EVOLUTIONARY MICROECONOMICS AND THE THEORY
OF EXPECTATIONS

ABSTRACT

This paper sketches a framework for the analysis of expectations in an evolutionary microeconomics. The core proposition is that expectations form a network structure, and that the geometry of that network will provide a suitable guide as to the dynamical behaviour of that network. It is a development towards a theory of the computational processes that construct the data set of expectations. The role of probability theory is examined in this context. Two key issues will be explored: (1) on the nature and stability of expectations when they form as a complex network; and (2), the way in which this may be modelled within a multi-agent simulation platform. It is argued that multi-agent simulation (a-life) techniques provide an expedient analytical environment to study the dynamic nature of mass expectations, as generated or produced objects, in a way that bridges micro and macroeconomics.

INTRODUCTION

Two elemental propositions underpin the emerging framework of evolutionary microeconomics: (1) that the economic system is a complex open adaptive system; and (2), that the essential behaviour of this system is endogenous change and self-transformation (in historical time). For the greatest part, self-styled evolutionary economists have concentrated their attentions upon representation and analysis of the evolutionary dynamic, an approach complicit with the core Schumpeterian notion that, in market-based economic systems, change is the normal state of affairs. Given this, it is perhaps curious that the question of how agent behaviour then reacts to the possibility and indeed *inevitability* of such change has remained an issue mostly peripheral to the theoretical scheme. There is a conspicuous gap in the research programme - there is no definitive evolutionary theory of expectations. This paper is an attempt to construct the first principles of an evolutionary theory of expectations.

The basic idea is twofold. From the perspective of the individual agent, we recognize that agents use internal models to process environmental signals into expectations. These *production systems* are represented in a multi-agent simulation framework as *schemata* (Holland 1995, Epstein and Axtell 1996), which are sets of strings that operate as a kind of algorithmic suite. Agents are heterogeneous and capable of learning (see, e.g., Arthur 1993, Lettau and Uhlig 1999, Riechmann 1999). But an evolutionary theory of expectations involves more than just a refinement on the theory of agent learning to incorporate expertise.

It must also account for the fact that expectation formation is a distributed process. Expectations form over a network ensemble as agents incorporate the expertise of selected other agents. The dynamics of expectations are the dynamics of a complex distributed network process. Alan Kirman (1997: 346-7) recently expressed the following view:

> What seems to me to be the most important factor to consider when studying economics in terms of networks is, not only how behaviour changes as agents interact through a network, but also how networks themselves evolve. …what one wants is that links should be reinforced by good experience and weakened by bad. Very little has been done in this area in economics.

Expectations are an instance of a network structure *par excellence*. The attitudes, beliefs, hypotheses and conjectures, our world-views and other such knowledge structures are products of local, discrete interactions. The standard approach is of course to conjure an information set with carefully defined set theoretic and topological properties before endowing it upon the agent. It usually passes without remark that this imparts a degree of epistemic precision to expectations that is simply untenable. A major problem with the standard abstraction is that it fails to account for the fact that information always has a discrete and local origin; when we have information it is because someone or something communicated it to us (deliberately or otherwise).[1] It is a small step from there to note that this process may well be highly complex, with many intermediate layers of filtering and amplification, all intermediated by schematic representation. The cognitive scientists assure us that this is indeed so.

An evolutionary microeconomic theory of expectations must therefore aim to synthesize this notion of complex heuristically defined, partially specialized agents situated in a variable network context. The crux of the framework that I propose is the notion that agents build internal models of the external environment. That much is orthodox. The heterodox (Austrian, Behavioural, Evolutionary) spin is that these models are autonomous *production systems* that undertake many functions besides expectation generation.[2] But to explore the implications of this, we need to first build a multi-agent simulation model; I do not do this. The objective of this paper is simply to consider some of the main issues in building such models with epistemic componentry.

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[1] Information implies a source, a signal, and a receiver (see Shannon and Weaver 1949).
[2] A production system is a generative grammar, which means that in the simplest case it is a set of rules, a working memory, and a rule interpreter. The rules are of the condition-action type (see Gilbert and Troitzsch 1999: 164).

This paper has five parts. First, we consider the meaning of expectations. Second, the relation between expectations and probability theory is examined. Third, the relationship between expectations and plans is considered, and fourth, the relationship between expectations and expertise in the context of a multi-agent modelling environment. Fifth, the architecture of an 'expectations generating program' is proposed. Conclusions conclude.

1.   WHAT ARE EXPECTATIONS?

Expectations are of course a Swedish invention, and crucial to any understanding of the nature of money.[3] However there is more to expectations than the theory of Swedish money. Keynes (1936, 1937) thought expectations important to explain mass movements in consumption and investment expenditure. Much of what Keynes said was subsequently assimilated into the mainstream (e.g. Hicks 1939), but not his theory of expectations, which was doggedly misunderstood (see, e.g., Townshend 1939 on this point). George Shackle (1938, 1958, 1972, 1979) thought that expectations presented a deep theoretical and epistemic challenge, but Shackle was always a bit too much for many economists, who by-and-large preferred not to be challenged epistemically. And so his writings were largely ignored, except for the other occasions upon when they were misread (see Earl 1998). Georgescu-Roegen's (1958, 1971) oblique study of expectations suffered similarly. Both clearly saw that the framework of positivism was ill-equipped to handle the epistemic and evolutionary-dynamic implications of expectations as cognitive by-product.[4] Katona (e.g. 1951, 1960) thought expectations might be measured, but others quickly recognized that it would involve a lot of empirical survey work, and, nefariously, smelled a bit too much like sociology. Debreu (1959) had elegantly solved the problem anyway by assuming markets all the way to the end of the universe. Lucas (1972) saw that expectations are a property of agents, and thus argued that all macroeconomics must incorporate this: he folded macroeconomics back into microeconomics along the seam of expectations.

The problem with expectations is trying to figure what they are and where to put them in a scheme of analysis. There are a number of options; all are perverse. If expectations are ignored completely, then economic analysis becomes hopelessly static. If they are treated as unambiguous data, then analysis becomes woefully deterministic. If they are fixed to rational

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[3] See: Wicksell (1898) *Interest and Prices*; Cassel (1928) 'The Rate of Interest, the Bank Rate, and the Stabilisation of Prices.' *Quarterly Journal of Economics*; Myrdal (1939) *Monetary Equilibrium*; Lindahl (1939) *Studies in the Theory of Money and Capital*.
[4] See Louçã (1997) on the broader implications surrounding this point.

agents, then macroeconomics collapses. If they are treated as conjectural inference, then optimisation collapses. If they are found to be unobservable and unmeasurable, then positivism collapses. If they are permitted, then imagination, learning and novelty must also be permitted. If they exist, then a mechanism for generating them must also exist. If they exist *en masse*, then this implies some distributed network or field though which they are coordinated. This implies interdependence, which suggests self-organisation and reflexivity. The point is this—the way in which expectations are treated is not an addendum to an otherwise well-defined analytical framework, but, rather, effectively determines the very nature of that framework. For this reason, expectations are a subject matter that arguably deserves a much deeper consideration than usually afforded.[5]

The way that agent expectations have generally come to be treated in the standard conceptions of economic theory and modelling is simply to fit a further vector to the information set. Mathematically, this is a conventional and relatively unsophisticated treatment that does not affect the hard core of the research programme, nor alters the formal perception of the geometry of economic space, which remains in $\mathbf{R}^n$ (see Potts 1999a, 1999b). This treatment is *prima facie* acceptable, as it takes the aspect we are sure of, namely that expectations are an input into the moment of choice, and suitably abstracts from the aspects we are not so sure of. These aspects encompass the nature of the production system that constructs expectations, the inputs into this process, and the geometry of the total network of interactions. Yet all of these aspects require explicit representation in a multi-agent framework.

In the standard mathematical treatment, an expectation is a construction built from elements of the theory of probability. The mathematical expectation of some variable X, denoted E(X), is a weighted average of *n* possible states of X with probabilities P_i used as weights, such that:

$$E(X) = \sum_{i=1}^{n} P_i X_i \tag{1}$$

At least three operations are required to map the concept of an expectation into economic theory. First, agents must construct a list space (**X**), which defines the set of all possibilities. Second, each of these possibilities must be weighted (**P**). Third, the array $\mathbf{E} = \mathbf{P(X)}$ must then

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[5] So long ago Schumpeter (1939: Vol. 1: 55) cautioned: 'We must discontinue the practice of treating expectations as if they were ultimate data and treat them as what they are—variables which it is our task to explain.'

be interpreted. From a computational perspective, this implies the existence of some *imagination/search* operator that constructs **X**, a *rational/mathematical* operator that subsequently constructs **P**, and a *cognitive/heuristic* operator that translates their product (**E**) into the space of behaviour via some framework of interpretation.

The construction of **X** is in an important sense unconstrained. For reasons of both bounded rationality and inductive insufficiency, it is impossible to know whether **X** is a complete list. In consequence, the individual elements in **X** need not actually have any formal relation to each other. However some necessary logical structure is imposed upon **P**: (1) there must be as many elements in **P** as there are in **X**; and (2), if expressed numerically, such that the weights are interpreted as probabilities, the elements in **P** must sum to unity. When this occurs, the outcome is an expectational array that can enter into decision-making (e.g. as in Subjective Expected Utility theory). Prior to this, we may delimit the various expectational schools of thought with respect to their definition of the two rudimentary constructs **P** and **X**.

The standard approach is of course the Rational Expectations (RE) framework,[6] which makes a number of simplifications. The essential assumption is that the construction of both **X** and **P** is a effortless, timeless, computable calculation of **E**. Furthermore, it is supposed that in the aggregate (and in the long run) all agents converge upon the same set of expectations.[7] It is implicitly assumed that there is nothing fundamentally interesting to be said about how agents actually calculate expectations. (Physicists once made the same sort of presumption about how particles experience forces.) By this presumption, then, the nature of these constructive methods and cognitive computational operations remain entirely within the so-called 'black-box' (cf. Leibenstein 1979). RE is not a theory of how expectations are formed, but rather is the theoretical conjecturing of an *ex ante* data set consistent with the assumption

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[6] The New Classical theory of expectations is properly considered a development over the neoclassical theory of expectations, which originally orientated expectation formation with the assumption that the future value of a variable will be some function of past values of the variable in question. Fisher (1930) defined an expected inflation proxy as a distributed lag from past values. Cagan (1956) introduced the concept of adaptive expectations using an exponentially declining lag structure. This sort of approach was refined to account for such things as error-correction (Meiselman, 1962), and a variable lag structure (Friedman 1959). Muth (1961), and later Lucas (1972), introduced the concept of agents using current models to make projections about future variables. In this scheme, future values of a variable were now related not to past values of the variable, but to contemporaneous values of other economic variables within a closed-form model. The orientation changed but the basic presumptions have remained the same. The (representative) agent was assumed to use an econometric model to form expectations rather than merely using some technique of extrapolation (see Hartley 1997). For the subjectivists (e,g, Savage 1954) the weights are not actually attached to events or variables but rather to 'states of the world' (see Lawson 1988: 47). Nevertheless, in all such cases the list can be exhaustively constructed and weights (as probabilities) attached to something. In passing, I note that George Richardson in fact recognized the RE concept prior to Muth. Richardson (1959: 229) writes: 'It is commonly believed that expectations are satisfactorily taken care of merely by assuming their functional dependence on other elements of the system. … Nevertheless, the propriety of this whole method of approach seems questionable. We are rightly accustomed to assume that economic agents generally act rationally on the basis of their beliefs; we ought similarly to assume that beliefs or expectations themselves are rational, that they are based, that is to say, on adequate evidence or information.'

of an exhaustive list of possible outcomes, each, ultimately, correctly weighted. The Austrian school and other Subjectivists deny that these weights (the **P**) are probabilities in the formal sense (e.g. see Lachmann 1976) but instead something more closely corresponding to anticipation or subjective belief (because it is impossible to know the probability distributions). The Post-Keynesians, following Keynes (1937) directly on this matter, agree that these weights are not objective probability measures and also doubt whether an exhaustive list can ever be constructed (e.g. Davidson 1982-3, 1991). For the Behavioural economists (see Earl 1992), the impossibility of list construction is a direct consequence of bounded rationality.[8] The extreme position on this is taken by Shackle (1954, 1972), who argues both that the list cannot be constructed and that weights cannot be meaningfully attached (also Loasby 1991). The theory of expectations thus ranges from the initial presumption that all possibilities can be defined (**X**) and weighted (**P**), to the suggestion that only **P** can be constructed, to the Shacklean argument that neither of these can be meaningfully constructed.

The evolutionary theory of expectations takes the Shacklean position with respect to list construction and weighting. The Shacklean posture is not a denial of the meaningfulness of forming expectations, but rather of the incorrectness of conceptualizing them in terms of well-defined (exhaustive, finite, linear) concepts of **X** and **P** (see Potts 1999b). There is a simple reason for this. The real world, as that which agents are attempting to form expectations about, is informationally complex and continually changing as an open system (see Collier and Hooker 1999), and in consequence we should not presume that the formation of expectations is a trivial task.[9]

There is not, at present, a plausible and functionally defined theory of the way in which agents adapt their expectations to a continually changing environment. It is a mistaken view to suppose that either the RE theory (a theory of information updating, Muth 1961) or the Behavioural arguments (various theories of individual cognitive learning[10]) provide, in themselves, a satisfactory theory of the evolutionary dynamics of expectations. The RE framework does not explain how expectations are formed nor how this process may evolve

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[7] So that given the same initial conditions and preferences, they would make the same choices.
[8] For a more orthodox concession of this point, see Sargent (1993).
[9] Consider Hicks' (1939: 124-5) qualifications to the treatment of expectations within his formal system: 'We generally interpret these expectations [of future prices] in a strict and rigid way, assuming that every individual has a definite idea of what he expects any price which concerns him to be in any future week. This method is of course excessively rigid, and actually errs in two different ways. For one thing, people's expectations are often not expectations of prices given to them from outside, but of market conditions, demand schedules for example. … Second, and perhaps more importantly, people rarely have *precise* expectations at all.'

with time. It is a framework that does nothing more than examine the consequences of functionally a complete and consistent information set. It does not consider how this might come to be, or whether it has any correspondence to the actual processes of information coordination. The Behavioural theories, while providing plausible accounts of processes, still lack a population perspective or account of how all expectations ultimately interact. The Post-Keynesian and Austrian treatments of expectations, with their mutual emphasis on fundamental uncertainty, realized as the impossibility of completing **X,** and problems of mass coordination do address the issues missing in the Behavioural analysis.[11] But at the same time they lack an explicit treatment of what is actually going on inside the 'black box'. An evolutionary theory of expectations is in this respect a highly synthetic approach, but one that ultimately ends up with an emergent product that is different from its components.

The essential point to recognize is that complex evolutionary systems (dynamical systems) of the sorts under consideration (agents, firms, governments, etc) are autonomous anticipative adaptive systems (see Holland and Miller 1991). For Holland (1995: 31-4), anticipative behaviour implies the existence of internal models of the local external environment.[12] These models form the basis and mechanism by which agents look ahead, and thus seek to adapt their behaviour to fit with the expected future environment. It is important, therefore, to distinguish between such things as short term *versus* long term adaptation (and by implication, short *versus* long term internal models), as well as to identify anticipations that are common to the population of agents and the level at which difference occurs. Yet in all such thinking one overarching point is clear: agents use models as the basis for anticipation or expectation. It follows that a general theory of expectations is necessarily a general theory of the nature of these internal models: of the way in which they are constructed; the heterogeneity and complexity of the population of them; and the way in which they both individually and collectively change. It is these models that we seek to understand, in terms of their individual existence as complex systems and the further fact that these complex systems themselves are elements in a higher level complex system made of the interactions between these models. And the nature and dynamics of this superset, that of mass expectations as a complex system, although of obvious significance to our understanding of macroeconomic dynamics, is nevertheless something we actually know very little about (cf. Potts 2000).

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[10] For example, see Katona (1959), Payne (1976), Kahneman and Tversky (1979), Loomes and Sugden (1982).
[11] Lucid statements of the respective positions can be found in Heiner (1983, 1989), Lawson (1988), Davidson (1991), O'Driscoll and Rizzo (1985), and Rosser (1999).

To summarise, the prime dimension of a theory of expectations is the construction of an array. This consists first of defining the set of possible events (constructed as either a finite set or on a continuum) and second, of weighting each of the possible events. The primary concerns that differentiate the different theoretical approaches to the theory of expectations (Neowalrasian, Neoclassical, Post-Keynesian, Austrian and Behavioural) are whether or not the array can be meaningfully defined as a closed set, and then either way, whether or not it can then be weighted in such a way that probabilities can be defined. The evolutionary theory of expectations takes the extreme view on this, and supposes that the set of possible events cannot be meaningfully defined as a closed set, and that it cannot be weighted in a way congruent with conventional probability measures. I will defend this position in the subsequent section.

From an evolutionary perspective of algorithmically defined agents embedded in a variable lattice (about which they construct internal models), it is apparent that there are several dimensions missing from the theory of expectations. In my view, there are three: (1) time horizons; (2) the importance (or otherwise) of being correct; and (3) the geometry of the network of expectations.

First, how far ahead do agents look-ahead into the future? Much study has been devoted to the measurement of lag-structures and to analysis of the consequence of infinite horizons, but there has been seemingly little concern with the functional structure of horizons. The reason for this over-sight is obvious: this hypothetical process does not naturally generate 'numerical data'. If it is sought it must be measured by the probing of intentions, which has no place in axiomatic models. The simplest conception of look-ahead horizons is to suppose that there are two different populations of expectations: short and long period. Short period expectations are focused about the direction of movement of a single variable, which computationally involves solving a closed form model. Long period expectations are those that involve factoring into account changes in the very nature of the economic system and suppose a very high degree of confidence in one's theoretical scheme for prediction. Keynes warned against the treatment of these sorts of expectations in the same way as those of the short period, and preferred instead to think of them in a Jungian manner, so to speak. Minsky (1975, 1982) developed this mass psychological posture as the locus of his Financial Instability Hypothesis. However in the evolutionary framework, the functional difference

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[12] Rational Expectations theory of course also presumes that agents use internal models. But this is more in the line of a story accompanying the theory (like the Walrasian Auctioneer) rather than a functional component of the theory *per se*.

between expectation horizons turns on whether expectations refer to either the elements or the connections of a system. That is, we distinguish between expectations about the movement of variables such as prices, and expectations about possible changes in the set of connections within the system.

A further dimension to consider is the relative importance of being correct in forming expectations. It is apparent that the measure of this is the degree to which the behaviour (e.g. the investment or contract) is reversible. Expectations matter most when decisions based upon them are irreversible in consequence of irrecoverable expenditure. Irreversibility calls forth strategic behaviours to deal with the possibility of expectations being confounded (e.g. to increase the liquidity of the asset portfolio, or to diversify the portfolio). However another way of viewing this phenomenon is to consider the way in which different decisions (about the holding of assets, say) fit within the particular lattice structure of an economic 'lifestyle', which we define as a coordinated set of behaviours. This is an important issue for both consumers and firms (see Earl 1990). Again, this conception focuses concern on the existence of key elements, and in this sense is congruent with the lexicographic models of consumer choice (Earl 1986). The evolutionary microeconomic framework develops this one step further in abstraction, and considers the graphical structure of these complex systems. It becomes apparent that irreversibility, and hence the importance of the expectation decision, has a microeconomic explanation in addition to the degree of imperfection of the second-hand asset market or contingent contracting. This is the measure of centrality of the specific element in the system (the 'graph', in the language of the theory).

To continue in this vein, we recognize that the density of connections in a network of expectations may vary over time, and, furthermore, that there may be frictions or costs associated with changing network connections. We generally conceive of activity in terms of measured output, so that during a recession there is less (growth in) economic activity. Does it follow that the connective geometry of the economic system, as the density of connections, changes during the course of fluctuations in economic output? I think this a reasonable hypothesis.[13] It is suggestive of threshold effects, such that as growth rates of the economy accelerate (or decelerate) then existing activity is simply speeded up (slowed down) along the

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[13] An anonymous referee has suggested an alternative hypothesis. It is that recessions are times of increased activity due to crisis effects (firms facing bankruptcy, workers facing redundancy) that force agents to re-think their internal models and plans. Note that this applies to expectational structures *within* agents. But the point above relates to structures *between* agents. However this does beg the question of how these two levels interact. My guess is that the linkage is the active component of a search model, in which if all is well (activity levels are high) then there will be little incentive to search for

same lines, but beyond some threshold changes in the connective geometry of the system occur. The estimate of this threshold will be a function of the frictions involved in changing existing connections or the costs involved in establishing new ones. If so, this will presumably have significant implications for the structure of the web of expectations. The question is whether there is anything important to discover about the path (and the dynamic properties) of some measure of this density. This may be useful in understanding the inter-linkage between phenomena such as growth and business cycles. We may suppose that recessions induce relatively low connectivity and prolonged booms have relatively high connectivity, then the implication would be that boom periods would be characterized by relatively more unstable expectational dynamics than periods of recession.

So what are expectations? If one is willing to accept the closure and transparency of all set concepts involved in the description of an economic system, then expectations are an array that may be appended to the information set possessed by each and all agents. If this total closure is not acceptable, due to problems of reflexivity and incompleteness, then expectations must be seen as the product of internal generative models of external networks which are themselves the product of these schematic models, and so on. And if it comes to pass that this is the essential structure of distributed expectations, then we must admit that classical probability theory has little to say on these matters (see also Palley 1993).

2.   ON THE RELATION OF PROBABILITY TO EXPECTATIONS

> There is no more common error than to assume that, because prolonged and accurate mathematical calculations have been made, the applications of the result to some fact of nature is absolutely certain.

<div align="center">A.N. Whitehead, An Introduction to Mathematics. (1911: 27)</div>

The variants of the standard framework of mathematical expectations[14] can be organized with respect to the different meanings attributed to the nature of the probability concept.

First, how does the concept of probability relate to knowledge?[15] It is notable that there is no definitive taxonomy of forms of relation. Tony Lawson (1988: 40), for instance, makes two

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other modes of activity. But below that threshold, agents may well be faced with crisis that involves re-assessment of plans and expectations and may well call for expertise (see section 4).

[14] Perhaps curiously, there seems to be no serious use made of other statistical measures of central tendency such as the mode (that which has occurred most often as the expectation) or median (the middle of a sample as the expectation).

[15] In an excellent study of the prehistory of the modern probability concept, which arrives with Pascal and Laplace, Hacking (1975) recounts how prior to around 1660 the meaning of the words 'probable' and 'probability' were associated with best opinion rather than best evidence. In this sense, the opinions of a good or approved authority (such as those of the Monarchy or the Church for instance) were more 'probable' to be correct than that of lesser opinion. Probability was a concept

overarching categorizations - (1) probability as a property of an objective external reality; and (2) probability as a property of agent's knowledge of this reality - by asking the question 'are probabilities understood as objects of knowledge, or merely as a form of knowledge?' This distinguishes the subjectivists (e.g. de Finetti, Savage, Ramsey), who locate probability as a property of internal knowledge, from the objectivists (e.g. von Mises, proponents of the RE hypothesis), who locate probability as a property of external reality. This duality between an epistemological and an aleatory meaning has been long recognized by historians and philosophers of probability (Hacking 1975: 11ff). Rudolf Carnap (1950: chapter 2) distinguished between what he termed *probability₁*, which refers to a logical relation between two sentences (hypothesis and evidence) interpreted as degree of confirmation, and *probability₂* which is the relative frequency in the long run of one property of some thing with respect to another. For Carnap, as for Keynes (1921), the first probability concept appeals to a logical analysis, the second probability concept appeals to an empirical analysis. The distinction turns upon whether probability is ultimately a logical or a mathematical relation.[16]

Both such interpretations are generally regarded as conforming to the formal laws of modern probability theory, which derives from the axiomitization of probability theory with measure theory (Kolmogorov 1933).[17] Kolmogorov's treatment is based upon an abstract space of elementary events (Ω, the probability space) and a σ-algebra of subsets of Ω (e.g. see von Plato 1994: 21ff). However because both interpretations are consistent, the axiomatic approach effectively circumvents rather than addresses the question of the ontological nature of the probability concept. This issue is perhaps somewhat esoteric in the context of, say, quantum physics or statistical mechanics, whereby it is ultimately immaterial whether the probability measures be attached to the events themselves or the investigators claim to knowledge of them. However it is of more immediate relevance when we address the status of agent expectations in the economic context, where the subjectivist epistemic interpretation finds itself broadly consistent with introspection. Indeed, in the writings of George Shackle

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associated with 'who said what' rather than with what was actually said or presented. Modern probability emerged, according to Hacking (1975: 35), when the concept of internal evidence replaced that of situated opinion. Evidence, in this sense, was a product of the 'low sciences' of medicine, alchemy and such like, which could not by their nature furnish demonstrative proof, but instead appealed to 'signs' (*ibid*: 39ff) that signified evidence. In this respect, Hume's 1739 *A Treatise of Human Nature*, which defined the problem of induction with reference to sequences of evidence, would not have been possible without this prior shift in the meaning of probability (degree of belief) from opinion to evidence.

[16] There are of course other ways of parsing the probability concept. Pollock (1989), for instance, distinguishes between 'epistemic probability' (a measure of how good our reasons are for various beliefs, and is associated with subjective probability) and 'physical probability' (which concerns the structure of the physical world independent of knowledge or opinion, and is associated with objective probability).

[17] Note that Bayesian probability theorists (e.g. de Finetti 1931, Savage 1954) reject the formalistic approach of measure theoretic probability.

(1954, 1972) we find staked out a radical subjectivist position that effectively denies the logic of any form of objective probability concept that has any relevance to the economic context of expectation formation (see Runde 2000). This radical subjectivist position, ascribed to by both the Austrian and Post-Keynesian schools of thought, is based upon the notion that empirical knowledge of the external environment is incomplete (there exists fundamental uncertainty). It follows that if the probability space itself is undefined (unbounded) then it is meaningless (c.f. impossible) to attach prior probabilities to imagined events. Shackle rejects the concept of probability and instates the concept of potential surprise, which is a non-additive and purely subjective measure that does not require the otherwise critical notion of closure that is imposed in any axiomatized scheme.

Nicolas Georgescu-Roegen (1971: 52ff) argues that the various "doctrines" of probability 'all in fact have the same objective: to order expectations with the aid of some numerical coefficient which each doctrine calls "probability".' He broadly distinguishes between the Subjectivistic, of which he is dismissive, and the Objectivistic, which defines the probability measure independent of the individual. He then insists, detailing the Frequentist and Classical formulations, that 'objective probability is basically a dialectical notion' of which the dialectical root is randomness. A dialectical concept is one which is 'surrounded by a penumbra within which they overlap with their opposites' (*ibid*: 45). Probability, as such, is not a discretely distinct concept but ranges continuously over the space between randomness and order.[18]

This leads us to concern with the relation between probability and the nature of the (supposedly objective) world to which the concept refers. In this the crucial distinction is between an ergodic and a non-ergodic system. This moves us from concern with the *ex ante* nature of the *space* over which a system is defined to the *ex post* nature of a system's *behaviour* within that space. A collection of systems forms an ergodic ensemble if the modes of behaviour found in any one system from time to time resemble its behaviour at other temporal periods and if the behaviour of any other system chosen at random also is like the one system. (In statistics, ergodicity is called stationarity.) In contrast, a non-ergodic system (one whose behaviour is non-ergodic) is one that is in certain crucial respects

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[18] Georgescu-Roegen's attention was primarily focussed upon the entropy law and the underpinning relation between macro-states of order and disorder and the flow of time. In this context, probability is defined in terms of the 'entropic minima' of some ensemble (maximal disorder) which is the 'most probable' state for the system to be in. The statistical mechanical interpretation of probability (Boltzmann's time average notion, which Georgesu-Roegen rejects) is related to the Frequentist approach (where the probability of an event is defined as: $p = \lim f_n$, for $n \rightarrow \infty$) by the definition of an ergodic system.

incomprehensible through observation either for lack of repetition or for lack of stability. The *Web Dictionary of Cybernetic and Systems* defines that 'Evolution and social processes involving structural changes are inherently non-ergodic.' Paul Davidson (1982-3) distinguishes between ergodic and non-ergodic decision making environments. This classifies environments into those in which the objective rules of probability do apply (a necessary condition is stationarity) and those in which they do not (a sufficient condition is non-stationarity). In a non-ergodic environment decisions are unique, irreversible, and potentially of crucial significance (Shackle 1972, Davidson 1991). In such circumstances, the ergodic 'laws' of probability simply do not apply.

The Austrian and Behavioural schools dispense with a direct account of probability and proceed to expectations with uncertainty and bounded rationality leading to rule governed behaviour. Expectations are then an application of such. The Austrian theory of expectations derives from the radical epistemic subjectivism of Hayek and Lachmann. For Dow (1985: 139-142), the Austrian theory of expectations is synonymous with Shackle (see also Lachmann 1976). However Dow points out that while the Austrian conception contains an implicit argument in convergence and stability of expectations, Shackle's Kaleidic framework invokes no such presumption. It is also noteworthy that the Austrian economists did not distinguish a theory of expectations from a theory of knowledge (e.g. Hayek 1945) until the 1970's. O'Driscoll and Rizzo (1985, 1986) distinguish between neoclassical spatialized Time (or Newtonian Time, in their words) and Subjectivist 'real' Time (a concept they associate with the thinking of Henri Bergson).[19] The existence of uncertainty makes optimizing behaviour impossible, and thus agents standardize their behaviour in terms of rules (see Bausor 1983, Heiner 1983).

In consequence of the existence of fundamental[20] uncertainty, the concept of probability is not a necessary corollary to the concept of an expectation. As O'Driscoll and Rizzo (1986: 258) note:

> The idea of genuine uncertainty implies both the endogeneity of the source of uncertainty and the perceived unlistability of all the potential outcomes of a course of action. Each of these features of uncertainty provides a basis for the adoption of rules.

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[19] Peters (1999) is a good introduction to the linkage between Austrian economics, uncertainty, and complexity.

[20] I sometimes feel compelled to use the prefix 'fundamental' to distinguish it from the game-theoretic usage, in which uncertainty means that you know the set of actions open to other agents, but do not know the associated probability distributions. With fundamental uncertainty, you have no idea even of what the possibilities are.

Such rule-based (i.e. algorithmic, heuristic) approaches to decision making have been simmering in the economics literature for some time now (Tversky and Kahneman 1974, Simon 1982) and have recently begun to flourish in the more mainstream literature (e.g. Campbell and Mankiw 1990, Rosenthal 1993, Lettau and Uhlig 1999). This is certainly a welcome advance, in that it moves towards furnishing economic models with plausible (observable and testable) psycho-foundations. But if decision-making can be comfortably and usefully perceived as heuristically generated (without reference to complete information sets or well-defined probability density functions), it seems reasonable to suppose that other internal cognitive actions can also be afforded such representation. In the following sections I submit that plans, expertise and expectation are all viably understood as the outcomes of *constrained generating procedures* (Holland 1998).

The economic agent in the multi-agent setting (*Hetero Economicus*, Potts 1999b) is defined as an algorithmic suite of such functions.[21] Some of these make decisions, others construct plans or embody expertise, and others still generate expectations. All such functions can be grouped together as a class and defined within an Object Orientated Design of an economic agent. In this setting expectations, decisions and other behaviours are generated rather than deduced, and so infer a procedural rather than substantitive rationality (Simon 1976). Within this setting, expectations are not necessarily related to an underlying probability construct.

3.   EXPECTATIONS AND PLANS

What do agents form expectations about, and for what purpose? The standard answer is that they form expectations about the probability distributions of future asset prices, and they use these to maximise subjective expected utility. But this misunderstands an important relationship, which George Richardson (1959: 224) has pointed out (see also Richardson 1960, 1971, 1972):

> It is obvious that no direct connection can exist between objective
> conditions and purposive activity; the immediate relationship is between
> *beliefs* about relevant conditions and *planned* activities which it may or may
> not prove possible to implement. [Italics in original]

In the evolutionary microeconomics, we suppose that agents also form expectations over the space of connections in the economic system, and that they order these in relation to

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[21] A useful metaphor is to think of the agent as a kind of operating system, with hard-wired rules of search, information structuring and modification, upon which runs a set of programs as heuristic procedures.

coordinated plans, which are of course proposals for future activity. The formation of expectations and the construction and coordination of plans are, in this way, two sides of the same coin. Hence, there are two reasons for constructing expectations. The first reason is as input data into calculation of optimal choice. Expectations of future prices serve this function, and serve to guide *immediate* actual activity. The other reason for forming expectations is to guide the coordination of plans for *future* actual or contingent activity. It is important to note that we (as economic agents) do not form expectations about everything. Equivalently, we do not make plans about everything. The demand for expectations comes from the need to construct plans, and insofar as plans are circumscribed, so too will be expectations.

Expectations and plans are interrelated constructs (they are both mental models), and each supposes the existence of the other. When the plans of individual agents are stable and widely known (or not), then it follows that expectations will also be stable (or not). Darley and Kauffman (1997) discuss this in the context of optimally complex (adaptively rational) models of other agent's actions and behaviour. Similarly, when an agent is not able to form suitable expectations of other agents' behaviours, then they will likely restrict their own planning horizon as a means of protection. This will feedback into the system of expectations engendering even further instability.[22]

Why do agents form plans? Several answers might be given. An important one that we ought not to dismiss is their psychological value. To have a plan (or a goal) is to be acting with purpose, which, to some degree, is an end in itself. When businesses do this, it is called strategic planning (e.g. writing mission statements, etc) and arguably serves much the same function. In general, though, a plan is a sequence of steps (conditionals) supposed sufficient to reach a target or goal or some such. These may be explicit and definite, as in forward contracts, or indefinite and unobservable as they exist in the minds of agents. Hirschman (1970) has argued that the expression, by 'voice', of these unobservables plays an important role in market coordination. We might also suppose that local clusters within networks (clubs, trade associations, families, etc) play a similarly important role (cf. Hazledine (1998) and Fukuyama (1999) on the 'social capital' underpinnings of such).

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[22] Potts (2000) observes that because of the increased uncertainty, agents will tend to increase the interdependence of expectations. It follows that the resultant network structure will, because of the increased density of feedback loops, exhibit more chaotic dynamics. Potts calls this 'the uncertainty principle of uncertainty.'

A multi-agent simulation platform in which agents have internal representations of the 'world' is also one in which agents' behaviour can be set explicitly as a function of internal goals and heuristic procedures for realizing those. This implies the existence of plans, as a framework for action, thus creating a demand for expectations over the space of the connections to which these refer.

4.   EXPECTATIONS AND EXPERTS

Expectations are a species of knowledge, which is a constructed or generated form of information, and thus of a rather different nature to what we might call data. One of the more spurious aspects of the representative agent fiction is total neglect of the way in which expectations are processed as a distributed network (see Hartley 1997, Kirman 1997, Fleith and Foster 1999, Potts 1999a), and therein consideration of how that network is structured.[23] For, as Keynes noted, when agents form expectations under conditions of informational complexity or uncertainty the reference point cannot be 'fundamentals' because it is precisely such fundamentals that are incalculable. Instead, agents will tend to look to the behaviour of others. But which others? If the set of 'others' is not chosen at random (although this could usefully be the benchmark modelling assumption), so that there is some discrimination at work, we might reasonably suppose that 'the others' are selected for some criterion of expertise. Agents will consult experts.[24] The expert is an agent with specialized knowledge.[25] We can represent this idea of expertise, as codified knowledge and rules for accessing that knowledge, with strings, which are also an easily computable and functionally transformable data structure. The agent will have a set of internal strings (*schemata*, Holland 1995) that are the product of experience and learning.

Supposing, then, that expertise can be represented as a set of strings with specific functional definition, how does this relate to expectation? Expectation is the output of internal schemata (models), but we must recognize that some of the inputs are the outputs of other agents'

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[23] There is a considerable body of literature and empiricism in sociology on the structure and dynamics of networks. See, in particular, the journal *Social Networks*.

[24] They are not entrepreneurs *per se*. Entrepreneurs are the *coordinators* of expectations, expertise and plans, moving between the boundaries of finance and law, and within the space of mass distributed expectations. (See Cooper *et al* (1995) and Zacharakis and Meyer (1998) on the nature of entrepreneurial decision making under uncertainty.)

[25] In the psychological literature on expertise (e.g. Bédard and Chi 1992), the distinction is made between the expert and the novice. For instance, experts know more about their subject domain than novices, experts' knowledge is better organized (Einhorn 1973), they perform tasks better than novices in domain related skills, and these skills often do not translate into unrelated domains (see also Wright and Bolger 1992). In Artificial Intelligence, the eponymous instance is the expert-system, which is an application program that makes decisions or solves problems in a particular field by using knowledge and analytical rules defined by experts in the field. An expert system is a program that combines databases, retrieval, interpretation and perception systems, knowledge representation schemes and inference engines.

schemata. It is a construction of expectations by means of expectations problem, if you will. But agents specialize, and as such while it may be useful to be an expert, it is equally useful to know to how find and identify one. Supposing this can be done, the computation of expectations involves the compiling of these distributed expert 'programs'. In the multi-agent simulation environment, such as *Swarm*, agents interact by sending messages to each other. This domain of message space defines the domain of interactions. The messages interact with what John Holland (1995) calls *Tags*. This is a useful terminology, and I will use it here (also Potts 1999b). Tags are the external representation of the internal schemata.

Expertise can be represented as a string, and one that is constructed by the process of learning. These will be specialized, and hence require coordination. Agents signal to others their expertise via tags, which are also represented as bit strings. Network structures of interaction will emerge, over which mass expectations flow. In this way, the structure of expectations in the evolutionary microeconomics consists of the distributed computation of sub-routines of expertise.

This integration of distributed expertise into the theory of expectations is clearly a concern that is emergent to a multiagent framework. The Neowalrasian framework is of course based upon the implicit presumption that agents use internal models (and specifically ergodic econometric models) to form expectations.[26] However there is no essential recognition that there exists a *population* of such agents, where each may differ in the model they use. By making use of the representative agent abstraction, the issue of where this 'model' actually exists (e.g. within agents or between them or both) is entirely evaded (see Davidson 1982-3, Kirman 1992). From this perspective, there is little to be said on such otherwise fundamentally important questions as, for instance, where the internal model came from, the processes by which it changes, and whether we indeed all have the same one. It is I think seriously misleading to conceive in this case of a 'representative agent' with a 'representative internal model'. For if we allow that agents have different internal models, this provides the sufficient conditions for a diversity of expectations irrespective of whether they have different input data or not. As such, it seems most reasonable to base our theorizing on the presumption that there will be a heterogeneous population of agents, each with different

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[26] In General Equilibrium models there is effectively only one representative agent, and thus no logical scope for interaction (reflexivity or feedback) between agents to effect expectations, and as such via an information updating procedure the subjective probability set ultimately converges to the objective probability set as a martingale. The crucial task in RE is that of correct weighting of **X** (with **P**) so as to identify the relevant initial conditions (which further requires a theory of Bayesian learning or information updating) that then *in toto* determine choice.

internal models. The key difference between such systems resides in their internal (and external[27]) connectivity.

5.   A MULTI-AGENT FRAMEWORK FOR ANALYSIS OF EXPECTATIONS

What is the relationship between the expectation, and the process by which the expectation is generated? Typically, expectations are defined as a class of information; specifically they are *ex ante* cognitive data. This of course presumes that there exists a method by which these are generated, but no explicit account is ever given (or required) in the theory. In the evolutionary microeconomics Object-Orientated Programming techniques are used to shift our attention to the process that generates expectations. (Which may be implemented within a multi-agent simulation platform such as *Swarm*[28].) An expectation is defined as an object, which is a way of encapsulating both data and the functions (or methods) that operate on the data. A general model of the interacting agent has been proposed in Potts (1999b) and we will use that to define the formal structure of an evolutionary algorithmic model of expectation formation . The agent, *hetero economicus*, is a complex system composed of three sub-systems:

- a set of resources and technology $\quad\quad\quad < \mathbf{V} : \Sigma \mathbf{S(A)} >$
- a set of control algorithms and schemata $\quad\quad < \mathbf{X} : \mathbf{P} : \mathbf{Y} >$
- a set of tags for interaction $\quad\quad\quad\quad\quad < \Omega^{+} : \Omega^{-} > < \pi^{+} : \pi^{-} > \equiv \mathbf{E(P)}$

$$\text{AGENT} = \boldsymbol{u} : < \mathbf{E(P)} > \ < \mathbf{V} : \Sigma \mathbf{S(A)} > \ < \mathbf{X} : \mathbf{P} : \mathbf{Y} >$$

This agent does, in effect, three things. (1) It explores the set of possible combinations that can be made with its current resource set. (2) It selects good combinations that can become building blocks in higher level systems. It makes extensive use of abstraction to do so. And (3), it engages in exchange and combination interactions with other agents for the dual purposes of obtaining inputs into its current process system, and to integrate itself into higher level systems. In other words, the agent identifies itself as an ensemble of resources (**V**). Specific combinations of these are technologies - **ΣS(A)**. The rules for making these structures become the fundamental dynamical operators in **P**. The system 'comes to life' when

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[27] Alan Kirman (1997: 340) notes that a 'good analogy would be to think of the economy as being like the brain, certain circuits are stimulated and reinforced, others never develop or fade away. The most important thing is that the individuals in the economy learn, not only about appropriate actions for them to take, but also about whom they should interact with.' For obvious reasons, representative agent models do not recognize this last aspect.
[28] For information on the *Swarm* simulation platform and Object Orientated Programming, see: http://www.swarm.org

P is capable of internal self-adaptation (**X**). The algorithmic basis for interaction is furnished in **Y**, and with the set of tags serving as the conditionals that intermediate interaction. The same suite of algorithms that generates plans, expertise, and other internal models generates expectations. This is a function of the internal control set <**X**, **P**, **Y**> which consists of three operators:

$$\mathbf{X} = \{X_1, X_2, ... X_q, ... X_r\} \qquad \text{// Operators defining change in strings}$$
$$\mathbf{P} = \{P_1, P_2, ... P_s, ... P_t\} \qquad \text{// Schematic preferences as strings}$$
$$\mathbf{Y} = \{\texttt{LIST: CONSTRUCT : RANK : SELECT}\} \qquad \text{// Platform of search algorithms}$$

The static and operational basis of the control function is the set of schematic preferences **P**, which is the set (or population) of rules that are active in behaviour. The set **X** is an internal system for modifying these rules. **Y** is the set of processes engaged in search and evaluation. The set of schematic preferences is a population of high-level search rules (theories, as such) that enable the boundedly rational agent to cope with very large information sets (that contain redundancy) and/or novelty (see Holland 1975, 1995, 1998; Scarf 1989). Schematic preferences are made of schemata, which are rules for conditional acceptance of the IF-THEN form. The individual rules (e.g. P_s in **P**) are symbol strings with combinations of specific and/or null sites. The symbol '#' represents the null sites, and thus enables agents to choose on the basis of partial mappings (as with incompletely specified, highly complex or novel situations). They enable agents to act in a changing, evolving world. (The extent to which schemata contain specific rather than null sites represents a measure of the 'expertise' of the agent.) These rules are internally modified by a set of bio-memetic (genetic) operators **X**. Learning occurs as **X** acts on **P**.

The evolutionary microeconomic theory of expectations can be stated thus. Expectations are generated from a WORLD MODEL, which is constructed within the set **P**.[29] Two classes of expectations are generated. The standard treatment is to presume that expectations are estimates of the probability distributions of a closed set of possible events. This is not the perspective taken here. Instead, expectations are estimates of the set of elements in the system, and the connections between them. Expectations are estimates (predictions) of the network geometry of the system. In this way, expectations are effectively statements about

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[29] The WORLD MODEL, skills and expertise, plans, and connections to other agents are all codes as sub-programs (strings) within **P**. The learning sub-routines and search algorithms are in **X** and **Y** respectively. Strictly speaking, **X** and **Y** could also be defined within **P**, but the distinction is made between an 'operating system' (**X** and **Y**) and the 'programs' that run on it (**P**).

the stability, robustness, exhaustiveness and generality of the WORLD MODEL, and hence are epistemic objects that attempt to penetrate uncertainty.

Furthermore, depending upon the performance of the WORLD MODEL, expertise is generated. How do we measure the performance of the agents' model? Obviously, entrepreneurs test their own internal models against reality. However, the problem is that they also need to evaluate other agents' models so as to value the output of those models. We can deal with this problem by supposing that agents signal their WORLD MODEL (expertise, expectations, plans) with representative *tags*.

From a multi-agent perspective (cf. a representative agent framework), the process by which expectations are generated can be schematically represented as below.

INPUTS [condition] $\rightarrow$ EXPECTATIONS GENERATING PROGRAM $\rightarrow$ OUTPUTS [action]

```
DEFINE EXPECTATIONS GENERATING PROGRAM:

   FOR EACH AGENT <control>:
       [STEP1]     DEFINE WORLD MODEL (P in Zⁿ)          // define space of strings
       [STEP2]     SET X, SET Y         // operators defining changes in strings, platform of search algorithms
       [STEP3]     INITIALIZE P = (Pᵢ  i=1, ..., n)          // set initial state of agent heuristics

   FOR EACH AGENT <resource>
       [STEP1]     DEFINE E, SET E       // define and set connections to other elements in Zⁿ

   FOR EACH AGENT <tags>
       [STEP1]     DEFINE E(P)          // define the set of external tags

BUILD EXPECTATIONS GENERATING PROGRAM
       [STEP1]     GET INPUTS CONTROL        // load strings
       [STEP2]     PROCESS EXPERTISE, DEFINE PLANS          // compute expertise and construct goals
       [STEP3]     SUBMIT E(P)              // construct tags for interaction
       [STEP4]     CONNECT WITH OTHER AGENT EXPERTISE        // connect with external tags
       [STEP5]     BUILD WORLD MODEL        // construct from internal and external sources a world model

COMPILE TO ACTION HEURISTICS
       [STEP1]     ACT IN V SPACE        // changes in set of elements
       [STEP2]     ACT IN E SPACE        // changes in set of connections

UPDATE
       [STEP1]  REDEFINE E        // update world model to account for changed state-space

LOOP
```

The construction and simulation of this model is a task that remains to be done.

The evolutionary microeconomic theory of expectations is framed within a multi-agent simulation platform. Each agent consists of a set of behavioural heuristics plus a system for modifying these as a rule-governed production system (an algorithmic rationality, e.g. Darley

and Kauffman 1997).[30] The agent uses this 'bio-mimetic' algorithmic suite to perform such actions as production, trading, search, making plans, sending signals, monitoring and processing information, forming alliances and generating expectations (see Potts 1999b). In the Object-Orientated Programming environment, agents are self-contained objects that can transform themselves and interact with other agents. The environment is a partially connected lattice. Resources (in this case expertise and plans) are distributed over this network.

6. CONCLUSION

Evolutionary microeconomics is a synthetic framework that both underpins and overarches the main heterodox schools of thought. In this paper, I have proposed a theory of expectations as generated from the evolutionary microeconomics. The nature of the argument is this. First, we account for two primitive notions: (1) that economic agents build internal models with which they analyze and structure their economic actions; and (2), that agents are individually nodes in a network of information flows and interactions. Second, we recognize that the internal cognitive processing systems and the external network systems are both complex systems, which invites us to re-think the economic meaning of the probability concept at the interface of these two systems. The classical probability framework is rejected (as is the Bayesian approach), and instead we suppose that decision-making under uncertainty (including expectation formation) is achieved with rule-based production systems. Third, it is recognized that a theory of expectations must include the actions of the agents and the network, as well as the interactions between both levels.

There are three crucial aspects to the evolutionary theory of expectations. The first is the expert, which is a specialized agent that has developed highly effective subroutines for processing information relating to some dimension of the economic system. All agents face the task of either becoming themselves an expert, or of finding and connecting themselves to one. The second aspect is the construction of goals and plans, as the structure of behaviour. Expectations act to mark out the dimensionality of this behaviour. The superstructure that contains this process is termed a WORLD MODEL. The third aspect of expectations was the dynamic structure of the environment. Many of these ideas have their origins deep in heterodox thought. The purpose of this paper is simply to view these notions afresh through

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[30] Lane *et al* (1996) refer to *Generative Relationships* (GRs) in a way that is congruent with the notion of production systems. They write (1996: 59): 'It is through their participation in GRs that economic agents come to understand their world and how to act in it. Moreover, GRs are the structure in and through which economic innovation takes place.'

the lens of a multi-agent simulation platform, which is a most expedient mode of analytical expression for the study of distributed emergent evolutionary processes.

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