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9

Environmental Policy and Firm Behavior

Abatement Investment and Location Decisions under Uncertainty and Irreversibility

Anastasios Xepapadeas

9.1 Introduction

A firm's response to changes in environmental policy is an issue that has drawn considerable attention in the environmental economics literature.[1] Questions that usually arise when environmental policy is introduced or changed are primarily associated with how firms react regarding their choices of investment in productive or abatement capital, the mix of relatively more or less polluting inputs, the choice of labor input or the decisions about research and development (R&D) expenses (process R&D or environmental R&D),[2] or what kind of decisions firms make regarding location choices.[3]

This paper focuses on these questions, and in particular it explores the behavior of polluting firms regarding expansion of abatement capital and location decisions in the presence of environmental policy. Environmental policy takes the form of emissions taxes or tradable emissions permits, and subsidies for the costs of expanding abatement capital. In this context, accumulated abatement capital can be interpreted as the stock of knowledge in pollution and abatement processes. This knowledge is useful in designing new "cleaner" products or better abatement processes.

One of the major factors affecting the responses of firms when a regulatory policy—in our case environmental policy—is introduced or changed

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1. See, e.g., Xepapadeas (1992, 1997a), Kort (1995), and Hartl and Kort (1996).

2. See, e.g., Xepapadeas (1992, 1997a), Kort (1995), Hartl and Kort (1996), and Carraro and Soubeyran (1996a, 1996b).

3. See, e.g., Markusen, Morey, and Olewiler (1993, 1995), Motta and Thisse (1994), Hoel (1994), Rauscher (1995), and Carraro and Soubeyran (1999).

is uncertainty regarding important parameters of the model. In particular for the case of firms responding to environmental policy, uncertainty could be associated with output price movements; that is, demand could be affected by stochastic shocks, or technological uncertainty could affect the efficiency of the abatement process. Another type of uncertainty that could be important is policy uncertainty, which in our case can be associated with stochastic movements of tradable emissions prices or unpredictable (from the firms' point of view) policy changes. In all these cases, the analysis of firms' behavior under uncertainty could be important not only in explaining the effects of environmental policy on abatement investment or relocation decisions, but also as a guide for exploring issues of optimal environmental policy design.

A second important factor affecting the same problem is the fact that firms' decisions regarding abatement investment and location have irreversibility characteristics. Thus abatement investment expenses are irreversible once they are incurred by the firms; movement to a new location when the costs of returning to the old location are sufficiently high is also an irreversible decision.

Since abatement investment is a dynamic process of accumulating abatement capital, and the type of uncertainty described undoubtedly embodies a time dimension since output or tradable emissions price evolves dynamically through stochastic processes, it follows that the analysis of firms' responses to environmental policy might be more realistically explored in a dynamic framework. In a dynamic setup, the interaction of uncertainty with the irreversibility characteristics of investment decisions or relocation decisions generates well-known option value issues.[4]

Thus, the purpose of the present paper is to explore abatement investment and location responses to environmental policy under uncertainty and irreversibility. The problem is analyzed in a dynamic setup, where uncertainty is modeled by Itô stochastic differential equations, by using optimal stopping methodologies. The idea is to define continuation intervals during which firms do not expand abatement capital or relocate, and intervals during which firms take the irreversible decision to undertake abatement investment expenses or relocate. The optimal stopping methodology will define a free boundary. When a state variable—which could be output price, the price of tradable emissions permits, or a technological coefficient—crosses the boundary, the irreversible decision to increase abatement capital or relocate is taken. The structure of the free boundary determines, therefore, the conditions under which the firm will invest in abatement capital or relocate.

Using this methodological approach, free boundaries are determined or

4. See Arrow and Fisher (1974), Fisher and Hanemann (1986, 1987), and Xepapadeas (1998) for related issues or Dixit and Pindyck (1994) for a more general treatment.

characterized for a variety of cases that include output price uncertainty, policy uncertainty expressed both in terms of continuous fluctuations of permit prices and unpredictable policy changes, and technological uncertainty. The advantage of this approach is that although a complex mathematical model is used, the emerging results regarding the structure of the free boundary are relatively simple and depend on estimable parameters. Thus it allows the analysis of firms' responses to environmental policy in a framework that combines uncertainty and irreversibility effects.

A second advantage of the approach is that when uncertainty is not associated with environmental policy parameters, but is either demand or technological uncertainty, the free boundaries are defined parametrically in terms of policy instruments, such as emissions taxes or abatement investment subsidies. This allows the meaningful performance of comparative statics regarding the irreversible decisions. Thus it is possible, given the parameters characterizing the stochastic processes associated with output price or technological uncertainty, to analyze how firms respond to changes in emissions taxes or abatement subsidies, regarding abatement investment or relocation decisions, by analyzing the shifts of the boundary.

Finally, the optimal stopping approach makes possible the design of optimal policies under uncertainty. The optimal policy is determined such that the firm's free boundary under the optimal policy is identical to a free boundary determined by maximizing the objective function of a regulator. In this case, the firm reaches decisions regarding abatement investment or relocation, given the stochastic movements of the state variables (output price or technological uncertainty), which are the same as the decisions that a regulator would have taken in the same stochastic environment. Thus this methodology introduces an approach to optimal policy design in which, under uncertainty, the target is not to choose the instrument such that the firms choose the same value of the variable of interest (e.g., abatement), but rather the target is to induce them to base their decision rule on the same rule that the regulator would have selected. In this case, the decision rule is determined by the free boundary.

9.2 Abatement Investment Decisions under Uncertainty

We assume an industry consisting of n identical firms producing in a small open economy. The firms behave competitively and sell their product in the world market where international competition prevails. We consider the representative firm producing at each instant of time output $q(t)$ at a cost determined by a cost function $c(q(t))$, with $c'(q) > 0$, $c''(q) > 0$. Output is sold in the world market at an exogenous world price $p(t)$.

The production of output generates emissions. Emissions per unit of output are determined by the function $E(t) = v(t)\, e(R(t))$, where $v(t) > 0$ is an efficiency parameter associated with the abatement process and $e(R(t))$

is a function of the accumulated abatement capital, up to time t.[5] A reduction in v indicates an improvement in the efficiency of the abatement process, while abatement capital, denoted by $R(t)$, is defined as

$$R(t) = \int_0^t r(s)ds,$$

where $r(s)$ is the abatement investment flow undertaken at instant of time s. This flow can, for example, represent resources devoted to the firm's lab in order to design cleaner processes at time s. It is assumed that $r(s) \geq 0$; thus, the abatement capital accumulation process is irreversible.[6] For the abatement process, we assume that

$$e(0) = 0, \quad e'(R) < 0, \quad e''(R) > 0,$$
$$\lim_{R\to 0} e'(R) < -\infty, \quad \lim_{R\to\infty} e'(R) = 0.$$

Therefore an increase in abatement capital reduces unit emissions at a decreasing rate, which means that diminishing returns in abatement capital are assumed. Thus, when the firm produces output $q(t)$, total emissions are defined as $v(t)\ e(R(t))\ q(t)$.

The cost for increasing the stock of accumulated abatement capital by ΔR is defined as $(1 - s)\ h\Delta R$, where h is the exogenous unit-abatement investment cost and $s \in [0, 1)$ is a subsidy potentially given by the government to cover some of the expenses for expanding abatement capital. Assume that the firm pays an exogenously determined emission tax $\tau(t)$. Then the tax payments are defined as $\tau(t)[\ v(t)\ e(R(t))\ q(t)]$.

Given this setup, the firm has to decide about output production and abatement investment. At each time the firm decides about the optimal output level given the stock of abatement capital. Thus output is regarded as an operating variable and output decisions can be regarded as short-run decisions, while abatement investment decisions are long-run decisions. The optimal choice of output for any given level of abatement capital determines a reduced-form instantaneous profit function, which can be defined as

$$(1) \quad \pi(p, v, \tau, R) = \max_q \{p(t)q(t) - c(q(t)) - \tau(t)[v(t)e(R(t))q(t)]\}.$$

The first-order conditions for the optimal output choice, assuming interior solutions and dropping t to simplify notation, are given as

5. A more general formulation would be to define the $e(\cdot)$ function as $e(R, R^{AG})$, where $R^{AG} = nR$, is aggregate abatement knowledge. In this case there could be positive spillovers from aggregate abatement capital to the individual abatement function. When firms consider aggregate knowledge as fixed, there is a divergence between the private return of abatement capital and the social return of abatement capital (Xepapadeas 1997b).

6. To simplify things, we ignore depreciation issues.

$$p - c'(q) - \tau v e(R) = 0,$$

with optimal output determined as

$$q^* = q^*(p, \tau, v, R).$$

Using the first-order conditions for the optimal output choice, we obtain the following short-run comparative static results:

$$\frac{\partial q^*}{\partial p} > 0, \quad \frac{\partial q^*}{\partial \tau} < 0, \quad \frac{\partial q^*}{\partial v} < 0, \quad \frac{\partial q^*}{\partial R} > 0.$$

Thus an increase in the tax rate or a reduction in the abatement efficiency (increase in v) reduces optimal output, while an increase in the stock of abatement capital increases optimal output. From the short-run comparative statics and the envelope theorem, we obtain the derivatives of the profit function as[7]

$$\frac{\partial \pi}{\partial p} = q^*(p, v, \tau, R), \quad \frac{\partial^2 \pi}{\partial p^2} = \frac{\partial q^*}{\partial p} > 0,$$

$$\frac{\partial \pi}{\partial v} = -(\tau e(R) q^*) < 0, \quad \frac{\partial^2 \pi}{\partial v^2} = -\tau e(R) \frac{\partial q^*}{\partial v} > 0,$$

$$\frac{\partial \pi}{\partial \tau} = -(v e(R) q^*) < 0, \quad \frac{\partial^2 \pi}{\partial \tau^2} = -v e(R) \frac{\partial q^*}{\partial \tau} > 0,$$

$$\frac{\partial \pi}{\partial R} = -\tau v e'(R) q^* > 0, \quad \frac{\partial^2 \pi}{\partial R^2} = -\tau v \left(e''(R) q^* + e'(R) \frac{\partial q^*}{\partial R} \right).$$

Thus the profit function is convex in prices for fixed (τ, v, R), decreasing in (τ, v), and increasing in R.

Uncertainty can be introduced into this model in three ways. First, it can be assumed that the world demand is affected by stochastic shocks giving rise to a geometric Brownian motion price process. In this case, output price is the exogenous state variable,

$$dp(t) = ap(t)dt + \sigma p(t) dz_p(t), \tag{2}$$

where $[z_p(t)]$ is a Wiener process,[8] and a and σ are constants. If the current price is a given constant $p(0) = p_o$, then the expected value of $p(t)$ is $E[p(t)] = p_o e^{at}$ and the variance of $p(t)$ is[9]

7. It is assumed that $[e''(R)q^* + e'(R)(q^*/R)] > 0$, so that the profit function is concave in R for fixed (p, τ, v). The concavity assumption requires sufficient curvature of the unit emissions function $e(R)$.

8. For definitions see Malliaris and Brock (1982).

9. See Dixit and Pindyck (1994).

$$V[p(t)] = p_\sigma^2 e^{2at}(e^{\sigma^2 t} - 1).$$

It should be noted that the Brownian motion assumption causes price to move away from its starting point. If, however, price is related to long-run marginal costs, then a better assumption about price movements could be a mean-reverting process. Under this assumption, price tends toward marginal costs in the long run and price movements can be modeled as

$$dp(t) = a(\tilde{p}(t) - p(t))p(t)dt + \sigma p(t)dz_p(t),$$

where $\tilde{p}(t)$ can be interpreted as long-run marginal costs.[10]

Second, it can be assumed that environmental efficiency evolves stochastically according to the geometric Brownian motion:

$$dv(t) = \gamma\tau(t)dt + \delta v(t)dz_v(t). \tag{3}$$

The interpretation of this type of uncertainty can be associated with the stochastic operating conditions of abatement equipment. It can also be associated with stochastic effects of the general level of abatement knowledge in the economy that is external to the firm, but can affect the firm's abatement efficiency through spillover effects.[11]

Third, it can be assumed that environmental regulation takes place through a system of tradable permits, in which case $\tau(t)$ can be interpreted as the competitive market price for permits, which can evolve stochastically according to the geometric Brownian motion:[12]

$$d\tau(t) = \eta\tau(t)dt + \omega\tau(t)dz_\tau(t). \tag{4}$$

Given the firm's instantaneous profit function (1), the next stage is to define the optimal abatement investment policy for the types of uncertainty described.

9.2.1 Abatement Investment Decisions under Price Uncertainty

Having optimally chosen the output level, the next step is to analyze the decision to undertake new abatement investment, denoted by ΔR, from the existing abatement capital level of R_0, under price uncertainty modeled by equation (2) and assuming that (τ, v, s, h) are fixed parameters. Consider the firm's decision to undertake new abatement investment by ΔR from the existing abatement capital level R_0; then the new abatement capital level becomes

10. If we consider entry and exit decisions in the world market, then an upper reflecting barrier $\bar{p}$ to the price movement can be considered. When price moves to the reflecting barrier, new entry is triggered, quantity increases, and price decreases.

11. Stochastic delays in the R&D processes can be modeled by assuming that v follows a Poisson process.

12. Then $v(t)\, e(R(t))\, q(t)$ can be interpreted as the excess demand for permits. The expected values and the variances for $v(t)$ and of $\tau(t)$ are defined in a similar way as for $p(t)$.

$$R_{0+} = R_0 + \Delta R.$$

The cost of this change in abatement capital is defined as

(5) $$(1 - s)h(R_{0+} - R_0).$$

In the model developed here, the optimal abatement investment strategy takes the form of a free boundary, $p = p(R; \tau, v, s, h)$, relating price and accumulation of abatement capital. This boundary is parametrically defined for the vector of parameters (τ, v, s, h). When observed price p^{ob} is less than $p(R; \tau, v, s, h)$, no abatement investment is undertaken, while when p^{ob} is greater than $p(R; \tau, v, s, h)$ enough abatement investment is undertaken in the current period to restore equality on the boundary. Changes in the parameter vector (τ, v, s, h) shift the boundary and can accelerate or decelerate abatement investment accumulation for any given price. Thus we can determine, by using comparative statics associated with the free boundary, the effects of environmental policy on the firm's decision rule regarding abatement investment.

Assume that the initial price is p_0 and the firm's initial abatement capital stock is R_0. Given a discount rate ρ, the firm seeks the nondecreasing process $R(t)$, which will maximize the present value of profits less the cost of development. The value function[13] associated with this problem can be written as

(6) $$V(p,R) = \max_R \mathscr{E}\int_0^\infty e^{-\rho t}\pi(p(t),v(t),\tau(t),R(t))dt,$$

subject to equation (2).

At each instant of time, the firm has two choices: to undertake the new abatement investment or not. The time interval when no new abatement investment is undertaken and the existing abatement stock is used to determine the unit emission coefficient, can be defined as the continuation interval. A stopping time is defined as a time $\mathscr{T}$ at which new abatement investment is undertaken.

Let $R^*(\mathscr{T})$ be the optimal development process at time $\mathscr{T}$. If $\mathscr{T}$ is a stopping time, then

(7) $$V(p,R;\tau,v) = \max_R \mathscr{E}\left[\int_0^{\mathscr{T}} e^{-\rho u}\pi(p(u),v,\tau,R(u))du + e^{-\rho\mathscr{T}}V(R^*(\mathscr{T}),p(\mathscr{T}))\right],$$

where $R^*(\mathscr{T})$ is the optimal process at time $\mathscr{T}$ (see Fleming and Soner 1993). Assume that in the time interval $[0, \theta]$, the firm undertakes no new

13. By the concavity of the profit function in R and the linear dynamics, the value function is also concave in R (Dixit and Pindyck 1994).

abatement investment, but keeps it constant at R_0. By the principle of dynamic programming, the value function should be no less than the continuation payoff in the interval [0, θ], plus the expected value after θ, or

$$(8)\quad V(p,R;\tau,v) \geq \mathscr{E}\left[\int_0^\theta e^{-\rho u}\pi(p(u),v(u),\tau(u),R(u))du + e^{-\rho\theta}V(R(\theta),p(\theta))\right],$$

with equality if R_0 is the optimal policy in [0, θ]. Applying Itô's lemma to the value function on the right-hand side of equation (8), dividing by θ, and taking limits as $\theta \to 0$, we find that the value function should satisfy[14]

$$(9)\qquad \rho V \geq \frac{1}{2}\sigma^2 p^2 V_{pp} + apV_p + \pi(p,v,\tau,R),$$

with equality if $R(t) = R_0$ in the interval [0, θ].

Consider now the decision to undertake abatement investment instantaneously by $\Delta R = R_{0+} - R_0$. Then from the definition of the optimal stopping time, we have

$$(10)\qquad V(p,R;\tau,v) \geq \mathscr{E}[V(R_{0+},p;\tau,v) - (1 - s)h(R_{0+} - R_0)].$$

Since the value function is concave in R, the optimal abatement investment flow can be obtained by maximizing the right-hand side of inequality (10). The necessary and sufficient condition for the optimal abatement investment choice is

$$(11)\qquad V_R(R,p;\tau,v) - (1 - s)h \leq 0,$$

with equality if $\Delta R > 0$.

Thus when no new abatement investment is optimal, inequality (9) is satisfied as equality, whereas when new abatement investment is optimal, inequality (11) is satisfied as equality. Combining inequalities (9) and (11), the Hamilton-Jacobi-Bellman (HJB) equation can be written as

$$(12)\quad \min\left\{\left[\rho V - \frac{1}{2}\sigma^2 p^2 V_{pp} - apV_p - \pi(p,v,\tau,R)\right], - [V_R - (1 - s)h]\right\} = 0.$$

The optimal free boundary will divide the (p, R) space into two regions: the "no new abatement investment" region, which we call region I, and the "new abatement investment" region, which we call region II.

In region I, the first term of the HJB equation (12) is 0, since $\Delta R = 0$,

14. Subscripts associated with the value function denote partial derivatives.

and the second term of the HJB equation is positive by inequality (11); while in region II, the second term of equation (12) is satisfied as 0 and $\Delta R > 0$. These conditions allow the determination of the value function and the free boundary as functions of the policy parameters.

PROPOSITION 1. *Given the structure of the model as defined above and a quadratic cost function* $c(q) = \frac{1}{2}cq^2$, *the value function and the free boundary are defined respectively as*

$$V(p,R) = A_1(R)p^{\beta_1} + \Pi(p,R;\tau,v),$$

$$V_R(p,R) = A_1'(R)p^{\beta_1} + \Pi_R(p,R;\tau,v) = (1-s)h, \quad A_1'(R) < 0,$$

$$p(R;\tau,v,s,h) = \frac{-\beta_1}{\beta_1 - 1}\frac{(\rho - a)}{\rho}\frac{[c\rho(1-s)h + \tau ve(R)e'(R)]}{\tau ve'(R)}.$$

PROOF. See the appendix.

The solution of the value function $V(p, R)$ indicates that the maximized expected value consists of the term $\Pi(p, R; \tau, v)$, which can be interpreted as the present value of net profits when abatement capital is kept constant, and the term $A_1(R)p^{\beta_1}$, which is the current value of the option to expand abatement capacity. When the firm increases abatement capital, it sacrifices the option value of the incremental abatement capacity; thus $A_1'(R) < 0$. Therefore an increase in abatement is desirable if its contribution to net profit $\Pi_R(p, R; \tau, v)$, realized through savings in emissions taxes less the cost of giving up the option to wait $A_1'(R)p^{\beta_1}$, equals the marginal expansion cost $(1 - s)h$. The free boundary $p(R; \tau, v, s, h)$ can be determined for estimated parameter values that characterize the price process, the cost structure, and the discount rate. Since $p(R) > 0$, the free boundary is defined for parameter values such that $c\rho(1 - s)\,h > |\,\tau ve(R)e'(R)\,|$. In order to describe the free boundary we have, by the assumptions on the unit emissions function, $p(0) > 0$ and $\lim_{R\to\infty} p(R) = +\infty$. Furthermore,

$$\frac{\partial p}{\partial R} = \frac{-\beta_1}{\beta_1 - 1}\frac{(\rho - a)}{\rho}\frac{[(\tau v)(e')^3 - c\rho(1-s)he'']}{\tau v(e')^2} > 0.$$

The free boundary is shown in figure 9.1. For any given level of abatement capital, random price fluctuations move the point (R, p) vertically upward or downward. If the point goes above the boundary, then new abatement investment is immediately undertaken so that the point shifts on the boundary. Thus optimal abatement capital accumulation proceeds gradually. In the terminology of Dixit and Pindyck (1994), this is a "barrier control" policy.

By inverting the free-boundary function $p(R; \tau, v, s, h)$, we can obtain the optimal boundary function $R^* = p^{-1}(p; \mathbf{z})$, which determines the optimal abatement investment boundary as a function of the state variable p and

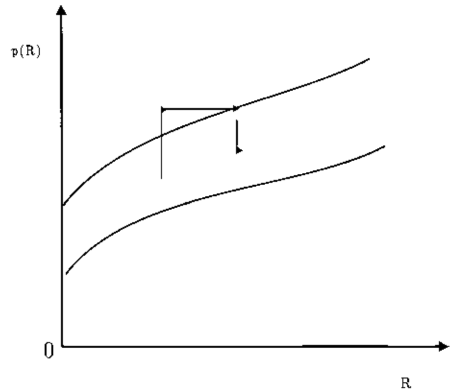

Fig. 9.1 Free boundary under price uncertainty

the vector **z** of the parameters of the problem. For price movements to the right of the boundary, new abatement investment is undertaken. If price stays on the left of the boundary, no new abatement investment is undertaken.

If price follows a mean reverting process, then the HJB equation for region I that corresponds to equation (12) becomes

$$(12') \qquad \rho V - \frac{1}{2}\sigma^2 p^2 V_{pp} - a(\tilde{p} - p)pV_p - \pi(p, R; \tau, v) = 0.$$

The steps for solving for the optimal boundary are the same as before; however, due to the more complicated structure of equation (12′), the analysis of the effects of mean reversion requires numerical solutions (see Dixit and Pindyck 1994).

9.2.2 The Impact of Changes in Policy Parameters

We can examine the shifts of the free boundary in response to changes in the tax parameter τ or the subsidy parameter s. These effects are determined as

$$\frac{\partial p}{\partial \tau} = \frac{\beta_1}{\beta_1 - 1}\frac{(\rho - a)}{\rho}\frac{c\rho(1 - s)h}{\tau^2 ve'} < 0,$$

$$\frac{\partial p}{\partial s} = \frac{\beta_1}{\beta_1 - 1}\frac{(\rho - a)}{\rho}\frac{c\rho h}{\tau ve'} < 0.$$

Thus an increase in the tax rate or the subsidy rate shifts the boundary downward and induces more abatement investment for any given price level, as is also shown in figure 9.1. An increase in the tax rate might not, however, increase abatement investment if there is a drop in prices below the boundary. This is because the reduction in equilibrium output and the consequent emissions reduction do not necessitate an increase in abatement. Abatement investment might increase at a future time when prices go up. This reveals that uncertainty affects the timing of the impact of environmental policy. At declining prices, a change in environmental policy might not induce any abatement investment. The impact of the policy might, however, be realized with a delay.

By performing the same type of comparative statics, we obtain

$$\frac{\partial p}{\partial v} < 0.$$

A reduction in abatement efficiency induces more abatement investment for any given price level.

9.2.3 Optimal Environmental Policy

In section 9.2.2 the tax and the subsidy parameters were treated as fixed. The analysis can, however, be extended to analyze the case of an environmental regulator who can choose the policy parameters optimally. Optimal policy choice is considered in the following way. From proposition 1, the free boundary that determines the profit-maximizing abatement investment depends on the tax and subsidy parameters. Consider the case of an environmental regulator that determines a socially optimal free boundary by explicitly taking into account environmental damages. An optimal environmental policy can then be defined by determining the values of the policy parameters such that the profit-maximizing free boundary will coincide with the socially optimal free boundary, as determined by the environmental regulator. Define a social profit function by

$$W(p, v, R) = \max_q [p(t)q(t) - c(q(t)) - D(v(t)e(R(t))q(t))],$$

where $D(v(t)e(R(t))q(t))$ is a strictly increasing and convex damage function. By following the steps in section 9.2.1, a free boundary that determines the socially optimal abatement investment under price uncertainty can be defined. Denote this free boundary by $p^s(R)$, and consider the free boundary defined in proposition 1 as a function of the policy parameters, or $p(R; \tau, s)$. An optimal environmental policy can be defined as the pair

$$(\tau^*, s^*) : p(R; \tau^*, s^*) = p^s(R).$$

A solution of the form $\tau^* = \zeta(s^*)$ will determine the trade-off between emissions taxes and abatement investment subsidies in the design of environmental policy.[15]

In the simplest possible case of constant marginal damages at the level d, the optimal trade-off is determined as $\tau^* = d(1 - s)$. The tax rule for

15. The relationship between the two policy instruments can be further elaborated to include budget-balancing schemes, where total tax revenues equal total subsidy expenses.

this case is very simple and, given the parameters of the model and output price observations, the regulator can determine the firms' responses regarding abatement investment.

It is interesting to note that under uncertainty and irreversibility, the optimal environmental policy equates the privately optimal and the socially optimal free boundaries and not the privately optimal and the socially optimal levels of the choice variables as in the case of optimal policy design under certainty.

9.3 Abatement Investment Decisions under Environmental Policy or Abatement Efficiency Uncertainty

When, under fixed prices, the environmental policy uncertainty is present in the form of stochastic evolution of prices for tradable emissions permits, or abatement efficiency is stochastic, then the mathematical treatment is similar, although the sources of uncertainty are different. Policy uncertainty can be regarded as uncertainty outside the firm, while abatement uncertainty can be regarded as internal to the firm. So although the mathematical results are the same, their interpretation and their policy implications are different.

In the case of policy uncertainty, the HJB equation can be written as

$$\min\left\{\left[\rho V - \frac{1}{2}\omega^2\tau^2 V_{\tau\tau} - \eta\tau V_\tau - \pi(\tau,R;p,v)\right], -[V_R - (1-s)h]\right\} = 0.$$

As before, the optimal free boundary will divide the (τ,R) space into two regions: the "no new abatement investment" region (region I) and the "new abatement investment" region (region II).

Proposition 2. *For the quadratic cost function defined in proposition 1, the value function and the free boundary are determined as*

$$V(\tau,R) = B_1(R)\tau^{\xi_1} + \phi(p,R;\tau,v), \quad B_1'(R) < 0,$$

$$V_R(\tau,R) = B_1'(R)\tau^{\xi_1} + \phi_R(p,R;\tau,v) = (1-s)h,$$

$$\tau(R) = \frac{-\xi_1}{2(\xi_1-2)}\,\frac{\Delta_1'(R)(\xi_1 - 1/\xi_1) + \sqrt{\Delta}}{\Delta_2'(R)},$$

$$\Delta = \left[\Delta_1'(R)\left(\frac{\xi_1-1}{\xi_1}\right)\right]^2 + 4\Delta_2'(R)\left(\frac{\xi_1-2}{\xi_1}\right)(1-s)h,$$

$$\Delta_1'(R) = -\frac{\rho v e'(R)}{c(\eta-\rho)} > 0, \quad \Delta_2'(R) = \frac{2ve(R)e'(R)}{2c(\omega^2+2\eta-\rho)}.$$

Proof. See the appendix.

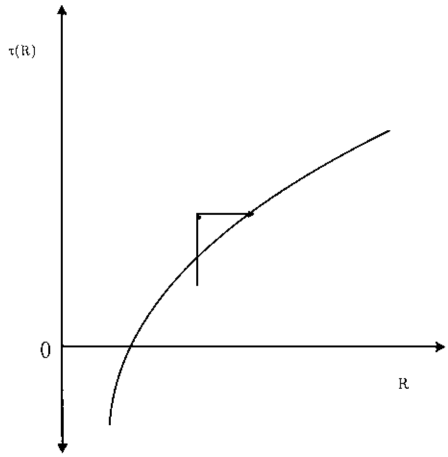

Fig. 9.2 Free boundary under policy uncertainty

The interpretations of the value function and the free boundary are similar to those under price uncertainty.

Using the assumptions about the unit-emissions function we have

$$\lim_{R\to 0} \tau(R) = -\infty, \quad \text{and} \quad \lim_{R\to\infty} \tau(R) = M > 0.$$

If the free boundary is monotonic, a property that can be checked by using specific functions, then its graph is shown in figure 9.2.

An increase of the policy parameter above the boundary induces more abatement investment. By inverting the $\tau(R)$ function, an optimal boundary function for abatement capital accumulation in terms of the policy parameter τ is defined as

$$R^* = \tau^{-1}(\tau, \mathbf{z}).$$

Policy uncertainty and abatement efficiency uncertainty can also be examined together by introducing the variable $z = \tau v$, with $\mathscr{E}(dz_\tau dz_v) = \rho_{\tau v} dt$. Using the fact that $\partial^2 z/\partial\tau^2 = \partial^2 z/\partial v^2 = 0$ and $\partial^2 z/\partial\tau v = 1$ we obtain

$$dz = (\gamma + \eta + \rho_{\tau v}\delta\omega) z dt + (\delta dz_v + \omega dz_\tau) z.$$

Thus changes in z have mean and variance

$$k_1 = \gamma + \eta + \rho_{\tau v}\delta\omega,$$

$$k_2 = \delta^2 + 2\rho_{\tau v}\delta\omega + \omega^2,$$

respectively. Following the same steps as before, the HJB equation is defined as

$$\min\left\{\left[\rho V - \frac{1}{2}k_2^2 z^2 V_{zz} - k_1 z V_z - \pi(z,R;p)\right], -[V_R - (1-s)h]\right\} = 0.$$

Then the free boundary can be defined in the context of correlated policy and technological uncertainty, as in the previous case of policy uncertainty.

9.3.1 Unpredictable Policy Changes

Policy uncertainty as analyzed previously is associated with continuous fluctuations of the tradable-emissions-permit price. It is possible, however, for a sudden change in policy due, for example, to an unexpected (from the firm's point of view) change in the supply of permits, to cause a discontinuous change in their price. In the context of our model, this unpredictable change introduces jump characteristics. Thus, while the usual fluctuations in prices are captured by the geometric Brownian motion, the sudden policy change should be captured by a Poisson process. Therefore, the price of permits is modeled by a mixed Brownian motion and jump process, or

$$d\tau = \eta\tau dt + \omega\tau dz_\tau + \tau dq^P,$$

where dq^P is the increment of a Poisson process with a mean arrival time of the change in the supply of permits λ. We further assume that the change in the supply of permits represents an increase and that this causes a fixed drop in the price[16] by a known percentage $\psi \in [0, 1]$ with probability 1, and that dz_τ and dq^P are independent.

To analyze this problem, the HJB equation is derived by using Itô's lemma for combined Brownian motion and jump process (Dixit and Pindyck 1994). Then the HJB equation can be written as

$$\min\left\{\left[\rho V - \frac{1}{2}\omega^2\tau^2 V_{\tau\tau} - \eta\tau V_\tau - \pi(\tau,R;p,v)\right] + \lambda[V((1-\psi)\tau) - V], -[V_R - (1-s)h]\right\} = 0.$$

The solution to the value function is

$$V(\tau,R,\psi) = B_1^\psi(R)\tau^{\xi_1^\psi} + \phi^\psi(\tau,R;p,v,\psi),$$

where ξ_1^ψ is the positive solution of the nonlinear equation (see Dixit and Pindyck 1994),

16. A fixed increase in the price can be treated symmetrically.

$$\frac{1}{2}\omega^2\xi^\psi(\xi^\psi - 1) + \eta\xi^\psi - (\rho + \lambda) + \lambda(1 - \psi)^{\xi^\psi} = 0.$$

Once a solution for ξ_1^ψ is obtained, then the free boundary can be obtained as before.

It should be noted that special cases of the general mixed Brownian motion and jump process model can be used to analyze specific cases. For example, if $\eta = \omega = 0$ is set and τ is interpreted as an emissions tax, then the same model can be used to analyze the implications of unpredictable changes in the emissions tax rates.

The analysis of policy uncertainty provides a general way of analyzing firms' reactions to environmental policy. Given the structure of the free boundary, which can be determined for estimated parameter values, the regulator can obtain the firms' reactions to a wide range of policy changes using a unified model.

9.4 Location Decisions

When we examine location decisions, the problem can also be defined as an optimal stopping problem. In the waiting or continuation region, the firm stays in its present location, pays the emissions tax, and follows the optimal abatement capital-accumulation path, $R^*(t)$, given uncertainty described by the evolution of the state variable (price, policy parameter, or technology parameter).

Suppose that the firm examines the possibility of relocation to a new location (country) where there is no environmental policy. Assume that the setup costs are fixed, F, and are incurred once at the time of relocation, that the cost function remains the same, and that there are no transportation costs.[17] Suppose that relocation takes place at time t_d; then the profit function for the firm that chooses optimally operating output is defined as

$$\begin{cases} \pi_d(p(t)) - F, & \text{for } t = t_d, \\ \pi_d(p(t)), & \text{for } t > t_d, \end{cases}$$

where $\pi_d(p) = \max_q[pq - c(q)]$.

Assuming that price uncertainty exists, then at each period of time the firm faces a binary choice:

1. Relocate and take the termination payoff defined as $W(p(t_d),F)) = \mathcal{E} \int_{t_d}^{\infty} e^{-\rho t}\pi_d(p(t))dt - F$.
2. Continue operation at the initial location for one period, choosing output and abatement investment optimally; receive the operating profits; and then consider another binary choice in the next period.

17. See the proofs of propositions 1 and 2 in the appendix for these conditions.

The Bellman equation for this problem can be written as

$$V(p) = \max\left\{\pi(p,\tau,v,R^*) + \frac{1}{1+\rho dt}\mathscr{E}[V(p+dp|p)], W(p,F)\right\}. \tag{13}$$

In the continuation region, the first term on the right-hand side is the largest. Using Itô's lemma on this term, we obtain the usual differential equation,

$$\rho V - \frac{1}{2}\sigma^2 p^2 V_{pp} - apV_p - \pi(p;R^*,\tau,v) = 0, \tag{14}$$

with solution

$$V(p(t);R^*,\tau,v) = K_1 p(t)^{\beta_1} + \Pi(p(t);R^*,\tau,v). \tag{15}$$

From the Bellman equation we have that at the critical relocation time t_d,

$$V(p(t_d);R^*,\tau,v) = W(p(t_d),F). \tag{16}$$

This is the value-matching condition. The smooth-pasting condition requires that[18]

$$V_p(p(t_d);R^*,\tau,v) = W_p(p(t_d),F). \tag{17}$$

Conditions (16) and (17) can be used to determine the constant K_1 and the free boundary $p = p^*(t_d)$. By inverting the boundary equation we obtain the optimal relocation time boundary function $t_d^* = p^{*-1}(p)$. This boundary determines the critical relocation time as a function of the observed price for given values of the parameters τ, v, and s.

Environmental policy uncertainty or abatement efficiency uncertainty can be treated in the same way. Suppose that policy uncertainty exists in the sense of stochastic permit prices. Then, following the same steps as before, the free boundary $\tau = \tau^*(t_d)$ is defined by the following conditions, using the quadratic cost function:

$$V(\tau(t);R^*,p,v) = \Lambda_1\tau(t)^{\xi_1} + \Phi(\tau(t);R^*,p,v),$$

$$V(\tau(t_d);R^*,p,v) = W(F), \quad \text{value matching},$$

$$W(p,F) = \int_0^\infty e^{-\rho t}\frac{p^2}{2c}dt - F = \frac{p^2}{2c\rho} - F, \quad p \text{ fixed},$$

$$V_\tau(\tau(t_d);R^*,p,v) = W_\tau(F) = 0, \quad \text{smooth pasting}.$$

Using these conditions, the free boundary $\tau^*(t)$ is implicitly defined by

18. Alternative assumptions could include the existence of a different environmental policy abroad, for example command and control regulation, or differences in the political systems that affect the stringency of environmental policy.

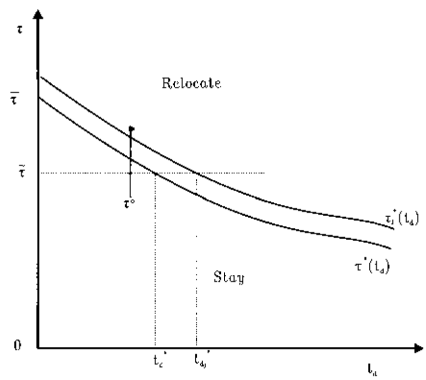

Fig. 9.3 Threshold policy parameter

$$(18)\qquad \frac{-\Phi_\tau(\tau^*(t);R^*,p,v)}{\xi_1}\tau^*(t) + \Phi(\tau^*(t);R^*,p,v) = \frac{p^2}{2cp} - F.$$

By inverting the boundary function $\tau^*(t_d)$, we obtain the optimal relocation time boundary function $t_d^* = \tau^{*-1}(\tau)$ in terms of the environmental policy parameter and the rest of the parameters of the problem. Optimal relocation implies the existence of a threshold policy parameter such that when the actual policy parameter crosses this threshold, relocation takes place. This result is stated in the following proposition.

PROPOSITION 3. *Let $\tau^o(t')$ be the observed environmental policy parameter at time t'. If $\tau^o(t') < \tau^*(t')$, then it is optimal to remain at the initial site. If $\tau^o(t') > \tau^*(t')$ it is optimal to relocate at time t'.*

PROOF. See the appendix.

This proposition implies that for any time t a threshold environmental policy parameter exists such that when the policy parameter exceeds the threshold, the firm moves to the new location. The relation between the threshold policy parameter and the optimal relocation time is shown in figure 9.3, where $\bar{\tau}$ indicates the permit price that induces immediate relocation. The lower the permit price, the further away the optimal relocation time is. When the permit price crosses the boundary, it is optimal to take the irreversible relocation decision. Similar analyses, although with different interpretations, can be applied to the case where uncertainty relates to abatement efficiency, or to correlated policy and abatement uncertainty.

The analysis suggests that since firms are identical, they will relocate simultaneously when the critical time arrives. If firms are heterogeneous regarding characteristics of production cost or abatement technologies, then the optimal relocation time will be different across firms. In this case, there will more than one boundary such as the one depicted in figure 9.3.

Suppose that a second boundary, $\tau_j^*(t_d)$, exists and, as shown in figure 9.3, the two boundaries do not intersect. Then the same permit price $\bar{\tau}$ implies different optimal relocation times.

9.4.1 Relocation Time Policy under Uncertainty

The free boundaries and the optimal relocation policy functions derived here can be used to provide useful information regarding the effects of exogenous shocks on the critical relocation time and describe a framework for designing a policy that could affect relocation time. This can be obtained by performing comparative static analysis of the optimal boundary function.

Assuming, again, identical firms to simplify things, consider the boundary function $t_d^* = p^{*-1}\,(p;\, \tau,\, v,\, s)$ and take the derivative:

$$\frac{\partial t_d^*}{\partial \tau} = \frac{\partial p^{*-1}(p)}{\partial \tau}.$$

This derivative determines the effect on the critical relocation time of an exogenous change in the emissions tax for any given price level. In the same way, the effects from changes of other parameters of the model on relocation time can also be defined.

Consider now the total differential,

$$dt_d^*(p) = \frac{\partial t_d^*}{\partial \tau} dt_d + \frac{\partial t_d^*}{\partial s} ds.$$

This differential can express the rate of substitution between the emissions tax rate and the abatement investment subsidy rate in order to produce a given change in the critical relocation time at any given price level. For example by setting $dt_d^*(p) = 0$, the marginal rate,

$$\frac{ds}{d\tau} = \frac{\partial t_d / \partial \tau}{\partial t_d / \partial s},$$

expresses the necessary increase in the abatement investment subsidy in order to keep the critical relocation time constant after an increase in the emissions tax for any given price. The changes in the policy parameters shift the free boundary $p = p^*(t_d)$ and enlarge or shrink the stay or relocate regions as shown in figure 9.4. The particular forms of the policy functions, the comparative static derivatives, and the marginal rates of substitution can be explored under the quadratic cost function assumption.

9.5 Concluding Remarks

The responses of firms to environmental policy regarding their abatement investment and location decisions have been analyzed in an analyti-

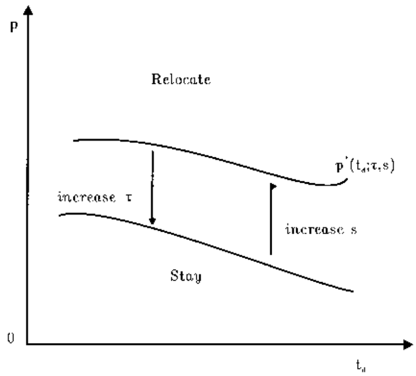

Fig. 9.4 Relocation time policy

cal framework characterized by uncertainty and irreversibilities. The optimal stopping time methodology adopted in this paper makes it possible to analyze firms' responses to environmental policy in terms of the impact that this environmental policy has on the barrier control policies followed by firms regarding their profit-maximizing decisions. The analysis of environmental policy impacts through their effects on barrier control policies makes it possible to view these impacts as shifts of a free boundary that determines firms' policies regarding abatement investment or relocation decisions. In this sense the approach developed in this paper can be regarded as another way of analyzing the effects of regulatory policy under conditions of uncertainty and irreversibility. Despite the mathematical complexity of the approach, the effects of regulatory policies on the free boundary are determined by parameters related to the stochastic process associated with uncertainty and the firms' production structure, which are in principle estimable.

It is also possible to use the optimal stopping time methodology in order to design optimal environmental policy under uncertainty and irreversibility, in the sense of choosing the policy parameters so that the free boundary, or equivalently the optimal policy function under profit maximization, coincides with the socially optimal free boundary (abatement-investment-policy function). The implication of this approach regarding optimal policy design under price uncertainty and irreversibility is that a regulator can in principle design a policy scheme consisting of two instruments: an emissions tax or a tradable permit system, and a subsidy on abatement investment. The policy scheme takes into account uncertainty through its dependence on the parameters of the price process and will induce individual firms to undertake the same output and abatement investment under uncertainty that a regulator would have undertaken. In this sense, the policy mix of emissions taxes (or emissions permits) and abatement investment subsidies will be welfare maximizing. It should be noticed that the function $\tau^* = \zeta(s^*)$, determining the optimal trade-off between taxes and subsidies, allows the regulator to determine the policy mix in order to obtain an optimal balance between the output-contracting pollution control by emissions taxes and pollution control through the subsidization of the accumulation of abatement capital.

A similar mix of emissions taxes (or emissions permits) and abatement investment subsidies can be used to affect location decisions. By linking location decisions to subsidies in emissions-reducing abatement investment, it was possible to derive rules relating to the amount of subsidy required in order not to accelerate relocation after the introduction of a stricter environmental policy. Given the function that determines optimal relocation time as a function of the observed price, an increase in emissions taxes may induce relocation of all or a subset of firms, depending on the heterogeneity assumptions, by bringing relocation time forward at the same price level. If relocation is not desirable, it may be prevented by an appropriate increase in the abatement subsidy. On the other hand, if price movements in the world market induce relocation, our results indicate that it could be prevented by an appropriate change of the policy mix, that is, changes in emissions taxes and/or abatement subsidies.

Further research could be directed toward the study of the relocation time when the country abroad follows a different environmental policy, or when the firms in the home country are heterogeneous. Further research could also be directed toward the study of the socially optimal relocation time. The optimal-stopping-time methodology could indicate the time at which it is socially desirable for a firm to relocate, and then help to design a policy scheme to prevent suboptimal relocation decisions.

Appendix

Proof of Proposition 1

From the HJB equation we obtain for region I

$$\rho V - \frac{1}{2}\sigma^2 p^2 V_{pp} - apV_p - \pi(p,R;\tau,v) = 0. \quad \text{(A1)}$$

The general solution of this second-order differential equation (A1) can be obtained as[19]

$$V(p,R) = A_1(R)p^{\beta_1} + A_2(R)p^{\beta_2} + \Pi(p,R;\tau,v),$$

where

19. The homogeneous part of this differential equation is an Euler equation.

$$\beta_1 = \frac{1}{2} - \frac{a}{\sigma^2} + \sqrt{\left(\frac{a}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} > 1$$

is the positive root; β_2 is the corresponding negative root of the fundamental quadratic,

$$Q = \frac{1}{2}\sigma^2\beta(\beta - 1) + a\beta - \rho = 0;$$

and $\Pi(p, R; \tau, v)$ is the particular solution. We need to disregard the negative root in order to prevent the value from becoming infinitely large when the price tends to 0; thus we set $A_2(R) = 0$ (see Dixit and Pindyck 1994). So the solution becomes

(A2) $$V(p,R) = A_1(R)p^{\beta_1} + \Pi(p,R;\tau,v).$$

In order to obtain tractable results we need a better specification of the particular solution. To obtain such a specification, we consider a quadratic cost function $c(q) = {}^1\!/\!{}_2 cq^2$; then the profit function becomes

$$\pi(p,\tau,v,R) = \frac{1}{2c}\{p^2 - 2\tau ve(R)p + [\tau ve(R)]^2\}.$$

Using the method of undetermined coefficients, we obtain the particular solution as

(A3) $$\Pi(p,R;\tau,v) = \Gamma_0 + \Gamma_1 p + \Gamma_2 p^2,$$

(A4) $$\Gamma_0 = -\frac{[\tau ve(R)]^2}{2c\rho}, \quad \Gamma_1 = -\frac{\tau ve(R)}{c(\rho - a)}, \quad \Gamma_2 = \frac{1}{2c(\rho - 2a - \sigma^2)},$$

$$\rho - a > 0, \quad \rho - 2a - \sigma^2 > 0.$$

In region II the second term of equation (12) is satisfied as 0 and $\Delta R >$ 0, or

(A5) $$V_R(p,R) - (1 - s)h = 0.$$

Solving equation (A5) for p in terms of R, we can write the yet unspecified boundary equation as $p = p(R)$. From equations (A2) and (A5) we can determine the unknown functions $A_1(R)$ and $p = p(R)$ using the value-matching and the smooth-pasting conditions.[20] The value-matching condition means that on the boundary separating the two regions, the two value functions should be equal. Then we have, combining equations (A2) and (A5) and substituting for p,

20. For a presentation of these conditions, see Dixit and Pindyck (1994).

(A6) $V_R(p,R) = A_1'(R)p^{\beta_1} + \Gamma_0'(R) + \Gamma_1'(R)p = (1-s)h, \quad p = p(R).$

The smooth-pasting condition means that the derivatives of the value functions with respect to p on the boundary are equal, or

(A7) $V_{Rp}(p,R) = \beta_1 A_1'(R)p^{\beta_1 - 1} + \Gamma_1'(R) = 0, \quad p = p(R).$

Combining equations (A6) and (A7), we can solve for the unknown functions $p(R)$ and $A_1'(R)$ to obtain

(A8) $$p(R) = \frac{\beta_1}{\beta_1 - 1}\frac{(1-s)h - \Gamma_0'(R)}{\Gamma_1'(R)},$$

(A9) $$A_1'(R) = -\left(\frac{\Gamma_1'(R)}{\beta_1}\right)[p(R)]^{1-\beta_1}.$$

Relationship (A8) is the equation of the free boundary, which can be written, after substituting for $\Gamma_0'(R)$ and $\Gamma_1'(R)$, as

(A10) $$p(R) = \frac{-\beta_1}{\beta_1 - 1}\frac{(a-\rho)}{\rho}\frac{[c\rho(1-s)h + \tau ve(R)e'(R)]}{\tau ve'(R)}.$$

Proof of Proposition 2

In region I, the first term of the HJB equation is 0, since $\Delta R = 0$, and the second term of the HJB equation is positive. Thus in region I,

$$\rho V - \frac{1}{2}\omega^2\tau^2 V_{\tau\tau} - \eta\tau V_\tau - \pi(\tau, R; p, v) = 0.$$

The general solution of this second-order differential equation can be obtained as before as

$$V(\tau, R) = B_1(R)\tau^{\xi_1} + B_2(R)\tau^{\xi_2} + \Phi(p, R; \tau, v),$$

where

$$\xi_1 = \frac{1}{2} - \frac{\eta}{\omega^2} + \sqrt{\left(\frac{\eta}{\omega^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\omega^2}} > 1$$

is the positive root; ξ_2 is the corresponding negative root of the fundamental quadratic,

$$Q = \frac{1}{2}\omega^2\xi(\xi - 1) + \eta\xi - \rho = 0;$$

and $\Phi(\tau, R; p, v)$ is the particular solution. As before, we set $B_2(R) = 0$, so the solution becomes

$$V(\tau, R) = B_1(R)\tau^{\xi_1} + \Phi(\tau, R; p, v).$$

Using again the quadratic cost function specification, $c(q) = \frac{1}{2}cq^2$, we obtain the particular solution as

$$\Phi(\tau, R; p, v) = \Delta_0 + \Delta_1\tau + \Delta_2\tau^2,$$

$$\Delta_0 = -\frac{p^2}{2c\rho}, \quad \Delta_1 = -\frac{pve(R)}{c(\eta - \rho)}, \quad \Delta_2 = \frac{[ve(R)]^2}{2c(\omega^2 + 2\eta - \rho)}.$$

In region II, the second term of the HJB equation is satisfied as 0 and $\Delta R > 0$, or

$$V_R(\tau, R) - (1 - s)h = 0.$$

The value-matching and smooth-pasting conditions imply that

(A11) $$V_R(\tau, R) = B_1'(R)\tau^{\xi_1} + \Delta_1'(R)\tau + \Delta_2'(R)\tau^2 = (1 - s)h, \quad \tau = \tau(R),$$

and

(A12) $$V_{R\tau}(\tau, R) = \xi_1 B_1'(R)\tau^{\xi_1 - 1} + \Delta_1'(R) + 2\Delta_2'(R)\tau = 0, \quad \tau = \tau(R),$$

respectively, where

$$\Delta_1'(R) = -\frac{\rho v e'(R)}{c(\eta - \rho)} > 0, \quad \Delta_2'(R) = \frac{2ve(R)e'(R)}{2c(\omega^2 + 2\eta + \rho)}.$$

Combining equations (A11) and (A12), we obtain a quadratic expression that implicitly defines $\tau(R)$ as

(A13) $$\Delta_2'(R)\left(\frac{\xi_1 - 2}{\xi_1}\right)\tau(R)^2 + \Delta_1'(R)\left(\frac{\xi_1 - 1}{\xi_1}\right)\tau(R) - (1 - s)h = 0.$$

By taking the positive root of equation (A13), the free boundary is defined as

$$\tau(R) = \frac{-\xi_1}{2(\xi_1 - 2)} \frac{\Delta_1'(R)[(\xi_1 - 1)/\xi_1] + \sqrt{\Delta}}{\Delta_2'(R)},$$

$$\Delta = \left[\Delta_1'(R)\left(\frac{\xi_1 - 1}{\xi_1}\right)\right]^2 + 4\Delta_2'(R)\left(\frac{\xi_1 - 2}{\xi_1}\right)(1 - s)h,$$

with $\xi_1 > 2$ for $\Delta > 0$.

Proof of Proposition 3

The Bellman equation is

$$V(\tau) = \max\left\{\pi(\tau; p, v, R^*) + \frac{1}{1 + \rho dt}\mathscr{E}[V(\tau + d\tau|\tau)], W(p, F)\right\}.$$

Following Dixit and Pindyck (1994), define $G(\tau) = V(\tau) - W$, $\tau + d\tau = \tau'$ and subtract W from both sides of the Bellman equation to obtain

$$\text{(A14)}\quad G(\tau) = \max\left\{0, \pi(\tau) - W + \frac{1}{1+\rho dt}\int V(\tau')d\Phi(\tau'|\tau)\right\}$$

$$= \max\left\{0, \pi(\tau) - W + \frac{1}{1+\rho dt}W + \frac{1}{1+\rho dt}\int G(\tau')d\Phi(\tau'|\tau)\right\}.$$

Since W does not depend on τ, we have that the expression,

$$m(\tau) = \pi(\tau) - W + \frac{1}{1 + \rho dt}W,$$

is decreasing in τ, since $dm/d\tau = d\pi/d\tau < 0$. The function $m(\tau)$ reflects the difference between waiting for one period before relocating and relocating right away. Since $m(\tau)$ is decreasing in τ, continuation—that is, no relocation—should be optimal when τ is low.

Assume that the cumulative distribution $\Phi(\tau' \mid \tau)$ of the future values of the policy parameter shifts uniformly to the left as τ increases, so that the disadvantages of an increase in the current value of the environmental policy parameter in the original location are unlikely to be reversed in the future. This assumption, along with the decreasing $m(\tau)$ function, implies that $G'(\tau) < 0$ (see Dixit and Pindyck 1994, app. B).

Therefore, the second argument of equation (A14) is decreasing in τ. Thus, a unique critical time $\tau^*(t')$ exists such that the second argument of equation (A14) is negative if and only if $\tau^o(t') > \tau^*(t')$. Then it is optimal to relocate (optimal to stop) at time t'. If $\tau^o(t') < \tau^*(t')$, then it is optimal to remain at the initial site (continue).

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Comment Charles D. Kolstad

This is a very interesting and impressive piece of work. Prof. Xepapadeas has tackled a very difficult question: In a dynamic context with irreversibilities in abatement investments and a pollution tax and abatement subsidy, as well as uncertainty, what is an optimal output and investment profile? The author examines three types of uncertainty: stochastic evolution of product price, stochastic evolution of the efficiency of abatement, and stochastic evolution of the pollution tax (or the price of pollution permits). Quite naturally, the author then asks what is the optimal environmental policy given this behavior of firms? Related to this issue, he closes the paper by asking what the optimal time is for firms to pull up stakes and move to a location with lighter environmental regulations.

While the paper provides an impressive use of stochastic calculus, the paper would be improved by refining or at least justifying some aspects of the model to provide a closer connection to realistic problems in pollution policy. For instance, of the three types of stochasticity, product-price evolution seems the most realistic and interesting. It is difficult to visualize the efficiency of abatement equipment following a random walk, sometimes increasing, sometimes decreasing. In the case of the level of a pollution tax, one would not expect it to wander around, since it is set by policymakers. On the other hand, it is plausible that the price of permits could follow a stochastic process, much as stocks do.

It would appear that one of the primary purposes of this paper is to investigate the importance of uncertainty and irreversibilities. Because of this, it is somewhat disappointing that the paper focuses on only one type of irreversibility—abatement capital investment irreversibility. There are several interesting extensions that would push this issue further, for instance, a stock of pollution whose level is nondiminishing, corresponding approximately to the stock of carbon dioxide in the atmosphere or nuclear wastes in salt domes.

Another interesting extension would be to look at the stock of knowledge (as, in fact, the author hints at in the opening paragraph and examined in an earlier version of this paper). This endogenization of the R&D process would be an important contribution to this literature. The problem, which is very difficult to overcome, is that there are spillovers among firms. Without spillovers, the problem is less interesting (although still potentially of some interest).

The treatment of location is one of the more unique aspects of this pa-

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per. The firm can choose to move to a regulation-free area, incurring a one-time fixed charge. The question is, when to do it? Basically, by paying a one-time charge, a firm can forever evade environmental regulation. This is really a shut-down and start-up question, which could be separated. When should a firm choose to shut down? When should a firm choose to restart in a pollution-free area, incurring a fixed start-up cost? This is an interesting problem, with many possible extensions.

One possible extension is to treat location as continuous, developing a Hotelling model of spatial competition. This would not be easy, but could be potentially very interesting. Where do firms locate in response to environmental regulations? How do regulations interact with spatial differences in pollution? Do environmental regulations provide an entry barrier? How do regulators compete with one another over space?

In summary, this is an ambitious paper that pushes our knowledge of firm behavior in a polluting environment subject to stochastic shocks. I would encourage the author to push this further, bringing more policy-relevant dimensions into the problem.