Volume Title: Output, Input, and Productivity Measurement

Volume Author/Editor: The Conference on Research in Income and Wealth

Volume Publisher: UMI

Volume ISBN: 0-870-14181-3

Volume URL: http://www.nber.org/books/unkn61-1

Publication Date: 1961

Chapter Title: Some Difficulties in the Concept of Economic Input

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Chapter URL: http://www.nber.org/chapters/c2229

Chapter pages in book: (p. 331 - 346)

Some Difficulties in the Concept of Economic Input

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THE concept of economic activity as an input-output process is perhaps the most basic concept of economics. Nevertheless it is vague, and curious difficulties emerge when an effort is made to specify the inputs and outputs involved and to define the nature of the transformation implied. These difficulties may arise because the general concept is necessary in the statement and solution of many different, though not wholly unrelated, problems. Each problem requires a tailormade set of concepts and it is unlikely that a general definition will cover the range of needs.

There are four major problems where the concept of an input-output process is important. First, there is the problem of income distribution. Aggregate real income is equated in some sense to aggregate output, and the latter has to be allocated to participants in the system. The pertinent question concerns the extent to which the individual's income share measures his "contribution," or input. Second, there is the problem of the allocation of resources among different occupations. Inputs are conceived of as distributed among different "industries," with their distribution determining the structure of output. Interest focuses on changes in the composition of output when inputs are shifted from one "use" to another. The third problem is that of the relations between the stocks and flows of the system. Input and output are regarded as flow variables rates per unit of time. Input flows into the capital stock, output emerges later. The set of problems associated with the Austrian theory of capital are involved in this concept. The fourth problem is that of gauging efficiency. Efficiency is always measured in some sense as a ratio of output to input; the higher the ratio the more efficient the production process.

In all of these problems, most of the difficulties arise because of the extreme heterogeneity of both inputs and outputs; a property which is not accidental, but basic. A system with only one kind of input and output would lack most of the significant features of actual economic organization. Some sort of production function could be postulated

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(though it is difficult to see how it could be anything but linear), and an efficiency ratio calculated, but the problem of income distribution, if it is not to be trivial, calls for at least two kinds of input. Similarly, the problem of resource allocation requires at least two kinds of output. Actually, there are numerous kinds of input and output, and it is the condensation of this multidimensioned structure into manageable proportions without loss of essential information which constitutes the major problem. Situations that arise because of the uncertainty of economic events make the problem of abstracting from reality yet more difficult. The same is true when the complex time relationships of input and output—the fact, for instance, that today's input may be associated with the output of many future dates while today's output may be associated with a series of earlier inputs—are taken into account.

The reduction of the many dimensions of input and output to a single measurable dimension can only be done by multiplying each diverse quantity by a valuation coefficient or "price." Here the familiar index number problem of the most appropriate set of valuation coefficients is encountered. But even if this problem is avoided by assuming no change in the price structure, there are serious difficulties connected with the "dollar value" concepts of input and output. From the point of view of the income distribution and allocation problems, input and output should be defined so that their measure is the same. The aim is to allocate all output as income to the providers of input, possibly with some provision for "surplus value" and transfer payments, and to allocate all input, however employed, to the various forms of output. In this context, therefore, it is important that input and output be construed as equal.

This concept is modified, though not drastically so, in the "simple" stock-flow or river model. The process is defined by some segment of a "river"—input is the substance (water) which enters the segment, output is the substance which leaves. Input now need not equal current output, though each unit of input can ultimately be tagged as output, just as each drop of water that enters a segment of a stream eventually leaves it. Any imbalance between flows implies an adjustment in stock. Thus, if in a year 100 million bushels of wheat are produced (input) and 90 million consumed (output), 10 million bushels have necessarily been added to the stock. This model becomes more complicated if an effort is made to apply it to the whole economy and to separate out profit or surplus value, as did Ricardo and Böhm-Bawerk. Input becomes restricted to "original factors" such as labor, or labor and land, and the value of output exceeds the value of the input *which gave rise to it*, because of the accrual of profit

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were fed into the stream from underground springs, and represent a
form of input that is added in the course of time.

When we turn to the efficiency concept, we find now that if the
concept is to mean anything, input and output must be defined so
that they are *not* equal. If all input is conserved as output, and all out-
put originates as input, the efficiency ratio (output per unit of input)
is always unity, and there can be no way of comparing the efficiency
of alternative processes. For the concept to be useful, it is necessary
to differentiate between significant and nonsignificant input or out-
put. When an engineer measures the efficiency of an engine by the
ratio of output of kinetic or other available energy to input of
chemical energy, he is implicitly assuming that the output of non-
kinetic energy is not significant or valuable. On this basis, a funda-
mental input-output equality, following the law of conservation is
expressed thus:

Energy input
$$= \text{Available energy output} + \text{Unavailable energy output}.$$

A "price system" indicative of significance is then applied to the out-
put. Since the valuation coefficients are 1 for available and 0 for
unavailable energy, the value of the output is equal to the amount of
available energy and the efficiency measure becomes:

$$\text{Efficiency} = \frac{\text{Value of output}}{\text{Value of input}} = \frac{\text{Available energy output}}{\text{Energy input}}.$$

An effort to express this concept in terms of accounting data en-
counters serious difficulties. Insofar as all revenue is imputed back to
some expenditure, measuring efficiency by the ratio of revenue (value
of output) to cost (value of input) involves the conservation problem
again. To circumvent this problem, expenditure is divided into cost
and profit, and the measure of efficiency becomes:

$$\text{Efficiency} = \frac{\text{Revenue}}{\text{Cost}} = \frac{\text{Cost} + \text{Profit}}{\text{Cost}} = 1 + \frac{\text{Profit}}{\text{Cost}}.$$

Even from the point of view of the individual firm this is a curious
and unsatisfactory measure. The investor sees an investment as a
series of payments into and out of a capital account, the in-payments
being just that, the out-payments consisting of dividends, interest,
and capital distributions. If the process involves a single in-payment
C and a single out-payment R one year later, then the efficiency ratio
is R/C, or $1 + (R - C)/C$, which again is $1 + \text{Profit}/\text{Cost}$, or the "force"
of interest. In this simple case $(R - C)/C$ is the rate of interest. How-
ever, if the in- and out-payments are spread along the time scale, the

ratio of out-payments to in-payments neglects the time-position of these quantities. For example, an investment of $100 which returned $105 after one year would not be regarded as equally efficient with one that yielded $105 ten years later. Of course, no matter how complex the series of in- and out-payments, it is always possible to derive an internal rate of return which is essentially the average rate of growth of capital over the life of the investment.[1]
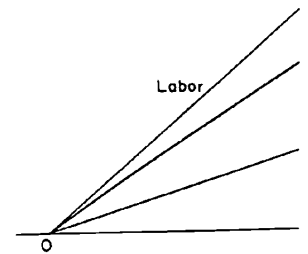
The tendency here is toward the "Austrian" notion of input as that which grows into output, and of the rate of growth as a measure of efficiency. However, great difficulties arise when this notion is applied to the economy as a whole. Is labor the only "significant" input, the only "original" factor, or is it necessary to construct, as Böhm-Bawerk tried to do, a melange of labor and land? Is capital merely an intermediate product, the embodiment of original factors on their journey toward realization in final product, or should it be included in an input measure? There are no single or simple answers to these questions; the concept must fit the task.

When per capita income is used to measure the economic efficiency of the system, the whole population is regarded as an input productive of the whole output. As a rough measure of performance this makes sense; if per capita income with a given price structure is $100 in country A and $1,000 in country B, it is likely that A is much poorer than B. However, per capita income is not the only test of economic efficiency and a rise in per capita income achieved by undesirably hard work and sacrificed leisure might represent a worsening of economic welfare. Furthermore, population efficiency is not the only significant resource-efficiency concept, although it is the most important. Land-efficiency (output per acre) is also of interest in certain cases; with a given population on a given land area an increase in per capita income implies an increase in per acre income, whether stemming from increased yield per acre of crops or a shift to more productive industrial employment. The concept of capital-efficiency is more difficult, but not meaningless: if the same income can be obtained with a smaller capital stock this is a clear gain. Here, capital efficiency is measured by the reciprocal of the average period of production, and a capital-saving improvement is one which shortens the production period, or enhances the capital-income ratio, for a given income.

These relationships can be illustrated by means of the familiar production function. In Chart 1 labor and capital are measured along OL and OC respectively, and product or income vertically

[1] See K. E. Boulding, "The Theory of a Single Investment," *Quarterly Journal of Economics*, May 1935, p. 475.

along ZY. If land or nat tion surface $OCYL$ will diminishing returns to represents the relation c

both land and labor, (income increases at a and land in constant pr usual diminishing retu

In Chart 2 three kin are distinguished. OY_1

Chart 1 in the plane assumed to be the m A shift from OY_1M_1 hardly an improvemer

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tion surface $OCYL$ will not be linearly homogeneous, but will exhibit
diminishing returns to scale—that is a curve such as OY, which
represents the relation of income to equal proportional increases in

CHART 1
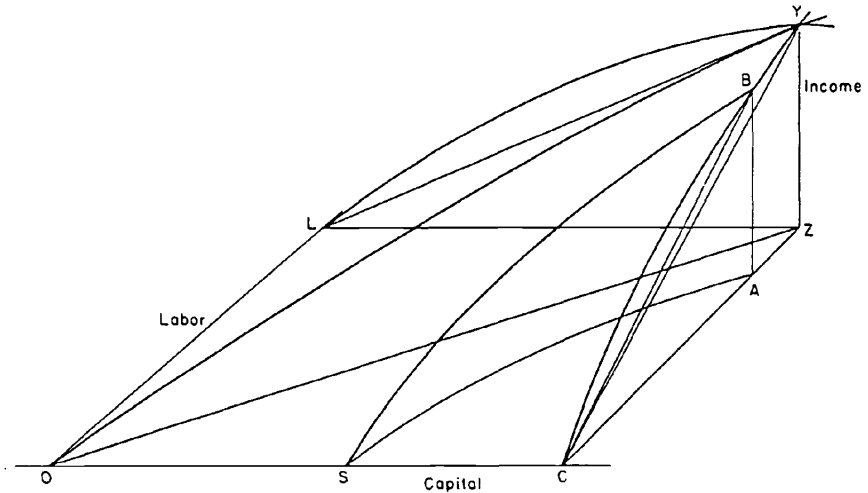
both land and labor, will curve downwards and eventually decline
(income increases at a decreasing rate with increasing doses of labor
and land in constant proportions). The curves LY and CY exhibit the
usual diminishing returns to capital and labor respectively.

In Chart 2 three kinds of "pure shift" in the production function
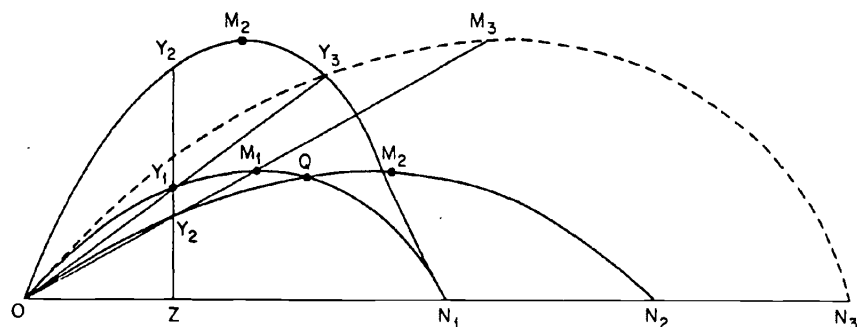are distinguished. $OY_1M_1N_1$ is a section of the production surface of

CHART 2

Chart 1 in the plane OZY. M_1 is the maximum output (income),
assumed to be the maximum of the production surface of Chart 1.
A shift from $OY_1M_1N_1$ to $OY_2M_2N_2$ is a pure horizontal shift. It is
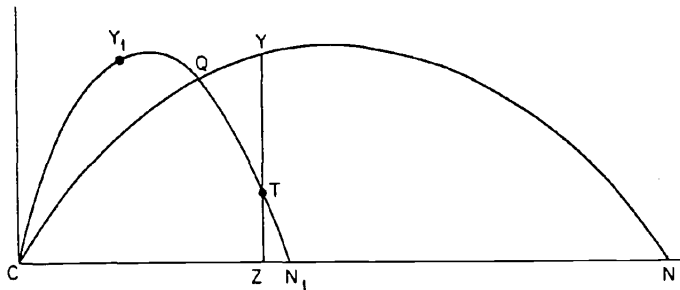hardly an improvement, since between O and Q it results in a lowering

of per capita income. However, it does push the maximum outwards and postpone the diminution of total output or the increase in labor and capital. It might be described as a pure land-saving shift; as far as the figure goes, it could represent either an increase in yields (income per acre) or an extension of the land area

By contrast a shift from OM_1N_1 to OM_2N_1 is a pure "labor-and-capital" saving shift, representing a simple rise in each vertical originate of the production function of Chart 1. This is undoubtedly an improvement as at each point of the field per capita income is increased. It is difficult, however, to distinguish unequivocally between labor saving and capital saving improvements. It might be supposed that a shift, which moved the maximum of the production surface of Figure 1 to a point with more labor but the same amount of capital, would be a labor-saving shift. If this were a pure horizontal shift, however, there would be a region (like OQ in Chart 2) where the

CHART 3

shift was actually a "disimprovement" with lower labor and capital efficiencies. Thus in Chart 3, CY, CZ are as in Chart 1: CYN is the section of the production surface in this plane. A pure labor-saving shift then means a shift to CY_1N_1: each quantity of product can now be produced with less labor than before. Now it is at large amounts of labor input, beyond the point of intersection of the two curves at Q, that the improvement becomes a "disimprovement." At the old position Z, output, and therefore the efficiency of both labor and capital, has actually fallen, from ZY to ZT. The difficulty arises because it is impossible to raise the production surface of Chart 1 without raising both labor and capital efficiencies. Thus at point Y in Chart 1 the labor efficiency (per capita income) is ZY/LZ and the capital efficiency (income-capital ratio) is ZY/CZ. Increasing ZY obviously increases both these fractions and in exactly the same proportion.

A third type of shift might be called a pure scale shift, represented by the move from $OY_1M_1N_1$ of Chart 2 to $OY_3M_3N_3$. Here a

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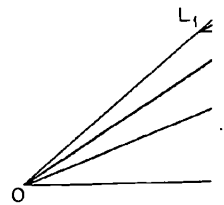be increased in the prop
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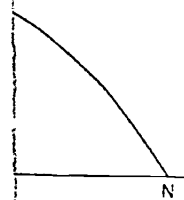
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constant proportion, OY_1/OY_3 is maintained, meaning that if there is a pure scale improvement in the ratio k, then both labor and capital can be increased in the proportion k without change in the efficiency of either. Thus a pure scale shift might be thought of as equally saving of all three factors.

This conceptual apparatus may illuminate the significance of the input concept from the point of view of efficiency studies. Suppose that history reveals, as it has in the United States, a rise in the total output of the society: to what extent can this be attributed to a rise in input, and to what extent is it attributable to a rise in the efficiency of the production process? The answer depends largely on the concept

CHART 4
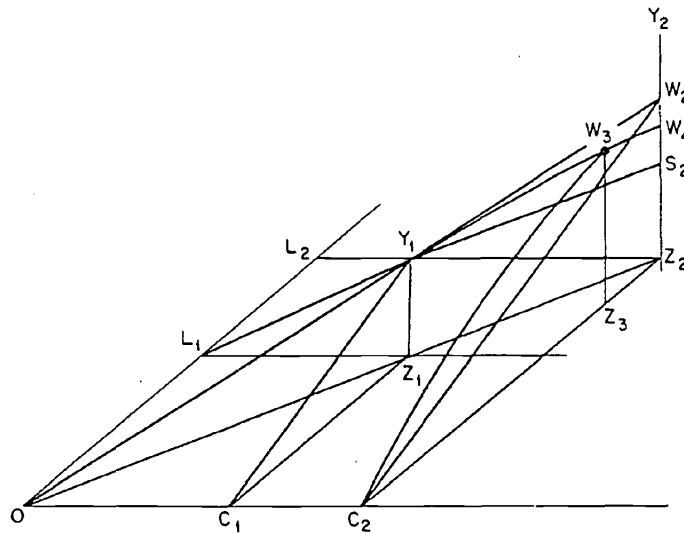
of input employed. In effect, a comparison between two dates in-volves a comparison of two points on two different production functions, like Y_1 and Y_2 in Chart 4. If the production function was known, one could say with some confidence how much of the rise in total output between state Y_1 and state Y_2 was due to a change in the inputs and how much to a change in the production function itself. Thus if OY_1W_2 is a line on the production surface cutting Z_2Y_2 in W_2, of the increase in output S_2Y_2, S_2W_2 is due to an increase in inputs and W_2Y_2 to an increase in efficiency. If the production function is linearly homogeneous in the case of the two inputs, and if the input increases in the same ratio, as from Z_1 to Z_2, the answer is wholly unambiguous. Whether some measure of the sum of the in-puts is used, or each is taken separately, the result is the same. For

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example, labor efficiency at point Z_1 is $Z_1 Y_1/C_1 Z_1$. If the labor efficiency had remained constant, output at state Z_2 would have been $Z_2 W_2$, and the increase due to increased efficiency would again be $W_2 Y_2$.

In practice, of course, inputs rarely increase in the same proportion. If capital, for instance, has increased faster than labor, the second position may be at a point such as Z_3, where $OZ_1 Z_3$ is not a straight line. Now even if the production function is linearly homogeneous, the point W_3 on the old production function cannot be found by simple geometric constructions; it is necessary to know the formula of the production curve $C_2 W_3 W_2$. Another complication is that the production surface itself is unlikely to be linearly homogeneous, especially if there are land or natural resource limitations. The production surface then may run not from Y_1 to W_2 but to W_4, showing diminishing returns to increases in labor-and-capital taken together. In this case, even if the two factors increase in the same proportion, the position of W_4 cannot be calculated without mathematical knowledge of the production function. It may be argued that if the land factor was included in the analysis there would be no diminishing returns to scale and no necessity for knowing the production curve: simple proportionality to total inputs could be used to decide what the total output would have been if there had been no change in efficiency. This method breaks down, however, when the factors have not increased in the same proportion. This assumption may not be wholly erroneous before 1890, but for the most part, labor and capital have expanded against a much less elastic land and resources barrier.

Thus far it has been assumed that labor and capital were measurable in homogeneous units. Dropping this assumption brings up the formidable problem of the significance of aggregate measures of labor and capital. In the case of labor it seems reasonable to start with hours of work performed but even this simple measure involves difficult decisions when it comes to separating labor from leisure. For instance, should the labor of the housewife and the self-employed be counted? If not, then a shift of these individuals into wage labor will lead to seriously misleading statistics. (The Russians seem frequently to have been guilty of this error.) On the other hand, assessing the "labor" time of a housewife necessarily involves difficult matters of judgment. The enormous rise of "do it yourself" projects suggests that an appreciable proportion of the labor input of this country is leisure time activity. Thus estimates of national input or product should ideally include the cooking or dinner of the housewife and the carpentry or furniture of her spouse. This is a difficult area where

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The problem of how to measure the *intensity* of labor has plagued theorists from the time of Adam Smith and the labor theory of value. Wages are a notoriously poor measure of labor intensity and many other factors have to be allowed for before there would be even a positive relation between hard work and high wages. Yet everybody senses a distinction between hard and light work—a distinction which cannot be measured merely by calorie output, for information throughout may be just as exhausting—and the observation that Americans work harder than the Portuguese or Ceylonese is not meaningless, however difficult it may be to quantify. Currently the most promising approach would seem to be to pursue some physiological leads: if a fairly simple measure of fatigue could be obtained and applied over a large population, the results would be interesting despite the fact that fatigue depends on many subtle psychological factors not necessarily closely related to physical exertion. In any event, the question "has an increase in output come about because of a rise in efficiency or because of a rise in the intensity of labor" deserves serious attention in view of the Marxist tendency to attribute an increase in product under capitalism to an increase in labor-intensity, and in view of the recurrent difficulties in industrial relations as a result of real or imaginary speed-ups.

Aggregate capital is even more difficult to measure than aggregate labor input. The composition and physical nature of capital changes constantly, and any measure of its aggregate becomes increasingly arbitrary as time goes on. How, for instance, can horses be compared with tractors, abacuses with IBM computers, cotton plants with nylon spinners? The most that can be done is to estimate the dollar value of the existing capital stock and compare this with a dollar estimate of national income. This capital-income ratio, however, is a measure of the period of production only in stationary equilibrium; in a dynamic society there may be wide divergences between the number of years of current income required to total the capital stock, and the average production period. Thus in the case of human capital (the population), the ratio of the total population to annual births or deaths is equal to the average age at death only in a static situation; if the population is growing or declining, or if the age distribution is changing rapidly, annual births (input) may differ from annual deaths (output), so that the capital-income ratio depends on the measure of income selected. Also, the capital-income ratio, however computed, will not in general correspond to the average age at death.

Nevertheless, the latter figure (the period of production) is of great importance not only for the human population but also for the population of goods. Generally speaking, the more durable the product of a given input, the better off the society. Of course this formulation disregards the problem of the optimum flexibility of structure: if either people or goods are too long lived, it is hard to replace them with younger and possibly superior items!

This argument has been inconclusive, and not perhaps much help to the statistician. However, the problems are very difficult, and the context, very general. In all problems of measurement, the fundamental question is, *what questions can be answered better* as a result of the measure devised. There is perhaps a tendency among statisticians to devise measures for their own sake, rather than with a particular purpose in view. The danger of the information system controlling the questions instead of vice versa is to be taken seriously.

COMMENT

Murray Kemp, McGill University

Kenneth Boulding's chief concern is the problem of allocating changes in output to changes in factor inputs and to shifts in the production function. He insists that nothing can be inferred from the data unless special assumptions are made about the form of the production function and the market structure. Even if constant returns to scale and perfect competition are assumed, it is impossible to allocate changes in output, except conceivably where factor proportions remain unchanged, or changes in input and output are very small. The production function may, of course, shift in any number of ways, and each type of shift produces its own effect on total output and on the relative marginal productivities of the factors. Boulding selects for detailed discussion three "pure" types of shift. He defines each one diagrammatically and then advances several statements, apparently intended as theorems, concerning their individual factor-saving (or factor-wasting) properties. Several of these theorems seem to be weak or at least in need of a more careful defense than Boulding has provided.

It is necessary at the outset to clarify the meaning of a factor-saving shift (innovation). Boulding remarks that "A pure labor-saving shift then means . . . each quantity of product can now be produced with . . . [less] labor than before." It is not clear whether this is intended as a definition or as a theorem based on some alternative but unstated concept of a labor-saving shift. However, it is the nearest that Boulding comes to a definition and I shall accept it as such. Then similar definitions of capital- and land-saving shifts can

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easily be formulated. The three shift types can be illustrated with some elementary algebra. Imagine that a single output P is a function of labor L, capital K, and land A:

$$p \cdot P(lL, kK, aA).$$

The lower-case letters stand for shift parameters which initially are set equal to unity. Then it is in keeping with Boulding's definition to associate a pure labor-saving shift with an increase in l and to identify capital- and land-saving shifts with increases in k and a, respectively. Three properties of pure factor-saving shifts are analytically important: (1) if P is a maximum, it is invariant under such shifts, (2) maximum (P) is attained after a shift with less of the factor economized but with the same quantities of the other factors. It follows from (1) and (2) that (3), a factor-saving shift, does not necessarily result in an increase in the output associated with a given vector of inputs, since the efficiency of the latter may be reduced. In particular, the preshift product-maximizing vector will be less efficient after the shift than before.

A pure output-increasing innovation, on the other hand, may be identified with an increase in p. For *any* vector of inputs the postshift output exceeds the preshift output and the efficiency of all inputs increases. In the special case when constant returns to scale prevail over the relevant range of input values, an output-increasing innovation is equivalent to a factor-saving innovation which is general and uniform in its incidence, that is, for which

$$\Delta l = \Delta k = \Delta a = \Delta p.$$

Boulding's analysis is in terms of a series of two-dimensional diagrams with "output" measured vertically and "labor and capital" measured horizontally. Four variables are collapsed into two by supposing that labor and capital are combined in fixed proportions and that land is fixed in amount.

His first type of shift, the "pure horizontal," involves increases, in the same proportion, of the amounts of labor and capital needed to produce a given output. This is the opposite of a labor- and capital-saving shift and might be called labor- and capital-*wasting*, with

$$\Delta l = \Delta k < 0.$$

Boulding's discussion is confusing largely, I suspect, because he is undecided whether it is total or per capita output that is being measured along the vertical axis. For example, he states that "... between O and Q [a pure horizontal shift] results in a lowering of *per capita* income. However, it does push the maximum outwards and postpone the diminution of *total output* . . ." (my italics). Assuming

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that total output is at issue, income per capita must fall uniformly and not simply between O and Q. I had also difficulty with the description of this type of shift as "pure land-saving" and with the remark that "as far as the figure goes, it could represent either an increase in 'yields' (income per acre) or an extension of the land area." In the scheme of definitions set out above, a land-saving shift is not equivalent to a labor-and-capital-wasting shift. A horizontal shift is land-saving only in the sense that, for given output and given land input, the ratio of land input to labor-and-capital input falls as the result of the shift.[1]

Boulding's second type of shift is described as "pure labor and capital saving" and is represented diagrammatically by "a simple rise in each vertical originate." However, a pure labor-and-capital-saving shift should be represented by a leftward compression of the curve, and while the latter usually implies an increase in the output associated with any particular input of labor-and-capital, such an increase is not inevitable. In particular, the increase will not take place for those values of capital-and-labor input in excess of the preshift output-maximizing input (where the latter exists).

Boulding describes his third type of shift as a "pure scale shift" and depicts it diagrammatically as a radial projection, in constant proportion, of all points on the total product curve. In symbolic terms:

$$\Delta p = -\Delta l = -\Delta k > o,$$
$$\Delta a = 0.$$

Thus, it is a combined output-increasing and labor-and-capital-wasting shift and involves a horizontal extension of the *average* product curve for labor-and-capital. To describe it as a "pure scale shift" is a little misleading; to infer, as Boulding does, that it is equally saving of all factors is incorrect.

I am sure that the propositions discussed here are regarded by Boulding as tangential to his major theme, with which I am in complete sympathy. Also, my comments may be unjust for they are based on definitions which I have read into a single paragraph which occurs *after* the section of the paper that I have criticized. I hope, therefore, that Boulding will make clear (1) whether the definitions are acceptable to him, and, if so, (2) the stage of his analysis at which they are to be adopted.

[1] A land-saving shift, in the sense of the above definition, involves: (1) for given output and given labor-and-capital input, a decrease in the ratio of land to labor-and-capital, but (2) for given output and given land input, an increase in the ratio of land to other inputs. Further, for given output and given land input, a land-saving shift does not call for an increase in labor-and-capital.

KARL BORCH, Europe
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KARL BORCH, European Productivity Agency

The main utility of a conference like this is probably that it makes one re-examine long-held beliefs. Thus I have asked myself, *what* is this productivity we want to measure, and *why* do we want to measure it?

Circular definitions are an obvious danger. Productivity is a ratio between output and inputs, but as Kenneth Boulding and others have pointed out, the value of all inputs must be equal to the value of total output. Similarly, the price paid for new capital equipment must be related to the prospective profits, and under certain reasonably plausible assumptions one can prove that the capital output ratio must be constant. If it fluctuates, it is because the expectations of the entrepreneurs have not been fulfilled.

I have gradually come to take a more pragmatic view, and to ask if the results obtained by measurement are useful—more specifically, do they enter into any conceivable decision models? Is there really any such justification for trying to find out the number of hours worked by a farmer, his wife, and children?

I discussed these questions in Yugoslavia awhile ago. The Marx-Lenin-Tito concepts of productivity are different from those that have been discussed here. In making productivity indexes, the Yugoslavs seek to allow for every disturbing factor; quality of the man-hour, investments in vocational training, etc. They arrive at an almost perfect index, the only drawback being that this paragon could go up even if the cruder index of output per man went down. We agreed that a productivity index was useful only if it measured the output per capita available for consumption and investment. We further agreed that the Yugoslav index was ideal if you wanted to reward efficient workers, managers, and officials with medals or titles like "hero of socialist labor," but entirely unsuitable if the rewards were to take the form of increased real wage income.

In twenty years European GNP per capita in constant prices will be about 50 per cent higher than it is today. Does this information have any real meaning or practical value? All that is signified is a gain in utility—which is not measurable. Since a 50 per cent increase in GNP will mean a substantial change in the composition of the total output, knowledge of the behavior of the aggregate can tell us nothing about capital requirements and the distribution of labor between sectors twenty years hence. Data like the trend of bushels of wheat per acre and tons of coal per man-shift are obviously useful in predicting and preparing for the future but I see no use for an aggregate estimate.

Over-all productivity clearly plays a fundamental part in economic analysis, but that does not mean that it is possible or necessary to

measure it. As a parallel, look at the velocity of circulation of money in the Quantity Theory. A generation ago economists thought this concept was of supreme importance, and tried to measure it. This turned out to be not only fruitless, but unnecessary to an understanding of the relationship between prices and monetary policy.

I am probably the only one here working for an institution which has "productivity" in its title, so it may seem strange that I should try to minimize the importance of this concept. However, I am less certain today than five years ago that productivity is the best approach to the complex problems of economic growth and technical progress. In this connection it may be worth mentioning that we are seriously considering renaming our institution, "The European Development Agency."

REPLY, MR. BOULDING

Kemp raises some interesting problems which, since they are peripheral to the main concerns of this conference, must receive much briefer treatment than they deserve. In part the difficulty is one of language. The terms "labor saving," "capital saving," or "land saving" correspond to very vague—though important—notions. Attempts to impart precision to these notions inevitably result in a multiplication of concepts. Such proliferation is necessary and useful as long as it does not provoke controversy about which of the proposed substitutes is the most "correct"; one precise concept may be useful for one purpose, one for another.

Kemp's definition of a factor-saving change in the production function—which amounts to a rescaling of the factor axis—is not the only way of sharpening a vague concept. Also it evades the basic difficulty that no change in the production function is possible which does not change the amount of product produced by a given combination of all factors, and which therefore changes, in a different sense at different points, the product-factor ratio for all factors. Furthermore, if the production function exhibits a maximum, there will be some combinations of inputs for which, say, a labor-saving change according to Kemp's definition, actually produces a diminution of product and hence a diminution of product per unit of labor. Thus while the type of change which Kemp describes is interesting, I am not sure that it deserves the appellation of factor-saving. I am not sure either that I am prepared to defend my substitute definitions. The real difficulty is that a factor-saving change can be defined only in relation to a given price or market structure; it cannot be adequately defined as a property of the production function alone. Thus a change in the production function which might be "labor saving"

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at one set of relative prices of labor and land might be "land saving" at another set, as defined by its impact on the proportions of factors actually used.

Turning to a general problem relating to the topic of this conference, I feel it is time for statisticians in general, and economic statisticians in particular, to recognize more explicitly the sociological, as well as the arithmetical, basis of their art. A statistic is an interesting number. For the most part this meeting has considered how one interesting number can be expressed as the product of two others— a value index as a price index times a quantity index, or an output index as an input index times a productivity index. There are infinite ways of achieving multiplicative disaggregation, but very few of these are interesting. If statistics is the science of interesting numbers more investigation is needed into what it is that makes numbers interesting, for interest is a property of the reader, not of the number. The attempt to find purely internal, mathematical justifications for statistical procedures is doomed to frustration, as it abstracts from the essentially sociological nature of the subject.

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