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Curvature-Constrained Estimates of Technical Efficiency and Returns to Scale for U.S. Electric Utilities

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Abstract: We estimate an input distance function for U.S. electric utilities under the assumption that non-negative variables associated with technical inefficiency are time-invariant. We use Bayesian methodology to impose curvature restrictions implied by microeconomic theory and obtain exact finite-sample results for nonlinear functions of the parameters (eg. technical efficiency scores). We find that Bayesian point estimates of elasticities are more plausible than maximum likelihood estimates, technical efficiency scores from a random effects specification are higher than those obtained from a fixed effects model, and there is evidence of increasing returns to scale in the industry.

Keywords: Input Distance Function, Stochastic Frontier, Bayes, Markov Chain Monte Carlo

1. Introduction

The stochastic frontier models proposed independently by Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977) provide the foundation for the econometric measurement of technical inefficiency. These early models were estimated using cross-section data under the assumption that non-negative random variables representing technical inefficiency were half-normally or exponentially distributed. Subsequently, efficiency and productivity researchers have considered several alternative distributional assumptions (see Stevenson , 1980; Greene, 1990) and developed a range of models for use with panel data (see Pitt and Lee, 1981; Schmidt and Sickles, 1984). Some panel data models allow technical inefficiency effects to vary across firms but not over time; others allow technical inefficiency effects to vary both across firms and over time. Empirical examples of these panel data models include Battese and Coelli (1988), Battese, Coelli and Colby (1989), Cornwell, Schmidt and Sickles (1990), Kumbhakar (1990), Battese and Coelli (1992) and Lee and Schmidt (1993).

Most of this empirical work has been conducted in a sampling theory (or frequentist) estimation framework (esp. maximum likelihood). More recently, econometricians have begun to estimate frontier models in a Bayesian framework. A comprehensive introduction to Bayesian methods in econometrics is provided by Koop (2003) and a comparison of sampling theory and Bayesian approaches to inference is provided by Poirier (1995). For stochastic frontier researchers, a particular advantage of the Bayesian approach is that it can provide exact finite sample results for nonlinear functions of the unknown parameters (eg. technical efficiency scores). In addition, Bayesians have formal mechanisms for incorporating non-sample information (eg. curvature restrictions implied by economic theory) into the estimation process.

Until recently, the Bayesian approach has not been widely used for estimating stochastic frontiers, partly because it involves the evaluation of analytically intractable integrals. However, recent advances in computer power and simulation methods for evaluating integrals have led to renewed interest in the approach. The most important algorithms used for stochastic frontier analysis are Markov Chain Monte Carlo (MCMC) algorithms such as the Gibbs sampler. MCMC algorithms have been used to estimate stochastic frontier models by van den Broeck et al (1994), Koop, Steel, Osiewalski (1995), Koop, Osiewalski and Steel (1997), Osiewalski and Steel (1998) and Koop and Steel (2001). More recently, O'Donnell and Coelli (2004) have shown how MCMC methods can be used to impose regularity conditions on the parameters of a translog output distance function.

Distance functions are one way of representing a well-behaved multi-input multi-output production technology. Empirical researchers find them useful because they do not require behavioral assumptions such as cost minimization or profit maximization. Among other things, this means distance functions can be used to represent the production technologies of most network industries (eg. utilities and transport) where regulation may make such assumptions inappropriate. Two types of distance function have received significant attention in the literature: an input distance function describes the maximum proportional contraction of the input vector that is possible without changing the output vector; an output distance function describes the degree to which a firm can expand its output vector, given an input vector. Distance functions have been estimated by Färe et al. (1993), Grosskopf et al. (1995), Coelli and Perelman (1999), Atkinson and Primont (2002) and O'Donnell and Coelli (2004), among others. In this paper we estimate an input distance function for U.S. electric utilities. An input distance function is estimated instead of an output distance function because electric utilities tend to adjust their inputs to produce exogenously-determined levels of electricity output.

Electricity is generated in the U.S. using a variety of different technologies and fuels. According to the Energy Information Administration (EIA), steam electric power generation accounted for 61 percent of U.S. electricity generated in 1998. Nuclear power generation is the second largest sector of the industry and accounts for nearly 19 percent of generation capacity. The remaining power generation is from hydroelectric (9 percent) and non-utility (12 percent) sources.

In recent years, Atkinson and Primont (2002) and Rungsuriyawiboon and Stefanou (2003) have used data from major investor-owned utilities to estimate the cost structure and other economic characteristics of steam electric power generators. Economic characteristics of nuclear power generators have been estimated by Krautmann and Solow (1988), Marshall and Navarro (1991) and Canterbery, Johnson, and Reading (1996), but these studies are somewhat dated and/or relatively limited in scope. In this paper we use panel data from 26 utilities over the period 1989 to 1998 to estimate characteristics of the nuclear power generation technology, including estimates of technical inefficiency. Thus, the results reported in this study complement the estimates reported by Atkinson and Primont (2002) and Rungsuriyawiboon and Stefanou (2003). Our study is timely because electricity restructuring and deregulation is now on the policy agenda in most states of the U.S.

The structure of the paper is as follows. We begin by defining the input distance function and its theoretical properties. We then consider the translog input distance function and the parametric restrictions implied by the properties of homogeneity, monotonicity and curvature. Next we consider Bayesian methods for estimating the model under the assumptions that variables representing technical inefficiency are either fixed or random. We then describe the data before presenting the empirical results. In the concluding section we make some comments on the benefits of using a Bayesian approach.

2. The Input Distance Function

We consider a multi-input, multi-output production technology where a firm uses a non-negative $K \times 1$ input vector $X = (X_1,...,X_K)'$ to produce a non-negative $M \times 1$ output vector $Q = (Q_1,...,Q_M)'$. The set of all technologically feasible input-output combinations is S = {(X, Q): X can produce Q}. We assume this production technology satisfies the standard properties discussed in Färe and Primont (1995).

The production technology can also be described in terms of input sets and distance functions. Specifically, the input set $L(Q)$ is the set of all input vectors, X, that can produce the output vector, Q:

$$L(Q) = \{ X : X \text{ can produce } Q\}. \tag{1}$$

The input distance function can be defined in terms of this input set as

$$D^I(X,Q) = \max\{\rho : (X/\rho) \in L(Q)\} \tag{2}$$

and gives the maximum proportional reduction in inputs that is possible without changing the output vector. Both the input set and the input distance function summarize all the economically-relevant characteristics of the production technology.

It is clear that if $X \in L(Q)$ then $D^I(X,Q) \geq 1$. Moreover, $D^I(X,Q) = 1$ if X belongs to the 'frontier' of the input set. Finally, the input distance function is non-decreasing, linearly homogenous and concave in X, and non-increasing and quasi-concave in Q. In this paper we aim to estimate the parameters of a translog input distance function in a manner consistent with these properties.

3. The Translog Functional Form

To estimate the parameters of an input distance function we must first specify a functional form. The translog functional form is a quadratic in logarithms and can provide a second-order approximation to an arbitrary functional form. When defined over *K* inputs and *M* outputs the translog input distance function is:

$$\ln D^I = \beta_0 + \sum_{k=1}^{K} \beta_k \ln X_k + \frac{1}{2}\sum_{k=1}^{K}\sum_{l=1}^{K} \beta_{kl} \ln X_k \ln X_l + \sum_{m=1}^{M} \alpha_m \ln Q_m + \frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{M} \alpha_{mn} \ln Q_m \ln Q_n + \sum_{k=1}^{K}\sum_{m=1}^{M} \gamma_{km} \ln X_k \ln Q_m \quad (3)$$

where the αs, βs and γs are unknown parameters to be estimated. The second-order parameters satisfy the identifying restrictions $\beta_{kl} = \beta_{lk}$ and $\alpha_{kl} = \alpha_{lk}$ for all *k* and *l* (these restrictions also ensure the function satisfies the symmetry restrictions implied by Young's Theorem).

4. Homogeneity, Monotonicity and Curvature Constraints

As we have seen, the input distance function is non-decreasing, linearly homogenous and concave in *X*, and non-increasing and quasi-concave in *Q*. These properties imply a number of equality and inequality constraints on the unknown parameters in the translog function (3).

Sufficient conditions for the function (3) to be homogeneous of degree one in inputs are

$$\sum_{k=1}^{K} \beta_k = 1; \ \sum_{k=1}^{K} \beta_{kl} = 0 \text{ for all } l\text{; and } \sum_{k=1}^{K} \gamma_{km} = 0 \text{ for all } m. \quad (4)$$

These are equality constraints on the unknown parameters and, because they do not involve the data, they are observation-invariant. In contrast, the non-increasing and non-decreasing properties of the distance function (ie. monotonicity properties) will be satisfied if and only if

$$\frac{\partial d^I}{\partial x_k} = \beta_k + \sum_{l=1}^{K} \beta_{kl} x_l + \sum_{m=1}^{M} \gamma_{km} q_m \geq 0 \quad (5)$$

and $$\frac{\partial d^I}{\partial q_m} = \alpha_m + \sum_{n=1}^{M} \alpha_{mn} q_n + \sum_{k=1}^{K} \gamma_{km} x_k \leq 0 \qquad (6)$$

where $d^I = \ln D^I$, $x_k = \ln X_k$ and $q_m = \ln Q_m$. These are inequality constraints involving both the parameters and the data, so they are observation-varying.

For the concavity property we let $H = \left[h_{kl} \right]$ be the Hessian matrix of the translog input distance function, with elements

$$h_{kl} \equiv \frac{\partial^2 D^I}{\partial X_k \partial X_l} = \left(\frac{D^I}{X_k X_l} \right) \left[\beta_{kl} + \frac{\partial d^I}{\partial x_k} \frac{\partial d^I}{\partial x_l} - \delta_{kl} \frac{\partial d^I}{\partial x_k} \right] \qquad (7)$$

where $\delta_{kl} = 1$ if $k = l$ and 0 otherwise. The function (3) will be concave in inputs over the nonnegative orthant if and only if H is negative semi-definite. In turn, H will be negative semi-definite if and only if every principal minor of odd order is non-positive and every principal minor of even order is non-negative (Simon and Blume, 1994, p.383).

Finally, for the quasi-concavity property we define the bordered Hessian matrix

$$F = \begin{bmatrix} 0 & f_1 & \cdots & f_M \\ f_1 & f_{11} & \cdots & f_{1M} \\ \vdots & \vdots & \cdots & \vdots \\ f_M & f_{1M} & \cdots & f_{MM} \end{bmatrix} \qquad (8)$$

where

$$f_m \equiv \frac{\partial D^I}{\partial Q_m} = \frac{D^I}{Q_m} \frac{\partial d^I}{\partial q_m} \qquad (9)$$

and $$f_{mn} \equiv \frac{\partial^2 D^I}{\partial Q_m \partial Q_n} = \left(\frac{D^I}{Q_m Q_n} \right) \left[\alpha_{mn} + \frac{\partial d^I}{\partial q_m} \frac{\partial d^I}{\partial q_n} - \delta_{mn} \frac{\partial d^I}{\partial q_m} \right] \qquad (10)$$

The function (3) will be quasi-concave in outputs over the nonnegative orthant if the first principal minor of F is negative and the remaining principal minors alternate in sign (Simon and Blume, 1994, p.530).

5. Elasticities of Scale and Substitution

Atkinson and Primont (2002) consider the duality between cost and input distance functions and define two quantities of economic interest. First, they show that a measure of returns to scale can be obtained from the input distance function as

$$RTS = -\frac{1}{\nabla_Q D^I(X,Q)'Q} \tag{11}$$

where $\nabla_Q D^I(X,Q)$ is the vector of first derivatives of the distance function with respect to outputs. Values of the RTS greater than one imply increasing returns to scale, values less than one imply decreasing returns to scale, while a value of one implies constant returns to scale.

Second, the first partial derivative of the log-distance with respect to the *k*-th log-input:

$$\frac{\partial d^I}{\partial x_k} = \frac{X_k}{D^I}\frac{\partial D^I}{\partial X_k} \tag{12}$$

can be interpreted as an implicit input value share for the *k*-th input. Ratios of these derivatives provide a unit-less measure of input substitutability.

6. Estimation

For estimation purposes it is convenient to impose the homogeneity constraints (4) by normalizing all inputs by the *K*-th input. Then the translog input distance function (3) can be rewritten as

$$\begin{aligned} -x_{Kit} = \beta_0 + \sum_{k=1}^{K-1}\beta_k x^*_{kit} + \frac{1}{2}\sum_{k=1}^{K-1}\sum_{l=1}^{K-1}\beta_{kl}x^*_{kit}x^*_{lit} + \sum_{m=1}^{M}\alpha_m q_{mit} + \frac{1}{2}\sum_{m=1}^{M}\sum_{n=1}^{M}\alpha_{mn}q_{mit}q_{nit} \\ + \sum_{k=1}^{K-1}\sum_{m=1}^{M}\gamma_{km}x^*_{kit}q_{mit} - u_{it}. \end{aligned} \tag{13}$$

where $x^*_{kit} = \ln\left(X_{kit}/X_{Kit}\right)$ is the k-th normalized input, $u_{it} = d^I_{it}$ is a non-negative term capturing the effects of technical inefficiency, and the subscripts i and t have been introduced to index firms and time periods respectively (i = 1, ..., N; t = 1, ..., T).

In this paper we assume $u_{it} = u_i$ (ie. the technical inefficiency effects are time-invariant) and write (13) in the more compact form:

$$y_{it} = \beta_0 + z'_{it}\phi + v_{it} - u_i \,, \tag{14}$$

where $y_{it} = -x_{Kit}$, z_{it} is a $P \times 1$ vector of the logarithms of normalized inputs and outputs and their cross-products, ϕ is $P \times 1$ vector of parameters, and $v_{it} \sim N\left(0, h^{-1}\right)$ is an independent normally distributed random error term introduced to represent statistical noise. Equation (14) is recognizable as a standard stochastic frontier model for panel data with time-invariant technical inefficiency effects. Models of this type are usually estimated under the assumption that the u_i are either fixed parameters (the so-called *fixed effects model*) or random variables (the *random effects model*).

Irrespective of whether the inefficiency effects are treated as fixed or random, the parameters of the distance function can be estimated in either a sampling theory or Bayesian framework. The optimal sampling theory point estimates are usually those that maximize the value of the likelihood function, while the optimal Bayesian point estimates are usually the means of posterior probability density functions (pdfs) that summarize all the post-sample information we have about the unknown parameters. Formally, Bayes's Theorem states that

$$p(\theta|\text{data}) \propto p(\text{data}\,|\theta)\, p(\theta) \tag{15}$$

where $p(\theta|\text{data})$ is the posterior pdf of the unknown parameter vector θ, $p(\text{data}\,|\theta)$ is the familiar likelihood function, $p(\theta)$ is a prior pdf summarizing all our pre-sample knowledge about θ, and $\propto$ is notation for "is proportional to". The joint posterior pdf is the main focus of Bayesian inference – it underpins the evaluation of competing hypotheses and point and interval estimation of all parameters. For example, when interest centres on a single element of θ, say θ_i, we obtain its marginal posterior pdf by integrating the joint posterior pdf with respect to all elements of θ other than θ_i. The mean of this marginal posterior pdf is the optimal Bayesian point estimate of the parameter (under quadratic loss).

Integrating multivariate joint posterior pdfs is often analytically difficult, if not impossible. In practice, this obstacle is overcome using Markov Chain Monte Carlo (MCMC) simulation techniques. Specifically, an MCMC posterior simulator is used to generate draws from the posterior, $p(\theta|\text{data})$, and this sample of draws is then used to estimate any interesting characteristics of the marginal pdfs (eg, means, standard errors). Details of particular MCMC simulators will be provided below.

The Fixed Effects Model

If the inefficiency component in equation (14) is treated as a fixed parameter then the T observations on the i-th firm can be compactly written:

$$y_i = \psi_i j_T + Z_i'\phi + v_i \tag{16}$$

where $\psi_i = \beta_0 - u_i$ is the i-th individual effect, $y_i = \left(y_{i1},...,y_{iT}\right)'$ and $v_i = \left(v_{i1},...,v_{iT}\right)'$ are $T\times 1$, $Z_i = \left(z_{i1},...,z_{iT}\right)'$ is $T\times P$, and j_T is a $T\times 1$ vector of ones. Moreover, the full set of NT observations can be written:

$$y = \left(I_N \otimes j_T\right)\psi + Z\phi + v = W\theta + v\ . \tag{17}$$

where $y = \left(y_1',...,y_N'\right)'$ and $v = \left(v_1',...,v_N'\right)'$ are $NT\times 1$, $Z = \left(Z_1',...,Z_N'\right)'$ is $NT\times P$, $W = \left(I_N \otimes j_T, Z\right)$ is $NT\times\left(N+P\right)$, $\psi = \left(\psi_1',...,\psi_N'\right)'$ is $N\times 1$ and $\theta = \left(\psi',\phi'\right)'$ is $\left(N+P\right)\times 1$.

The sampling theory approach to estimating the parameters of this fixed effects model is quite straightforward – sampling theorists simply find the parameter values that maximize the likelihood function:

$$p(y|\theta,h) \propto h^{NT/2} \times \exp\left\{-0.5h\left[\upsilon\hat{h}^{-1} + \left(\theta-\hat{\theta}\right)' W'W\left(\theta-\hat{\theta}\right)\right]\right\} \tag{18}$$

where $\hat{\theta} = (W'W)^{-1}W'y$ is the least squares estimator, $\hat{h}^{-1} = \left[\left(y-W\hat{\theta}\right)'\left(y-W\hat{\theta}\right)\right]\Big/\upsilon$ and $\upsilon = N(T-1)-P$. The maximum likelihood estimator of θ is clearly the least squares estimator.

Bayesian estimation of the model is slightly more complicated. In this paper we adopt the prior pdfs:

$$p(\theta) \propto I(\theta \in R) \tag{19}$$

and $$p(h) \propto h^{-1} \tag{20}$$

where $I(\theta \in R)$ is an indicator function and R is the region of the parameter space where the economic regularity constraints implied by economic theory hold. Thus, $I(\theta \in R)$ will take the value one if the stochastic input distance function satisfies the monotonicity and curvature conditions discussed in Section 4, and will take the value zero otherwise. Equations (19) and (20) are noninformative apart from the regularity constraints provided by economic theory.

Equations (19) and (20) imply a joint prior pdf of the form

$$p(\theta,h) \propto h^{-1} \times I(\theta \in R) \tag{21}$$

Bayes's Theorem (ie. equation 15) is used to combine this joint prior with the likelihood function (18) to obtain the joint posterior pdf

$$\begin{aligned} p(\theta,h|y) &\propto p(y|\theta,h)\, p(\theta,h) \\ &\propto h^{NT/2-1} \times \exp\left\{-0.5h\left[\upsilon\hat{h}^{-1} + \left(\theta-\hat{\theta}\right)' W'W\left(\theta-\hat{\theta}\right)\right]\right\} \times I(\theta \in R). \end{aligned} \tag{22}$$

The parameter h is of little interest (Bayesians refer to it as a "nuisance" parameter) so it is usually integrated out of (22) to yield the marginal posterior pdf for θ:

$$p(\theta|y) \propto \left\{(y - W\theta)'(y - W\theta)\right\}^{-NT/2} \times I(\theta \in R). \tag{23}$$

If we are interested in characteristics of the marginal posterior pdf of a particular element of θ then we need to further integrate (23) using MCMC simulation. The particular MCMC algorithm we use in this paper is a random-walk Metropolis-Hastings (M-H) algorithm – details are available in Chen, Shao and Ibrahim (2000). To implement the M-H algorithm we use a multivariate normal proposal density with covariance matrix equal to a tuning scalar multiplied by the maximum likelihood estimate of the covariance matrix of the parameters. Following the work of Roberts, Gelman and Gilks (1997), the tuning scalar is set so that the optimal acceptance rate lies between 0.23 and 0.45.

MCMC draws from the posterior pdf (23) can be used to estimate the marginal posterior pdf of functions of θ, including the following measure of relative technical efficiency:

$$RTE_i = \exp\left(\min_j\left(\psi_j\right) - \psi_i\right) = \exp\left(\min_j\left(u_j\right) - u_i\right). \tag{24}$$

RTE_i represents the efficiency of firm i relative to the most efficient firm in the sample, and its value ranges between 0 and 1.

The Random Effects Model

We can also treat the inefficiency component in equation (14) as a random variable. In this case the T observations on the i-th firm can be compactly written as:

$$y_i = X_i\beta + v_i - u_i j_T, \tag{26}$$

where $X_i = (j_T, Z_i)$ is $T \times (P+1)$ and $\beta = (\beta_0, \phi')'$ is $(P+1) \times 1$. The full set of NT observations can then be written:

$$y = X\beta + v - (I_N \otimes j_T)u , \tag{27}$$

where $X = (X_1', ..., X_N')'$ is $NT \times (P+1)$ and $u = (u_1, ..., u_N)'$ is $N \times 1$.

Various distributional assumptions for the inefficiency term have been used in the literature. In this paper we assume the u_i are i.i.d. exponential random variables with pdf

$$p(u_i | \lambda^{-1}) = f_G(u_i | \lambda, 2) = \lambda^{-1} \exp(-\lambda^{-1} u_i) \tag{25}$$

The likelihood function for this exponential model can be maximized numerically using programs such as LIMDEP (Greene, 2002). Following estimation, the technical efficiencies of each firm in each year can be estimated by substituting the parameter estimates into an expression for the conditional expectation E{ $\exp(-u_i) | (v_{it} - u_i)$ }. Since u_i is a non-negative random variable, these technical efficiency estimates lie between 0 and 1, with a value of 1 indicating full technical efficiency.

For Bayesian analysis we specify the following prior pdfs for the unknown parameters β, h and λ:

$$p(\beta) \propto I(\beta \in R) \tag{28}$$

$$p(h) \propto h^{-1}. \tag{29}$$

and $$p(\lambda^{-1}) \propto f_G(\lambda^{-1} | -1/\ln(\tau^*), 2) \propto \exp(\ln(\tau^*)/\lambda). \tag{30}$$

where τ^* is the prior median of the efficiency distribution. The prior (30) is a proper prior (ie. integrates to one) that ensures the posterior pdf is also proper – see Fernandez, Osiewalski and Steel (1997). Treating the inefficiency effects as unknown parameters, the joint prior pdf is

$$p(\beta, h, u, \lambda^{-1}) \propto p(\beta)\, p(h)\, p(u | \lambda^{-1})\, p(\lambda^{-1})$$

$$= h^{-1} \times I(\beta \in R) \times f_G\left(\lambda^{-1} \middle| -1/\ln\left(\tau^*\right), 2\right) \times \prod_{i=1}^{N} f_G\left(u_i \middle| \lambda, 2\right) \quad (31)$$

As usual, this prior pdf can be combined with the likelihood function to obtain a posterior pdf $p\left(\beta, h, u, \lambda^{-1} \middle| y\right)$ and, once more, we can obtain marginal posterior pdfs using MCMC simulation. In the case of the random effects model it is convenient to do posterior simulation using a Gibbs sampler with data augmentation – see Chen, Shao and Ibrahim (2000). This involves drawing iteratively from the following conditional posterior pdfs:

$$p\left(\lambda^{-1} \middle| y, \beta, h, u\right) \propto f_G\left(\lambda^{-1} \middle| N+1/\left(u'j_N - \ln(\tau^*)\right), 2(N+1)\right) \quad (32)$$

$$p\left(h \middle| y, \beta, u, \lambda^{-1}\right) \propto f_G\left(h \middle| NT/\left[y - X\beta + \left(I_N \otimes j_T\right)u\right]'\left[y - X\beta + \left(I_N \otimes j_T\right)u\right], NT\right) \quad (33)$$

$$p\left(\beta \middle| y, h, u, \lambda^{-1}\right) \propto f_N\left(\beta \middle| b, h^{-1}\left(X'X\right)^{-1}\right) \times I\left(\beta \in R\right), \quad (34)$$

$$p\left(u_i \middle| y, \beta, h, \lambda^{-1}\right) \propto f_N\left(u_i \middle| \bar{y}_i - \bar{X}_i'\beta + \left(Th\lambda\right)^{-1}, \left(Th\right)^{-1}\right) \times I\left(u_i \geq 0\right), \quad (35)$$

where $b = \left(X'X\right)^{-1} X'\left[y + \left(I_N \otimes j_T\right)u\right]$ is $\left(P+1\right) \times 1$, $\bar{X}_i = (1/T)j_T{}'X_i$ is $1 \times \left(P+1\right)$ and $\bar{y}_i = \left(1/T\right)j_T'y_i$ is a scalar. Draws from these pdfs can be shown to converge to draws from the posterior pdf $p\left(\beta, h, u, \lambda^{-1} \middle| y\right)$.

Drawing random numbers from the conditional posteriors (32) and (33) is straightforward using standard non-iterative methods. Drawing from the conditional posteriors (34) and (35) can be accomplished using accept-reject methods or using an M-H algorithm. Following estimation, draws on the u_i can be used to obtain estimates of (characteristics) of the marginal posterior pdf of the measure of technical efficiency:

$$TE_i = \exp(-u_i) \quad (36)$$

7. Data

We estimated the fixed and random effects models using panel data from 1989-1998 on 26 major investor-owned U.S. nuclear power electric utilities. The primary sources of the data were the EIA, the Federal Energy Regulatory Commission (FERC) and the Bureau of Labor Statistics (BLS). Each record in the data set contains measurements on firm output and input quantities for nuclear power production. Input variables included fuel, an aggregate of labor and maintenance, and capital. The output variable was net nuclear power generation in megawatt-hours (mwh).

An implicit quantity index for fuel was calculated as the ratio of fuel cost to a Tornqvist price index for uranium. Uranium prices in dollars per pound U_3O_8 equivalent were downloaded from EIA webpages.

An implicit quantity index for labor and maintenance was obtained by dividing the aggregate cost of labor and maintenance by a cost-share-weighted Tornqvist price index for labor and maintenance. Data on labor and maintenance costs were obtained by subtracting fuel expenses from total nuclear power production expenses. The price of labor was measured using a company-wide average wage rate. The price of maintenance and other supplies was measured using a price index of electrical supplies.

The capital input was measured using estimates of capital costs as discussed in Considine (2000). The base year value of the capital stock was estimated using the replacement cost of base and peak load capacity. This estimate was then updated in subsequent years based upon the value of capital additions and retirements to the nuclear power plant.

Table 1 presents average annual electricity production over the period 1989-1998 for each firm in our sample. Average production over the period ranges from a low of 1.60 million mwh by Delmarva Power and Light to 61.8 million mwh by Commonwealth Edison Co. Average production across all firms is 11.2 million mwh with a standard deviation of 12.4

million mwh. There are 14 firms with average output below the sample mean, and one firm with output that is nearly five times larger than the average.

Prior to estimation, all variables were scaled to have unit means. Thus, the estimated first-order coefficients of the distance function can be interpreted as elasticities of distance with respect to inputs evaluated at the sample means. Fuel was used as the normalizing input (see equation 13).

8. Results

For purposes of comparison, we began by estimating the fixed effects and random effects models using the method of maximum likelihood (ML). The two sets of ML estimates were obtained using LIMDEP (Greene, 2002) and are presented in the left-hand-columns of Table 2. The fixed and random effects models yielded similar estimates of elasticities evaluated at the variable means – the estimates from the fixed effects model were 0.67, 0.45 and -0.12 for labor and maintenance, capital and fuel respectively; the random effects estimates were 0.64, 0.45 and -0.1. The negative estimates of the fuel elasticity are theoretically implausible and this caused us to immediately focus our attention on the regularity constraints (ie. monotonicity and curvature).

The (observation-varying) monotonicity and curvature constraints discussed in Section 4 were examined at each data point in the sample of 260 observations. The results are summarized in Table 3. Point estimates of elasticities were found to be inconsistent with the monotonicity property at more than 190 data points, with most of these violations being associated with the fuel input. Point estimates of the relevant Hessian and bordered Hessian matrices were found to be inconsistent with curvature at more than 230 data points. Thus, we conclude that unconstrained maximum likelihood estimates of the parameters are inconsistent with economic theory, and any measures of returns to scale or technical efficiency derived from them will be unreliable. Consequently, in the remainder of this paper we restrict our attention to the Bayesian results.

The Bayesian estimation approach allows us to incorporate pre-sample information into the estimation process and thereby guarantee that estimated posterior pdfs for all economic quantities of interest (ie. elasticities, measures of returns to scale, and technical efficiency scores) are consistent with the regularity properties implied by economic theory. Monotonicity- and curvature-constrained Bayesian point estimates of the parameters are reported in the right-hand columns of Table 2. The estimated posterior moments reported in Table 2 are the means and standard deviations of samples of size 10,000 generated using the MCMC techniques described in Section 6. To estimate the random effects model we followed van den Broeck et al. (1994) and Koop et al. (1995) by setting the prior median of the efficiency distribution, τ^*, to 0.875. An M-H algorithm with 4 sub-iterations was used to draw from the conditional pdf (34).

Bayesian point estimates of the parameters of the fixed and random effects models are very similar (the fixed and random effects estimates of β_{12} might be opposite in sign, but both estimates are very close to zero with relatively large estimated standard errors). By construction, the input elasticites evaluated at the variable means (and, for that matter, every point in the sample) are correctly-signed – elasticity estimates from the fixed effects model are 0.69, 0.27 and 0.04 for labor and maintenance, capital and fuel respectively; the random effects estimates are 0.62, 0.33 and 0.04. These estimated input elasticities are quite different from estimates reported in the literature for alternative electricity generation technologies. For example, Atkinson and Primont (2002) report estimates of 0.15, 0.17 and 0.68 for steam electric power generators.

Table 4 presents estimated posterior means, standard errors and 90% coverage regions for the technical efficiency scores of each firm in the sample. The technical efficiency scores were obtained using equations (24) and (36). A 90% coverage region is the shortest interval containing 90% of the area under a pdf, and is analogous to a 90% confidence interval. The technical efficiency scores obtained from the fixed effects model are generally lower than those from the random effects model – the scores from the fixed effects model range from 0.615 to 0.935 with an average of 0.776, while those from the random effects model range from a low of 0.903 to 0.977 with an average of 0.953. There are approximately 10 firms with estimated technical efficiency scores below the average. Estimates obtained from the random effects model exhibit less variation across firms than those from the fixed effects model. Estimates obtained from the fixed effects model are consistent with efficiency scores reported by Atkinson and Primont (2002) and Rungsuriyawiboon and Stephanou (2003).

Table 5 presents estimated posterior means, standard errors and 90% coverage regions for the labor and maintenance input elasticity evaluated at the average input level of each firm. The fixed effects model yields elasticity estimates that range from a low of 0.685 to a high of 0.718 with an average of 0.698; the random effects model yields estimates that range from 0.587 to 0.678 with an average of 0.633. Table 6 presents a similar set of results for the capital input. The fixed effects model yields capital input elasticities that range from 0.217 to 0.284 with an average of 0.256; the random effects model yields estimates that range from 0.253 to 0.382 with an average of 0.318. The estimated posterior standard errors from the random effects model are generally lower than those of the fixed effects model.

Our MCMC samples were also used to estimate a measure of returns to scale for each firm. Posterior means, standard errors and 90% coverage regions for the returns to scale measure given by equation (11) (evaluated at average input and output levels for each firm) are presented in Table 7. The fixed effects model yielded RTS estimates ranging from 1.045 to 1.171 with an average of 1.100; the random effects model yielded estimates ranging from 1.085 to 1.169 with an average of 1.111. These point estimates suggest that all nuclear-power generators operate in the region of increasing RTS. This result is consistent with the findings of Rungsuriyawiboon and Stephanou (2003). However, a few of the coverage regions span one, suggesting there is positive probability that some utilities operate in regions of constant or decreasing RTS.

Further insights into likely and unlikely values of parameters, elasticities, technical efficiency scores and RTS measures can be obtained by examining estimated marginal posterior pdfs. For illustrative purposes, several estimated pdfs are presented in Figures 1 and 2. In Figure 1 we present the posterior pdfs for the three input elasticities and the measure of returns to scale for Ohio Edison (Firm 16) in 1998. In Figure 2 we present estimated pdfs for Commonwealth Edison (Firm 7) in the same year. Panels (a) to (c) in both figures show that for these firms (and year) there is zero probability that input shares lie outside the unit interval (by construction). Panel (d) in Figure 1 shows that there is zero probability that Ohio Edison is operating in the region of constant or decreasing returns to scale, while panel (d) in Figure 2 shows that Commonwealth Edison is operating in this region with probability 0.15 (using the fixed effects model) or 0.27 (random effects). The asymmetry evident in the pdfs suggests that frequentist interval estimation and hypothesis testing procedures predicated on the assumption of asymptotic normality are inappropriate in this case.

9. Conclusions

Distance functions are appropriate representations of multi-input multi-output technologies in network industries where behavioral assumptions such as profit maximization are often untenable. The input distance function is particularly appropriate in the electric utility industry where firms adjust inputs to produce exogenously-determined levels of electricity output. Unfortunately, empirical estimates of the parameters of input distance functions are often inconsistent with regularity constraints (eg. monotonicity and curvature) implied by economic theory. As a consequence, interesting functions of the estimated parameters (eg. technical efficiency scores, elasticities, measures of returns to scale) are unreliable and may lead to perverse conclusions regarding economic behavior. For example, monotonicity violations imply that increasing the inputs used to produce a given level of output will *increase* measured efficiency.

One solution to this problem is to use Bayesian methodology to impose monotonicity and curvature restrictions on the parameters of the model. In this paper we estimated input distance functions for U.S. nuclear power generators under the assumptions that inefficiency effects were either fixed or random. Results from the two models were economically plausible (by construction) and, in this respect, much preferred to estimates obtained using the method of maximum likelihood.

Our results were summarized in terms of estimated characteristics of marginal posterior pdfs for parameters, elasticities, technical efficiency scores and measures of returns to scale. The fixed effects and random effects models yielded similar estimates of elasticities and measures of returns to scale. However, the fixed effects model yielded estimates of technical efficiency that were generally lower than estimates obtained using the random effects model – similar findings in other empirical contexts have been reported and explained by several authors, including Kim and Schmidt (2000) and O’Donnell and Coelli (2004).

One of the advantages of the Bayesian approach is that it is possible to obtain exact finite sample results concerning any (linear or nonlinear) functions of the parameters. Thus, for example, we were able to estimate finite sample pdfs for measures of returns to scale and use these pdfs to determine that, for most firms, there is zero probability they are operating in regions of constant or decreasing returns to scale. Our estimated pdfs were often asymmetric, suggesting that sampling theory procedures for testing hypotheses and constructing confidence intervals are inappropriate in this case.

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Table 1: Average Outputs (1989-1998)

No	Utility Name	State	Average Output (10^6 mwh)
1	Southern Company	AL	24.4
2	Arizona Public Service	AZ	6.70
3	Entergy Corporation	AR	19.0
4	Baltimore Gas & Electric	MD	10.0
5	Carolina Power & Light	NC	16.9
6	Centerior Energy Corp	OH	11.6
7	Commonwealth Edison	IL	61.8
8	Consolidated Edison-NY	NY	5.40
9	Consumers Energy	MI	4.60
10	Delmarva Power & Light	DE	1.60
11	Duke Power	NC	34.9
12	Florida Power and Light	FL	19.3
13	General Public utilities	NJ	10.2
14	Kansas City Power and Light	KS	3.90
15	Kansas Gas and Electric	KS	4.20
16	Ohio Edison	OH	7.00
17	Pacific Gas and Electric	CA	15.7
18	Pennsylvania Power and Light	PA	13.6
19	Public Service Co of NM	NW	2.10
20	Public Service Electric & Gas	NJ	16.2
21	San Diego Gas and Electric	CA	3.30
22	Southern California Edison	CA	15.9
23	Union Electric	MO	7.30
24	United Illuminating	CT	2.40
25	Virginia Electric and Power	VA	21.7
26	Wisconsin Electric Power	WI	6.70
	Mean		11.2
	Standard deviation		12.4

Table 2: Estimated Parameters[a]

Parameter	Maximum Likelihood				Bayes			
	Fixed Effects		Random Effects		Fixed Effects		Random Effects	
	Estimate	Est. Asy. St. Error	Estimate	Est. Asy. St. Error	Estimate	Est. St. Error	Estimate	Est. St. Error
ψ_1	0.155	(0.110)	-	-	0.194	(0.109)	-	-
ψ_2	0.226	(0.098)	-	-	0.243	(0.099)	-	-
ψ_3	0.123	(0.100)	-	-	0.153	(0.104)	-	-
ψ_4	0.067	(0.099)	-	-	0.094	(0.122)	-	-
ψ_5	-0.051	(0.094)	-	-	-0.013	(0.126)	-	-
ψ_6	0.020	(0.096)	-	-	0.098	(0.092)	-	-
ψ_7	-0.020	(0.096)	-	-	0.054	(0.134)	-	-
ψ_8	-0.113	(0.095)	-	-	-0.039	(0.128)	-	-
ψ_9	-0.025	(0.097)	-	-	0.055	(0.128)	-	-
ψ_{10}	0.014	(0.093)	-	-	0.051	(0.105)	-	-
ψ_{11}	-0.034	(0.096)	-	-	0.026	(0.095)	-	-
ψ_{12}	0.021	(0.095)	-	-	0.059	(0.109)	-	-
ψ_{13}	0.027	(0.097)	-	-	0.044	(0.100)	-	-
ψ_{14}	0.008	(0.095)	-	-	0.059	(0.117)	-	-
ψ_{15}	-0.091	(0.093)	-	-	-0.009	(0.122)	-	-
ψ_{16}	0.049	(0.097)	-	-	0.077	(0.110)	-	-
ψ_{17}	-0.192	(0.098)	-	-	-0.179	(0.129)	-	-
ψ_{18}	-0.165	(0.096)	-	-	-0.146	(0.131)	-	-
ψ_{19}	-0.022	(0.099)	-	-	-0.026	(0.114)	-	-
ψ_{20}	-0.112	(0.096)	-	-	-0.069	(0.133)	-	-
ψ_{21}	0.067	(0.097)	-	-	0.107	(0.110)	-	-
ψ_{22}	0.075	(0.096)	-	-	0.098	(0.133)	-	-
ψ_{23}	0.058	(0.093)	-	-	0.069	(0.121)	-	-
ψ_{24}	0.166	(0.097)	-	-	0.130	(0.097)	-	-
ψ_{25}	-0.076	(0.095)	-	-	-0.011	(0.120)	-	-
ψ_{26}	0.092	(0.099)	-	-	0.182	(0.120)	-	-
β_0	-	-	0.247	(0.041)	-	-	-0.021	(0.039)
β_1	0.669	(0.070)	0.636	(0.065)	0.694	(0.077)	0.623	(0.044)
β_2	0.447	(0.082)	0.451	(0.061)	0.265	(0.070)	0.334	(0.045)
β_{11}	0.191	(0.107)	0.347	(0.134)	0.008	(0.053)	0.017	(0.055)
β_{12}	-0.051	(0.079)	-0.217	(0.114)	0.005	(0.051)	-0.005	(0.053)
β_{22}	-0.146	(0.119)	-0.157	(0.186)	-0.023	(0.052)	-0.012	(0.055)
α_1	-0.933	(0.031)	-0.885	(0.039)	-0.924	(0.028)	-0.910	(0.027)
α_{11}	0.004	(0.037)	0.076	(0.068)	-0.024	(0.027)	-0.008	(0.030)
γ_{11}	-0.094	(0.052)	-0.295	(0.073)	-0.005	(0.036)	-0.022	(0.028)
γ_{21}	0.137	(0.053)	0.350	(0.095)	0.011	(0.037)	0.029	(0.028)

[a] Subscripts on β coefficients refer to inputs: 1 = labor and maintenance; 2 = capital; q = output.

Table 3: ML Regularity Violations [a]

Model	Number of Concavity Violoations	Number of Monotonicity Volations			
		Fuel	Labor and Maintenance	Capital	Total
Fixed Effects	239	178	0	16	194
Random Effects	252	199	1	4	201

[a] Total number of observations = 260

Table 4: Estimated Technical Efficiency Scores

Firm	Fixed Effects			Random Effects		
	Estimate	Est. St. Error	90% Coverage Region	Estimate	Est. St. Error	90% Coverage Region
1	0.892	0.087	[0.742, 1.000]	0.931	0.103	[0.823, 0.994]
2	0.935	0.072	[0.785, 1.000]	0.903	0.105	[0.763, 0.988]
3	0.856	0.089	[0.709, 1.000]	0.940	0.102	[0.841, 0.996]
4	0.808	0.089	[0.660, 0.969]	0.947	0.107	[0.857, 0.997]
5	0.725	0.078	[0.599, 0.866]	0.960	0.116	[0.894, 0.998]
6	0.810	0.081	[0.674, 0.948]	0.959	0.110	[0.887, 0.997]
7	0.776	0.087	[0.637, 0.925]	0.960	0.109	[0.883, 0.998]
8	0.708	0.087	[0.588, 0.858]	0.967	0.120	[0.906, 0.998]
9	0.779	0.103	[0.611, 0.959]	0.960	0.109	[0.886, 0.997]
10	0.773	0.080	[0.642, 0.897]	0.959	0.110	[0.879, 0.998]
11	0.754	0.083	[0.611, 0.890]	0.961	0.114	[0.888, 0.998]
12	0.780	0.092	[0.632, 0.939]	0.956	0.108	[0.875, 0.997]
13	0.767	0.077	[0.642, 0.897]	0.952	0.108	[0.868, 0.996]
14	0.779	0.079	[0.659, 0.921]	0.961	0.112	[0.886, 0.998]
15	0.728	0.085	[0.610, 0.884]	0.964	0.116	[0.893, 0.998]
16	0.793	0.085	[0.658, 0.937]	0.941	0.109	[0.839, 0.995]
17	0.615	0.072	[0.515, 0.740]	0.977	0.131	[0.931, 0.999]
18	0.636	0.077	[0.509, 0.756]	0.977	0.125	[0.926, 0.999]
19	0.716	0.077	[0.595, 0.836]	0.961	0.117	[0.882, 0.998]
20	0.685	0.074	[0.579, 0.815]	0.967	0.119	[0.895, 0.998]
21	0.817	0.076	[0.700, 0.945]	0.944	0.111	[0.838, 0.995]
22	0.810	0.088	[0.684, 0.991]	0.938	0.110	[0.835, 0.997]
23	0.789	0.101	[0.634, 1.000]	0.955	0.108	[0.867, 0.996]
24	0.836	0.085	[0.692, 0.983]	0.942	0.106	[0.851, 0.997]
25	0.727	0.079	[0.610, 0.865]	0.965	0.115	[0.904, 0.997]
26	0.881	0.091	[0.722, 1.000]	0.927	0.107	[0.794, 0.991]
Mean	0.776	0.084	[0.642, 0.916]	0.953	0.112	[0.869, 0.997]

Table 5: Estimated Labor/Maintenance Elasticity

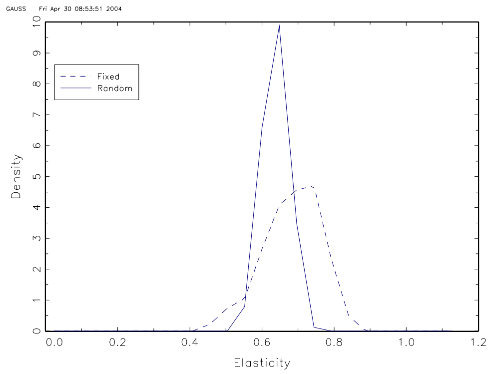
Firm	Fixed Effects			Random Effects		
	Estimate	Est. St. Error	90% Coverage Region	Estimate	Est. St. Error	90% Coverage Region
1	0.688	0.092	[0.532, 0.822]	0.609	0.056	[0.512, 0.696]
2	0.686	0.071	[0.555, 0.788]	0.625	0.048	[0.543, 0.699]
3	0.703	0.083	[0.557, 0.829]	0.627	0.051	[0.539, 0.707]
4	0.703	0.078	[0.563, 0.818]	0.637	0.052	[0.549, 0.719]
5	0.686	0.089	[0.536, 0.813]	0.610	0.053	[0.522, 0.696]
6	0.689	0.077	[0.547, 0.799]	0.621	0.052	[0.530, 0.698]
7	0.685	0.122	[0.502, 0.874]	0.587	0.071	[0.474, 0.704]
8	0.703	0.074	[0.568, 0.813]	0.641	0.053	[0.555, 0.728]
9	0.708	0.060	[0.595, 0.796]	0.650	0.040	[0.583, 0.712]
10	0.718	0.059	[0.620, 0.810]	0.678	0.056	[0.576, 0.763]
11	0.699	0.099	[0.537, 0.851]	0.613	0.059	[0.513, 0.708]
12	0.703	0.086	[0.557, 0.830]	0.624	0.050	[0.540, 0.705]
13	0.694	0.074	[0.565, 0.796]	0.625	0.042	[0.555, 0.694]
14	0.693	0.063	[0.573, 0.789]	0.642	0.051	[0.551, 0.717]
15	0.704	0.063	[0.586, 0.798]	0.651	0.050	[0.564, 0.724]
16	0.692	0.067	[0.568, 0.784]	0.629	0.041	[0.560, 0.691]
17	0.689	0.085	[0.548, 0.808]	0.613	0.046	[0.536, 0.691]
18	0.705	0.076	[0.561, 0.821]	0.636	0.054	[0.543, 0.713]
19	0.701	0.073	[0.574, 0.815]	0.654	0.056	[0.563, 0.748]
20	0.693	0.083	[0.552, 0.811]	0.617	0.046	[0.538, 0.693]
21	0.712	0.056	[0.609, 0.798]	0.662	0.044	[0.582, 0.729]
22	0.694	0.080	[0.554, 0.809]	0.621	0.047	[0.540, 0.695]
23	0.711	0.070	[0.580, 0.818]	0.655	0.059	[0.549, 0.739]
24	0.691	0.088	[0.536, 0.821]	0.659	0.089	[0.489, 0.783]
25	0.697	0.089	[0.540, 0.833]	0.620	0.058	[0.521, 0.709]
26	0.707	0.065	[0.581, 0.803]	0.648	0.046	[0.567, 0.715]
Mean	0.698	0.078	[0.561, 0.813]	0.633	0.053	[0.542, 0.714]

Table 6: Estimated Capital Elasticity

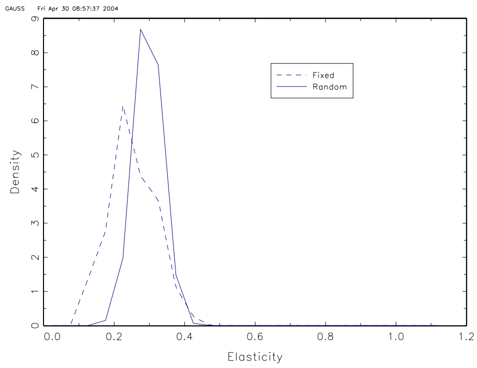
Firm	Fixed Effects			Random Effects		
	Estimate	Est. St. Error	90% Coverage Region	Estimate	Est. St. Error	90% Coverage Region
1	0.281	0.087	[0.142, 0.431]	0.358	0.058	[0.266, 0.456]
2	0.268	0.063	[0.173, 0.382]	0.326	0.046	[0.255, 0.404]
3	0.257	0.078	[0.128, 0.393]	0.331	0.053	[0.248, 0.417]
4	0.244	0.071	[0.131, 0.365]	0.308	0.053	[0.224, 0.393]
5	0.275	0.082	[0.150, 0.417]	0.349	0.053	[0.263, 0.436]
6	0.275	0.072	[0.165, 0.407]	0.340	0.053	[0.259, 0.432]
7	0.284	0.117	[0.096, 0.467]	0.382	0.072	[0.258, 0.492]
8	0.236	0.067	[0.131, 0.349]	0.295	0.053	[0.208, 0.381]
9	0.235	0.055	[0.148, 0.328]	0.290	0.043	[0.220, 0.359]
10	0.217	0.061	[0.121, 0.318]	0.253	0.059	[0.159, 0.353]
11	0.267	0.094	[0.111, 0.425]	0.351	0.062	[0.252, 0.452]
12	0.255	0.080	[0.124, 0.393]	0.331	0.052	[0.249, 0.416]
13	0.261	0.066	[0.159, 0.380]	0.327	0.041	[0.260, 0.395]
14	0.258	0.055	[0.172, 0.357]	0.306	0.049	[0.233, 0.393]
15	0.244	0.057	[0.155, 0.342]	0.293	0.050	[0.217, 0.381]
16	0.262	0.058	[0.173, 0.368]	0.321	0.039	[0.259, 0.385]
17	0.270	0.078	[0.152, 0.403]	0.344	0.046	[0.265, 0.420]
18	0.255	0.071	[0.138, 0.381]	0.321	0.056	[0.238, 0.412]
19	0.224	0.065	[0.121, 0.330]	0.268	0.054	[0.178, 0.358]
20	0.267	0.077	[0.147, 0.400]	0.340	0.047	[0.263, 0.417]
21	0.229	0.052	[0.146, 0.319]	0.276	0.047	[0.200, 0.354]
22	0.267	0.074	[0.149, 0.398]	0.338	0.048	[0.262, 0.417]
23	0.242	0.066	[0.135, 0.359]	0.295	0.061	[0.203, 0.398]
24	0.268	0.085	[0.140, 0.423]	0.294	0.089	[0.171, 0.469]
25	0.268	0.084	[0.131, 0.416]	0.343	0.060	[0.251, 0.442]
26	0.241	0.060	[0.145, 0.345]	0.297	0.048	[0.221, 0.376]
Mean	0.256	0.072	[0.142, 0.381]	0.318	0.054	[0.234, 0.408]

Table 7: Estimated Measures of Returns to Scale

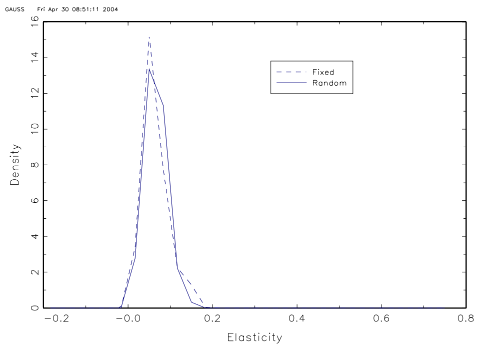
Firm	Fixed Effects			Random Effects		
	Estimate	Est. St. Error	90% Coverage Region	Estimate	Est. St. Error	90% Coverage Region
1	1.062	0.043	[0.992, 1.134]	1.085	0.046	[1.001, 1.159]
2	1.101	0.045	[1.043, 1.173]	1.109	0.034	[1.060, 1.171]
3	1.079	0.038	[1.018, 1.142]	1.099	0.040	[1.029, 1.165]
4	1.110	0.061	[1.035, 1.202]	1.125	0.047	[1.054, 1.207]
5	1.075	0.045	[1.005, 1.154]	1.099	0.044	[1.024, 1.170]
6	1.080	0.035	[1.025, 1.139]	1.090	0.032	[1.040, 1.146]
7	1.045	0.066	[0.943, 1.161]	1.091	0.077	[0.951, 1.209]
8	1.127	0.072	[1.042, 1.239]	1.140	0.053	[1.061, 1.232]
9	1.124	0.054	[1.062, 1.206]	1.130	0.041	[1.066, 1.203]
10	1.163	0.087	[1.060, 1.282]	1.145	0.073	[1.039, 1.270]
11	1.062	0.050	[0.984, 1.145]	1.092	0.055	[0.993, 1.179]
12	1.080	0.042	[1.015, 1.152]	1.105	0.043	[1.031, 1.178]
13	1.092	0.038	[1.038, 1.159]	1.109	0.034	[1.058, 1.168]
14	1.117	0.050	[1.056, 1.200]	1.111	0.039	[1.051, 1.180]
15	1.122	0.051	[1.059, 1.206]	1.119	0.042	[1.054, 1.191]
16	1.101	0.039	[1.051, 1.164]	1.110	0.031	[1.067, 1.164]
17	1.078	0.040	[1.014, 1.150]	1.102	0.041	[1.033, 1.169]
18	1.087	0.034	[1.033, 1.143]	1.097	0.033	[1.044, 1.152]
19	1.171	0.128	[1.042, 1.348]	1.169	0.087	[1.048, 1.323]
20	1.079	0.039	[1.017, 1.148]	1.102	0.040	[1.034, 1.168]
21	1.137	0.061	[1.066, 1.224]	1.132	0.049	[1.057, 1.217]
22	1.079	0.036	[1.022, 1.139]	1.097	0.036	[1.035, 1.159]
23	1.113	0.045	[1.050, 1.186]	1.109	0.040	[1.049, 1.180]
24	1.134	0.075	[1.028, 1.269]	1.098	0.070	[1.002, 1.231]
25	1.071	0.044	[1.001, 1.146]	1.092	0.045	[1.014, 1.164]
26	1.115	0.046	[1.058, 1.185]	1.120	0.037	[1.064, 1.185]
Mean	1.100	0.052	[1.029, 1.184]	1.111	0.046	[1.037, 1.190]



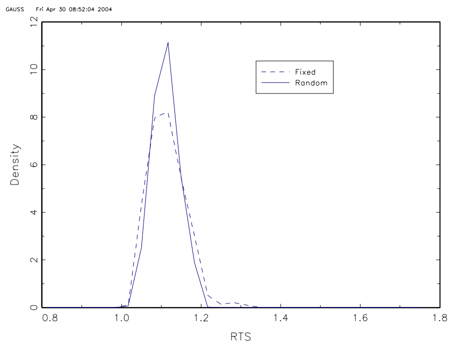
(a) Labor Elasticity



(b) Capital Elasticity

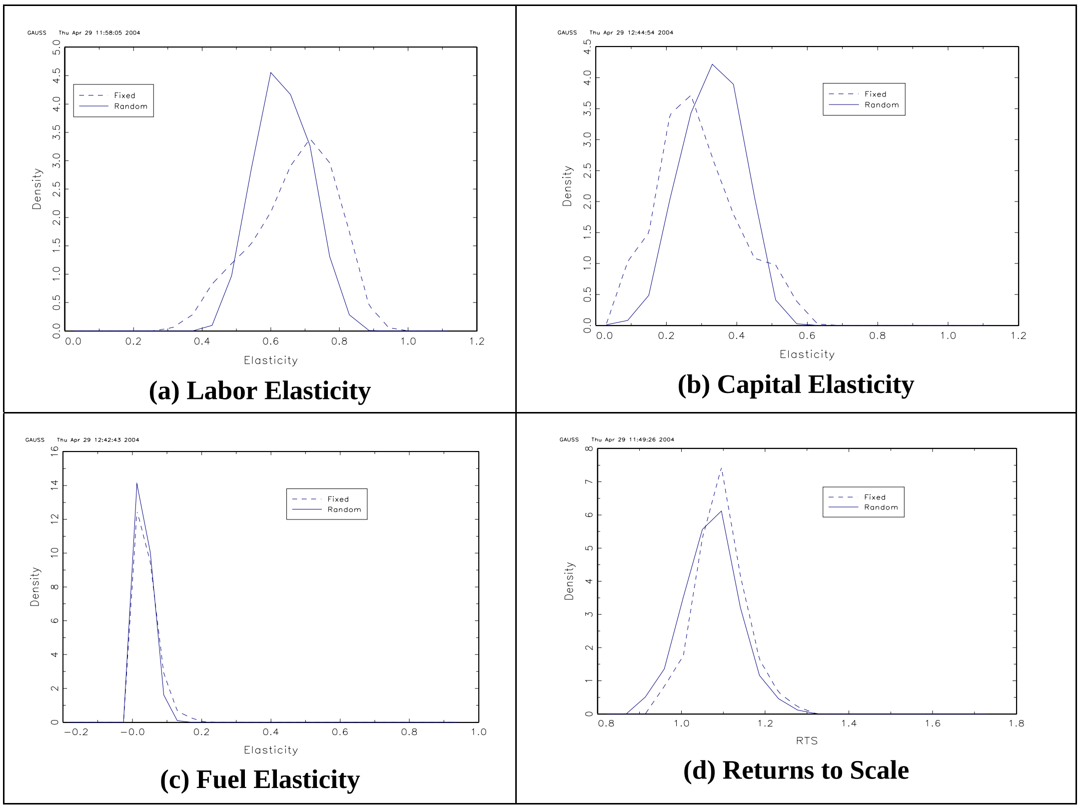


(c) Fuel Elasticity



(d) Returns to Scale

Figure 1: Estimated Marginal Posterior PDFs for Ohio Electric in 1998



(a) Labor Elasticity

(b) Capital Elasticity

(c) Fuel Elasticity

(d) Returns to Scale

Figure 2: Estimated Marginal Posterior PDFs for Commonwealth Edison in 1998