the early equilibrium to be dominated.

In words, a large probability of releasing relevant information induces the leader to delay her investment, in order to grasp large benefits from the increased market dimension during her limited cost-advantage period. This effect proves to be strong enough to sustain – even for a relatively small spillover size – the equilibrium in which the leader invests after $\underline{z}$.

A larger σ enhances quite significantly the threshold $\theta(\lambda)$. The intuition for this result is simple: an increase in uncertainty has the usual effects on the follower's optimal choices: it delays the thresholds $\underline{z}$ and $\bar{z}$.[20] The increase in the follower's value of waiting, delaying his investment triggers, not only increases the follower's payoff, but it also benefits the leader's value: in particular, she enjoys, in the early stages of the game, a longer cost-advantage period, because the follower's optimal policy dictates that he invest – upon information disclosure – at $\underline{z}$. This obviously acts in favour of the subgame perfectness of the early equilibrium.

We now portray a minor innovation. In this case, the cost reduction is modest, so that it is not convenient for the leader to sizably expand her production at the follower's expenses. Hence, the leader, even when she is the unique innovator, does not enjoy profits so much larger than those of the follower. Accordingly, to depict a minor innovation, besides normalizing $\Pi_0 = 1$ as before, we assume that $\Pi_1^h = 1.21$, $\Pi_1^l = 0.9025$, and $\Pi_2 = 1.1025$.[21]

Figure 7 shows the threshold $\theta(\lambda)$ for a minor innovation, again for $\sigma \in \{0.03, 0.1\}$.

[Figure 7 about here]

When θ is low, it is the simultaneous investment that prevails. In fact, consider first that the simultaneous investment strategy is optimal once the market dimension – and hence the potential increase in profits due to the innovation – have reached high values. In this case, an innovation leader cannot emerge because the rival would

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[20] As for $\underline{z}$, the effect can be verified analytically from Eqs. (3) and (12), following the usual steps expounded in Dixit and Pindyck (1994).

[21] In this case, the values in the main text are consistent with Cournot competition in the final product market, when the innovation size amounts to 0.05 of the market dimension.

immediately copy her decision. The existing literature suggests that the simultaneous investment equilibrium is subgame perfect when the size of the innovation is small, because the per-period first-innovator profits are not significant, which avoids preemptive behaviors, and hence an equilibrium in which a leader invests in $z_L < \underline{z}$.

When the follower's best reply is to invest immediately upon information disclosure (i.e., for $z_t \in [\underline{z}, \bar{z})$), an higher θ reduces, for a given λ, the difference between the leader's and the follower's payoffs. While the negative effect on the leader is limited, because the cost advantage period is essentially governed by λ (unless z_t is "very close" to $\bar{z}$), the follower significantly benefits from the lower investment cost. This increases the follower's value function, postponing the candidate equilibrium point $\tilde{z}$, which acts against the subgame perfectness of the simultaneous equilibrium.[22]

In words, in the interval $z_t \in [\underline{z}, \bar{z})$, the presence of a spillover induces the leader to delay her investment; in fact, she is aware that it is not in the follower's interest to preempt her, because he knows that by waiting he will obtain a reduction in the sunk cost. This increases the firms' expected values in this candidate equilibrium, which therefore tends to dominate the simultaneous solution.

An increase in λ benefits the follower while harming the leader's payoff. Hence, it acts in favour of the subgame perfectness of the equilibrium in $[\underline{z}, \bar{z})$, which therefore dominates for lower values of the spillover parameter.

In this case also, a larger σ enhances appreciably the threshold $\theta(\lambda)$. The intuition for this result is simple: an increase in uncertainty delays the threshold $z_S = \max\{z', \bar{z}\}$.[23] The increase in z_S benefits both firms' values, which obviously acts in favour of the subgame perfectness of the simultaneous investment equilibrium.

The evaluation of our result involves a thorny issue, namely the assessment of the actual size of the spillover parameter.

Some early literature (see Mansfield et al., 1981) suggests that the ratio of the imitator's cost to that of the first innovator is 0.65; more recent contributions estimate the role of technological externalities from production functions. Los and Verspagen (2000) find the role of "external R&D" to be extremely important for U.S. manufacturing firms. Actually, they find an elasticity of output to external R&D on the order of 0.5-0.6. Ornaghi (2006) estimates that in Spain the elasticity of output with respect to "technological spillovers" is on the order of 0.2 of the elasticity of output to *own* R&D.[24]

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[22]Even if Figure 1 has been drawn for an high cost reduction, it may be helpful to visualize the effects of an higher θ on the value functions.

[23]As for z', the effect can be verified analytically from Eqs (10), and (12), following Dixit and Pindyck (1994).

[24]In both papers, the technological spillover variable is a weighted sum of the R&D expenditures

These data are suggestive of the fact that the role of inter-firm spillovers is actually relevant; they also support the view that there is quite a significant inter-sectoral variation of the importance of spillovers. Accordingly, we believe that our results apply at least to the industrial sectors in which, due to the geographical or technological proximity of the producers, the spillovers are likely to be relevant.

5 Optimal subsidization

To underscore the policy relevance of our results, we present an exercise in which a benevolent planner chooses the optimal tax/subsidization rate of investment in a duopoly characterized by the elements we have depicted so far.

In dealing with this issue, we have adopted a second-best perspective: for us, neither the number of firms acting in the market nor the way they compete in the second-stage quantity game lies within the regulatory power of the benevolent planner. Hence, what this non-omnipotent planner chooses is the timing of innovation, which is affected via the subsidy (or the tax) on investment.[25] The planner's decisions are based on welfare; in particular for our simulation we use the welfare levels – computed à la Marshall – that can be obtained under the Cournot decentralized solution, for the market described in Appendix 3. The instantaneous welfare levels are discounted at the same rate, r, that is used by firms.

The details concerning the computation of the welfare function are provided in Appendix 1. Here, we analyze the consequence of the changes in the investment triggers that are induced by a proportional subsidization of the fixed investment cost. In our exercises, subsidy levels are decided upon at time 0, and they are left unchanged thereafter. In particular, we focus on the case of a major innovation introduced in a not-very-volatile sector ($\sigma = 0.03$). Figure 8 shows the welfare-maximizing subsidization rates for $\theta > \theta(\lambda)$, the parameters configuration being the one used to generate Figure 6.

[Figure 8 about here]

The equilibrium in $[\underline{z}, \bar{z})$ implies that the optimal policy requires a substantial public intervention in favour of the investment activity.[26] When this equilibrium

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of the firms belonging to a specific sector.

[25] This approach is standard in the literature: see, e.g., Hoppe (2000) and Weeds (2002). The first best equilibrium for an omnipotent planner implies the presence of only one firm: whenever there are non-decreasing returns in the innovation size or probability, it is optimal to have only one firm to innovate and cover the entire market at the marginal (post-innovation) cost.

[26] When the equilibrium is of the second-mover advantage type, an increase in θ or in λ calls for a modest increase in the subsidy rate because the effect of these parameters on the equilibrium investment trigger is weak (refer to Figure 5, Panel (c)).

is subgame perfect, an increase in uncertainty – delaying the equilibrium – calls for higher subsidization rates, a result that applies independently of agents' risk aversion.

Figure 9 is drawn for comparison, and it shows the optimal subsidization rate called for by a "preemptive equilibrium" when it exists. In other words, Figure 9 shows the optimal subsidization rate that applies for θ below the threshold $\theta(\lambda)$. It also shows the optimal subsidization that would have applied had the strategy of investing at z_L been subgame perfect for the leader, even for $\theta > \theta(\lambda)$.

[Figure 9 about here]

The fact that the optimal policy portrayed in Figure 9 implies the taxation of the investment is not surprising. In such an equilibrium configuration, the first mover invests "very soon" to avoid being preempted, and the R&D investment is socially excessive, so that it must be delayed via taxation (see again Fudenberg and Tirole, 1985, but also Riordan, 1992, and others). Notice that our result implies that the optimal tax rate is virtually independent from λ. This happens because – in the early equilibrium – when the leader invests, she is "virtually sure" that the follower obtains the spillover before $\underline{z}$, and hence the trigger point z_L is "almost independent" from λ.

When we consider the case of a minor innovation, the results are less striking because in this case it is the simultaneous equilibrium than tends to prevail with low spillover. The collusive flavour of this equilibrium implies underinvestment, which calls for positive subsidization. In this case, our result implies that the policies aimed at stimulating R&D have to be less sizeable than suggested before because the underinvesting equilibrium in $[\underline{z}, \bar{z})$ is closer to the social optimum than the simultaneous equilibrium.

6   Concluding remarks

What drives the result in our model is not the fact that an increasing spillover progressively postpones the leader adoption date in the "early" equilibrium. While this happens, the crucial aspect is that a different equilibrium of the dynamic game emerges. In fact, for low – and hence realistic – spillover, we find a subgame-perfect equilibrium in which the leader invests much later. Actually, she delays her investment until the stochastic fundamental is high enough that the follower invests as soon as he obtains the spillover.

The model could be extended in various ways. First, the spillover size parameter, and the probability of benefiting from the spillover could be endogenized, while al-

27

ternative stochastic processes for profits could be assumed, such as those exhibiting mean reversion. These assumptions would generate similar qualitative results.

The paper has focused on the symmetric duopoly case. If firms' costs are instead allowed to differ, the identities of the leader and of the follower could be defined, with the more efficient firm receiving a greater payoff. In this case, it would be interesting to analyze how the Pawlina and Kort (2006) sequential equilibrium affects the selection of the subgame-perfect equilibrium. We leave this point for future research.

An increase in the number of firms is problematic. As explained by Fudenberg and Tirole (1985), with three identical firms the equilibrium selection is complicated by the fact that the first mover's payoff can be discontinuous. Moreover, even in the simplest case, i.e. when the two followers obtain the spillover at the same time, the number of candidate equilibria that may emerge increases substantially. For example, there is a candidate equilibrium in which the leader delays her investment until the demand is high enough that the first of the two followers invests upon benefiting from the spillover, and the third innovator enters sequentially. Alternatively, the two followers may invest at the same time. In addition to the previous candidate equilibria, it is possible that two simultaneous entries occur, while the third firm waits for the realization of the spillover. The specific assumptions about the relative size of the profits obtained by the three firms determine which equilibrium is subgame perfect. Notice however, that, whenever one of the above equilibria is subgame perfect, two important results of our analysis are preserved: the presence of the spillover implies realistic entry lags, and it reduces the difference in the firms' value function, and therefore in the firms' market betas.

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8 Appendix 1: Details on the value functions

8.1 The follower has obtained the spillover

Following the usual approach, we reformulate the Bellman equation (2) as follows:

$$F^d(z_t) = \max\left\{\Pi_1^l z_t dt + E_t\left[F^d(z_{t+dt})e^{-rdt}\right], \frac{\Pi_2}{r-\alpha}z_t - (1-\theta)I\right\}.$$

We guess that for $z_t \in (0,\underline{z})$ the follower's maximum value function is

$$F^d(z_t) = C_1 z_t + D_1 z_t^\gamma,$$

where C_1, D_1, and γ are undetermined coefficients, while the threshold $\underline{z}$ is determined endogenously.

Ito's Lemma guarantees that, for $z_t < \underline{z}$,

$$E_t\left[F^d(z_{t+dt})e^{-rdt}\right] = F^d(z_t) + \frac{\partial F^d(z_t)}{\partial z_t}\alpha z_t dt + \frac{\partial^2 F^d(z_t)}{\partial z_t^2}\frac{\sigma^2}{2}z_t^2 dt - rF^d(z_t)dt.$$

Following the strategy commonly used in the literature, we now exploit the expression above in Eq. (1), and we use our guess to obtain that, for $z_t \in (0,\underline{z})$,

$$0 = \Pi_1^l z_t + \left(C_1 + \gamma D_1 z_t^{\gamma-1}\right)\alpha z_t + \gamma(\gamma-1)D_1 z_t^\gamma \frac{\sigma^2}{2} - r(C_1 z_t + D_1 z_t^\gamma).$$

The above equation implies that $C_1 = \frac{\Pi_1^l}{r-\alpha}$, and that γ is the positive root of:[27]

$$\gamma\alpha + \gamma(\gamma-1)\frac{\sigma^2}{2} - r = 0. \tag{12}$$

The usual value-matching and smooth-pasting conditions determine D_1 and $\underline{z}$:

$$\begin{cases} \frac{\Pi_1^l}{r-\alpha}\underline{z} + D_1\underline{z}^\gamma = \frac{\Pi_2}{r-\alpha}\underline{z} - (1-\theta)I \\ \frac{\Pi_1^l}{r-\alpha} + \gamma D_1\underline{z}^{\gamma-1} = \frac{\Pi_2}{r-\alpha} \end{cases}.$$

It is immediate to verify that the system above yields $\underline{z}$ as in (3), and that $D_1 = \frac{(1-\theta)I}{\gamma-1}\underline{z}^{-\gamma}$.

8.2 The follower has not obtained the spillover

We consider first the follower's optimal behavior for $z_t \in [\underline{z}, \infty)$. In this case, the Bellman equation is

$$F(z_t) = \max\left\{\Pi_1^l z_t dt + \lambda\left[\frac{\Pi_2}{r-\alpha}z_t - (1-\theta)I\right]dt + (1-\lambda dt)E_t\left[F(z_{t+dt})e^{-rdt}\right],\right.$$
$$\left.\frac{\Pi_2}{r-\alpha}z_t - I\right\}. \tag{13}$$

The second addendum on the right-hand side of the equation above comes from the fact that, with probability λdt, the follower benefits from the informational spillover, which triggers an immediate investment.

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[27] The negative root of the quadratic equation must be discarded because its use would imply that $\lim_{z_t\to 0} F^d(z_t) \neq 0$.

We guess that for $z_t \in [\underline{z}, \bar{z})$ — i.e., when the optimal strategy is to wait – the follower's maximum value function is

$$F(z_t) = A_2 + C_2 z_t + E_2 z_t^{\beta_1} + G_2 z_t^{\beta_2}, \tag{14}$$

where A_2, C_2, E_2, G_2, β_1, and β_2 are undetermined coefficients and the threshold $\bar{z}$ must be determined endogenously.

We apply Ito's Lemma to $E_t \left[F(z_{t+dt}) e^{-(r+\lambda)dt} \right]$, we use the resulting expression into Eq. (13), and we use (14) to obtain that, for $z_t \in [\underline{z}, \bar{z})$,

$$0 = \Pi_1^l z_t + \lambda \left[\frac{\Pi_2}{r-\alpha} z_t - (1-\theta)I \right] + \left(C_2 + \beta_1 E_2 z_t^{\beta_1 - 1} + \beta_2 G_2 z_t^{\beta_2 - 1} \right) \alpha z_t +$$

$$+ \left[\beta_1 (\beta_1 - 1) E_2 z_t^{\beta_1 - 2} + \beta_2 (\beta_2 - 1) G_2 z_t^{\beta_2 - 2} \right] z_t^2 \frac{\sigma^2}{2} +$$

$$-(r+\lambda)(A_2 + C_2 z_t + E_2 z_t^{\beta_1} + G_2 z_t^{\beta_2}).$$

The above equation implies: $A_2 = -\frac{\lambda}{r+\lambda}(1-\theta)I$, and $C_2 = \frac{(r-\alpha)\Pi_1^l + \lambda \Pi_2}{(r+\lambda-\alpha)(r-\alpha)}$; β_1, and β_2 are the roots of[28]

$$\beta \alpha + \beta(\beta - 1) \frac{\sigma^2}{2} - (r+\lambda) = 0. \tag{15}$$

To pin down the undetermined E_2, G_2, and the threshold $\bar{z}$, we can exploit the value-matching and smooth-pasting conditions at $\bar{z}$. This gives:

$$\begin{cases} -\frac{\lambda}{r+\lambda}(1-\theta)I + \frac{(r-\alpha)\Pi_1^l + \lambda \Pi_2}{(r+\lambda-\alpha)(r-\alpha)} \bar{z} + E_2 \bar{z}^{\beta_1} + G_2 \bar{z}^{\beta_2} = \frac{\Pi_2}{r-\alpha} \bar{z} - I \\ \frac{(r-\alpha)\Pi_1^l + \lambda \Pi_2}{(r+\lambda-\alpha)(r-\alpha)} + \beta_1 E_2 \bar{z}^{\beta_1 - 1} + \beta_2 G_2 \bar{z}^{\beta_2 - 1} = \frac{\Pi_2}{r-\alpha} \end{cases}. \tag{16}$$

Of course, we need to postpone the determination of E_2, G_2, and $\bar{z}$, until when we are able to identify a third equation, completing system (16).

When $z_t \in (0, \underline{z})$, the maximum value function for the follower solves

$$F(z_t) = \Pi_1^l z_t dt + \lambda \left[\frac{\Pi_1^l}{r-\alpha} z_t + \frac{(1-\theta)I}{\gamma - 1} \left(\frac{z_t}{\underline{z}} \right)^\gamma \right] dt + (1 - \lambda dt) E_t \left[F(z_{t+dt}) e^{-rdt} \right]. \tag{17}$$

Our tentative solution for the follower's maximum value function in the interval $z_t \in (0, \underline{z})$ is

$$F(z_t) = C_3 z_t + D_3 z_t^\gamma + E_3 z_t^{\beta_1}, \tag{18}$$

where C_3, D_3, and E_3 are undetermined coefficients, while γ and β_1 are pinned down by the quadratic equations (12) and (15), respectively.[29]

Our guess (18) readily gives:

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[28] In this case, the negative root of the quadratic equation cannot be discarded because we are considering an interval, $z_t \in [\underline{z}, \bar{z}]$, that does not contain 0.

[29] The negative roots of equation (15) must obviously be discarded, since the limit, for $z_t \to 0$, of the maximum value function defined by the Bellman equation (17) must be 0. It is easy to verify that γ must actually fulfill equation (12).

$$0 = \Pi_1^l z_t + \lambda \left[\frac{\Pi_1^l}{r - \alpha} z_t + \frac{(1 - \theta)I}{\gamma - 1} \left(\frac{z_t}{z} \right)^\gamma \right] + \left(C_3 + \gamma D_3 z_t^{\gamma - 1} + \beta_1 E_3 z_t^{\beta_1 - 1} \right) \alpha z_t +$$

$$+ \left[\gamma(\gamma - 1) D_3 z_t^{\gamma - 2} + \beta_1(\beta_1 - 1) E_3 z_t^{\beta_1 - 2} \right] z_t^2 \frac{\sigma^2}{2} - (r + \lambda) \left(C_3 z_t + D_3 z_t^\gamma + E_3 z_t^{\beta_1} \right).$$

The above equation implies that $C_3 = \frac{\Pi_1^l}{r - \alpha}$, and that $D_3 = \frac{(1-\theta)I}{\gamma - 1} z^{-\gamma}$.

At z, due to the follower optimizing behavior, the value-matching and smooth-pasting conditions between the maximum value functions (14) and (18) must apply. This yields

$$\begin{cases} \frac{\Pi_1^l}{r - \alpha} z + \frac{(1-\theta)I}{\gamma - 1} + E_3 z^{\beta_1} = -\frac{\lambda}{r + \lambda}(1 - \theta)I + \frac{(r - \alpha)\Pi_1^l + \lambda \Pi_2}{(r + \lambda - \alpha)(r - \alpha)} z + E_2 z^{\beta_1} + G_2 z^{\beta_2} \\ \frac{\Pi_1^l}{r - \alpha} + \gamma \frac{(1-\theta)I}{\gamma - 1} z^{(-1)} + \beta_1 E_3 z^{\beta_1 - 1} = \frac{(r - \alpha)\Pi_1^l + \lambda \Pi_2}{(r + \lambda - \alpha)(r - \alpha)} + \beta_1 E_2 z^{\beta_1 - 1} + \beta_2 G_2 z^{\beta_2 - 1} \end{cases}.$$

$$(19)$$

The four equations in (16) and (19) determine E_2, E_3, G_2, and the threshold $\bar{z}$.

8.3 Value of a leader who has invested

As a preliminary to the determination of the leader's value of investing, it is convenient to analyze her value of *having already invested*, when the follower has already obtained the spillover.

In the interval $z_t \in (0, z)$ the maximum value function can be obtained starting from its recursive form:

$$\bar{L}^d(z_t) = \Pi_1^h z_t dt + E_t \left[\bar{L}^d(z_{t+dt}) e^{-rdt} \right], \tag{20}$$

where a bar above the maximum value function denotes that the leader has already sunk the investment cost, and the superscript d underscores that the leader is assumed to be facing a follower that has already obtained the informational spillover. We guess that

$$\bar{L}^d(z_t) = C_5 z_t + D_5 z_t^\gamma, \tag{21}$$

where C_5, and D_5 are undetermined coefficients, while γ is the positive root of Eq. (12).

Using the standard procedure, we apply Ito's Lemma to $E_t \left[\bar{L}^d(z_{t+dt}) e^{-rdt} \right]$, we use the resulting expression in Eq. (20), and we exploit the tentative solution (21) to obtain

$$0 = \Pi_1^h z_t + \left(C_5 + \gamma D_5 z_t^{\gamma - 1} \right) \alpha z_t + \left[\gamma(\gamma - 1) D_5 z_t^{\gamma - 2} \right] z_t^2 \frac{\sigma^2}{2} - r(C_5 z_t + D_5 z_t^\gamma),$$

which gives: $C_5 = \frac{\Pi_1^h}{r - \alpha}$. The still-undetermined coefficient D_5 is obtained by means of a value-matching condition. At z, the value of being the leader given that the informational spillover has occurred, is identical to the expected stream of profits obtained when both the firms have sunk the fixed cost. In fact, there the follower is investing. Accordingly, at z, we have $\bar{L}^d(z) = F(z) + (1 - \theta)I$, and hence:

$$\frac{\Pi_1^h}{r - \alpha} z + D_5 z^\gamma = \frac{\Pi_2}{r - \alpha} z,$$

so that: $D_5 = \frac{\Pi_2 - \Pi_1^h}{r - \alpha} \underline{z}^{1-\gamma}$.

Hence, the value for a leader that has sunk the cost is:

$$\bar{L}^d(z_t) = \begin{cases} \frac{\Pi_1^h}{r-\alpha} z_t + \frac{\Pi_2 - \Pi_1^h}{r-\alpha} \underline{z} \left(\frac{z_t}{\underline{z}}\right)^\gamma, & z_t \in (0, \underline{z}) \\ \frac{\Pi_2}{r-\alpha} z_t, & z_t \in [\underline{z}, \infty) \end{cases}. \tag{22}$$

The interpretation for the value function above is straightforward. When $z_t \geq \underline{z}$, the follower invests upon information revelation, and the leader's payoff is given by the flows of future duopoly profits, discounted at the growth-adjusted rate $r - \alpha$. If, instead, $z_t < \underline{z}$, the follower delays his investment, and the leader enjoys – for a period of time of stochastic length – a cost advantage guaranteeing her the instantaneous profit $\Pi_1^h z_t$. The second addendum in the first line of Eq. (22) corrects the discounted profits value $\Pi_1^h z_t/(r - \alpha)$, taking account of the future reduction of instantaneous profits to $\Pi_2 z_t$ that takes place at $\underline{z}$.

8.4 Value of the investment for the leader

We first determine the leader's maximum value of investing in state $z_t \in (0, \underline{z})$. In this interval, the leader knows that the follower – even when the informational spillover has occurred – does not invest until z_t has reached $\underline{z}$. Hence, the leader enjoys the instantaneous profit $\Pi_1^h z_t$, which explains the first addendum on the right-hand side of the equation below. The second addendum comes from the fact that, with probability λdt, the follower benefits from the informational spillover but does not invest, so that the leader's maximum value function jumps to what is prescribed by the first line in Eq. (22). The third addendum is explained by the fact that, with probability $(1 - \lambda dt)$ there is no information revelation, and hence the leader obtains $\bar{L}(z_{t+dt})$.

Accordingly, the leader's maximum value is the solution of

$$L(z_t) = \Pi_1^h z_t dt + \lambda \left[\bar{L}^d(z_t)\right] dt + (1 - \lambda dt) E_t \left[\bar{L}(z_{t+dt})\right] e^{-rdt} - I.$$

Having determined $\bar{L}^d(z_t)$ as in the first line of Eq. (22), we exploit the fact that the value of having invested is

$$\bar{L}(z_t) = L(z_t) + I, \tag{23}$$

and we reformulate the leader's maximum value function as the solution of

$$L(z_t) = \Pi_1^h z_t dt + \lambda \left[\frac{\Pi_1^h}{r - \alpha} z_t + \frac{\Pi_2 - \Pi_1^h}{r - \alpha} \underline{z} \left(\frac{z_t}{\underline{z}}\right)^\gamma\right] dt + (1 - \lambda dt) E_t \left[L(z_{t+dt}) - I\right] e^{-rdt} - I. \tag{24}$$

Our tentative solution for the leader's value of investing is

$$L(z_t) = C_6 z_t + D_6 z_t^\gamma + E_6 z_t^{\beta_1} - I. \tag{25}$$

As usual, C_6, D_6, and E_6 are undetermined coefficients, while we shall verify that γ and β_1 are the positive roots of the quadratic equations (12) and (15), respectively.[30]

Applying Ito's Lemma to $E_t \left[\bar{L}(z_{t+dt})e^{-rdt}\right]$, using the resulting expression into

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[30] As before, we discard the negative roots of equation (15).

Eq. (24), and exploiting equations (22), (25), and (23), we obtain

$$0 = \Pi_1^l z_t + \lambda \left[\frac{\Pi_1^h}{r-\alpha} z_t + \frac{\Pi_2 - \Pi_1^h}{r-\alpha} \underline{z} \left(\frac{z_t}{\underline{z}} \right)^\gamma \right] + \left(C_6 + \gamma D_6 z_t^{\gamma-1} + \beta_1 E_6 z_t^{\beta_1 - 1} \right) \alpha z_t +$$

$$+ \left[\gamma(\gamma - 1) D_6 z_t^{\gamma-2} + \beta_1(\beta_1 - 1) E_6 z_t^{\beta_1 - 2} \right] z_t^2 \frac{\sigma^2}{2} +$$

$$- (r+\lambda)(C_6 z_t + D_6 z_t^\gamma + E_6 z_t^{\beta_1}),$$

which implies that $C_6 = \frac{\Pi_1^h}{r-\alpha}$, and that $D_6 = \frac{\Pi_2 - \Pi_1^h}{r-\alpha} \underline{z}^{1-\gamma}$; it is easy to verify that γ, and β_1 fulfill equations (12) and (15). Notice that E_6 is still to be determined.

At $\underline{z}$, due to the leader's optimizing behavior, a value-matching, and a smooth-pasting conditions must apply between the maximum value functions (25), and the one that shall be valid in $[\underline{z}, \bar{z})$.

We consider then the interval $z_t \in [\underline{z}, \bar{z})$. In this case, the leader knows that the market dimension is high enough to justify the immediate follower's investment upon information leakage.

Hence, we formulate the leader's maximum value of investing in state z_t as

$$L(z_t) = \Pi_1^h z_t dt + \lambda \left(\frac{\Pi_2}{r-\alpha} z_t \right) dt + (1 - \lambda dt) E_t \left[\bar{L}(z_{t+dt}) \right] e^{-rdt} - I. \qquad (26)$$

In the equation above, the second addendum on the right-hand side comes from the fact that, with probability λdt, the follower benefits from the spillover and invests, so that the leader's instantaneous profit falls to the duopoly level (and stays there forever). The third addendum expresses the fact that, with probability $(1 - \lambda dt)$ there is no information revelation, and hence the leader investing at z_t still enjoys her cost advantage.

Our tentative solution for the leader's value of investing is

$$L(z_t) = C_4 z_t + E_4 z_t^{\beta_1} + G_4 z_t^{\beta_2} - I, \qquad (27)$$

and hence, by Eq. (23), we have that $\bar{L}(z_t) = C_4 z_t + E_4 z_t^{\beta_1} + G_4 z_t^{\beta_2}$. C_4, E_4, and G_4 are coefficients to be determined, while β_1 and β_2 are the roots of Eq. (15).[31]

We apply Ito's Lemma to $E_t \left[\bar{L}(z_{t+dt}) e^{-rdt} \right]$, we use the resulting expression in Eq. (26), and we use the tentative solutions (27)-(23) to obtain

$$0 = \Pi_1^l z_t + \lambda \left(\frac{\Pi_2}{r-\alpha} z_t \right) + \left(C_4 + \beta_1 E_4 z_t^{\beta_1 - 1} + \beta_2 G_4 z_t^{\beta_2 - 1} \right) \alpha z_t +$$

$$+ \left[\beta_1(\beta_1 - 1) E_4 z_t^{\beta_1 - 2} + \beta_2(\beta_2 - 1) G_4 z_t^{\beta_2 - 2} \right] z_t^2 \frac{\sigma^2}{2} +$$

$$- (r+\lambda)(C_4 z_t + E_4 z_t^{\beta_1} + G_4 z_t^{\beta_2}).$$

The above equation implies that $C_4 = \frac{(r-\alpha)\Pi_1^h + \lambda \Pi_2}{(r+\lambda-\alpha)(r-\alpha)}$ and that β_1 and β_2 actually are the roots of Eq. (15).

Notice that, if the leader invests at $\bar{z}$, the follower immediately reacts by following suit. Hence, the two firms' values are the same, which provides the value-matching

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[31] Again, the negative root of the quadratic equation must not be discarded because we are concerned with the interval, $z_t \in [\underline{z}, \bar{z})$.

condition $L(\bar{z}) = F(\bar{z})$ that is part of the system pinning down the undetermined coefficients E_4, and G_4.[32] The value-matching condition at $\bar{z}$ is

$$\frac{(r-\alpha)\Pi_1^h + \lambda\Pi_2}{(r+\lambda-\alpha)(r-\alpha)}\bar{z} + E_4\bar{z}^{\beta_1} + G_4\bar{z}^{\beta_2} - I = \frac{\Pi_2}{r-\alpha}\bar{z} - I \tag{28}$$

As already remarked, at $\underline{z}$ a value-matching and a smooth-pasting conditions between the maximum value functions (25), and (27) must apply. This yields

$$\begin{cases} \frac{\Pi_1^h}{r-\alpha}\underline{z} + \frac{\Pi_2-\Pi_1^h}{r-\alpha}\underline{z} + E_6\underline{z}^{\beta_1} - I = \frac{(r-\alpha)\Pi_1^h+\lambda\Pi_2}{(r+\lambda-\alpha)(r-\alpha)}\underline{z} + E_4\underline{z}^{\beta_1} + G_4\underline{z}^{\beta_2} - I \\ \frac{\Pi_1^h}{r-\alpha} + \gamma\frac{\Pi_2-\Pi_1^h}{r-\alpha} + \beta_1 E_6\underline{z}^{\beta_1-1} = \frac{(r-\alpha)\Pi_1^h+\lambda\Pi_2}{(r+\lambda-\alpha)(r-\alpha)} + \beta_1 E_4\underline{z}^{\beta_1-1} + \beta_2 G_4\underline{z}^{\beta_2-1} \end{cases}. \tag{29}$$

The three equations in (28) and (29) determine E_4, E_6, and G_4, as in (8).

8.5 Maximum value function for the simultaneous investment problem

Following the usual approach, we reformulate the Bellman equation (9) as follows:

$$S(z_t) = \max\left\{\Pi_0 z_t dt + E_t\left[S(z_{t+dt})e^{-rdt}\right], \frac{\Pi_2}{r-\alpha}z_t - I\right\}.$$

We guess that, for $z_t \in (0, z_S)$, it is optimal for the firms to delay their investment. In this case, the tentative solution for their maximum value function is

$$S(z_t) = C_7 z_t + D_7 z_t^\gamma,$$

where C_7, D_7, and γ are undetermined coefficients, while the threshold z_S must be determined endogenously, taking into account the constraint $z_S \geq \bar{z}$.

Following our usual strategy, we exploit the Ito differential for $E_t\left[S(z_{t+dt})e^{-rdt}\right]$, and our guess above to reformulate Eq. (9) – for $z_t \in (0, z_S)$ – as

$$0 = \Pi_0 z_t + \left(C_7 + \gamma D_7 z_t^{\gamma-1}\right)\alpha z_t + \gamma(\gamma-1)D_7 z_t^\gamma \frac{\sigma^2}{2} - r(C_7 z_t + D_7 z_t^\gamma).$$

The above equation implies that $C_7 = \frac{\Pi_0}{r-\alpha}$, and that γ is the positive root of Eq. (12) (as usual, the negative root of that quadratic equation must be discarded).

Assuming for the moment that $z' \geq \bar{z}$, we determine D_7, and $z_S = z'$ by means of the usual value-matching and smooth-pasting conditions. These give

$$\begin{cases} \frac{\Pi_0}{r-\alpha}z' + D_7 z'^\gamma = \frac{\Pi_2}{r-\alpha}z' - I \\ \frac{\Pi_0}{r-\alpha} + \gamma D_7 z'^{\gamma-1} = \frac{\Pi_2}{r-\alpha} \end{cases}.$$

It is immediate to verify that the system above determines $z_S = z'$ as in Eq. (10), and $D_7 = \left(\frac{\Pi_2-\Pi_0}{r-\alpha}z' - I\right)z'^{(-\gamma)} = \frac{I}{\gamma-1}z'^{(-\gamma)}$. Notice that the maximum value function

$$S(z_t) = \frac{\Pi_0}{r-\alpha}z_t + \frac{I}{\gamma-1}\left(\frac{z_t}{z'}\right)^\gamma$$

---

[32] Because at $\bar{z}$ there is no optimal choice on the part of the leader, there is no corresponding smooth-pasting condition in this case (see Weeds, 2002).

gives the expected present discounted value of investing at z', conditional on being at z_t. (Refer to Dixit and Pindyck, 1994.)

When $z' \geq \bar{z}$, this qualifies the solution. When $z' < \bar{z}$, the constraint $z_S \geq \bar{z}$ is binding, and the two competitors are not free to choose when to invest. Accordingly, the smooth-pasting condition does not apply, and the solution is determined by the value matching condition

$$\frac{\Pi_0}{r - \alpha}\bar{z} + D_7\bar{z}^\gamma = \frac{\Pi_2}{r - \alpha}\bar{z} - I,$$

which gives: $D_7 = \left(\frac{\Pi_2 - \Pi_0}{r - \alpha}\bar{z} - I\right)\bar{z}^{(-\gamma)}$. The value for D_7 that applies when $z' \geq \bar{z}$ is $\left(\frac{\Pi_2 - \Pi_0}{r - \alpha}\bar{z} - I\right)\bar{z}^{(-\gamma)}$; accordingly, the maximum value function for the simultaneous investment problem can be written compactly as in (11).

8.6 The Social Welfare function

The welfare levels depend on the number of firms that have already sunk the cost. Let $M_i z_t$ be the instantaneous welfare level that is obtained when $i = 0, 1, 2$ firms have already invested, and when the market dimension variable takes the value z_t.

Social value of the leader's investment at $\tilde{z} \in [\underline{z}, \bar{z})$. Consider the case in which the leader has invested, while the follower has not. For $z_t \in [\tilde{z}, \bar{z})$, the follower shall invest immediately after he enjoys the spillover (or he shall invest at $\bar{z}$ if the fundamental gets there before the information disclosure takes place). Hence, when one firm has already invested but the information leakage has not occurred, the welfare $W_1(z_t)$, for $z_t \in [\tilde{z}, \bar{z})$, is given by

$$W_1(z_t) = M_1 z_t dt + \lambda dt \left[\frac{M_2}{r - \alpha}z_t - (1 - \theta)I\right] + (1 - \lambda dt)E_t\left[W_1(z_{t+dt})e^{-rdt}\right], \quad (30)$$

where the second addendum on the right-hand side comes from the fact that with probability λdt the follower benefits from the informational spillover and invests because $z_t \geq \underline{z}$, so that the instantaneous welfare jumps to $M_2/(r - \alpha)$. With probability $(1 - \lambda dt)$, there is no information revelation, and hence no investment.

Our guess for $W_1(z_t)$ is

$$W_1(z_t) = F + Gz_t + Hz_t^{\beta_1},$$

where F, G, and H are undetermined coefficients, while β_1 is the positive root of Eq. (15).

Using our standard procedure, we apply Ito's Lemma to $E_t\left[W(z_{t+dt})e^{-rdt}\right]$, we use the resulting expression into Eq. (30), and we exploit the tentative solution above to obtain

$$0 = \frac{(r - \alpha)M_1 + \lambda M_2}{r - \alpha}z_t - \lambda(1 - \theta)I + \alpha Gz_t + \alpha\beta_1 Hz_t^{\beta_1} +$$

$$+ \beta_1(\beta_1 - 1)Hz_t^{\beta_1}\frac{\sigma^2}{2} - (r + \lambda)(F + Gz_t + Hz_t^{\beta_1}),$$

which gives $F = -\frac{\lambda}{r + \lambda}(1 - \theta)I$, and $G = \frac{(r - \alpha)M_1 + \lambda M_2}{(r - \alpha)(r + \lambda - \alpha)}$. The still-undetermined coefficient H is obtained by means of a value-matching condition. In fact, at $\bar{z}$, the

social value of the leader's having invested is identical to the social value when the
follower also invests, net of its cost. Accordingly, we have:

$$W_1(\bar{z}) = -\frac{\lambda}{r+\lambda}(1-\theta)I + \frac{(r-\alpha)M_1 + \lambda M_2}{(r-\alpha)(r+\lambda-\alpha)}\bar{z} + H\bar{z}^{\beta_1} = \frac{M_2}{r-\alpha}\bar{z} - I,$$

which yields $H = \bar{z}^{(-\beta_1)}\left[\frac{M_2-M_1}{r-\alpha+\lambda}\bar{z} - \frac{r+\theta\lambda}{r+\lambda}I\right].$

Consider now the social value of the leader's investment, when it is still to be
performed. In this case, $z_t \in (0,\tilde{z})$ – i.e., when no firm has invested, the welfare
function is

$$W_0(z_t) = M_0 z_t dt + E_t\left[W_0(z_{t+dt})e^{-rdt}\right], \tag{31}$$

and the solution we propose is

$$W_0(z_t) = Nz_t + Pz_t^{\gamma}.$$

From the above tentative solution, where N and P are undetermined coefficients
and γ is given by Eq. (12), we readily obtain

$$0 = M_0 z_t + \alpha N z_t + \alpha\gamma P z_t^{\gamma} + \gamma(\gamma-1)Pz_t^{\gamma}\frac{\sigma^2}{2} - r(Nz_t + Pz_t^{\gamma}),$$

which gives: $N = M_0/(r-\alpha)$.

To pin down the coefficient P, notice that, at $\tilde{z}$, the social value of the future
investments must be equal to the value of the first investment, net of its cost, which
implies

$$N\tilde{z} + P\tilde{z}^{\gamma} = F + G\tilde{z} + H\tilde{z}^{\beta_1} - I,$$

and therefore,

$$P = \tilde{z}^{(-\gamma)}\left\{\frac{\lambda(M_2-M_0) + (r-\alpha)(M_1-M_0)}{r+\lambda-\alpha}\tilde{z} - \frac{r+\lambda(2-\theta)}{r+\lambda}I + \right.$$
$$\left. + \left[\frac{\lambda(M_2-M_1)}{r+\lambda-\alpha}\bar{z} - \frac{r+\theta\lambda}{r+\lambda}I\right]\left(\frac{\tilde{z}}{\bar{z}}\right)^{\beta_1}\right\}.$$

Collecting the above result, one obtains the following welfare function

$$W(z_t) = \begin{cases} \frac{M_0}{r-\alpha}z_t + Pz_t^{\gamma} & z_t \in (0,\tilde{z}) \\ -\frac{\lambda}{r+\lambda}(1-\theta)I + \frac{(r-\alpha)M_1+\lambda M_2}{(r+\lambda-\alpha)(r-\alpha)}z_t + \left[\frac{\lambda(M_2-M_1)}{r+\lambda-\alpha} - \frac{r+\theta\lambda}{r+\lambda}I\right]\left(\frac{z_t}{\bar{z}}\right)^{\beta_1} & z_t \in [\tilde{z},\bar{z}) \end{cases},$$

which has been used to generate Figure 8.

Social value of the leader's investment at $z_L \in (0, \underline{z})$. Following the logic of
the previous Sub-section, it is easy to obtain that, for $z_t \in [\underline{z},\bar{z})$, the social value of
the leader's investment is

$$W_1(z_t) = -\frac{\lambda}{r+\lambda}(1-\theta)I + \frac{(r-\alpha)M_1 + \lambda M_2}{(r+\lambda-\alpha)(r-\alpha)}z_t + \left[\frac{\lambda(M_2-M_1)}{r+\lambda-\alpha}\bar{z} - \frac{r+\theta\lambda}{r+\lambda}I\right]\left(\frac{z_t}{\bar{z}}\right)^{\beta_1},$$

which applies only if the follower has not benefited from the spillover while $z_t \in [z_L, \underline{z})$.

When $z_t \in [z_L, \underline{z})$, the social value of the investment sunk by the leading firm is

$$W_1(z_t) = M_1 z_t dt + \lambda W_1^d(z_t)dt + (1 - \lambda dt)E_t\left[W_1(z_{t+dt})e^{-rdt}\right], \qquad (32)$$

where $\tilde{W}_1(z_t)$ is the social value of the investment performed by the leader when the information has been *dis*closed, but the follower has not invested yet. Hence, the second addendum on the right-hand side comes from the fact that, with probability λdt, the follower benefits from the informational spillover but does not invest.

It is now be easy to show that

$$W_1^d(z_t) = \frac{M_1}{r - \alpha}z_t + \left[\frac{M_2 - M_1}{r - \alpha}\underline{z} - (1 - \theta)I\right]\left(\frac{z_t}{\underline{z}}\right)^{\gamma}.$$

Our tentative solution for Eq. (32) is

$$W_1(z_t) = Qz_t + Rz_t^{\beta_1} + Sz_t^{\gamma},$$

where obviously Q, R, and S are undetermined coefficients, and γ and β_1 are given, respectively, by Eqs. (12) and (15). From the above tentative solution, we readily obtain $Q = \frac{M_1}{r-\alpha}$, and $S = \left[\frac{M_2-M_1}{r-\alpha}\underline{z} - (1 - \theta)I\right]\underline{z}^{-\gamma}$. As for R, we notice that, at $\underline{z}$, the value-matching condition

$$Q\underline{z} + R\underline{z}^{\beta_1} + S\underline{z}^{\gamma} = F + G\underline{z} + H\underline{z}^{\beta_1}$$

must hold. This readily gives

$$R = \underline{z}^{(-\beta_1)}\left\{\left[\frac{M_2 - M_1}{r + \lambda - \alpha}\bar{z} - \frac{r + \theta\lambda}{r + \lambda}I\right]\left[\left(\frac{\underline{z}}{\bar{z}}\right)^{\beta_1} - \frac{\underline{z}}{\bar{z}}\right] - \theta I\right\}.$$

Finally, we shall determine the social welfare when no firm has invested – i.e., for $z_t \in (0, z_L]$. In this case, the welfare function is given again by (31), and the solution we propose is

$$W_0(z_t) = Tz_t + Uz_t^{\gamma}.$$

It is easy to see that $T = M_0/(r - \alpha)$; as for U, we need to exploit the value-matching condition

$$Tz_L + Uz_L^{\gamma} = Qz_L + Rz_L^{\beta_1} + Sz_L^{\gamma} - I,$$

which requires that, at the leader's investment trigger z_L, the social value of the future investments must be equal to the net value of the first investment.

Some calculation gives:

$$U = z_L^{-\gamma}\left\{\frac{M_1 - M_0}{r - \alpha + \lambda}z_L + \left\{\left[\frac{M_2 - M_1}{r - \alpha + \lambda}\bar{z} - \frac{r + \theta\lambda}{r + \lambda}I\right]\left[\left(\frac{\underline{z}}{\bar{z}}\right)^{\beta_1} - \frac{\underline{z}}{\bar{z}}\right] - \theta I\right\}\left(\frac{z_L}{\underline{z}}\right)^{\beta_1} +\right.$$

$$\left. + \left[\frac{M_2 - M_1}{r - \alpha}\underline{z} - (1 - \theta)I\right]\left(\frac{z_L}{\underline{z}}\right)^{-\gamma} - I\right\}.$$

In sum, when the leader's optimal decision is to invest at $z_L < \underline{z}$, the social welfare function is

$$
W(z_t) = \begin{cases}
\frac{M_0}{r-\alpha} z_t + U z_t^{\gamma} & z_t \in (0, z_L) \\
\begin{aligned}\frac{M_1}{r-\alpha} + \left\{ \left[\frac{M_2-M_1}{r-\alpha+\lambda} \bar{z} - \frac{r+\theta\lambda}{r+\lambda} I \right] \left[\left(\frac{z}{\bar{z}} \right)^{\beta_1} - \frac{z}{\bar{z}} \right] - \theta I \right\} \left(\frac{z_t}{\underline{z}} \right)^{\beta_1} + \\ + \left[\frac{M_2-M_1}{r-\alpha} \underline{z} - (1-\theta) I \right] \left(\frac{z_t}{\underline{z}} \right)^{\gamma} \end{aligned} & z_t \in [z_L, \underline{z}) \\
\frac{(r-\alpha)M_1+\lambda M_2}{(r+\lambda-\alpha)(r-\alpha)} z_t + \left[\frac{\lambda(M_2-M_1)}{r+\lambda-\alpha} - \frac{r+\theta\lambda}{r+\lambda} I \right] \left(\frac{z_t}{\bar{z}} \right)^{\beta_1} - \frac{\lambda}{r+\lambda}(1-\theta) I & z_t \in [\underline{z}, \bar{z})
\end{cases},
$$

which has been used to generate Figure 9.

9   Appendix 2: Proofs

Proof of Lemma 1.

$\left[\frac{\gamma(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)} - \frac{\beta_1 r}{(\beta_1-1)(r+\lambda)} \right] > 0$ implies $\left[\frac{\gamma(r-\alpha)}{(\gamma-1)r} - \frac{\beta_1(r+\lambda-\alpha)}{(\beta_1-1)(r+\lambda)} \right] > 0$. Define $F(\lambda, \sigma^2) = \left[\frac{\gamma(r-\alpha)}{(\gamma-1)r} - \frac{\beta_1(r+\lambda-\alpha)}{(\beta_1-1)(r+\lambda)} \right]$, and notice that $F(0, \sigma^2) = 0$, since, in this case $\beta_1 = \gamma$. Now compute

$$
\frac{\partial F(\lambda, \sigma^2)}{\partial \lambda} = \frac{1}{(\beta_1-1)^2(r+\lambda)^2} \left[(r+\lambda-\alpha)(r+\lambda) \frac{\partial \beta_1}{\partial \lambda} - \alpha\beta_1(\beta_1-1) \right].
$$

If $\frac{\partial F(\lambda, \sigma^2)}{\partial \lambda} > 0$, then $F(\lambda, \sigma^2) > 0$; hence, we now show that $\frac{\partial F(\lambda, \sigma^2)}{\partial \lambda} > 0$. From Eq. (15), it is immediate to obtain

$$
\frac{\partial \beta_1}{\partial \lambda} = \frac{2}{2\alpha + (2\beta_1-1)\sigma^2}.
$$

Accordingly,

$$
\frac{\partial F(\lambda, \sigma^2)}{\partial \lambda} = \frac{2(r+\lambda-\alpha)(r+\lambda) - \alpha\beta_1(\beta_1-1)\left[2\alpha + (2\beta_1-1)\sigma^2 \right]}{(\beta_1-1)^2(r+\lambda)^2 \left[2\alpha + (2\beta_1-1)\sigma^2 \right]}.
$$

Because the denominator of the above expression is positive, $\frac{\partial F(\lambda, \sigma^2)}{\partial \lambda} > 0$ if $G(\lambda, \sigma^2) = 2(r+\lambda-\alpha)(r+\lambda) - \alpha\beta_1(\beta_1-1)\left[2\alpha + (2\beta_1-1)\sigma^2 \right] > 0$.

Hence, we now study $G(\lambda, \sigma^2)$. Using the fact that $\sigma^2 = \frac{2(r+\lambda-\alpha\beta_1)}{\beta_1(\beta_1-1)}$ (exploit Eq. (15)), we obtain

$$
G(\lambda, \sigma^2) = 2(r+\lambda-\alpha)(r+\lambda) - 2\alpha \left[\alpha\beta_1(\beta_1-1) + (r+\lambda-\alpha\beta_1)(2\beta_1-1) \right];
$$

that is

$$
G(\lambda, \sigma^2) = 2(r+\lambda-\alpha)(r+\lambda) - 2\alpha \left[(2\beta_1-1)(r+\lambda) - \alpha\beta_1^2 \right].
$$

Notice, first, that $\lim_{\sigma^2 \to 0} G(\lambda, \sigma^2) = 0$ because $\lim_{\sigma^2 \to 0} \beta_1 = \frac{r+\lambda}{\alpha}$; hence, if $\frac{\partial G(\lambda, \sigma^2)}{\partial \sigma^2} > 0$, then $G(\lambda, \sigma^2) > 0$, and for $\sigma^2 \in (0, \infty)$, $\lambda \in (0, \infty)$.

Because $\frac{\partial G(\lambda, \sigma^2)}{\partial \sigma^2} = -4\alpha \left(r+\lambda-\alpha\beta_1 \right) \frac{\partial \beta_1}{\partial \sigma^2}$, since we have that $r+\lambda-\alpha\beta_1 > 0$ (refer again to Eq. (15)), and $\frac{\partial \beta_1}{\partial \sigma^2} < 0$, the proof is completed. ∎

Proof of Proposition 2.

i) Because, by Lemma 1, $\left[\frac{\gamma(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)} - \frac{\beta_1 r}{(\beta_1-1)(r+\lambda)} \right] > 0$, we have that $\lim_{\bar{z} \to 0}$

$l.h.s.(6) = \infty$, and that $\lim_{\bar{z} \to \infty} l.h.s.(6) = 0$. Notice, moreover, that $\frac{\partial (l.h.s.(6))}{\partial \bar{z}} < 0$, and that $\frac{\partial^2 (l.h.s.(6))}{(\partial \bar{z})^2} > 0$, because $\beta_2 < 0$. The right-hand side of Eq. (6) is linear and increasing in $\bar{z}$. Hence, $\bar{z}$ is unique.

ii) Substitute $\underline{z}$ as given by Eq. (3) in Eq. (6), and let $\theta \to 0$ to obtain $\bar{z} = \frac{\gamma(r-\alpha)}{(\gamma-1)(\Pi_2 - \Pi_1^l)} I (= \underline{z})$.

iii) Observe that $\lim_{\theta \to 1} l.h.s.(6) = \lim_{\theta \to 1} r.h.s.(6)$, and that $\lim_{\theta \to 1} l.h.s.(6) = 0$. Accordingly, we need $\lim_{\theta \to 1} r.h.s.(6) = 0$, which implies $\lim_{\theta \to 1} \bar{z} = \frac{\beta_1(r+\lambda-\alpha)}{(\beta_1-1)(\Pi_2-\Pi_1^l)} I$. From Eq. (3), it is immediate to obtain $\lim_{\theta \to 1} \underline{z} = 0$.

iv) Notice that

$$r.h.s.(6)|_{\bar{z}=\underline{z}} = \left[\frac{\gamma(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)}(1-\theta) - \frac{\beta_1(r+\theta\lambda)}{(\beta_1-1)(r+\lambda)} \right] I,$$

while

$$l.h.s.(6)|_{\bar{z}=\underline{z}} = \left[\frac{\gamma(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)} - \frac{\beta_1 r}{(\beta_1-1)(r+\lambda)} \right] (1-\theta)I.$$

Hence, for $\theta > 0$ and $\lambda > 0$,

$$r.h.s.(6)|_{\bar{z}=\underline{z}} < l.h.s.(6)|_{\bar{z}=\underline{z}},$$

which – together with the facts mentioned in the proof for Part *i*) – proves the result. ∎

Proof of Corollary 3.
Recall that $\beta_2 < 0$; hence, the fact that $E_2, G_2 > 0$ is obvious from Lemma 1. As for E_3, consider that

$$E_3 = \frac{\Pi_2 - \Pi_1^l}{\beta_1(r+\lambda-\alpha)} \bar{z}^{1-\beta_1} - \frac{r(1-\theta)I}{(\beta_1-1)(r+\lambda)\underline{z}^{\beta_1}} - \frac{\beta_2}{\beta_1} G_2 \bar{z}^{\beta_2-\beta_1} + \frac{\beta_2-1}{\beta_1-1} G_2 \underline{z}^{\beta_2-\beta_1}.$$

Exploiting Eq. (3), the above expression may be written as:

$$\begin{aligned}
E_3 = {} & \frac{\Pi_2 - \Pi_1^l}{\beta_1(r+\lambda-\alpha)} \left(\bar{z}^{1-\beta_1} - \underline{z}^{1-\beta_1} \right) + \\
& + \left[\frac{\gamma(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)} - \frac{\beta_1 r}{(\beta_1-1)(r+\lambda)} \right] \frac{(1-\theta)I}{\beta_1 \underline{z}^{\beta_1}} + \\
& - \frac{\beta_2}{\beta_1} G_2 \left(\bar{z}^{\beta_2-\beta_1} - \underline{z}^{\beta_2-\beta_1} \right) - \frac{\beta_2}{\beta_1} G_2 \underline{z}^{\beta_2-\beta_1} + \frac{\beta_2-1}{\beta_1-1} G_2 \underline{z}^{\beta_2-\beta_1},
\end{aligned}$$

which, using the definition for G_2 in (5), simplifies to:

$$E_3 = \frac{\Pi_2 - \Pi_1^l}{\beta_1(r+\lambda-\alpha)} \left(\bar{z}^{1-\beta_1} - \underline{z}^{1-\beta_1} \right) - \frac{\beta_2}{\beta_1} G_2 \left(\bar{z}^{\beta_2-\beta_1} - \underline{z}^{\beta_2-\beta_1} \right).$$

If $\bar{z}$ were equal to $\underline{z}$, we would have $E_3 = 0$. Notice, moreover, that

$$\frac{\partial E_3}{\partial \bar{z}} = (1-\beta_1) \frac{\Pi_2 - \Pi_1^l}{\beta_1(r+\lambda-\alpha)} \bar{z}^{-\beta_1} - \frac{(\beta_2-\beta_1)\beta_2}{\beta_1} G_2 \bar{z}^{\beta_2-\beta_1-1} < 0.$$

∎

Proof of Lemma 4.

$\left[1 - \frac{(\beta_1 - 1)(r - \alpha)}{(\gamma - 1)(r + \lambda - \alpha)}\right] > 0$ requires $(\gamma - 1)(r + \lambda - \alpha) > (\beta_1 - 1)(r - \alpha)$. γ is the

positive root of Eq. (12), so that $\gamma - 1 = -\frac{\alpha}{\sigma^2} - \frac{1}{2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}$, while, Eq.

(15) implies $\beta_1 - 1 = -\frac{\alpha}{\sigma^2} - \frac{1}{2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r + \lambda)}{\sigma^2}}$. Hence, the above inequality
can be written as

$$\left[-\frac{\alpha}{\sigma^2} - \frac{1}{2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}\right](r + \lambda - \alpha) >$$

$$> \left[-\frac{\alpha}{\sigma^2} - \frac{1}{2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r + \lambda)}{\sigma^2}}\right](r - \alpha).$$

When $\lambda = 0$, the left- and the right-hand sides of the expression above are identical.

Notice, however, that the first derivative with respect to λ of the left-hand side is positive, while the second derivative is nought. As for the right-hand side of the expression above, the first derivative is positive, while the second one is negative. Hence, to prove the lemma, it suffices to show that the derivative of the left-hand side – evaluated at $\lambda = 0$ – is higher than the derivative of the right-hand side – i.e., that

$$\left[-\frac{\alpha}{\sigma^2} - \frac{1}{2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}\right] > \left[\sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}\right]^{-1} \frac{(r - \alpha)}{\sigma^2}.$$

Multiplying both sides by $\left[\sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}\right]$, and rearranging, we obtain:

$$\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2} > \frac{(r - \alpha)}{\sigma^2} + \left(\frac{\alpha}{\sigma^2} + \frac{1}{2}\right)\sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}},$$

which readily becomes:

$$\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{r + \alpha}{\sigma^2} > \left(\frac{\alpha}{\sigma^2} + \frac{1}{2}\right)\sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}.$$

Squaring both sides of the above expression gives

$$\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^4 + \left(\frac{r + \alpha}{\sigma^2}\right)^2 + 2\left(\frac{r + \alpha}{\sigma^2}\right)\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 >$$

$$> \left[\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\alpha}{\sigma^2}\right]\left[\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}\right].$$

Some simplifications give:

$$\left(\frac{r + \alpha}{\sigma^2}\right)^2 > \frac{4\alpha r}{\sigma^4},$$

which is always verified, since $r > \alpha$. ∎

Proof of Theorem 5.

i) Define $\Delta(z_t) \equiv L(z_t) - F(z_t)$. When $\theta \to 0$, *Proposition* 2 assures that $\underline{z} = \bar{z}$. Moreover, from (8), and (5), we compute that $E_6 = E_3 = 0$; hence, $\Delta(z_t)$, for $z_t \in (0, \bar{z})$, is

$$\Delta(z_t) = \frac{\Pi_1^h - \Pi_1^l}{r - \alpha} z_t + \left(\frac{\Pi_2 - \Pi_1^h}{r - \alpha} \underline{z} - \frac{I}{\gamma - 1} \right) \left(\frac{z_t}{\underline{z}} \right)^{\gamma} - I.$$

In this case, $\lim_{z_t \to 0} \Delta(z_t) = -I$, $\Delta(\underline{z}) = 0$ (taking advantage of (3)), and

$$\arg\max \Delta(z_t) = \left(\frac{(\gamma - 1)\left(\Pi_1^h - \Pi_1^l\right)\left(\Pi_2 - \Pi_1^l\right)}{\gamma I (r - \alpha)\left[\Pi_2 - \Pi_1^l + \gamma(\Pi_1^h - \Pi_2)\right]} \right)^{\frac{1}{\gamma - 1}} \underline{z}^{\frac{\gamma}{\gamma - 1}}.$$

It is easy to show that whenever Assumption A1 is fulfilled $\arg\max \Delta(z_t) < \underline{z}$. Moreover, we have that $\max \Delta(z_t) > 0$. In fact, $\Delta(\underline{z}) = 0$, and $\left.\frac{\partial \Delta(z_t)}{\partial z_t}\right|_{z_t = \underline{z}} < 0$. This proves that, when $\theta \to 0$, the equilibrium for $z_t \in (0, \underline{z} = \bar{z})$ is preemptive. We argue, by continuity, that a preemptive candidate equilibrium exists with the leader investing in $z_t \in (0, \underline{z})$ for values for θ in a (right) interval of 0.

ii) Proposition 2, part iii), shows that $\lim_{\theta \to 1} \bar{z} = 0$. Accordingly, when $\theta \to 1$, the function $\Delta(z_t)$ in $z_t \in [0, \bar{z}]$ is

$$\Delta(z_t) = \frac{\Pi_1^h - \Pi_1^l}{r + \lambda - \alpha} z_t + (E_4 - E_2)z_t^{\beta_1} + (G_4 - G_2)z_t^{\beta_2} - \frac{r + \lambda}{r + \lambda} I,$$

which is obtained by making use of the results in (7) and (4). Exploiting (8) and (5), we can write

$$E_4 - E_2 = \bar{z}^{\beta_2 - \beta_1} \left(\frac{\beta_2}{\beta_1} G_2 - G_4 \right) - \frac{\bar{z}^{1 - \beta_1}}{r + \lambda - \alpha} \left(\Pi_1^h - \Pi_2 + \frac{\Pi_2 - \Pi_1^l}{\beta_1} \right).$$

Taking advantage of the expression above, we obtain:

$$\frac{\partial \Delta(z_t)}{\partial z_t} = \frac{\Pi_1^h - \Pi_1^l}{r + \lambda - \alpha} + \beta_2(G_4 - G_2)z_t^{\beta_2 - 1} +$$
$$+ \left[\bar{z}^{\beta_2 - \beta_1}(\beta_2 G_2 - \beta_1 G_4) - \frac{\beta_1 \bar{z}^{1 - \beta_1}}{r + \lambda - \alpha} \left(\Pi_1^h - \Pi_2 + \frac{\Pi_2 - \Pi_1^l}{\beta_1} \right) \right] z_t^{\beta_1 - 1}.$$

Hence, we have that

$$\left.\frac{\partial \Delta(z_t)}{\partial z_t}\right|_{z_t = \bar{z}} = \frac{\Pi_1^h - \Pi_1^l}{r + \lambda - \alpha} + \beta_2(G_4 - G_2)\bar{z}^{\beta_2 - 1} +$$
$$+ \bar{z}^{\beta_2 - 1}(\beta_2 G_2 - \beta_1 G_4) - \frac{\beta_1}{r + \lambda - \alpha} \left(\Pi_1^h - \Pi_2 + \frac{\Pi_2 - \Pi_1^l}{\beta_1} \right),$$

which boils down to

$$\left.\frac{\partial \Delta(z_t)}{\partial z_t}\right|_{z_t = \bar{z}} = \frac{\left[\Pi_1^h - \Pi_1^l - \beta_1(\Pi_1^h - \Pi_2) - \left(\Pi_2 - \Pi_1^l\right)\right]}{r + \lambda - \alpha} + (\beta_2 - \beta_1)G_4\bar{z}^{\beta_2 - 1}.$$

We now substitute out G_4 using (8), and we obtain

$$\frac{\partial \Delta(z_t)}{\partial z_t}\bigg|_{z_t=\bar{z}} = (\Pi_1^h - \Pi_2) \left\{ \frac{1-\beta_1}{r+\lambda-\alpha} - \frac{1-\gamma}{r-\alpha}\left[1 - \frac{(\beta_1-1)(r-\alpha)}{(\gamma-1)(r+\lambda-\alpha)}\right]\left(\frac{\bar{z}}{\underline{z}}\right)^{\beta_2-1}\right\},$$

where we have exploited Eq. (3). The above expression can be written as:

$$\frac{\partial \Delta(z_t)}{\partial z_t}\bigg|_{z_t=\bar{z}} = \frac{(\Pi_1^h - \Pi_2)(1-\beta_1)}{r+\lambda-\alpha}\left\{1 + \left[1 - \frac{(\gamma-1)(r+\lambda-\alpha)}{(\beta_1-1)(r-\alpha)}\right]\left(\frac{\bar{z}}{\underline{z}}\right)^{\beta_2-1}\right\}.$$

Lemma 4 guarantees that both addenda inside the big curly brackets are positive, so that the derivative is negative. Because $\Delta(z_t)$ is continuous, $\Delta(0) = -I$, and $\Delta(\bar{z}) = 0$, the fact $\partial\Delta(z_t)/\partial z_t|_{z_t=\bar{z}} > 0$ guarantees that there exists at least one root for $\Delta(z_t) = 0$ in $z_t \in (0, \bar{z})$. We argue, by continuity, that a root for $\Delta(z_t) = 0$ exists in $z_t \in [\underline{z}, \bar{z})$ for values for θ in a (left) interval of 1. This guarantees the existence of an equilibrium with the leader investing in $[\underline{z}, \bar{z})$. ∎

10 Appendix 3: A Cournot interpretation for payoffs and welfare levels

Consider an industry composed of two firms, i and j, which, in each (infinitesimally short) period, are involved in a two-stage interaction: first they decide whether to innovate or not, and then they compete *à la* Cournot. The firms' horizon is infinite, and market demand is linear and equal to $P = a\sqrt{z_t} - bQ$, where P is the market clearing price and $Q = q_i + q_j$ is the total quantity supplied.

Each firm has a unit cost of production $c\sqrt{z_t}$. The assumption that both the market dimension parameter a, and the unit cost c are influenced by the same disturbance is widely used in the literature (Huisman and Kort, 2004; Pawlina and Kort, 2006; Cooper, 2006; Moretto, 2008). In fact, it greatly simplifies the analysis. To avoid excessive analytical intricacies, several other contributions admit only a few possible demand levels, or ignore variable cost (see, e.g., Grenadier, 1996; Nielsen, 2002). We think that the approach we follow is the optimal compromise between analytical tractability and "realism".

In each period t, firm i (and j) decides whether to invest in R&D or not. This investment immediately yields a cost-reducing process innovation, which shrinks the unit production cost by an amount $x\sqrt{z_t}$, with $x < c$. Hence, firm i's post–innovation production cost is $C(q_i) = (c-x)q_i\sqrt{z_t}$.

Each firm's payoff depends not only on its adoption date but also on that of its rival. If both firms have not invested up to period t, their individual profits in the Cournot subgame at t are those of the pre–innovation stage; i.e.,

$$\Pi_0 z_t = \frac{A^2}{9b}z_t, \tag{33}$$

where $A = a - c$. The subscript indicates the number of firms that have innovated at time t. The instantaneous welfare (computed *à la Marshall* as the sum of consumers' and producers' surpluses) is then equal to

$$M_0 z_t = \frac{4}{9}\frac{A^2}{b}z_t. \tag{34}$$

If instead only one firm, say firm i, invests in R&D at t, it benefits from an efficiency advantage, and obtains a higher market share. The market price at t

decreases in comparison with the pre-innovation level, while the individual profits become:

$$\Pi_1^h z_t = \frac{(A + 2x)^2}{9b} z_t; \Pi_1^l z_t = \frac{(A - x)^2}{9b} z_t, \tag{35}$$

where the superscript h denotes variables pertaining to the firms that have already invested, while l refers to the firms that have not innovated yet. Notice that $\Pi_1^h > \Pi_1^l$, $\Pi_1^h > \Pi_0$, and $\Pi_1^l < \Pi_0$, as required by Assumption 1. Because $q_j^l = \frac{A-x}{3b}$, to preserve the duopolistic structure characterizing our market we need to assume that $A > x$. This hypothesis implies that, in a Cournot environment, the cost-reducing innovation is non-drastic. In case of asymmetric behavior at t, welfare is

$$M_1 z_t = \frac{8A(A + x) + 11x^2}{18b} z_t, \tag{36}$$

with $M_1 > M_0$.

Finally, we need to compute the outcomes when both firms have innovated at t. In this case, being more efficient, they both produce more than in the *status quo*; therefore, the market price is lower. Individual profits at t are

$$\Pi_2 z_t = \frac{(A + x)^2}{9b} z_t. \tag{37}$$

Obviously, $\Pi_1^h > \Pi_2$, as required by Assumption 1. Notice, moreover, that the difference between Π_1^h and Π_2 is increasing in x: when only one firm enjoys a cost advantage, she obtains a larger market share while benefiting from an higher price-to-cost margin.

When both firms have innovated, the social welfare is

$$M_2 z_t = \frac{4(A + x)^2}{9b} z_t, \tag{38}$$

with $M_2 > M_1$.

When firms simultaneously invest in R&D, individual profits rise from (33) to (37), and welfare jumps from (34) to (38). Alternatively, firms may behave asymmetrically, so that there are both an innovation leader and a follower. Under these circumstances individual profits first change from (33) to (35) (and welfare from (34) to (36)) and then from (35) to (37) (and welfare from (36) to (38)).
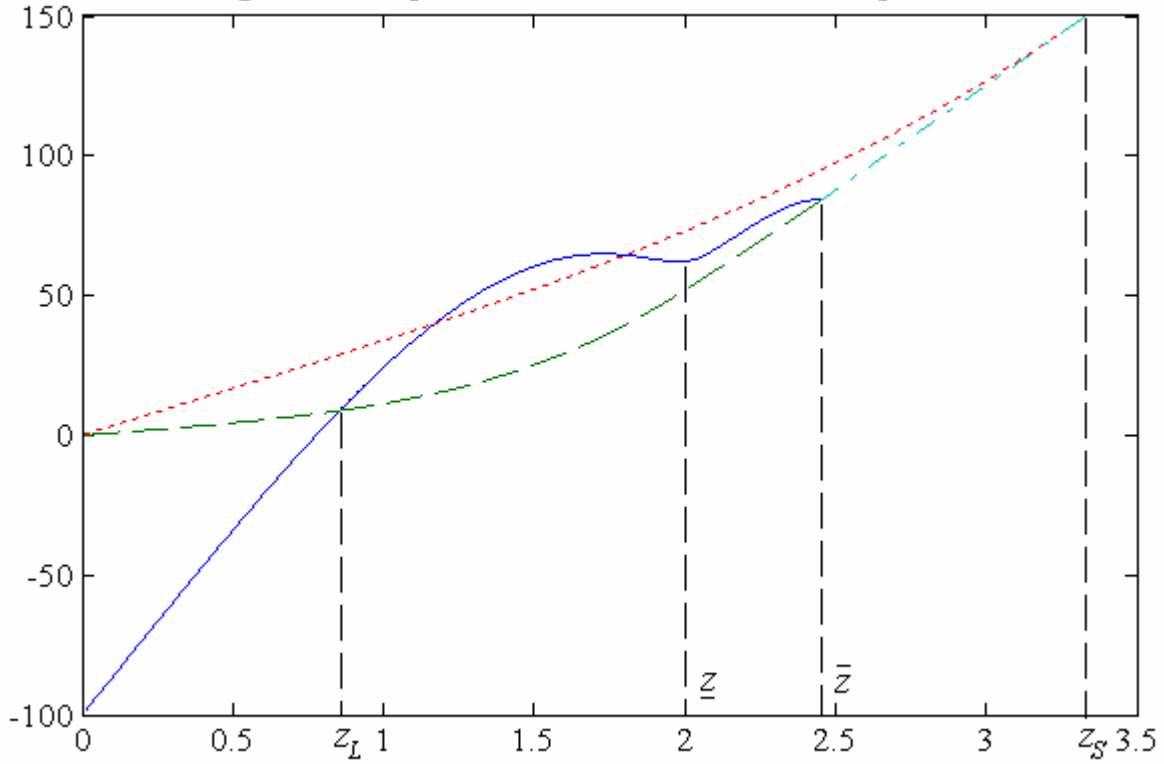
Figure 1: Maximum value functions
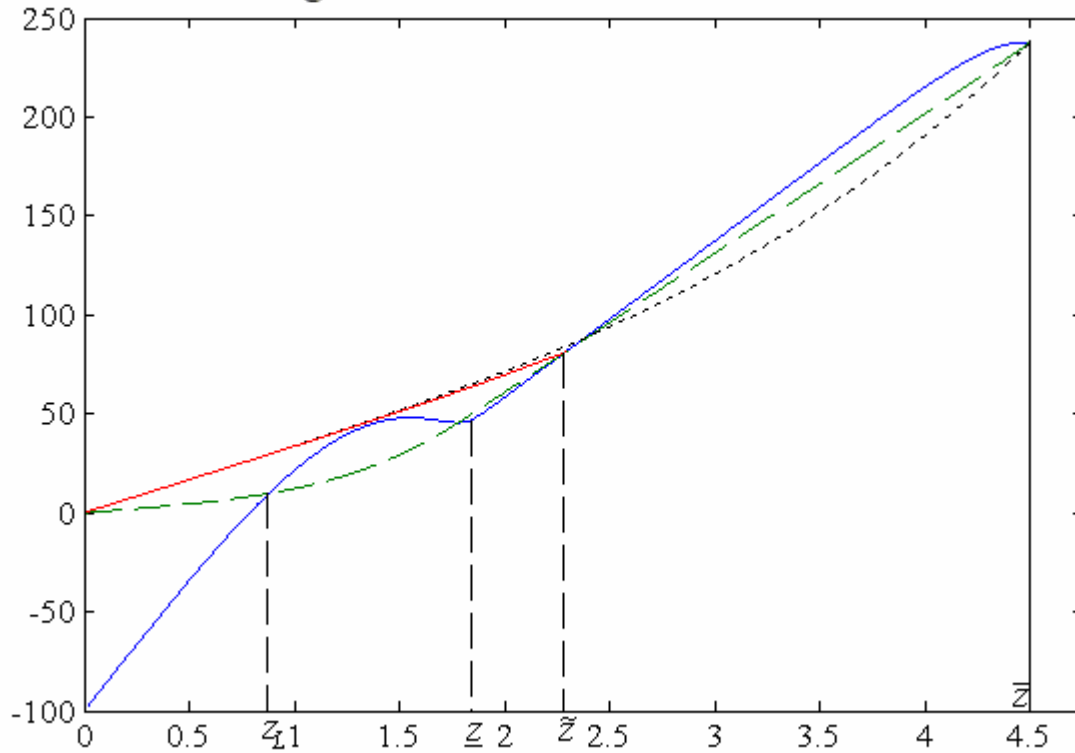
Leader's (continuous line) and follower's (dashed line) value functions for $r = 0.04$, $\alpha = 0.01$, $\sigma = 0.03$, $\theta = 0.12$, $\lambda = 0.40$, $I = 100$, $\Pi_0 = 1$, $\Pi_1^h = 4$, $\Pi_1^l = 0.25$, and $\Pi_2 = 2.25$. The dotted line represents both $S(z_t)$, and the discounted value of $L(\bar{z}) = F(\bar{z})$. The continuous line ending at $\tilde{z}$ represents the discounted value of $L(\tilde{z}) = F(\tilde{z})$.

Figure 2: Candidate equilibria in $(0, \bar{z})$
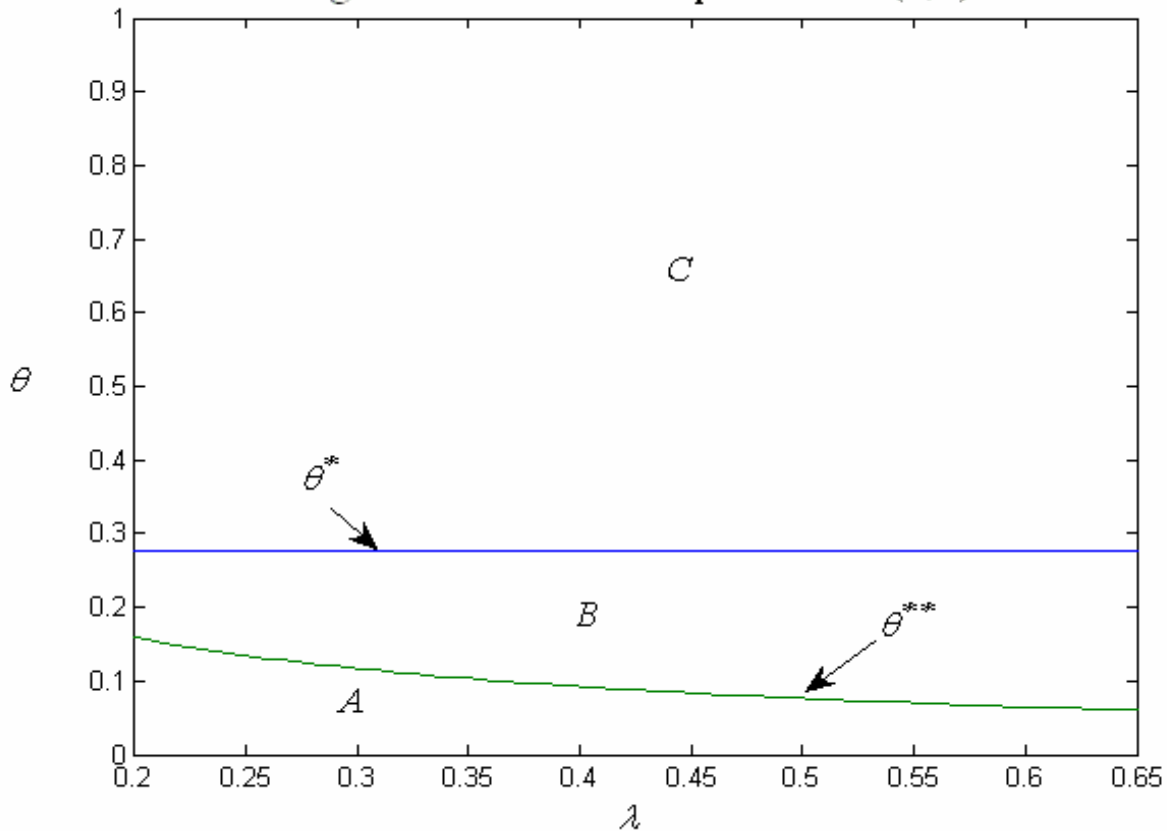
Existence regions for the candidate equilibria in $z_t \in (0, \bar{z})$ for $r = 0.04$, $\alpha = 0.01$, $\sigma = 0.03$, $\theta = 0.12$, $\lambda = 0.40$, $I = 100$, $\Pi_0 = 1$, $\Pi_1^h = 4$, $\Pi_1^l = 0.25$, and $\Pi_2 = 2.25$.

Figure 3: Equilibrium selection - low spillover

Leader's (continuous line) and follower's (dashed line) value functions for $r = 0.04$, $\alpha = 0.01$, $\sigma = 0.03$, $\theta = 0.02$, $\lambda = 0.40$, $I = 100$, $\Pi_0 = 1$, $\Pi_1^h = 4$, $\Pi_1^l = 0.25$, and $\Pi_2 = 2.25$. The dotted line represents $S(z_t)$, and the dotted-dashed line represents the value of a non-optimal simultaneous investment.

Figure 4: Equilibrium selection - high spillover
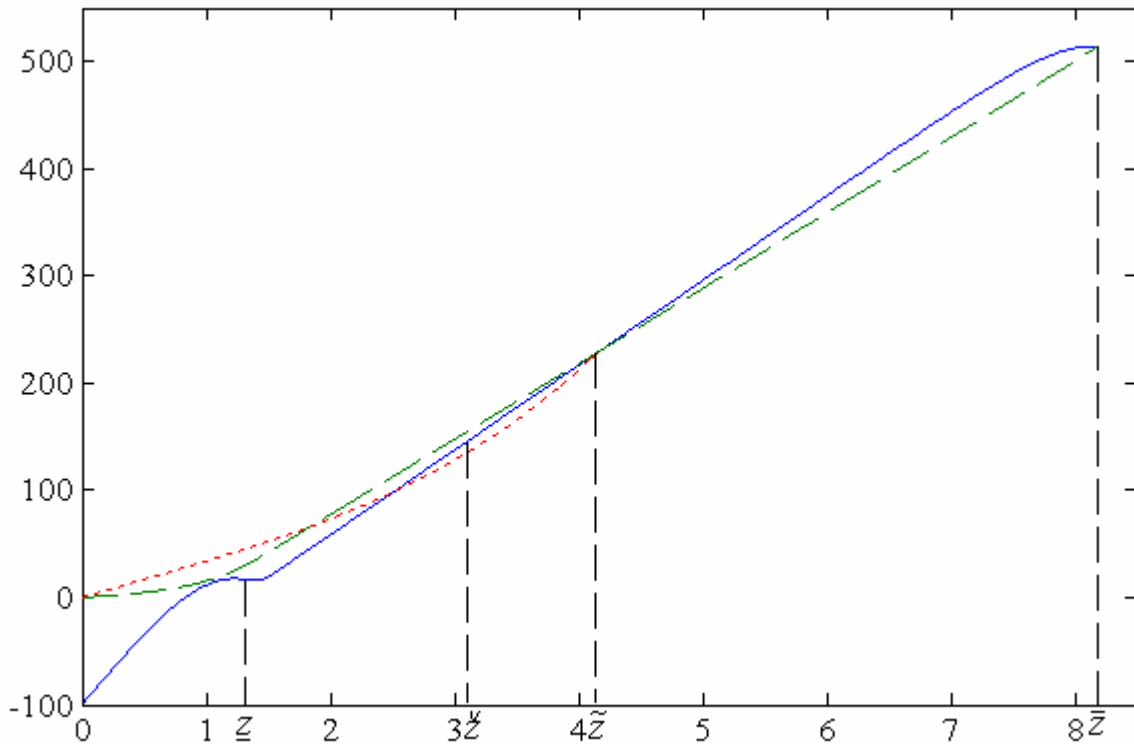
Leader's (continuous line) and follower's (dashed line) value functions for $r = 0.04$, $\alpha = 0.01$, $\sigma = 0.03$, $\theta = 0.30$, $\lambda = 0.40$, $I = 100$, $\Pi_0 = 1$, $\Pi_1^h = 4$, $\Pi_1^l = 0.25$, and $\Pi_2 = 2.25$. The dotted line represents the discounted value of $L(\tilde{z}) = F(\tilde{z})$.

Figure 5

Panel (a):
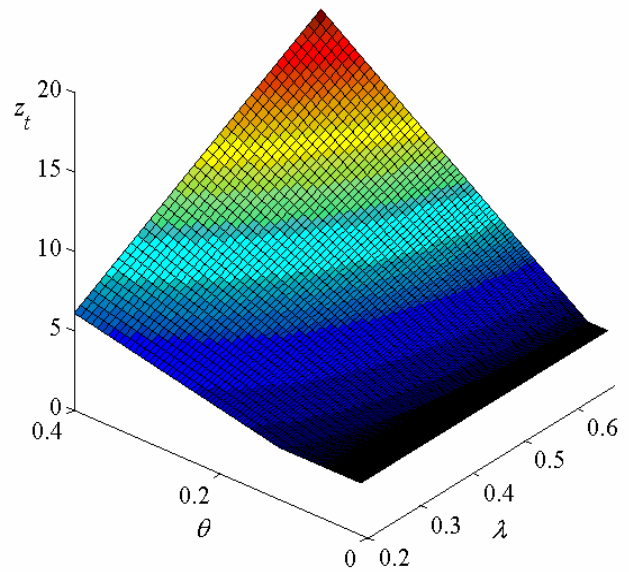Follower's threshold - preemptive equilibrium
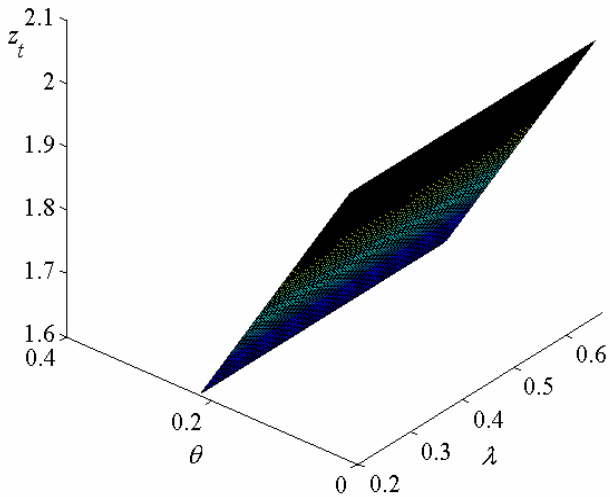
Panel(b):
Leader's threshold - preemptive equilibrium
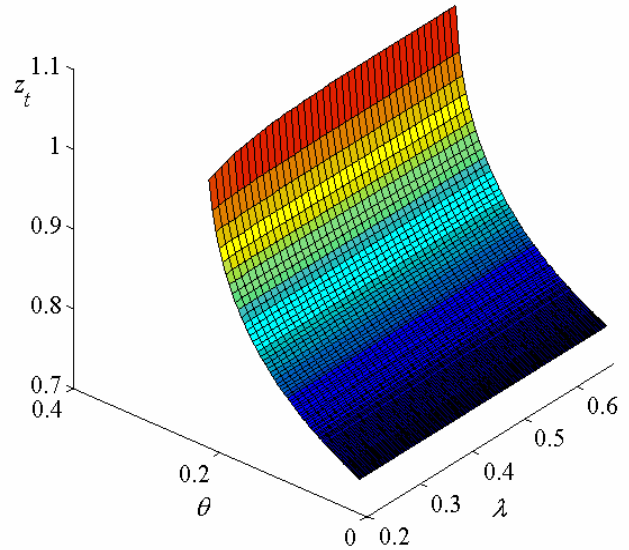
Panel (c):
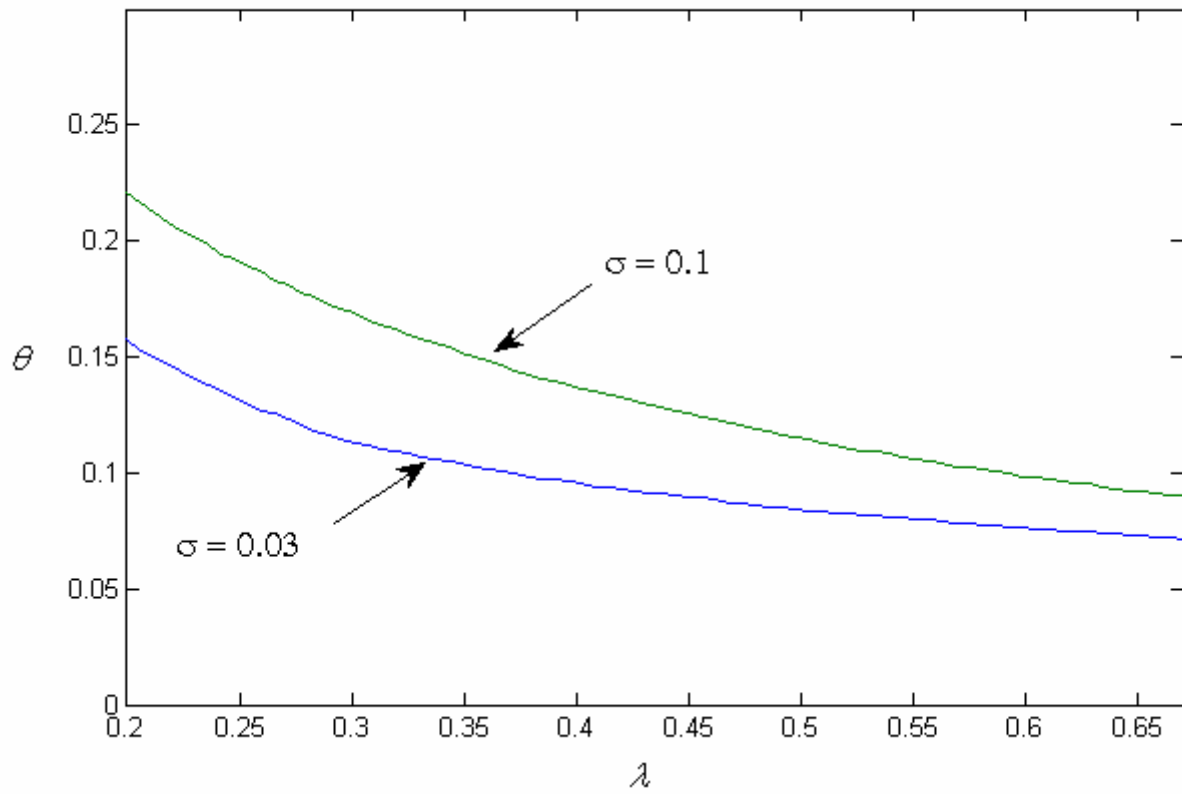Leader's threshold - intermediate equilibrium
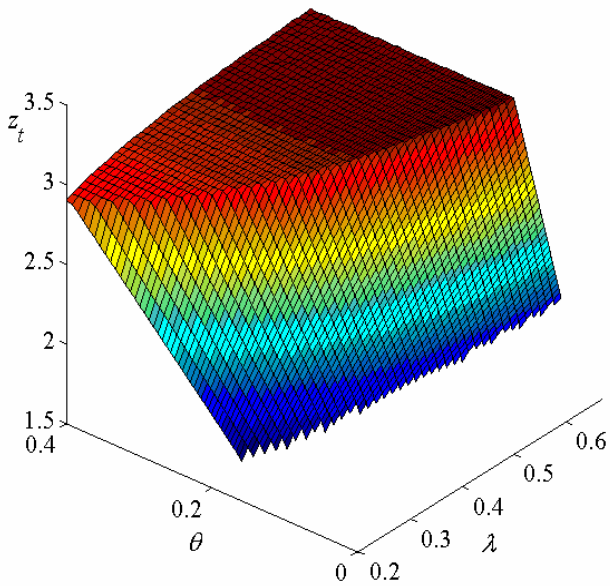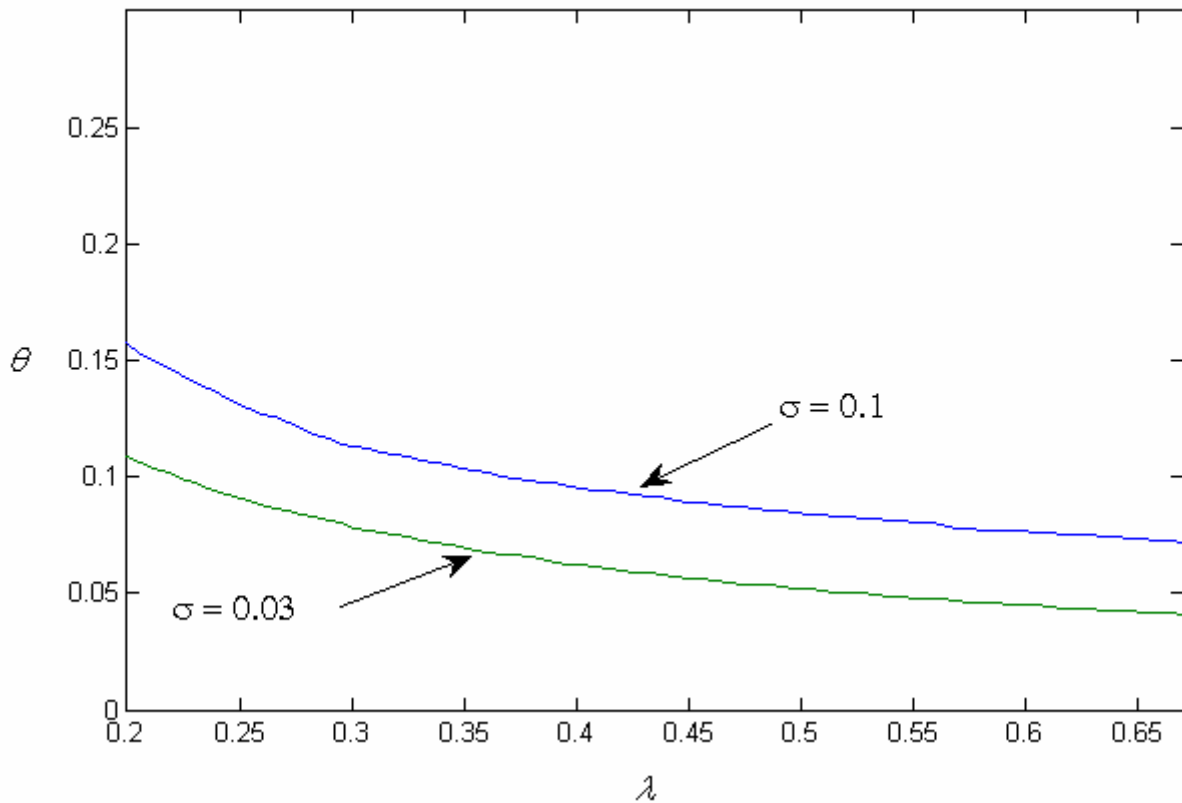
Panel (d):
Simultaneous equilibrium

Candidate equilibria as a function of θ and λ for $r = 0.04$, $\alpha = 0.01$, $\sigma = 0.03$, $I = 100$, $\Pi_0 = 1$, $\Pi_1^h = 4$, $\Pi_1^l = 0.25$, and $\Pi_2 = 2.25$.

Figure 6: Major innovation

In the areas above the $\theta(\lambda)$ frontiers, the leader delays her investment at least up to $\underline{z}$.

Figure 7: Minor innovation

In the areas above the $\theta(\lambda)$ frontiers, the leader delays her investment at least up to $\underline{z}$.
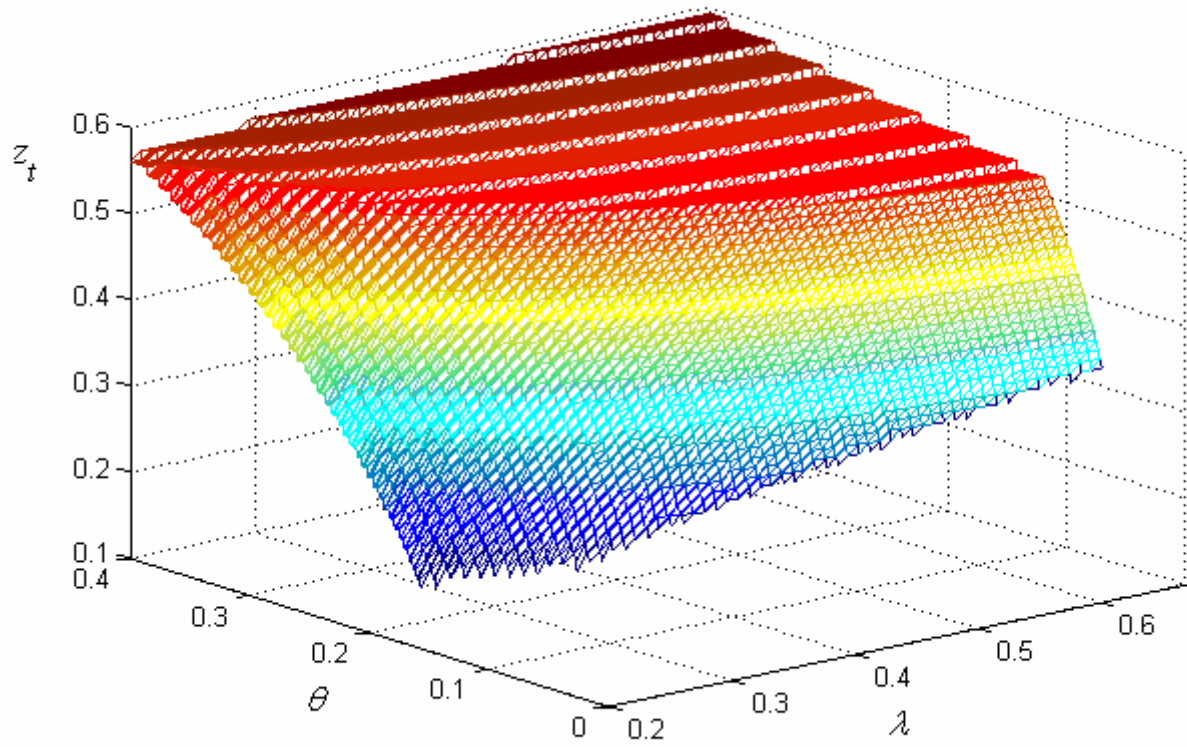
Figure 8: Optimal subsidization rates

Figure 9: Optimal subsidization rates - Early equilibrium