GREThA
Groupe de Recherche en
Économie Théorique et Appliquée

A Theory of Regret and Information

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Cahiers du GREThA

n° 2011-15

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Une théorie du regret et de l'information

Résumé

Nous proposons un modèle général de préférences qui prend en compte la modélisation du regret. En confrontant les fonctions d'utilité usuelles (fonction d'utilité additive et fonction d'utilité multiplicative) à ce modèle, nous en déduisons certaines propriétés que ces fonctions doivent présenter pour être conformes à notre modèle de préférences. Par ailleurs, le regret étant intrinsèquement lié à la notion d'information sur les choix qui n'ont pas été faits, nous généralisons notre modèle afin qu'il s'adapte à toute structure informationnelle. Nous montrons alors que moins la structure informationnelle est fine, plus l'utilité d'un individu, qui ressent du regret, est élevée. Ce résultat veut dire qu'un individu préfère ne pas être exposé, ex post, à de l'information sur les choix qu'il n'a pas fait. Nous étudions aussi la valeur de l'information en considérant deux cas : celui de la flexibilité où l'information peut être utilisée par l'individu pour faire son choix et celui de la non-flexibilité où l'information arrive, ex post, après le choix. Nous montrons que la valeur de l'information est toujours négative en l'absence de flexibilité et qu'elle peut aussi être négative lorsqu'il y a flexibilité.

Mots-clés : regret, information, choix en incertain, aversion au risque bivariée.

A Theory of Regret and Information

Abstract

Following Quiguin (1994), we propose a general model of preferences that accounts for individuals' regret concerns. By confronting the commonly-accepted additive and multiplicative regret utility functions to this model, we establish certain characteristics that these utility functions require to be in conformity with our preferences model. Equally, as regret is intrinsically related to the concept of information about the foregone alternatives, we generalize our framework so that it can accomodate any information structure. We show that the less informative that structure is, the higher the utility of a regretful individual. This result means that an individual prefers not to be exposed to ex post information about the foregone alternatives. We also focus on information value, and consider two cases. That of flexibility, where information arrives before the choice and can be used to determine the optimal strategy; that of non-flexibility, where information arrives after the choice. We show that information value is negative when there is no flexibility, and that it can also be negative when there is flexibility.

Keywords: regret, information, choice under uncertainty, bivariate risk aversion.

JEL: D03, D81, D82

Reference to this paper: GABILLON Emmanuelle, 2011, "A Theory of Regret and Information", *Cahiers du GREThA*, n°2011-15, http://ideas.repec.org/p/grt/wpegrt/2011-15.html.

1 Introduction

Most people consider that regret is the most intense of all negative emotions and that, next to anxiety, it is the most frequent emotion according to the empirical study of Saffrey, Summerville and Roese (2008). In economics, regret is of particular interest because it has a significant impact on the theory of choice. As Zeelenberg and Pieters (2007) observe, 'all other negative emotions can be experienced without choice, but regret cannot'. It is a counterfactual emotion (Kahneman and Miller 1986), which can occur when an individual compares the result of his choice to what he would have obtained had he made another decision. This conterfactual and negative emotion, when anticipated, plays a role in decision-making. In order to model regret it is necessary to move away from the axioms of von Neumann and Morgenstern (1944) (VNM), since preferences ordering depends on the entire set of alternatives. Early work by Bell (1982 and 1983) and Loomes and Sugden (1982) dealt with pairwise choices. Generalization has been pursued in more recent articles (Loomes and Sugden 1987 and Sugden 1993), which propose regret theories in which the choice set contains more than two alternatives. Sugden (1993), in particular, proposes a set of axioms implying a general regret theory. Quiguin (1994), following Loomes and Sugden (1987) and Sudgen (1993), promotes a utility function that depends on two payoffs: that of the chosen strategy and that of the *ex post* best strategy. The *ex post* best payoff is the reference payoff (or reference point) against which regret is evaluated. Quiguin shows that preferences are not manipulable when they are represented by a utility function which satisfies the Irrelevance of Statewise Dominated Alternatives property (ISDA property). This property states that the withdrawal of a statewise dominated strategy from the choice set does not modify the most preferred strategy. In this paper, we use the utility function of Quiguin to develop two new research paths. The first path is based on the observation that the Quiguin general utility function needs to satisfy additional assumptions in order to be operational and to constitute an adequate representation of regretful preferences. We thus propose a series of properties needing, in our opinion, to be satisfied by a regret-utility function. The second research path springs from the observation that regret is intrinsically related to the idea of information as regards the alternatives. The intensity of regret that an individual may feel depends on the information he has about the foregone alternatives. We thus propose a general framework which allows us to consider any information structure, and then use this framework to examine the value accorded by regretful individuals to information.

In the first part of the paper we propose a general model of preferences which takes into account individuals' regret concerns. We use the Quiguin utility function to propose a unified framework in which any regret-related question can be studied. To the best of our knowledge, this clarification work has not so far been carried out, so that each author uses his own assumptions. We address, in a particular, the question of modeling risk preferences when the utility function is bivariate, since the Quiguin regret-utility function depends on the outcome of two strategies. The question of multivariate risk aversion has already been investigated by M. Marinacci and L. Montrucchio (2005), Eeckhoudt, Rey and Schlesinger (2007) and A. Müller and M. Scarsini (2011). We observe that these approaches cannot be directly applied to a regret-utility function, and show how they need to be modified to be coherent with our framework. We use, in particular, both the supermodular function and separatively inframodular function concepts. We also introduce *reference point risk aversion* (RPRA) which refers to the concavity of the regret-utility function with respect to the *ex post* best outcome. RPRA is introduced by Khrämer and Stone (2008), who consider the decision-making of a regretful individual in a dynamic context. However, our definition of reference point is somewhat different. In Khrämer and Stone, the reference point refers to the preference valuation of a payoff, whereas our refrence point definition focuses on the payoff itself. Moreover, Khrämer and Stone's reference point is the utility obtained from the highest *ex post* payoff of the unchosen strategies. Our own reference point is the highest *ex post* payoff of all the strategies, which means that, unlike Khrämer and Stone, we exclude rejoicing. As Quiguin (1994) shows, rejoicing is not compatible with preferences satisfying the ISDA property. Lastly, Khrämer and Stone consider an additive-regret utility function, whereas our model deals with a general utility function. Having outlined a set of properties characterizing a general regret-utility function, we analyse the usual utility functions: the additive regret-utility function introduced by Bell (1982) and taken up by Braun and Muermann (2004), and the multiplicative regret-utility function introduced by Quiguin (1994). We derive a certain number of characteristics needed for the multiplicative utility function to be in conformity with our general preferences model.

The rest of the paper starts from the observation that much of regret theory is established under perfect information, where the results of the unchosen alternatives are perfectly observable. Since perfect information is a particular case, we propose a general framework which can acommodate any information structure about the foregone alternatives. Bell (1983) was the first to consider imperfect information about the outcomes of the unchosen alternatives in a model in which alternatives were independent. Imperfect information is also to be found in Khrämer and Stone (2008). In our paper, we adopt Khrämer and Stone's modeling of information structure. The results of the unchosen strategies are not observable for the decision maker, who can only observe the outcome of his own strategy. The outcome, however, includes both a payoff and a signal. The individual infers a certain amount of information about the unchosen strategies, not only from his observation of the payoff, but also from that of the signal. This broader approach enables any information structure to be dealt with.

We use our general framework to compare different information structures. We show that the expected utility of a regretful individual decreases as the information structure becomes finer, in the sense of Blackwell (1951). This result means that an individual prefers to minimize his exposure to *ex post* information about the foregone alternatives. We also assess the impact of regret on the willingness to pay for information. Under VNM axioms, information value is always positive with, in the worst case, the information being useless and of no value (see Gollier 2001, for example). In this paper, however, we show that information can be harmful when people experience regret. We consider two cases: the non-flexibility case in which information only arrives after the choice, and that of flexibility, in which information arrives before the choice and can be used to determine the choice. We show that information value is negative under non-flexibility and can also be negative under flexibility.

In the non-flexibility case, obtaining information about an unchosen strategy can lead to regret when the choice cannot be modified. People systematically dislike obtaining information which cannot be used to modify the choice. We show that the RPRA property is necessary to establish this result.

On the theoretical side, the idea of information harmfulness has already been considered by Krähmer and Stone (2008), who identify different forces that shape the behaviour of an individual. One of these forces is a tendency to behave conservatively. In their model, a regretful agent can be reluctant to modify his behaviour, fearing that he might regretfully discover that he would have been better off if he had done it before. The agent sticks to past choices, even if there are some indications to show that switching would be payoff maximizing. Conservative behaviour, highlighted by Krähmer and Stone, underlines the harmfulness of information. The agent is conservative because he fears information about foregone past actions. This is perfectly in line with our result concerning the negative value of information.

On the experimental side, our result conforms with that of Zeelenberg et al. (1996). The authors perform an experiment where they set up two risky lotteries to which participants are indifferent. Indifference as regards the two lotteries is established when there is no feedback on the foregone lottery. Stated otherwise, people exclusively obtain feedback on the lottery of their choice. One of the two lotteries is relatively risky, the other relatively safe (the probability of winning is higher but the gain is lower). Zeelenberg et al. modify the feedback context and observe the behavioural consequences. When people know that the result of the risky lottery will be revealed, they are no longer indifferent to the two lotteries, tending to prefer the risky one. They abandon the safe lottery because they try to protect themselves against the regret which may arise from having information about the foregone lottery (information about the risky lottery if they choose the safe lottery). Zeelenberg et al. show that 'regret aversion' induces risk-seeking behaviour (when people anticipate feedback on the risky lottery), or risk-avoiding behaviour (when people anticipate feedback on the safe lottery). These types of behaviour, which consist in avoiding information about the foregone lottery, are consistent with our result. Information about foregone alternatives is utility decreasing. The experimental investigation of Zeelenberg et al. can thus be interpreted as an empirical justification of the RPRA property needed for our result. Other experimental studies (Josephs, Larrick, Steele and Nisbett 1992, Larrick and Boles 1995, Ritov 1996, Inman and Zeelenberg 1998, Zeelenberg and Pieters 2004, Humphrey, Mann and Starmer 2005), also reveal the sensitivity of choices to the feedback context, and demonstrate that people try to protect themselves against information they could have obtained by making a different choice.

When there is flexibility, information is used to determine the optimal choice. At first sight, it might be thought that information is useful per se, and could not be harmful. However, we show that, for a regretful individual, information value can be negative. The explanation of this result is less intuitive than in the non-flexibility case. Information affects expected utility levels through two different channels. First, information modifies probabilities: an individual who receives information uses it to revise his beliefs about the available strategies. Under VNM axioms, this probability revision is the only channel through which information modifies expected utility levels and choices. Let us call this channel the *probability effect*. But, when a regretful individual is brought into the picture, information modifies expected utility levels via another channel. In this case, information modifies expected regret: good news about a given strategy can be bad news about other strategies[1]. For example, a signal which indicates good news about a particular strategy can increase the regret that an individual anticipates feeling if he were to choose another strategy. This channel, which we call the *regret effect*, explains why information value can be negative when there is flexibility. In order to understand this better, let us now consider a regretful individual who has the choice between two risky and independent alternatives, X and Y, where X denotes his optimal strategy. Let us now assume that the individual receives a perfect signal about Y. If the signal indicates bad news, X remains the optimal strategy. But if the signal indicates good news, let us consider the case where the *probability effect* is too weak to make Y the optimal strategy (the signal is not very good news). This means that, without the *regret effect*, X would remain the optimal choice. The story would come to an end,

[1] This is true even if the strategies are independent. At this particular point, we do not examine the probability effect.

and the expected utility of the individual, who anticipates obtaining the signal, would be unchanged. Now let us assume that the *regret effect*, which decreases the expected utility from choosing X, is strong enough to make Y the optimal strategy, despite the weakness of the *probability effect*. Strategy Y becomes optimal, not because the expected utility of Y becomes higher than the expected utility of X, but because the expected utility of X decreases. Good news about Y increases the regret that the individual anticipates feeling if he were to choose strategy X. In this example, in aggregate, the expected utility of the individual, who anticipates receiving the signal, decreases. The information value in this case is, therefore, negative. In the body of the article we give exact conditions under which information value is negative when there is flexibility.

The paper is organized as follows. In Section 2, we introduce a model of preferences that takes into account individuals' regret concerns, outlining a set of properties needed to be satisfied by a regret-utility function. Section 3 examines the usual regret-utility functions in the light of Section 2 properties. Section 4 generalizes the model introduced in Section 2 to any information structure. Section 5 is dedicated to the study of information value.

2 The model

Uncertainty is represented by a state space $\Omega = \{1, ..., S\}$ and a probability distribution $(\pi_1, ..., \pi_S)$ on Ω. Let Φ denote a set of $N + 1$ risky alternatives, with a risky alternative Y_n being an S-tuple of state-contingent outcomes $Y_n = (y_{n1}, ..., y_{nS})$. Following Quiguin (1994), in each state, a regret-utility function (r-utility function) depends on the payoff of the chosen strategy and on the highest realized payoff (among the $N + 1$ risky alternatives). If we denote the chosen strategy by $X = (x_1, ..., x_S)$, and the unchosen strategies by $Y_1 ... Y_N$, the expected r-utility obtained by selecting X, is written

$$\sum_{s=1}^{S} \pi_s u\left(x_s, r_s\right) \tag{1}$$

with $r_s = \max\left\{x_s, y_{1s}, .., y_{ns}, ., y_{Ns}\right\}$.

Here, we exclude the feeling of rejoicing when the agent learns that he has chosen the best strategy. Rejoicing has been investigated by G. Loomes and R. Sugden (1982) in a two-choice model and, in a more general setting, by G. Loomes and R. Sugden (1987) and R. Sugden (1993). Generalization to a choice set containing any number of alternatives generates a class of utility functions that depends on the results of all the risky alternatives: $u\left(x_s, y_{1s}, .., y_{ns}, ., y_{Ns}\right)$. However, Quiguin (1994) shows that, among these utility functions, the one which satisfies the ISDA property takes the form $u\left(x_s, r_s\right)$. With this form, rejoicing is eliminated from preferences. In this article, we follow Quiguin (1994), but consider that the functional form $u\left(x_s, r_s\right)$ is general and cannot be directly operational. It must satisfy some additional properties in order to constitute an adequate representation of regretful preferences. This section is thus dedicated to presenting a set of properties appropriate to an r-utility function. In the rest of the article, we assume that $u\left(x_s, r_s\right)$ satisfies these properties. This gives us a framework within which to study how a regretful individual evaluates information.

In order to develop our series of properties, we alleviate our notations by omitting the reference to the state of the world s. We thus rewrite the r-utility function as $u\left(x, r\right)$.

Under condition $r = x$ there is no feeling of regret, since the chosen strategy coincides with the *ex post* best strategy. The level of satisfaction $u\left(x, x\right)$ is, thus, not affected by any feeling of regret, and can be related to the 'choiceless utility function' of Loomes and Sudgen (1982). The authors define this utility function as 'the utility that an individual would derive from the consequence x without having chosen it'. This utility is the satisfaction derived from payoff x, independently of the idea of choice. In what follows, we retain the same terminology as Loomes and Sugden, calling function $u(x,x)$ the choiceless utility function (c-utility function). This represents the utility obtained from payoff x, unaffected by emotions. As we will see in this section and in Section 4, the c-utility function plays an essential role in the definition of regret.

Let $u_1(x,r)$ denote $\frac{\partial u(x,r)}{\partial x}$ and $u_2(x,r)$ denote $\frac{\partial u(x,r)}{\partial r}$. The same rule applies for the notations $u_{11}(x,r)$, $u_{22}(x,r)$, $u_{12}(x,r)$ and so on. We begin by introducing the properties of the c-utility function $u(x,x)$:

P1a. The c-utility is increasing $\frac{\partial u(x,x)}{\partial x} = u_1(x,x) + u_2(x,x) \geq 0$

P1b. The c-utility is concave $\frac{\partial^2 u(x,x)}{\partial x^2} = u_{11}(x,x) + u_{12}(x,x) + u_{21}(x,x) + u_{22}(x,x) \leq 0$

In order to better understand $P1a$ and $P1b$, let us imagine that, out of the set of available alternatives, there is one which gives the best payoff, whatever the state of the world. Choosing this *dominant strategy* ensures not having any feeling of regret, but does not protect against the payoff risk. We thus assume that an r-individual is averse to the payoff risk of a dominant strategy ($P1b$). Moreover, we assume that an r-individual utility increases with a dominant strategy payoff ($P1a$).

As the c-utility is an increasing function under $P1a$, we are now able to characterize the reference point and the feeling of regret:

Definition 1 *The reference point is the ex post payoff which maximises the c-utility function.*

The reference point, with respect to which regret is computed, is based on the c-utility function criterion. This concept will be used and generalized throughout the paper.

Definition 2 *Regret occurs as soon as the c-utility level generated by the reference point exceeds that of the chosen strategy payoff.*

There is regret as soon as $u(x,x)$ is lower than $u(r,r)$ or, under P1, as soon as r>x.

Let us now expose the properties of the r-utility:

P2a. The r-utility increases with x $u_1(x,r) \geq 0$

P2b. The r-utility decreases with r $u_2(x,r) \leq 0$

Property $P2b$, which is necessary for regret modeling, states that the r-utility decreases with the *ex post* best outcome. The payoff x being given, as the reference point increases, regret increases and utility decreases.

We now try to define the risk preferences of a regretful individual (r-individual). Since there is no unanimously accepted definition of bivariate risk aversion, we formulate the hypotheses best adapted to our regret-modeling objective. In order to do so, we consider two possible bivariate outcomes: $(\underline{x},\underline{r})$ and $(\overline{x},\overline{r})$ with $\underline{x} \leq \overline{x}$ and $\underline{r} \leq \overline{r}$.

P3. The r-utility is supermodular

$$u(\overline{x},\overline{r}) + u(\underline{x},\underline{r}) \geq u(\overline{x},\underline{r}) + u(\underline{x},\overline{r})$$

Property $P3$, as it is formulated, can be interpreted as follows: an individual prefers the 50-50 lottery $[(\overline{x},\overline{r}),(\underline{x},\underline{r})]$ to the 50-50 lottery $[(\overline{x},\underline{r}),(\underline{x},\overline{r})]$. An individual has a supermodular r-utility function if he prefers a 50-50 gamble where he can either have a high payoff with high regret, or a low payoff with low regret, rather than the negative correlation version of this game where payoff and regret are negatively correlated[2]. In other words, we assume that the risk preferences of an r-individual are characterized by *positive correlation loving*. This property is akin to the definitions of correlation loving given by Eeckhoudt, Rey and Schlesinger (2007). Although the authors do not specifically indicate the nature of the sign they use for their correlation, it seems clear that they call 'correlation averse' an individual who is a *negative correlation lover*[3], and 'correlation lover' an individual who is a *positive correlation lover*[4]. In this paper, as our utility function decreases with its second argument, it is the assumption of *positive correlation loving* which is called for.

It should also be noted that $P3$ can be rewritten as

$$u(\overline{x},\overline{r}) - u(\underline{x},\overline{r}) \geq u(\overline{x},\underline{r}) - u(\underline{x},r) \tag{2}$$

Starting from the above equation, it is easy to demonstrate that an r-utility function is characterized by positive correlation loving if and only if its cross second derivatives are positive. Thus $P3$ can be reformulated as

P3.* The *r-utility is supermodular

$u_{12}(x,r) = u_{21}(x,r) \geq 0$

When talking about risk preferences, we should also consider the following property:

P4* The *r-utility is separately inframodular

$\forall r, \forall \underline{x} \leq \overline{x}, \forall h \geq 0, u(\underline{x}+h,r) - u(\underline{x},r) \geq u(\overline{x}+h,r) - u(\underline{x},r)$

[2] It is easy to show that he also prefers positive correlation to the independent version of the game in which x and r are not correlated.

[3] Or who is *positive correlation averse.*

[4] Or who is a *negative correlation averse.*

$$\forall x, \forall \underline{r} \leq \overline{r}, \forall h \geq 0, u(x, \underline{r} + h) - u(x, \underline{r}) \geq u(x, \overline{r} + h) - u(x, \overline{r})$$

This could also be expresssed in the following terms :

P4a. The r-utility exhibits payoff risk aversion $u_{11}(x, r) \leq 0$

P4b. The r-utility exhibits reference point risk aversion $u_{22}(x, r) \leq 0$

Property $P4a$, unlike $P4b$, does not require any particular explanation. Property $P4b$ states that the r-utility function is concave with respect to the reference point. That is to say, we assume that an r-individual is *reference point risk averse*. We recognize that the concept of RPRA has to be investigated by means of experimental approaches in order to be justified. However, as we will see in Section 5.1, the results that we obtain under this assumption are in line with the available experimental studies in psychology on regret and information.

The definition of r-individual risk aversion, founded on properties $P3$ and $P4$, can be compared with the definition of multivariate risk aversion given by A. Müller and M. Scarsini (2011). The authors define multivariate risk aversion as the property of inframodularity which characterizes a function whose increments are decreasing (see also M. Marinacci and L. Montrucchio 2005). This constitutes a generalization of the one-dimensional concavity according to which a function has non-increasing differences [5]. It can be shown that a multivariate function is inframodular if and only if it is submodular (the reverse property of $P3$) and separately inframodular. In our paper, submodularity would not be a reasonable assumption, because the utility function decreases with regret. Thus, unlike A. Müller and M. Scarsini, we assume supermodularity (property $P3$). We also assume that the r-utility function is separately inframodular, which leads to payoff risk aversion and to RPRA.

[5] However, in the multi-dimensional case, inframodularity and concavity are two independent concepts.

To summarize, unlike A. Müller and M. Scarsini, we consider that inframodularity cannot characterize multivariate risk aversion when the utility function decreases with at least one of its arguments. We propose to characterize the risk preferences of a regretful individual as being supermodular[6] and separately inframodular.

To the best of our knowledge, the influence of the reference point on payoff risk aversion is still an open question, and so is the payoff influence on RPRA. Nevertheless, since regret increases with the reference point, it seems reasonable for us to assume that payoff risk aversion does not decrease with the reference point. Likewise, we assume that RPRA does not increase with payoff. In other words, we assume that the Arrow-Pratt absolute risk aversion coefficient $-\frac{u_{11}(x,r)}{u_1(x,r)}$ does not decrease with y, and that $-\frac{u_{22}(x,r)}{u_2(x,r)}$ does not decrease with x.

P5a. Payoff risk aversion does decrease with the reference point

$$u_{112}(x,r)\, u_1(x,r) - u_{12}(x,r)\, u_{11}(x,r) \leq 0$$

P5b. Reference point risk aversion does not increase with payoff

$$u_{221}(x,r)\, u_2(x,r) - u_{21}(x,r)\, u_{22}(x,r) \geq 0$$

Although $P5a$ and $P5b$ impose certain restrictions as to the form of the r-utility function, the multiplicative form $u(x,r) = w(x)\,\phi(r)$, or the following additive form $u(x,r) = w(x) + \phi(r)$, satisfy these properties. These two forms are special cases where x-risk aversion is independent of r, and r-risk aversion is independent of x.

Under $P2$, $P3$ and $P4$, property $P5a$ implies that $u_{112}(x,r) \leq 0$. This means that an r-individual is cross prudent as regards payoff (see Eeckhoudt,

[6] See Meyer and Strulovici (2010) for an analysis of supermodularity.

Rey and Schlesinger 2007). He prefers the 50-50 lottery $(x+\widetilde{\varepsilon},\underline{r})$ to the 50-50 lottery $(x+\widetilde{\varepsilon},\overline{r})$, where $\widetilde{\varepsilon}$ is a zero mean payoff random variable. Similarly, the combined set of $P2$, $P3$, $P4$ and $P5b$ implies that $u_{221}(x,r)\geq 0$, which means that an r-individual is cross prudent in regret (reference point). Our derivatives have the opposite sign to that of Eeckhoudt, Rey and Schlesinger for the reason given above: the r-utility function decreases with r.

In the next section, we examine the usual regret-utility functions in the light of $P1$ to $P5b$. However, in the other sections of the paper, it is not necessary to have the complete set of properties to obtain our results (for example, we never use $P5a$ and $P5b$). That is why, under each proposition, we indicate the specific properties needed to obtain the result.

3 The usual regret-utility functions

In the literature, two types of utility functions are used to model regret. The first, most commonly-used type, which exhibits additive regret, was introduced by Bell (1982), and by Braun and Muermann (2004). The second type, which exhibits multiplicative regret, was introduced by Quiguin (1994). Both types satisfy the ISDA property.

3.1 Additive regret

In the additive form, a regret function is added to the c-utility function as follows:

$$u(x,r)=v(x)-kg(v(r)-v(x)) \tag{3}$$

where $v'(.)\geq 0$, $v''(.)\leq 0$, $k>0$, $g(.)\geq 0$, $g'(.)\geq 0$ and $g''(.)\geq 0$.

Function $v(.)$ is the c-utility function, and function $g(.)$ is the regret function. It is easy to verify that the additive r-utility function satisfies properties

$P1$ to $P4a$. Property $P4b$, that is to say RPRA, is not necessarily satisfied. The additive utility function accords priority to the concept of *regret risk aversion* by assuming the convexity of the regret function $g(.)$. The idea of regret risk aversion, though more intuitive, is not necessarily more appropriate than that of RPRA. A justification of $P4b$ is given in Section 5.1. It should be noted that $P5a$ and $P5b$ are also not necessarily satisfied by the additive r-utility function.

3.2 Multiplicative regret

The multiplicative r-utility function has the following expression:

$$u(x,y) = w(x)\,\phi(r) \tag{4}$$

With the multiplicative form, the choice between strategy X_1 and strategy X_2 is determined by the sign of $\sum_{s=1}^{S} \pi_s \left[w(x_{1s}) - w(x_{2s})\right] \phi(r_s)$. As Quiguin (1994) observes, the effect of regret is to attach different weights to the different states. Moreover, Quiguin expects $\phi(r)$ to be increasing since he considers that, in the above-mentioned expression, states with high potential for regret should be weighted more heavily relative to their probability than states with low potential for regret.

In the light of $P1$ to $P5b$, we determine the exact characteristics of the functions $w(x)$ and $\phi(r)$. In order to simplify the presentation of our results, we rewrite the expression of $u(x,r)$, replacing $w(x)$ by $-v(x)$:

$$u(x,r) = -v(x)\,\phi(r) \tag{5}$$

Our results are summarized in Proposition 1 and Proposition 2:

Proposition 1 *The multiplicative r-utility function is negative.*

Proof. It is easy to verify that $P2$ and $P3$ are satisfied, either when $v(x) \geq 0$ and $\phi(r) \geq 0$, or when $v(x) \leq 0$ and $\phi(r) \leq 0$. In both cases, $u(x,r) = -v(x)\phi(r) \leq 0$. ■

In what follows we assume, without loss of generality, that $v(x) \geq 0$ and $\phi(r) \geq 0$.

Proposition 2 *The multiplicative r-utility function is characterized by:* $v'(x) \leq 0$, $v''(x) \geq 0$ *and* $\phi'(r) \geq 0$ *and* $\phi''(r) \geq 0$.

Proof. $P2 \Longrightarrow v'(x) \leq 0$ and $\phi'(r) \geq 0$. $P4 \Longrightarrow \phi''(r) \geq 0$ and $v''(x) \geq 0$. ■

Our results are consistent with the intuition of Quiguin (1994) since, given our set of properties, we find that function $\phi(r)$ increases. However, we note that this result only makes sense when combined with the result of Proposition 1. When the payoff level x is given, the r-utility should decrease when r increases (because regret increases). This is effectively the case when two conditions are met: $\phi(r)$ is an increasing function, and the multiplicative r-utility is negative.

We give two examples of multiplicative r-utility functions which satisfy $P1-P5b$:

Example 1 *When* $v(x) = e^{-\gamma x}$ *and* $\phi(r) = e^{kr}$*, the multiplicative r-utility function is*

$$u(x,r) = -e^{-\gamma x + kr} \tag{6}$$

When $\gamma \geq k \geq 0$*, the above r-utility function satisfies* $P1 - P5b$*.*

Example 2 *When* $v(x) = x^{-\gamma}$ *and* $\phi(r) = r^k$*, the multiplicative r-utility function is*

$$u(x,r) = -x^{-\gamma} r^k \tag{7}$$

When $\gamma \geq k \geq 1$*, the above r-utility function satisfies* $P1 - P5b$*.*

4 Regret and information structures

We call *decision stage* the time of the choice, and *learning stage* the time of uncertainty resolution. In this section, we focus on the information available at the learning stage.

Equation (1) implicitly assumes a particular *ex post* information structure. The r-individual observes not only the realization of the chosen strategy x_s but also the realizations of all the unchosen strategies: $\{y_{1s}, ..., y_{Ns}\}$. He thus learns the best outcome $r_s = \max\{x_s, y_{1s}, ..., y_{N-1s}\}$, and experiences regret when $x_s < r_s$. We refer to this information structure as the *perfect information structure* because, at the learning stage, the agent has perfect information about the *ex post* best outcome. It is easy to imagine many different alternative information structures: for example, the opposite case in which the agent learns the result of his chosen strategy but does not observe the result of any other strategy. In this case, at the learning stage, the agent does not know what the best outcome is. Does that mean that he does not feel any regret? We do not think so. Imagine that the outcome of the chosen strategy is very low. The agent might feel regret at not having chosen another strategy. Consequently, there is still a reference point, but it cannot be equal to r_s, since it is not observable.

In order to introduce different *ex post* information structures in our model, we choose to abandon the states of the world approach. We now assume that the payoff of a risky alternative Y_n is a random variable which takes its values in the support $W_{Y_n} \subset \mathbb{R}$. The agent now chooses from $N+1$ random variables or lotteries. Throughout the rest of the paper, we denote random variables by capital letters, and their typical realizations by small letters.

In order to define the reference point for any information structure, we need to consider the agent's information at the learning stage. We now assume, as in Khrämer and Stone (2008), that a strategy's outcome is made up of both a payoff and the realization of a signal about the foregone alternatives. Let S_X denote the strategy X information structure $(X - IS)$, which is characterized by the set of signals $\{S_x, x \in W_X\}$, with S_x denoting the signal conditional to the realization x of strategy X. Let $\Omega_{S_x} \subset \mathbb{R}^m$, with $m > 0$ denoting the support of S_x. The signal S_x represents the *ex post* information structure if the chosen strategy X takes the value x. At the learning stage, an unchosen risky alternative payoff Y_n is thus characterized by an *ex post* probability distribution, given the realization of both the signal s_x and the payoff x. We should also recall at this point that the feeling of regret is based on the c-utility (see Definition 2). We thus compute the *ex post* certainty equivalent of each strategy payoff Y_n using the c-utility function. The *ex post* certainty equivalent of a foregone lottery Y_n satisfies

$$u\left(EC_{Y_n}^{x,s_x}, EC_{Y_n}^{x,s_x}\right) = E\left[u\left(y_n, y_n\right) \mid x, s_x\right] \tag{8}$$

where the operator $E\left[. \mid x, s_x\right]$ represents the mathematical expectation conditional to the information at the learning stage. The certainty equivalent of the chosen lottery is equal to the realization of the lottery

$$EC_x^{x,s_x} = x \tag{9}$$

We are now able to give a general definition of the reference point:

Definition 3 *The reference point r_x is the highest ex post certainty equivalent:* $r^{x,s_x} = Max\left\{x, EC_{Y_1}^{x,s_x}, .., EC_{Y_n}^{x,s_x}, .., EC_{Y_N}^{x,s_x}\right\}$

Definition 3 generalizes Definition 1 given in Section 2. Regret is still defined using the c-utility function $u(x, x)$. At the learning stage, regret occurs when the c-utility obtained from x is lower than the highest expected c-utility which could be obtained from the foregone strategies. In other words, regret is to be found when $u(x,x)$ is lower than $u(r^{x,s_x}, r^{x,s_x})$ or, alternatively, when $x < r^{x,s_x}$. If it turns out that x is the best payoff at the learning stage (given the individual's information), then $r^{x,s_x} = x$, which means that regret is absent.

Let $f(y_1, ..., y_N | x, s_x)$ denote the density of $y_1, ..., y_N$ conditional on $X = x$ and on $S_x = s_x$. We now introduce a new definition:

Definition 4 *Let S_X^1 and S_X^2 denote two different $X - IS$. The second of these, S_X^2, is more informative than S_X^1 if*

$$\forall x \in W_X, \forall (y_1, ..., y_N) \in \prod_{n=1}^{N} W_{Y_n}, \quad f\left(y_1, ..., y_N | x, s_x^2, s_x^1\right) = f\left(y_1, ..., y_N | x, s_x^2\right)$$

This definition states s_x^2 is a sufficient statistic for $\left(s_x^1, s_x^2\right)$. It is an adaptation of Blackwell's concept of 'garbling' to our framework[7]. S_X^2 is more informative than S_X^1 if, for some x, S_x^1 is obtained by garbling the messages coming from S_x^2 (and if S_x^1 and S_x^2 are identical for the other values of x). In other words, for some x, S_x^1 is a stochastic transformation of S_x^2. As the stochastic transformation is independent of $y_1, ..., y_N$, information is lost through the transformation. For some x, S_x^2 gives, therefore, more information than S_x^1 about the foregone alternatives: S_x^2 induces a finer partition of the unchosen alternatives' support $\prod_{n=1}^{N} W_{Y_n}$ than S_x^1. When S_x^1 garbles S_x^2, the signal realization s_x^2 is a sufficient statistic for $\left(s_x^1, s_x^2\right)$.

We now define an information structure IS as a set of strategy information structures:

$$IS = \{S_X, S_{Y_1}, ..., S_{Y_N}\}$$

An information structure represents the *ex post* informative context that an r-individual is faced with when he makes his choice. In order to compare different information structures, we introduce the following definition:

Definition 5 *An information structure IS^2 is X-finer than an information*

[7] See, Malueg (1980) and Gollier (2001) for a presentation of the Blackwell (1951) theorem.

structure IS^1*, if* S_X^2 *is more informative than* S_X^1*, provided all the other* $Y_n - IS$*,* $n = 1...N$ *remain the same.*

An information structure IS^2 *is finer than an information structure* IS^1*, if* $\forall X \in \Phi$*,* S_X^2 *is more informative than* S_X^1*.*

We obtain the following proposition:

Proposition 3 *Let* X *denote the optimal strategy of an r-individual under the information structure* IS^1 *or under the information structure* IS^2*. If* IS^2 *is an* X*-finer information structure than* IS^1*, the r-individual prefers* IS^1 *to* IS^2*.*

Proof. The proof uses $P1a$, $P1b$, $P2b$ and $P4b$. See Appendix 1. ■

According to Proposition 3, an r-individual prefers to minimize his exposure to *ex post* information about the foregone alternatives. $P4b$, the property of RPRA, is central to this result. As suggested by the reference point expression given in Definition 3, the reference point fluctuates with the signal and, put simply, the finer the information, the riskier the reference point.

We also obtain the following proposition:

Proposition 4 *If the information structure* IS^2 *is finer than* IS^1*, any r-individual prefers* IS^1 *to* IS^2*.*

Proof. The proof uses $P1a$, $P1b$, $P2b$ and $P4b$. See Appendix 1. ■

Proposition 4 states that any r-individual prefers to live in the least *ex post* informative context. Under the VNM axioms, Proposition 3 and Proposition 4 would not hold. An individual would be indifferent to $IS^1$and IS^2 since he would only be concerned with his own payoff strategy.

Let us now call the uninformative information structure the situation in which all the signals are uninformative, or the situation in which a strategy outcome is limited to a payoff. We obtain the following corollary:

Corollary 1 *An r-individual prefers the uninformative information structure to any other information structure.*

As the uninformative information structure is coarser than any other information structure, this result is a direct consequence of Proposition 3 and Proposition 4. The preference, in Corollary 1, can be weak. For example, if Y denotes an unchosen strategy under the uninformative information structure, the r-individual is indifferent about the uniformative information structure and a Y-finer one. A Y-finer information structure improves the feedback context of strategy Y and decreases the expected r-utility that would have been obtained from this strategy. However, a Y-finer information structure has no impact on the expected r-utility of the chosen strategy X. On the contrary, an r-individual strictly prefers the uninformative information structure to a finer one or even to an X-finer one[8].

5 Regret and information value

In this section, we study the value of a signal S which gives information about the future realizations of the risky alternatives. After the signal, at the learning stage, each risky alternative Y_n is characterized by a conditional probability distribution, given the payoff of the chosen strategy x and the observed signals, s_x and s.

We first consider the case without flexibility. Information S arrives at the learning stage, after the choice has been made, and cannot be used to modify the choice. We then consider the case of flexibility in which information S arrives at the decision stage, and can be used to modify the choice. In both cases, information value is computed before the decision stage. Information value is

[8] Unless there exists a strategy X' such that the r-individual is indifferent to X and X' under the uninformative information structure. In that case, he can protect himself against feedback by choosing X'.

positive when the observation of the signal increases the expected r-utility: in other words, information has positive value when the agent is ready to pay for it. Under the VNM axioms, information has no value when there is no flexibility, because it cannot be used to modify the choice. On the contrary, when there is flexibility, information value is positive as soon as it allows, with a positive probability, the agent to modify his choice (see, for example, Gollier 2001). In what follows, we want to see how these results are modified with an r-utility function.

5.1 No flexibility

Let us consider an r-individual making his choice in the information structure background $IS = \{S_X, S_{Y_1}, ..., S_{Y_N}\}$. Strategy X denotes his optimal strategy under IS. Let us now consider a signal S which makes IS X-finer:

$$\forall x \in W_X, \forall (y_1, ..., y_N) \in \prod_{n=1}^{N} W_{Y_n}, \quad f(y_1, ..., y_N | x, s_x, s) = f(y_1, ..., y_N | x, s) \tag{10}$$

The agent receives the signal at the learning stage, after the choice has been adopted. When the agent receives the signal, the reference point has the following expression:

$$r^{x,s_x,s} = Max\left\{x, EC_{Y_1}^{x,s_x,s}, .., EC_{Y_n}^{x,s_x,s}, ., EC_{Y_N}^{x,s_x,s}\right\} \tag{11}$$

with $u\left(EC_{Y_n}^{x,s_x,s}, EC_{Y_n}^{x,s_x,s}\right) = E\left[u(y_n, y_n) | x, s_x, s\right]$.

When the signal S provides additional information about the unchosen strategies $Y_1 ... Y_N$, Equation (11) shows that, given the values of x and s_x, the reference point fluctuates with the signal. If $V(S)$ denotes the value of information, we obtain the following corollary:

Corollary 2 *Under non-flexibility (or when information does not modify the* *optimal choice),* $V(S) \leq 0$*.*

Introducing a signal X-refines the information structure and decreases the expected r-utility of the regretful individual. Corollary 2 directly results from Proposition 3.

By adding a new risk to the reference point, the signal decreases the expected r-utility under RPRA. We have

$$E_{x,s}\left[u\left(x, r^{x,s_x,s}\right)\right] \leq E_x\left[u\left(x, r^{x,s_x}\right)\right] \tag{12}$$

and thus, under $P1a$, there exists $v \leq 0$ (information value) such that

$$E_{x,s}\left[u\left(x, r^{x,s_x,s}\right)\right] = E_x\left[u\left(x+v, r^{x,s_x}+v\right)\right] \tag{13}$$

Equation (13) means that the r-individual must be paid to accept the information. Moreover, under flexibility, when choice X remains optimal whatever the value taken by the signal, the result still holds. Information increases the risk on the reference point without allowing any other choice to be made. Corollary 2 contrasts with what is obtained under VNM axioms: information value is negative for Corollary 2, whereas it is equal to zero under VNM axioms.

In what follows, we continue to assume that there is no flexibility. The choice cannot be modified after information is received. However, the choice can be modified, before the signal, when the r-individual learns that he will obtain some information. This leads to the following proposition:

Proposition 5 *Under non-flexibility, it can be optimal for an r-individual to modify his choice when he learns that he will receive additional information about some of the foregone strategies.*

Proof. *See Example* 3. ■

We now give an example in which an r-individual, who has the choice between two independent risky alternatives $\Phi = \{X, Y\}$, chooses strategy X. But, in this example, learning that he will obtain some information about Y incites him to change his choice from X to Y, in order to insure himself against the reference point risk. Under VNM axioms, the fact of receiving information in the future never modifies the optimal choice. We thus obtain another distinction between *regret behaviour* and *VNM behaviour*. In all the examples given in this paper, the r-utility function is the multiplicative r-utility function $u(x, y) = -e^{-x}e^{\frac{1}{2}r}$.

Example 3 *Let us consider a set of two risky alternatives* $\Phi = \{X, Y\}$*. The risky alternative* X *takes the value* 1*, and the value* 2*, with equal probabilities. The risky alternative* Y *takes the value* 0.8*, and the value* 2.5*, with equal probabilities. We consider an uninformative information structure in which the r-individual does not observe the realization of the foregone strategy at the learning stage. We then consider a perfect signal about strategy* Y*. Our results are summarized in the following table:*

Z	$E[u(z,z)]^*$	EC_Z^{**}	$E[u(z,r^z)]^\dagger$	$E[u(z,z^{z,s})]^\ddagger$
X	-0.487	1.438	-0.568	-0.683
Y	-0.487	1.438	-0.604	-0.604

* *Expected c-utility*

** *Certainty equivalent computed with the c-utility when there is no signal*

† *Expected r-utility when there is no signal*

‡ *Expected r-utility when the individual anticipates the signal*

As column † shows, X *is the optimal strategy when there is no signal. The comparison between lines* 2 *and* 3 *in column ‡ allows us to conclude that the introduction of a future signal on* Y *makes* Y *more attractive than* X*. We also note that, on line* 2*, the comparison between column † and column ‡ illustrates Proposition* 3*. The details of the computation are given in Appendix* 2*.*

On the theoretical side, this result is close to the underlying mechanism that explains the conservative behaviour identified by Krähmer and Stone (2008) in their two-period approach. The authors consider two strategies generating i.i.d payoffs[9] and show that, when the signal contained in a strategy is independent of the strategy payoffs, the individual prefers the less informative strategy (in the Blackwell sense) in order to minimize his exposure to *ex post* information. This result explains why, at the second period, the individual might be tempted to stick to his first period choice in order to ignore what he would have obtained had he made another decision at the first period. We notice that, since past actions are not modifiable, Khrämer and Stone's individual is in a non-flexibility situation as regards his first period choice.

On the experimental side, our results about regret and information are confirmed by the study of Zeelenberg et al. (1996) which shows that people tend to avoid having information about foregone alternatives. In their paper, Zeelenberg et al. use the term 'regret aversion'. Together with many others, they employ 'regret aversion' to qualify people who may feel regret: this corresponds to our $P2b$. We show here, however, that $P2b$ alone is not sufficient to obtain a result consistent with the experiments of Zeelenberg et al. What is lacking is $P4b$, RPRA, which is both necessary and central to our results. Consequently, even though more empirical facts would be needed to deal fully with this issue, the study of Zeelenberg et al. can be interpreted as an experimental justification of $P4b$.

5.2 Flexibility

Let us now consider the case of flexibility, where the signal takes place prior to the choice being adopted, and the decision can be adapted to the information. For the sake of simplicity, we consider an uninformative information structure:

[9] independently and identically distributed payoffs.

a strategy outcome is limited to a payoff. We distinguish two channels through which the signal affects the expected utility obtained from a strategy:

1. First, the individual revises his beliefs about the probability distributions of the strategies correlated to the signal, and the expected utilities of these strategies are then modified. We call this channel the *probability effect*.

2. Secondly, the signal can modify the regret that people anticipate feeling when they choose a strategy. We call this channel the *regret effect*. For example, a good signal on strategy Y can decrease the expected r-utility from strategy X, because choosing X can expose to feeling more regret than before (the regret of not having chosen Y). The anticipated regret associated with the choice of strategy Y can also be modified. This effect is specific to regret theory.

Let us now consider a particular r-individual. Let X denote his optimal strategy according to the c-utility function criterion. Using the c-utility function criterion amounts to saying that X is the optimal strategy if we do not take into account the *regret effect*. Let us now consider a signal S which has the particular feature of not modifying his optimal strategy (if the choice were still made on the basis of the c-utility function). Whatever the value s of the signal S, X would remain the optimal choice. This can be written as

$$\forall s, \forall Y_n \in \Phi, E\left[u\left(x,x\right)\right|s] \geq E\left[u\left(y_n,y_n\right)\right|s] \tag{14}$$

Moreover, since the signal does not modify the optimal strategy, the expected c-utility when the agent anticipates obtaining the signal is equal to his expected c-utility when there is no signal. As the c-utility function behaves like the VNM utility function, this is tantamount to saying that we have a signal that would have no value under VNM axioms.

Let us assume now that, when there is no signal, X is also the optimal strategy according to the r-utility function criterion and that

$$E\left[u\left(x,r^{x}\right)\right]=E\left[u\left(x,x\right)\right] \tag{15}$$

Strategy X is a *dominant strategy*: choosing X ensures having no regret. Under an uninformative information structure, this assumption does not necessarily imply that X always offer the highest payoff. What it does signify is that X is always the *ex post* best strategy given the r-individual's information at the learning stage. When the information structure is uninformative, the information, at the learning stage, is limited to the observation of the chosen strategy payoff.

We obtain the following proposition:

Proposition 6 *With flexibility and an uniformative information structure, if* X *is a dominant strategy, and* S *is information that would have no value under VNM axioms, then* $V(S) \leq 0$.

Proof. *The proof uses P2b. See Appendix* 3. ■

Let $X_s \in \Phi$ denote the chosen strategy when the value of the signal is s, and x_s a realization of X_s. Let $E\left[u\left(x_s, r^{x_s,s}\right)\right]$ denote the expected r-utility under flexibility. The above proposition states that $E\left[u\left(x_s, r^{x_s,s}\right)\right]$ is lower than $E\left[u\left(x,r^{x}\right)\right] = E\left[u\left(x,x\right)\right]$. Once again, our result differs from what is obtained under VNM axioms, where information value cannot be negative. This result might also seem somewhat surprising, because flexibility allows an individual to use information in an optimal way. In order to illustrate the above proposition and understand its underlying mechanisms we give, in what follows, an example in which the value of information is negative under flexibility assumption. We should stress that, in this example, the r-individual uses the information. He chooses, in an optimal way, his strategy conditionally to the signal value (X is not always optimal). Although, under VNM axioms, the information would be of no use, a regretful agent adapts his strategy to the signal. However, despite its usefulness for a regretful agent, the signal is globally harmful.

Example 4 *Let us consider a set of two independent risky alternatives,* $\Phi = \{X, Y\}$*. The risky alternative* X *takes the value* 1*, and the value* 2*, with equal probabilities. The risky alternative* Y *takes the value* 0.5*, and the value* 1.4 *with equal probabilities. We consider an uninformative information structure in which the agent does not observe the realization of the foregone alternative at the learning stage. Column* † *in the table below shows that strategy* X *is the optimal strategy. We then consider a perfect signal on strategy* Y*. The agent receives the signal at the decision stage and uses it to determine his best choice. Our results are given below:*

Z	$E[u(z,z)]^*$	EC_Z^{**}	$E[u(z,r^z)]^\dagger$	$E[u(z_s, r^{z_s,s})]$
X	-0.487	1.438	-0.487	-0.497
Y	-0.638	0.900	-0.876	

Comparison between the expected r-utility (i) with the signal $E\left[u\left(z_s, r^s_{z_s}\right)\right]$ *and (ii) without signal* $E[u(x, r^x)]$ *shows that, in this example, information value is negative, even if there is flexibility. Without the signal,* X *is the optimal choice. The agent, when he chooses* X*, does not expect to feel regret because* X *is always higher than the certainty equivalent of* Y*. As strategy* X *is a dominant strategy, the expected r-utility is equal to the expected c-utility:* -0.487*.*

Now when the agent receives a perfect signal on Y*, computation establishes that strategy* X *remains optimal when* $Y = 0.5$*. The agent still does not feel any regret and his expected r-utility is the same as before, that is to say* -0.487*. Thus, everything depends on what happens when* $Y = 1.4$*. When the agent learns that* $Y = 1.4$*, choosing* X *can now expose him to some regret because* X

can be lower than Y. This is the regret effect of the signal. Even if the signal is about strategy Y, it affects the expected r-utility obtained from strategy X. We find that the expected r-utility from choosing X decreases, and computation gives Y as the optimal choice. However, we find that the r-utility from choosing $Y = 1.4$ is equal to -0.506, which is lower than -0.487. This means that, if the expected r-utility obtained from X had not decreased, Y would not have become optimal. The probability effect of the signal is not sufficient, in itself, to make Y optimal. The strength of regret effect explains why the r-individual switches from strategy X to strategy Y, while the weakness of the probability effect explains why this switching results in a decrease of the utility.

To summarize: when $Y = 0.5$, strategy X remains optimal, and the level of utility is the same as before (when the agent does not receive the signal). When $Y = 1.4$, strategy Y becomes optimal, but the level of utility is lower than before. On average, the expected r-utility when the agent anticipates obtaining the signal is equal to $\frac{1}{2}(-0.487) + \frac{1}{2}(-0.506) = -0.497 < -0.487$. We conclude that, under flexibility, the expected r-utility of the agent, who anticipates receiving a perfect signal on Y, is lower than his expected r-utility without information. The information value is negative. The details of the computation are given in Appendix 4.

The above example allows us to make a comment about our modeling of regret. When the signal gives $Y = 1.4$, the r-utility obtained from choosing Y is written as follows (see Appendix 4):

$$E\left[u\left(y, r^{y,s}\right)\middle| s\right] = -e^{-1.4}e^{\frac{1.438}{2}} = -0.506 \tag{16}$$

Since $Y = 1.4$ is lower than $EC_X = 1.438$, the reference point, in the expression of the r-utility, is equal to EC_X. At first sight, a reference point higher than Y expresses regret. The r-utility obtained from choosing Y is lower than the c-utility from choosing Y. But we know that strategy Y is riskless (Y is equal to 1.4), and cannot generate some regret, when the outcome of the foregone strategy is not observable. When the r-individual chooses Y, he knows the exact value of Y. Moreover, he learns nothing new at the learning stage. Thus, there is no reason to feel any regret at having chosen Y. The r-utility is lower than the c-utility for another reason: choosing is painful because it implies giving up some opportunities. When there is no possibility of choice, the r-utility is equal to the c-utility. This level of utility represents the pure satisfaction of receiving a gain equal to 1.4. But, when an r-individual has the choice between $Y = 1.4$ and strategy X, even if he chooses Y, his r-utility is lower because he knows that he might have obtained a higher payoff with strategy X. The reference point here does not reflect a feeling of regret, but illustrates the fact that receiving $Y = 1.4$, or choosing $Y = 1.4$, does not generate the same satisfaction. In order to clarify this point, let us take a simple example. Imagine that we receive 100€. Obviously, we are happy about that. Now, imagine that we have the choice between receiving 100€ and playing in a lottery where we can earn 1000€ or nothing. If we choose to receive 100€ we are happy, but our level of satisfaction is lower than before because we know that we might, if we had chosen the lottery, have earned 1000€.

6 Conclusion

Using the utility function proposed by Quiguin (1994), we have proposed a general model of regretful preferences and confronted the usual regret utility functions to this model. We have highlighted some characteristics that these utility functions require in order to be in conformity with our preferences model. Moreover, we have emphasized that information is a key concept in regret theory, and have developed a model of regret which accomodates any information structure. Using the criterion of Blackwell (1951), we have classified the information structures according to a regretful individual's preferences. We have shown that he prefers a coarser information structure to a finer one. Our framework has also served as a basis for studying the concept of information value when agents are regretful. We have shown that information value is always negative when there is no flexibility. We have also shown that information value can be negative under flexibility.

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Appendix 1

We can rewrite the reference point (see Definition 1) as

$$r^{x,s_x} = Max\left\{x, EC_{Max}^{x,s_x}\right\} \tag{17}$$

with $EC_{Max}^{x,s_x} = Max\left\{EC_{Y_1}^{x,s_x}, .., EC_{Y_n}^{x,s_x}, .., EC_{Y_N}^{x,s_x}\right\}$ a random variable which fluctuates with S_x when x is given.

In order to demonstrate Proposition 3, we must show the following inequality:

$$E\left[u\left(x, Max\left(x, EC_{Max}^{x,s_x^2}\right)\right)\right] \leq E\left[u\left(x, Max\left(x, EC_{Max}^{x,s_x^1}\right)\right)\right] \tag{18}$$

First, note that $\forall x \in W_X, \forall n = 1...N$,

$$u\left(EC_{Y_n}^{x,s_x^2}, EC_{Y_n}^{x,s_x^2}\right) = E\left[\left.u\left(y_n, y_n\right)\right| x, s_x^2\right] \tag{19}$$

Thus

$$E\left[\left.u\left(EC_{Y_n}^{x,s_x^2}, EC_{Y_n}^{x,s_x^2}\right)\right| x, s_x^1\right] = E\left\{\left. E\left[\left.u\left(y_n, y_n\right)\right| x, s_x^2\right]\right| x, s_x^1\right\} \tag{20}$$

Now, IS^2 being X-finer than IS^1, we have $E\left\{\left. E\left[\left.u\left(y_n, y_n\right)\right| x, s_x^2\right]\right| x, s_x^1\right\} = E\left[\left.u\left(y_n, y_n\right)\right| x, s_x^1\right]$ and thus

$$E\left[\left.u\left(EC_{Y_n}^{x,s_x^2}, EC_{Y_n}^{x,s_x^2}\right)\right| x, s_x^1\right] = E\left[\left.u\left(y_n, y_n\right)\right| x, s_x^1\right] \tag{21}$$

Now, since $E\left[u\left(y_n,y_n\right)\middle|\, x,s_x^1\right]=u\left(EC_{Y_n}^{x,s_x^1},EC_{Y_n}^{x,s_x^1}\right)$, we obtain

$$E\left[u\left(EC_{Y_n}^{x,s_x^2},EC_{Y_n}^{x,s_x^2}\right)\middle|\, x,s_x^1\right]=u\left(EC_{Y_n}^{x,s_x^1},EC_{Y_n}^{x,s_x^1}\right) \tag{22}$$

Now, $P1b$ implies that

$$E\left[u\left(EC_{Y_n}^{x,s_x^2},EC_{Y_n}^{x,s_x^2}\right)\middle|\, x,s_x^1\right]\leq u\left(E\left(EC_{Y_n}^{x,s_x^2}\middle|\, x,s_x^1\right),E\left(EC_{Y_n}^{x,s_x^2}\middle|\, x,s_x^1\right)\right) \tag{23}$$

Thus, we finally obtain that

$$u\left(EC_{Y_n}^{x,s_x^1},EC_{Y_n}^{x,s_x^1}\right)\leq u\left(E\left(EC_{Y_n}^{x,s_x^2}\middle|\, x,s_x^1\right),E\left(EC_{Y_n}^{x,s_x^2}\middle|\, x,s_x^1\right)\right) \tag{24}$$

And $P1a$ implies that $\forall x\in W_X,\forall n=1...N$,

$$EC_{Y_n}^{x,s_x^1}\leq E\left(EC_{Y_n}^{x,s_x^2}\middle|\, x,s_x^1\right) \tag{25}$$

Let us put this result aside and come back to it later.

Secondly, we note that

$$E\left[u\left(x,Max\left(x,EC_{Max}^{x,s_x^2}\right)\right)\right]=E\left\{E\left[u\left(x,Max\left(x,EC_{Max}^{x,s_x^2}\right)\right)\middle|\, x,s_x^1\right]\right\} \tag{26}$$

Thus $P4b$ implies that

$$E\left[u\left(x,Max\left(x,EC_{Max}^{x,s_x^2}\right)\right)\right]\leq E\left[u\left(x,E\left(Max\left(x,EC_{Max}^{x,s_x^2}\right)\middle|\, x,s_x^1\right)\right)\right] \tag{27}$$

But as function $Max\left(x,.\right)$ is convex when x is given, we also have

$$E\left(Max\left(x,EC_{Max}^{x,s_x^1}\right)\middle|\, x,s_x^2\right)\geq Max\left(x,E\left(EC_{Max}^{x,s_x^1}\middle|\, x,s_x^2\right)\right) \tag{28}$$

Thus $P2b$ implies

$$E\left[u\left(x, Max\left(x, EC_{Max}^{x,s_x^2}\right)\right)\right] \leq E\left[u\left(x, Max\left(x, E\left(EC_{Max}^{x,s_x^2}\middle| x, s_x^1\right)\right)\right)\right] \quad (29)$$

Equation (25) and $P2b$ allow us to conclude that

$$E\left[u\left(x, Max\left(x, EC_{Max}^{x,s_x^2}\right)\right)\right] \leq E\left[u\left(x, Max\left(x, EC_{Max}^{x,s_x^1}\right)\right)\right] \quad (30)$$

If X denotes the optimal strategy under IS^1, we have shown here that switching from IS^1 to IS^2 decreases the expected r-utility that the r-individual obtains from strategy X. Moreover, even if choosing another strategy becomes optimal for him, this will not let him have the same expected utility as under IS^1. If X denotes the optimal strategy under IS^2, we have shown here that switching from IS^2 to IS^1 increases the expected r-utility that the r-individual obtains from strategy X. In both cases, the r-individual prefers IS^1 to IS^2.

The proof of Proposition 4 is identical. IS^2 being finer than IS^1, we have $\forall X \in \Phi$, $E\left\{ E\left[u\left(y_n, y_n\right)\middle| x, s_x^2\right]\middle| x, s_x^1\right\} = E\left[u\left(y_n, y_n\right)\middle| x, s_x^1\right]$ and thus we finally obtain

$$\forall X \in \Phi, E\left[u\left(x, Max\left(x, EC_{Max}^{x,s_x^2}\right)\right)\right] \leq E\left[u\left(x, Max\left(x, EC_{Max}^{x,s_x^1}\right)\right)\right] \quad (31)$$

The maximum expected utility that any r-individual can reach under IS^2 is lower than under IS^1.

Appendix 2

First, we compute the expected c-utilities of X and Y:

$$E\left[u\left(x,x\right)\right] = -\frac{1}{2}\left[e^{-\frac{1}{2}} + e^{-\frac{2}{2}}\right] = -0.48720505$$

$$E\left[u\left(y,y\right)\right]=-\frac{1}{2}\left[e^{-\frac{0.8}{2}}+e^{-\frac{2.5}{2}}\right]=-0.478412421$$

From that, we can easily compute that $EC_X = 1.438140393$ and $EC_Y = 1.47456422$.

The expected r-utilities under imperfect information are

$$E\left[u\left(x,r^x\right)\right]=-\frac{1}{2}\left[e^{-2}e^{\frac{2}{2}}+e^{-1}e^{\frac{1.47456422}{2}}\right]=-0.56841912$$

$$E\left[u\left(y,r^y\right)\right]=-\frac{1}{2}\left[e^{-2.5}e^{\frac{2.5}{2}}+e^{-0.8}e^{\frac{1.438140393}{2}}\right]=-0,604381613$$

Under imperfect information, the agent prefers strategy X.

Let us now consider the situation in which the agent obtains a perfect signal on Y at the learning stage. At the learning stage, the agent knows both the realization of X and the realization of Y. Thus, for each couple of values (x, y), the reference point is $r^{x,s} = Max\left(x, y\right)$ and the expected r-utility from choosing X becomes

$$E\left[u\left(x,r^{x,s}\right)\right]=-\frac{1}{4}\left[e^{-2}e^{\frac{2}{2}}+e^{-2}e^{\frac{2.5}{2}}+e^{-1}e^{\frac{2.5}{2}}+e^{-1}e^{\frac{1}{2}}\right]=-0.682700518 \tag{32}$$

The expected r-utility from choosing Y is unchanged since there is no signal on X:

$$E\left[u\left(y,r^{y,s}\right)\right]=-0.604381613 \tag{33}$$

We thus have $E\left[u\left(x,r^{x,s}\right)\right] < E\left[u\left(y,r^{y,s}\right)\right]$. Anticipating the signal on strategy Y, the agent changes his strategy from X to Y in order to insure himself against the risk on the reference point generated by the signal.

Appendix 3

We should recall that X denotes the optimal strategy when there is no signal. Moreover, we assume that X does not generate *ex post* regret (See Equation 15). We also assume that, without the *regret effect*, X would remain the optimal strategy whatever the signal value (See Equation 14).

Let us now consider a regretful individual. Let Ω denote the set in which signal S takes its value. Let $\Omega_1 \subset \Omega$ denote the subset containing the value of S such that X remains optimal ($\forall s \in \Omega_1,\ X_s = X$).

Since $r^{x,s} \geq x$, $P2b$ implies that

$$E\left[u\left(x, r^{x,s}\right)\middle| s\right] \leq E\left[u\left(x, x\right)\middle| s\right] \tag{34}$$

We thus have

$$\forall s \in \Omega_1, E\left[u\left(x_s, r^{x_s,s}\right)\middle| s\right] \leq E\left[u\left(x, x\right)\middle| s\right] \tag{35}$$

Let $\Omega_2 \subset \Omega$ denote the subset containing the value of S such that X is no longer optimal. Let Y_{ns} denote the optimal startegy ($\forall s \in \Omega_2,\ X_s = Y_{ns}$).

Since $r^{y_{ns},s} \geq y_{ns}$, $P2b$ implies that

$$E\left[u\left(y_{ns}, r^{y_{ns},s}\right)\middle| s\right] \leq E\left[u\left(y_{ns}, y_{ns}\right)\middle| s\right] \tag{36}$$

The above equation and Equation (14) imply that

$$\forall s \in \Omega_2, E\left[u\left(x_s, r^{x_s,s}\right)\middle| s\right] \leq E\left[u\left(x, x\right)\middle| s\right] \tag{37}$$

Equations (35) and (37) imply that

$$\forall s \in \Omega, E\left[u\left(x_s, r^{x_s,s}\right)\middle| s\right] \leq E\left[u\left(x, x\right)\middle| s\right] \tag{38}$$

Thus

$$E\left[u\left(x_s, r^{x_s,s}\right)\right] \leq E\left[u\left(x, x\right)\right] \tag{39}$$

The expected r-utility, when the agent anticipates the signal, is lower than his expected r-utility without the signal. Thus, under $P1a$, there exists $v \leq 0$ such that

$$E\left[u\left(x_s, r^{x_s,s}\right)\right] = E\left[u\left(x+v, x+v\right)\right] \tag{40}$$

The information value is negative.

Appendix 4

First, we compute the expected c-utilities and certainty equivalents of X and Y:

$$E\left[u\left(x, x\right)\right] = -\frac{1}{2}\left[e^{-\frac{1}{2}} + e^{-\frac{2}{2}}\right] = -0.487205 \text{ and } EC_X = 1.4381404 \tag{41}$$

$$E\left[u\left(y, y\right)\right] = -\frac{1}{2}\left[e^{-\frac{0.5}{2}} + e^{-\frac{1.4}{2}}\right] = -0.637693 \text{ and } EC_Y = 0.8997964 \tag{42}$$

The expected r-utilities when there is no signal are

$$E\left[u\left(x, r^x\right)\right] = -\frac{1}{2}\left[e^{-1}e^{\frac{1}{2}} + e^{-2}e^{\frac{2}{2}}\right] = -0.487205 \tag{43}$$

$$E\left[u\left(y, r^y\right)\right] = -\frac{1}{2}\left[e^{-0.5}e^{\frac{1.4381404}{2}} + e^{-1,4}e^{\frac{1.4381404}{2}}\right] = -0.8755324 \tag{44}$$

As can be seen, whatever the value of Y, the agent feels some regret because Y is always lower than EC_X. On the contrary, the agent does not feel regret with strategy X. Lottery X is chosen under imperfect information, since $E\left[u\left(x, r^x\right)\right] > E\left[u\left(y, r^y\right)\right]$.

Let us assume that, at the decision stage, the agent receives a perfect signal on strategy Y. He chooses strategy X_s, which maximizes his expected r-utility, given the value of the signal.

When the agent learns that $y = 0.5$, the expected r-utilities become

$$E\left[u\left(x, r^{x,s}\right)\middle| s\right] = -\frac{1}{2}\left[e^{-1}e^{\frac{1}{2}} + e^{-2}e^{\frac{2}{2}}\right] = -0.487205 \tag{45}$$

$$E\left[u\left(y, r^{y,s}\right)\middle| s\right] = -e^{-0.5}e^{\frac{1.4381404}{2}} = -1.2449187 \tag{46}$$

Thus, when $y = 0.5$, $X_s = X$.

When the agent learns that $y = 1.4$, the expected r-utilities become

$$E\left[u\left(x, r^{x,s}\right)\middle| s\right] = -\frac{1}{2}\left[e^{-1}e^{\frac{1.4}{2}} + e^{-2}e^{\frac{2}{2}}\right] = -0.5543488 \tag{47}$$

$$E\left[u\left(y, r^{y,s}\right)\middle| s\right] = -e^{-1.4}e^{\frac{1.4381404}{2}} = -0.5061461 \tag{48}$$

Thus, when $y = 1.4$, $X_s = Y$. Learning that $y = 1.4$ increases utility obtained from strategy Y, and decreases utility obtained from strategy X (when $x = 1$, the agent feels regret because $x < 1.4$).

Before receiving the signal, the expected r-utility is thus

$$E\left[u\left(x_s, r^{x_s,s}\right)\right] = \frac{1}{2}\left[-0.487205 - 0.5061461\right] = -0.4966755 < E\left[u\left(x, r^x\right)\right] \tag{49}$$

Under flexibility, the expected r-utility when the agent anticipates perfect information about Y is lower than when he anticipates not having information about Y. The information value is, therefore, negative.

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