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AN EFFICIENT TEST OF FISCAL SUSTAINABILITY

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Abstract

We suggest a multivariate efficient test of the 'strong' fiscal sustainability hypothesis, based on Horvath and Watson's (1995) cointegration test when cointegration vectors are pre-specified. Using data for a set of developed and developing economies, we show that, unlike our procedure, conventional methodologies tend to penalize the sustainability hypothesis.

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1 Introduction

A stable and sustainable long-term relationship between government expenditures and revenues is a key requirement for macroeconomic stability. Given its relevance, this issue has attracted a great deal of attention, with particular emphasis on testing empirically whether or not a given country's fiscal stance is sustainable. There is, however, a contradiction between the predictions of empirical models, which point to a significant degree of unsustainability across different countries, and the relative scarcity of episodes of full-scale defaults. Therefore, it is of great importance to reassess empirical methodologies dealing with the analysis of fiscal sustainability. In this paper, we show that once an appropriate testing method is put to use, the paradoxical findings of earlier literature virtually disappear.

Tests of fiscal sustainability are commonly based on the government's intertemporal budget constraint (IBC) in its present value form. Given (in real terms) government expenditures G, revenues R, public debt B and the interest rate i, the government's one-period budget constraint is written as

$$G_t + (1+i)B_{t-1} = R_t + B_t.$$

A similar condition holds for periods $t+1$, $t+2$, ... with forward substitution yielding the IBC

$$B_t = \sum_{j=0}^{\infty} \Pi_{k=1}^{j}(1+i_{t+k})^{-1}(R_{t+j} - G_{t+j}) + \lim_{j\to\infty} \Pi_{k=1}(1+i_{t+k})^{-1}B_{t+j}, \qquad (1)$$

implying that current government debt B_t must be financed by the present value of future primary surpluses[1]. Assuming that interest rates are stationary, the above expression can

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[1]Ruling out Ponzi games and therefore the second, asymptotic term should converge to 0.

be conveniently rewritten for empirical purposes as

$$GG_t - R_t = \sum_{j=0}^{\infty} \delta^{j-1}(\Delta R_{t+j} - \Delta GG_{t+j} + i\Delta B_{t+j-1}) \tag{2}$$

where GG_t is now government expenditures inclusive of interest payments, with discount factor $\delta = (1+r)^{-1}$.

Given that the variables GG_t and R_t usually display non-stationary behaviour, this provides a statistical framework for testing sustainability. Indeed, fiscal sustainability implies that revenues and expenditures must be cointegrated, if GG_t and R_t are $I(1)$ processes. In practice, this amounts to estimate the generic regression equation

$$R_t = a + bGG_t + u_t \tag{3}$$

and, depending on the cointegration vector $[1, -b]$ obtained, we may have three possible scenarios for sustainability analysis:

- 'Strong' sustainability, if and only if the $I(1)$ processes R_t and GG_t are cointegrated and $b = 1$;

- 'Weak' sustainability when R_t and GG_t are cointegrated, but $0 < b < 1$: a smaller than 1 long-run elasticity of revenue relative to expenditure may signal debt default;

- Unsustainability, when $b \leq 0$, implying that deficits are being accumulated at a rate greater than the growth rate in the economy and the IBC is therefore violated.

Thus, the common procedure in the literature[2] is to apply cointegration tests to (3) (see Haug, 1991, Hakkio and Rush, 1991, Ahmed and Rogers, 1995). This usually involves two stages: i) test for cointegration, assuming the cointegration vector is unknown; ii)

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[2]Another possibility it to test for a unit root in B_t (Hamilton and Flavin, 1986 and Wilcox, 1989).

if cointegration is found, proceed with estimation, with cointegration maintained both under the null and the alternative, with a 'restricted' cointegration vector arising from the first step.

However, as pointed out by Horvath and Watson (1995), in this situation the usual tests are inefficient. These authors derived a testing procedure for the case when the cointegration vector is known, which allows for substantial gains in power when compared to standard procedures that do not impose a cointegration vector. Its computation is straightforward, as it is based on a Wald test of the error correction term in a Vector Error Correction Model (VECM). Therefore, we depart from, and thus contribute to, the literature by using the Horvath-Watson efficient test of the 'strong' sustainability hypothesis, when the cointegration vector is pre-specified as $[1, -1]$.

The theoretical restriction implied by the IBC also suggests an alternative, stricter test of the 'strong' sustainability hypothesis, obtained by testing the stationarity of the primary surplus/deficit $PS_t = R_t - GG_t$. However, Horvath and Watson (1995) show that a multivariate cointegration approach can lead to efficiency gains over the univariate unit root tests if the error terms of GG_t and R_t are correlated. This is likely to be the case, as shocks affecting the expenditure and the revenue sides are likely to be highly correlated.

Therefore, there seems to be a compelling case for the use of this procedure. The caveat of this test is, naturally, that its relative power will suffer if the variables are cointegrated with a cointegrating vector different form the pre-specified one, namely the case of 'weak' sustainability. We argue, however, that given the implications of the latter, the 'strong' hypothesis should be the benchmark case when assessing fiscal sustainability.

The next section describe the Horvath-Watson testing procedure. We then analyse the

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fiscal regimes of 6 countries using conventional unit root and cointegration methodologies and contrast these results with those obtained with the efficient test of Horvath and Watson (1995). A summary concludes.

2 Testing for cointegration when the cointegration vector is pre-specified

The setup for the derivation of the test is similar to the reduced rank procedure based on a Gaussian VAR

$$Y_t = d_t + X_t$$

$$X_t = \sum_{i=1}^{p} \Pi_i X_{t-i} + \varepsilon_t$$

where Y_t and X_t are $n \times 1$ variables, d_t is a deterministic term (possibly including time trends) and ε_t is normally distributed with covariance matrix Σ_ε. We can rewrite the above system in vector error-correcting form as

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{p-1} \Phi_i \Delta X_{t-i} + \varepsilon_t \tag{4}$$

where $\Pi = -I_n + \sum_{i=1}^{p} \Pi_i$.

As in Johansen (1988), a test for $r = rank(\Pi)$ can be developed for the hypotheses

$$H_o \quad : \quad rank(\Pi) = r = r_o$$

$$H_a \quad : \quad rank(\Pi) = r = r_o + r_a, \qquad r_a > 0.$$

We follow the notation of Horvath and Watson (1995), so that the alternative hypothesis contains r_a, the number of additional cointegrating vectors that are present under the

alternative. We can partition the ranks according to the number of (un)known cointegration vectors, that is, $r_o = r_{o_k} + r_{o_u}$ and $r_a = r_{a_k} + r_{a_u}$, with the subscripts k and u indicating 'known' and 'unknown', respectively.

In order to derive the test statistic, we need to factor the matrix Π as $\Pi = \delta\alpha'$, so that δ and α are $n \times r$ matrices of full column rank and the columns of α give the cointegration vectors. As above, these matrices can be partitioned into $\alpha = (\alpha_o \alpha_a)$ and $\delta = (\delta_o \delta_a)$ and, to reflect the knowledge of the cointegration vector, $\alpha_a = (\alpha_{a_k} a_{a_u})$ and $\delta_a = (\delta_{a_k} \delta_{a_u})$, so that the r_{a_k} columns of α_{a_k} are the additional known cointegration vectors under the alternative H_a. This implies that $\Pi X_{t-1} = \delta_o(\alpha_o' X_{t-1}) + \delta_a(\alpha_a' X_{t-1})$.

In our case, given that we have a bivariate relationship, we will be testing H_o v H_a in the case where $r_o = 0$ (i.e., no cointegration) and $r_a = r_{a_k} = 1$, since we have a single, pre-specified cointegration vector $[1, -1]$ implied by IBC. Thus, the model can be rewritten as (ignoring d_t for notational convenience)

$$\Delta Y_t = \delta_{a_k}(a_{a_k}' Y_{t-1}) + \beta Z_t + \varepsilon_t,$$

where $\beta = (\Phi_1 \Phi_2 ... \Phi_{p-1})$ and $Z_t = (\Delta Y_{t-1}' \Delta Y_{t-2}' ... \Delta Y_{t-p+1}')$. Let $Y = [Y_1 Y_2 ... Y_T]'$, $\Delta Y = Y - Y_{-1}$, $Z = [Z_1 ... Z_T]$, $\varepsilon = [\varepsilon_1 ... \varepsilon_T]$ and $M_Z = [I - Z(Z'Z)^{-1}Z']$. The Wald statistic for H_o against H_a is

$$W = [vec(\Delta Y' M_z Y_{-1} \alpha_{a_k})]'[(\alpha_{a_k}' Y_{-1}' M_z Y_{-1} \alpha_{a_k})^{-1} \otimes \hat{\Sigma}_\varepsilon^{-1}][vec(\Delta Y' M_z Y_{-1} \alpha_{a_k})] \quad (5)$$

where $\hat{\Sigma}_\varepsilon^{-1}$ is the OLS (MLE, given the Gaussianity assumption) estimator of Σ ($\hat{\Sigma}_\varepsilon = T^{-1}\hat{\varepsilon}'\hat{\varepsilon}$) and $(\Delta Y' M_z Y_{-1} \alpha_{a_k})(\alpha_{a_k}' Y_{-1}' M_z Y_{-1} \alpha_{a_k})^{-1}$ is the OLS (MLE) estimator of δ_{a_k}.

Horvath and Watson (1995) show that the above statistic has an asymptotic distribution that depends on Wiener processes. Critical values were obtained by simulation

and tabulated by the authors. In our empirical application, we allow for a constant term in the VECM, to reflect the fact that the variables may contain trends. Thus, critical values for our case can be found when $n - r_{o_u} = 2$, $r_{o_k} = r_{a_u} = 0$, $r_{a_k} = 1$ and for Case 2, with critical values 13.73. 10.18 and 8.30, for the 1%, 5% and 10% significance levels, respectively (see Table 1 of Horvath and Watson, 1995, pp. 996-998).

3   Empirical analysis

For illustration purposes, we test the fiscal sustainability of a variety of developed and developing countries, namely the Bahamas, Finland, France, South Africa, Thailand and the United States. We use quarterly data for the relevant variables (in real terms), spanning from 1975 to 2005 and collected from the International Financial Statistics database. While initial studies have focused on developed economies (see Payne, 1997, for example), increasing attention has been devoted to the fiscal stance of developing countries (see Kalyoncu, 2005). Empirical evidence is ambiguous, suggesting that the case of 'weak' sustainability is very common, particularly for developing economies. This section revisits this evidence, first using standard testing procedures, then applying the Horvath-Watson test.

Preliminary unit root tests confirmed that government revenues and expenditures for all countries appear to follow $I(1)$ processes[3], thus suggesting that cointegration is the appropriate framework to assess the sustainability of these fiscal regimes. In order to implement the univariate approach for testing 'strong' sustainability, we employ the Augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and the Elliot-Rothenberg-Stock

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[3]Results available upon request.

(ERS) unit root tests, both on the levels and in first-differences of the primary surplus PS_t series (with a constant term included, lag lengths and bandwidths of the Bartlett kernel automatically selected based on the Schwarz Information Criterion). Results are presented in Table 1.

< Insert Table 1 here >

We find that the series for primary surplus in the Bahamas and France display stationary and, hence, sustainable behaviour. The picture is less clear for the USA, given that the PP test does not reject the null of a unit root. On the other hand, Finland, South Africa and Thailand appear to be on an unsustainable path, given that all tests fail to reject the null of non-stationarity.

However, it could be argued that this approach lacks flexibility, given that it implicitly imposes the cointegration vector $[1, -1]$. Also, there may be efficiency gains in resorting to a multivariate testing framework, using the joint dynamics of expenditures and revenues. We next explore cointegration inference involving these two variables, by estimating the cointegration regression (3) and testing whether $b = 1$ or $0 < b < 1$.

We employ a residual-based approach to testing cointegration, i.e., we first estimate (3) and then ascertain whether the estimated equilibrium errors are stationary or not, by means of the cointegration counterparts of the ADF and PP tests (denoted as AEG and PO). There are no efficiency losses in pursuing a single-equation route when compared to the multi-equation method of Johansen (1988), as we are studying a bivariate relationship with only one potential cointegration vector. Thus, for conciseness, we consider the standard OLS estimator of b, as well as the dynamic OLS (DOLS) efficient estimator

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of Stock and Watson (1993), which augments the cointegrating regression with p lags and leads[4] of the differenced explanatory variable, in order to correct for second-order biases usually associated with the simple OLS estimator.

< Insert Table 2 here >

We observe from Table 2 that, in general, the OLS estimates tend to be further away from 1 that the corresponding DOLS estimates ($\hat{b}_{DOLS}$). Considering the estimates alone, this would imply that the Bahamas, Finland and France would be classified as 'weakly' sustainable, with the remaining countries to be considered 'strongly' sustainable. If one looks at the DOLS results, however, all countries display estimates very close to the 'strong' sustainability benchmark, with the exception of Finland, with $\hat{b} = 0.824$.

Note that this analysis is conditional on the existence of cointegration between expenditures and revenues. Looking at the residual-based tests with OLS residuals, one would conclude that, according to the AEG test, Thailand, Finland and the USA would fail to meet the sustainability criteria, given that the statistic fails to reject the null of no cointegration. Interestingly, however, the Phillips-Ouliaris test indicates that only the US would not be sustainable.

If we consider instead tests based on the DOLS estimator, the AEG would point to unsustainability for all countries with the exception of France. The PO test, on the other hand, would add South Africa and the Bahamas to the latter. Therefore, a contradiction seems to emerge: by employing a theoretically more appealing estimator, it appears that the case for sustainability is weakened, although the point estimates suggest that the

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[4]We determine p by testing down the significance of the extra leads and lags, starting from $p = 4$.

cointegration vector is indeed $[1, -1]$. We therefore employ the efficient test of Horvath and Watson (1995) to try to disentangle this issue.

We test the rank of matrix Π in (4) using the Wald statistic (5) described in the previous section. In the case at hand, the null hypothesis if $H_0 : r = 0$, that is, no cointegration, against $H_1 : r = 1$, with cointegration vector $[1, -1]$. This entails estimating the VAR in vector-error correction form. We establish the number of lags to be included using the SIC criterion.

The results of the test are displayed in the rightmost column of Table 2 (under H-W Wald test). It is interesting to notice that the null hypothesis of no cointegration is rejected quite comfortably, at the 1% significance level, for all countries. This suggests that these countries pursue a strongly sustainable fiscal policy. It appears that the results of conventional methodologies tend to penalize the sustainability hypothesis, even when the estimated b is close to 1. This could be explained by the fact that the inefficiency of conventional tests may lead to loss of power of unit root and cointegration tests and, therefore, that the null hypothesis of no cointegration is rejected less often than it should.

4   Conclusion

This paper revisited the empirical evidence on the implications of a government's Intertemporal Budget Constraint, using a multivariate efficient test of the 'strong' sustainability hypothesis, based on the test of Horvath and Watson (1995). This framework is more efficient than both univariate and standard cointegration tests, as it accounts for the likely correlation between innovations to revenues and expenditures and it incorporates the appropriate theoretical restriction on the cointegration vector. When the Horvath-Watson

test is employed, the empirical support for the 'strong' sustainability hypothesis is quite convincing, with the null of no cointegration being rejected at the 1% significance level for all countries.

References

[1] Ahmed, S. and Rogers, J. H.(1995), Government budget deficits and trade deficits Are present value constraints satisfied in long-term data?, *Journal of Monetary Economics,* 36, 351-374.

[2] Hakkio, C. S. and Rush, M. (1991), Cointegration and Government Borrowing Constraints: Evidence for the United States, *Journal of Business and Economic Statistics*, 9, 429-445.

[3] Hamilton J. D. and Flavin M. A. (1986), On the Limitations of Government Borrowing. A framework for Testing, *American Economic Review*, 76, 808-819.

[4] Haug, A. A. (1991), Cointegration and Government Borrowing Constraints: Evidence for the United States, *Journal of Business and Economic Statistics*, 9, 97-101.

[5] Horvath, M. T. K. and Watson, M. W. (1995), Testing For Cointegration When Some of the Cointegrating vectors are Prespecified, *Econometric Theory*, 11, 984-1014.

[6] Johansen, S. (1988), Statistical Analysis of Cointegration Vectors, *Journal of Economic Dynamics and Control*, 12, 231-254.

[7] Kalyoncu, H. (2005), Fiscal policy sustainability: test of intertemporal borrowing constraints, *Applied Economics Letters*, 12, 957-962.

[8] Payne, J. E. (1997), International evidence on the sustainability of budget deficits, *Applied Economics Letters*, 4, 775-779.

[9] Stock, J. H. and Watson, M. W. (1993), A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems, *Econometrica*, 61, 783-820.

[10] Wilcox D. (1989), The Sustainability of Government Deficit: Implications of the Present-value Borrowing Constraint, *Journal of Money Credit and Banking*, 21 291-306.

5   Appendix

Table 1: Unit root tests for the Primary Surplus series

	Levels			First-difference		
Countries	ADF	PP	ERS	ADF	PP	ERS
Bahamas	-4.381^{**}	-4.454^{**}	1.001^{**}	-10.608^{**}	-15.013^{**}	0.226^{**}
Finland	-1.859	-2.358	-3.657	-11.242^{**}	-16.210^{**}	0.259^{**}
France	-7.846^{**}	-8.350^{**}	0.465^{**}	-9.251^{**}	-14.647^{**}	0.946^{**}
South Africa	-1.133	-1.467	20.411	-5.514^{**}	-10.104^{**}	2.489^{*}
Thailand	-1.713	-3.371	4.476	-12.044^{**}	-16.147^{**}	0.267^{**}
United States	-3.022^{*}	-0.842	0.021^{**}	-2.502	-15.505^{**}	6.654

Note: * significant at 5%, ** significant at 1%

Table 2: Cointegration analysis

Countries	AEG	PO	$\hat{b}$	AEG_{DOLS}	PO_{OLS}	$\hat{b}_{OLS}$	HW-Wald test
Bahamas	-3.460^{*}	-12.575^{**}	0.899 (0.042)	-3.324	-9.395^{**}	0.972 (0.036)	30.516^{**}
Finland	-2.015	-3.555^{*}	0.813 (0.042)	-1.609	-2.866^{**}	0.824 (0.046)	15.553^{**}
France	-7.671^{**}	-8.229^{**}	0.825 (0.067)	-4.353^{**}	-6.956^{**}	1.013 (0.059)	29.181^{**}
South Africa	-3.670^{*}	-7.049^{**}	0.929 (0.035)	-2.836	-3.761^{*}	0.975 (0.031)	14.270^{**}
Thailand	-1.725	-3.378^{*}	1.005 (0.034)	-1.672	-2.519	1.008 (0.033)	14.602^{**}
United States	-2.469	-2.736^{**}	0.969 (0.026)	-2.499	-2.196	0.978 (0.026)	46.194^{**}

Note: see notes to Table 1; standard errors in brackets