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8 What Should the Monetary Authority Do When Prices Are Sticky?

Robert G. King and Alexander L. Wolman

8.1 Introduction

Practical macroeconomics gives a simple and direct answer to the question in the title of this chapter: monetary policy should regulate aggregate demand to stabilize output and inflation. Stabilizing output is presumed to eliminate the "Okun gaps" that arise from changes in aggregate demand when prices are sticky. Low and stable inflation is widely viewed as an important policy goal: high and variable inflation is taken to increase relative price variability as well as increasing other costs of production and exchange. To determine how to balance Okun gaps against costs of inflation—either in level or variability—it is necessary to assume a loss function for the monetary policy authority. While the specific form of the loss function plays a key role in determining the details of optimal monetary policy, a general presumption is that optimal policy involves variability in both inflation and real economic activity. From this standard perspective, a monetary policy that is directed principally toward stabilizing the price level—as proposed in the Mack bill—appears obviously inefficient.[1]

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1. The Mack bill refers to S.611, a bill introduced before the 105th Congress on 17 April 1997, which would "require the Board of Governors of the Federal Reserve System to focus on price stability in establishing monetary policy."

In this chapter, we provide a simple yet fully articulated macroeconomic model where this intuition is incorrect. In our model economy, macroeconomic equilibrium is inefficient because producers have market power and Okun gaps can arise as a result of price stickiness. However, the monetary authority should nevertheless make the price level the sole objective of monetary policy. Further, the price level stabilization policy is optimal in a very specific sense: it maximizes the utility of the representative individual in the economy. To derive this result, we draw on two broad areas of recent literature.

First, we use the standard public finance approach to policy analysis. While this approach is little used in practical macroeconomics, it is being increasingly applied in dynamic macroeconomic theory.[2] In general, the public finance approach focuses on identifying distortions and measuring the resulting costs to individuals, which are sometimes called "Harberger triangles." Optimal policy then involves trading off various distortions—minimizing the sum of the Harberger triangles—given the available policy instruments. Practical macroeconomics has tended to deviate from the public finance approach because the conventional wisdom—famously articulated by James Tobin (1977)—is that "it takes a heap of Harberger triangles to fill an Okun gap." Okun gaps were seen by Tobin and many others as fundamentally different phenomena, not amenable to being studied with public finance tools because they did not involve microeconomic distortions.

Second, we draw on recent developments in "New Keynesian" macroeconomics that provide a microstructure for sticky prices and thus facilitate the unified approach to policy analysis. Our model economy contains two central New Keynesian features: an explicit modeling of imperfect competition in product markets and an optimizing approach to sticky prices.[3] We then embed these price-setting mechanisms in a dynamic general equilibrium model, of the form studied in real business cycle research. This "new neoclassical synthesis" framework is more and more widely used for the positive analysis of the business cycle but is just beginning to be employed for the study of optimal monetary policy.[4] Models using the framework have a well-defined Okun gap that can fluctuate through time with aggregate demand, but Harberger-type analysis is nonetheless the appropriate way to identify distortions and characterize optimal policy. This is because Okun gaps can be interpreted as arising from microeconomic distortions. Specifically, with some prices fixed, nominal distur-

2. Notable contributions include Lucas and Stokey (1983), Judd (1985), Chamley (1986), and Chari, Christiano, and Kehoe (1991).

3. The imperfect competition approach to product markets has been developed by a number of authors, notably Blanchard and Kiyotaki (1987) and Rotemberg (1987). The optimizing approach to sticky prices has also been an area of extensive research, with some important contributions being Calvo (1983), Rotemberg (1982), and Yun (1996).

4. See Goodfriend and King (1997) for a summary of these ongoing developments in business cycle modeling. There is a much smaller literature on optimal monetary policy in this class of models, which includes Ireland (1995, 1996, 1997), Rotemberg and Woodford (1997; chap. 2 of this volume), Yun (1994, chap. 4), and Aiyagari and Braun (1997).

bances affect the markup of price over marginal cost. The markup is the economy's terms of trade between output and inputs and is a key distortion that can be influenced by real and nominal shocks.[5] In particular, changes in output that result from nominal disturbances are always accompanied by changes in the markup.

The approach that we take differs from those taken in other chapters in this volume. While we are making progress on building a small-scale, fully articulated macroeconomic model that can be used for the twin purposes of explaining postwar U.S. data and conducting simulations of alternative policy rules, we do not yet have a specific quantitative model that we use to identify the main sources of economic fluctuations. For this reason, we cannot ask the questions that are posed in many other chapters, such as "What is the trade-off between inflation variability and output variability, under alternative specifications of the interest rate rule?" However, we can study optimal policy within a basic macroeconomic model that captures central features of a broad class of models. Our analysis of optimal policy is centered on questions that are related to those in other chapters:

What is the optimal monetary policy response to a particular structural shock, such as a productivity shock?
What are the implications of this optimal policy response for output, inflation, and interest rates?

As a by-product of answering these questions, we also learn about the optimal long-run rate of inflation. Our findings can be summed up in remarkably simple terms: both in the long run and in response to higher frequency shocks, optimal monetary policy involves stabilizing the price level. Intuitively, it is perhaps not surprising that sticky prices make it optimal for the price level not to vary. After all, if the price level never changes, then in a sense it does not matter whether prices are sticky. By studying a simple model in detail in this chapter, we work toward understanding the broader circumstances in which this intuition is correct. While we focus on productivity shocks in our discussion, we have applied our approach to aggregate demand shocks (government purchase shocks and preference shocks) and to money demand shocks: all of these shocks lead to the same simple message about the importance of stabilizing the price level.

Our model does imply that there is an optimal interest rate rule, which takes a simple form. Since the price level never changes under optimal policy, the nominal rate set by the monetary authority must track the underlying real rate that would prevail under price flexibility. We show how to make this interest rate rule consistent with price level determinacy by incorporating a simple

5. This theme is developed in more detail in Goodfriend and King (1997). The idea that *all* effects of monetary policy in "new synthesis" models can be interpreted as relative price distortions appears initially in Woodford's (1995) comments on Kimball (1995).

specification of how the monetary authority would respond to deviations of the price level from its path under optimal policy.

Relative to other chapters in this volume, the approach we take is most similar to that of Rotemberg and Woodford in chapter 2. Both chapters use as their analytical framework an optimizing sticky price model; both also use the representative agent's expected utility as the welfare criterion. Where we differ from our compatriots is in the treatment of steady state distortions. Rotemberg and Woodford assume that fiscal or other mechanisms eliminate the monopolistic competition distortion in steady state, so that distortions only arise out of steady state. We assume that monetary policy is the only tool available for combating distortions, in or out of steady state. Further, we find that the policy of stabilizing the price level is optimal even when there are large steady state distortions.

The chapter proceeds as follows. In section 8.2 we lay out the basic model, which features staggered price setting. In section 8.3, we illustrate the nonneutralities that occur in our model, including the effects of sustained (steady state) inflation and the effects of various monetary shocks. Section 8.4 discusses the nature of constraints on the monetary authority, which we interpret as constraints on allocations that a social planner can choose. In section 8.5, we lay out the nature of the (real) optimum problem for that social planner. In section 8.6, we determine that the steady state solution to this problem involves real allocations that would be achieved in a market economy only under a zero inflation rate, a result that we call a modified golden rule for monetary policy. In section 8.7, we discuss the nature of optimal allocations in an economy with productivity shocks, under the assumption that the monetary authority can credibly commit to future actions; optimal allocations again involve price level stability. However, imperfect competition means that there are temptations for the monetary authority to abandon the price level policy, and we explore this issue quantitatively in section 8.8. Section 8.9 discusses the nature of an interest rate rule that would achieve the optimal outcomes. Section 8.10 concludes.

8.2 A Macromodel with Staggered Price Setting

The macroeconomic model assumes that final product prices are set optimally by monopolistically competitive firms, which satisfy all demand at posted prices. The model is in the tradition of Taylor (1980), in that price setting is staggered: each firm sets its price for J periods with $1/J$ of the firms adjusting each period. In common with Taylor's model, monetary policy matters because stickiness in individual prices gives rise to stickiness in the price level, and hence to nonneutrality.[6] In our exposition of this model, we focus on the case in which there is two-period pricing setting, but we discuss extensions

6. Taylor (1980) focused on nominal rigidity in wages rather than prices. The methods that are used in this paper could also be applied to such environments.

for larger J and richer time-dependent price-setting schemes at various places below.

The model is designed to be representative of recent work on the new neoclassical synthesis, in that New Keynesian–style price stickiness is introduced into an economy with otherwise neoclassical features including intertemporal optimization on the part of households and firms. However, five features deserve special attention. First, we abstract from capital accumulation to simplify the analysis as much as possible.[7] Second, we assume that the production function for all final products is constant returns in the single variable factor, labor, to approximate the relationship between output and input in a more realistic model in which firms can simultaneously vary labor and capacity utilization (see Dotsey, King, and Wolman 1997). Third, we use a form of preferences for the representative agent that allows for an arbitrarily high labor supply elasticity in response to monetary disturbances, while retaining a zero response of labor input to productivity disturbances if prices are flexible. This permits our model to generate large and persistent effects of money on output.[8] Fourth, we abstract from money demand distortions associated with positive nominal interest rates (triangles under the money demand curve). There are two motivations for this assumption. Empirically, transactions balances increasingly bear interest, so that this abstraction is increasingly more realistic. Theoretically, this assumption allows us to focus completely on the effects of monetary policy that operate through sticky prices. Fifth, we abstract from fiscal policy, by assuming that lump-sum taxes and transfers are available to offset changes in the money supply. The joint determination of optimal monetary and fiscal policy with sticky prices is an interesting issue, and one on which considerable progress has been made by Yun (1994). We focus on monetary policy in order to clearly exposit the basic implications of price stickiness and also because we think that current policy structures assign the task of stabilization policy to the monetary authority in most countries.

As exposited in this section, the model is one in which monopolistically competitive firms set their prices every two periods, but the conclusions we reach apply to arbitrary patterns of staggered price setting. There is a continuum of these firms, and they produce differentiated consumption goods using as the sole input labor provided by consumers at a competitive wage. Consum-

7. In the case of two-period price setting, this simplication should allow us to derive analytical results for our model, but we have not yet worked these out.

8. Models with sticky prices have the generic feature that persistence of the real effect of a monetary shock is almost completely determined by the change in the incentive to adjust price induced by the shock. Typical preference specifications generate large changes in this incentive, and hence real effects that do not persist. In contrast, the preferences used here create only small incentives to adjust, and hence persistent effects of the shock. We use this preference specification because we believe that monetary shocks do have persistent real effects empirically, arising from economic mechanisms that enhance the supply responsiveness of the economy to nominal disturbances. Some New Keynesian economists—such as Ball and Romer (1990) and Jeanne (1998)—would argue that this preference specification is proxying for institutional features of the labor market, such as efficiency wages, that are ultimately responsible for enhancing nonneutrality and persistence.

ers are infinitely lived and purchase consumption goods using income from their labor and income from firms' profits. Consumers also must hold money in order to consume, although we assume that money bears a near competitive rate of interest, so that there are no distortions associated with money demand.

8.2.1 Consumers

Consumers have preferences over a consumption aggregate (c_t) and leisure ($1 - n_t$) given by

$$(1) \qquad \sum_{t=0}^{\infty} \beta^t u(c_t, n_t; a_t),$$

where the flow utility is

$$(2) \qquad u(c_t, n_t; a_t) = \ln\left(c_t - \frac{a_t\theta}{1+\gamma} n_t^{1+\gamma}\right).$$

In this specification and below, a_t is a random preference shifter that also acts as a productivity shock.[9]

Microstructure of Consumption

As in Blanchard and Kiyotaki (1987) and Rotemberg (1987), we assume that every producer faces a downward-sloping demand curve with elasticity ε. With a continuum of firms, the consumption aggregate is an integral of differentiated products

$$c_t = (\int c(\omega)^{(\varepsilon-1)/\varepsilon} d\omega)^{\varepsilon/(\varepsilon-1)}$$

as in Dixit and Stiglitz (1977).

Focusing on the case in which prices are fixed for just two periods, and noting that all producers that adjust their prices in a given period choose the same price, we can write the consumption aggregate as

$$(3) \qquad c_t = c(c_{0,t}, c_{1,t}) = \left(\frac{1}{2} c_{0,t}^{(\varepsilon-1)/\varepsilon} + \frac{1}{2} c_{1,t}^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)},$$

where $c_{j,t}$ is the quantity consumed in period t of a good whose price was set in period $t - j$. The constant elasticity demands for each of the goods take the form

$$(4) \qquad c_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} c_t,$$

9. It may seem unusual to have the same random variable shifting both preferences and technology. In order for eq. (2) to be consistent with balanced growth occurring through growth in technology, it must be that the preference and productivity shifters grow at the same rate. We also assume that they vary together over the business cycle.

where $P_{j,t}$ is the nominal price at time t of any good whose price was set j periods ago and P_t is the price index at time t, which is given by

$$(5) \qquad P_t = \left(\frac{1}{2}P_{0,t}^{1-\varepsilon} + \frac{1}{2}P_{1,t}^{1-\varepsilon}\right)^{1/(1-\varepsilon)}.$$

Intertemporal Optimization

Consumers choose contingency plans for consumption demand and labor supply to maximize expected utility (1), subject to an intertemporal budget constraint,

$$(6) \qquad c_t + \frac{M_t}{P_t} + v_t s_t + \frac{B_t}{P_t} = z_t s_{t-1} + v_t s_{t-1} + w_t n_t + (1 + R_{t-1}^M)\frac{M_{t-1}}{P_t} + (1 + R_{t-1})\frac{B_{t-1}}{P_t}.$$

In this expression, the uses of the individual's date t wealth are consumption (c_t), acquisition of money balances (M_t/P_t) and nominal one-period bonds (B_t/P_t), and purchases of shares s_t in the representative firm at price v_t. The sources of wealth are current labor income $w_t n_t$, the value of previous-period money balances $(1 + R_{t-1}^M)(M_{t-1}/P_t)$ and maturing bonds $(1 + R_{t-1})(B_{t-1}/P_t)$, and the value of previous-period asset holdings, including current profits (z_t). The interest rate on bonds is endogenous, while the monetary authority pays interest on money at a rate R_t^M marginally below R_t. In terms of this asset structure, there are two additional points to be made. First, the representative firm is an average (portfolio) of firms that set prices at different prior periods. Second, in equilibrium, asset prices must adjust so that $s_t = 1$.

The first-order conditions for the household's optimal choice problem for the allocation of consumption and leisure over time are[10]

$$(7) \qquad 0 = \beta^t E_0\left(\frac{\partial u(c_t, n_t, a_t)}{\partial c_t} - \lambda_t\right),$$

$$(8) \qquad 0 = \beta^t E_0\left(\frac{\partial u(c_t, n_t, a_t)}{\partial n_t} + \lambda_t w_t\right).$$

In these expressions, λ_t measures the utility value of a unit of real income at date t (the multiplier on the asset accumulation constraint), and the marginal utility of consumption is equated to it. The marginal disutility of work is equated to $w_t\lambda_t$, where w_t is the real wage rate.

10. In appendix A of this paper, we describe the representative agent's choice problem in detail, with money demand motivated by a transactions time requirement. In this extended setting, conditions (7) and (8) are limiting versions when there is a small marginal cost of transacting, as we assume in the body of the paper.

Although there is intertemporal choice of consumption and leisure, the assumed form of preferences means that there is a simple labor supply function, which is related solely to the real wage rate at date t. To derive this labor supply function, note that the utility function implies that

$$\frac{\partial u(c_t, n_t, a_t)}{\partial c_t} = \left(c_t - \frac{a_t \theta}{1+\gamma} n_t^{1+\gamma} \right)^{-1},$$

$$\frac{\partial u(c_t, n_t, a_t)}{\partial n_t} = -a_t \theta n_t^{\gamma} \left(c_t - \frac{a_t \theta}{1+\gamma} n_t^{1+\gamma} \right)^{-1}.$$

Equating the marginal rate of substitution between leisure and consumption to the real wage yields a labor supply function with a constant wage elasticity (equal to $1/\gamma$):

$$n_t = \left(\frac{w_t}{a_t \theta} \right)^{1/\gamma}. \tag{9}$$

Labor supply is raised by the real wage and lowered by the productivity shifter, so there is no trend growth in hours worked when the real wage and productivity have a common trend.

Money Demand

The representative consumer must hold enough money to cover the quantity of his purchases:

$$M_t = kP_t c_t. \tag{10}$$

We think of this money demand function as a limiting case that applies when money is interest bearing, when there is a satiation level of cash balances (k) per unit of consumption, and the interest rate on money is close to the market rate. In such a setting, we should be able to ignore the "triangles under the demand curve for money" in order to focus on other costs of inflation.[11]

Other Financial Assets

Consumers also hold a diversified portfolio of shares in the monopolistically competitive firms, which pays dividends equal to the firms' monopoly profits. The real and nominal interest rates in this economy are governed by Fisherian principles. The real interest rate, r_t, must satisfy

11. Wolman's (1997) estimates of a "transactions technology"–based money demand function indicate the presence of a satiation level of cash balances per unit of consumption. They also indicate that most of the welfare gains from reducing average inflation from 5 percent to the Friedman rule are gained by making inflation zero.

(11) $$E_t\left(\beta\frac{\lambda_{t+1}}{\lambda_t}(1 + r_t)\right) = 1,$$

and the nominal interest rate, R_t, must satisfy

(12) $$E_t\left(\beta\frac{\lambda_{t+1}}{\lambda_t}\cdot\frac{P_t}{P_{t+1}}(1 + R_t)\right) = 1.$$

8.2.2 Firms

Each firm produces with an identical technology that is linear in labor and subject to random variation in productivity:

(13) $$c_{j,t} = a_t n_{j,t},$$

where $n_{j,t}$ is the labor input employed in period t by a firm whose price was set in period $t - j$. Given the price that a firm is charging, it hires enough labor to meet the demand for its product at that price. Firms that do not adjust their prices in a given period can thus be thought of as passive, whereas firms that adjust their prices do so optimally.

Firms set their prices to maximize the present discounted value of their profits. Given that it has a relative price $p_{j,t} = P_{j,t}/P_t$, real profit $z_{j,t}$ for a firm of type j is

(14) $$z_{j,t} = p_{j,t}a_t n_{j,t} - w_t n_{j,t},$$

that is, revenue less cost. To derive later results, it is useful to define real marginal cost ψ_t, which is equal to w_t/a_t in our setting. Then profit for a firm of type j is

(15) $$z_{j,t} = p_{j,t}^{-\varepsilon}c_t(p_{j,t} - \psi_t),$$

using the requirement that demand equal output ($c_{j,t} = p_{j,t}^{-\varepsilon}\, c_t = a_t n_{j,t}$).

Optimal Pricing without Price Stickiness

If prices were fully flexible, then a familiar set of expressions would govern optimal pricing in this constant elasticity, constant marginal cost world. Optimal monopoly pricing is illustrated in figure 8.1. Panel 8.1a shows the demand curve for the firm, $p_{j,t}^{-\varepsilon}c_t$, under the assumption that the level of aggregate demand is unity ($c_t = 1$) and the demand elasticity is four ($\varepsilon = 4$). Panel 8.1b shows profit as a function of the relative price. The relative price that maximizes profit is given by

$$p_t^* = \frac{\varepsilon}{\varepsilon - 1}\psi_t.$$

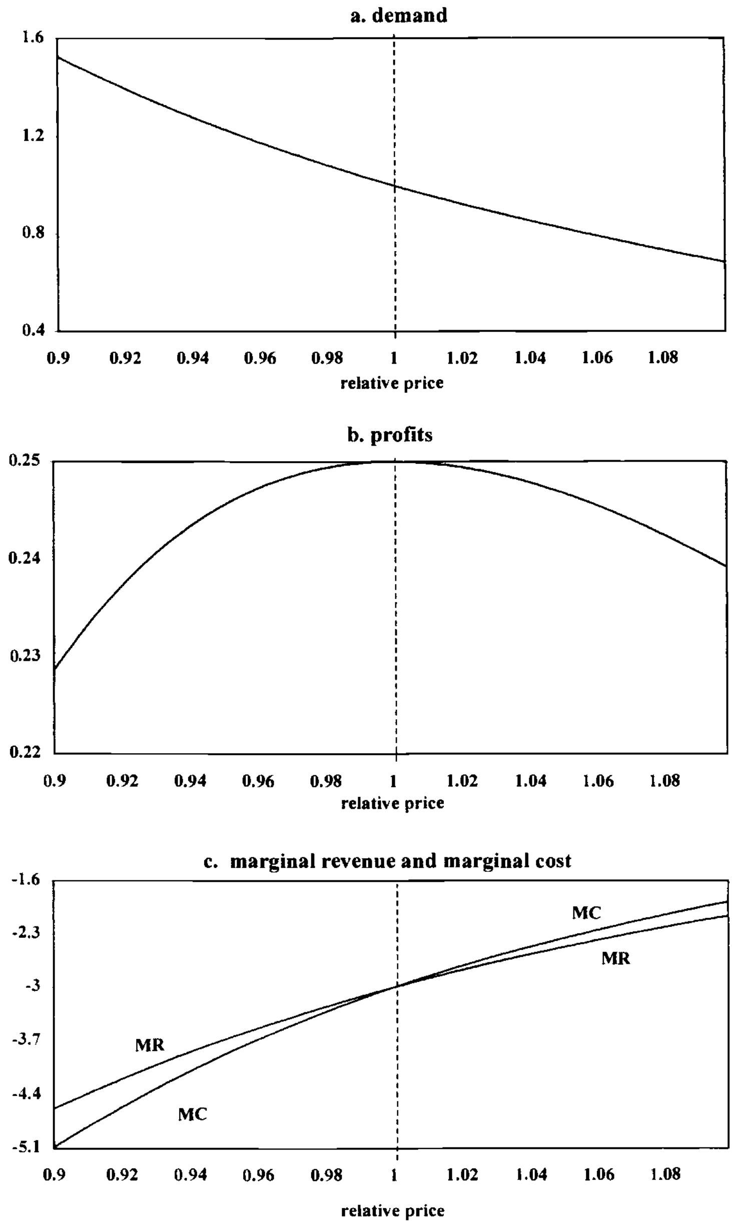

Fig. 8.1 Optimal pricing

In drawing the diagram, we have assumed real marginal cost is $(\varepsilon - 1)/\varepsilon$ so that the optimal relative price is unity.

Panel 8.1c shows the marginal revenue and marginal cost schedules that are relevant when the monopoly problem is written with price as the decision variable, rather than quantity as in the standard textbook presentation. Marginal revenue is negative, as the elasticity of demand must exceed one for the profit maximization problem to make sense (marginal revenue is $(1 - \varepsilon)p_{j,t}^{-\varepsilon}c_t$). For low levels of the relative price, marginal revenue exceeds marginal cost $(-\varepsilon p_{j,t}^{-\varepsilon-1}c_t\psi_t)$ and it is desirable to raise the price. Correspondingly, for high levels of the relative price, marginal revenue is less than marginal cost and profits could be increased by lowering the relative price.

Our model economy is one in which there are substantial real consequences of the market power shown in figure 8.1. In constructing this figure and in conducting simulations, the assumption that the demand elasticity is 4 implies a steady state markup of 1.33 when there is no price stickiness (or zero inflation). Combined with a highly elastic supply of labor ($\gamma = 0.10$), this implies that output is about 6 percent of its efficient level.[12] While this level of distortion is certainly too high, the extreme assumption does serve to stress that our results on optimal policy are valid for economies in which there are substantial departures of output from its efficient level and large effects of money on output.

Optimal Price Setting When Prices Are Sticky

Maximization of present value implies that a firm chooses its current relative price taking into account the effect on current and expected future profits. When it sets its nominal price at t, the firm knows that its relative price will move through time according to

$$(16) \qquad p_{1,t+1} = \frac{p_{0,t}}{\Pi_{t+1}},$$

where Π_{t+1} is the gross inflation rate between t and $t + 1$ ($\Pi_{t+1} = P_{t+1}/P_t$). That is: if there is positive inflation, the firm expects that its relative price will fall as a result of the fact that it has a nominal price that is fixed for two periods.[13]

The optimal relative price must balance effects on profits today and tomorrow, given that inflation erodes the relative price. Formally, an optimal relative price satisfies

12. The marginal rate of substitution, $-D_2u(c_t, n_t, a_t)/D_1u(c_t, n_t, a_t)$ is equal to $a_t\theta n_t^{\gamma}$ and the real wage rate is equal to a_t/μ, where μ is the average markup of price over marginal cost. Accordingly, the ratio of labor to its efficient level is given by $n_t/n^* = \mu^{-1/\gamma}$. The calculation in the text assumes that $\gamma = 0.10$ and $\mu = 1.33$.

13. If the nominal price that the firm charges is $P_{0,t} = P_{1,t+1}$, then $p_{0,t} = P_{0,t}/P_t$ and $p_{1,t+1} = P_{1,t+1}/P_{t+1} = (P_{0,t}/P_t)(P_t/P_{t+1})$.

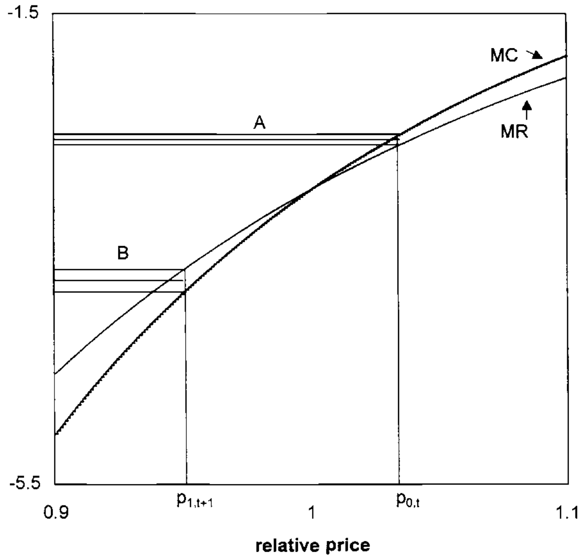

Fig. 8.2 Optimal pricing with stickiness

$$(17) \quad 0 = \lambda_t \frac{\partial z(p_{0,t}, c_t, \psi_t)}{\partial p_{0,t}} + \beta E_t \left(\lambda_{t+1} \frac{\partial z(p_{1,t+1}, c_{t+1}, \psi_{t+1})}{\partial p_{1,t+1}} \cdot \frac{1}{\Pi_{t+1}} \right).$$

Multiplying equation (17) by $p_{0,t}$ yields a more symmetric form of the efficiency condition that will be convenient for deriving optimal policy:

$$(18) \quad 0 = \lambda_t p_{0,t} \frac{\partial z(p_{0,t}, c_t, \psi_t)}{\partial p_{0,t}} + \beta E_t \left(\lambda_{t+1} p_{1,t+1} \frac{\partial z(p_{1,t+1}, c_{t+1}, \psi_{t+1})}{\partial p_{1,t+1}} \right).$$

Figure 8.2 shows two central aspects of this efficiency condition (this figure repeats panel c of figure 8.1 but adds some additional information). First, the marginal profit terms in equation (18) are gaps between marginal revenue and marginal cost at t and $t + 1$: with price stickiness, the firm can no longer make these both zero, but it can choose its price to balance the gaps through time. Second, figure 8.2 shows a firm setting a relative price p_0 that is too high relative to the static optimum—in the sense that marginal revenue is less than marginal cost—and a related price p_1 that is too low. This would be an optimal policy, for example, in a steady state situation of sustained inflation. Efficient price setting equates the boxes marked A and B, which correspond to the consumption value of profits lost at t, which is $p_{0,t} \cdot \partial z(p_{0,t}, c_t, \psi_t/\partial p_{0,t}$ in equation (18), and the profits lost at $t + 1$, which is $p_{1,t+1} \cdot \partial z(p_{1,t+1}, c_{t+1}, \psi_{t+1})/\partial p_{1,t+1}$.[14]

14. Technically, we must have $\lambda_t = \lambda_{t+1}$ and $\beta = 1$, as well as $\Pi > 1$ to use our diagram exactly.

As in Taylor's (1980) model, there is a forward-looking form of the price equation that can be developed. Using the expressions for marginal revenue and marginal cost, we can show that the optimal price is

$$(19) \qquad p_{0,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{j=0}^{1} \beta^j E_t[\lambda_{t+j} \psi_{t+j} (P_{t+j}/P_t)^{\varepsilon} c_{t+j}]}{\sum_{j=0}^{1} \beta^j E_t[\lambda_{t+j} (P_{t+j}/P_t)^{\varepsilon-1} c_{t+j}]}.$$

Thus equation (18) implicitly links the current relative price chosen by a price-setting firm to current and future marginal cost, as well as expected inflation and interest rates. Further, near zero inflation, this expression is well approximated by

$$\log p_{0,t} = \log\left(\frac{\varepsilon}{\varepsilon - 1}\right) + \frac{1}{1 + \beta}\log \psi_t$$
$$+ \frac{\beta}{1 + \beta}(E_t \log \psi_{t+1} + E_t \log P_{t+1} - \log P_t),$$

which is even more like Taylor's forward-looking specification. Approximately, therefore, an adjusting firm sets a relative price that is a weighted average of the real marginal cost it expects over the next two periods, with a correction for the effect of expected inflation.

8.3 Influences of Money on Economic Activity

In this section, we discuss how monetary policy affects economic activity in our model. Before getting into the details, however, it is useful to think about why a monetary authority might seek to influence economic activity in this type of model. The rationale is as follows. Since there is monopolistic competition, there is a positive markup of price over marginal cost even without sticky prices. The markup is an inefficiency, as it implies that the resource cost of producing the marginal good is below the utility benefit of consuming the marginal good.[15] As stressed by Mankiw (1990) and Romer (1993), it is consequently desirable to have policies that expand aggregate economic activity, if such policies are feasible. Sticky prices give the monetary authority some ability to alter the markup and to thus expand or contract economic activity, although we will see below that there are severe limitations on that ability in the long run. Sticky prices also introduce the possibility of a second distortion, namely, variation in the relative prices of the differentiated products. Because the differentiated products are produced using a common production technology, efficiency dictates that their relative prices should be identical. However,

15. An alternative, equally valid interpretation is that the marginal product of labor exceeds the marginal payment to labor, as measured by the real wage.

since some nominal prices are prohibited from adjusting in a given period, relative prices will not be identical for all goods if firms that are able to adjust their nominal prices choose to do so. This situation will occur, for example, if there is a nonzero inflation rate.

8.3.1 Effects of Steady Inflation

The effect of steady inflation on the two distortions—the relative price distortion and the markup distortion—can be seen by referring to the equations for the price index and the optimal price of an adjusting firm. The relative price distortion is minimized (eliminated, in fact) at a zero inflation rate, and the markup distortion is minimized at a slightly positive inflation rate. The fact that these distortions are not invariant to steady state inflation means that the model exhibits nonsuperneutrality, or, in the lingo of sticky price models, a nonvertical long-run Phillips curve. As will be shown below, however, the long-run Phillips curve is nearly vertical. Furthermore, there is a negative relationship between inflation and output over most positive inflation rates. We analyze the effect of steady inflation on relative price distortions first and then turn to its effect on the markup distortion.

Relative Price Distortions

To begin, if the gross inflation rate is $\Pi \equiv P_t/P_{t-1}$, then the ratio of relative prices of adjusting and nonadjusting firms is $\Pi = P_{0,t}/P_{0,t-1}$. That is, with positive and steady inflation an adjusting firm in the current period sets its nominal price at a level Π times greater than a firm that adjusted in the previous period. Further, from equation (5), the price index in steady state is

$$P_t = \left(\frac{1}{2}P_{0,t}^{1-\varepsilon} + \frac{1}{2}\Pi^{\varepsilon-1}P_{0,t}^{1-\varepsilon}\right)^{1/(1-\varepsilon)}, \tag{20}$$

so the ratio P_0/P—which can be thought of as a measure of the variation in relative prices—is increasing in inflation (since $P_0/P = [(1 + \Pi^{\varepsilon-1})/2]^{1/(\varepsilon-1)}$).

Variation in relative prices implies that there is a gap between potential and actual consumption, where potential consumption is the maximum quantity of the consumption aggregate that can be obtained from a given level of technology and labor input. Since the aggregator, $c(c_{0,t}, c_{1,t})$, is concave and symmetric, potential consumption corresponds to equal quantities of each of the different goods. However, equal quantities will be *chosen* by individuals only if there is no variation in relative prices, and since zero inflation equates relative prices, it achieves potential consumption. Mathematically, equations (3) and (4) can be manipulated to show that in steady state, the ratio of actual to potential consumption is given by

$$\frac{c}{c^{\mathrm{p}}} = \left(\frac{1}{2}\right)^{1/(\varepsilon-1)} \frac{(1 + \Pi^{\varepsilon-1})^{\varepsilon/(\varepsilon-1)}}{1 + \Pi^{\varepsilon}}, \tag{21}$$

which uses the fact that potential consumption is simply the linear index of consumption, $c^p = c_0/2 + c_1/2$. Expression (21) confirms that with price stability ($\Pi = 1$), actual and potential consumption are equal, whereas the relative price variation induced by inflation makes actual consumption less than potential. The relative price distortion is quite small at low inflation rates.

Average Markup Distortion

The effect of inflation on the average markup can be seen by combining the price index with the steady state version of the optimal price equation (19). In our economy, as in others with marginal cost that is common across firms, the average markup (μ) is simply the inverse of real marginal cost. Using this fact, the optimal price equation becomes

$$P_0 = P\frac{\varepsilon}{\varepsilon - 1}\left(\frac{1}{\mu}\right)\left(\frac{1 + \beta\Pi^{\varepsilon}}{1 + \beta\Pi^{\varepsilon-1}}\right). \tag{22}$$

When we combine this expression with the price index, $P = P_0[(1 + \Pi^{\varepsilon-1})/2]^{1/1-\varepsilon}$, the average markup is

$$\mu = \frac{\varepsilon}{\varepsilon - 1}\left(\frac{1 + \Pi^{\varepsilon-1}}{2}\right)^{1/(1-\varepsilon)}\left(\frac{1 + \beta\Pi^{\varepsilon}}{1 + \beta\Pi^{\varepsilon-1}}\right). \tag{23}$$

The inflation rate therefore affects the average markup, with the "static" markup $\varepsilon/(\varepsilon - 1)$ resulting from zero inflation ($\Pi = 1$).

There are two components in the inflation-markup link. Higher inflation makes adjusting firms choose a higher markup when they do adjust, but it also makes the markup of nonadjusting firms erode more severely. In general, for high enough inflation the effect on adjusting firms dominates, so that higher inflation is associated with a higher markup. While these counteracting effects are qualitatively interesting, figure 8.3a shows that they are small in the following sense. When $\varepsilon = 4$, the steady state markup is minimized at an extremely low inflation rate (approximately 1 percent annually), and this result carries over to any reasonable degree of market power (higher or lower than $\varepsilon = 4$); furthermore, for a given value of ε, the markup is essentially insensitive to inflation. That is: over the inflation rates shown in figure 8.3, which are between -1 and 2 percent at an annual rate, the markup changes only at the fifth digit (see the markup scale in fig. 8.3a).[16]

Since the relative price distortion is eliminated at zero inflation, it follows that steady state welfare is maximized at some inflation rate between zero and that which minimizes the markup. We define $\hat{\Pi}$ to be the inflation rate that

16. Goodfriend and King (1997) perform a similar calculation with a model of four-quarter price stickiness and a wider range of inflation rates. They find that the average markup rises more rapidly at higher rates of inflation.

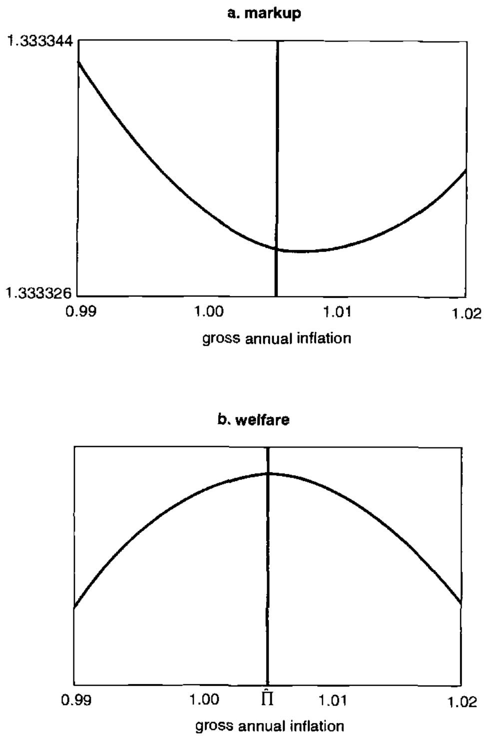

Fig. 8.3 Steady state

maximizes steady state welfare; it is illustrated in figure 8.3b, which plots steady state welfare as a function of inflation.

8.3.2 Dynamic Response to Monetary Disturbances

There are two central implications of New Keynesian macroeconomic models beginning with Taylor (1980). First, there can be large and persistent effects of changes in money. Second, it is crucial to specify the dynamic nature of the monetary change and the corresponding beliefs that agents hold. To display these crucial implications in our context, we close the model by adding a policy rule. For expository purposes, we begin by assuming an exogenous process for the money growth rate and examining responses to two types of shocks. First, we look at a persistent increase in the money growth rate, but one that ultimately dies away so that trend inflation is unaffected. Second, we look at a permanent decrease in the money growth rate, which results in a permanent reduction in the inflation rate. Then, because monetary policy in the industrialized countries is generally conducted with an interest rate instrument, we ex-

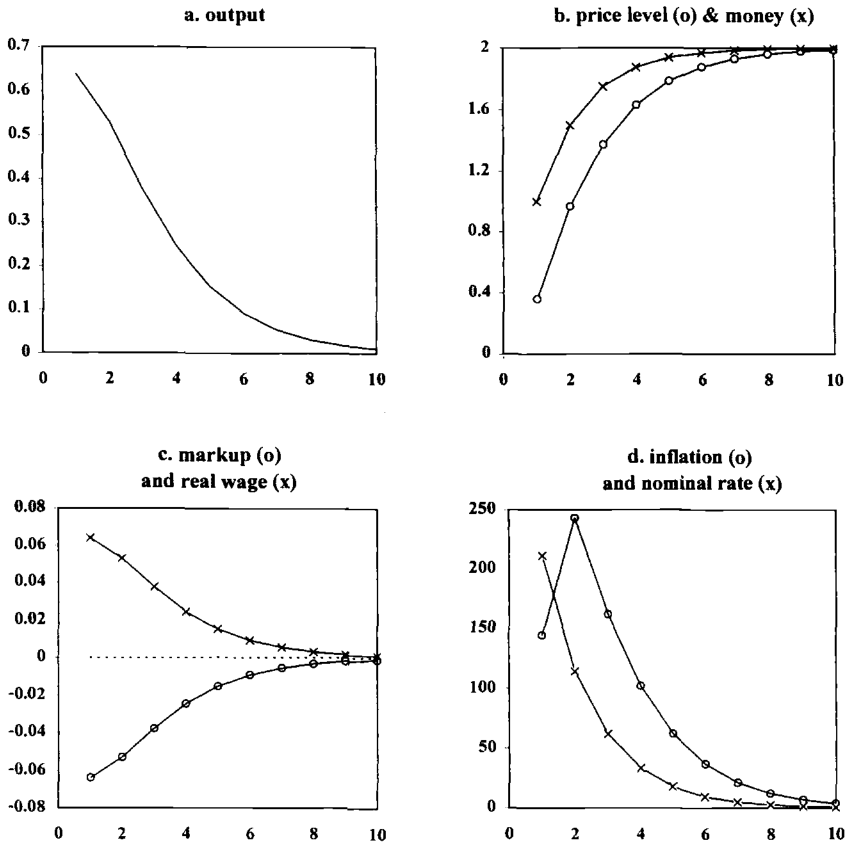

Fig. 8.4 Unanticipated, persistent increase in money growth

Note: x-Axes in quarters; *y*-axes in percentage deviations from steady state, except panel d, which is in basis points at an annual rate.

amine the effect of a temporary policy shock when policy is given by an interest rate feedback rule.

Effects of a Persistent Increase in Money Growth

As illustrated in figure 8.4, an unanticipated persistent increase in the money growth rate generates a persistent increase in output, as money growth exceeds inflation for several periods. (The policy shock corresponds to a 1 percent increase in money on impact and ultimately a 2 percent increase in the level of money).[17] There are two complementary ways of thinking about the output

17. Technically, the driving process for money is $\log M_t - \log M_{t-1} = \frac{1}{2}(\log M_{t-1} - \log M_{t-2}) + \xi_t$, with ξ_t being a sequence of random shocks.

expansion, which begins with an immediate increase in output of about two-thirds of a percent and then dies away through time. First, the monetary expansion produces an increase in aggregate demand, which is accommodated by firms that are holding prices fixed (the strength of this aggregate demand effect can be measured as the distance between the money and price responses). Second, the increase in output reflects the monetary authority's ability to change the extent of distortions in the economy and relative prices. In particular, the average markup falls in the face of a monetary expansion. Since the average markup is the reciprocal of real marginal cost ($\mu = P/(P\psi) = 1/\psi$) and since marginal cost is the real wage divided by labor productivity ($\psi_t = w_t/a_t$), there is a corresponding rise in the real wage that stimulates aggregate supply. Given an assumed high-amplitude response of labor supply to wages, this decline in the markup results in a significant expansion of aggregate output. On net, the monetary authority has stimulated the economy by driving down the markup temporarily, taking advantage of preset prices.

One notable feature of this dynamic response, as in Taylor (1980), is that the persistence in real variables is much greater than the duration of fixed prices. Recent research by Chari, Kehoe, and McGrattan (1996) has stressed that this persistence in real output requires that increases in output carry with them only small increases in marginal cost. In our example economy in figure 8.4 the labor supply is highly elastic, which translates into low elasticity of marginal cost with respect to output. More specifically, the elasticity of real marginal cost with respect to output is γ, and figure 8.4 was generated with $\gamma = 0.1$.[18] We use this specification throughout the current analysis, as we think that a more complete macroeconomic model with variable capacity utilization and indivisible labor may generate a large and persistent effect of monetary shocks.[19]

Another notable feature of this response is the behavior of the nominal interest rate, which exhibits a persistent increase, reflecting mainly the behavior of expected inflation. It is a general feature of sticky price models that persistent increases in the money supply generate persistent increases in expected inflation, and usually the nominal interest rate. This result is troubling if one believes that monetary policy is appropriately modeled by an exogenous money growth rate, as it conflicts with the conventional wisdom that expansionary monetary policy involves a decrease in the nominal interest rate. We will see below that if policy is modeled as an interest rate feedback rule, then expansionary (contractionary) shocks do generate decreases (increases) in the nominal interest rate.

18. In contrast to the preference specification used here, the more standard form ($\ln c - \theta n^{1+\gamma}$) implies an elasticity of real marginal cost with respect to output that is at least unity, due to the income effect on labor supply.

19. See Dotsey et al. (1997) for an analysis along these lines.

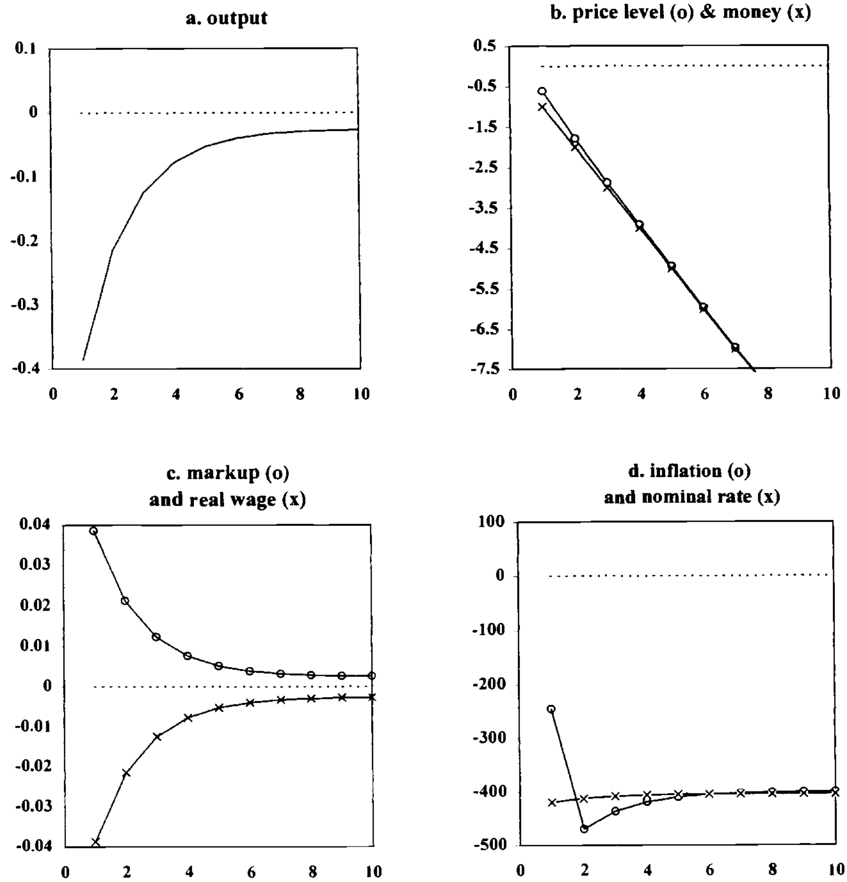

Fig. 8.5 Unanticipated, permanent decrease in money growth
Note: For axis units, see note to fig. 8.4.

Effects of a Permanent Decrease in Inflation

Figure 8.5 contains impulse response functions for an unanticipated permanent decrease in the money growth rate. Aside from the sign difference, these responses look similar to figure 8.4, confirming the standard notion that disinflation is costly with sticky prices. However, it is important to stress two features of this policy shock. First, the magnitude of the temporary aggregate contraction is smaller, about half as large as in figure 8.4. Second, the sustained deflation also has a long-run effect on relative prices and relative quantities as individuals substitute across goods. The initial steady state inflation rate in figure 8.5 is zero, so a permanent decrease in inflation raises the markup permanently (see eq. [23]).

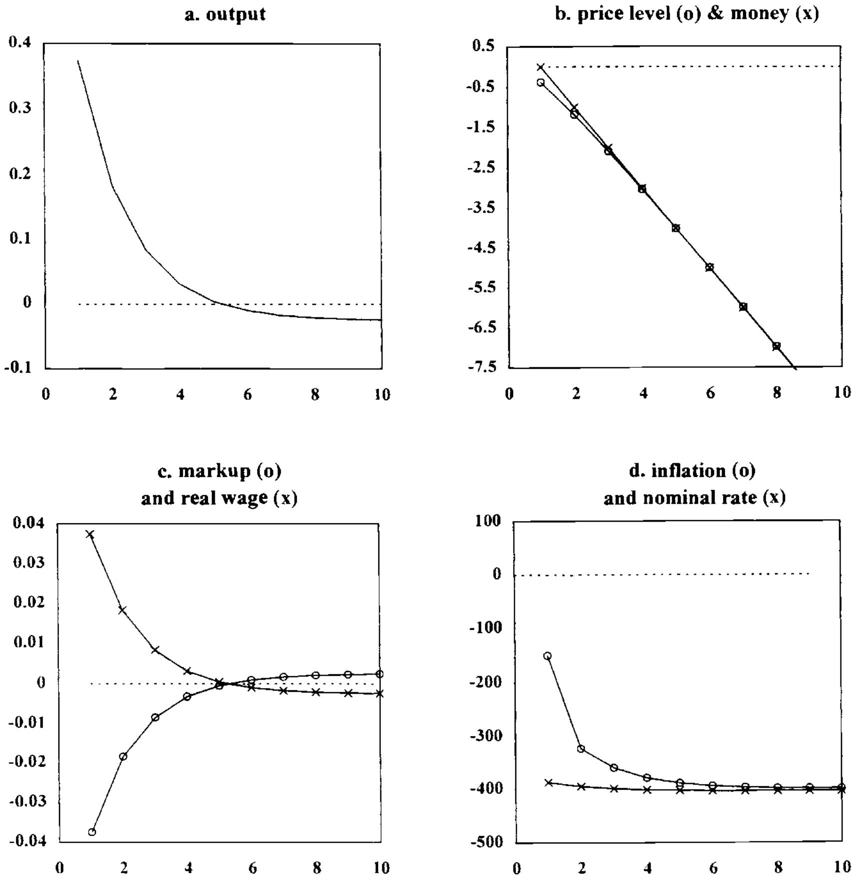

Fig. 8.6 Anticipated, permanent decrease in money growth
Note: For axis units, see note to fig. 8.4.

Ball (1995) has stressed that sticky price models generally imply that there is a very different effect of a credible anticipated disinflation: our model is no exception. As illustrated in figure 8.6, an anticipated deflation is expansionary: consumption actually rises for several periods. This expansion occurs because firms act in advance of the policy change, lowering their relative prices now because they know that future inflation will be lower. Since figure 8.6 assumes that the starting point is zero inflation, agents will be in a worse steady state in the long run (the markup will be slightly higher), with lower consumption and higher leisure. However, during the transition they benefit from a *lower* markup and corresponding higher level of output. As with figure 8.5, however, it is again the case that the effect of a sustained deflation is smaller than the effect of a temporary change in the money growth rate, even though both disturbances are normalized to have the same (1 percent) effect on money in the initial period. That is: the forward-looking price setting built into this model implies that the effects of anticipated and unanticipated changes in the trend inflation rate are smaller than the effects of shocks that are not permanent.

Effects of an Interest Rate Shock

It is straightforward to analyze how the model behaves in response to policy shocks when policy is given by an interest rate rule, which is the more relevant case empirically. Suppose that instead of an exogenous money growth rate, the monetary policy rule specifies that

$$R_t = f\cdot(\ln P_t - \ln \overline{P}_t) + e_t, \tag{24}$$

where R_t is the nominal interest rate, $\overline{P}_t$ is a target price level that either is constant or grows at a constant rate, f is a positive coefficient that describes the feedback from price level deviations to nominal interest rate changes, and e_t is a random variable that follows a stochastic process known to agents in the economy. Figure 8.7 illustrates the response of key variables to an unanticipated decrease in e_t, assuming that e_t follows an AR(1) process with autoregressive coefficient 0.5. Such a decrease in R is an expansionary policy shock: while the nominal interest rate falls 100 basis points on impact and then climbs back to its steady state value, the money supply behaves almost as a mirror image, rising on impact by about 0.6 percent, and then falling back to its steady state level. Because the policy rule involves feedback from the price level instead of inflation, the money supply and the price level both return to their steady state levels.[20] It may seem puzzling at first that an expansionary policy shock generates an increase in the nominal interest rate with a money supply rule, but a decrease with an interest rate rule. The resolution lies in the behavior of the money supply in these two cases. With the money supply rule, an expansionary shock involves an initial increase in the money supply that is amplified over time, at a decreasing rate, until a steady state with a higher quantity of money has been reached. In contrast, with the interest rate feedback rule, an expansionary shock involves an initial increase in money that is *reversed* over time. The money rule thus leads to an increase in expected inflation following the initial unexpected rise in the price level, whereas the interest rate rule leads to a decrease in expected inflation.

8.4 Constraints on Monetary Policy

We now turn to the central topic of the paper, optimal monetary policy. That is, we turn to describing how the monetary authority should behave, having already described how the model economy behaves given specific monetary

20. A similar rule with feedback from inflation rather than the price level would generate base drift, permanent changes in the price level and money supply.

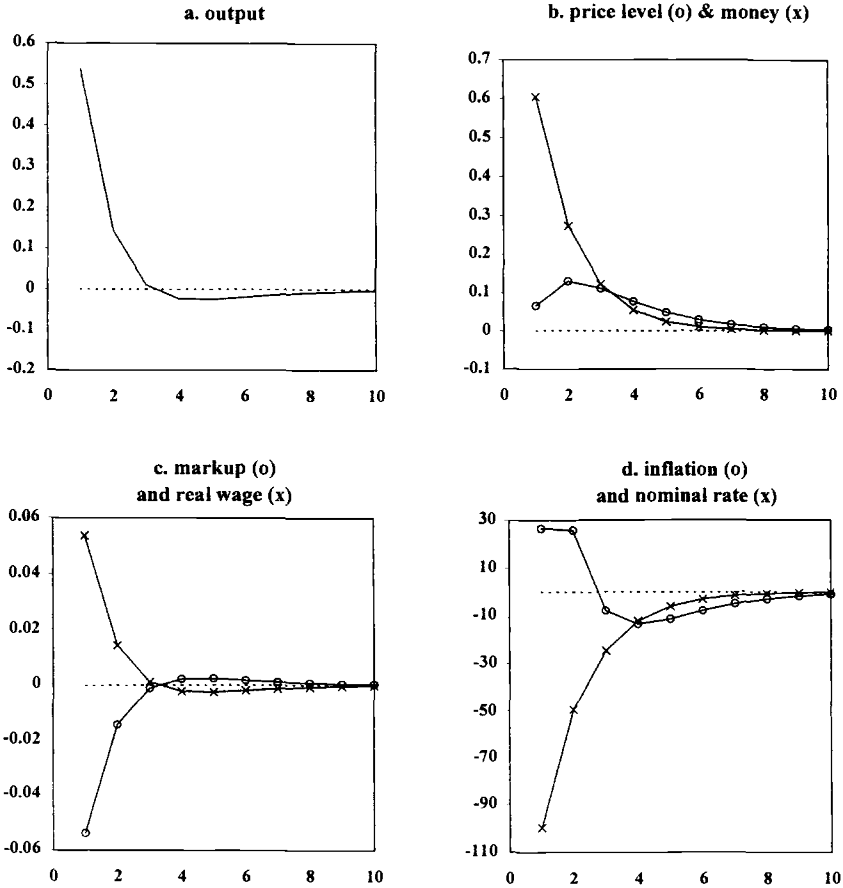

Fig. 8.7 Persistent shock to an interest rate rule
Note: For axis units, see note to fig. 8.4.

policy actions. To keep the analysis as simple as possible, we assume that there is a single shock to macroeconomic activity—an aggregate shock to productivity—in this section. Later we discuss generalizing the analysis to accommodate other types of shocks. The objective of optimal policy is to maximize welfare, that is, the representative agent's lifetime utility. Absent monopolistic competition and sticky prices, laissez faire would be an optimal policy, with any rate of inflation yielding the same welfare level.

In our model, however, monopolistic competition and sticky prices are key features. In principle, the policymaker would like to offset the effects of monopolistic competition and sticky prices; specifically, it would like to make relative prices behave as if these two frictions were absent. In pursuing this objective, the policymaker is constrained in two ways. First, the economy has exogenous technology and limited resources. Second, firms are monopolistically competitive with sticky prices, and in setting prices they take into account the demand curves and marginal costs they expect to face. Firms' expectations, in turn, depend on their beliefs about how policy is conducted.

We assume that policy is conducted with commitment; the monetary authority chooses a state-contingent plan and sticks with it, even though the plan is time inconsistent. The policymaker in our model does not have access to a full set of fiscal policy instruments. With such instruments it would be possible to achieve a first-best allocation; production subsidies financed by lump-sum taxation would succeed in offsetting the effects of imperfect competition. Alternatively, as in Yun (1994), the combined fiscal and monetary authorities could be constrained to raise revenue with distorting taxes, so that the first-best allocation might not be achievable, but a combined Ramsey taxation approach applied. Rotemberg and Woodford, in chapter 2 of this volume, assume that fiscal instruments are available to eliminate steady state distortions, leaving to monetary policy the job of business cycle stabilization. We assume that there are no fiscal interventions available, and we impose no fiscal constraints on the monetary authority's plans.[21] That is: our objective is to isolate principles of optimal monetary policy without a complicating discussion of fiscal issues.

Resource Constraints

The constraints involving technology and resources are given by the production functions

(25) $$c_{j,t} \leq a_t n_{j,t} \qquad \text{for } j = 0, 1,$$

the consumption aggregator

(26) $$c_t \leq \left(\frac{1}{2} c_{0,t}^{(\varepsilon-1)/\varepsilon} + \frac{1}{2} c_{1,t}^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)},$$

and agents' limited time endowments

(27) $$1 \geq n_t = \frac{1}{2} n_{0,t} + \frac{1}{2} n_{1,t}.$$

Implementation Constraint

The monetary authority must choose quantities that are consistent with monopolistic competition and sticky prices. In the current model with two-period price stickiness, the single constraint involving firms' price setting is as follows: quantities must be consistent with the fact that any firm adjusting its price will do so optimally. We call this constraint an "implementation constraint" because it describes how the monetary authority is constrained by the

21. As mentioned previously, the monetary authority does have lump-sum taxes available to finance monetary policy actions.

fact that it must induce sticky price, monopolistically competitive firms to implement its chosen quantities.

The most convenient formulation of the implementation constraint is based on the requirement that the present value of marginal profits from a price change is zero (as in eq. [18]):

$$\lambda_t p_{0,t} \frac{\partial z_{0,t}}{\partial p_{0,t}} + \beta E_t \lambda_{t+1} p_{1,t+1} \frac{\partial z_{1,t+1}}{\partial p_{1,t+1}} = 0.$$

It is then straightforward to rewrite the implementation constraint in terms of real quantities only, without any nominal magnitudes or relative prices:

$$(28) \qquad x(c_{0,t}, c_t, n_t, a_t) + \beta E_t x(c_{1,t+1}, c_{t+1}, n_{t+1}, a_{t+1}) = 0,$$

where x is a function that involves real quantities only, as we discuss next. To derive this version of the constraint on the monetary authority, we use the demand function ($c_{j,t} = p_{j,t}^{-\varepsilon} c_t$) to eliminate relative prices. We eliminate real marginal cost by using its definition as w_t/a_t and then use the equality between the real wage and the marginal rate of substitution between consumption and leisure to eliminate the real wage. In the current model, the x function reduces to[22]

$$(29) \qquad x(c_{j,t} c_t, n_t, a_t) = \lambda_t c_t \left[(1 - \varepsilon) \left(\frac{c_{j,t}}{c_t} \right)^{1-1/\varepsilon} + \varepsilon \theta n_t^{\gamma} \left(\frac{c_{j,t}}{c_t} \right) \right].$$

In what sense is equation (28) a constraint on the policymaker? As a simple example, suppose the policymaker wanted to raise output through a monetary injection in period t. One rationale for the expansion might be that output is inefficiently low: since some firms cannot adjust their prices, an increase in aggregate demand is desirable because it will raise output toward the efficient level. However, firms that do adjust their prices—both in current and future periods—behave according to equation (19), raising their prices in the face of an increase in demand. In this way the behavior of price-setting firms constrains (but does not eliminate) the monetary authority's ability to manipulate real quantities.

8.5 The Real Policy Problem

To determine optimal monetary policy, we first determine choices for real activity that are optimal subject to the resource and implementation constraints. Subsequently, we determine the behavior of nominal variables and relative prices that are consistent with these real quantities. While unusual in macroeconomics, this two-step practice is common in public finance and other

22. As above, we use λ_t to denote the marginal utility of consumption in period t. As such, λ_t is a function of c_t, n_t, and a_t.

areas of applied general equilibrium analysis. To make this problem relatively easy to state formally, we also ignore expectations throughout this section; in related work using a complete contingent markets approach, we have found that similar efficiency conditions describe optimal policy under uncertainty.

Optimal policy under commitment can be found by writing a restricted social planner's problem that involves maximizing expression (2) with a choice of sequences for $c_{0,t}$, $c_{1,t}$, c_t, $n_{0,t}$, $n_{1,t}$, n_t, subject to conditions (25)–(28). The Lagrangian for this problem is

$$
\begin{aligned}
L = & \sum_{t=0}^{\infty} \beta^t u(c_t, n_t, a_t) \\
& + \sum_{t=0}^{\infty} \beta^t \phi_t [x(c_{0,t}, c_t, n_t, a_t) + \beta x(c_{1,t+1}, c_{t+1}, n_{t+1}, a_{t+1})] \\
& + \sum_{t=0}^{\infty} \beta^t \Lambda_t [c(c_{0,t}, c_{1,t}) - c_t] \\
& + \sum_{t=0}^{\infty} \beta^t \Omega_t \left(n_t - \frac{1}{2} n_{0,t} - \frac{1}{2} n_{1,t} \right) \\
& + \sum_{t=0}^{\infty} \beta^t [\rho_{0,t}(a_t n_{0,t} - c_{0,t}) + \rho_{1,t}(a_t n_{1,t} - c_{1,t})].
\end{aligned} \tag{30}
$$

We will discuss the various multipliers in this Lagrangian as they appear in the analysis below.

8.5.1 Background to the Policy Problem

The optimal policy problem described by equation (30) is a *restricted* planner's problem because any social planner worth his or her salt would not be constrained by staggered price setting. To see how our specific restriction plays a role, we can imagine a series of problems with weaker restrictions.

First, if we were to remove constraint (28), the problem would be that of a planner constrained only by tastes and technology, as typically defined. The optimal allocations would make consumption at both types of firms equal (since the aggregator is concave and symmetric in these quantities). Further, the unrestricted social planner would choose aggregate consumption and work effort so that the marginal rate of substitution

$$
-\frac{\partial u(c_t, n_t, a_t)}{\partial n_t} \Bigg/ \frac{\partial u(c_t, n_t, a_t)}{\partial c_t}
$$

equaled the marginal rate of transformation a_t.

Second, consider an intermediate planner, who cannot overcome monopolistic competition but can force firms to change their prices each period. Instead of facing constraint (28), such a planner would face the constraint $x(c_t, c_t, n_t, a_t) = 0$. Symmetry and concavity of the aggregator c would again make it unambiguous that optimal policy would equate consumption across the differentiated products. However, the planner could no longer equate the marginal rate of substitution to a_t because this would not respect the implementation constraint; that is, it would be inconsistent with monopolistic competition. Considering this intermediate problem reveals that if monetary policy can cause consumption of all the goods to be equated, it will have "undone" the price stickiness. We saw above, however, that it is price stickiness—combined with imperfect competition—that gives the monetary authority some real leverage in the economy. The monetary authority's optimality conditions must then effectively involve balancing the distortions arising from price stickiness against the ability to affect real quantities.

8.5.2 Optimality Conditions

The first-order conditions for equation (30) can be written in the following time-invariant form if we define an artificial multiplier ϕ_{-1} to enter several of the constraints at date $t = 0$; we return later to a more detailed discussion of the role of this multiplier. Optimal choice of the labor input at each type of firm implies

$$(31) \qquad 0 = \frac{\partial L}{\partial n_{j,t}} = \beta^t\left(\rho_{j,t}a_t - \frac{1}{2}\Omega_t\right) \qquad \text{for } j = 0, 1,$$

and optimal choice of the consumption levels from each type of firm implies

$$(32) \qquad 0 = \frac{\partial L}{\partial c_{j,t}} = \beta^t\left(\Lambda_t\frac{\partial c(c_{0,t}, c_{1,t})}{\partial c_{j,t}} - \rho_{j,t} + \phi_{t-j}\frac{\partial x(c_{j,t}, c_t, n_t, a_t)}{\partial c_{j,t}}\right) \quad \text{for } j = 0, 1.$$

Optimal choice of aggregate consumption and labor requires

$$(33) \qquad 0 = \frac{\partial L}{\partial c_t} = \beta^t\left(\frac{\partial u(c_t, n_t, a_t)}{\partial c_t} - \Lambda_t + \sum_{j=0}^{1}\phi_{t-j}\frac{\partial x(c_{j,t}, c_t, n_t, a_t)}{\partial c_t}\right)$$

and

$$(34) \qquad 0 = \frac{\partial L}{\partial n_t} = \beta^t\left(\frac{\partial u(c_t, n_t, a_t)}{\partial n_t} + \Omega_t + \sum_{j=0}^{1}\phi_{t-j}\frac{\partial x(c_{j,t}, c_t, n_t, a_t)}{\partial n_t}\right).$$

In addition to these quantity efficiency conditions, the first-order conditions include the constraints themselves, (25)–(28), which all will hold with equality.

Note that for $t = 0$ we have introduced ϕ_{-1} in equations (32)–(34), whereas it is not present in equation (30). As in Kydland and Prescott (1980), the purpose of introducing this artificial multiplier is to have a convenient mathematical expression for the policymaker's problem in a world of commitment, as we discuss further below. With this multiplier in place, the efficiency conditions for a successor government at date $t + 1$ would take exactly the same form as those efficiency conditions for the current government that occur for $t + 1$, $t + 2$, and so forth.

8.5.3 General Implications

Some of the first-order conditions for our restricted social planning problem look just like those for an unrestricted social planner. For example, optimal labor allocations equate the utility-denominated price of a unit of each type of good ($\rho_{j,t}$) to a measure of unit cost, the utility-denominated value of labor (Ω_t) divided by productivity (a_t).

However, the other conditions reflect the implementation constraint's effect on the restricted social planner's behavior. Comparing these first-order conditions to the analogues from the decentralized formulation of the model provides some insight into the optimal policy problem. First, however, it is necessary to say a word about ϕ_t. The multiplier ϕ_t is the shadow value of decreasing a price-setting firm's marginal present discounted profits with respect to relative price. Because the planner would like firms to have positive marginal profits, the multiplier is negative.[23] For consumption and labor, the individual's first-order conditions were

$$0 = \beta^t \left(\frac{\partial u(c_t, n_t, a_t)}{\partial c_t} - \lambda_t \right), \tag{37}$$

$$0 = \beta^t \left(\frac{\partial u(c_t, n_t, a_t)}{\partial n_t} - \lambda_t w_t \right).$$

As in section 8.2 above, λ_t is the shadow value of a unit of consumption to the individual, whereas Λ_t is the value the *planner* attaches to a marginal unit of consumption. In a competitive economy these objects are identical. With monopolistic competition, the planner values the marginal unit of consumption more than an individual does because higher aggregate consumption alleviates the monopoly pricing constraint (28). A marginal unit of labor input has a similar

23. Note that the second set of terms in eq. (30), which is written there as

$$+\sum_{t=0}^{\infty} \beta^t \phi_t [x(c_{0,t}, c_t, n_t, a_t) + \beta x(c_{1,t+1}, c_{t+1}, n_{t+1}, a_{t+1})], \tag{35}$$

can alternatively be written as

$$-\sum_{t=0}^{\infty} \beta^t \phi_t [0 - x(c_{0,t}, c_t, n_t, a_t) - \beta x(c_{1,t+1}, c_{t+1}, n_{t+1}, a_{t+1})]; \tag{36}$$

zero is the "bound" on marginal profits, and $-\phi$ is the value of relaxing that bound.

effect ($\Omega_t > -\partial u(c_t, n_t, a_t)/\partial n_t$) because labor supply dictates that higher labor input corresponds to a higher real wage and hence a lower markup.

The first-order conditions for c_j do not have as simple an analogue in the decentralized problem, but they can be easily understood as describing how the policymaker equates appropriately defined marginal rates of substitution and transformation between c_0 and c_1. Rewriting equation (32) as

$$(38)\quad \Lambda_t \frac{\partial c(c_{0,t}, c_{1,t})}{\partial c_{j,t}} + \phi_{t-j} \frac{\partial x(c_{j,t}, c_t, n_t, a_t)}{\partial c_{j,t}} = \rho_{j,t} \qquad \text{for } j = 0,\ 1,$$

and dividing equation (38) for c_0 by (38) for c_1 yields

$$(39)\quad \frac{\Lambda_t \dfrac{\partial c(c_{0,t}, c_{1,t})}{\partial c_{0,t}} + \phi_t \dfrac{\partial x(c_{0,t}, c_t, n_t, a_t)}{\partial c_{0,t}}}{\Lambda_t \dfrac{\partial c(c_{0,t}, c_{1,t})}{\partial c_{1,t}} + \phi_{t-1} \dfrac{\partial x(c_{1,t}, c_t, n_t, a_t)}{\partial c_{1,t}}} = \frac{\rho_{0,t}}{\rho_{1,t}}.$$

The left-hand side is the policymaker's marginal rate of substitution between $c_{0,t}$ and $c_{1,t}$; it describes how much $c_{0,t}$ the policymaker would forgo to gain a marginal increase in $c_{1,t}$. The decrease in $c_{0,t}$ has two effects. First, it mechanically decreases the index of consumption, and marginal consumption is valued at Λ_t. Second, it affects marginal profits ($\partial x(c_{0,t}, c_t, n_t, a_t)/\partial c_{0,t}$), and *marginal* marginal profits are valued at ϕ_t. Increasing $c_{1,t}$ has similar effects, except that the change in marginal profits is valued with last period's multiplier, reflecting the importance of firms who set their prices in that period. Under commitment, the policymaker takes into account the effect current-period policy actions have on previous-period decisions. The right-hand side of equation (39) is the marginal rate of transformation between $c_{0,t}$ and $c_{1,t}$; it describes how much $c_{1,t}$ could be produced using the resources freed up by a marginal decrease in the production of $c_{0,t}$.

8.5.4 Time Invariance and Time Consistency

We are interested in optimal allocations that arise when the policy authority can fully commit to follow through on a plan that is optimal, that is, the solution to a maximization problem such as that discussed above. This focus raises a set of interrelated conceptual and technical issues.

Technically, an unusual aspect of the restricted social planning problem is that there is a forward-looking constraint (28), as in Aiyagari and Braun (1997). This is reflected in the form of the efficiency conditions (33) and (34) for choices at date t, which involve the lagged multiplier ϕ_{t-1}. Intuitively, the presence of this lagged multiplier originates from the fact that for any date $t > 0$, a change in c_t affects the pricing decision of firms in period $t - 1$. In fact, we can rewrite the implementation constraint terms in the Lagrangian as follows,

$$\sum_{t=0}^{\infty} \beta^t \phi_t [x(c_{0,t}, c_t, n_t, a_t) + \beta x(c_{1,t+1}, c_{t+1}, n_{t+1}, a_{t+1})]$$
$$= \phi_0 x(c_{0,0}, c_0, n_0, a_0) + \sum_{t=1}^{\infty} \beta^t [\phi_t x(c_{0,t}, c_t, n_t, a_t) + \phi_{t-1} x(c_{1,t}, c_t, n_t, a_t)]. \quad (40)$$

This rewriting stresses that there is an asymmetry attached to the initial start-up period. Above, we eliminated this asymmetry by including a lagged multiplier, so that the first term on the right-hand side of equation (40) is implicitly written as $\phi_t x(c_{0,t}, c_t, n_t, a_t) + \phi_{t-1} x(c_{1,t}, c_t, n_t, a_t)$ for $t = 0$. Including the lagged multiplier yields a time-invariant system. Time invariance is desirable from a computational point of view, since it allows us to employ standard fixed-coefficient linear rational expectations solution methods to calculate the solution to the restricted social planning problem.

Conceptually, if the policy authority is free to reformulate its optimal plan on a period-by-period basis, then there is a problem of time inconsistency of optimal plans as in Kydland and Prescott (1977) and Barro and Gordon (1983). It is then not sensible to formulate the optimal policy problem as we have, which is based on the assumption that the monetary authority can choose a sequence of binding actions for every period. Our implicit assumption is that the policy authority is required to commit to a state-contingent plan in period zero and follow it in all subsequent periods. Following Kydland and Prescott (1980), we view the introduction of the artificial multiplier as a device corresponding to the focus of our investigation: we want to consider the behavior of an economy after the effects of an initial "start-up" period have worn away. That is, we are looking at a stochastic steady state in which a monetary authority has long been following an optimal monetary policy. We can calculate the effect of an initial start-up period by setting the lagged multiplier ϕ_{-1} to zero and studying the resulting paths of economic activity from this initial condition. Computing the magnitude of welfare in this situation, relative to one that starts with ϕ_{-1} equal to its steady state value, provides a measure of the temptation for a policymaker to renege on a previously chosen plan.

We divide our discussion of optimal monetary policy into two questions. First, what pattern of real quantities should the monetary authority pick as its long-run objective and what does this imply about the optimal rate of inflation? Second, how should quantities vary in response to productivity shocks and what does this imply about the relationship of nominal variables to the business cycle?

8.6 A Monetary Modified Golden Rule

In section 8.3, we noted that relative price distortions are minimized at zero inflation, and the smallest average markup is achieved at a low but positive inflation rate. Not surprisingly, then, the highest steady state flow of momentary utility is achieved at a low but positive inflation rate, defined as $\hat{\Pi}$ in subsection 8.3.1. However, this low but positive inflation rate is not the optimal policy in steady state. Instead, the steady state of the solution to the optimal policy problem is zero inflation. The distinction between these two optimal policy problems is subtle but important: if the monetary authority is constrained to choosing a constant inflation rate, it would choose $\hat{\Pi}$, but if unconstrained, it would choose a path that ended in a steady state of *zero* inflation.

Henceforth, we will refer to this surprising result as a modified monetary golden rule. We now substantiate it by examining the constraints and first-order conditions, and showing that optimal policy mandates $c_{0,t} = c_{1,t} = c_t$, which is only consistent with equilibrium under zero inflation. It is straightforward to impose a steady state on equations (25)–(28) and thereby show that it is feasible for a steady state to have $c_{0,t} = c_{1,t} = c_t$ and $x_{j,t} = 0$ for all j. To show that this is also desirable for the social planner, we must examine the first-order conditions. The crucial condition is (39), repeated here in its steady state form:

$$\frac{\Lambda_t \dfrac{\partial c(c_0, c_1)}{\partial c_0} + \phi \dfrac{\partial x(c_0, c, n, a)}{\partial c_0}}{\Lambda_t \dfrac{\partial c(c_0, c_1)}{\partial c_1} + \phi \dfrac{\partial x(c_1, c, n, a)}{\partial c_1}} = \frac{\rho_0}{\rho_1}. \tag{41}$$

where variables without time subscripts denote steady state values.

To determine whether it is desirable for the steady state to have $c_0 = c_1 = c$, we need to know whether equation (41) is satisfied by these values. As discussed previously, the right-hand side of this expression is always unity since the utility cost of producing each good is identical: $\rho_0 = \rho_1 = \Omega/a$. The symmetry of the aggregator function implies that $\partial c(c_0, c_1)/\partial c_0 = \partial c(c_0, c_1)/\partial c_1 = 1/2$. Further, the effect of consumption associated with today's price setters on today's implementation constraint, $\partial x(c_0, c, n, a)/\partial c_0$, is just the same as the effect of consumption associated with yesterday's price setters on yesterday's implementation constraint, $\partial x(c_1, c, n, a)/\partial c_1$. Hence, the left-hand side is also equal to unity when $c_0 = c_1 = c$. Imposing $c_0 = c_1 = c$ on equations (31)–(34) in steady state then implies unique values for the key endogenous variables.

This result may seem very special, but it can be shown to generalize to many other related environments, including models with multiperiod price setting and with randomly timed adjustments by individual firms of a very rich form (as in Calvo 1983; Levin 1991). Rather than pursue these extensions, we concentrate on the intuition behind this general result in the simple case at hand.

As the title of this section suggests, there is an analogy between the suboptimality of $\hat{\Pi}$ and the suboptimality of the golden rule in growth models. It is suboptimal to maintain a capital stock corresponding to the highest sustainable constant consumption in the one-sector growth model because, in the transition to a lower capital stock, consumption and hence utility can be increased. Eventually, at the modified golden rule steady state, consumption will be lower into the infinite future. But the fact that future utility is discounted makes it optimal to move from the golden rule to the modified golden rule. In our model, it is suboptimal to maintain $\hat{\Pi}$, the constant inflation rate that yields highest welfare, because, in the transition to a lower inflation rate, consumption and hence utility can be increased. Eventually, and in particular in the new steady state, the markup will be higher and consumption and utility lower, but in the early stages of the transition, utility is higher. The fact that future utility is discounted makes it optimal to undertake this transition.

One perspective on the optimality of zero inflation comes from looking back at figure 8.6, which shows the expansionary effect of an anticipated disinflation. Given that the steady state solution has zero inflation and that solution is saddle-path stable, it must be that if a policymaker wakes up in a world with low but positive inflation, it is optimal to disinflate. Figure 8.6 shows exactly this pattern: a disinflation that is announced in advance generates increases in period utility for several periods, followed in the long run by a decrease. The benefit of the transition comes because it involves a lower markup; adjusting firms lower their relative prices in anticipation of the slowdown in money growth. With a lower markup, the real wage is higher and consumption and utility are higher, given that productivity is unchanged.[24]

The above argument is straightforward in implying that optimal policy has a steady state with lower inflation than $\hat{\Pi}$, but by itself it leaves unanswered the following interrelated questions. First, what is it about zero inflation that makes the argument for an announced disinflation invalid once zero is reached? Second, how can it be that the specific optimality of zero inflation does not depend on parameter values, in particular the discount factor? Zero inflation is special in that it involves elimination of the relative price distortion. By reducing inflation toward zero, the monetary authority earns a benefit from bringing relative prices into line. By further reducing inflation, the monetary authority incurs a cost in terms of relative price distortions.

As for the invariance of the zero inflation result under changes in the discount factor, the missing link in this puzzle is that $\hat{\Pi}$ itself depends on β. From equation (23), one can show that $\hat{\Pi}$ converges to unity from above as β converges to unity from below.[25] If β is high, there is little incentive to announce a disinflation because the long-run increase in the markup—and corresponding decrease in welfare—is not discounted very much. However, high β also implies that $\hat{\Pi}$ is close to unity, so the disinflation consistent with optimal policy

24. Of course the figure is generated using particular parameters. The result that zero inflation is the steady state of the solution to the optimal policy problem does not depend on parameters within the class of models we consider.

25. From eq. (23), the constant gross inflation rate that minimizes the markup converges to unity with the discount factor. Because unity is also the inflation rate that eliminates the relative price distortion, it follows that $\hat{\Pi}$ converges to unity with the discount factor ($\hat{\Pi}$ is always between unity and the inflation rate that minimizes the markup).

is very small. In contrast, if the discount factor is low, optimal policy involves a larger disinflation in the long run ($\hat{\Pi}$ is higher), but the long run is discounted more heavily.

Thus far our explanation of the zero-inflation result has been focused on why it would be optimal to disinflate to get to zero inflation. Earlier, however, we showed that positive monetary shocks were expansionary, which suggests that the monetary authority would actually choose to leave an initial steady state of price stability for positive inflation. This is incorrect because in steady state any inflation is expected. Accordingly, the planner who wants to have an expansion at t must pay for it at $t - 1$ in terms of effects on the implementation constraint, which is sufficiently costly that he chooses to forgo his leverage on the average markup.

8.7 Optimal Stabilization Policy

We now turn our attention to describing the behavior of real economic activity under optimal stabilization policy, specifically the response of economic activity to productivity disturbances. With a steady state that equates consumption across firms of different types (as would occur under price stability), we can analyze optimal stabilization policy by log-linearizing the system of equations (25)–(28) and (31)–(34) in the neighborhood of this steady state. The resulting linear system is described fully in appendix B. It is straightforward to use that linear system to analyze the optimal response of real quantities to a productivity shock. That optimal response turns out to involve equality of all relative prices, which translates into zero inflation just as in steady state. In what follows, we describe how to show this result and discuss the mechanics of how monetary policy can achieve it.

8.7.1 Real Dynamics under Optimal Policy

As one might expect, an important equation for understanding optimal policy is the optimal pricing equation or, equivalently, the implementation constraint (28). In its linearized form, that equation is

$$dx_{0,t} + \beta \cdot dx_{1,t+1} = 0,$$

and the linearized x functions are given by[26]

$$dx_{j,t} = \lambda c[(1 - \varepsilon)(1 - 1/\varepsilon) + \varepsilon\theta n^{\gamma}](dc_{j,t}/c - dc_t/c) + \gamma\varepsilon\theta\lambda c n^{\gamma}(dn_t/n), \quad \text{for } j = 0, 1. \tag{42}$$

26. Note that there are extra terms involving deviations in c and λ which vanish because they are multiplied by zero, the steady state value of x: at the steady state, adjusting and nonadjusting firms charge the same price, and it is the price that sets "marginal profits" equal to zero period by period.

If a productivity shock is not to optimally induce price variation, it must not cause marginal profits to deviate from zero for adjusters or nonadjusters. Referring to equation (42), this would mean holding x_j at zero (so $dx_j = 0$) and holding $c_j = c$. For these requirements to be mutually consistent, it must be that labor input does not respond to the shock. For the preferences in equation (2), it is in fact the case that labor input will not respond to productivity shocks as long as the markup does not respond (see eq. [9], and recall that the markup is a_t/w_t). And from equation (19), an unchanged constant markup ($= 1/\psi_t$) is consistent with adjusting firms not changing their relative prices if the price level is constant (under these conditions the numerator and denominator of eq. [19] cancel once real marginal cost is factored out of the numerator).

So far this line of reasoning does not prove that a constant price level is part of the optimal response to a productivity shock, but it does suggest a constructive method of proof. First, conjecture zero response of labor input, and responses of c_t and $c_{j,t}$ exactly equal to the change in productivity. Next refer to equation (39); if symmetry with respect to c_0 and c_1 is maintained, as it will be according to the conjecture, then it must be that $\phi_t = \phi_{t-1} = \phi$. Expand the conjecture, then, to include zero response of ϕ_t. Confirming that the conjecture is correct requires some tedious algebra that we will not reproduce, but conceptually it is straightforward. Simply impose the conjecture on the linearized equations in appendix B, and verify that those equations are satisfied. They are.

In response to a productivity shock, then, the monetary authority should accommodate so that the price level is unchanged and firms continue to maximize profits on a period-by-period basis. Figure 8.8 displays impulse response functions under optimal monetary policy for a serially correlated ($\rho = 0.9$), positive productivity shock. Those responses are identical to what would be found in a real business cycle model. With the price level constant, the nominal interest rate behaves identically to the real interest rate. Because it essentially tracks expected consumption growth, the real interest rate falls initially and then gradually rises back to its steady state level.

We have thus verified that the constant inflation, constant markup policy conjectured to be optimal by Goodfriend and King (1997) and King and Wolman (1996) is in fact optimal. However, the fact that there are no money demand distortions in the current framework implies that *zero* inflation is optimal. Our demonstration of the desirability of a constant price level proceeds differently from that of Rotemberg and Woodford (1997) in that we do not assume that a combined fiscal and monetary authority has overcome the underlying monopolistic competition distortions in the economy.

8.7.2 Optimal Monetary Policy

Because the real effects of monetary policy work through relative prices in this model, they can be interpreted as working through relative quantities. In fact, it is possible to fully describe the real outcomes under optimal monetary policy without any discussion of nominal variables, although we chose not to

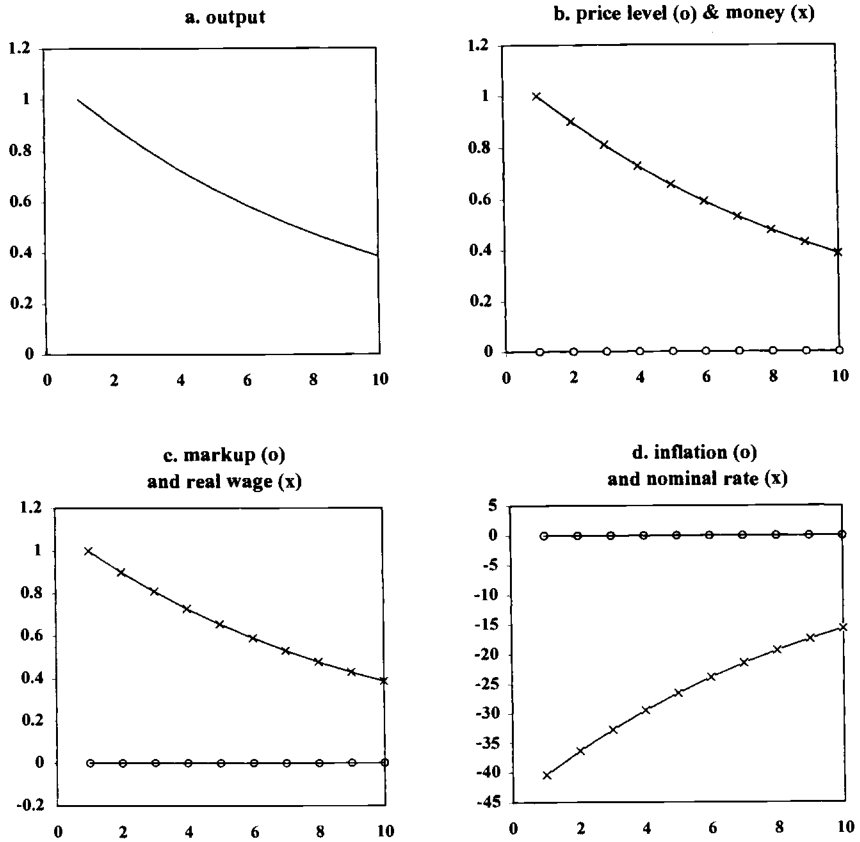

Fig. 8.8 Persistent productivity shock with optimal policy response
Note: For axis units, see note to fig. 8.4.

pursue this expository strategy in the previous subsection. In practice, however, monetary policy operates through nominal variables such as the nominal interest rate, the money stock, and the price level. This subsection details how to reinterpret optimal policy in terms of the nominal variables that would produce our optimal allocations in (monopolistically competitive) general equilibrium.

The state variables of the model include prices set by firms in previous periods, which are relevant since these firms are unable to change their prices. In the two-period case, the only relevant historical information is $P_{1,t} = P_{0,t-1}$. The real policy problem provides optimal quantities at date t, $c_{0,t}$ and $c_{1,t}$. "Decentralizing" this optimal policy requires that the relative nominal prices satisfy

$$P_{0,t}/P_{1,t} = (c_{0,t}/c_{1,t})^{-1/\varepsilon}. \tag{43}$$

Since $P_{1,t}$ is predetermined, equation (43) implies a unique level of $P_{0,t}$, and thus the price level is uniquely determined under optimal policy by equation (5), as $P_t = (\frac{1}{2}P_{0,t}^{1-\varepsilon} + \frac{1}{2}P_{1,t}^{1-\varepsilon})^{1/(1-\varepsilon)}$. Given the price index and the level of real activity, equation (10) dictates that the quantity of money must be

$$M_t = kP_tc_t .$$

The real interest rate must satisfy the real Fisher equation (11), which links it to the marginal utility of consumption,

$$1 + r_t = \left(\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\right)^{-1},$$

and the nominal interest rate must satisfy equation (12),

$$1 + R_t = \left[\beta E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}\right)\right]^{-1},$$

so that we can also determine the relevant interest rates under optimal policy. Combining these equations with the result that $P_t = P_{t-1}$, we find that the real interest rate r_t is equal to the nominal interest rate R_t.

Since there is a predetermined price level from the previous period, P_{t-1}, and it is optimal to maintain that price level in the current period, there is a simpler way to provide a monetary interpretation of optimal policy. With all real variables determined by the solution to the real policy problem, the money supply is then given from equation (10) as

$$M_t = kP_{t-1}c_t .$$

In this setting, as in the more general case, the fact that yesterday's price level is an observable, predetermined variable provides the anchor needed to produce determinacy.

8.7.3 Extensions to Multiperiod Price Setting

Our analysis has focused on the two-period case of Taylor-style staggering for concreteness. However, it is easy to extend the analysis of optimal policy in two directions. First, one can determine an optimal policy for multiperiod price setting with two minor modifications of the approach that we developed above: it is necessary to (1) modify the implementation constraint to $0 = E_t \sum_{j=0}^{J-1}\beta^j x(c_{j,t+j}, c_{t+j}, n_{t+j}, a_{t+j})$ in the J-period case and (2) introduce additional constraints on real quantities at date t of the form $(c_{1,t}/c_{2,t})^{-1/\varepsilon} = (c_{0,t-1}/c_{1,t-1})^{-1/\varepsilon}$.[27] An important consequence of these modifications is that more lags of the multiplier ϕ are added to the dynamic system. Second, we can incorporate randomly timed adjustments by individual firms of a very rich form (as in Levin's 1991 extension of the Calvo 1983 framework). In this case the distribution of

27. We provide an example of this structure in section 8.8 below.

firms would no longer be uniform with respect to time since last price adjustment, and consequently the implementation constraint would have unequal (declining) weights on $x_{j,t+j}$, $j = 0, 1, \ldots, J-1$.

8.8 Temptations for the Monetary Authority

While we have demonstrated that a policy of pegging the price level is optimal under commitment, there are temptations for the monetary authority to deviate from this plan. In this section, we provide two examples of how this temptation might arise and quantify its magnitude.

8.8.1 Starting Up

The most basic temptation is that associated with "starting up" the policy of pegging the price level. For example, if a rule like the Mack bill were adopted, one option would be for Congress to allow the Federal Reserve System two years to choose a price level that it would peg, perhaps so that it could get appropriate policy procedures in place.

In our model economy, this would correspond to an optimal monetary policy problem with the initial multiplier ϕ_0 set equal to zero; all lagged product prices and quantities would be given by history, but since optimal policy under commitment was not followed in the past, the lagged multiplier would not be given by history. (Technically, it would then be appropriate for the monetary authority to ignore the lagged multiplier, that is, set it to zero.) The results of simulating optimal policy in this setting are shown in figure 8.9 for a model with four-quarter price setting.[28]

In the initial period, the money stock more than doubles, and with a majority of prices fixed, this yields a huge increase in output (almost as large as the increase in the money stock). There is a jump in the price level, and the corresponding high inflation rate is maintained for the four quarters until all firms have had the opportunity to adjust their prices. In this transitional period the money supply is decreasing, however. Nominal interest rates are low during the transition; this is reconciled with the high expected inflation through anticipated decreases in consumption. In terms of the representative agent's preferences, the transition yields the equivalent of a 1.6 percent per year permanent increase in consumption. In other words, the temptation for the monetary authority to ignore ϕ_0 in *any* period would be large. The fact that the model is parameterized with a high labor supply elasticity and a high markup is directly responsible for the size of the temptation; the higher the labor supply elasticity, the greater the consequences of a given markup. With this parameterization, steady state output is far below what it would be with perfect competition, so

28. In the four-quarter price-setting case, there are three lagged multipliers that must be set to zero. As described in subsection 8.7.3, when prices may be fixed for more than two periods, lagged ratios of real quantities are also relevant state variables.

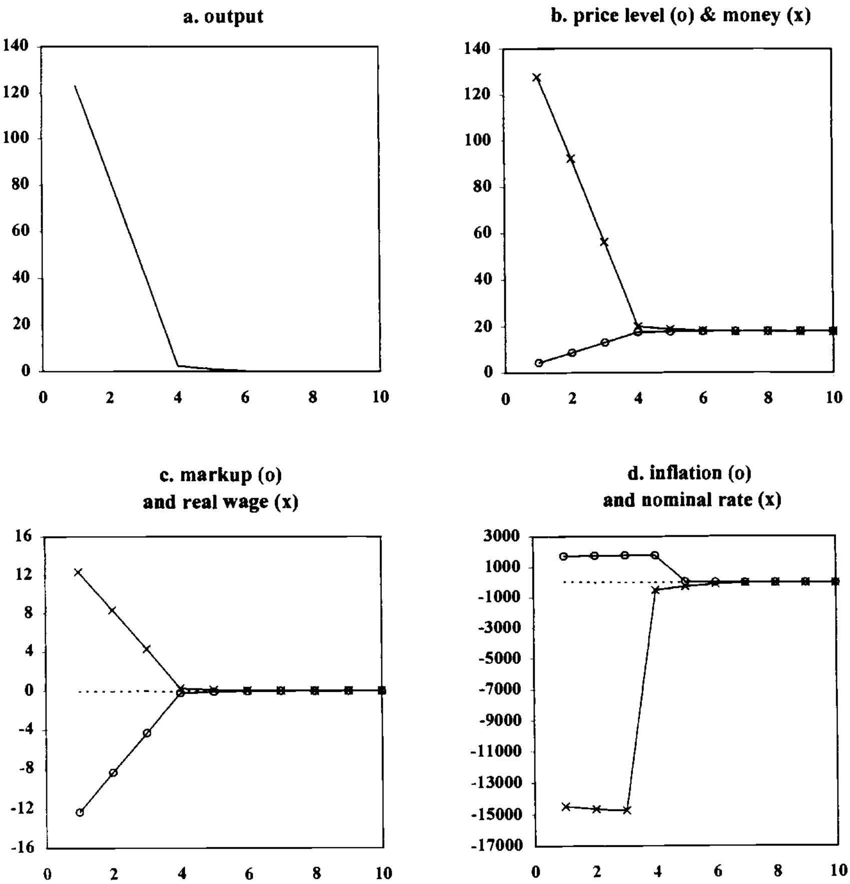

Fig. 8.9 Exploiting initial conditions with four-quarter staggering
Note: For axis units, see note to fig. 8.4.

the monetary authority can generate large movements in output on a temporary basis.

8.8.2 An Unusual Shock

In section 8.7, we assumed that the monetary authority had to determine how it would respond to shocks before they occurred. This assumption was at the heart of our analysis of how monetary policy should respond to a productivity shock: we were interested in how the monetary authority should respond as part of a *policy.* An alternative is to determine how the monetary authority should respond to a shock that is unusual, in the sense that it is unexpected and viewed as never to recur. In this case, the monetary authority should exploit initial conditions, setting lagged ϕs to zero.

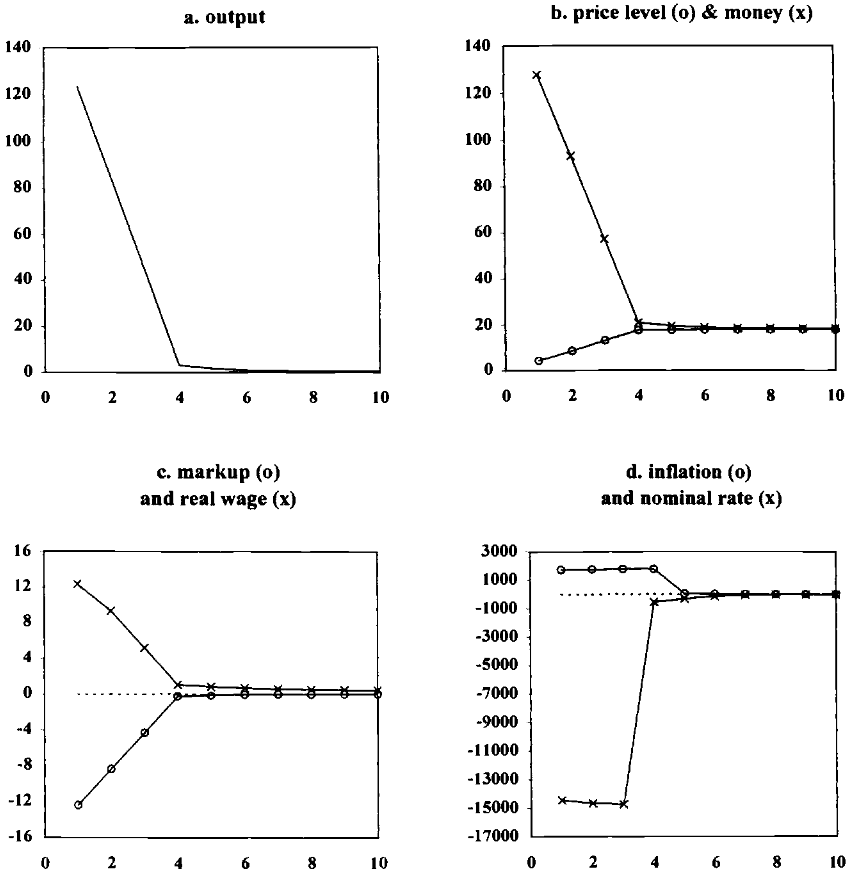

Fig. 8.10 Exploiting initial conditions in the face of a productivity shock
Note: For axis units, see note to fig. 8.4.

The results are shown in figure 8.10. A productivity shock is now accompanied by a burst of inflation, and in fact the inflationary response associated with $\phi_0 = 0$ dominates the response associated with the productivity shock. Another way of seeing this point is to look across some prior figures: the response to an unusual shock in figure 8.10 looks much more like the figure 8.9 case of starting up, where the only deviation from steady state initial conditions is in ϕ_0, than it looks like the optimal policy response in figure 8.7, where the only deviation from steady state initial conditions is in the productivity shock.

How are we to interpret these temptations to deviate from optimal policy under commitment? To begin, the fact that the temptations are so large hinges on the magnitude of the economy's existing distortions. To the extent that our parameterization of those distortions is proxying for other features of actual economies, one would want to better understand those other features before passing final judgment on the size of the temptations. Conditional on the large temptation being accurate, a natural interpretation is that it points to the need for some commitment device, with an act of Congress being a natural example. Without a commitment device, one is led to consider whether optimal policy under commitment can be sustained through reputation effects or trigger strategies. Ireland (1997) has investigated this issue in a sticky price model where optimal policy can be sustained through reputation; whether our model admits such equilibria is an open question.

8.9 Interest Rate Rules and Economic Activity

In contrast to other chapters in this volume, our work has concentrated on determining optimal monetary policy rather than on making comparisons across alternative monetary policy rules, with some specific emphasis on interest rate rules following Taylor (1993). In this section, we discuss two aspects of interest rate rules and macroeconomic activity, suggesting how our analysis could be extended to bring it more into line with the other research reported in this volume.

8.9.1 An Optimal Interest Rate Rule

Our characterization of optimal monetary policy implies that there should be zero inflation, so that nominal and real interest rates are identical. Optimal monetary policy therefore can be implemented through an interest rate rule of the form

$$R_t = r_t^* + f\cdot(\ln P_t - \ln \bar{P}), \tag{44}$$

where r_t^* is the real interest rate determined by the real Fisher equation (11) and the optimal quantity response. As with the interest rate rule that we used to study the response of the economy to interest rate shocks (24), the optimal interest rate rule (44) also involves a positive response of the nominal rate to deviations of the price level from its target. This response assures that there is a determinate price level under optimal policy but otherwise is unimportant since with $R_t = r_t^*$ it follows that $f\cdot(\ln P_t - \ln \bar{P}) = 0$.

Implementing optimal policy, however, requires knowing how the underlying real interest rate—the real interest rate that would obtain if prices were flexible—responds to shocks. To illustrate that this is a nontrivial problem for a monetary authority, figure 8.11 shows the response of the real interest rate under three different assumptions about the productivity process. The first is the first-order scheme for which we described optimal policy in figure 8.8; the second makes productivity a second-order autoregression, $\ln a_t = 1.3 \ln a_{t-1} - 0.4 \ln a_{t-2} + e_t$; and the third assumes that productivity is difference stationary, $\ln a_t = 1.3 \ln a_{t-1} - 0.3 \ln a_{t-2} + e_t$. All of these specifications can be

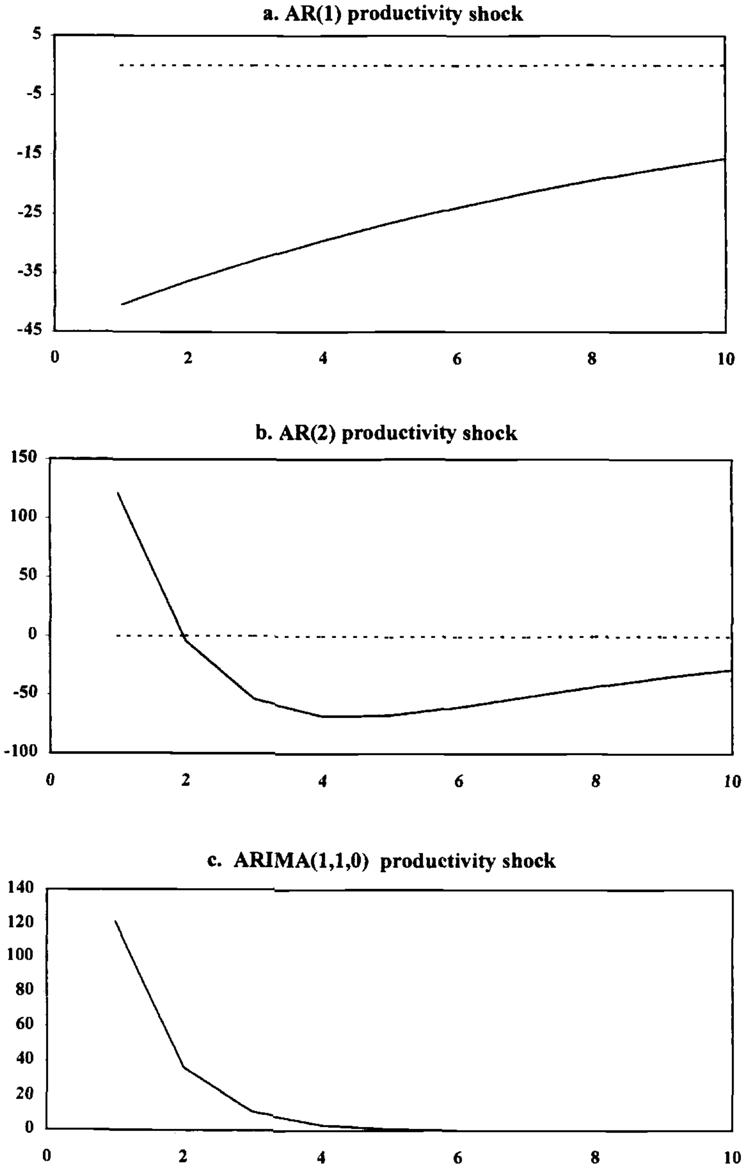

Fig. 8.11 Behavior of the real interest rate under optimal policy
Note: x-Axes in quarters; *y*-axes in basis points.

captured as special cases of the second-order stochastic difference equation $\ln a_t = \zeta_1 \ln a_{t-1} - \zeta_2 \ln a_{t-2} + e_t$.

Our model economy is very simple: there is no investment and labor does not respond to productivity shocks under the optimal allocation, so output and consumption track productivity perfectly. This simplicity also means that the real interest rate approximately tracks the expected growth rate of productivity,[29]

$$r_t^* = E_t(\ln a_{t+1} - \ln a_t) = [(\zeta_1 - 1)\ln a_t - \zeta_2 \ln a_{t-1}].$$

Under our basic specification, productivity (and hence output) is a stationary first-order autoregression, so the real interest rate depends negatively on the level of output ($\zeta_1 - 1 < 0$ and $\zeta_2 = 0$ imply that $r_t^* = (\zeta_1 - 1) \ln a_t$). By contrast, under our third specification, there is *no* connection between the level of the real interest rate and the *level* of output: productivity *growth* is a stationary first-order autoregression ($\zeta_1 = 1.3$ and $\zeta_2 = 0.3$) so that the real interest rate rises when output *growth* is unexpectedly strong, $r_t^* = 0.3 (\ln a_t - \ln a_{t-1})$. Finally, the second specification combines an initial period of positive growth effects with a subsequent period of negative growth effects, and thus aspects of each of the other specifications.

From the standpoint of a central bank using an interest rate instrument, the difficulty is that optimal policy requires knowledge of the structure of the economy if there is a single type of productivity shock and of the type of shock that is currently occurring if there are multiple shocks.

8.9.2 Comparison of Alternative Rules

To compare alternative monetary policy rules with the optimal rule that we have discussed, it is necessary to take a stand on the details of the structure of the economy, including the internal mechanisms and forcing processes. Then one can calculate the stationary level of lifetime utility (1) under the optimal and alternative rules.[30] As the analysis of Rotemberg and Woodford in chapter 2 of this volume indicates, the results of policy analysis along these lines depend importantly on the particular structure of the economy; we do not pursue such analysis here because we are not yet willing to take a stand on the details of that structure.

29. This expression is exact under optimal monetary policy, since this makes employment constant and the marginal utility of consumption is then

$$D_1 u(c_t, n_t, a_t) = \left(c_t - a_t \frac{\theta}{1+\gamma} n_t^{1+\gamma}\right)^{-1} = a_t^{-1}\left(n - \frac{\theta}{1+\gamma} n^{1+\gamma}\right)^{-1}.$$

It would be exact under all policies if the preference specification were $u = \ln c - \theta n^{1+\gamma}$.

30. This approach is similar to that of Rotemberg and Woodford (chap. 2 of this volume), who look at policies in terms of their effect on the stationary level of momentary utility (1).

8.10 Conclusions

This chapter provides a basic example of the analysis of optimal monetary policy in an environment with imperfect competition and sticky prices. The general approach resembles that traditional in public finance rather than practical macroeconomics. That is: we derive optimal policy by maximizing the welfare of the economy's representative agent, subject to resource constraints and an additional condition that summarizes the implications of imperfect competition and sticky prices for what the monetary authority can feasibly select. We show that a policy of stabilizing the price level is optimal in two respects. First, the average rate of inflation should be zero. Second, the price level should not vary with the business cycle.

While this result was obtained in a very simple economy, the methods applied in this paper are capable of extending the analysis of monetary policy well beyond settings in which there is a single shock (to productivity), strong assumptions about preferences (implying constant demand elasticities and zero optimal labor response to productivity shocks), a single factor of production (labor), a single location of real distortions (imperfect competition in the commodity market), and complete information about underlying shocks. We thus outline some directions in which it is important for this research to be extended.

Multiple Shocks. Within the simple model without capital, there are two important directions of extensions. First, one would like to understand how the economy responds to "aggregate demand" shocks, such as changes in government demand for final output and exogenous changes in the timing of consumption decisions by households. We have undertaken these extensions and find that the policy of smoothing the price level continues to be optimal. Second, it is important to think about the effects of energy shocks. One direct interpretation is that these are productivity shocks, in which case we already have the answer. But an alternative approach would be to add energy as an input to final consumption that was in exogenous supply and was sold by flexible price firms. One would no longer expect that optimal monetary policy would smooth all measures of the price level (in particular, not the final consumption deflator) but rather an index corresponding to the prices of imperfectly competitive sticky price firms.

Alternative Preference Specifications. Our example economy incorporated strong—constant elasticity utility—assumptions about the preferences of individuals for differentiated products and for the trade-off between consumption and leisure. Public finance theory teaches us that the exact optimality of tax equalization (across products) or tax smoothing (over time) typically requires constant elasticities. Our general analytical approach does not require this set of assumptions, which we used because they are simple and conventional in the literature. It would be useful to explore how alternative preference specifications would alter our conclusions about the optimal rate of inflation and the optimal cyclical variation in the price level.

Richer Production Structure. In our view, a successful positive model of the business cycle requires the endogenous determination of investment and capacity utilization. Thus it is important to determine the nature of optimal monetary policy when the production structure is enriched along these lines.

Richer Pricing Structure. Our analysis has concentrated entirely on time-dependent pricing. It would be useful to extend the analysis to state-dependent pricing, and it seems feasible to use the framework of Dotsey, King, and Wolman (forthcoming) for this purpose.

Additional Sources of Distortions. Many economists believe that the labor market does not clear in the way that we have specified in this chapter, but that (1) there is market power on the part of firms and workers, or (2) there are incentive problems arising from incomplete information about worker characteristics or effort. For example, Romer (1993) argues that additional "real rigidities" along these lines are a necessary ingredient of a successful business cycle theory, with implications for the nature of optimal monetary policy. The general approach developed in our research appears capable of handling extensions to additional distortions, say along efficiency wage lines, which would introduce another implementation constraint into the real policy problem. Exploring the implications of these frictions for optimal monetary policy seems feasible and fascinating.

Incomplete Information. Our analysis is conducted under the assumption that the monetary authority has full current information about the shocks that are impinging on the economy. McCallum (1997) has stressed that monetary policy rules should be operational, in the sense of respecting the informational constraints on the monetary authority. It is important to extend our analysis to situations of incomplete information, and we believe that it is feasible to do so.

More General Specification of Policy. We have restricted the monetary authority to following deterministic rules. However, it may be optimal to employ a randomized rule because of the presence of distortions (technically, the form of the implementation constraint). For example, the monetary authority might choose a policy that randomly expanded the economy but was accompanied by a commitment to disinflate whenever expansions occurred.[31] Recent analy-

31. The possible desirability of this sort of policy was suggested to us by Athanasios Orphanides.

ses of optimal fiscal policies, such as that of Bassetto (1997), permit the policy authority to follow such randomized strategies and determine whether they are part of an optimal plan.

This is a lengthy list of open topics, but in principle each topic can be analyzed with the same basic approach used in the current paper. That approach uses standard tools of public finance to analyze optimal policy in what have now become standard models of monopolistic competition and sticky prices.

Appendix A
The Household's Choice Problem

This appendix provides more detail on the household's dynamic choice problem for aggregate consumption and labor supply that leads to equations (7) and (8). We also use this appendix to sketch out the incorporation of a "shopping time" approach to money demand along the lines of King and Wolman (1996), when there is interest-bearing money. The household's optimization problem can be written in dynamic programming form as

$$v(m_{t-1}, b_{t-1}, s_{t-1}, \sigma_t) = \max_{(c_t, n_t, s_t, m_t, b_t)} \{u(c_t, n_t, a_t) + \beta E v(m_t, b_t, s_t, \sigma_{t+1} | \sigma_t)\},$$

where the relevant aggregate state variables at date t are σ_t. (We write conditional expectations such as $E\{v(m_t, b_t, s_t, \sigma_{t+1}) | \sigma_t\}$ more compactly as $E_t v(m_t, b_t, s_t, \sigma_{t+1})$ below.) The maximization takes place subject to the budget constraint (6), which we write as

$$c_t + m_t + v_t s_t + b_t + w_t h(m_t / c_t) =$$

$$z_t s_{t-1} + v_t s_{t-1} + w_t n_t + (1 + R^M_{t-1})(P_{t-1}/P_t) m_t + (1 + R_{t-1})(P_{t-1}/P_t) b_{t-1}.$$

In these expressions, $m_t = M_t / P_t$ is current real balances, $b_t = B_t / P_t$ is current real bonds, and $h(m_t / c_t)$ is the amount of time spent in transactions activity. Forming a Lagrangian

$$\begin{aligned} L_t = {} & \{u(c_t, n_t, a_t) + \beta E_t v(m_t, b_t, s_t, \sigma_{t+1})\} \\ & + \lambda_t [z_t s_{t-1} + v_t s_{t-1} + w_t n_t + (1 + R^M_{t-1})(P_{t-1}/P_t) m_{t-1} \\ & + (1 + R_{t-1})(P_{t-1}/P_t) b_{t-1} - c_t - m_t - v_t s_t - b_t - w_t c_t h(m_t / c_t)], \end{aligned}$$

we find that the first-order conditions are

$$\frac{\partial u(c_t, n_t, a_t)}{\partial c_t} = \lambda_t \left[1 - w_t \frac{m_t}{c_t^2} h'\left(\frac{m_t}{c_t}\right)\right],$$

$$\frac{\partial u(c_t, n_t, a_t)}{\partial n_t} = -\lambda_t w_t,$$

$$\lambda_t\left[1 + \frac{w_t}{c_t}h'\left(\frac{m_t}{c_t}\right)\right] = \beta E_t\frac{\partial v(m_t, b_t, s_t, \sigma_{t+1})}{\partial m_t} = \beta E_t\left(\lambda_{t+1}(1 + R_t^M)\frac{P_t}{P_{t+1}}\right),$$

$$\lambda_t = \beta E_t\frac{\partial v(m_t, b_t, s_t, \sigma_{t+1})}{\partial b_t} = \beta E_t\left(\lambda_{t+1}(1 + R_t)\frac{P_t}{P_{t+1}}\right),$$

$$\lambda_t v_t = \beta E_t\frac{\partial v(m_t, b_t, s_t, \sigma_{t+1})}{\partial s_t} = \beta E_t[\lambda_{t+1}(z_{t+1} + v_{t+1})],$$

where the final equalities on the right-hand sides of the final three equations arise from standard "envelope theorem" arguments, such as those found in Stokey and Lucas (1989).

These five conditions have the following interpretations. The first two conditions are requirements for efficient consumption and labor supply. The third is the requirement for efficient holdings of real money balances, that is, the model's implicit money demand function. The fourth is the efficiency condition for holding of nominal bonds, that is, the nominal Fisher equation in the text. The fifth is the efficiency condition for holding risky assets, such as the equities in our model.

The nature of the real demand for money implicit in these equations can be highlighted by combining the third and fourth equations to yield

$$\lambda_t\left(\frac{w_t}{c_t}h'\left(\frac{m_t}{c_t}\right)\right) = \beta E_t\left(\lambda_{t+1}(R_t^M - R_t)\frac{P_t}{P_{t+1}}\right).$$

We now turn to thinking about the limiting form of these equations that obtains when R_t^M approaches R_t; that is, the real cost of holding money goes to zero. Then the demand for money approaches

$$m_t = kc_t,$$

where k is a constant such that $h'(k) = 0$, which represents the satiation level of real cash balances.

Appendix B
Linearized Equations for Optimal Policy

This appendix contains the linearized equations of the model with optimal policy. For convenience we first reproduce the optimal policy problem. Then we list the first-order conditions in their true and linearized forms. In the linearized equations, $\hat{s}_t$ denotes the percentage deviation of s from its steady state value, whereas ds_t denotes the level deviation from steady state. Unsubscripted endogenous variables denote steady state values.

The Lagrangian for the optimal policy problem is

$$
\begin{aligned}
L = & \sum_{t=0}^{\infty} \beta^t u(c_t, n_t, a_t) \\
& + \sum_{t=0}^{\infty} \beta^t \phi_t [x(c_{0,t}, c_t, n_t, a_t) + \beta x(c_{1,t+1}, c_{t+1}, n_{t+1}, a_{t+1})] \\
& + \sum_{t=0}^{\infty} \beta^t \Lambda_t [c(c_{0,t}, c_{1,t}) - c_t] \\
& + \sum_{t=0}^{\infty} \beta^t \Omega_t (n_t - 0.5 n_{0,t} - 0.5 n_{1,t}) \\
& + \sum_{t=0}^{\infty} \beta^t [\rho_{0,t}(a_t n_{0,t} - c_{0,t}) + \rho_{1,t}(a_t n_{1,t} - c_{1,t})].
\end{aligned} \tag{B1}
$$

The first-order conditions are as follows: for labor input at firms with prices set in periods t and $t-1$

$$0 = \frac{\partial L}{\partial n_{j,t}} = \beta^j (\rho_{j,t} a_t - 0.5\Omega_t) \quad \text{for } j = 0, 1, \tag{B2}$$

$$0 = \hat{a}_t + \hat{\rho}_{j,t} - \hat{\Omega}_t \quad \text{for } j = 0, 1; \tag{B3}$$

for consumption of goods with prices set in periods t and $t-1$

$$0 = \frac{\partial L}{\partial c_{j,t}} = \beta^t \left(\Lambda_t \frac{\partial c(c_{0,t}, c_{1,t})}{\partial c_{j,t}} - \rho_{j,t} + \phi_{t-j} \frac{\partial x(c_{j,t}, c_t, n_t, a_t)}{\partial c_{j,t}} \right) \quad \text{for } j = 0, 1, \tag{B4}$$

$$
\begin{aligned}
0 = & \Lambda \left(0.5 \hat{\Lambda}_t + c \sum_{i=0}^{1} \frac{\partial^2 c(\cdot)}{\partial c_i \partial c_j} \hat{c}_{j,t} \right) - \rho_j \hat{\rho}_{j,t} + \frac{\partial x(\cdot)}{\partial c_j} d\phi_{t-j} \\
& + \phi \left(\frac{\partial^2 x(\cdot)}{\partial c_j^2} c \hat{c}_{j,t} + \frac{\partial^2 x(\cdot)}{\partial c_j \partial c} c \hat{c}_t + \frac{\partial^2 x(\cdot)}{\partial c_j \partial n} n \hat{n}_t + \frac{\partial^2 x(\cdot)}{\partial c_j \partial a} a \hat{a}_t \right);
\end{aligned} \tag{B5}
$$

for the consumption index

$$0 = \frac{\partial L}{\partial c_t} = \beta^t \left(\frac{\partial u(c_t, n_t, a_t)}{\partial c_t} - \Lambda_t + \sum_{i=0}^{1} \phi_{t-j} \frac{\partial x(c_{it}, c_t, n_t, a_t)}{\partial c_t} \right), \tag{B6}$$

(B7)
$$0 = cU_{cc}\hat{c}_t + nU_{cn}\hat{n}_t + aU_{ca}\hat{a}_t - \Lambda\hat{\Lambda}_t + \frac{\partial x(\cdot)}{\partial c}\sum_{i=0}^{1} d\phi_{t-i}$$
$$+ \phi c \sum_{i=0}^{1} \frac{\partial^2 x(\cdot)}{\partial c_i \partial c}\hat{c}_{i,t} + 2\phi\left(\frac{\partial^2 x(\cdot)}{\partial c^2} c\hat{c}_t + \frac{\partial^2 x(\cdot)}{\partial c \partial n} n\hat{n}_t + \frac{\partial^2 x(\cdot)}{\partial c \partial a} a\hat{a}_t\right);$$

and for total labor input

(B8)
$$0 = \frac{\partial L}{\partial n_t} = \beta^t\left(\frac{\partial u(c_t, n_t, a_t)}{\partial n_t} + \Omega_t + \sum_{j=0}^{1} \phi_{t-j}\frac{\partial x(c_{j,t}, c_t, n_t, a_t)}{\partial n_t}\right),$$

(B9)
$$0 = cU_{cn}\hat{c}_t + nU_{nn}\hat{n}_t + aU_{an}\hat{a}_t + \Omega\hat{\Omega}_t + \frac{\partial x(\cdot)}{\partial n}\sum_{i=0}^{1} d\phi_{t-i}$$
$$+ \phi c \sum_{i=0}^{1} \frac{\partial^2 x(\cdot)}{\partial n \partial c_i}\hat{c}_{i,t} + 2\phi\left(\frac{\partial^2 x(\cdot)}{\partial n \partial c} c\hat{c}_t + \frac{\partial^2 x(\cdot)}{\partial n^2} n\hat{n}_t + \frac{\partial^2 x(\cdot)}{\partial n \partial a} a\hat{a}_t\right).$$

The constraints involve technology,

(B10)
$$c_{j,t} = a_t n_{j,t} \quad \text{for } j = 0, 1,$$

(B11)
$$0 = \hat{c}_{j,t} - \hat{a}_t - \hat{n}_t;$$

the consumption aggregator,

(B12)
$$c_t = \left(\frac{1}{2}c_{0,t}^{(\varepsilon-1)/\varepsilon} + \frac{1}{2}c_{1,t}^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)},$$

(B13)
$$0 = -\hat{c}_t + \frac{1}{2}\sum_{i=0}^{1}\hat{c}_{i,t};$$

time,

(B14)
$$\hat{n}_t = \frac{1}{2}n_{0,t} + \frac{1}{2}n_{1,t},$$

(B15)
$$0 = -\hat{n}_t + \frac{1}{2}\sum_{i=0}^{1}\hat{n}_{i,t};$$

and optimal price setting

(B16)
$$x(c_{0,t}, c_t, n_t, a_t) + \beta x(c_{1,t+1}, c_{t+1}, n_{t+1}, a_{t+1}) = 0,$$

(B17)
$$0 = dx_{0,t} + \beta \cdot dx_{1,t+1}.$$

In these equations, $x(c_{i,t}, c_t, n_t, a_t)$ is the change in period t profits associated with a marginal price change in period $t - i$, assuming that the nominal price chosen in period $t - i$ is in effect in period t:

(B18) $$x(c_{j,t}, c_t, n_t, a_t) \equiv \lambda_t c_t \left[(1 - \varepsilon)\left(\frac{c_{j,t}}{c_t}\right)^{1-1/\varepsilon} + \varepsilon\theta n_t^{\gamma} \frac{c_{j,t}}{c_t} \right],$$

(B19) $$dx_{j,t} = \lambda c[(1 - \varepsilon)(1 - 1/\varepsilon) + \varepsilon\theta n^{\gamma}](\hat{c}_{j,t} - \hat{c}_t) + \lambda c \gamma \varepsilon \theta n^{\gamma} \hat{n}_t;$$

and λ_t is the marginal utility of consumption:

(B20) $$\lambda_t \equiv \frac{\partial u(c_t, n_t, a_t)}{\partial c_t} = \left(c_t - \frac{a_t \theta}{1 + \gamma} n_t^{1+\gamma} \right)^{-1},$$

(B21) $$\hat{\lambda}_t = -\lambda c \hat{c}_t + a\theta\lambda n^{1+\gamma} \hat{n}_t + \frac{a\theta\lambda n^{1+\gamma}}{1 + \gamma} \hat{a}_t.$$

The above linearizations make use of the facts that in steady state, $\partial x(\cdot)/\partial a = 0$ and $\partial c(\cdot)/\partial c_j = 0.5$.

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Comment Benjamin M. Friedman

Robert King and Alexander Wolman have written a highly appropriate paper for a conference on monetary policy rules. Their paper usefully anchors, both methodologically and substantively, one end of the intellectual spectrum under debate here. King and Wolman argue rigorously for a rigorous rule of price stability. Their paper is sharp and clear on both counts. The monetary policy rule that they find optimal in the model they present differs from many of the "rules" considered in other papers at this conference in that (1) it is a genuine *rule,* the effects of which center on credible commitment by the central bank, not merely an indicative guide or "rule of thumb," and (2) it focuses on stabilization of prices, to the exclusion of any direct concern for smoothing output, interest rates, or other variables.

Moreover, the paper is forthright—comprehensively so—about the limitations inherent in the framework of analysis it deploys. As a result, a perfectly accurate response to many of my remarks as discussant would be "We know that, and in fact on page such and such we talked about that issue ourselves." But while that response would be true, it would miss the point. Being forthright about a model's limitations does not render them without force. In my comments on the paper I shall first explain why I find the authors' case for a price stability rule unpersuasive, then discuss several features of the underlying model that merit attention even though they do not bear centrally on the paper's basic recommendation, and finally step aside from the paper as a whole to pose two somewhat more general questions about a price stability rule for monetary policy.

Why the Paper's Case for a Price Stability Rule Is Unpersuasive

The analysis that King and Wolman carry out in this paper is impressive and serious. The model that they construct embodies many useful properties—not least an explicit recognition that prices are sticky and a real attempt to represent, internally, the monopolistically competitive process that makes them so. (The model also renders the inflation rate persistent.) Nevertheless, I do not recognize in their model the key features of the actual monetary policy envi-

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ronment that lead me to believe that a strict price stability rule would be a poor way to conduct monetary policy. There are three main reasons.

First, the central bank in the world of King and Wolman's paper anticipates and understands all shocks. (In fact, King and Wolman consider only one kind of shock, a simultaneous shock to preferences and productivity, but that is another matter.) Much of what makes monetary policy making hard, with or without a rule, is precisely that central bankers cannot know that a shock is coming in advance, and once it occurs they usually do not know just what kind of shock it is. In the specific context of price stability, the hard questions that therefore arise are whether, and if so, when and how, to return to a preestablished price path, once the economy is knocked off of it. Here that never happens. Because policymakers can anticipate each shock, and act as it occurs, the aggregate price level never gets knocked off of its stable trajectory in the first place.

Second, while prices in the King-Wolman model are sticky, nominal wages are not. As a result, when a productivity shock occurs, not only is there no impediment to reaching supply-equals-demand equilibrium in the labor market, but workers and firms can reach the market-clearing real wage with *any* price level that the central bank chooses to set. (In King and Wolman's model the productivity shock that shifts labor demand also shifts labor supply in an exactly offsetting way, and so the quantity of labor input remains unaffected, but the real wage does have to move.) This structure is exactly the opposite of the more familiar story in which *nominal wages* are sticky, and part of the job of monetary policy is to use *price level* adjustment to move the real wage toward its postshock market-clearing value. King and Wolman take pains to analyze carefully the real implications, in their monopolistic competition setting, of relative prices on different goods being out of line. By contrast, what seems to me an even more important relative price—the real wage—is, by assumption, never out of line.

Third, King and Wolman never explicitly show the value of commitment to the price stability rule they espouse. In a brief but much to the point section of the paper, they do show that "there are temptations for the monetary authority to deviate from this plan," and that if a shock arises that is "unexpected and viewed as never to recur," the optimal one-shot response is "a burst of inflation" that raises output. But what is the *cost* of this departure from price stability? Barro and Gordon (1983), in their paper that did much to stimulate interest in commitment to a monetary policy rule, used a simple model of reputation effects to address this question. King and Wolman show that the temptation is large but then leave the matter at that.

Further Peculiarities of the Model and Its Use

In addition to the flexibility of money wages, the odd (but here rather harmless) assumption that productivity shocks exactly mirror shocks to individuals' labor-leisure preferences, and the absence of any shocks at all to either money demand or real aggregate demand, two further aspects of King and Wolman's analysis merit specific comment. Neither is central to their principal argument, but in light of the focus of this conference each bears attention.

First, the argument made here for a stable price path with *zero* inflation, as opposed to a price path with a modest upward tilt, amounts to turning Feldstein's (1979) familiar argument on its head. In King and Wolman's model, the long-run average inflation rate matters in two ways. Relative price distortions, which are strictly welfare reducing because they lead to suboptimal allocations among different consumption goods, are eliminated by *zero* inflation. But the distortion due to firms' average markup of price over production cost, which is also welfare reducing in that it depresses aggregate output, is minimized not at zero inflation but at an inflation rate that is small but *positive*. Not surprisingly, King and Wolman find that for reasonable parameter values the relative price distortion is quantitatively unimportant. Why, then, doesn't the optimal price trajectory slope upward at the rate that minimizes the welfare loss due to the markup?

Their answer is a reverse Feldstein argument: Suppose that the economy is already at the markup-minimizing positive inflation rate. Because of the staggered price setting that underlies the stickiness of prices, a preannounced transition to lower inflation—say, zero—temporarily *raises* output. In the new state of zero inflation that prevails after the transition, the average markup is permanently higher, and therefore output and consumption are permanently lower. But, King and Wolman argue, the fact that this permanent reduction of output occurs only in the future, while the surge of output associated with the transition happens immediately, means that expected utility, appropriately discounted over the infinite future, is enhanced.

It is not clear how they can make this call without knowing the magnitudes of the temporary output increase and the permanent output reduction, the specific discount rate, and so on. Most obviously, in the limit as the discount rate goes to zero, *no* temporary increase in output, no matter how large, can offset the utility-reducing effect of even a small permanent reduction of output thereafter. Moreover, by analogy with what Feldstein has argued, taking into account that the larger permanent average markup and therefore the smaller permanent average output will occur in a growing economy would further bias the answer toward simply keeping inflation at the positive rate that minimizes the markup. But at the least, a reader familiar with Feldstein's (1979) paper will find the logic here familiar, albeit with the crucial signs reversed. Ironically, in King and Wolman's setting, applying Feldstein's logic with *Feldstein's* signs would mean that the King-Wolman model resembles the recent analysis by Akerlof, Dickens, and Perry (1996) in providing an argument that a low but positive long-run inflation rate is preferable to zero in that it minimizes output-depressing distortions.

One additional feature of King and Wolman's analysis merits specific comment. Although they never write down a money demand function, or provide much other detail about the nature of asset markets in their model, they do assume that people must hold money to buy consumption goods and that the nonmoney asset is an equity security. They choose to disregard the familiar welfare loss associated with "triangles under the money demand function," however, on the ground that nowadays most of what people use as money bears a "competitive return." I certainly sympathize with the view that, at least at the inflation rates that seem relevant in most western industrialized countries today, the money demand triangles that over the years have been such an obsession in much of welfare theoretic analysis of monetary policy are uninteresting.

But if the story here is not just that this matter is too small to bother with, but rather that money bears a "competitive return" in the sense that the return on money equals the return on the alternative asset, that leaves open the question of what determines how much money people hold. Because much of King and Wolman's analysis of monetary policy begins with the central bank's varying money supply, presumably a well-behaved money *demand,* however derived, must in the end be integral to their story.

Further Thoughts on a Stable Price Rule for Monetary Policy

I shall conclude with two somewhat broader questions about the rationale and the design of a stable price rule for monetary policy.

First, on the rationale: Does anybody still think time inconsistency is a problem that needs solving in the monetary policy of the world's major economies? Two decades ago, when high and rising inflation rates stood out as the chief economic problem in the majority of industrialized countries, it was at least plausible—though even then hardly a sure thing—to suggest that this inflation was a consequence of a policy-making framework based on discretionary actions by the central bank. If so, then the gain from restricting that discretion was potentially large. But by now most industrialized countries have succeeded in slowing their inflation to very low levels, indeed approximately zero for practical purposes in some countries. More to the point, many countries, including in particular the United States, have done so under formal policy-making institutions no different from what they had before. Even some of the countries that have introduced formal inflation targets (and that is not always the same as a genuine, committed policy rule) have done so only *after* achieving the crucial turnaround in their inflation problems.

This is not to say that the analysis of time inconsistency by Kydland and Prescott (1977), Barro and Gordon (1983), and others was logically wrong. But I believe it shows that it *was* wrong to conclude from that analysis that committing the central bank to a monetary policy rule was required to resolve the time-inconsistency problem. Maybe, as Barro and Gordon themselves suggested, the central bank's own awareness of reputation effects has provided the solution. Perhaps, following Rogoff (1985), the appointment of "conservative" central bankers has been the answer. There remains much room for research and debate about how different industrialized countries have solved their respective inflation problems. But the fact remains that most have done so. Before seriously considering committing monetary policy to a price stability rule, therefore—or, for that matter, any other rule—we ought at least to know what is the problem that commitment to a rule is supposed to solve. High inflation due to time inconsistency is no longer a satisfactory answer.

Last, a question about the design of a price stability rule if there were to be one: I conjecture that, especially in the United States, there is a trade-off between adhering to a genuine long-run price-targeting path, in the sense that bygones are *not* bygones and departures from the path *are* corrected (and in contrast to a strategy that accepts past mistakes and therefore under which the price level has infinite long-run variance), and aiming at a price path with zero slope. In other words, monetary policy can eliminate price level "base drift," or it can aim at zero inflation, but it cannot do both.

The reason is that, unlike in King and Wolman's model, actual central banks cannot anticipate all disturbances, and so from time to time the actual price level will depart from whatever is the targeted trajectory. Some of those departures will be on the low side, some on the high side. But whenever actual prices are above the targeted path, if that path is horizontal then returning to it requires that prices fall absolutely. By contrast, if the specified path is upward sloping—for example, at 2.5 percent per annum as in the case of the Bank of England's target—returning to it simply requires that for a while prices increase less rapidly than the path does, or perhaps even remain unchanged.

Would the Federal Reserve, as a part of its publicly announced policy strategy, deliberately seek falling prices? Should it do so? Unless the answers to these questions are yes—and I doubt that they are—then the most that monetary policy can do with respect to prices and inflation is either aim at a horizontal price path but let bygones be bygones (especially on the upside), or hold to a long-run price path without base drift but do so for a path with upward slope.

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Discussion Summary

Frederic Mishkin strongly disagreed with Friedman and made three points on why time consistency might not work in the future. First, the Federal Reserve has an excellent chairman at the moment and an Open Market Committee (FOMC) that does understand policy. Second, the administration has been very supportive of the Federal Reserve, even when it raises interest rates. Third, the economy has had very favorable shocks, which means that the political pressure on the Federal Reserve has not been as severe as it could have been with less favorable supply shocks. Part of what has happened is that central banks have dealt with time inconsistency by being more transparent, by making themselves more accountable, and by being more explicit about numerical inflation goals. *Friedman* replied that there was no disagreement since Mishkin just said that central banks are handling their problems on their own and do not need a rule to which to commit. *Tom Sargent* asked Friedman what he meant by "they are handling it on their own." The paper is sharp in its definition of the game that is being played. "Handling it on their own" sounds as if Friedman was predicting the same outcome from a different game, in which both sides choose sequentially. In this model, however, the outcome will not be the same and the appeal to reputation is not going to help for reasons isolated by Chari, Christiano, and Kehoe (1996). *Bob Hall* remarked that Kydland and Prescott (1977) and Barro and Gordon (1983) are must-reads for central bankers everywhere in the world. *Lars Svensson* also cautioned that these time-inconsistency issues should not be dismissed easily. Ten years ago, time-consistency issues were still very relevant in countries other than the United States. *William Poole* also noted that there was no inflation problem in 1963 and many members of the profession ridiculed the Eisenhower administration's 1950s campaign against "creeping inflation." Everything that Ben Friedman just said had already been said back in 1963.

Svensson wondered about the absence of any mechanism that would make it possible for the central bank to be committed to its rule. *Robert King* suggested that one answer to this question is that this might be a rule formulated outside the central bank, such as a law passed by U.S. Congress. *Ben McCallum* remarked that the discussion about the desirability of a monetary policy rule passed by Congress is tricky. McCallum's view on this issue is that a rule, mandating inflation to be the Federal Reserve's primary objective, passed by Congress, would make the Fed more independent.

Michael Woodford noted that in his paper with Rotemberg the first best solution also involves complete price stability. Achieving that first best may involve driving the nominal interest negative, which leads to a trade-off between price level and interest rate stabilization in order to have low average inflation consistent with the zero nominal interest rate bound. In this framework, some kinds of real shocks to aggregate demand can be easily added to the model without changing the results, such as government spending shocks, stochastic shocks to the rate of time preference, or changing preferences over consumption versus saving in the private sector. With these real shocks, the first-best equilibrium is still stable prices that undo the distortions associated with the reasons for price stickiness. Introducing nominal wage inflexibility does change the conclusion. If the relevant nominal inflexibility was in wages, complete nominal wage stability would probably be first best undoing that distortion, meaning that in response to a technology shock the price level would move. In particular, with a negative supply shock prices should be allowed to go up, but only by a certain amount once. So there will not be persistent inflation, which is quite different from the monetary policy responses to supply shocks in the 1970s.

Robert King broadly agreed with the comments made by Friedman about the sensitivity of the price-level-targeting result to the nature of nominal rigidities. But he argued that the methods of the paper should be applied to models with alternative nominal rigidities, such as sticky wages, to determine the extent of such policy sensitivity.

On the time-consistency issue, King made the observation that there was uncertainty about whether Alan Greenspan would continue as chairman of the Federal Reserve or would be replaced by a new chairman who was less concerned with low inflation. During the intense public discussion of this topic, the long rate of interest went up 100 basis points, indicating that the time-inconsistency issue was still unresolved.

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