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AN ECONOMETRIC APPROACH TO ESTIMATING SUPPORT PRICES AND MEASURES OF PRODUCTIVITY CHANGE IN PUBLIC HOSPITALS[1]

by

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Abstract: In industry sectors where market prices are unavailable it is common to represent multiple-input multiple-output production technologies using distance functions. Econometric estimation of such functions is complicated by the fact that more than one variable in the function may be endogenous. In such cases, maximum likelihood estimation can lead to biased and inconsistent estimates of the model parameters and associated measures of firm performance. We solve the problem by using linear programming to construct a quantity index. The distance function is then written in the form of a conventional stochastic frontier model where the explanatory variables are unambiguously exogenous. We use this approach to estimate productivity indexes and support (or shadow) prices for a sample of Australian public hospitals. We decompose the productivity index into several measures of environmental change and efficiency change. We find that the productivity effects of improvements in input-oriented technical efficiency have been largely offset by the effects of deteriorations in the production environment over time.

KEYWORDS: endogeneity, distance functions, shadow prices, efficient prices, technical efficiency, scale and mix efficiency.

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1. INTRODUCTION

Australian public hospitals are established under state and territory legislation to provide medical and healthcare services to all 'public patients' (i.e., persons who meet national Medicare eligibility criteria). The funding of public hospitals is shared mainly[2] between Commonwealth, State and Territory governments. In 2010 the Commonwealth Government announced that it will "become the majority funder of the Australian public hospital system. The Government will fund: 60 percent of the efficient price of every public hospital service provided to public patients; …. and over time, up to 100 per cent of the efficient price of 'primary health care equivalent' outpatient services provided to public hospital patients. In return … the Commonwealth will require the states to commit to system wide reform to improve public hospital governance, performance and accountability" (Commonwealth of Australia, 2010, p. 27)[3]. To implement its reform agenda, the Government announced it will appoint an independent 'umpire' to set nationally efficient prices that will "strike a balance between reasonable access, clinical safety, efficiency and fiscal considerations" (Commonwealth of Australia, 2010, p. 70).

This paper uses econometric methodology to estimate efficient prices and measures of performance for a sample of Australian public hospitals. The paper seeks to inform the Government's hospital reform agenda in three ways. First, it shows how data on output and input quantities alone (i.e., no prices) can be used to estimate the productive performance of individual hospitals operating in different production environments (e.g., urban, rural). Second, it shows how differences in hospital productivity that are not accounted for by differences in the production environment can be attributed to differences in various measures of efficiency (e.g., scale efficiency). Finally, it shows how quantity data can be combined with total cost data to estimate upper bounds on the efficient prices of hospital outputs produced in different production environments. Thus, the paper provides a framework within which the Government's independent umpire can determine efficient prices that "reflect the actual cost of providing hospital services, and developments in best practice" (Commonwealth of Australia, 2010, p. 71).

Our methodology involves the estimation of Shephard (1953) distance functions. A feature of the methodology is that it does not require any assumptions concerning the optimizing behavior of hospital managers (e.g., cost minimization) or the degree of competition in input or output markets. Nor does it require any data on input prices or input cost shares. Indeed, lack of reliable input price data is the main reason we chose a distance function approach[4].

The econometric approach to estimating output and input distance functions typically involves factoring out one of the outputs or inputs and estimating the resulting equation using conventional stochastic frontier estimation methods (e.g., Productivity Commission, 2010; Barbetta, Turati and Zago, 2001; Ferrari, 2006; Morrison Paul, 2002; and Dervaux et al., 2004). An unsatisfactory feature of this approach is that some of the outputs and inputs that are not factored out may be correlated with the composite error term, leading to biased and inconsistent estimates. The problem is sometimes referred to as the 'endogeneity problem' (e.g., Roibas and Arias, 2004) Our approach to the problem involves the construction of a quantity index. When we factor this quantity index out of the distance function we are left with a conventional stochastic production frontier model where the explanatory variables are uncorrelated

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2 A small portion (< 7% in 2007-2008) of public hospital funding comes from insurance funds and other non-government sources.

3 In February 2011 the Commonwealth Government announced that it will fund a lower percentage of efficient prices.

4 If input prices are available then an estimated cost function will give the minimum cost (i.e., efficient price) of providing a vector of outputs. The first derivatives of the cost function with respect to individual outputs will give the efficient prices of those outputs under marginal cost pricing – see O'Donnell and Nguyen (2011).

with the error term. Our approach overcomes some practical disadvantages associated with alternative sampling theory and Bayesian approaches.

The outline of the paper is as follows. In Section 2 we assume the production technology can be represented by input and/or output distance functions that allow for variable returns to scale. In Section 3 we show how the input distance function can be used within the aggregate quantity framework of O'Donnell (2008) to define a measure of productivity change and several input-oriented measures of efficiency (change). Section 4 discusses alternative econometric approaches to estimating the parameters of distance functions and associated measures of firm performance. Section 5 shows how linear programming methods can be used construct a quantity index that makes our preferred econometric approach operational. Section 6 describes the data and variables used in the empirical analysis. Section 7 reports estimates of efficient output prices and measures of productivity and efficiency (change) for a sample of Australian public hospitals. We conclude the paper in Section 8 with a summary of our main findings.

2. THE PRODUCTION TECHNOLOGY

We represent the production technology using the separable transformation function

(1) $$T(x,q,z) = g(q) - \exp\left(\gamma' z + \eta \ln h(x)\right) \leq 0$$

where $\eta > 0$ and $\gamma \in \Re^M$ are unknown parameters, $x \in \Re_+^K$ is a vector of input quantities, $q \in \Re_+^J$ is a vector of output quantities, and $z \in \Re^M$ is a vector of exogenous variables measuring characteristics of the production environment. We assume the functions $g(.)$ and $h(.)$ are both non-negative, non-decreasing and linearly homogeneous. Among other things, these properties mean η can be interpreted as the elasticity of scale. Technically-feasible but technically-inefficient input-output combinations are defined by $T(x,q,z) < 0$. Technically-efficient production plans are defined by $T(x,q,z) = 0$.

Production technologies can also be represented using Shephard (1953) output and input distance functions. The output distance function gives the reciprocal of the largest factor by which the output vector can be scaled up while holding the input vector fixed. The input distance function gives the largest factor by which the input vector can be scaled down while holding the output vector fixed. In the case of the technology defined by (1), the logarithms of the output and input distance functions are[5]

(2) $$\ln D_O(x,q,z) = \ln g(q) - \gamma' z - \eta \ln h(x) \leq 0 \quad \text{and}$$

(3) $$\ln D_I(x,q,z) = \ln h(x) + \delta' z - \eta^{-1} \ln g(q) \geq 0$$

where $\delta \equiv \eta^{-1}\gamma$. The assumed properties of $g(.)$ and $h(.)$ imply the output distance function is

O.1 nonincreasing in inputs: $D_O(x_0,q,z) \geq D_O(x_1,q,z)$ for $x_1 \geq x_0$,

O.2 nondecreasing in outputs: $D_O(x,q_1,z) \geq D_O(x,q_0,z)$ for $q_1 \geq q_0$,

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5 If $\delta = D_O(x,q)$ then $T(x,q/\delta) = g(q/\delta) - \exp\left(\gamma' z + \eta \ln h(x)\right) = 0$ which can be solved for $\ln \delta = \ln g(q) - \gamma' z - \eta \ln h(x)$. If $\rho = D_I(x,q)$ then $T(x/\rho, q) = g(q) - \exp\left(\gamma' z + \eta \ln h(x/\rho)\right) = 0$ which can be solved for $\ln \rho = \ln h(x) + \eta^{-1}\gamma' z - \eta^{-1} \ln g(q)$.

O.3 linearly homogenous in outputs: $D_O(x,\lambda q,z)=\lambda D_O(x,q,z)$ for $\lambda>0$,

while the input distance function is

I.1 nondecreasing in inputs: $D_I(x_1,q,z)\geq D_I(x_0,q,z)$ for $x_1\geq x_0$,

I.2 nonincreasing in outputs: $D_I(x,q_1,z)\leq D_I(x,q_0,z)$ for $q_1\geq q_0$, and

I.3 linearly homogenous in inputs: $D_I(x,q,z)=\lambda D_I(\lambda x,q,z)$ for $\lambda>0$.

If the technology exhibits constant returns to scale (i.e., $\eta=1$) then $D_O(x,q,z)=D_I(x,q,z)^{-1}$.

Production technologies can also be represented using other functions, including cost, revenue and profit functions. For example, the cost function is defined as

(4) $$c(w,q,z)=\min_x\left\{w'x:T(x,q,z)\leq 0\right\}$$

where $w\in\Re_+^K$ is a vector of input prices. The cost function gives the minimum cost of producing q when input prices are w and the production environment is characterised by z. Distance functions and cost functions can both be used to define important measures of efficiency (change).

3. MEASURES OF PRODUCTIVITY AND EFFICIENCY (CHANGE)

In this section we introduce a firm subscript i and a time subscript t into the notation and let $x_{it}=(x_{1it},\ldots,x_{Kit})'$, $q_{it}=(q_{1it},\ldots,q_{Jit})'$, $z_{it}=(z_{1it},\ldots,z_{Mit})'$ and $w_{it}=(w_{1it},\ldots,w_{Kit})'$ denote vectors of input quantities, output quantities, environmental variables and input prices for firm i in period t $(i=1,\ldots,N;\ t=1,\ldots,T)$. We follow O'Donnell (2008, 2010a) and measure the total factor productivity (TFP) of the firm as:

(5) $$TFP_{it}\equiv\frac{Q_{it}}{X_{it}}$$ (TFP)

where $Q_{it}\equiv Q(q_{it})$ is an aggregate output, $X_{it}\equiv X(x_{it})$ is an aggregate input, and $Q(.)$ and $X(.)$ are non-negative, non-decreasing and linearly homogeneous aggregator functions. O'Donnell (2008, 2010a) shows how aggregate outputs and inputs can also be used to define several output- and input-oriented measures of efficiency. For example, in a production environment characterized by z_{it} :

(6) $$ITE_{it}\equiv\frac{\bar{X}_{it}}{X_{it}}=D_I(x_{it},q_{it},z_{it})^{-1}$$ (input-oriented technical efficiency)

(7) $$ISE_{it}\equiv\frac{Q_{it}/\bar{X}_{it}}{\tilde{Q}_{it}/\tilde{X}_{it}}$$ (input-oriented scale efficiency)

(8) $$IME_{it}\equiv\frac{\hat{X}_{it}}{\bar{X}_{it}}$$ (input-oriented mix efficiency)

(9) $$ISME_{it} \equiv \frac{Q_{it} / \bar{X}_{it}}{Q_{it}^{*} / X_{it}^{*}}$$ (input-oriented scale-mix efficiency) and

(10) $$CAE_{it} \equiv \frac{c(w_{it}, q_{it}, z_{it})}{W_{it} \bar{X}_{it}}$$ (cost-allocative efficiency)

where $\bar{X}_{it} \equiv X_{it} D_I (x_{it}, q_{it}, z_{it})^{-1}$ is the minimum aggregate input possible when using a scalar multiple of x_{it} to produce q_{it}; $\hat{X}_{it}$ is the minimum aggregate input possible using *any* input vector to produce q_{it}; $\tilde{Q}_{it}$ and $\tilde{X}_{it}$ are the aggregate output and input obtained when TFP is maximized subject to the constraint that the output and input vectors are scalar multiples of q_{it} and x_{it} respectively; Q_{it}^{*} and X_{it}^{*} are the aggregate output and input associated with the output-input combination that maximizes TFP; and $W_{it} \equiv w_{it}' x_{it} / X_{it}$ is an implicit aggregate input price. Input-oriented technical efficiency (ITE) is a measure of the increase in TFP (or the reduction in cost) that is possible by holding the output vector fixed and scaling down the input vector; input-oriented scale efficiency, mix efficiency and scale-mix efficiency (ISE, IME and ISME) are all measures of the increases in TFP that are possible by moving around the frontier surface to capture economies of scale and/or scope; and cost-allocative efficiency is a measure of the reduction in cost that is possible by moving around the frontier surface in order to reach the least-cost input vector capable of producing the output vector. These and other (residual) input-oriented measures of efficiency (and a set of analogous output-oriented measures) are discussed in more detail in O'Donnell (2008, 2010a).

If TFP is defined by (5) then the index that compares the TFP of firm *i* in period *t* with the TFP of firm 1 in period 1 is

(11) $$TFP_{11,it} \equiv \frac{TFP_{it}}{TFP_{11}} = \frac{Q_{it} / X_{it}}{Q_{11} / X_{11}} = \frac{Q_{11,it}}{X_{11,it}}$$

where $Q_{11,it} \equiv Q_{it} / Q_{11}$ and $X_{11,it} \equiv X_{it} / X_{11}$ are output and input quantity indexes respectively. O'Donnell (2008, 2010a) uses the term 'multiplicatively-complete' to describe TFP indexes that can be written in terms of aggregate outputs and inputs as in equation (11). He also shows how any multiplicatively-complete TFP index can be decomposed into recognisable measures of environment change and efficiency change. For example, equations (6) to (9) imply:

(12) $$TFP_{11,it} = \left(\frac{TFP_{it}^{*}}{TFP_{11}^{*}} \right) \left(\frac{ITE_{it}}{ITE_{11}} \right) \left(\frac{ISME_{it}}{ISME_{11}} \right)$$

where $TFP_{it}^{*} \equiv Q_{it}^{*} / X_{it}^{*}$ is the maximum TFP that is possible in a production environment characterized by z_{it}. The first term on the right-hand side of (12) is a measure of changes in the production environment. In the special case where $z_{it} = z(t)$ (i.e., the production environment only changes with the passage of time) this first term corresponds to common notions of technical change. The remaining terms in equation (12) are measures of technical efficiency change and scale-mix efficiency change. The technical efficiency change component captures changes in productivity as firms move towards or away from the production frontier. The scale-mix efficiency change component measures changes in productivity as firms move around the frontier surface.

In the special case where the technology is given by (1) and the (non-negative, non-decreasing and linearly homogeneous) functions $g(.)$ and $h(.)$ are used as output and input aggregator functions, all output-input combinations are fully mix-efficient and the TFP index defined above decomposes into measures of environment change, technical efficiency change, and pure scale efficiency change. Mathematically, if $Q(.) = g(.)$ and $X(.) = h(.)$ then

$$(13) \qquad TFP_{11,it} = \left(\frac{\exp(\gamma' z_{it})}{\exp(\gamma' z_{11})}\right)\left(\frac{D_I(x_{it}, q_{it}, z_{it})^{-1}}{D_I(x_{11}, q_{11}, z_{11})^{-1}}\right)\left(\frac{h(x_{it})D_I(x_{it}, q_{it}, z_{it})^{-1}}{h(x_{11})D_I(x_{11}, q_{11}, z_{11})^{-1}}\right)^{\eta-1}$$

where the interpretations of the components are obvious. If the technology also exhibits constant returns to scale (i.e. $\eta = 1$) then the last component in (13) disappears and TFP change is plausibly attributed to environment change and technical efficiency change only. The index defined by (13) satisfies important axioms and tests from index number theory, including an identity axiom and a transitivity test. The identity axiom says that if two firms use the same inputs to produce the same outputs then the TFP index equals one. The transitivity test means that a direct comparison of the TFP of two firms/periods will yield the same estimate of TFP change as an indirect comparison through a third firm/period (i.e., $TFP_{11,it} = TFP_{11,hs}TFP_{hs,it}$).

Equations (12) and (13) are input-oriented decompositions of TFP change. Analogous output-oriented decompositions are also available. Irrespective of the orientation or the aggregator functions used to construct the TFP index, decomposing the index into measures of environment change and efficiency change involves estimating the production technology represented by (1) to (3).

4. ECONOMETRIC MODELS

Econometric estimation of the production technology involves approximating the unknown functions $g(.)$ and $h(.)$. Common parametric approximations to $h(.)$ include:

$$(14) \qquad \ln h(x_{it}) = \sum_{k=1}^{K} \beta_k \ln x_{kit} + \varepsilon_{it} \qquad \text{(Cobb-Douglas)}$$

$$(15) \qquad \ln h(x_{it}) = \rho^{-1} \ln\left(\sum_{k=1}^{K} \lambda_k^{\rho} x_{kit}^{\rho}\right) + \varepsilon_{it} \qquad \text{(CES)}$$

$$(16) \qquad h(x_{it}) = \sum_{k=1}^{K}\sum_{h=1}^{K} \theta_{kh}(x_{kit}x_{hit})^{1/2} + \varepsilon_{it} \qquad \text{(Generalised Leontief) and}$$

$$(17) \qquad h(x_{it}) = \sum_{k=1}^{K} \phi_k x_{kit} + \varepsilon_{it} \qquad \text{(linear)}$$

where

$$(18) \qquad \beta_k \geq 0 \text{ for } k = 1,\dots,K \text{ and } \sum_{k=1}^{K} \beta_k = 1;$$

$$(19) \qquad \lambda_k \geq 0 \text{ for } k = 1,\dots,K \text{ and } \rho > 1;$$

$$(20) \qquad \theta_{kh} = \theta_{hk} \geq 0 \text{ for } h,k = 1,\dots,K,$$

$$(21) \qquad \phi_k \geq 0 \text{ for } k = 1,\dots,K,$$

and ε_{it} is an error of approximation. The constraints (18) to (21) guarantee that *h*(.) is non-negative, non-decreasing and linearly homogeneous as assumed in Section 2. Similar approximations are available for *g*(.). The Cobb-Douglas and translog approximations have been widely used in the hospital efficiency literature: Vitaliano and Toren (1996), Yong and Harris (1999), Chirikos and Sear (2000), Frech and Mobley (2000), Folland and Hofler (2001), Linna, Hakkinen and Magnussen (2006), Herr (2008) and Diaz and Sanchez (2008) have all used the Cobb-Douglas form; Smet (2007), Rosko and Proenca (2005), Carey (2003), McKay, Deily and Dorner (2002) and Deily, McKay and Dorner (2000) have all used the translog form. A smaller number of studies have used the linear form (e.g., Jacobs, 2001; Street 2003; and Street and Jacobs, 2002) and the Generalized Leontief form (e.g., Li and Rosenman, 2001).

The first step towards estimating the production technology usually involves selecting parametric approximations to both *g*(.) and *h*(.). The second step then involves rewriting either the output or input distance function in the form of a conventional stochastic frontier model. The choice of whether to work with the output or input distance function usually depends on whether outputs or inputs are regarded as endogenous. If inputs (outputs) are regarded as endogenous then one of the inputs (outputs) is typically chosen as the dependent variable and the remaining inputs (outputs) are used as explanatory variables. For example, if inputs are regarded as endogenous and if *g*(.) and *h*(.) are both approximated by linearly homogenous Cobb-Douglas functions then the input distance function (3) can be written in the form:

$$(22) \qquad -\ln x_{Kit} = \delta' z_{it} + \sum_{k=1}^{K-1} \beta_k \ln(x_{kit} / x_{Kit}) - \sum_{j=1}^{J} \alpha_j \ln q_{jit} + v_{it} - u_{it}$$

where

$$(23) \qquad \alpha_j \geq 0 \text{ for } j = 1,\ldots,J \text{ and } \sum_{j=1}^{J} \alpha_j = \eta^{-1}.$$

The error term v_{it} in (22) arises from the use of Cobb-Douglas functions to approximate *g*(.) and *h*(.). The error term $u_{it} \equiv \ln D_I(x_{it}, q_{it}, z_{it}) = -\ln ITE_{it} \geq 0$ is a technical inefficiency effect. Equation (22) is in the form of the conventional stochastic frontier model of Aigner, Lovell and Schmidt (1977).

Deriving econometric models such as (22) involves an asymmetric treatment of the log-inputs – one log-input is arbitrarily selected as the endogenous dependent variable while all other log-inputs are treated as exogenous explanatory variables. Maximum likelihood estimation of such models is possible in the restrictive special case where a linear function of all *K* endogenous log-inputs is exogenous[6]. In most other cases at least two log-inputs will be correlated with the error terms and maximum likelihood estimation may yield biased and inconsistent estimates. The problem is known as the ‘endogeneity problem’.

One solution to the problem is to estimate the frontier model using the generalized method of moments (GMM). An advantage of the GMM approach is that it does not require any assumptions concerning the shapes of the distributions of the error terms (e.g., normal, half-normal). A disadvantage of the approach is that, in practice, plausible moment conditions can be difficult to find. Results may also be sensitive to the choice of weight matrix used to form the GMM criterion function. GMM methodology has been used to estimate hospital production technologies by Bradford et al. (2001) and Biorn et al. (2002).

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[6] Coelli (2000, p. 10-16) shows that it is also justified in the case where firms minimise costs and there are no approximation errors or other sources of statistical noise (i.e., when the frontier is deterministic).

An alternative Bayesian solution to the endogeneity problem has been developed by Fernandez, Koop and Steel (2000). An advantage of the Bayesian approach is that it can be used to draw exact finite sample inferences concerning (nonlinear functions of) the unknown parameters. A disadvantage of the approach is that, in practice, it can be computationally intensive. Empirical examples include Fernandez et al. (2000) and O'Donnell (2011).

This paper solves the endogeneity problem at the very first step: instead of selecting parametric approximations to both *g*(.) and *h*(.), we use linear programming (LP) methods to estimate one of *g*(.) or *h*(.) and then select a parametric approximation to the other. The second step still involves rewriting either the output or input distance function in the form of a conventional stochastic frontier model. However, unlike the approach that led to equation (22), our approach yields a frontier model where all the explanatory variables are unambiguously exogenous. For example, if inputs are endogenous, outputs are exogenous, and *g*(.) is approximated by the same Cobb-Douglas function as the one used to derive equation (22), the input distance function (3) takes the form

$$(24) \qquad -\ln \hat{h}_{it} = \delta' z_{it} - \sum_{j=1}^{J} \alpha_j \ln q_{jit} + v_{it} - u_{it}$$

where $\hat{h}_{it}$ is an LP estimate of $h_{it} \equiv h(x_{it})$. The error term v_{it} is now associated with the use of $\hat{h}_{it}$ to approximate h_{it} and a Cobb-Douglas function to approximate *g*(.). By way of further example, if outputs are endogenous, inputs are exogenous, and *h*(.) is approximated by the Generalised Leontief function (16), the output distance function (2) takes the form:

$$(25) \qquad \ln \hat{g}_{it} = \gamma' z_{it} + \sum_{k=1}^{K} \sum_{h=1}^{K} \kappa_{kh} (x_{kit} x_{hit})^{1/2} + v_{it} - u_{it}$$

where $\kappa_{kh} = \eta\theta_{kh}$ and $u_{it} \equiv -\ln D_O(x_{it}, q_{it}, z_{it}) \geq 0$ is now an output-oriented technical inefficiency effect. The error term v_{it} now accounts for the facts that $\hat{g}_{it}$ is an LP estimate of $g_{it} \equiv g(q_{it})$ and equation (16) has been used to approximate *h*(.). In contrast to equation (22), equations (24) and (25) are stochastic frontier models in which all the explanatory variables are exogenous.

5. QUANTITY INDEXES

Models such as (24) and (25) are commonplace in the empirical economics literature. For example, every stochastic production frontier model that uses (the logarithm of) a Laspeyres, Paasche or Lowe[7] output quantity index to form the dependent variable is a model of this type. These particular indexes are members of the class of linear estimators (or filters) defined by

$$(26) \qquad \hat{g}_{it} = \mu_{it}' q_{it}$$

where $\mu_{it} = (\mu_{1it}, \ldots, \mu_{Jit})' \geq 0$. The Laspeyres, Paasche and Lowe quantity indexes are obtained by selecting

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[7] The properties of Laspeyres and Paasche quantity indexes are well-known. For a discussion of Lowe quantity indexes, see O'Donnell (2010b).

(27) $\mu_{jit} = \dfrac{p_{j00}}{p'_{00}q_{00}}$ (Laspeyres)

(28) $\mu_{jit} = \dfrac{p_{jit}}{p'_{it}q_{00}}$ (Paasche) and

(29) $\mu_{jit} = \dfrac{\bar{p}_j}{\bar{p}'_j q_{00}}$ (Lowe)

where $p_{it} = (p_{1it},...,p_{Jit})' \geq 0$ denotes the vector of output prices faced by firm *i* in period *t* and $\bar{p} = (\bar{p}_1,...,\bar{p}_J)' \geq 0$ is a vector of arbitrary reference prices (e.g., sample average prices). Similarly, Laspeyres, Paasche and Lowe input quantity indexes are linear estimators of the form

(30) $\hat{h}_{it} = \phi'_{it} x_{it}$

where $\phi_{it} = (\phi_{1it},...,\phi_{Kit})' \geq 0$. Logarithms of these quantity indexes have been used to measure health care outputs and inputs and/or to construct the dependent variables in stochastic frontier models by Mai (2004), Ibiwoye (2010), Yu and Ariste (2008) and Yu (2011). Other common quantity indexes (e.g., Fisher, Tornquist, Hicks-Moorsteen) can be viewed as nonlinear estimators.

This paper estimates $g_{it} \equiv g(q_{it})$ and $h_{it} \equiv h(x_{it})$ using the linear estimators defined by (26) and (30). We intend to apply the methodology in an empirical context where prices are unavailable, so we select the weight vectors in (26) and (30) using linear programming.

To compute $\hat{g}_{it}$ we let $U = \max(\hat{g}_{11},...,\hat{g}_{NT}) < \infty$ denote the maximum value of $\hat{g}_{it}$ in the sample. Computing $\hat{g}_{it}$ is simply a matter of selecting $\mu_{it} \geq 0$ to maximise $\hat{g}_{it} = \mu'_{it} q_{it} \leq U$. Aside from the non-negativity constraint, the only constraints on μ_{it} are that if it is used to compute an estimate of $g_{hs} \equiv g(q_{hs})$ for any $h \in \mathbb{Z}^N_{++}$ or $s \in \mathbb{Z}^T_{++}$ (i.e., for any observation in the sample) then that estimate should also be no greater than *U*. The linear program (LP) is:

(31) $\hat{g}_{it} = \max\limits_{\mu_{it}, U} \quad \mu'_{it} q_{it}$

(32) s.t. $\mu'_{it} q_{hs} \leq U$ for $h = 1,...,N$ and $s = 1,...,T$

(33) $U = 1$

(34) $\mu_{it} \geq 0.$

The constraint (33) is a normalizing constraint that allows $\hat{g}_{it}$ to be interpreted as a quantity index[8]. It also identifies a unique solution to the problem in the same way that normalising constraints are used to identify unique solutions to data envelopment analysis (DEA) problems. Indeed, for practical purposes, it is useful to consider the standard DEA problem for estimating technical efficiency under the assumption of no technical change and constant returns to scale[9] (e.g., O'Donnell, 2010a, p. 543, eq. 6.5):

[8] Let $(s,h) = \arg\max_{s,h}\{\hat{g}_{hs} \geq \hat{g}_{it}$ for all $h, i \in \mathbb{Z}^N_{++}$ and $s,t \in \mathbb{Z}^T_{++}\}$. Then $\hat{g}_{hs} = U = 1$ and $Q_{hs,it} = \hat{g}_{it} / \hat{g}_{hs} = \hat{g}_{it}$ is an index that compares the outputs of firm *i* in period *t* with the outputs of firm *h* in period *s*.

[9] In the DEA literature there is a convention to suppress subscripts indicating that μ_{it} may vary from one observation (or LP) to the next – see O'Donnell (2010a, p. 542) .

(35) $TE_{it} = \max_{\mu_{it},\upsilon_{it}} \quad \mu'_{it} q_{it}$

(36) s.t. $\mu'_{it} q_{hs} \le \upsilon'_{it} x_{hs}$ for $h = 1,...,N$ and $s = 1,...,T$

(37) $\upsilon'_{it} x_{hs} = 1$

(38) $\mu_{it}, \upsilon_{it} \ge 0.$

Observe that this problem collapses to the problem given by (31) to (34) whenever inputs are the same for all firms in all time periods (i.e.., $x_{hs} = x_{it}$ for $i,h = 1,...,N$ and $s,t = 1,...,T$). Thus, it is possible (and convenient) to compute $\hat{g}_{it}$ by constructing an artificial dataset in which all the input values are replaced by a constant, and then solving the standard DEA problem under the assumption of no technical change and constant returns to scale. Importantly, the 'technical efficiency' estimates produced using this artificial data set are estimates of $g_{it} \equiv g(q_{it})$ and should not be interpreted as measures of technical efficiency.

In a similar way, we let $L = \min(\hat{h}_{11},...,\hat{h}_{NT}) > 0$ and compute a value of $\hat{h}_{it}$ by selecting $\phi_{it} \ge 0$ to minimise $\hat{h}_{it} = \phi'_{it} x_{it} \ge L$. The LP is:

(39) $\hat{h}_{it} = \min_{\phi_{it},L} \quad \phi'_{it} x_{it}$

(40) s.t. $\phi'_{it} x_{hs} \ge L$ for $h = 1,...,N$ and $s = 1,...,T$

(41) $L = 1$

(42) $\phi_{it} \ge 0.$

Again, it is convenient to compute $\hat{h}_{it}$ by constructing an artificial data set in which all the output values are replaced by a constant. We then solve the standard DEA LP under the assumption of no technical change and constant returns to scale. The reciprocals of the 'technical efficiency' estimates obtained using this artificial dataset are estimates of $h_{it} \equiv h(x_{it})$ and should not be interpreted as (reciprocals of) measures of efficiency.

6. DATA AND VARIABLES

We apply the methodology to data on J = 3 outputs, K = 3 inputs and $M - 1 = 2$ environmental variables drawn from the InfoBank and Casemix databases of Queensland Health. The dataset is a balanced panel covering N = 116 public hospitals in the state of Queensland over the T = 9 financial years (i.e., years ended 30 June) from 1996 to 2004. The vectors of outputs, inputs and environmental variables are:

$$q_{it} = (OUTP_{it}, WESC_{it}, WEMC_{it})'$$
$$x_{it} = (MO_{it}, NURS_{it}, BEDS_{it})' \quad \text{and}$$
$$z_{it} = (1, t, REGION_{it})'$$

where

$OUTP_{it}$ = the number of outpatient occasions of service (for firm i in period t). Outpatients are patients who are not formally admitted to hospital. Outpatient services include emergency department visits as well as pathology, radiology, speech therapy and family planning examinations, consultations, treatments and services.

$WESC_{it} =$ the number of weighted episodes of surgical care. An episode of care is a period of care provided to an admitted hospital patient and characterised by a single treatment type. Episodes of surgical care are those that involve an operating room procedure. Weights are assigned to different types of surgical procedures according to an Australian Refined system of Diagnosis Related Groups (AR-DRGs).

$WEMC_{it} =$ the number of weighted episodes of medical care. These episodes of care do not involve any type of procedure. Weights are also assigned to different types of medical care under the AR-DRGs system.

$MO_{it} =$ the number of full-time equivalent (FTE) medical officers. Medical officers include both staff medical officers (usually a mix of general practitioners and specialists) and visiting medical officers (specialists only).

$NURS_{it} =$ the number of full-time equivalent (FTE) nurses. Nurses include registered, enrolled, clinical and assistant nurses.

$BEDS_{it} =$ the number of beds (a measure of the capital input).

$REGION_{it} = 1$ for the two hospitals (Cooktown and Weipa) located in the tropical Cape York region; = 0 otherwise.

Descriptive statistics for all output, input and environmental variables are reported in Table 1. Unfortunately, we were unable to find any hospital-specific measures of output quality (e.g., re-admission rates) or input quality (e.g., patient characteristics) for use in the analysis. One of the advantages of the econometric approach to efficiency analysis is that any errors associated with omitting these types of variables can be subsumed into the idiosyncratic error term.

7. RESULTS

We treat inputs as endogenous and estimate the preferred model given by equation (24) (hereafter referred to as Model 24) and the conventional model given by equation (22) (hereafter referred to as Model 22). In both cases, the set of all NT observations can be written in the compact form:

$$y = X\beta + \upsilon - u \tag{43}$$

where $\upsilon = (\upsilon_{11}, \upsilon_{12}, ..., \upsilon_{NT})'$ and the remaining definitions are obvious, although it is worth noting that X is $NT \times (M + J + K - 1)$ in the case of Model 22 and $NT \times (M + J)$ in the case of Model 24. To estimate both models we assume the idiosyncratic and technical inefficiency errors are normal and half-normal respectively: $\upsilon \sim N(0, \sigma_\upsilon^2 I_{NT})$

and $u \sim N^{+}(0, \sigma_u^2 I_{NT})$ where I_{NT} denotes an identity matrix of order *NT*. This section reports maximum likelihood estimates of the unknown parameters and associated economic quantities of interest.

Parameters

Estimates of the parameters are reported in Table 2. Most[10] of the estimates obtained using Model 22 are qualitatively similar to those obtained using Model 24. To avoid repetition, we focus on the estimates obtained using our preferred model, Model 24. Using this model, the estimated elasticity of scale is $\hat{\eta} = 1.1997 > 1$ (and significantly different from one) indicating that the technology exhibits increasing returns to scale. The estimated coefficient of the time trend is $\hat{\delta}_1 = -0.0322 < 0$ (and significantly different from zero) indicating that the sector experienced technical regress over the sample period. Indeed, the annual rate of technical regress is estimated to be $\partial \ln \hat{D}_O(x_{it}, q_{it}, z_{it}) / \partial t = \hat{\gamma}_1$ $\approx -0.0322 \times 1.1997 = -3.9\%$. In the present context, technical regress means that advances in medical technology have not been fast enough or large enough to offset the effects of factors that cause deteriorations in the hospital production environment (e.g., increased resistance to antibiotics[11]) – this phenomenon is sometimes referred to as the Red Queen effect[12]. Finally, the estimate of $\lambda = \sigma_u / \sigma_\upsilon$ is significantly different from zero indicating there is technical inefficiency in this dataset.

Productivity and Efficiency (Change)

Table 3 reports estimates of (the components of) the productivity index defined by (13). This particular table compares the performance of selected hospitals in selected years with the performance of hospital 1 in 1996. Hospital 1 is a large urban hospital providing a full range of healthcare services, including a 24-hour emergency department, intensive care, coronary care, day surgery, oncology and hospice respite services. The measures of performance reported in Table 3 are

(44) $$TFP_{11,it} = ENV_{11,it} \times EFF_{11,it}$$ (TFP change)

(45) $$ENV_{11,it} = TIME_{11,it} \times REGION_{11,it}$$ (change in the production environment) and

(46) $$EFF_{11,it} = ITE_{11,it} \times ISE_{11,it}$$ (efficiency change)

where

(47) $$TIME_{11,it} = \frac{\exp(\gamma_1 t)}{\exp(\gamma_1)}$$ (change in the production environment over time)

(48) $$REGION_{11,it} = \frac{\exp(\gamma_2 REGION_{it})}{\exp(\gamma_2 REGION_{11})}$$ (change in the production environment across regions)

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10 Model 24 does not involve the parameters β_1 or β_2. Model 22 yields $\hat{\beta}_1 = \text{st.error}(\hat{\beta}_1) = 0$ because the constraint $\beta_1 \geq 0$ is binding.

11 For information on the severity of the problem of antibiotic resistance, see Miller and Miller (2011).

12 The term is a reference to the Red Queen's race in Lewis Carroll's *Through the Looking-Glass*. The Red Queen said " it takes all the running you can do, to keep in the same place. If you want to get somewhere else, you must run at least twice as fast as that!"

(49) $$ITE_{11,it} = \frac{D_I(x_{it}, q_{it}, z_{it})^{-1}}{D_I(x_{11}, q_{11}, z_{11})^{-1}}$$ (input-oriented technical efficiency change) and

(50) $$ISE_{11,it} = \left(\frac{h(x_{it}) D_I(x_{it}, q_{it}, z_{it})^{-1}}{h(x_{11}) D_I(x_{11}, q_{11}, z_{11})^{-1}} \right)^{\eta-1}.$$ (input-oriented scale efficiency change)

The measures given by (47) and (48) were evaluated by replacing the unknown parameters γ_1 and γ_2 with their maximum likelihood estimates. The numerator and denominator in equation (49) were evaluated separately using the Battese and Coelli (1988) technical efficiency estimator. The aggregate outputs $h(x_{it})$ and $h(x_{11})$ in equation (50) were evaluated separately using predictions from equation (14) (in the case of Model 22) or $\hat{h}_{it}$ and $\hat{h}_{11}$ (in the case of Model 24). Estimates of the components (47) to (50) were then used to compute estimates of (44) to (46).

Interpretation of the index numbers reported in Table 3 is straightforward. For example, the row corresponding to observation 929 reveals the following: hospital 1 was only 2.5% less productive in 2004 than it had been in 1996 ($TFP_{11,1T} = 0.9758$); a 33% improvement in the efficiency of hospital 1 ($EFF_{11,1T} = 1.3287$) was not enough to offset the effects of a 27% deterioration in the production environment ($ENV_{11,1T} = 0.7344$); and by far the largest contribution to the estimated efficiency improvement of hospital 1 came through technical efficiency improvement ($ITE_{11,1T} = 1.3054$). We conclude that the technical efficiency of hospital 1 increased (distance to the frontier decreased), not so much because the hospital used fewer inputs to produce the same outputs, but because the frontier moved inwards and in 2004 was much closer to the point where hospital 1 was operating than it had been in 1996. Some hospitals were unable to maintain their productivity levels in the face of this deterioration in the production environment. For example, the rows corresponding to observations 116 and 1044 reveal that hospital 116 experienced the same deterioration in the production environment and yet was only half as productive in 2004 as it had been in 1996: $TFP_{h1,hT} = TFP_{h1,11} \times TFP_{11,hT} = TFP_{11,h1}^{-1} \times TFP_{11,hT} = 0.7397^{-1} \times\ 0.3826 = 0.5172$ for $h = 116$. The calculations in this last example exploit the facts that the TFP index defined by (13) satisfies all economically-relevant axioms and tests from index number theory, including a transitivity test and a time and space reversal test. Transitivity is especially important in the present context – it means it is possible to compare the performance of all hospitals in all periods with the performance of an arbitrary hospital h in an arbitrary period s by simply dividing all the rows in the table by the row corresponding to hospital h in period s. For example, Table 3 compares the performance of selected hospitals in selected years with the performance of hospital 1 in 1996. If we were interested in comparing the performance of these hospitals with the performance of hospital 107 in 1996, say, then we would divide every row in the table by the row for observation 107.

The observation-by-observation results and the descriptive statistics reported at the bottom of Table 3 reveal that most of the productivity and efficiency estimates obtained using Model 22 are qualitatively similar to those obtained using Model 24. To get a clearer picture of the similarities between the two sets of estimates, Figure 1 presents distributions of productivity and efficiency indexes for the $N = 116$ hospitals in 1996. Observe that most hospitals tended to be less productive and less scale efficient, but more technically efficient, than hospital 1 (in that year). They tended to be less productive because they were less scale efficient, and they tended to be less scale efficient because they were smaller than hospital 1 and (we estimate that) the technology everywhere exhibits increasing returns to scale.

Point and interval estimates of levels of input-oriented technical efficiency are reported in Table 4. The results obtained using Model 24 indicate that the input-oriented technical efficiency of hospital 1, for example, increased from 0.57 to 0.74 over the sample period (i.e., as we saw earlier, $ITE_{11,1T} = ITE_{1T} / ITE_{11} = 0.7441 / 0.5701 = 1.3054$). One of the payoffs from estimating the distance function in an econometric framework is that it is straightforward to construct confidence intervals for estimated efficiency scores. Table 4 reports 95% confidence interval limits computed using the results of Horrace and Schmidt (1996). Again, the interpretation of these confidence intervals is straightforward. For example, we are 95% confident that in 2004 the technical efficiency of hospital 1 was between 50% and 98%.

Efficient Prices

One of the advantages of estimating the production technology in a parametric framework is that we can obtain analytical expressions for the partial derivatives of the output distance function with respect to output quantities. These derivatives are revenue-deflated support (or shadow) prices. If the technology is represented by (1) then the revenue-deflated support prices for firm *i* in period *t* are

(51) $$\frac{p^*_{kit}}{p'_{it}q_{it}} = \frac{\partial \ln g(q_{it})}{\partial q_{kit}} D_O(x_{it}, q_{it}, z_{it}) = \frac{\partial \ln g(q_{it})}{\partial q_{kit}} \frac{1}{D_I(x_{it}, q_{it}, z_{it})^{\eta}} \quad \text{for } k = 1,...,K.$$

The minimum (i.e., efficient) prices that would need to have been paid to hospital *i* in period *t* so that it could have produced q_{it} and still run a balanced budget are

(52) $$p^*_{kit} = \frac{\partial \ln g(q_{it})}{\partial q_{kit}} \frac{c(w_{it}, q_{it}, z_{it})}{D_I(x_{it}, q_{it}, z_{it})^{\eta}} \quad \text{for } k = 1,...,K.$$

To estimate these prices we observe that if *g*(.) is approximated by the same Cobb-Douglas function that was used to derive Models 22 and 24 then $\partial \ln g(q_{it}) / \partial q_{kit} = \alpha_k / q_{kit}$ and

(53) $$p^*_{kit} = \left(\frac{c(w_{it}, q_{it}, z_{it})}{w'_{it}x_{it}D_I(x_{it}, q_{it}, z_{it})^{-1}} \right) \left(\frac{\alpha_k}{q_{kit}} \frac{w'_{it}x_{it}}{D_I(x_{it}, q_{it}, z_{it})^{1+\eta}} \right) \quad \text{for } k = 1,...,K.$$

The first term in parentheses on the right-hand side of (53) is the measure of cost-allocative efficiency defined by equation (10). In this paper we assume that all hospitals are cost-allocatively efficient (i.e., $CAE_{it} = 1$) and estimate efficient prices by evaluating the second term on the right-hand side of (53). Estimated efficient prices for a subset of hospitals are reported in Table 5. If hospitals are not cost-allocatively efficient (i.e., if $CAE_{it} < 1$) then the estimated prices reported in Table 5 are upper bounds on the prices that would have allowed these hospitals to produce the same outputs and run balanced budgets. Irrespective of levels of cost-allocative efficiency, relative efficient prices are

(54) $$\frac{p^*_{kit}}{p^*_{jit}} = \frac{\alpha_k}{\alpha_j} \frac{q_{jit}}{q_{kit}} \quad \text{for } j,k = 1,...,K.$$

Estimates of these relative prices are also reported in Table 5. The large variations in estimated support price ratios reported in Table 5 can be attributed to large variations in hospital output ratios. In turn, large variations in output ratios reflect the fact that Australian public hospitals have a legal obligation to provide medical and healthcare services to all public patients, which is to say that outputs are determined exogenously, not by hospitals attempting to maximise revenue in a competitive market environment.

8. SUMMARY

Expenditure on public hospitals is the largest single component of Australian Commonwealth, state and territory government recurrent expenditure on health. In 2010 the Commonwealth government announced that it will become the majority funder of the efficient price (i.e., cost) of hospital and outpatient services provided to public patients. This paper shows how econometric methods can be used to estimate the minimum (i.e., economically efficient) prices of individual hospital services provided to public patients in different locations at different points in time. Our approach can be implemented without data on input prices or input cost shares, and without any overly restrictive assumptions concerning hospital optimising behaviour or the structure of product markets. However, data on input and output quantities is needed in order to estimate a distance function representation of the hospital production technology.

Estimating distance functions is complicated by the fact that some of the explanatory variables in the estimating equation may be correlated with the error term. Our solution to the so-called endogeneity problem involves the use of linear programming methods to construct a quantity index. This quantity index allows us to re-write the distance function in the form of a conventional stochastic frontier model in which all the explanatory variables are exogenous.

The methodology was applied to data covering 116 Queensland public hospitals over the period 1996 to 2004. The estimated parameters of the distance function were used to estimate spatially- and temporally-transitive indexes of total factor productivity change. The decomposition methodology of O'Donnell (2008) was then used to decompose these indexes into two measures of environment change (i.e., time and region) and two measures of efficiency change (i.e., technical efficiency and scale efficiency). We found that the productivity effects of improvements in input-oriented technical efficiency tended to be offset by deteriorations in the hospital production environment. We conclude that improvements in technical efficiency have been caused mainly by an inward shift in the production frontier and not by any significant changes in input or output levels – the frontier has been moving closer to the hospitals rather than the hospitals moving closer to the frontier. Factors that contribute to such movements in the frontier include increasing resistance to antibiotics.

The estimated parameters of the distance function were also used to estimate support (i.e., efficient) prices for individual hospital outputs. Large variations in these estimated support prices were plausibly due to large variations in the outputs themselves. These estimated support prices can be viewed as upper bounds on the hospital- and output-specific prices that an independent ‘umpire’ might set in order to meet the Commonwealth government’s hospital efficiency and fiscal objectives.

Table 1. Variables

VARIABLE	MEAN	ST. DEV.	MINIMUM	MAXIMUM
OUTP	64227	121710	806	1012000
WESC	2276.9	6340.4	0.1	44825.0
WEMC	3862.9	7573.4	85.5	51536.0
MO	29.26	78.52	0.20	606.95
NURS	114.58	257.52	6.09	1864.20
BEDS	80.22	148.22	2	1138
t	5.00	2.58	1	9
REGION	0.02	0.13	0	1

Table 2. Parameters

PARAMETER	VARIABLE	MODEL 24 - PREFERRED			MODEL 22 - CONVENTIONAL		
		ESTIMATE	ASY. ST. ERROR	ASY. T-RATIO	ESTIMATE	ASY. ST. ERROR	ASY. T-RATIO
$\delta_0 = \eta^{-1}\gamma_0$	CONSTANT	4.7095	0.1260	37.3860	2.0741	0.1265	16.4010
$\delta_1 = \eta^{-1}\gamma_1$	t	-0.0322	0.0044	-7.2830	-0.0160	0.0044	-3.6244
$\delta_2 = \eta^{-1}\gamma_2$	REGION	0.1252	0.0819	1.5284	0.2265	0.0797	2.8414
β_1	MO	-	-	-	0.0000	0.0000	0.0000
β_2	NURS	-	-	-	0.7271	0.0307	23.6590
α_1	OUTP	0.0551	0.0208	2.6497	0.0009	0.0194	0.0467
α_2	WESC	0.0684	0.0082	8.3286	0.0620	0.0076	8.1800
α_3	WEMC	0.7100	0.0249	28.4957	0.7051	0.0237	29.6998
η		1.1997	0.0170	70.7241	1.3021	0.0206	63.1731
σ_υ		0.5240	0.0223	23.4750	0.4774	0.0200	23.9180
λ		1.9173	0.2676	7.1657	1.7652	0.2398	7.3599

Table 3. The Components of TFP Change

Obs	Hospital	Year	MODEL 24 - PREFERRED									MODEL 22 - CONVENTIONAL								
			$Q_{11,it}$	$X_{11,it}$	$TFP_{11,it}$	$ENV_{11,it}$	$EFF_{11,it}$	$TIME_{11,it}$	$REGION_{11,it}$	$ITE_{11,it}$	$ISE_{11,it}$	$Q_{11,it}$	$X_{11,it}$	$TFP_{11,it}$	$ENV_{11,it}$	$EFF_{11,it}$	$TIME_{11,it}$	$REGION_{11,it}$	$ITE_{11,it}$	$ISE_{11,it}$
1	1	1996	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	1996	0.5313	0.6569	0.8088	1	0.8088	1	1	0.8986	0.9001	0.5175	0.6226	0.8313	1	0.8313	1	1	0.9686	0.8583
3	3	1996	0.7642	0.7031	1.0869	1	1.0869	1	1	1.1366	0.9562	0.742	0.6593	1.1254	1	1.1254	1	1	1.2061	0.9331
4	4	1996	1.9113	2.1429	0.8919	1	0.8919	1	1	0.8008	1.1139	2.0367	2.1582	0.9437	1	0.9437	1	1	0.8001	1.1794
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
107	107	1996	0.0484	0.0571	0.8478	1.1621	0.7296	1	1.1621	1.2382	0.5892	0.0445	0.0593	0.7502	1.3429	0.5586	1	1.3429	1.2315	0.4536
108	108	1996	0.0262	0.0329	0.7961	1	0.7961	1	1	1.4593	0.5455	0.0232	0.0411	0.5637	1	0.5637	1	1	1.3503	0.4175
109	109	1996	0.0337	0.041	0.8218	1	0.8218	1	1	1.4449	0.5687	0.0331	0.0562	0.5892	1	0.5892	1	1	1.2993	0.4535
110	110	1996	0.0232	0.0331	0.7001	1	0.7001	1	1	1.31	0.5345	0.0232	0.0455	0.5109	1	0.5109	1	1	1.2229	0.4177
111	111	1996	0.0133	0.0261	0.5089	1	0.5089	1	1	1.0445	0.4872	0.0121	0.0314	0.3855	1	0.3855	1	1	1.0732	0.3592
112	112	1996	0.3739	0.4337	0.862	1	0.862	1	1	1.0154	0.8489	0.3553	0.4341	0.8183	1	0.8183	1	1	1.0404	0.7865
113	113	1996	0.0233	0.0288	0.8072	1	0.8072	1	1	1.5093	0.5348	0.0217	0.0384	0.5649	1	0.5649	1	1	1.3737	0.4112
114	114	1996	0.0124	0.0288	0.4306	1	0.4306	1	1	0.8943	0.4814	0.0123	0.0351	0.3495	1	0.3495	1	1	0.9699	0.3603
115	115	1996	0.0175	0.0257	0.6821	1	0.6821	1	1	1.3376	0.51	0.013	0.0264	0.4911	1	0.4911	1	1	1.3461	0.3648
116	116	1996	0.0236	0.0319	0.7397	1	0.7397	1	1	1.3802	0.5359	0.0164	0.0307	0.534	1	0.534	1	1	1.3862	0.3853
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
929	1	2004	0.817	0.8372	0.9758	0.7344	1.3287	0.7344	1	1.3054	1.0179	0.8163	0.814	1.0027	0.8469	1.184	0.8469	1	1.1942	0.9915
930	2	2004	0.6454	0.8108	0.7959	0.7344	1.0838	0.7344	1	1.1074	0.9787	0.6123	0.6914	0.8856	0.8469	1.0456	0.8469	1	1.1274	0.9275
931	3	2004	0.6421	0.7087	0.9061	0.7344	1.2338	0.7344	1	1.2617	0.9779	0.6309	0.6447	0.9785	0.8469	1.1554	0.8469	1	1.2371	0.934
932	4	2004	1.7638	2.4324	0.7251	0.7344	0.9874	0.7344	1	0.8534	1.157	1.8717	2.2427	0.8346	0.8469	0.9854	0.8469	1	0.8198	1.202
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
1035	107	2004	0.0396	0.0626	0.6324	0.8534	0.7411	0.7344	1.1621	1.2356	0.5998	0.0394	0.0685	0.576	1.1374	0.5064	0.8469	1.3429	1.1047	0.4584
1036	108	2004	0.0178	0.0298	0.596	0.7344	0.8115	0.7344	1	1.5078	0.5382	0.0167	0.0361	0.4625	0.8469	0.5461	0.8469	1	1.3579	0.4022
1037	109	2004	0.0266	0.0514	0.5181	0.7344	0.7055	0.7344	1	1.2253	0.5757	0.0265	0.0615	0.4313	0.8469	0.5093	0.8469	1	1.1375	0.4478
1038	110	2004	0.0176	0.0319	0.5527	0.7344	0.7526	0.7344	1	1.3999	0.5376	0.0185	0.0431	0.4296	0.8469	0.5073	0.8469	1	1.2317	0.4119
1039	111	2004	0.0093	0.0308	0.3032	0.7344	0.4128	0.7344	1	0.8539	0.4835	0.0089	0.0342	0.2602	0.8469	0.3072	0.8469	1	0.8841	0.3475
1040	112	2004	0.3336	0.4235	0.7878	0.7344	1.0727	0.7344	1	1.2232	0.8769	0.3108	0.3896	0.7977	0.8469	0.9419	0.8469	1	1.1885	0.7925
1041	113	2004	0.0204	0.0376	0.5417	0.7344	0.7376	0.7344	1	1.3394	0.5507	0.0187	0.0418	0.4486	0.8469	0.5297	0.8469	1	1.2823	0.4131
1042	114	2004	0.0119	0.0303	0.3911	0.7344	0.5326	0.7344	1	1.0584	0.5032	0.0113	0.0327	0.345	0.8469	0.4073	0.8469	1	1.109	0.3673
1043	115	2004	0.0188	0.0419	0.4484	0.7344	0.6105	0.7344	1	1.1237	0.5433	0.0171	0.0424	0.4031	0.8469	0.476	0.8469	1	1.1773	0.4043
1044	116	2004	0.0147	0.0385	0.3826	0.7344	0.5209	0.7344	1	0.9986	0.5216	0.0131	0.0364	0.3597	0.8469	0.4248	0.8469	1	1.1174	0.3801

MEAN	0.2954	0.3323	0.7167	0.8636	0.8323	0.8612	1.0027948	1.254	0.6717	0.2993	0.3277	0.6112	0.927	0.6606	0.9216	1.0059121	1.1624	0.5778
MINIMUM	0.0064	0.018	0.2072	0.7344	0.2695	0.7344	1	0.5644	0.4315	0.0061	0.0143	0.1419	0.8469	0.1641	0.8469	1	0.4558	0.3087
MAXIMUM	4.8226	5.4545	1.2337	1.1621	1.4101	1	1.1621	1.6723	1.3582	5.0957	5.1548	1.2231	1.3429	1.3217	1	1.3429	1.4585	1.4947

Table 4. Levels of Input-Oriented Technical Efficiency

Obs	Hospital	Year	MODEL 24 - PREFERRED			MODEL 22 - CONVENTIONAL		
			2.5% limit	ITE	97.5% limit	2.5% limit	ITE	97.5% limit
1	1	1996	0.3666	0.5701	0.8415	0.4197	0.6358	0.9066
2	2	1996	0.3288	0.5123	0.7615	0.4056	0.6158	0.8857
3	3	1996	0.4202	0.6479	0.9256	0.5262	0.7668	0.9805
4	4	1996	0.2928	0.4565	0.6795	0.3336	0.5087	0.7435
:	:	:	:	:	:	:	:	:
107	107	1996	0.4645	0.7059	0.962	0.5424	0.783	0.984
108	108	1996	0.5935	0.8319	0.9917	0.6365	0.8585	0.9938
109	109	1996	0.5827	0.8237	0.9908	0.5915	0.8261	0.9907
110	110	1996	0.4999	0.7468	0.9768	0.5368	0.7775	0.9829
111	111	1996	0.3836	0.5954	0.873	0.454	0.6824	0.9448
112	112	1996	0.3724	0.5788	0.8528	0.4383	0.6615	0.9296
113	113	1996	0.6348	0.8604	0.9942	0.6602	0.8734	0.995
114	114	1996	0.3272	0.5098	0.7579	0.4062	0.6167	0.8866
115	115	1996	0.5148	0.7625	0.9808	0.6324	0.8558	0.9936
116	116	1996	0.5395	0.7868	0.9856	0.6739	0.8813	0.9955
:	:	:	:	:	:	:	:	:
929	1	2004	0.4975	0.7441	0.976	0.5189	0.7592	0.9786
930	2	2004	0.4083	0.6313	0.9109	0.4814	0.7168	0.9637
931	3	2004	0.4756	0.7193	0.9677	0.5461	0.7865	0.9847
932	4	2004	0.3121	0.4865	0.7238	0.3419	0.5212	0.7615
:	:	:	:	:	:	:	:	:
1035	107	2004	0.4633	0.7044	0.9613	0.4697	0.7024	0.9567
1036	108	2004	0.6334	0.8595	0.9941	0.644	0.8633	0.9942
1037	109	2004	0.4586	0.6985	0.9585	0.4868	0.7232	0.9665
1038	110	2004	0.5519	0.7981	0.9875	0.5425	0.7831	0.9841
1039	111	2004	0.3123	0.4868	0.7242	0.369	0.5621	0.8186
1040	112	2004	0.4576	0.6973	0.958	0.5156	0.7557	0.9776
1041	113	2004	0.5158	0.7635	0.981	0.5782	0.8153	0.9893
1042	114	2004	0.389	0.6033	0.8821	0.4719	0.7051	0.9581
1043	115	2004	0.4149	0.6406	0.9194	0.509	0.7485	0.9755
1044	116	2004	0.3661	0.5693	0.8405	0.4762	0.7104	0.9608

MEAN	0.4918	0.7149	0.9249	0.5190	0.7391	0.9408
MINIMUM	0.2063	0.3217	0.479	0.19	0.2898	0.4239
MAXIMUM	0.8409	0.9533	0.9987	0.7703	0.9273	0.9978

Table 5. Efficient Prices (Model 24)

Obs	Hospital	Year	p1	p2	p3	p2/p1	p3/p1
1	1	1996	5.77	115.94	1170.67	20.09	202.84
2	2	1996	8.41	229.07	1870.27	27.23	222.32
3	3	1996	27.17	714.91	1713.41	26.31	63.05
4	4	1996	3.66	34.33	435.95	9.38	119.09
:	:	:	:	:	:	:	:
107	107	1996	195.47	51204.19	42574.03	261.95	217.8
108	108	1996	199.3	754131.27	89649.6	3783.85	449.82
109	109	1996	41.1	25946.73	15595.44	631.33	379.46
110	110	1996	141.11	19639.23	21793.87	139.18	154.45
111	111	1996	56.81	62697.26	33493.37	1103.64	589.57
112	112	1996	4.97	445.01	1704.12	89.53	342.85
113	113	1996	101.47	256282.48	32510.5	2525.58	320.38
114	114	1996	225.64	75711.48	38865.37	335.54	172.25
115	115	1996	72.67	984576.25	52121.09	13547.81	717.19
116	116	1996	89.29	647335.96	43447.84	7249.66	486.58
:	:	:	:	:	:	:	:
929	1	2004	0.93	24.87	226.6	26.62	242.53
930	2	2004	1.01	28.94	238.25	28.56	235.11
931	3	2004	3.52	55.48	297.95	15.77	84.66
932	4	2004	0.46	4.71	69.07	10.18	149.08
:	:	:	:	:	:	:	:
1035	107	2004	4.11	3061.65	1788.61	744.08	434.69
1036	108	2004	3.14	20831.39	1874.83	6630.03	596.71
1037	109	2004	3.69	66054.29	1055.01	17898	285.86
1038	110	2004	9.4	8189.66	2592.64	871.01	275.74
1039	111	2004	5.51	5257.48	2758.53	954.76	500.95
1040	112	2004	0.5	48.7	163.42	97.73	327.94
1041	113	2004	10.24	72015.5	2413.79	7033.18	235.74
1042	114	2004	12.38	76403.57	2816.56	6171.09	227.49
1043	115	2004	8.16	74789.13	2336.04	9166.21	286.31
1044	116	2004	6.67	19018.37	2641.98	2852.94	396.32

MEAN	64.511954	252426.59	12936.979	4427.4296	279.24872
MINIMUM	0.01	0.29	2.51	9.23	41.41
MAXIMUM	2839.41	45775782	465221.7	185447.33	1804.71

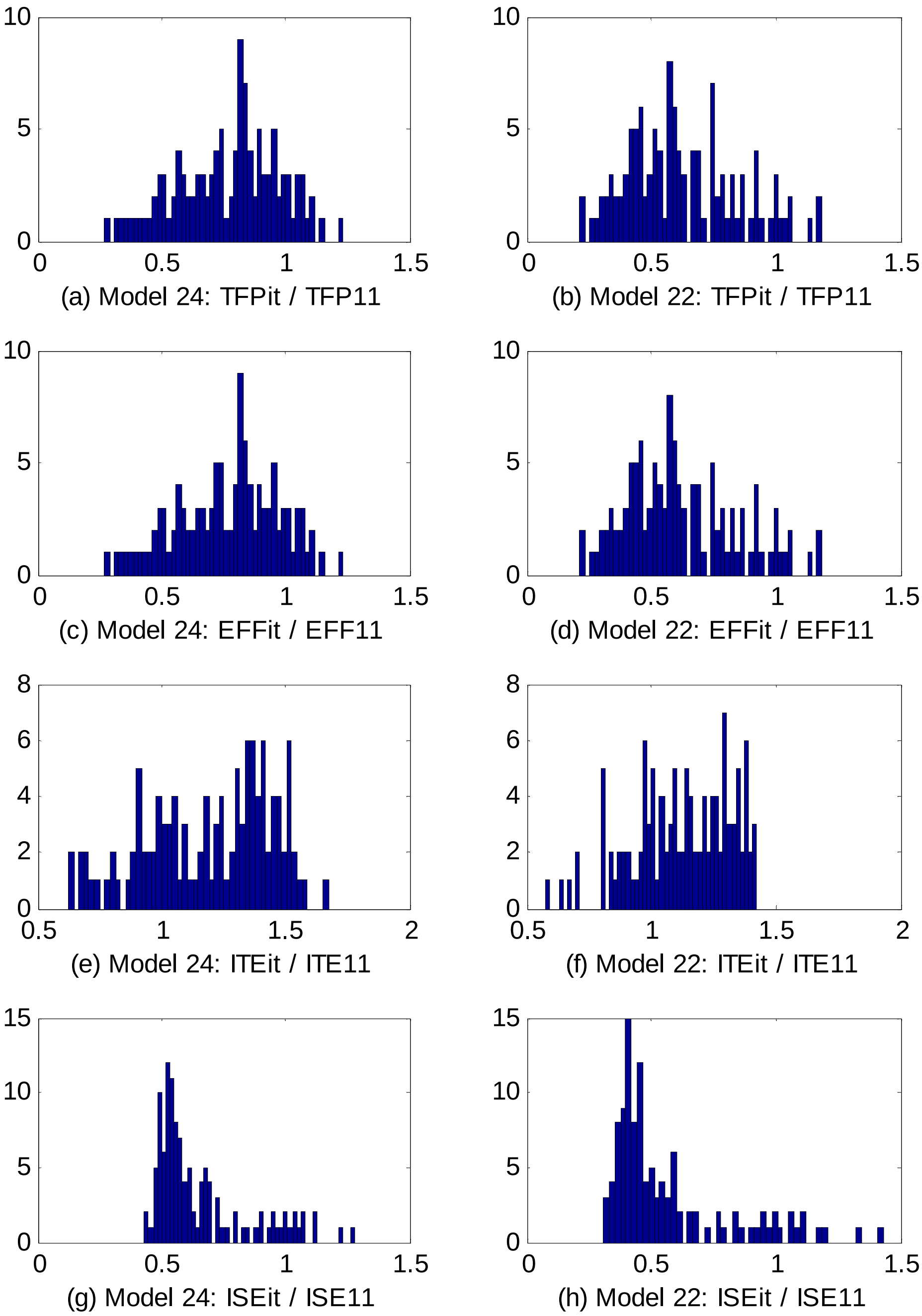

Figure 1. Distribution of TFP and Efficiency Indexes for $N = 116$ firms in 1996

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